

EECS 762 HW5 Luke Dercher

5.8.1.4 parenthesize the following

$$(a) \lambda x. \lambda y. y \lambda x =$$

$$\lambda x. \lambda y. x = (\lambda x. (\lambda y. x))$$

$$(b) x x x = (x(x(x)))$$

$$(c) x \lambda x. x x = (x(\lambda x. x(x)))$$

5.8.2

1. compute free vars for the following

$$(a) x y \lambda x. x y$$

$$fv(x y \lambda x. x y)$$

$$= fv(x) \cup fv(y) \cup fv(\lambda x. x / \{x\}) \cup fv(y)$$

$$= fv(x) \cup fv(y) \cup \{x\} \cup \{y\}$$

$$= \{x\} \cup \{y\} = \{x, y\}$$

$$(b) \lambda x. y x x$$

$$fv(\lambda x. y x x)$$

$$= fv(\lambda x. y) \cup \{x\} \cup fv(x) \cup fv(x)$$

$$= \{y\} \cup \{x\} \cup \{x\} = \{y, x\}$$

$$(c) \lambda x. (\lambda y. y) y \lambda x. x$$

$$fv(\lambda x. (\lambda y. y) y \lambda x. x)$$

$$= fv(\lambda x. (\lambda y. y) y / \{y\}) \cup fv(\lambda x. x / \{x\})$$

$$= fv(\lambda y. y) \cup fv(y)$$

$$= \{y\}$$

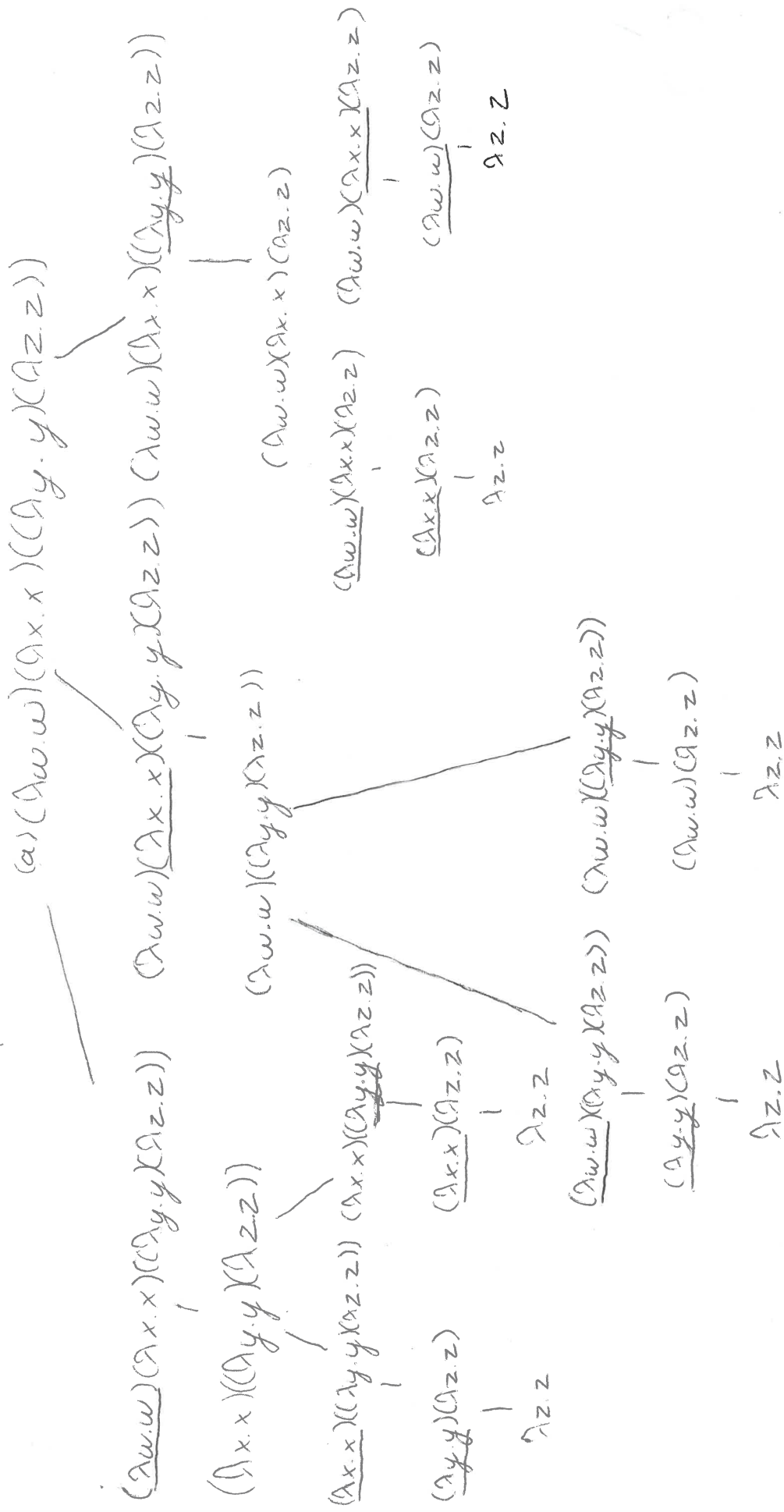
5.8.2.2 perform subst.

