Learning Decision-Focused Uncertainty Sets for Robust Optimization

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Joint work with

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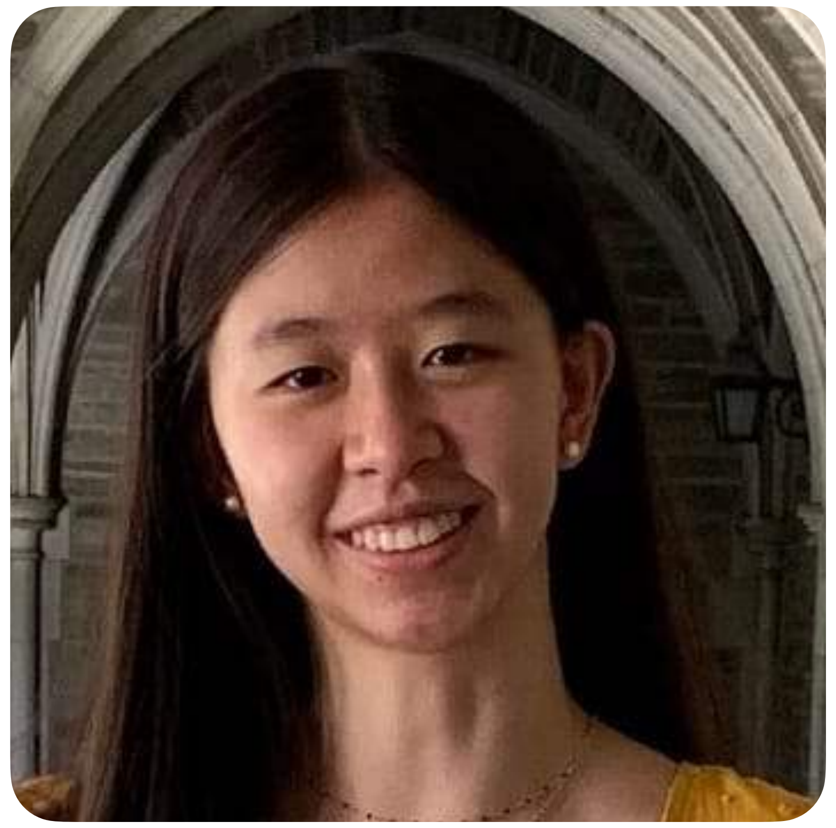
MIT



Annie Liang
Princeton ORFE

Intelligent systems make high-stakes decisions in an uncertain world

Intelligent systems make high-stakes decisions in an uncertain world

self-driving taxis

Intelligent systems make high-stakes decisions in an uncertain world

self-driving taxis

energy grids

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self-driving taxis

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finance

Intelligent systems make high-stakes decisions in an uncertain world

self-driving taxis

energy grids

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robotics

Warm-up example

Portfolio optimization with penalty on trades

objective

$$-u^T x + \lambda \|x - x^{\mathrm{prev}}\|_1$$

Portfolio optimization with penalty on trades

uncertain returns

$$u \sim \mathcal{N}(\mu, \Sigma)$$

$$\mu = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}, \ \Sigma = \begin{bmatrix} 0.5 & -0.3 \\ -0.3 & 0.4 \end{bmatrix}$$

objective

$$-u^T x + \lambda \left\| x - x^{\text{prev}} \right\|_1$$

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objective

$-u^T x + \lambda \|x - x^{\text{prev}}\|_1$

previous holdings

$x^{\text{prev}} \sim \text{Dir}(\alpha)$

$\alpha = (2.5, 1)$

Portfolio optimization with penalty on trades

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investments decisions

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investments decisions

robust problem
reformulation

$$\begin{array}{ll} \text{minimize} & t + \lambda \|x - x^{\text{prev}}\|_1 \\ \text{subject to} & -u^T x \le t \quad \forall u \in \mathcal{U}(\theta) \\ & \mathbf{1}^T x = 1, \quad x \ge 0 \end{array}$$

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robust problem reformulation

$$\begin{array}{ll} \text{minimize} & t + \lambda \|x - x^{\text{prev}}\|_1 \\ \text{subject to} & -u^T x \le t \quad \forall u \in \mathcal{U}(\theta) \\ & \mathbf{1}^T x = 1, \quad x \ge 0 \end{array}$$

uncertainty set

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uncertainty set

how do we pick the uncertainty set?

Mean-variance vs reshaped uncertainty sets

parameters

$$\theta = (A, b)$$

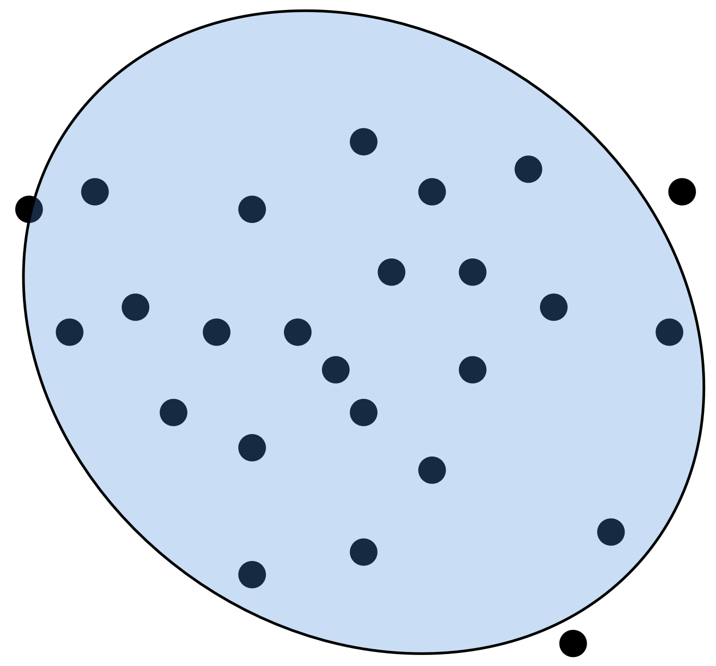
Mean-variance vs reshaped uncertainty sets

parameters
$$\theta = (A, b)$$

mean-variance set

$$\mathcal{U}^{\mathrm{mv}}(\theta) = \{u = \hat{\mu} + \hat{\Sigma}^{1/2} z \mid \|z\|_2 \le \rho\} = \{b^{\mathrm{mv}} + A^{\mathrm{mv}} z \mid \|z\|_2 \le \rho\}$$

empirical
mean and covariance



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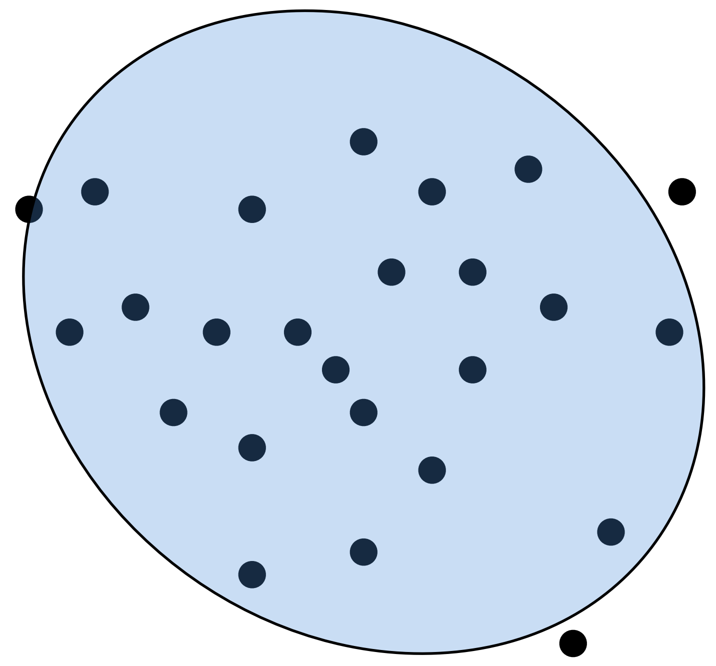
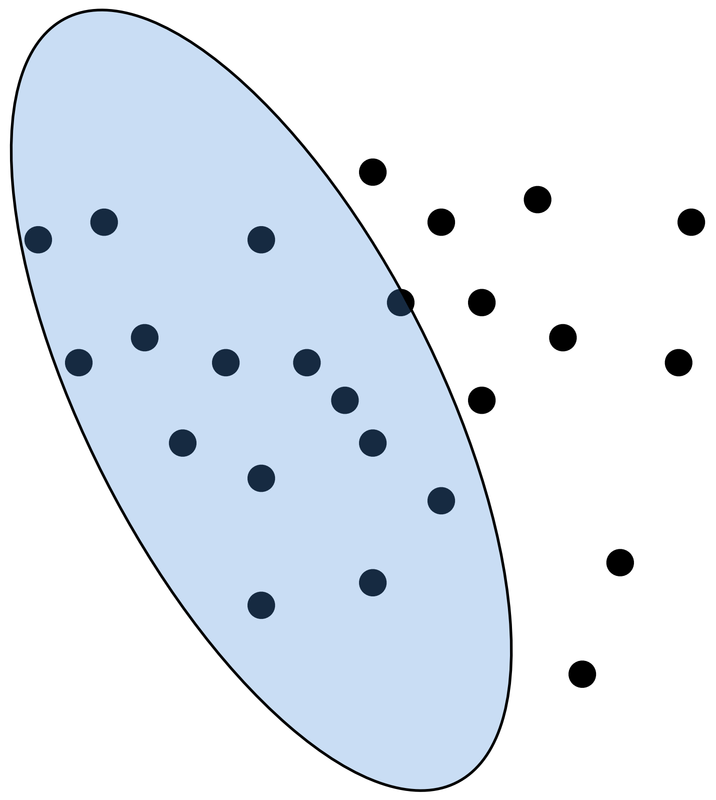


reshaped uncertainty set

$$\mathcal{U}^{\mathrm{re}}(\theta) = \{u = b^{\mathrm{re}} + A^{\mathrm{re}} z \mid \|z\|_2 \leq \rho\}$$



Mean-variance vs reshaped uncertainty sets

parameters

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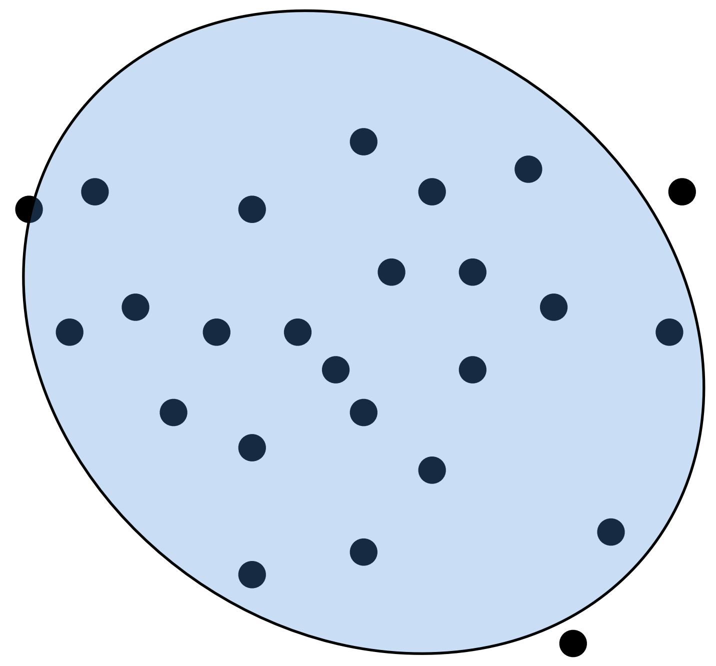
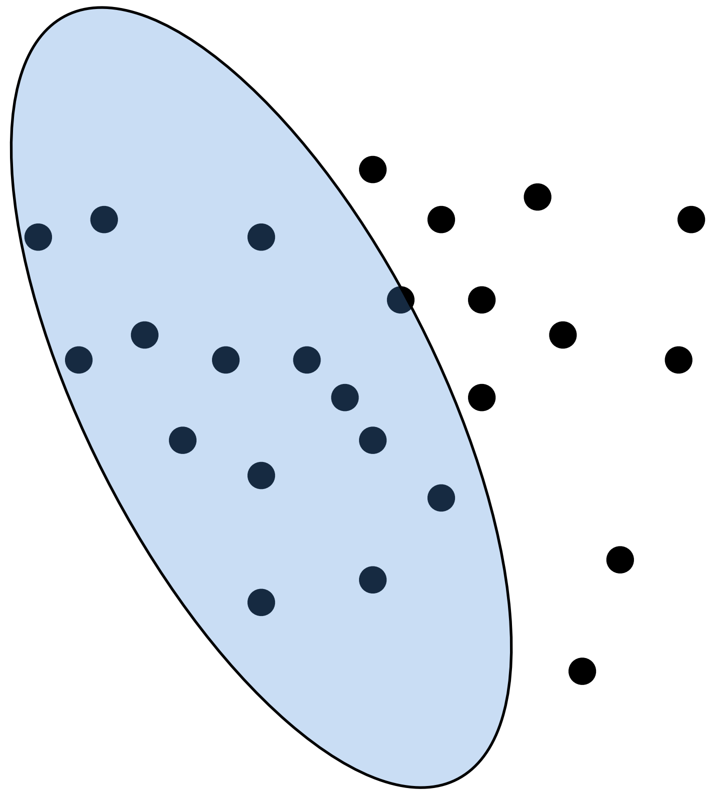




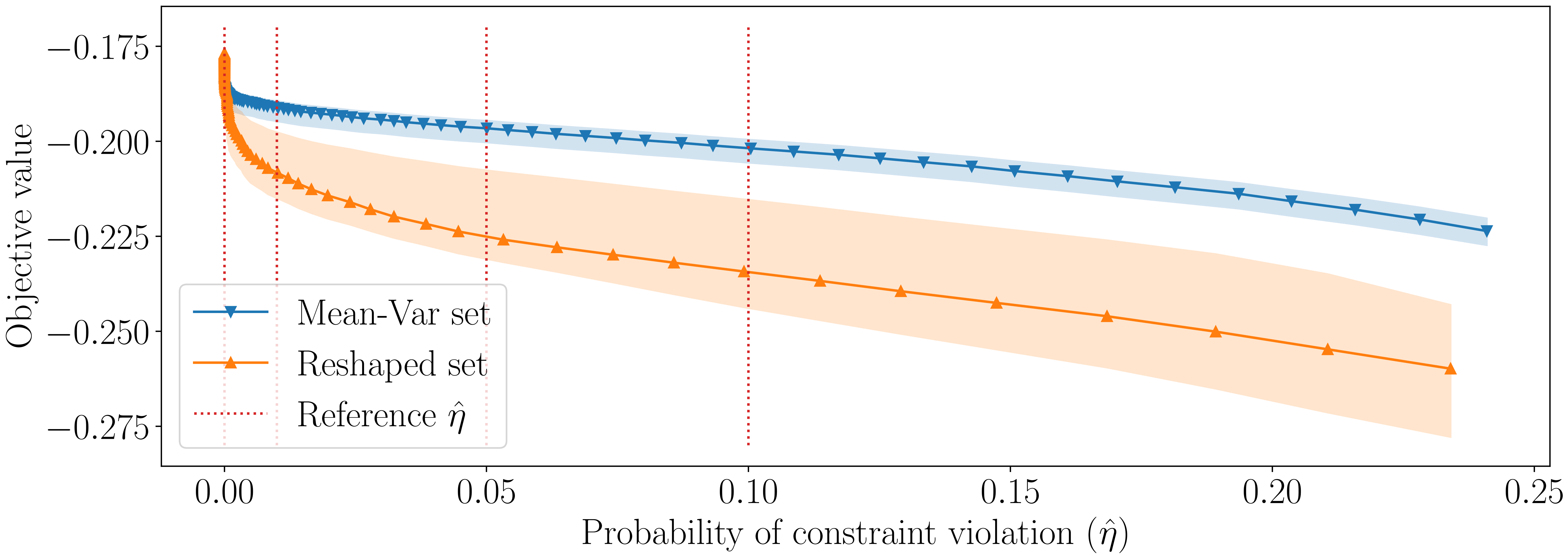
reshaped uncertainty set

$$\mathcal{U}^{\mathrm{re}}(\theta) = \{u = b^{\mathrm{re}} + A^{\mathrm{re}} z \mid \|z\|_2 \leq \rho\}$$

can the reshaped set
do better?

