Appendix A

Theorem 1 *In the cloud node game model, let $\mathcal{B}_k$ denote the total transaction pool in round k, and $(\psi^*_{i,k}, i \in C)$ constitute the equilibrium strategy for cloud node i, where:*

$$\psi^*_{i,k} = \begin{cases} max(\mathcal{B}_k) & If\ \zeta_{max} \geq \eta \\ \varnothing & If\ \zeta_{max} < \eta \end{cases} \tag{A1}$$

Proof According to the profit function of cloud node i in round k, we have:

$$\begin{aligned} \pi_{c_{i,k}} &= \alpha_i \sum_{tx_{i'j} \in \psi_{i,k}} \zeta_{i'j}\sigma_{i'j} - \sum_{l \in C} \alpha_l \sum_{tx_x \in \psi_{l,k}} \sigma_x \eta \\ &= \alpha_i \sum_{tx_{i'j} \in \psi_{i,k}} (\zeta_{i'j} - \eta)\sigma_{i'j} - \sum_{l \in C, l \neq i} \alpha_l \sum_{tx_x \in \psi_{l,k}} \sigma_x \eta \end{aligned}$$

It is not difficult to observe that the strategy of cloud node i can only determine the value of $\alpha_i \sum_{tx_{i'j} \in \psi_{i,k}} (\zeta_{i'j} - \eta)\sigma_{i'j}$. Therefore, there are two cases:

- If $\zeta_{\max} \geq \eta$, then $\zeta_{i'j} - \eta \geq 0$, and $\alpha_i \sum_{tx_{i'j} \in \psi_{i,k}} (\zeta_{i'j} - \eta)\sigma_{i'j} \geq 0$. In this case, cloud node i selects $\psi_{i,k} = \max(\mathcal{B}_k)$, which means it chooses transactions with the highest unit byte fee from the transaction pool to include in the block.
- If $\zeta_{\max} < \eta$, then $\zeta_{i'j} - \eta < 0$, and $\alpha_i \sum_{tx_{i'j} \in \psi_{i,k}} (\zeta_{i'j} - \eta)\sigma_{i'j} < 0$. In this case, cloud node i selects $\psi_{i,k} = \varnothing$, indicating that it does not select any transactions and temporarily suspends block generation.

$\square$

Appendix B

For user node $i \in U$, its average transaction waiting time τ_i is:

$$\tau_i = \begin{cases} \dfrac{\lambda_{i_1}}{\Phi - \sum_{l \in U} \lambda_{l_1}} + \sum_{q=2}^{I} \dfrac{\Phi \lambda_{i_q}}{(\Phi - \sum_{x=1}^{q-1} \sum_{l \in U} \lambda_{l_x})(\Phi - \sum_{x=1}^{q} \sum_{l \in U} \lambda_{l_x})} & \text{If } \zeta_1 > \zeta_2 > ... > \zeta_I \geq \eta \\ \dfrac{\lambda_{i_1}}{\Phi - \sum_{l \in U} \lambda_{l_1}} + \sum_{q=2}^{Y} \dfrac{\Phi \lambda_{i_q}}{(\Phi - \sum_{x=1}^{q-1} \sum_{l \in U} \lambda_{l_x})(\Phi - \sum_{x=1}^{q} \sum_{l \in U} \lambda_{l_x})} & \text{If } \zeta_1 > \zeta_2 > ...\zeta_Y \geq \eta > ... > \zeta_I \\ \dfrac{\lambda_{i_1}}{\Phi - \sum_{l \in U} \lambda_{l_1}} & \text{If } \zeta_1 \geq \eta > \zeta_2 > ... > \zeta_I \\ \infty & \text{If } \eta > \zeta_1 > \zeta_2 > ... > \zeta_I \end{cases} \tag{B2}$$

Proof If for all $q \in I$, $\zeta_q < \eta$, then regardless of the unit byte transaction fee chosen by user node i, transactions will not be packaged into blocks by cloud nodes. Therefore, it is evident that the average transaction waiting time τ_i for user node i is ∞ at this point.

Now, consider the case where $\zeta_1 \geq \eta > \zeta_2 > \ldots > \zeta_I$. In this scenario, only when user node i chooses the unit byte fee ζ_1 to generate transactions, will transactions be selected by cloud nodes for packaging, as discussed in subsequent proof sections. All other transactions can be neglected, effectively reducing the transaction pool $\mathcal{B}$ to only contain transactions with a unit byte fee of ζ_1.

The average transaction waiting time τ_i for user i in this scenario can be further expressed as:

$$\tau_i = \lim_{t \to \infty} \frac{\sum_{j=1}^{txn_{i_1}} \mathbb{E}[\tau_{i_1,j}]}{t} = \lim_{t \to \infty} \frac{\sum_{j=1}^{txn_{i_1}} \mathbb{E}[\tau_{i_1,j}]}{txn_{i_1}} \lim_{t \to \infty} \frac{txn_{i_1}}{t}$$

Here, txn_{i_1} represents the total number of transactions generated by user node i with a unit byte fee of ζ_1, and $\mathbb{E}[\tau_{i_1,j}]$ is the mathematical expectation of the transaction waiting time corresponding to the j-th transaction.

