

13. 1.  $z = x^3 - 3x^2y + 4x^3y^2 - y^3$  2.  $z = \frac{3x}{y}$

$$\frac{\partial z}{\partial x} = 3x^2 - 6xy + 12x^2y^2$$

$$\frac{\partial z}{\partial y} = -3x^2 + 8x^3y - 3y^2$$

3.  $z = \frac{y-3x}{x+4y}$

$$\frac{\partial z}{\partial x} = \left( \frac{y-3x}{x+4y} \right)' = \frac{(y-3x)' \cdot (x+4y) - (x+4y)' \cdot (y-3x)}{(x+4y)^2} = \frac{-3x - 12y - y + 3x}{(x+4y)^2} = \frac{-13y}{(x+4y)^2}$$

$$\frac{\partial z}{\partial y} = \left( \frac{y-3x}{x+4y} \right)' = \frac{(y-3x)' \cdot (x+4y) - (x+4y)' \cdot (y-3x)}{(x+4y)^2} = \frac{x+4y - 4y + 12x - 13}{(x+4y)^2} = \frac{13x-13}{(x+4y)^2}$$

4.  $z = e^{-\frac{x}{y}}$

$$\frac{\partial z}{\partial x} = e^{-\frac{x}{y}} \cdot -\frac{1}{y} = -\frac{e^{-\frac{x}{y}}}{y}$$

$$\frac{\partial z}{\partial y} = e^{-\frac{x}{y}} \cdot \frac{x}{y^2} = \frac{x e^{-\frac{x}{y}}}{y^2}$$

$$5. z = \ln(2x-y)$$

$$\frac{\partial z}{\partial x} = \frac{2}{2x-y}$$

$$\frac{\partial z}{\partial y} = -\frac{1}{2x-y}$$

(1/4)

$$1. z = \frac{x-2y}{x+y}$$

$$M(2, -1)$$

$$\frac{\partial z}{\partial x} = \frac{(x-2y)' \cdot (x+y) - (x+y)' \cdot (x-2y)}{(x+y)^2} = \frac{x+y-x+2y}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

$$\frac{\partial z}{\partial x} (2, -1) = \frac{-3}{(2-1)^2} = -3$$

$$\frac{\partial z}{\partial y} = \frac{(x-2y)' \cdot (x+y) - (x+y)' \cdot (x-2y)}{(x+y)^2} = \frac{-2x-2y-x+2y}{(x+y)^2} = \frac{-3x}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} (2, -1) = \frac{-6}{(2-1)^2} = -6$$

$$2. z = e^{\frac{3x}{y}}$$

$$M(1, 1)$$

$$\frac{\partial z}{\partial x} = e^{\frac{3x}{y}} \cdot \frac{3}{y} = \frac{3e^{\frac{3x}{y}}}{y}$$

$$\frac{\partial z}{\partial x} (1, 1) = 3e^3$$

$$\frac{\partial z}{\partial y} = -\frac{e^{\frac{3x}{y}} \cdot 3x}{y^2}$$

$$\frac{\partial z}{\partial y} (1, 1) = -3e^3$$

$$3. z = \ln(x^2+y^2)$$

$$M(2, -2)$$

$$\frac{\partial z}{\partial x} = \frac{2x}{x^2+y^2}$$

$$\frac{\partial z}{\partial x} (2, -2) = \frac{1}{2}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = -\frac{1}{2}$$

$$4. z = \frac{y}{x} + x$$

$$M(1, -2)$$

$$\frac{\partial z}{\partial x} = -\frac{y}{x^2} + 1$$

$$\frac{\partial z}{\partial x} (2, -1) = 3$$

$$\frac{\partial z}{\partial y} = \frac{1}{x}$$

$$\frac{\partial z}{\partial y} = 1$$



15)

$$1. z = \frac{y}{x+y} \quad M(2; -1)$$

$$\frac{\partial z}{\partial x} = \left( y \cdot \frac{1}{x+y} \right)' = \frac{-y}{(x+y)^2} \quad \frac{\partial z}{\partial x} (2; -1) = \frac{-(-1)}{(2-1)^2} = 1$$

$$\frac{\partial z}{\partial y} = \frac{y'(x+y) - (x+y)'y}{(x+y)^2} = \frac{x+y-y}{(x+y)^2} = \frac{x}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} (2; -1) = \frac{2}{(2-1)^2} = 2$$

$$dz = dx + 2dy$$

$$2. z = \sin(x^2 + 2y)$$

$$x=1 \quad y=2 \\ dx=0,1 \\ dy=0,2$$

$$z'_x = \cos(x^2 + 2y) \cdot 2x = 0,56$$

$$z'_y = 2\cos(x^2 + 2y) = 0,56$$

$$dz = 0,56 \cdot 0,1 + 0,56 \cdot 0,2 = 0,168$$

$$3. z = e^{\frac{x}{2y}}$$

$$x=2 \quad y=1$$

$$dx=0,2 \quad dy=0,1$$

$$z'_x = e^{\frac{x}{2y}} \cdot \frac{1}{2y} = \frac{e}{2}$$

$$z'_y = e^{\frac{x}{2y}} \cdot \left( -\frac{x}{2y^2} \right) = -e$$

$$dz = \frac{e}{2} \cdot 0,2 + (-e) \cdot 0,1 = 0$$

$$4. z = \ln(2x+y) \quad M(1; 0)$$

$$z'_x = \frac{1}{2x+y} \cdot 2 = 1$$

$$z'_y = \frac{1}{2x+y} = \frac{1}{2}$$

$$dz = dx + \frac{dy}{2}$$