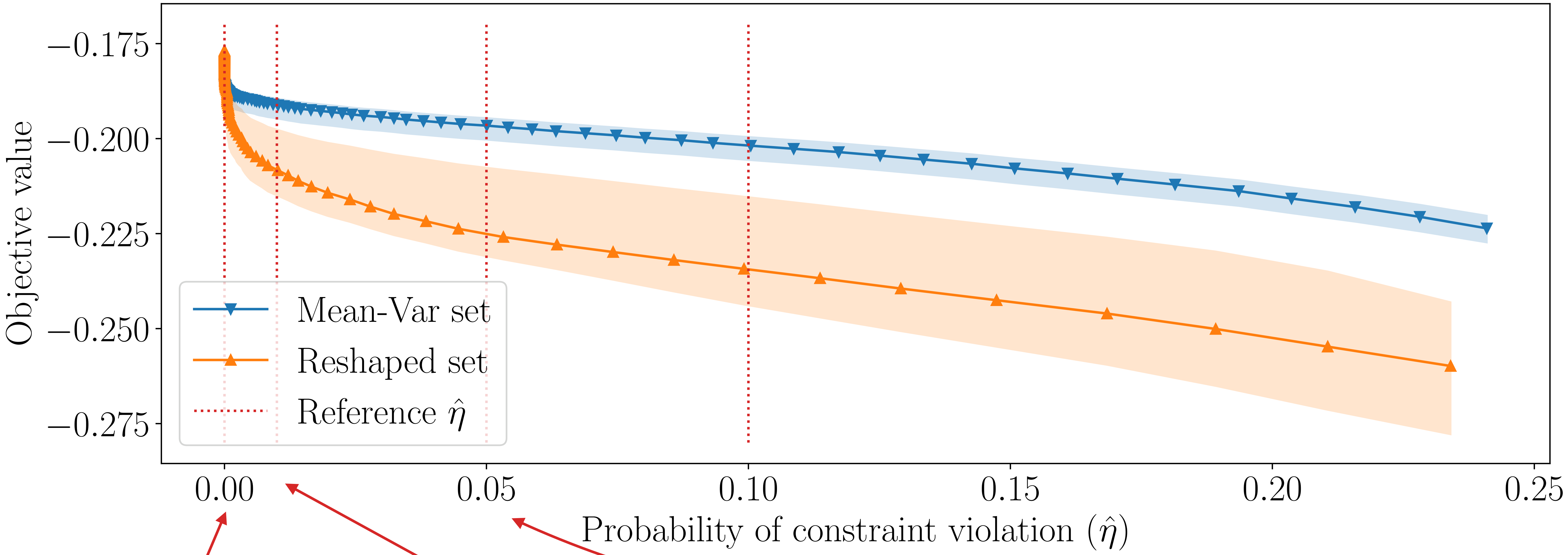
Reshaped set performs much better

pareto curves
for varying size
ρ



Reshaped set performs much better

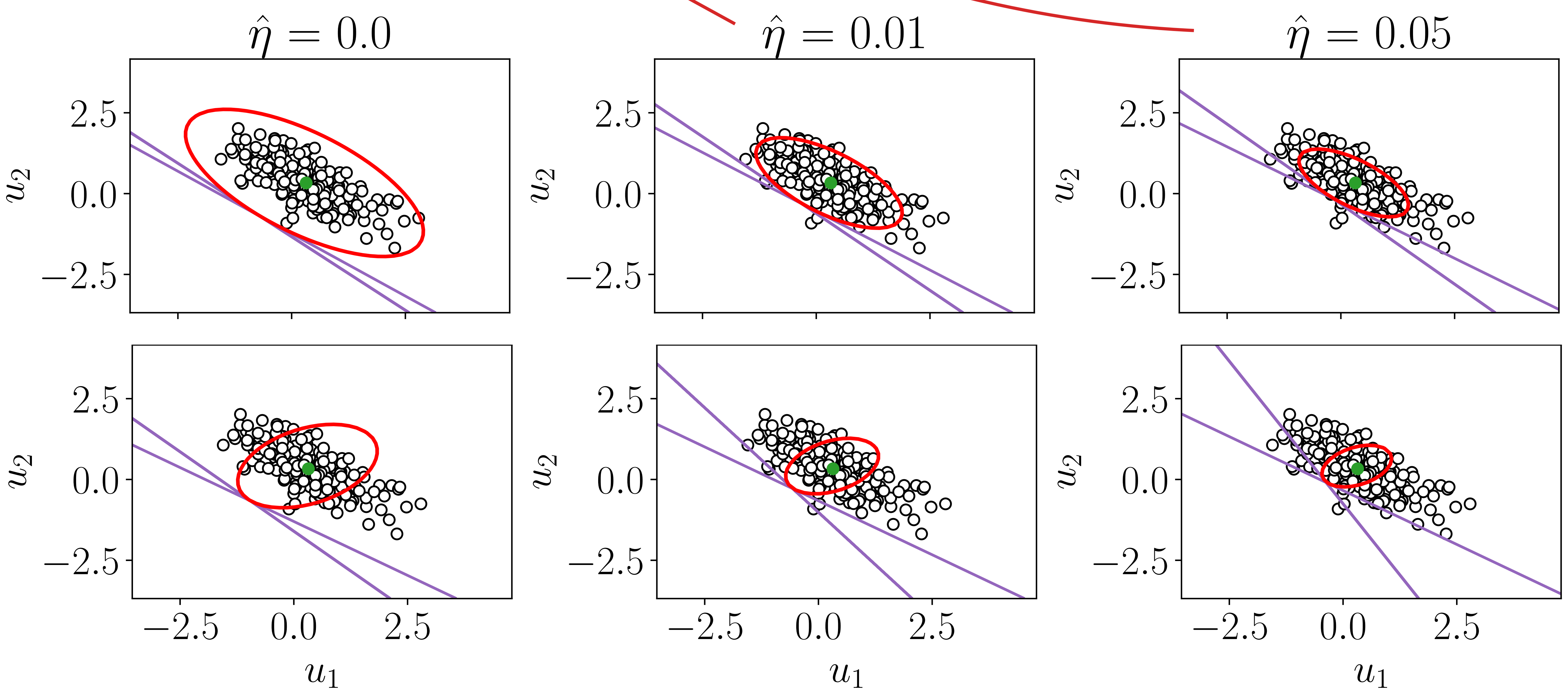
pareto curves for varying size ρ



level curves $-u^T x^\star - t^\star = 0$

mean-variance $\mathcal{U}^{\rm mv}(\theta)$

reshaped $\mathcal{U}^{\rm re}(\theta)$



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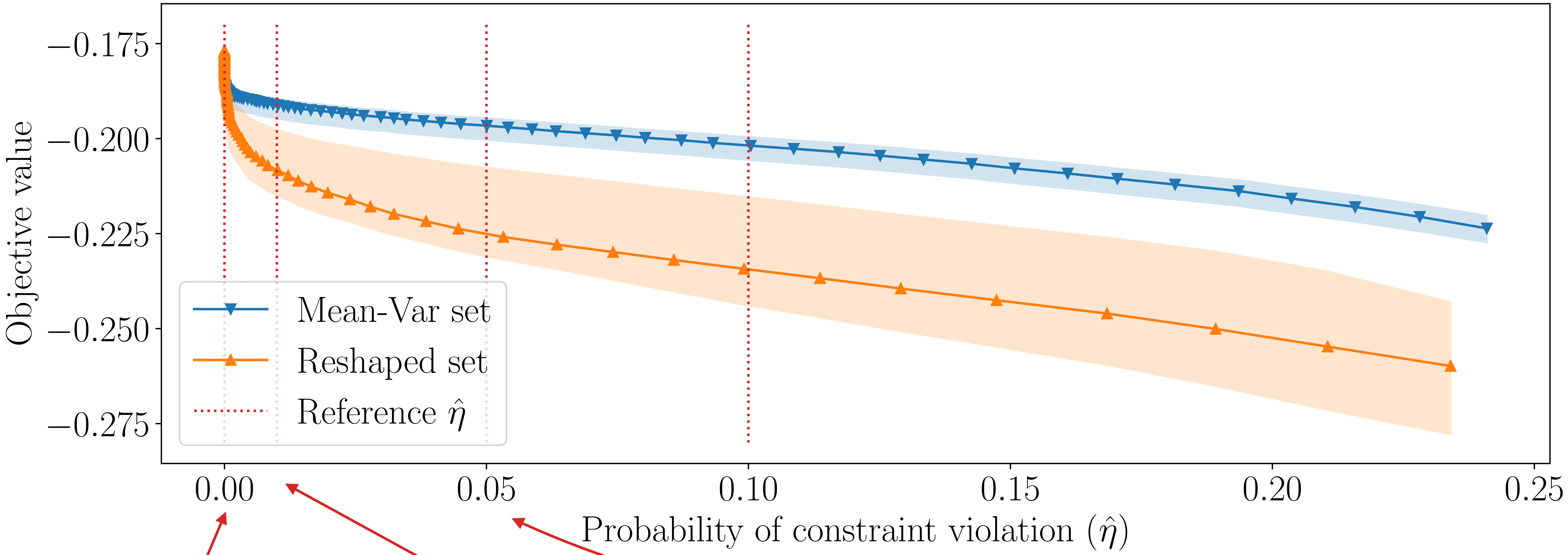
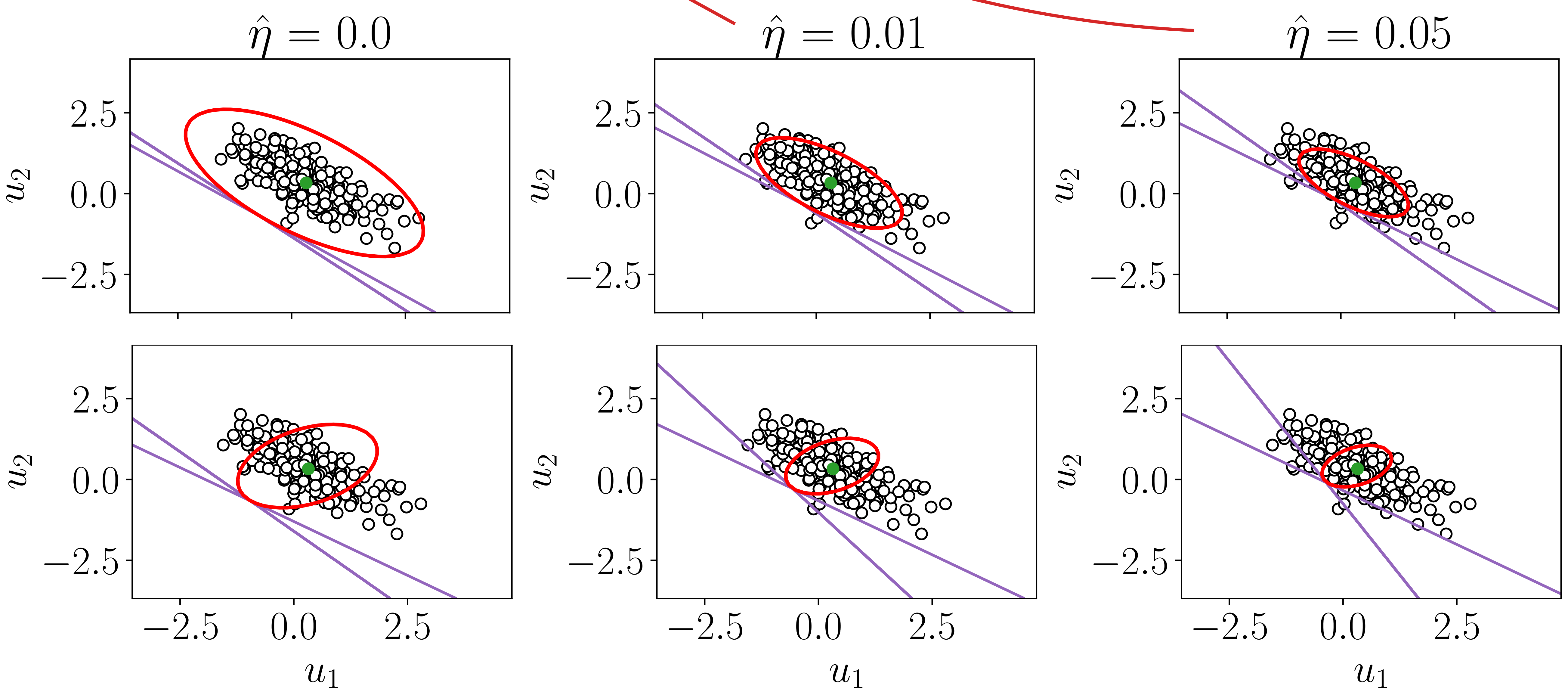


level curves
$-u^T x^\star - t^\star = 0$

mean-variance
$\mathcal{U}^{\rm mv}(\theta)$

reshaped
$\mathcal{U}^{\rm re}(\theta)$



how can we find the reshaped set?

Problem setup

Decision-making with uncertain constraints

parametric robust optimization

$$
\begin{aligned}
x(\theta, y) \in \quad & \text{argmin} && f(x, y) \\
& \text{subject to} && g(x, u, y) \leq 0 \quad \forall u \in \mathcal{U}(\theta)
\end{aligned}
$$

Decision-making with uncertain constraints

parametric robust optimization

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decisions

Decision-making with uncertain constraints

parametric robust optimization

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decisions

uncertain parameter

Decision-making with uncertain constraints

parametric robust optimization

family/context parameter

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decisions

uncertain parameter

probabilistic guarantees

$$
\mathbf{P}_{(u,y)}\big(g(x(\theta, y), u, y) \leq 0\big) \geq 1 - \eta
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Decision-making with uncertain constraints

parametric robust optimization

family/context parameter

$$x(\theta, y) \in \quad \text{argmin} \quad f(x, y)$$
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decisions

uncertain parameter

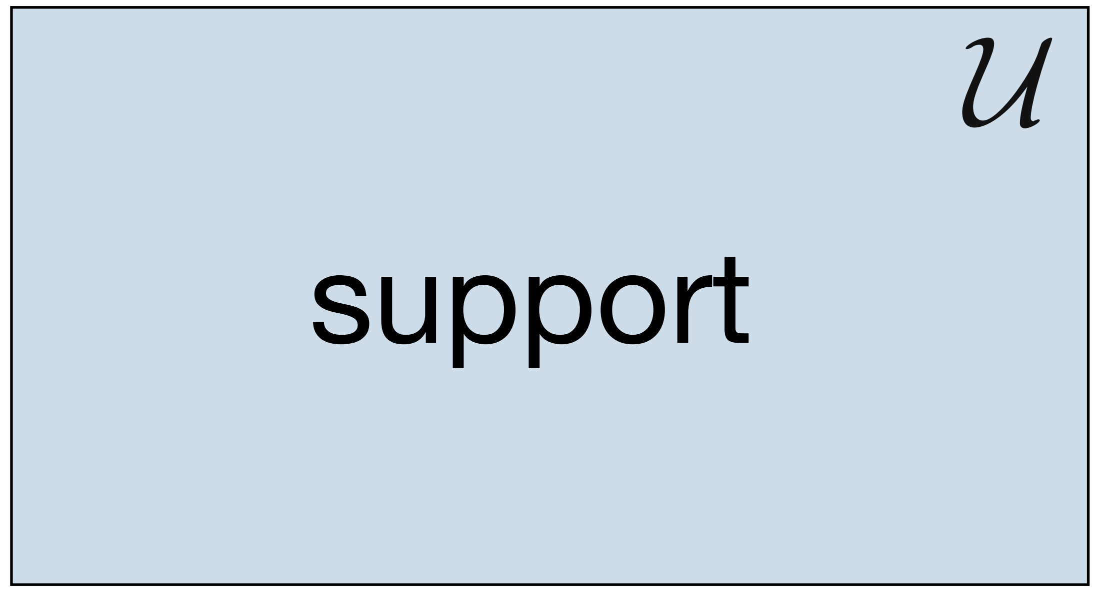
probabilistic guarantees

$$\mathbf{P}_{(u,y)}\big(g(x(\theta, y), u, y) \leq 0\big) \geq 1 - \eta$$

Can we construct a set that ensures the probabilistic guarantees?