According to the limit distribution in preemptive priority $M/M/1$ queueing systems in queueing theory, and considering that the transaction generation rate within the system for unit byte fee ζ_1 is $\sum_{l\in U}\lambda_{l_1}$, we have:

$$\lim_{t\to\infty}\frac{\sum_{j=1}^{txn_{i_1}}\mathbb{E}[\tau_{i_1,j}]}{txn_{i_1}}=\lim_{t\to\infty}\mathbb{E}[\tau_{i_1,txn_{i_1}}]=\frac{1}{\Phi-\sum_{l\in U}\lambda_{l_1}}$$

The transaction generation follows a Poisson distribution, hence:

$$\lim_{t\to\infty}\frac{txn_{i_1}}{t}=\lambda_{i_1}$$

Therefore, for the case of $\zeta_1\geq\eta>\zeta_2>...>\zeta_I$, the average transaction waiting time τ_i for user i is:

$$\tau_i=\lim_{t\to\infty}\frac{\sum_{j=1}^{txn_{i_1}}\mathbb{E}[\tau_{i_1,j}]}{t}=\lim_{t\to\infty}\frac{\sum_{j=1}^{txn_{i_1}}\mathbb{E}[\tau_{i_1,j}]}{txn_{i_1}}\lim_{t\to\infty}\frac{txn_{i_1}}{t}=\frac{\lambda_{i_1}}{\Phi-\sum_{l\in U}\lambda_{l_1}}$$

Next, let's consider the case where $\zeta_1>\zeta_2>...\zeta_Y\geq\eta>\zeta_{Y+1}>...>\zeta_I$. In this scenario, the unit byte fee available for user i to choose from are ζ_q for all $q\in[1,Y]$. Any transactions corresponding to unit byte fee less than ζ_Y will not be considered by the cloud nodes. We have already obtained the average transaction waiting time for user i when generating transactions with ζ_1 as the unit byte fee in the previous proof. Now, let's discuss the case when user i generates transactions with ζ_q $(q\geq 2)$ as the unit byte fee.

Let $n_b(\zeta_q)$ denote the number of transactions with byte fee strictly greater than ζ_q, and $n_e(\zeta_q)$ denote the number of transactions with byte fee equal to ζ_q. According to the fact that transactions with byte fee strictly greater than ζ_q have higher priority than those with byte fee equal to ζ_q, and the relevant formulas for the $M/M/1$ queue, we have:

$$\mathbb{E}[n_b(\zeta_q)]=\frac{\sum_{x=1}^{q-1}\sum_{l\in U}\lambda_{l_x}}{\Phi-\sum_{x=1}^{q-1}\sum_{l\in U}\lambda_{l_x}}$$

$$\mathbb{E}[n_b(\zeta_q)]+\mathbb{E}[n_e(\zeta_q)]=\frac{\sum_{x=1}^{q}\sum_{l\in U}\lambda_{l_x}}{\Phi-\sum_{x=1}^{q}\sum_{l\in U}\lambda_{l_x}}$$

Combining the above formulas with Little's Law, we can conclude that:

$$\lim_{t\to\infty}\mathbb{E}[\tau_{i_q,txn_{i_q}}]=\frac{\mathbb{E}[n_e(\zeta_q)]}{\sum_{l\in U}\lambda_{l_q}}=\frac{\mathbb{E}[n_b(\zeta_q)]+\mathbb{E}[n_e(\zeta_q)]-\mathbb{E}[n_b(\zeta_q)]}{\sum_{l\in U}\lambda_{l_q}}=\frac{\Phi}{(\Phi-\sum_{x=1}^{q-1}\sum_{l\in U}\lambda_{l_x})(\Phi-\sum_{x=1}^{q}\sum_{l\in U}\lambda_{l_x})}$$

Similarly, we have:

$$\lim_{t\to\infty}\frac{\sum_{j=1}^{txn_{i_q}}\mathbb{E}[\tau_{i_q,j}]}{t}=\lim_{t\to\infty}\frac{\sum_{j=1}^{txn_{i_q}}\mathbb{E}[\tau_{i_q,j}]}{txn_{i_q}}\lim_{t\to\infty}\frac{txn_{i_q}}{t}=\frac{\Phi\lambda_{l_q}}{(\Phi-\sum_{x=1}^{q-1}\sum_{l\in U}\lambda_{l_x})(\Phi-\sum_{x=1}^{q}\sum_{l\in U}\lambda_{l_x})}$$

