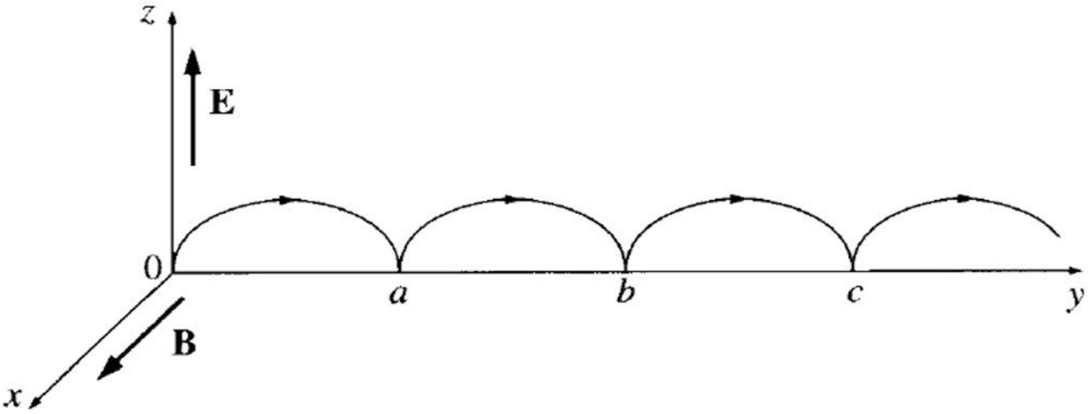
Wave and Electromagnetic

Exercise 1. We have a point change that moving in electromagnetic field where in initial case the point Q is $t = 0s, v_o = 0m/s, x_o = 0, y_o = 0, z_o = 0$. Find the differential equation of this particle is cycloid.



Solution 1. Differential Equation of Partical.

By Principle of Fundemental of Dynamic(PFD):

$$\sum \vec{f} = m\vec{a}$$

$$\Rightarrow m\vec{a} = q(\vec{E} + \vec{v} \wedge \vec{B})$$

In coordinate Cartesien $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$:

$$\Rightarrow m \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = q \left[\begin{pmatrix} 0 \\ 0 \\ E_z \end{pmatrix} + \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \wedge \begin{pmatrix} B_x \\ 0 \\ 0 \end{pmatrix} \right]$$

$$\Rightarrow \begin{cases} ma_x = 0, & (1) \\ ma_y = qv_zB_x, & (2) \\ ma_z = q(E_z - v_yB_x). & (3) \end{cases}$$

where $a_x = \ddot{x}, a_y = \ddot{y}, a_z = \ddot{z}$ and the motion in the plan $(o, \vec{u}_y, \vec{u}_z)$:
for equation(2):

$$m\ddot{y} = q\dot{z}B_x$$

$$\Rightarrow d\dot{y} = \frac{qB_x}{m}\dot{z}dt = \frac{qB_x}{m}dz$$

$$\dot{y} = \frac{qB_x}{m}z + c^{te}$$

at $t = 0s \Rightarrow \dot{y}(0) = 0$ then $c^{te} = 0 \Rightarrow \dot{y} = \dfrac{qB_x}{m}z \quad (i)$
for equation(3)

$$m\ddot{z} = q(E_z - \dot{y}B_x)$$

$$\Rightarrow m\ddot{z} = q(E_z - B_x\frac{qB_x}{m}z)$$

$$\ddot{z} + \left(\frac{qB_x}{m}\right)^2 z = \frac{qE_z}{m}$$

, where this equation is the oscillation harmonic in form: $\ddot{z} + \omega^2 z = \dfrac{qE_z}{m}$ with $\omega = \dfrac{qB_x}{m}$

the solution of this equation is: $z(t) = c_1 \cos\omega t + c_2 \sin\omega t + \dfrac{qE_z}{m\omega^2}$

at $t = 0s \Rightarrow z(0) = 0 \Rightarrow c_1 = -\dfrac{qE_z}{m\omega^2}$ and we take derivative on the both side on z(t) respect to t ,so $\dot{z} = -c_1\omega \sin\omega t + c_2\omega\cos\omega t$ at $t = 0s, \dot{z}(0) = 0 \Rightarrow \dot{z} = 0$

Thus ,$z(t) = -\dfrac{qE_z}{m\omega^2}\cos\omega t + \dfrac{qE_z}{m\omega^2} = \dfrac{qE_z}{m\omega^2}(1 - \cos\omega t)$