Picking the uncertainty set is difficult

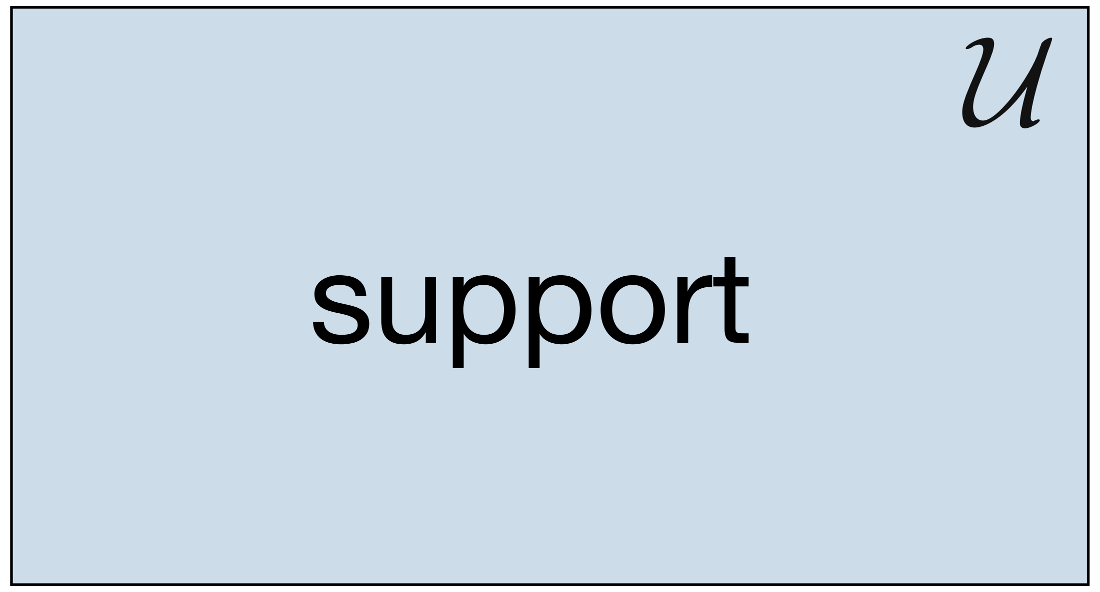
Worst-case approach



❌ Very conservative

Picking the uncertainty set is difficult

Worst-case approach



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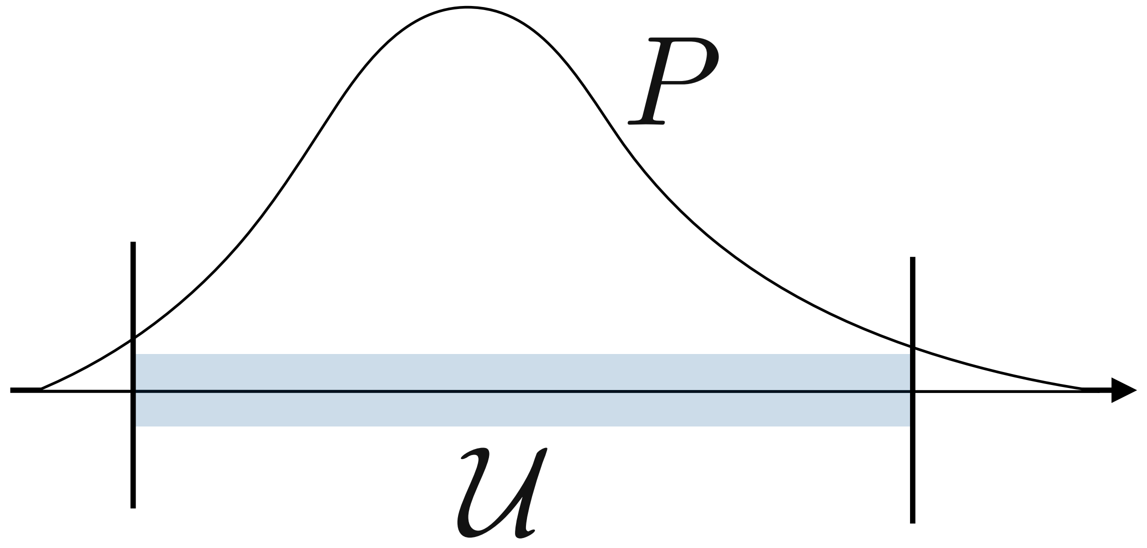
Probabilistic approach



❌ nobody knows P

"Probabilistic Guarantees in Robust Optimization", D. Bertsimas, D. den Hertog, and J. Pauphilet (2019)

Picking the uncertainty set is difficult

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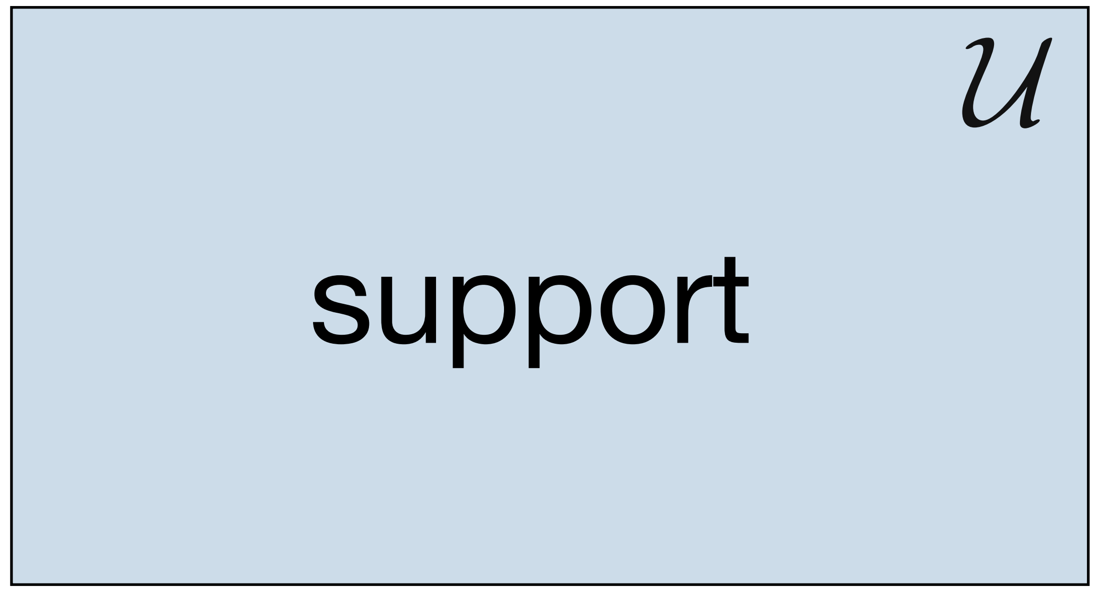

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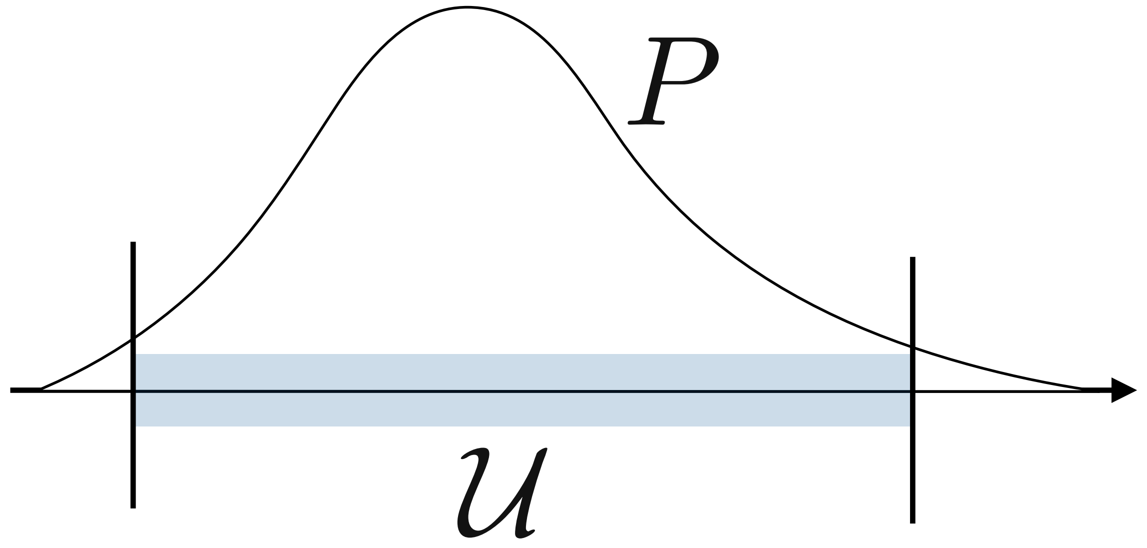


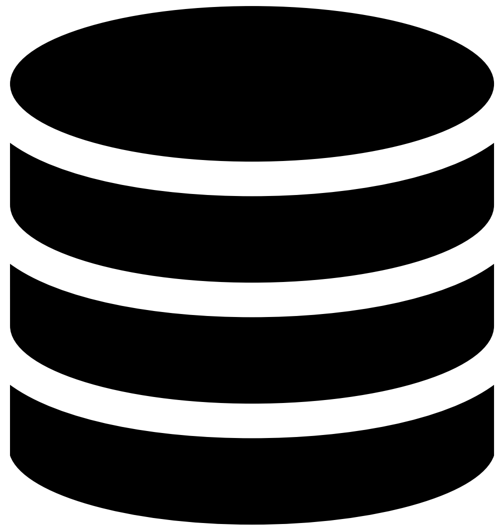
❌ nobody knows P

Data-driven approach

Can we use data to construct uncertainty sets?

“Probabilistic Guarantees in Robust Optimization”, D. Bertsimas, D. den Hertog, and J. Pauphilet (2019)

Data-driven methods for robust optimization



Hypothesis testing

D. Bertsimas, V. Gupta,
and N. Kallus (2014)

Data-driven methods for robust optimization

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Wasserstein Distributionally Robust Optimization

P. M. Esfahani and D. Kuhn. (2018).

D. Bertsimas, S. Shtern, B. Sturt (2022)

I. Wang, C. Becker, B. Van Parys, and B. Stellato (2023)

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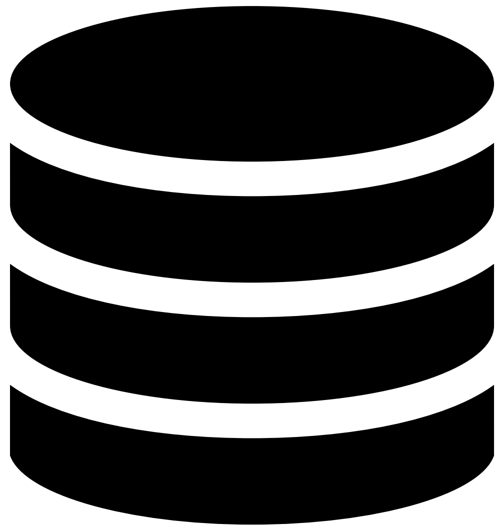

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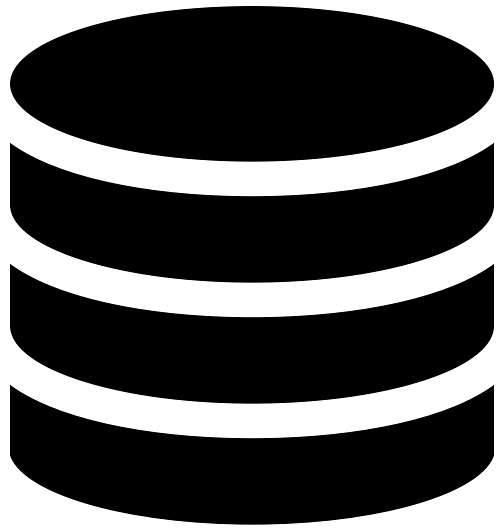

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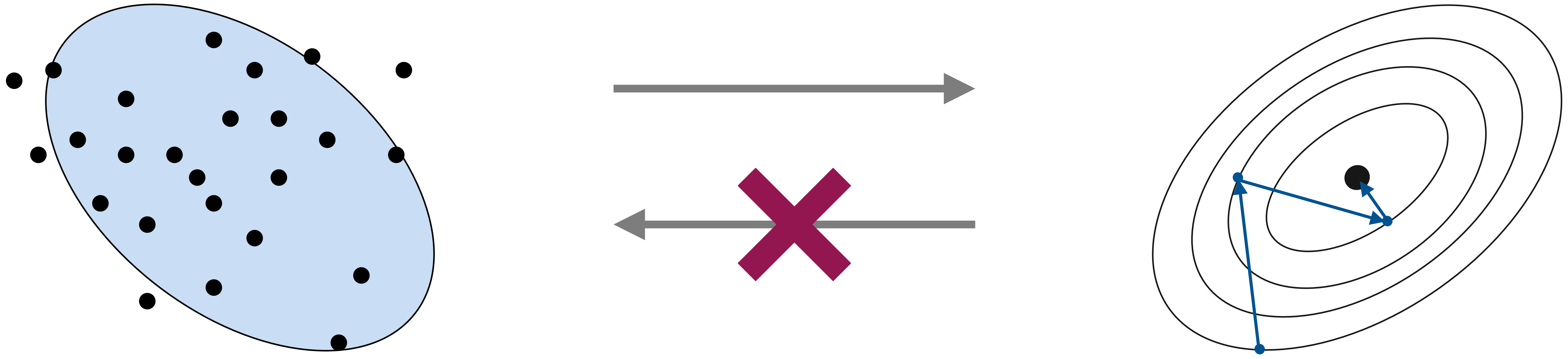
Most approaches decouple

Uncertainty set construction

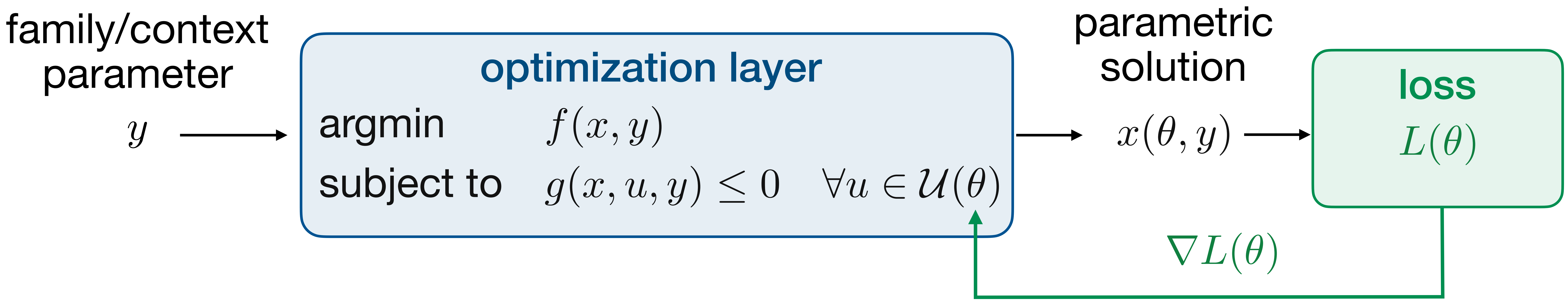
Downstream optimization task



high coverage requirement

$$\mathbf{P}(u \in \mathcal{U}) \geq 1 - \epsilon$$

Leveraging the solution to tune the uncertainty sets



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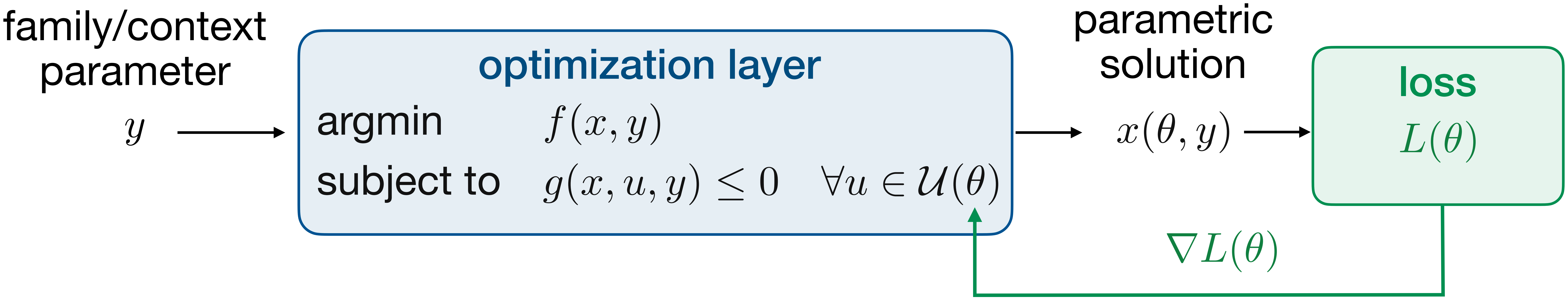