Therefore, for the case where $\zeta_1>\zeta_2>...\zeta_Y\geq\eta>\zeta_{Y+1}>...>\zeta_I$, the average transaction waiting time τ_i for user i is given by:

$$\begin{aligned}\tau_i&=\lim_{t\to\infty}\frac{\sum_{q=1}^{Y}\sum_{j=1}^{txn_{i_q}}\mathbb{E}[\tau_{i_q,j}]}{t}=\sum_{q=1}^{Y}\lim_{t\to\infty}\frac{\sum_{j=1}^{txn_{i_q}}\mathbb{E}[\tau_{i_q,j}]}{txn_{i_q}}\lim_{t\to\infty}\frac{txn_{i_q}}{t}\\&=\frac{\lambda_{i_1}}{\Phi-\sum_{l\in U}\lambda_{l_1}}+\sum_{q=2}^{Y}\frac{\Phi\lambda_{i_q}}{(\Phi-\sum_{x=1}^{q-1}\sum_{l\in U}\lambda_{l_x})(\Phi-\sum_{x=1}^{q}\sum_{l\in U}\lambda_{l_x})}\end{aligned}$$

For the case where $\zeta_1>\zeta_2>...>\zeta_I\geq\eta$, the proof process is similar to the case of $\zeta_1>\zeta_2>...\zeta_Y\geq\eta>\zeta_{Y+1}>...>\zeta_I$, and the final result is obtained as follows:

$$\tau_i=\frac{\lambda_{i_1}}{\Phi-\sum_{l\in U}\lambda_{l_1}}+\sum_{q=2}^{I}\frac{\Phi\lambda_{i_q}}{(\Phi-\sum_{x=1}^{q-1}\sum_{l\in U}\lambda_{l_x})(\Phi-\sum_{x=1}^{q}\sum_{l\in U}\lambda_{l_x})}$$

□

Appendix C

In the user node game model, suppose $\boldsymbol{\lambda}_H^*$ represents the equilibrium strategy of high-value user nodes, and $\boldsymbol{\lambda}_L^*$ represents the equilibrium strategy of low-value user nodes. Together, they constitute the equilibrium strategy of all users in the system, where:

$$\boldsymbol{\lambda}_{H/L}^* = \begin{cases} \lambda_{H/L}(\zeta_1, U_H^{net}, U_L^{net}) & \text{If } \varpi(\zeta_2, U_H^{net}, U_L^{net}) > \zeta_1\sigma \\ \lambda_{H/L}(\zeta_i, U_H^{net}, U_L^{net}) & \text{If } \varpi(\zeta_{i+1}, U_H^{net}, U_L^{net}) > \zeta_i\sigma, \varpi(\zeta_i, U_H^{net}, U_L^{net}) \leq \zeta_{i-1}\sigma \\ \lambda_{H/L}(\zeta_I, U_H^{net}, U_L^{net}) & \text{If } \varpi(\zeta_I, U_H^{net}, U_L^{net}) \leq \zeta_{I-1}\sigma \end{cases} \tag{C3}$$

$$\lambda_H = \begin{cases} 0 & \text{If } U_H^{net} \leq \zeta_i\sigma + \dfrac{\xi}{\Phi} \\ h_1 & \text{If } U_H^{net} > \zeta_i\sigma + \dfrac{\xi}{\Phi}, U_L^{net} \leq \zeta_i\sigma + \dfrac{\xi}{\Phi - h_1 n_H^{net}} \\ min(\dfrac{\Phi}{n_H^{net} + n_L^{net}}, \dfrac{(\Phi - h_2)[h_2(U_H^{net} - \zeta_i\sigma) - \xi]}{n_H^{net}[h_2(U_H^{net} - \zeta_i\sigma) - \xi) + n_L^{net}[h_2(U_L^{net} - \zeta_i\sigma) - \xi)]}) & \text{If } U_L^{net} > \zeta_i\sigma + \dfrac{\xi}{\Phi - h_1 n_H^{net}} \end{cases}$$

$$\lambda_L = \begin{cases} 0 & \text{If } U_L^{net} \leq \zeta_i\sigma + \dfrac{\xi}{\Phi - h_1 n_H^{net}} \\ \dfrac{\Phi - n_H^{net}\Lambda_H}{n_L^{net}} - \dfrac{\xi(n_L^{net} - 1) + \sqrt{\xi^2(n_L^{net} - 1)^2 + 4n_L^{net}(U_L^{net} - \zeta_i\sigma)\xi(\Phi - n_H^{net}\Lambda_H)}}{2N_{net}^{l2}(U_L^{net} - \zeta_i\sigma)} & \text{If } U_L^{net} > \zeta_i\sigma + \dfrac{\xi}{\Phi - h_1 n_H^{net}} \end{cases}$$

$$h_1 = min(\frac{\Phi}{n_H^{net} + n_L^{net}}, \frac{\Phi}{n_H^{net}} - \frac{\xi(n_H^{net} - 1) + \sqrt{\xi^2(n_H^{net} - 1)^2 + 4\xi\Phi n_H^{net}(U_H^{net} - \zeta_i\sigma)}}{2n_H^{net^2}(U_H^{net} - \zeta_i\sigma)})$$

