EECS 762

HW4 Luke Dercher

4.5.1 ex. 2

$$\Delta_3 = \dfrac{x := x-1,\ y := y-2 \ \sigma_2 \Downarrow \{x \mapsto 0, y \mapsto 2\} \qquad y := y-2 \ \{x \mapsto 0, y \mapsto 2\} \Downarrow \sigma_3}{x := x-1;\ y := y-2 \ \sigma_2 \Downarrow \sigma_3}$$

$$\Delta_2 = \dfrac{\sigma_2 \vDash x > 0 \qquad \dfrac{\Delta_3}{x := x-1,\ y := y-2 \ \sigma_2 \Downarrow \{x \mapsto 0, y \mapsto 4\}} \qquad \dfrac{\sigma_3 \nvDash x > 0}{\text{while } x>0 \text{ do } (x := x-1,\ y := y-2)\ \sigma_3 \Downarrow \sigma'}}{\text{while } x>0 \text{ do } (x := x-1,\ y := y-2)\ \sigma_2 \Downarrow \sigma'}$$

$$\sigma_3 = \{x \mapsto 0, y \mapsto 4\}$$

$$\Delta_1 = \dfrac{x := x-1,\ \sigma_1 \Downarrow \{x \mapsto 1, y \mapsto 3\} \qquad y := y-2 \ \{x \mapsto 1, y \mapsto 3\} \Downarrow \sigma_2}{x := x-1;\ y := y-2 \Downarrow \{x \mapsto 1, y \mapsto 2\}}$$

$$\sigma_1 = \{x \mapsto 2, y \mapsto 3\} \qquad \sigma_2 = \{x \mapsto 1, y \mapsto 2\}$$

$$\dfrac{\{x \mapsto 2, y \mapsto 3\} \vDash x > 0 \qquad \dfrac{\Delta_1}{x := x-1,\ y := y-2 \Downarrow \{x \mapsto 1, y \mapsto 2\}} \qquad \dfrac{\Delta_2}{\text{while } x>0 \text{ do } (x := x-1,\ y := y-2)\ \{x \mapsto 1, y \mapsto 2\} \Downarrow \sigma'}}{\text{while } x>0 \text{ do } (x := x-1,\ y := y-2)\ \{x \mapsto 2, y \mapsto 3\} \Downarrow \sigma'}$$

4.5.2 ex.2

$$\Delta_7 = \dfrac{\dfrac{}{\text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_4 \to y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_4} \qquad \dfrac{\Delta_8}{y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_4 \overset{*}{\to} \text{skip } \sigma'}}{\text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_4 \overset{*}{\to} \text{skip } \sigma'}$$

$$\Delta_6 = \dfrac{\dfrac{\dfrac{x:=x-1\ \sigma_3 \to \text{skip } \sigma_4}{x:=x-1; y:=y\cdot 2\ \sigma_3 \to \text{skip } y:=y\cdot 2\ \sigma_4}}{x:=x-1; y:=y\cdot 2 \text{ while } x>0 \text{ do } \lambda\ \sigma_3 \to \text{skip}; y:=y\cdot 2 \text{ while } x>0 \text{ do } \lambda\ \sigma_4} \qquad \dfrac{\Delta_7}{\text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_4 \overset{*}{\to} \text{skip } \sigma'}}{\lambda \text{ while } c \text{ do } \lambda\ \sigma_3 \overset{*}{\to} \text{skip } \sigma'}$$

$$\sigma_4 = \{x \mapsto 0, y \mapsto 2\}$$

$$\Delta_5 = \dfrac{\dfrac{\sigma_3 \models x>0}{\text{while } c \text{ do } \lambda\ \sigma_3 \to \lambda \text{ while } c \text{ do } \lambda\ \sigma_3} \qquad \dfrac{\Delta_6}{\lambda \text{ while } c \text{ do } \lambda\ \sigma_3 \overset{*}{\to} \text{skip } \sigma''}}{\text{while } c \text{ do } \lambda\ \sigma_3 \overset{*}{\to} \text{skip } \sigma''}$$

$$\Delta_4 = \dfrac{\dfrac{}{\text{skip while } c \text{ do } \lambda\ \sigma_3 \to \text{while } c \text{ do } \lambda\ \sigma_3} \qquad \dfrac{\Delta_5}{\text{while } c \text{ do } \lambda\ \sigma_3 \overset{*}{\to} \text{skip } \sigma'}}{\text{skip while } c \text{ do } \lambda\ \sigma_3 \overset{*}{\to} \text{skip } \sigma'}$$

$$\Delta_3 = \dfrac{\dfrac{y:=y\cdot 2\ \sigma_2 \to \text{skip } \sigma_3}{y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_2 \to \text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_3} \qquad \dfrac{\Delta_4}{\text{skip while } c \text{ do } \lambda\ \sigma_3 \overset{*}{\to} \text{skip } \sigma'}}{y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_2 \overset{*}{\to} \text{skip } \sigma'}$$

