Functions

Definition 1. A function $f : A \to B$ is:

1. *Injective* if whenever $f(x) = f(y)$, then $x = y$;
2. *Surjective* if for every y there is some x such that $f(x) = y$;
3. *Bijective* if it is both injective and surjective; and,
4. *Invertable* if there is some function $g : B \to A$ such that $g \circ f = I_A$ and $f \circ g = I_B$ (where I_X is the identity function from X to X).

Problem 1. Restate Definitions 1.1, 1.2, and 1.4 using the syntax of $FO(\mathbb{Z})$.

Problem 2. Suppose that f has two inverses g and h. (That is, each of g and h meet the conditions in Definition 1.4.) Show that $g = h$.

Problem 3 ($\star$)**.** Show that if f has an inverse, then f is bijective.

Relations

Definition 2. A relation R is:

1. *Reflexive* if for every x, $x\ R\ x$;
2. *Symmetric* if whenever $x\ R\ y$, then $y\ R\ x$;
3. *Transitive* if whenever $x\ R\ y$ and $y\ R\ z$, then $x\ R\ z$.

A relation that is reflexive, symmetric, and transitive is called an *equivalence* relation.

Problem 4. Show (or give a counterexample) that if R and S are equivalence relations, then so is $R \cap S$.

Problem 5. Show (or give a counterexample) that if R and S are equivalence relations, then so is $R \cup S$.

$FO(\mathbb{Z})$

Section 1.13.1 exercises 1–3.