$$h_2 = \frac{\sqrt{\xi^2(n_H^{net} + n_L^{net} - 1)^2 + 4\xi\Phi[n_H^{net}(U_H^{net} - \zeta_i\sigma) + n_L^{net}(U_L^{net} - \zeta_i\sigma)]}}{2[n_H^{net}(U_H^{net} - \zeta_i\sigma) + n_L^{net}(U_L^{net} - \zeta_i\sigma)]} + \frac{\xi(n_H^{net} + n_L^{net} - 1)}{2[n_H^{net}(U_H^{net} - \zeta_i\sigma) + n_L^{net}(U_L^{net} - \zeta_i\sigma)]}$$

$$\varpi(\zeta_I, U_H^{net}, U_L^{net}) = max(\zeta_i\sigma + \frac{\xi[(n_H^{net} - 1)\Lambda_H + n_L^{net}\Lambda_L]}{\Phi(\Phi - n_H^{net}\Lambda_H - n_L^{net}\Lambda_L)}, U_L^{net} - \frac{\xi}{\Phi} - \frac{\xi\Lambda_L(2\Phi - n_H^{net}\Lambda_H - n_L^{net}\Lambda_L])}{\Phi(\Phi - n_H^{net}\Lambda_H - n_L^{net}\Lambda_L)^2})$$

Proof Consider a unit byte fee vector $\boldsymbol{\zeta}$ satisfying the condition $\zeta_1 > \zeta_2 > \ldots > \zeta_I \geq \eta$.[1] Combining the user node's payoff function and the average transaction waiting time derived, we can obtain the unit-time payoff function for user nodes as follows:

$$\pi_{u_i} = \sum_{l \in U, l \neq i} \sum_{q=1}^{I} \lambda_{l_q}\sigma C_{xx} + \lambda_{i_1}(U_{net} - \zeta_1\sigma - \frac{\xi}{\Phi - \sum_{l \in U}\lambda_{l_1}}) + \sum_{q=2}^{I}(U_{net} - \zeta_q\sigma - \frac{\xi\Phi}{(\Phi - \sum_{x=1}^{q-1}\sum_{l \in U}\lambda_{l_x})(\Phi - \sum_{x=1}^{q}\sum_{l \in U}\lambda_{l_x})}) \tag{C4}$$

Based on Equation C4, the goal of user nodes is always to maximize their own payoff. Suppose there exists a high-net-worth user node i at this moment, with all other users in the system already at equilibrium. The optimization problem for user i is as follows:

[1] In fact, in the system designer stage, the design principle for unit byte fee is $\forall i \in [1, I], \zeta_i \geq n_c\eta$, where n_c represents the number of inner-layer cloud nodes. Therefore, this assumption is reasonable.

$$
\begin{aligned}
max \quad & \pi_{u_i} = \lambda_{i_1}(U_H^{net} - \zeta_1\sigma - \frac{\xi}{\Phi - \lambda_{i_1} - (n_H^{net}-1)\lambda_{H_1}^* - n_L^{net}\lambda_{L_1}^*}) + \\
& \sum_{q=2}^{I}(U_H^{net} - \zeta_q\sigma - \frac{\xi\Phi}{[\Phi - \sum_{x=1}^{q-1}(\lambda_{i_x} + (n_H^{net}-1)\lambda_{H_x}^* + n_L^{net}\lambda_{L_x}^*)][\Phi - \sum_{x=1}^{q}(\lambda_{i_x} + (n_H^{net}-1)\lambda_{H_x}^* + n_L^{net}\lambda_{L_x}^*)]}) \\
s.t. \quad & \sum_{q=1}^{I}\lambda_{i_q} \leq \frac{\Phi}{n_H^{net} + n_L^{net}} \\
var. \quad & \lambda_{i_q} \geq 0, q = 1, 2, ..., I
\end{aligned}
\tag{C5}
$$

Where $\lambda_{H_x}^*$ and $\lambda_{L_x}^*$ respectively denote the transaction generation rates of high-net-worth user nodes and low-net-worth user nodes at equilibrium, given the unit byte cost ζ_x.

The optimization problem above is a constrained inequality optimization. We can obtain the optimal solution by defining the Lagrangian function and introducing the Karush–Kuhn–Tucker (KKT) conditions. The Lagrangian function constructed for Problem C5 is as follows:

$$
L(\boldsymbol{\lambda}_i, \boldsymbol{\nu}_z) = \pi_{u_i} + \nu_{i_0}(\frac{\Phi}{n_H^{net} + n_L^{net}} - \sum_{q=1}^{I}\lambda_{i_q}) + \nu_{i_1}\lambda_{i_1} + ... + \nu_{i_I}\lambda_{i_I}
$$

Where $\boldsymbol{\lambda}_i = (\lambda_{i_1}, \lambda_{i_2}, ...\lambda_{i_I})$, Lagrange multiplier $\boldsymbol{\nu}_z = (\nu_{i_0}, \nu_{i_1}, ...\nu_{i_I})$.