$$\sigma_3 = \{x \mapsto 1, y \mapsto 2\}$$

$$\Delta_2 = \dfrac{\dfrac{}{\text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \sigma_2 \to y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_2} \qquad \dfrac{\Delta_3}{y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_2 \overset{*}{\to} \text{skip } \sigma'}}{\text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \sigma_2 \overset{*}{\to} \text{skip } \sigma'}$$

$$\Delta_1 = \dfrac{\dfrac{\dfrac{x:=x-1\ \sigma_1 \to \text{skip } \{x \mapsto 1, y \mapsto 1\}}{x:=x-1, y:=y\cdot 2\ \sigma_1 \to \text{skip } y:=y\cdot 2\ \{x \mapsto 1, y \mapsto 1\}}}{x:=x-1; y:=y\cdot 2 \text{ while } x>0 \text{ do } \lambda\ \sigma_1 \to \text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \{x \mapsto 1, y \mapsto 1\}} \qquad \dfrac{\Delta_2}{\text{skip } y:=y\cdot 2 \text{ while } c \text{ do } \lambda\ \sigma_2 \overset{*}{\to} \text{skip } \sigma'}}{\lambda \text{ while } c \text{ do } \lambda\ \sigma_1 \overset{*}{\to} \text{skip } \sigma'}$$

$$\sigma_2 = \{x \mapsto 1, y \mapsto 1\}$$

$$\dfrac{\dfrac{\sigma_1 \models x>0}{\text{while } c \text{ do } \lambda\ \sigma_1 \to \lambda \text{ while } c \text{ do } \lambda\ \sigma_1} \qquad \dfrac{\Delta_1}{\lambda \text{ while } c \text{ do } \lambda\ \sigma_1 \overset{*}{\to} \text{skip } \sigma'}}{\underbrace{\text{while } x>0}_{c} \text{ do } \underbrace{(x:=x-1, y:=y\cdot 2)}_{\lambda} \underbrace{\{x \mapsto 2, y \mapsto 1\}}_{\sigma_1} \overset{*}{\to} \sigma'}$$

$$\sigma_5 = \sigma'$$

$$\Delta_{10}\ \frac{\sigma_6 \not\models x > 0}{\text{while } c \text{ do } \downarrow \sigma_5 \rightsquigarrow \text{skip } \sigma_6} \qquad \frac{\sigma_5 \rightsquigarrow \text{skip } \sigma'}{\text{skip } \sigma_6 \rightsquigarrow \text{skip } \sigma'}$$
$$\text{while } c \text{ do } \downarrow \sigma_5 \rightsquigarrow \text{skip } \sigma'$$

$$\Delta_9 = \frac{\text{skip while } c \text{ do } \downarrow \sigma_5 \rightsquigarrow \text{while } c \text{ do } \downarrow \sigma_5 \qquad \text{while } c \text{ do } \downarrow \sigma_5 \rightsquigarrow \text{skip } \sigma'}{\text{skip while } c \text{ do } \downarrow \sigma_5 \rightsquigarrow \text{skip } \sigma'}$$

$$\sigma_5 = \{x \mapsto 0, y \mapsto 43\}$$

$$\Delta_8 = \frac{y := y \cdot 2 \rightsquigarrow \text{skip } \sigma_5 \qquad \text{skip while } c \text{ do } \downarrow \sigma_5 \rightsquigarrow \text{skip } \sigma'}{y := y \cdot 2 \text{ while } c \text{ do } \downarrow \sigma_4 \rightsquigarrow \text{skip } \sigma'}$$

4.6.2

3-rule system

① $\dfrac{a\,Ra'}{a\,R^*a'}$ ② $\dfrac{a_1 R^* a_2 \quad a_2 R^* a_3}{a_1 R^* a_3}$ ③ $\dfrac{}{aR^*a'}$

2-rule

① $\dfrac{a_1 R a_2 \quad a_2 R^* a_3}{a_1 R^* a_3}$ ② $\dfrac{}{a\,R^*a}$

observe: $\dfrac{a_1 R a_2 \quad a_2 R^* a_3}{a_1 R^* a_3} = \dfrac{\dfrac{a_1 R a_2}{a_1 R^* a_2} \quad a_2 R^* a_3}{a_1 R^* a_3}$

1. obersee rule 1 of the two rule system is an application of 3-rule system rule 3 to rule 2.

2. $\dfrac{a\,Ra'}{a\,R^*a'} = \dfrac{a\,Ra_0' \quad \overline{a_0' R^* a'}}{a\,R^*a'}$

3. Assume $\overline{a_1 R^* a_2} \quad \overline{a_2 R^* a_3}$

show $\overline{a_1 R^* a_3}$

if $\dfrac{a_1 Ra_2' \quad \overline{a_2 R^* a_3}}{a_1 R^* a_3}$ = T by IH

Therefore $\dfrac{}{a_1 R^* a_3}$

4. Since the 2-rule system is derivable from the three rule system & vice versa, the two are equivalent.