Main idea

Use differentiable optimization to automatically learn *shape* and *size*

Connections with Contextual Optimization

Contextual Optimization

$$x(y) \in \underset{x}{\text{argmin}}\, \mathbf{E}_{\mathbf{P}(u|y)}(f(x,u))$$

[D. Bertsimas and N. Kallus (2020)], [Elmachtoub and Grigas (2022)],
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“A Survey of Contextual Optimization Methods for Decision Making under Uncertainty”,
U. Sadana, A. Chenreddy, E. Delage, A. Forel, E. Frejinger, T. Vidal (2023)

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Conditional Robust Optimization

$$x(y) \in \operatorname*{argmin}_{x} \max_{u \in \mathcal{U}(y)} f(x,u)$$

[A.R. Chenreddy, N. Bandi, E. Delage (2022)], [S. Ohmori (2021)],
[E. Persak, M. F. Anjos (2023)], [C.Sun, L. Liu, X. Li (2023)]

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Differences from our work

- Distribution/uncertainty set depends on y
- High coverage requirements $\mathbf{P}(u \in \mathcal{U}(y)) \geq 1 - \epsilon$
- Limited focus on uncertain constraints

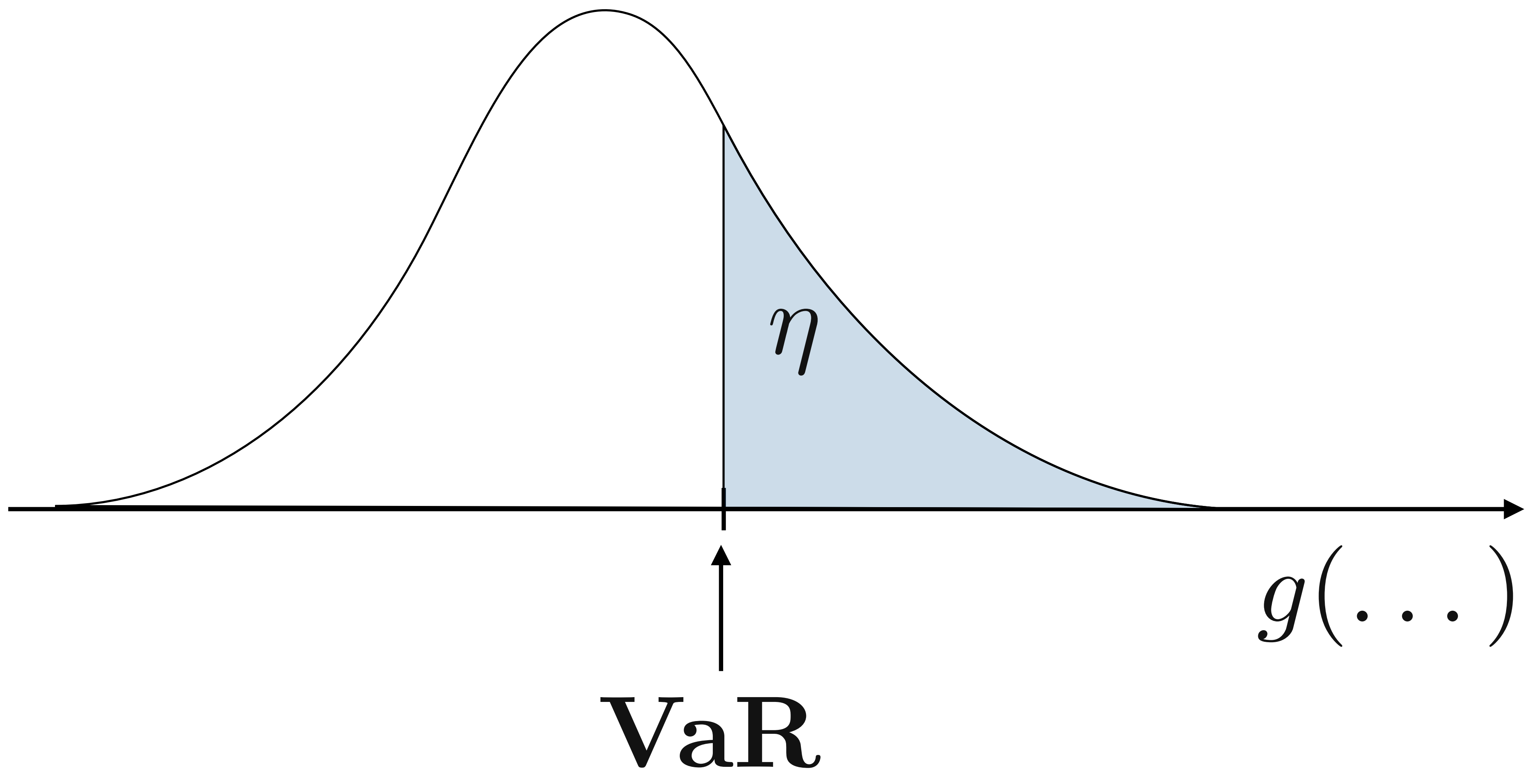
Learning problem formulation

Enforcing probabilistic guarantees with CVaR

probabilistic guarantees

$$\mathbf{P}_{(u,y)}\big(g(x(\theta,y),u,y) \leq 0\big) \geq 1-\eta$$

same as $\mathbf{VaR}(g(\dots),\eta) \leq 0$



Enforcing probabilistic guarantees with CVaR

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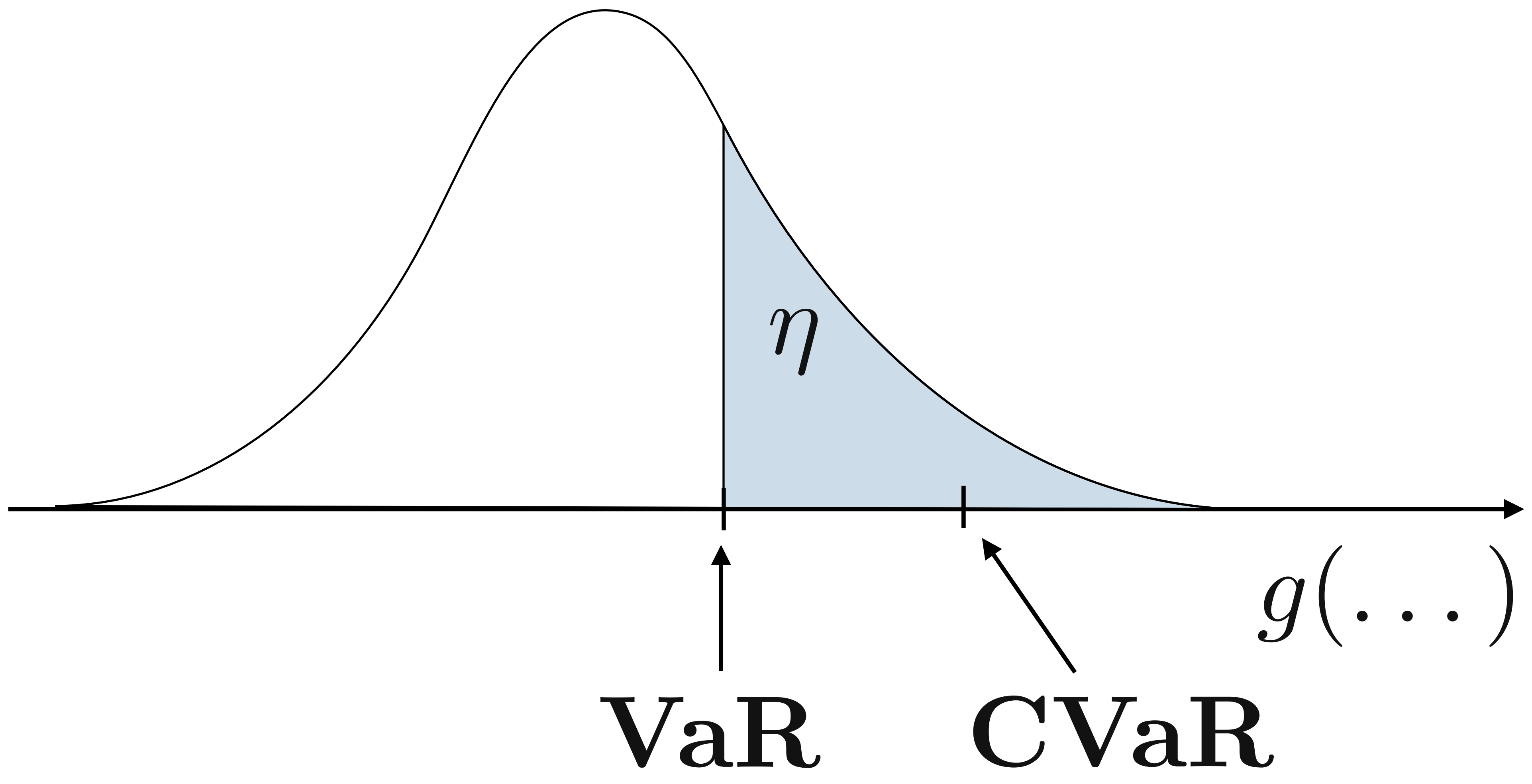
tractable approximation

$$\mathbf{CVaR}(g(x(\theta, y), u, y), \eta) \leq 0$$

same as $\mathbf{VaR}(g(\dots), \eta) \leq 0$



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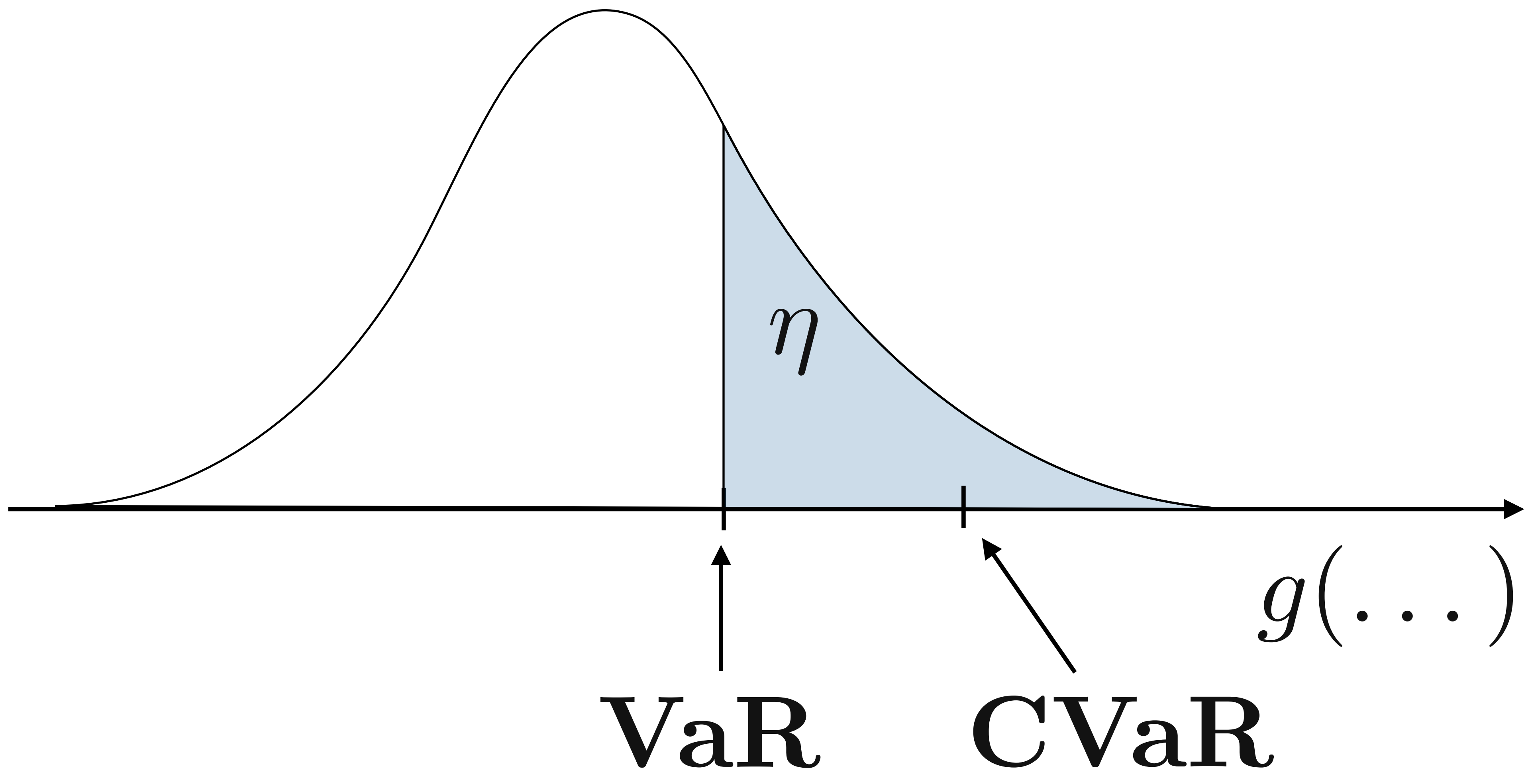


tractable approximation

$$\mathbf{CVaR}(g(x(\theta, y), u, y), \eta) \leq 0$$

turn into constraint

$$\mathbf{E}_{(u,y)}\left(\frac{(g(x(\theta, y), u, y) - \alpha)_+}{\eta} + \alpha\right) \leq 0$$