The KKT condition is as follows:

$$
\begin{cases}
\sum_{q=1}^{I}\lambda_{i_q}^* \leq \frac{\Phi}{n_H^{net} + n_L^{net}} \\
\nu_{i_q}\lambda_{i_q}^* = 0 \\
\frac{\partial \pi_{u_i}}{\partial \lambda_{i_q}}|_{\lambda_{i_q} = \lambda_{i_q}^*} + \nu_{i_q} - \nu_{i_0} = 0 \\
\nu_{i_0}(\sum_{q=1}^{I}\lambda_{i_q}^* - \frac{\Phi}{n_H^{net} + n_L^{net}}) = 0 \\
\lambda_{i_q}^* \geq 0 \\
\nu_{i_q} \geq 0, q = 0, 1, ..., I
\end{cases}
$$

For the discussion of the above KKT conditions, since π_{u_i} achieves its optimal value at the transaction generation rate $\lambda_{i_q}^*, \forall q = 1, 2, ...I$, we discuss different cases for the value of q, and the general formula is as follows:

$$
\begin{cases}
\frac{\partial \pi_{u_i}}{\partial \lambda_{i_q}}|_{\lambda_{i_q} = \lambda_{i_q}^*} = 0, & \lambda_{i_q}^* \in [0, \frac{\Phi}{n_H^{net} + n_L^{net}}] \\
\frac{\partial \pi_{u_i}}{\partial \lambda_{i_x}}|_{\lambda_{i_x} = \lambda_{i_x}^*} \leq 0, & \lambda_{i_x}^* = 0, x \in [1, I], x \neq i
\end{cases}
$$

Solving the above general formula, we can obtain the equilibrium strategy for high net worth users. For low net worth users, the analysis process is similar to that of high net worth users.

This completes the proof.

□

Appendix D

The optimal solution for the unit byte fee vector $\boldsymbol{\zeta}$ and unit byte compensation $\boldsymbol{C}$ determined by the system designer are as follows, which can cover the total expenses of the system's cloud nodes while achieving optimal social welfare.

Vector of unit byte transaction fee $\boldsymbol{\zeta}$=$(\zeta_1, \zeta_2, ..., \zeta_I)$, $\forall q \in [1, I]$, we have

$$
\zeta_q = n_c\eta + \frac{(I-q)\xi}{\Phi\sigma}
$$

Vector of unit byte compensation $\boldsymbol{C} = (C_{HH}, C_{HL}, C_{LH}, C_{LL})$ satisfies:

- If $U_H \leq n_c\eta\sigma + \dfrac{\xi}{\Phi}$

$$\begin{cases}(n_H-1)C_{HH}+n_LC_{HL}=0\\ n_HC_{LH}+(n_L-1)C_{LL}=0\end{cases}$$

- If $U_H > n_c\eta\sigma + \dfrac{\xi}{\Phi}$

$$\begin{cases}(n_H-1)C_{HH}+n_LC_{HL}=\dfrac{U_H}{\sigma}-\zeta_I-\dfrac{\xi[\Phi-(n_H-1)g_1-n_Lg_2]}{(\Phi-n_Hg_1-n_Lg_2)^2\sigma}\\ n_HC_{LH}+(n_L-1)C_{LL}=\dfrac{U_L}{\sigma}-\zeta_I-\dfrac{\xi[\Phi-n_Hg_1-(n_L-1)g_2]}{(\Phi-n_Hg_1-n_Lg_2)^2\sigma}\end{cases}$$

Where:

$$\begin{cases}g_1=min(\dfrac{\Phi}{n_H+n_L},\dfrac{1}{n_H}[\Phi-\sqrt{\dfrac{\Phi\xi}{U_H-n_c\eta\sigma}}])\\ g_2=\begin{cases}0 & \text{If } U_L<n_c\eta\sigma+\dfrac{\xi(n_H+n_L)^2}{\Phi n_L^2}\\ \dfrac{\Phi}{n_H+n_L}-\dfrac{1}{n_L}\sqrt{\dfrac{\Phi\xi}{U_L-n_c\eta\sigma}} & \text{Others}\end{cases}\end{cases}$$

Proof We first derive the equilibrium strategy for user transaction generation when the system achieves optimal social welfare without minimum unit byte fee constraints and compensation. Then, with the inclusion of minimum fee constraints and compensation, we demonstrate that the user transaction generation strategy remains consistent with the unconstrained case.

Firstly, without considering compensation, assuming the unit byte fee vector $\boldsymbol{\zeta}$ satisfies $\zeta_1>\zeta_2>...>\zeta_I \geq \eta$, the formula for social welfare is as follows:

