Section 2.10.3 exercises 1–2.

**Textbook Problem 1.** Determine if the following equations are monotonic and continuous

$$f(n) = \{0 \text{ if } n \in \mathbb{N} \text{ is even, } 1 \text{ if } n \in \mathbb{N} \text{ is odd, } \omega \text{ if } n = \omega \tag{1}$$

$$f(n) = \{2 * n \text{ if } n \in \mathbb{N} \text{ , } \omega \text{ if } n = \omega \tag{2}$$

**Textbook Solution 1.** 1. This function is not monotonic nor is it continuous. Why? ex. $f(4) = 0$, $f(3) = 1$. In this case we have $x < y$ but not $f(x) < f(y)$. The function is not continuous since it can only output 0, 1, or $\omega$.

2.This function is monotonic. If we have an $x < y$, $2 * x < 2 * y$, therefore $f(x) < f(y)$. This function is also continuous. This is because for all n, f(n) has a value, and no skips or jumps take place in the function at any time.

Section 2.10.4 exercise 1.

**Textbook Problem 2.** consider while command while $x \neq y$ do $x = x + 1$. Write out first three approximations.

**Textbook Solution 2.** the first approximation is the following

$\perp_f(\phi) = \perp$

second:

$F(\perp_f(\phi)) = \{\phi$ if $\phi(x) = \phi(y)$, $\perp_f(\phi[x := x + 1]) = \perp$ otherwise

third:

$F(F(\perp_f(\phi))) = \{\phi$ if $\phi(x) = \phi(y)$, $\perp_f(\phi[x := x + 1]) = \perp$ if x = y - 1, $\perp_f(\phi[x := [x + 1] + 1]) = \perp$ otherwise

Section 2.11.3 exercises 1 and 2 ($\star$)

**Textbook Solution 3.** (a) elements of $B \mapsto A$ are as follows

(0,1),(1,0),(0,2),(2,0)(1,2),(2,1),(0,0),(1,1),(2,2)

(b) The elements of $B \mapsto A$ that are related by pointwise ordering are the following

A(0,1) $\mapsto$ B(0,1), A(0,1) $\mapsto$ B(0,2), A(1,0) $\mapsto$ B(1,0), A(1,0) $\mapsto$ B(2,0)

(c) the elements that are monotonic functions from $(B, \subseteq_B)$ to $(A, \subseteq_A)$ are the following

$\{0 \mapsto 0, 1 \mapsto 1\}$, $\{0 \mapsto 0, 1 \mapsto 2\}$, $\{0 \mapsto 1, 1 \mapsto 1\}$, $\{0 \mapsto 1, 1 \mapsto 2\}$, $\{0 \mapsto 2, 1 \mapsto 1\}$, $\{0 \mapsto 2, 1 \mapsto 2, \}$

(d) A monotonic function from $B \mapsto A$ to $B \mapsto A$ is the following