Enforcing probabilistic guarantees with CVaR

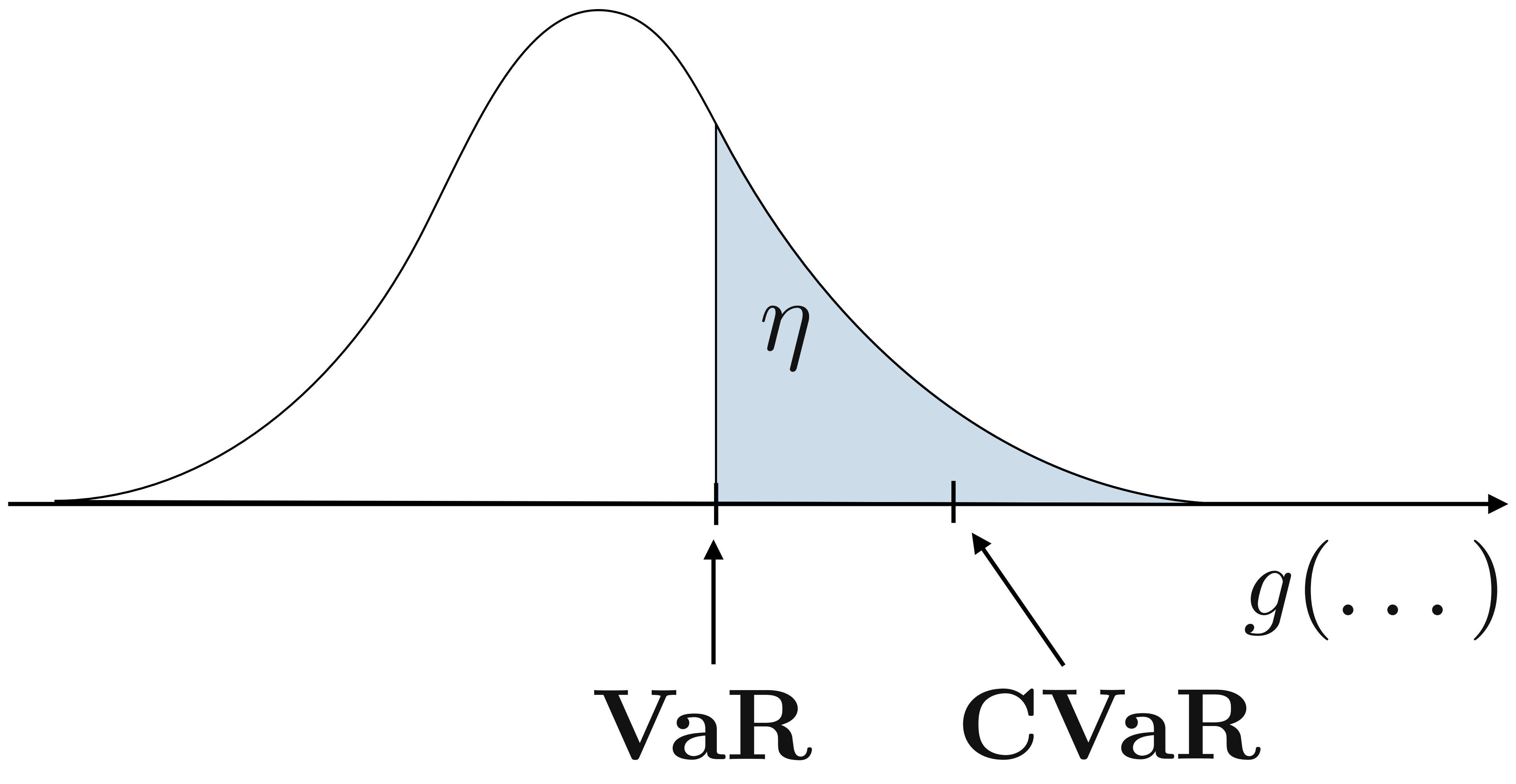
probabilistic guarantees

$$\mathbf{P}_{(u,y)}\big(g(x(\theta,y),u,y)\leq 0\big)\geq 1-\eta$$

same as $\mathbf{VaR}(g(\dots),\eta)\leq 0$

tractable approximation

$$\mathbf{CVaR}(g(x(\theta,y),u,y),\eta)\leq 0$$



turn into constraint

$$\mathbf{E}_{(u,y)}\left(\frac{(g(x(\theta,y),u,y)-\alpha)_+}{\eta}+\alpha\right)\leq 0 \xrightarrow{w=(u,y)} \mathbf{E}_w\left(h(\alpha,\theta,w)\right)\leq \kappa$$

threshold

Stochastic bilevel optimization to learn the uncertainty set

loss

$$\ell(\theta, w) = f(x(\theta, y), y)$$

training problem

$$\begin{array}{ll} \text{minimize} & \mathbf{E}_w[\ell(\theta, w)] \\ \text{subject to} & \mathbf{E}_w[h(\alpha, \theta, w)] \leq \kappa \end{array}$$

CVaR constraint

$$h(\alpha, \theta, w) = \frac{(g(x(\theta, y), u, y) - \alpha)_+}{\eta} + \alpha$$

decision variables
θ, α

random variables
$w = (u, y)$

Stochastic bilevel optimization to learn the uncertainty set

loss

$$\ell(\theta, w) = f(x(\theta, y), y)$$

training problem

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CVaR constraint

$$h(\alpha, \theta, w) = \frac{(g(x(\theta, y), u, y) - \alpha)_+}{\eta} + \alpha$$

decision variables
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inner robust problem

$$\begin{array}{lll} x(\theta, y) \in & \text{argmin} & f(x, y) \\ & \text{subject to} & g(x, u, y) \leq 0 \quad \forall u \in \mathcal{U}(\theta) \end{array}$$

we must reformulate the infinite dimensional constraints

Uncertainty set parameters enter nicely in the reformulation

$$
\begin{array}{ll}
\text{minimize} & f(x, y) \\
\text{subject to} & g(x, u, y) \leq 0 \quad \forall u \in \mathcal{U}(\theta)
\end{array}
$$

Uncertainty set parameters enter nicely in the reformulation

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learned parameters

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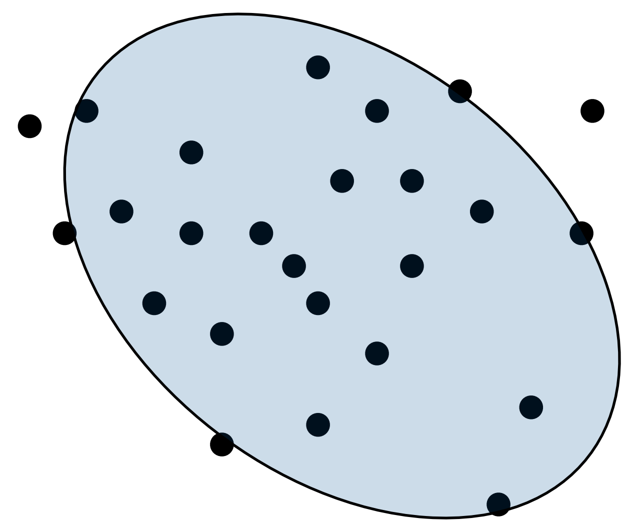
learned parameters

Example: ellipsoidal set

$$\mathcal{U}(\theta) = \{u = b + Az \mid \|z\|_2 \leq 1\}$$



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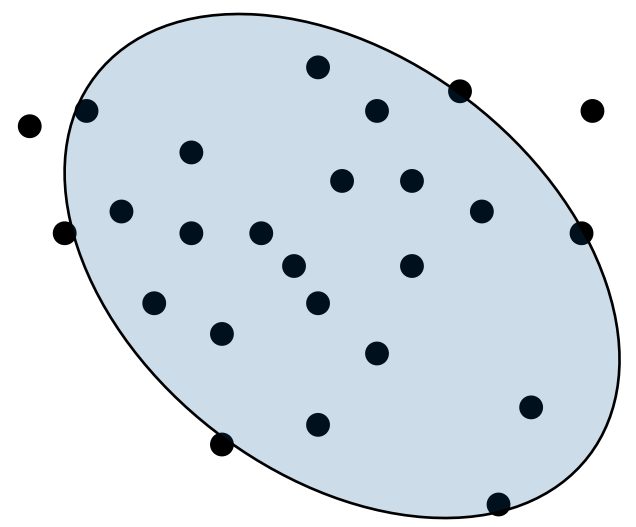
$$\theta = (A, b)$$



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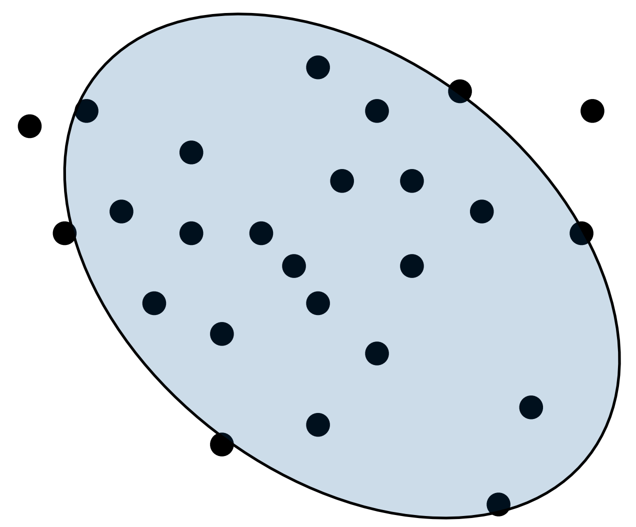

linear constraint

$$g(x, u, y) = (y + Pu)^T x \leq 0, \quad \forall u \in \mathcal{U}(\theta)$$

Uncertainty set parameters enter nicely in the reformulation

$$\begin{array}{ll} \text{minimize} & f(x, y) \\ \text{subject to} & g(x, u, y) \leq 0 \quad \forall u \in \mathcal{U}(\theta) \end{array} \longleftarrow$$

learned parameters

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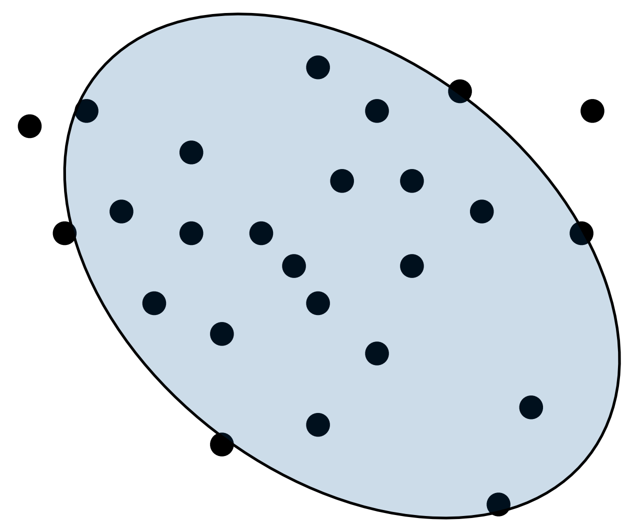

linear constraint

$$g(x, u, y) = (y + Pu)^T x \leq 0, \quad \forall u \in \mathcal{U}(\theta)$$

robust counterpart

$$y^T x + b^T P x + \|A^T P^T x\|_2 \leq 0$$

Solution algorithm

Constrained learning problem

$$F(\theta)$$

$$\begin{array}{ll} \text{minimize} & \boxed{\mathbf{E}_w[\ell(\theta, w)]} \\ \text{subject to} & \boxed{\mathbf{E}_w[h(\alpha, \theta, w)]} \leq \kappa \end{array}$$

$$H(\alpha, \theta)$$

Constrained learning problem

$$\begin{array}{ll} \text{minimize} & \underbrace{\mathbf{E}_w[\ell(\theta, w)]}_{F(\theta)} \\ \text{subject to} & \underbrace{\mathbf{E}_w[h(\alpha, \theta, w)]}_{H(\alpha,\theta)} \leq \kappa \end{array}$$

introduce slack variable $s \geq 0$

reformulated training problem

$$\begin{array}{ll} \text{minimize} & \mathbf{E}_w[\ell(\theta, w)] \\ \text{subject to} & \mathbf{E}_w[h(\alpha, \theta, w)] + s = \kappa \\ & s \geq 0 \end{array}$$

decision variables

θ, α, s

Stochastic augmented Lagrangian method

penalty

augmented Lagrangian

$$L(\alpha, \theta, s, \lambda, \mu) = F(\theta) + \lambda(H(\alpha, \theta) + s - \kappa) + \frac{\mu}{2}\|H(\alpha, \theta) + s - \kappa\|^2$$

multiplier

Stochastic augmented Lagrangian method

augmented Lagrangian

penalty

$$L(\alpha, \theta, s, \boxed{\lambda}, \boxed{\mu}) = F(\theta) + \boxed{\lambda}(H(\alpha, \theta) + s - \kappa) + \frac{\boxed{\mu}}{2}\|H(\alpha, \theta) + s - \kappa\|^2$$

multiplier

for $k = 1, \ldots, k_{\max}$ **do**

$G^k \leftarrow \hat{\nabla} L(\alpha^k, \theta^k, s^k, \lambda^k, \mu^k)$ **implicit differentiation**

$(\theta^{k+1}, \alpha^{k+1}, s^{k+1}) \leftarrow (\theta^k, \alpha^k, s^k) - tG^k$

$s^{k+1} \leftarrow (s^{k+1})_+$

if $\|H(\alpha^{k+1}, \theta^{k+1}) + s^{k+1} - \kappa\|_2 \leq \tau H_{\text{best}}$ **then**

$\lambda^{k+1} \leftarrow \lambda^k + \mu^k(H(\alpha^{k+1}, \theta^{k+1}) + s^{k+1} - \kappa)$

$\mu^{k+1} \leftarrow \mu^k$

$H_{\text{best}} \leftarrow \|H(\alpha^{k+1}, \theta^{k+1}) + s^{k+1} - \kappa\|_2$

else

Choose $\mu^{k+1} > \mu^k$

$\lambda^{k+1} \leftarrow \lambda^k$

Stochastic augmented Lagrangian method

augmented Lagrangian

penalty

$$L(\alpha, \theta, s, \lambda, \mu) = F(\theta) + \lambda(H(\alpha, \theta) + s - \kappa) + \frac{\mu}{2}\|H(\alpha, \theta) + s - \kappa\|^2$$

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- $s^{k+1} \leftarrow (s^{k+1})_+$

  update primal variables
- **if** $\|H(\alpha^{k+1}, \theta^{k+1}) + s^{k+1} - \kappa\|_2 \leq \tau H_{\text{best}}$ **then**
  - $\lambda^{k+1} \leftarrow \lambda^k + \mu^k(H(\alpha^{k+1}, \theta^{k+1}) + s^{k+1} - \kappa)$
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Stochastic augmented Lagrangian method

augmented Lagrangian

penalty

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    Choose $\mu^{k+1} > \mu^k$
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update primal variables

update multiplier
tighten tolerance

Stochastic augmented Lagrangian method

augmented Lagrangian

penalty

$$L(\alpha, \theta, s, \lambda, \mu) = F(\theta) + \lambda(H(\alpha,\theta) + s - \kappa) + \frac{\mu}{2}\|H(\alpha,\theta) + s - \kappa\|^2$$

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update multiplier
tighten tolerance

else

Choose $\mu^{k+1} > \mu^k$

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update penalty

Stochastic gradients rely on KKT differentiation

family/context parameter

y $\longrightarrow$

optimization layer

argmin	$f(x, y)$
subject to	$g(x, u, y) \leq 0 \quad \forall u \in \mathcal{U}(\theta)$

$\longrightarrow$ $x(\theta, y)$

parametric solution

Stochastic gradients rely on KKT differentiation

family/context parameter

$y \longrightarrow$

optimization layer

argmin $\quad f(x, y)$

subject to $\quad g(x, u, y) \leq 0 \quad \forall u \in \mathcal{U}(\theta)$

parametric solution

$\longrightarrow x(\theta, y)$

Implicit differentiation

$\hat{\nabla}_\theta L$ *depends on jacobian* $J_x(\theta)$

Stochastic gradients rely on KKT differentiation

family/context parameter

$y \longrightarrow$

optimization layer

argmin	$f(x, y)$
subject to	$g(x, u, y) \leq 0 \quad \forall u \in \mathcal{U}(\theta)$

parametric solution

$\longrightarrow x(\theta, y)$

Implicit differentiation

$\hat{\nabla}_\theta L$ *depends on jacobian* $J_x(\theta)$

(obtained by differentiating through the KKT optimality conditions)

“Differentiable Convex Optimization Layers”, A. Agrawal, B. Amos, S. Barratt, S. Boyd, S. Diamond, and J. Zico Kolter (2019)
“Differentiating Through a Conic Program”, A. Agrawal, S. Barratt, S. Boyd, E. Busseti, W. M. Moursi (2019)

Convergence analysis

(informal)

Convergence analysis

(informal)

Theorem (chain rule works)

if inner robust problem
- convex and conic
- has unique solution

$\longrightarrow$ $x(\theta, y)$ is path-differentiable with conservative jacobian $J_x(\theta) \neq \emptyset$

$\longrightarrow$ L is path-differentiable

Concepts of path-differentiability -> "Nonsmooth Implicit Differentiation for Machine Learning and Optimization", J. Bolte, T. Le, E. Pauwels, A. Silveti-Falls (2021)

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Theorem (locally optimal solutions)

if sequence of penalty parameters μ^k is bounded $\longrightarrow$ the algorithm converges almost surely to a feasible solution $(\alpha^\star, \theta^\star, s^\star)$