$$\begin{aligned}\vartheta^{'} &= \sum_{k\in C}\pi_{c_i}+\sum_{i\in U}\pi_{u_i}\\ &=\sum_{i\in U}\lim_{t\to\infty}\frac{\sum_{j=1}^{tx(i)}\zeta_{ij}\sigma_{ij}-n_c\sigma_{ij}\eta}{t}+\sum_{i\in U}\lim_{t\to\infty}\frac{\sum_{j=1}^{tx(i)}U_x-\zeta_{ij}\sigma_{ii}-\xi\tau_{ij}}{t}\\ &=\sum_{i\in U}\lim_{t\to\infty}\frac{txn_i}{t}\lim_{t\to\infty}\frac{\sum_{j=1}^{tx(i)}R_x}{txn_i}-\lim_{t\to\infty}\frac{\xi\sum_{i\in U}txn_i}{t}\lim_{t\to\infty}\frac{\sum_{i\in U}\sum_{j=1}^{tx(i)}\tau_{ij}}{\sum_{i\in U}txn_i}-\sum_{i\in U}\lim_{t\to\infty}\frac{txn_i}{t}\lim_{t\to\infty}\frac{\sum_{j=1}^{tx(i)}n_c\sigma_{ij}\eta}{txn_i}\\ &=\sum_{i\in U}\sum_{q=1}^{I}\lambda_{i_q}U_x-\xi\sum_{i\in U}\sum_{q=1}^{I}\lambda_{i_q}\frac{1}{\Phi-\sum_{i\in U}\sum_{q=1}^{I}\lambda_{i_q}}-\sum_{i\in U}\sum_{q=1}^{I}\lambda_{i_q}n_c\hat{\sigma}\eta\\ &=\sum_{i\in U_H}\sum_{q=1}^{I}\lambda_{H_q}(U_H-n_c\hat{\sigma}\eta)+\sum_{i\in U_L}\sum_{q=1}^{I}\lambda_{L_q}(U_L-n_c\hat{\sigma}\eta)-\frac{\xi(\sum_{i\in U_H}\sum_{q=1}^{I}\lambda_{H_q}+\sum_{i\in U_L}\sum_{q=1}^{I}\lambda_{L_q})}{\Phi-(\sum_{i\in U_H}\sum_{q=1}^{I}\lambda_{H_q}+\sum_{i\in U_L}\sum_{q=1}^{I}\lambda_{L_q})}\end{aligned}$$

Where txn_i is the number of transactions generated by user i, and $\hat{\sigma}$ is the average transaction size.

From the above equation, the social welfare function is a function of the user transaction generation rates. We denote (Λ_H,Λ_L) as $(\sum_{i\in U_H}\sum_{q=1}^{I}\lambda_{H_q},\sum_{i\in U_L}\sum_{q=1}^{I}\lambda_{L_q})$. At this point, the optimization problem becomes:

$$\begin{aligned} max \quad & \vartheta^{'}=\Lambda_H(U_H-n_c\hat{\sigma}\eta)+\Lambda_L(U_L-n_c\hat{\sigma}\eta)-\frac{\xi(\Lambda_H+\Lambda_L)}{\Phi-\Lambda_H-\Lambda_L}\\ s.t. \quad & 0\leq\Lambda_H\leq\frac{\Phi n_H}{n_H+n_L},0\leq\Lambda_L\leq\frac{\Phi n_L}{n_H+n_L}\\ var. \quad & \Lambda_H,\Lambda_L\end{aligned} \tag{D6}$$

The above problem is also a convex optimization problem with inequality constraints, similar to the optimization problem (D6), and can be solved by defining the Lagrangian function and introducing the KKT conditions. The Lagrangian function constructed is as follows:

$$L(\boldsymbol{\Lambda},\boldsymbol{\nu})=\vartheta^{'}(\Lambda_H,\Lambda_L)+\nu_1(\frac{\Phi n_H}{n_H+n_L}-\Lambda_H)+\nu_2(\frac{\Phi n_L}{n_H+n_L}-\Lambda_L)+\nu_3\Lambda_H+\nu_4\Lambda_L$$

Where $\boldsymbol{\nu}=(\nu_1,\nu_2,\nu_3,\nu_4)$ is the Lagrange multiplier.

The KKT condition is as follows:

$$\begin{cases} 0\leq\Lambda_H^*\leq\dfrac{\Phi n_H}{n_H+n_L}, 0\leq\Lambda_L^*\leq\dfrac{\Phi n_L}{n_H+n_L} \\ \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_H}|_{\Lambda_H=\Lambda_H^*}-\nu_1+\nu_3=0 \\ \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_L}|_{\Lambda_L=\Lambda_L^*}-\nu_2+\nu_4=0 \\ \nu_1(\dfrac{\Phi n_H}{n_H+n_L}-\Lambda_H)=\nu_2(\dfrac{\Phi n_L}{n_H+n_L}-\Lambda_L)=\nu_3\Lambda_H=\nu_4\Lambda_L=0 \\ \nu_1,\nu_2,\nu_3,\nu_4\geq 0 \end{cases}$$

Since

$$\begin{cases} \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_H}=U_H-n_c\hat{\sigma}\eta-\dfrac{\Phi\xi}{(\Phi-\Lambda_H-\Lambda_L)^2} \\ \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_L}=U_L-n_c\hat{\sigma}\eta-\dfrac{\Phi\xi}{(\Phi-\Lambda_H-\Lambda_L)^2} \end{cases}$$

And $U_H>U_L$, thus $\frac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_H}>\frac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_L}$, therefore, in discussing the Karush-Kuhn-Tucker (KKT) conditions, there are three possible cases in total:

- $$\begin{cases} \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_H}|_{\Lambda_H=\Lambda_H^*}\leq 0, \qquad \Lambda_H^*=0 \\ \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_L}|_{\Lambda_L=\Lambda_L^*}<0, \qquad \Lambda_L^*=0 \end{cases}$$

  When $U_H\leq n_c\hat{\sigma}\eta$, satisfying the above conditions, the solution is obtained as $(\Lambda_H^*,\Lambda_L^*)=(0,0)$, indicating that no users in the system generate transactions, achieving the optimal social welfare.