$$(a) [x/y](\lambda z. zy) = \lambda z. zx$$

$$(b) [(x x)/x](\lambda z. x y z) = \lambda z. (x x) y z$$

$$(c) [(z x)/x](\lambda z. x z) = \lambda w. (z x) w$$

5.8.3.5 List all possible Beta reductions



5. 3. 5 (P)

$$\begin{array}{c}
 (\lambda x.\lambda y.x)((\lambda x.x)(\lambda y.y)) \\
 | \\
 (\lambda x.\lambda y.x)((\lambda x.x)(\lambda y.y)) \quad (\lambda x.\lambda y.x)((\lambda x.x)(\lambda y.y)) \quad (\lambda x.\lambda y.x)((\lambda x.x)(\lambda y.y)) \\
 | \qquad \qquad \qquad | \qquad \qquad \qquad | \\
 \lambda y.(\lambda x.x)(\lambda y.y) \quad \lambda y.(\lambda x.x)(\lambda y.y) \quad \lambda y.(\lambda x.x)(\lambda y.y) \\
 | \qquad \qquad \qquad | \qquad \qquad \qquad | \\
 \lambda y.(\lambda y.y) \quad \lambda y.(\lambda y.y) \quad \lambda y.(\lambda y.y)
 \end{array}$$

$$\begin{aligned} & (ax \cdot ay \cdot x)(ax \cdot x)(ay \cdot y)) \\ & \frac{(ax \cdot ay \cdot x)(ax \cdot x)(ay \cdot y))}{1} \quad (ax \cdot ay \cdot x)(\underline{(ax \cdot x)(ay \cdot y)}) \\ & ay \cdot (\underline{(ax \cdot x)})(ay \cdot y) \quad (ax \cdot ay \cdot x)(ay \cdot y) \\ & ay \cdot (ay \cdot y) \quad ay \cdot (ay \cdot y) \end{aligned}$$

5.8.4.2 for each reduction step write down the corresponding context

$$a) \lambda x. (\lambda y. yy) ((\lambda z. zz) x) \rightsquigarrow \lambda x. (\lambda y. yy) (xx)$$

$$C = \lambda x. (\lambda y. yy) *$$

$$b) (\lambda x. x) ((\lambda y. yy) \lambda z. (\lambda w. w) z) \rightsquigarrow (\lambda x. x) ((\lambda y. yy) \lambda z. z)$$

$$C = (\lambda x. x) ((\lambda y. yy) \lambda z. *)$$

$$c. (\lambda x. x) (\lambda y. y) \lambda z. zz \rightsquigarrow (\lambda y. y) \lambda z. zz$$

$$C = * \lambda z. z$$

5.9.1.

$$(\lambda x. xx) (\lambda x. xxx)$$