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if inner robust problem
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→ $x(\theta, y)$ is path-differentiable with conservative jacobian $J_x(\theta) \neq \emptyset$

→ L is path-differentiable

Theorem (locally optimal solutions)

if sequence of penalty parameters μ^k is bounded → the algorithm converges almost surely to a feasible solution $(\alpha^\star, \theta^\star, s^\star)$

in addition, if $(\alpha^\star, \theta^\star, s^\star)$ satisfies:
- LICQ
- second-order optimality conditions

→ it is a locally optimal solution

Concepts of path-differentiability -> "Nonsmooth Implicit Differentiation for Machine Learning and Optimization", J. Bolte, T. Le, E. Pauwels, A. Silveti-Falls (2021)

Finite-sample probabilistic guarantees via threshold

optimality tolerance

target confidence

dimension of x

number of samples of u

Finite-sample probabilistic guarantees via threshold

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dimension of x

number of samples of u

threshold constraint

$$\mathbf{E}_w[h(\alpha^\star, \theta^\star, w)] \leq \kappa$$

Finite-sample probabilistic guarantees via threshold

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Implies

Finite-sample probabilistic guarantee

$$\mathbf{P}^{N \times J}\left(\mathbf{P}_{(u,y)}(g(x, u, y) \leq 0) \geq 1 - \eta \quad \forall x \in \mathcal{X}\right) \geq 1 - \beta$$

Ingredients

- Tail bounds
- $\mathbf{CVaR} \geq \mathbf{VaR}$

Finite-sample probabilistic guarantees via threshold

optimality tolerance

target confidence

dimension of x

number of samples of u

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Implies

Finite-sample probabilistic guarantee

$$\mathbf{P}^{N\times J}\left(\mathbf{P}_{(u,y)}(g(x, u, y) \leq 0) \geq 1 - \eta \quad \forall x \in \mathcal{X}\right) \geq 1 - \beta$$

$\Rightarrow$ it holds also for $x(\theta^\star, y)$

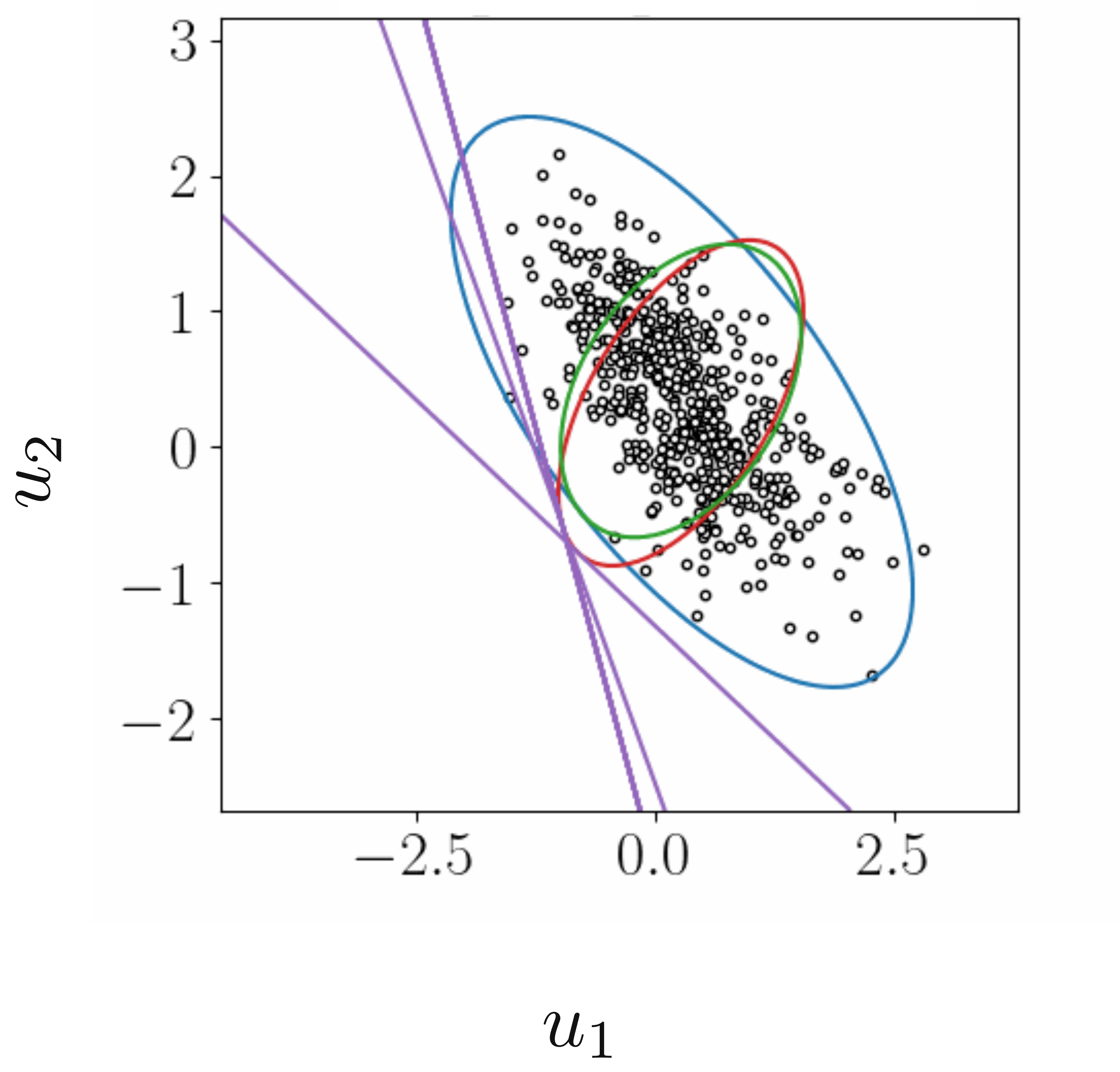
Ingredients

- Tail bounds
- $\mathbf{CVaR} \geq \mathbf{VaR}$

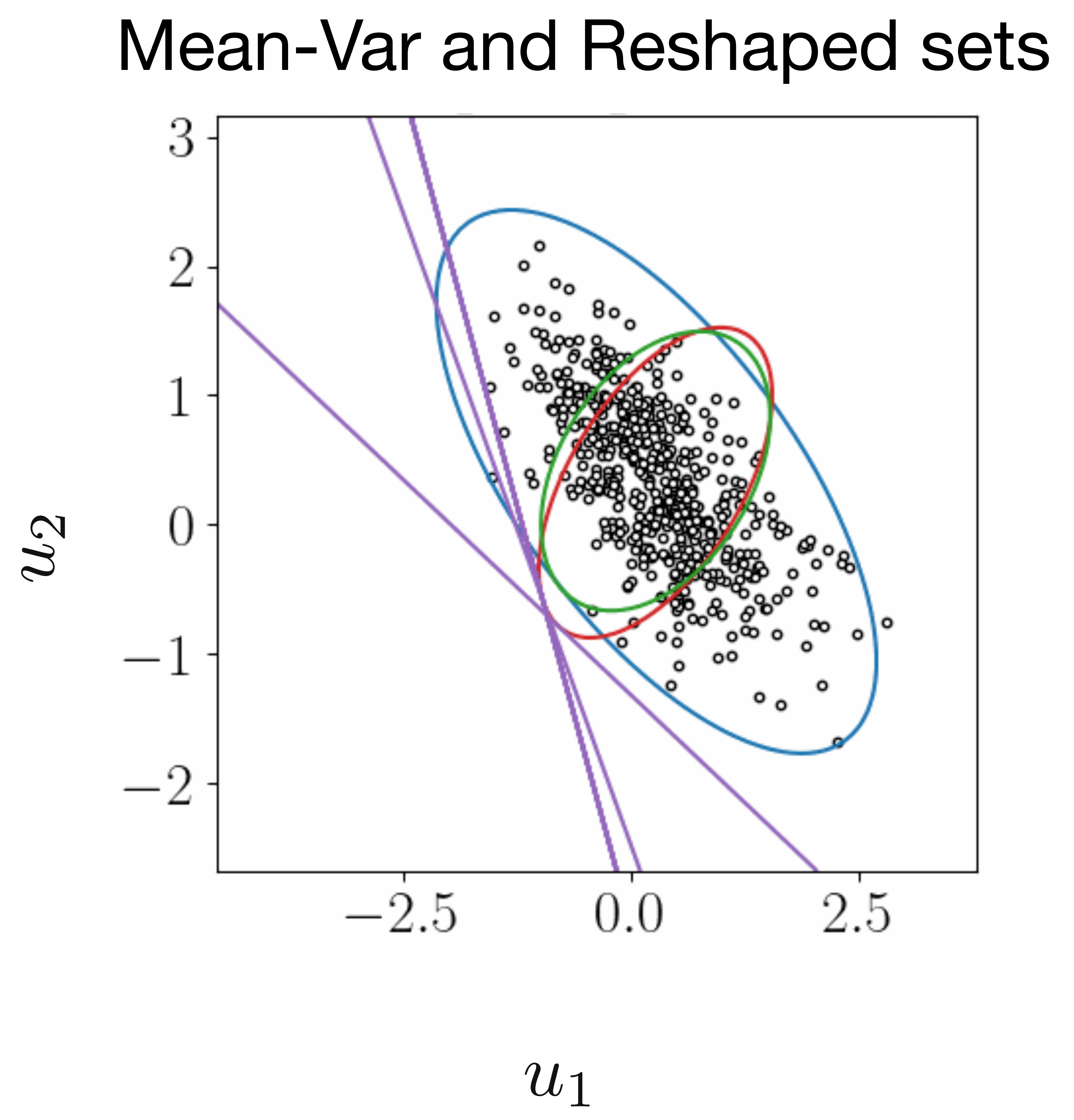
Numerical examples

Training the uncertainty set for the portfolio example

Mean-Var and Reshaped sets



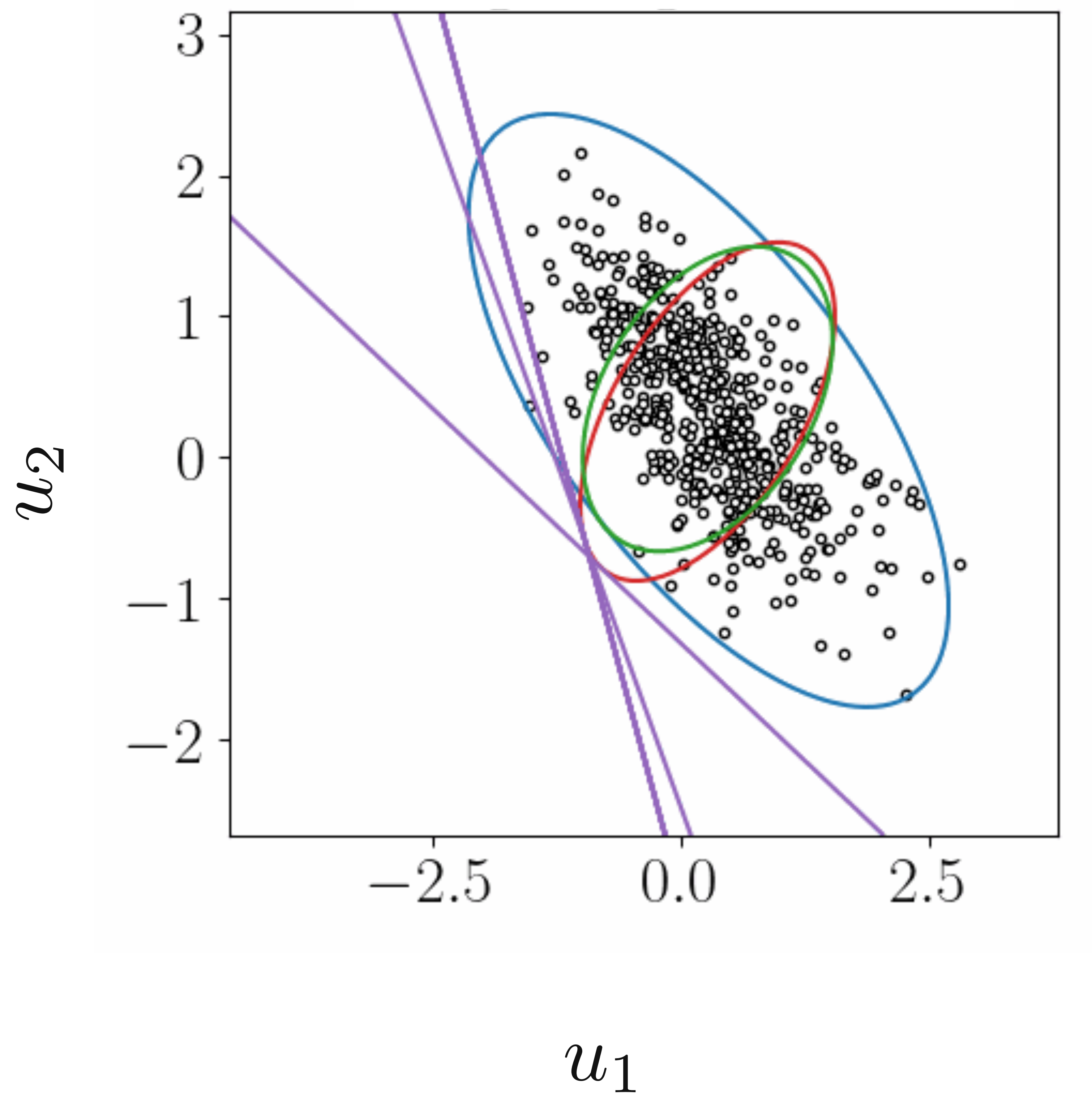
Training the uncertainty set for the portfolio example



Training the uncertainty set for the portfolio example

$$
\begin{array}{ll}
\text{minimize} & t + \lambda \|x - x^{\text{prev}}\|_1 \\
\text{subject to} & -u^T x \le t \quad \forall u \in \mathcal{U}(\theta) \\
& \mathbf{1}^T x = 1, \quad x \ge 0
\end{array}
$$

Mean-Var and Reshaped sets



Training the uncertainty set for the portfolio example

$y = x^{\text{prev}}$
previous holdings

$$\begin{array}{ll} \text{minimize} & t + \lambda \|x - \boxed{x^{\text{prev}}}\|_1 \\ \text{subject to} & -u^T x \le t \quad \forall u \in \mathcal{U}(\theta) \\ & \mathbf{1}^T x = 1, \quad x \ge 0 \end{array}$$