- $$\begin{cases} \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_H}|_{\Lambda_H=\Lambda_H^*}\geq 0, \qquad \Lambda_H^*\in[0,\dfrac{\Phi n_H}{n_H+n_L}] \\ \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_L}|_{\Lambda_L=\Lambda_L^*}<0, \qquad \Lambda_L^*=0 \end{cases}$$

  When $U_H\geq n_c\hat{\sigma}\eta$, and $U_L<n_c\hat{\sigma}\eta+\frac{\xi(n_H+n_L)^2}{\Phi n_L^2}$, satisfying the above conditions, the solution is obtained as $(\Lambda_H^*,\Lambda_L^*)=(\min(\frac{\Phi n_H}{n_H+n_L},\Phi-\sqrt{\frac{\Phi\xi}{U_H-n_c\hat{\sigma}\eta}}),0)$, indicating that in the system, high-utility users generate transactions with $\Lambda_H^*=\frac{1}{n_H}\min(\frac{\Phi n_H}{n_H+n_L},\Phi-\sqrt{\frac{\Phi\xi}{U_H-n_c\hat{\sigma}\eta}})$, while low-utility users do not generate transactions, achieving optimal social welfare.

- $$\begin{cases} \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_H}|_{\Lambda_H=\Lambda_H^*}>0, \qquad \Lambda_H=\dfrac{\Phi n_H}{n_H+n_L} \\ \dfrac{\partial\vartheta^{'}(\Lambda_H,\Lambda_L)}{\partial\Lambda_L}|_{\Lambda_L=\Lambda_L^*}\geq 0, \qquad \Lambda_L\in[0,\dfrac{\Phi n_H}{n_H+n_L}] \end{cases}$$

When $U_H > U_L \geq n_c\hat{\sigma}\eta + \dfrac{\xi(n_H+n_L)^2}{\Phi n_L^2}$, satisfying the aforementioned conditions, the solution is obtained as $(\Lambda_H^{,}\Lambda_L^{)} = (\dfrac{\Phi n_H}{n_H+n_L}, \dfrac{\Phi n_L}{n_H+n_L} - \sqrt{\dfrac{\Phi\xi}{U_L - n_c\hat{\sigma}\eta}})$. In this scenario, high-utility users in the system generate transactions with $\Lambda_H^{=}\dfrac{\Phi}{n_H+n_L}$, while low-utility users generate transactions with $\Lambda_L^{=}\dfrac{\Phi}{n_H+n_L} - \dfrac{1}{n_L}\sqrt{\dfrac{\Phi\xi}{U_L - n_c\hat{\sigma}\eta}}$, achieving optimal social welfare.

Now, we derive the optimal user transaction generation strategy under the presence of minimum fee constraints and compensation. Let the unit byte fee vector be denoted as $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, ..., \zeta_I)$, and the unit byte compensation as $\boldsymbol{C} = (C_{HH}, C_{HL}, C_{LH}, C_{LL})$, set as follows:

$$\begin{cases} \zeta_I = n_c\eta \\ \forall q \in [1, I-1], \zeta_q\sigma + \frac{\xi}{\Phi} > \max(U_H - [(n_H-1)C_{HH} + n_L C_{HL}]\sigma, \\ \qquad\qquad U_L - [n_H C_{HH} + (n_L-1)C_{HL}]\sigma, n_c\sigma\eta) \\ (n_H-1)C_{HH} + n_L C_{HL} = \dfrac{U_H}{\sigma} - \zeta_I - \dfrac{\xi[\Phi - (n_H-1)\Lambda_H^* - n_L\Lambda_L^*]}{(\Phi - n_H\Lambda_H^* - n_L\Lambda_L^*)^2\sigma} \\ n_H C_{LH} + (n_L-1)C_{LL} = \dfrac{U_L}{\sigma} - \zeta_I - \dfrac{\xi[\Phi - n_H\Lambda_H^* - (n_L-1)\Lambda_L^*]}{(\Phi - n_H\Lambda_H^* - n_L\Lambda_L^*)^2\sigma} \end{cases} \tag{D7}$$

In this case, we have $\varpi(\zeta_I, U_H^{net}, U_L^{net}) \leq \zeta_{I-1}\sigma$. By constructing the KKT conditions as before, we can solve for the user transaction generation strategy as follows:

$$\zeta_{h_I} = \begin{cases} 0, & \text{If } U_H < n_c\sigma\eta + \frac{\xi}{\Phi} \\ \min(\frac{\Phi}{n_H+n_L}, \frac{1}{n_H}(\Phi - \sqrt{\frac{\Phi\xi}{U_H - n_c\hat{\sigma}\eta}})), & \text{If } U_H \geq n_c\sigma\eta + \frac{\xi}{\Phi} \end{cases}$$

$$\zeta_{l_I} = \begin{cases} 0, & \text{If } U_L < n_c\sigma\eta + \frac{\xi(n_H+n_L)^2}{\Phi n_L^2} \\ \frac{\Phi}{n_H+n_L} - \frac{1}{n_L}\sqrt{\frac{\Phi\xi}{U_L - n_c\sigma\eta}}, & \text{If } U_L \geq n_c\sigma\eta + \frac{\xi(n_H+n_L)^2}{\Phi n_L^2} \end{cases}$$

Hence, it can be inferred that when there are minimum fee constraints and compensation, the user's transaction generation strategy is consistent with the unconstrained condition, achieving optimal social welfare. Additionally, with $\zeta_1 > \zeta_2 > ... > \zeta_I \geq \eta$, the transaction fee paid by the system's user nodes can cover the total expenses of the entire system's cloud nodes, promoting sustainable development of the system.

When $U_H \leq n_c\sigma\eta$, for optimal social welfare, users in the system do not generate transactions. According to Equation D7, the mechanism parameters are set as follows:

$$\begin{cases} \zeta_I = n_c\eta \\ \forall q \in [1, I-1], \zeta_q \geq n_c\eta \\ (n_H-1)C_{HH} + n_L C_{HL} = \dfrac{U_H}{\sigma} - \zeta_I - \dfrac{\xi}{\Phi\sigma} \\ n_H C_{LH} + (n_L-1)C_{LL} = \dfrac{U_L}{\sigma} - \zeta_I - \dfrac{\xi}{\Phi\sigma} \end{cases}$$

Thus, we have:

$$\zeta_q = \frac{(I-q)\xi}{\Phi\sigma} n_c\eta$$

$$\begin{cases} (n_H-1)C_{HH} + n_L C_{HL} = 0 \\ n_H C_{LH} + (n_L-1)C_{LL} = 0 \end{cases}$$

When $U_H > n_c\sigma\eta$ and $U_L < n_c\sigma\eta + \frac{\xi(n_H+n_L)^2}{\Phi n_L^2}$, high-value users in the system generate transactions at a rate of $\Lambda_H^* = \frac{1}{n_H}\min(\frac{\Phi n_H}{n_H+n_L}, \Phi - \sqrt{\frac{\Phi\xi}{U_H - n_c\sigma\eta}})$, while low-value users do not generate transactions. According to Equation **??**, the mechanism parameters are set as follows:

$$\begin{cases} \zeta_I = n_c\eta \\ \forall q \in [1, I-1], \zeta_q \geq n_c\eta \\ (n_H - 1)C_{HH} + n_L C_{HL} = \dfrac{U_H}{\sigma} - \zeta_I - \dfrac{\xi[\Phi - (n_H - 1)\Lambda_H^*]}{(\Phi - n_H\Lambda_H^*)^2\sigma} \\ n_H C_{LH} + (n_L - 1)C_{LL} = \dfrac{U_L}{\sigma} - \zeta_I - \dfrac{\xi[\Phi - n_H\Lambda_H^*]}{(\Phi - n_H\Lambda_H^*)^2\sigma} \end{cases}$$

Thus, we have:

$$\zeta_q = \frac{(I-q)\xi}{\Phi\sigma} n_c\eta$$

$$\begin{cases} (n_H - 1)C_{HH} + n_L C_{HL} = \dfrac{U_H}{\sigma} - \zeta_I - \dfrac{\xi[\Phi - (n_H - 1)\Lambda_H^*]}{(\Phi - n_H\Lambda_H^*)^2\sigma} \\ n_H C_{LH} + (n_L - 1)C_{LL} = \dfrac{U_L}{\sigma} - \zeta_I - \dfrac{\xi}{(\Phi - n_H\Lambda_H^*)\sigma} \end{cases}$$

When $U_H > U_L \geq n_c\sigma\eta + \frac{\xi(n_H+n_L)^2}{\Phi n_L^2}$, all users in the system participate in transaction generation. Mechanism parameters are set as in Equation **??**, with unit byte fee set to $\zeta_q = \frac{(I-q)\xi}{\Phi\sigma} n_c\eta$, and unit byte compensation satisfying:

$$\begin{cases} (n_H - 1)C_{HH} + n_L C_{HL} = \dfrac{U_H}{\sigma} - \zeta_I - \dfrac{\xi[\Phi - (n_H - 1)\Lambda_H^* - n_L\Lambda_L^*]}{(\Phi - n_H\Lambda_H^* - n_L\Lambda_L^*)^2\sigma} \\ n_H C_{LH} + (n_L - 1)C_{LL} = \dfrac{U_L}{\sigma} - \zeta_I - \dfrac{\xi[\Phi - n_H\Lambda_H^* - (n_L - 1)\Lambda_L^*]}{(\Phi - n_H\Lambda_H^* - n_L\Lambda_L^*)^2\sigma} \end{cases}$$

This completes the proof. □