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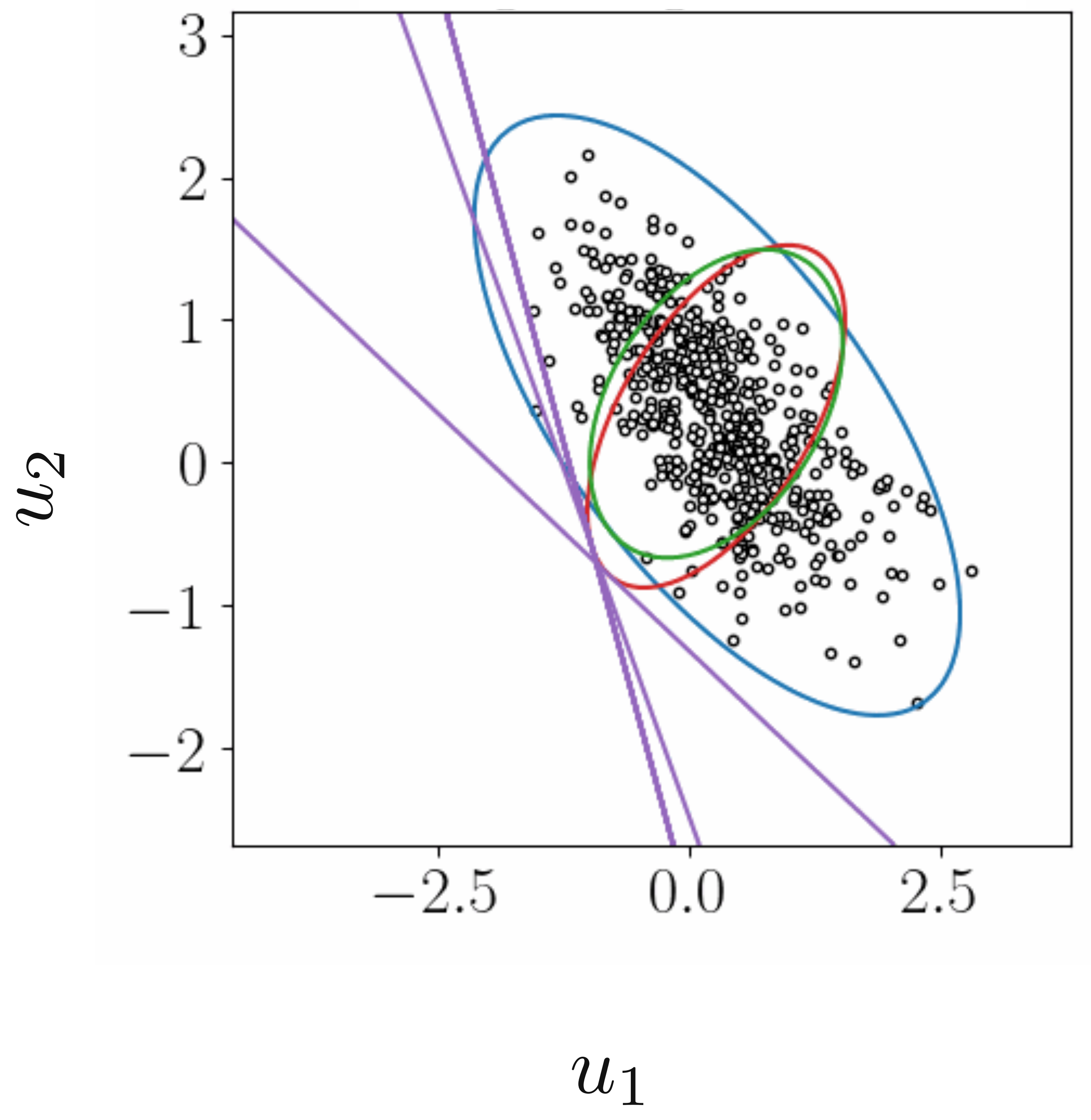

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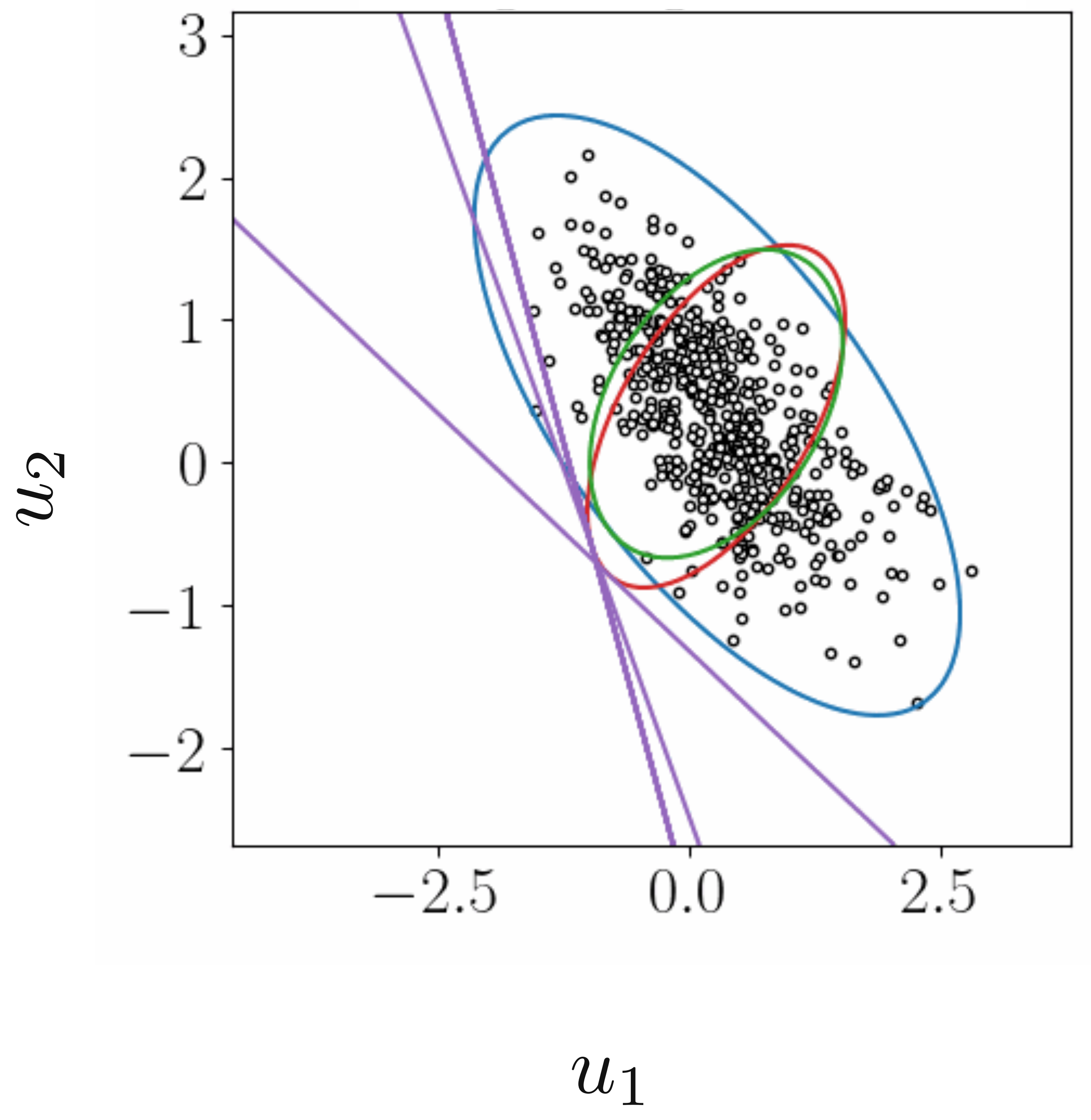

constraint level curves
$-u^T x(\theta, y) - t(\theta, y) = 0$

Inventory management as two-stage robust optimization

n retail points

“Pareto Efficiency in Robust Optimization” D. A. Iancu, N. Trichakis (2014)

Inventory management as two-stage robust optimization

n retail points

uncertain demand $d = \bar{d} + Qu$

uncertain market factors $u \in \mathbf{R}^m$

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two-stage decisions

stocking decisions $s \in \mathbf{R}^n$

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sales decisions $w(d) \in \mathbf{R}^n$

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\begin{array}{ll}
\text{minimize} & \tau \\
\text{subject to} & (t+h)^T s - y^T w(u) \le \tau, \quad \forall u \in \mathcal{U}(\theta) \\
& w(u) \le s, \quad \forall u \in \mathcal{U}(\theta) \\
& w(u) \le \bar{d} + Qu, \quad \forall u \in \mathcal{U}(\theta) \\
& \mathbf{1}^T s = C \\
& 0 \le s \le c
\end{array}
$$

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transportation and holding costs

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\end{array}
$$

(minimize worst-case costs)

"Pareto Efficiency in Robust Optimization" D. A. Iancu, N. Trichakis (2014)

Inventory management as two-stage robust optimization

n retail points

uncertain demand $d = \bar{d} + Qu$

uncertain market factors $u \in \mathbf{R}^m$

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transportation and holding costs

sale prices

$$
\begin{array}{lll}
\text{minimize} & \tau & \\
\text{subject to} & (t+h)^T s - y^T w(u) \leq \tau, \quad \forall u \in \mathcal{U}(\theta) & \textit{(minimize worst-case costs)} \\
& w(u) \leq s, \quad \forall u \in \mathcal{U}(\theta) & \textit{(sell less than stocked items)} \\
& w(u) \leq \bar{d} + Qu, \quad \forall u \in \mathcal{U}(\theta) & \\
& \mathbf{1}^T s = C & \\
& 0 \leq s \leq c &
\end{array}
$$

“Pareto Efficiency in Robust Optimization” D. A. Iancu, N. Trichakis (2014)

Inventory management as two-stage robust optimization

n retail points

uncertain demand $d = \bar{d} + Qu$ ← uncertain market factors $u \in \mathbf{R}^m$

two-stage decisions

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& w(u) \leq s, \quad \forall u \in \mathcal{U}(\theta) & \textit{(sell less than stocked items)} \\
& w(u) \leq \bar{d} + Qu, \quad \forall u \in \mathcal{U}(\theta) & \textit{(sell less than demand)} \\
& \mathbf{1}^T s = C & \\
& 0 \leq s \leq c &
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\begin{array}{lll}
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& w(u) \leq s, \quad \forall u \in \mathcal{U}(\theta) & \textit{(sell less than stocked items)} \\
& w(u) \leq \bar{d} + Qu, \quad \forall u \in \mathcal{U}(\theta) & \textit{(sell less than demand)} \\
& \mathbf{1}^T s = C & \textit{(total units of product available)} \\
& 0 \leq s \leq c &
\end{array}
$$

Inventory management as two-stage robust optimization

n retail points

uncertain demand $d = \bar{d} + Qu$

uncertain market factors $u \in \mathbf{R}^m$

two-stage decisions

stocking decisions $s \in \mathbf{R}^n$

sales decisions $w(d) \in \mathbf{R}^n$

transportation and holding costs

sale prices

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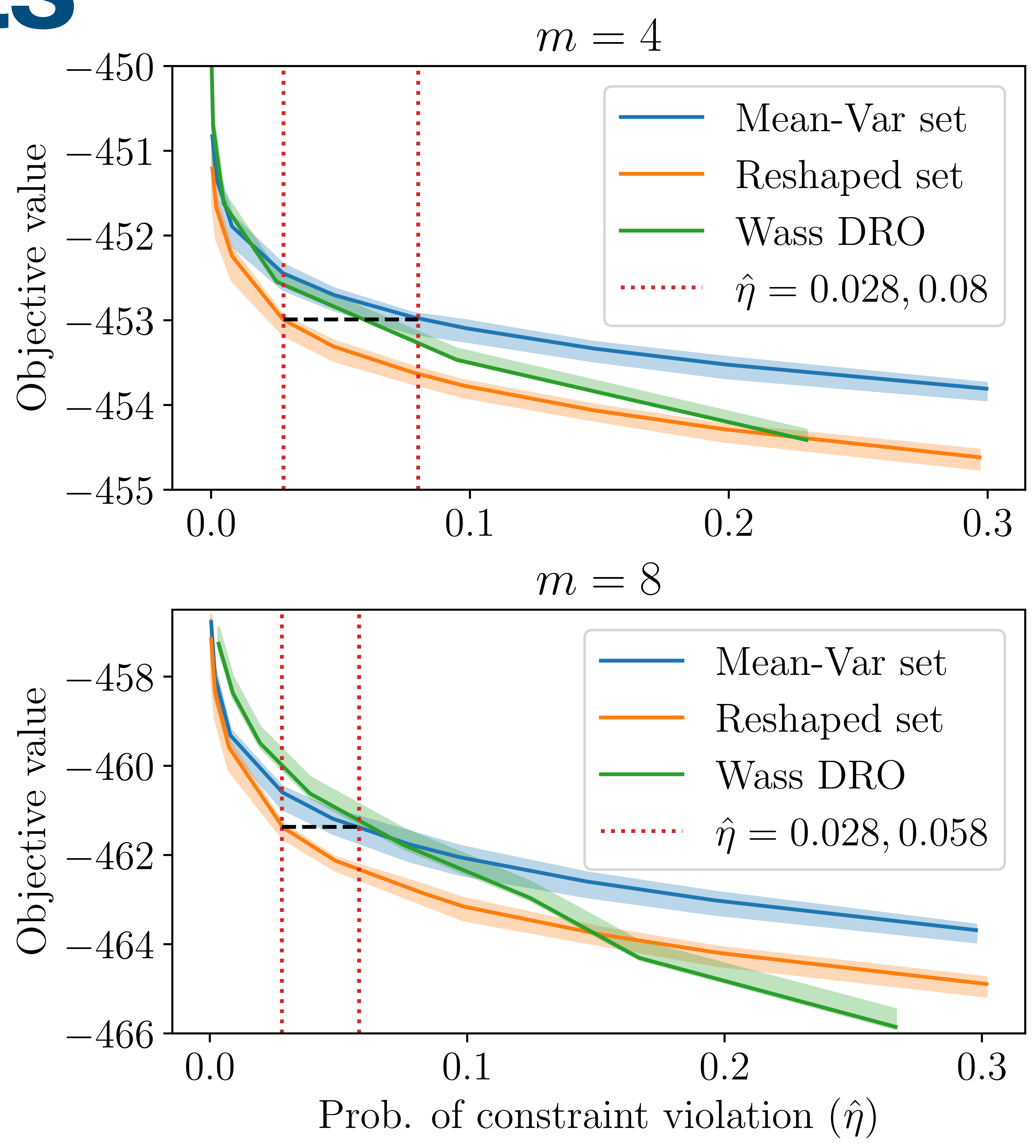
& w(u) \le \bar{d} + Qu, \quad \forall u \in \mathcal{U}(\theta) & \textit{(sell less than demand)} \\
& \mathbf{1}^T s = C & \textit{(total units of product available)} \\
& 0 \le s \le c & \textit{(maximum stock volume)}
\end{array}
$$

“Pareto Efficiency in Robust Optimization” D. A. Iancu, N. Trichakis (2014)

Inventory management results

u size				$m = 4$			
Method	LROPT	LRO-T$_{0.03}$	LRO-T$_{0.05}$	MV-RO$_{0.03}$	MV-RO$_{0.05}$	W-DRO$_{0.03}$	W-DRO$_{0.05}$
Obj.	-451.67	-452.99	-453.31	-452.44	-452.70	-452.54	-452.77
$\hat{\eta}$	0.002	0.0278	0.0471	0.0277	0.0476	0.0252	0.0451
$\hat{\beta}$	0	0.2	0.3	0.3	0.25	0	0.1
t	0.00203	0.00212	0.00206	0.00201	0.00209	0.336	0.315
u size				$m = 8$			
Obj.	-459.49	-461.27	-462.06	-460.62	-461.18	-459.49	-460.62
$\hat{\eta}$	0.0068	0.0257	0.0458	0.0257	0.0477	0.0195	0.0390
$\hat{\beta}$	0	0	0	0	0.06	0.2	0.2
t	0.00623	0.00630	0.00613	0.00634	0.00619	0.910	0.932



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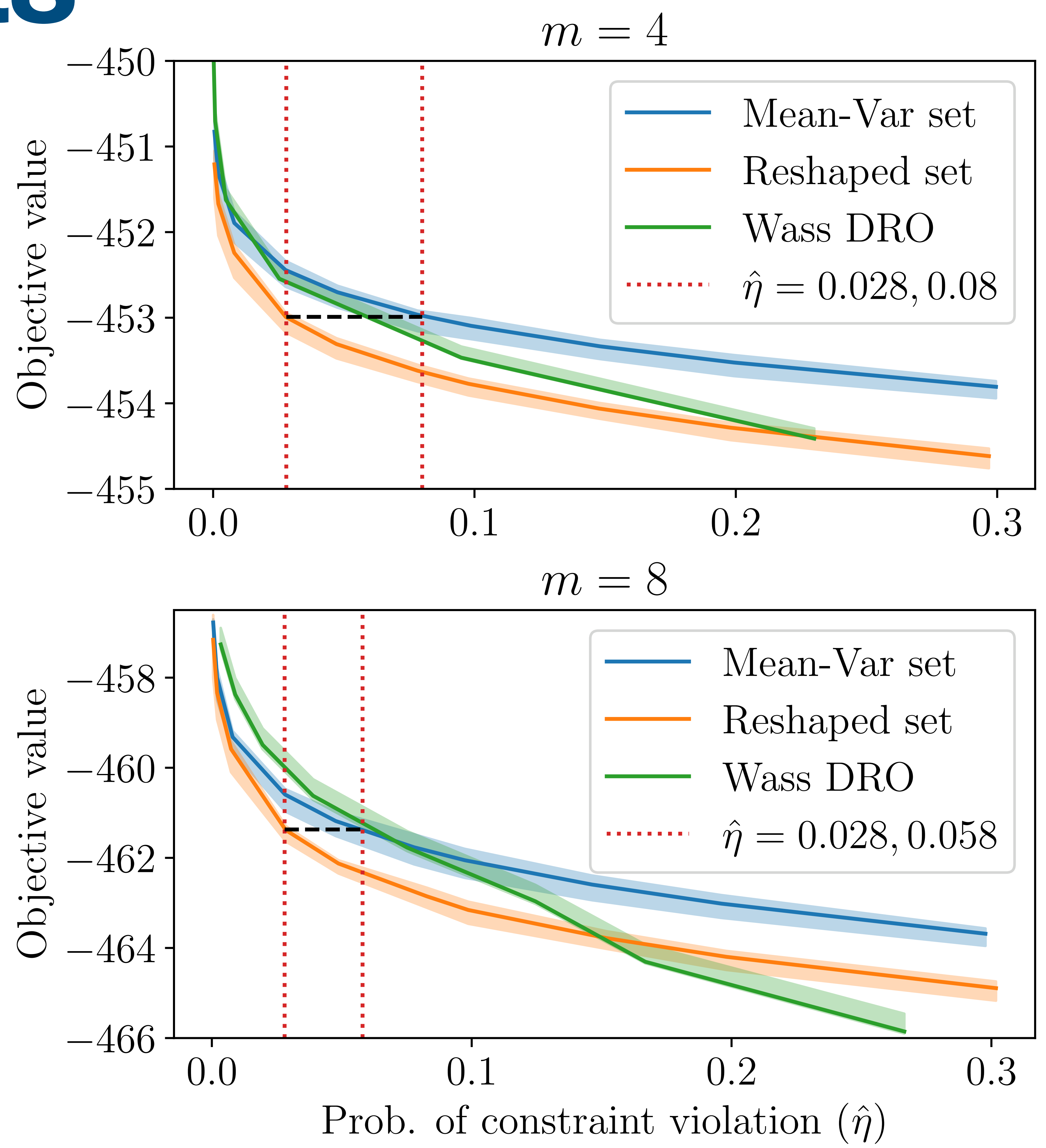

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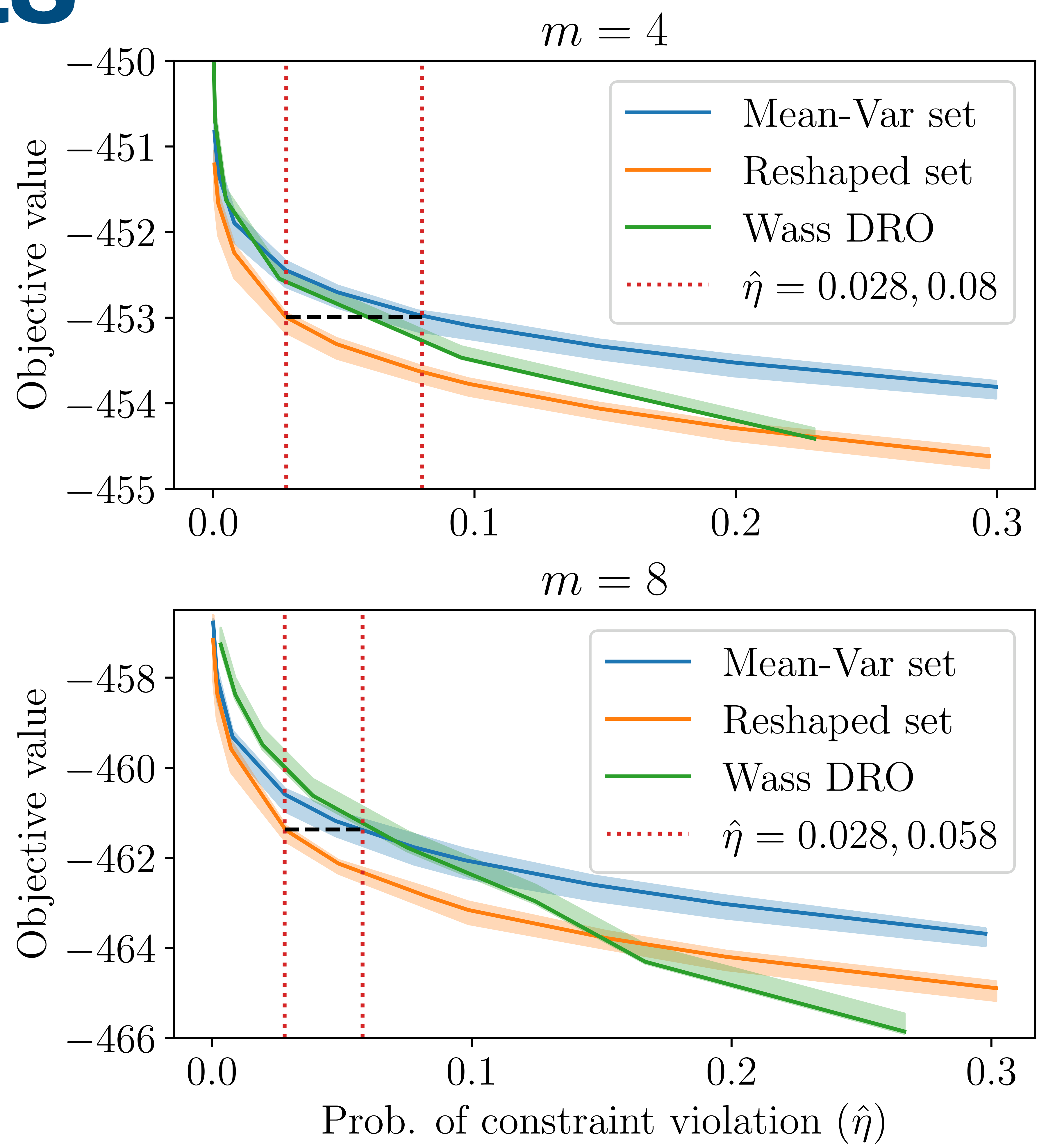

Learning decision-focused uncertainty sets for robust optimization

- Optimize **shape and size** of uncertainty sets
- **Bi-level optimization** formulation
  - CVaR constraint
  - Differentiable optimization to compute derivatives
  - Probabilistic guarantees
- **Improvements over RO and DRO formulations**

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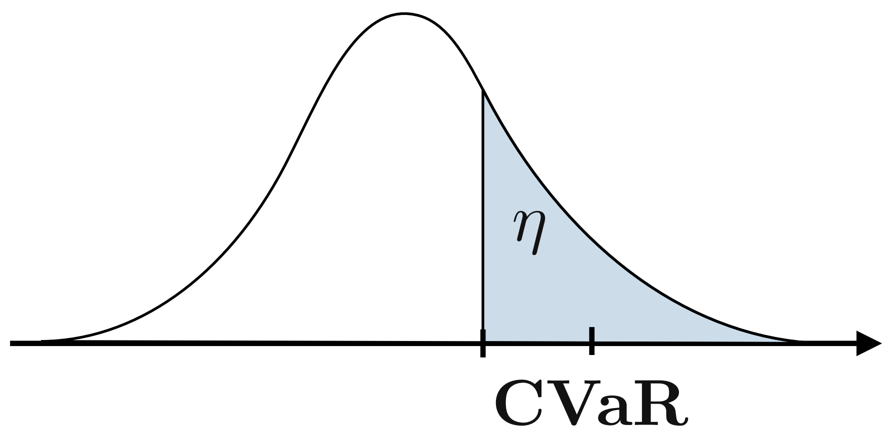

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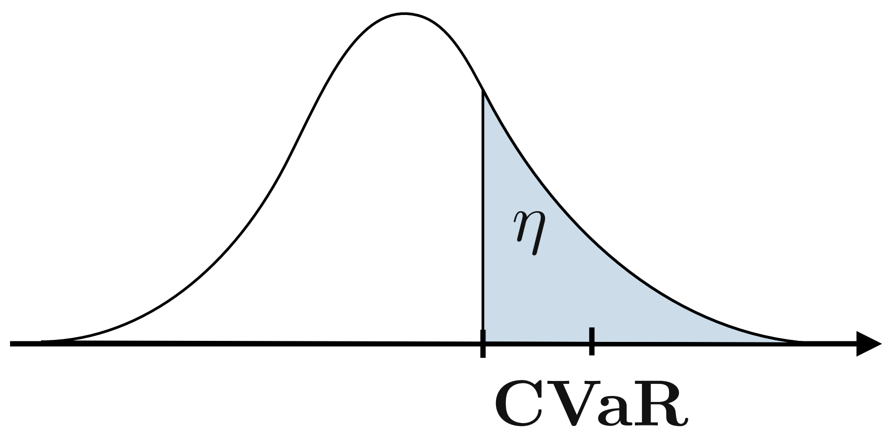

optimization layer

$$y \longrightarrow \begin{array}{ll} \text{argmin} & f(x,y) \\ \text{subject to} & g(x,u,y) \leq 0 \quad \forall u \in \mathcal{U}(\theta) \end{array} \longrightarrow x(\theta, y)$$

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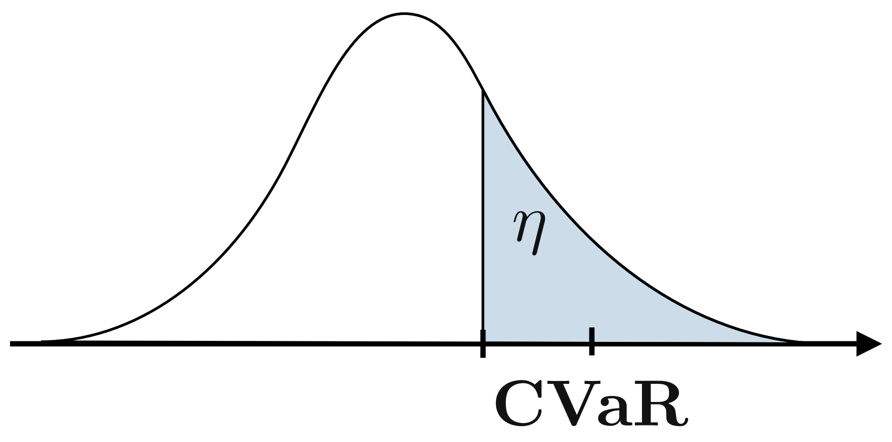

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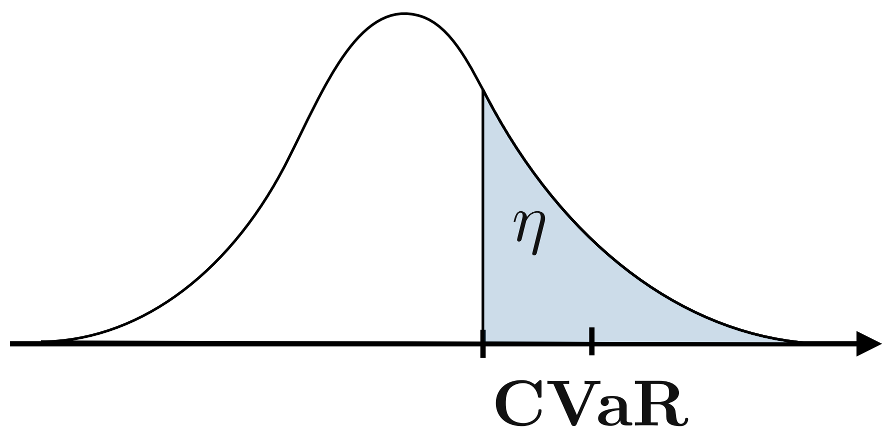

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https://github.com/stellatogrp/lropt

Learning for Robust Optimization
I. Wang, C. Becker, B. Van Parys, and B. Stellato
arxiv.org: 2305.19225, 2023

⚠️ will be updated soon!

LROPT software package (WIP)

It can be *hard to dualize* robust optimization problems

…not to mention *finding the right uncertainty set!*

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LROPT package

github.com/stellatogrp/lropt

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2. Automatically tune uncertainty sets (using cvxpylayers)

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$$
\begin{array}{ll}
\text{minimize} & x^T P x + y^T x \\
\text{subject to} & (a + Bu)^T x \leq d, \quad \forall u \in \mathcal{U}
\end{array}
$$

$$\mathcal{U} = \{u = b + Az \mid \|z\|_2 \leq 1\}$$

```
unc_set = lropt.Ellipsoidal(u_data)
u = lropt.UncertainParameter(n,
                             uncertainty_set=unc_set)
x = cp.Variable(n)
y = cp.Parameter(n)
constraints = [(a + B@u) @ x <= d]
objective = cp.Minimize(cp.quad_form(P, x) + y @ x)
problem = lropt.RobustProblem(objective,
                              constraints)
problem.train()
```

Jun 27–28, 2024, Princeton University

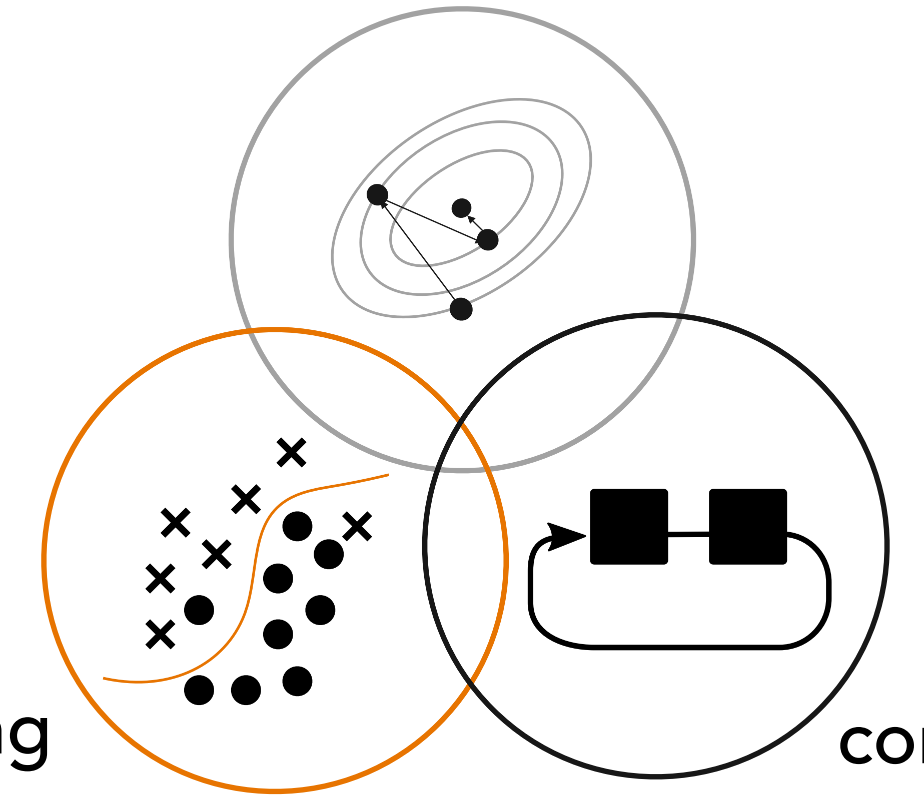
PRINCETON UNIVERSITY

Princeton Workshop on

Optimization, Learning, and Control

The Princeton Workshop on Optimization, Learning, and Control is a single-track workshop highlighting the latest research advances across these disciplines. Its main goal is to foster new interactions and lay the groundwork for new collaborations. The workshop will include a poster session for junior researchers.

optimization



learning

control

Important dates

- **March 3**: Deadline for poster abstract submission and travel support application
- **May 1**: Registration opens
- **June 1**: Registration deadline

Contacts

- **Website**: stellato.io/olc
- **Organizer**: Bartolomeo Stellato, Princeton University — stellato.io
- **Questions**: olc24@princeton.edu

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Anuradha Annaswamy
MIT

Francesco Borrelli
UC Berkeley

Sarah Dean
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Elad Hazan
Princeton University

Andrea Lodi
Cornell Tech

Robert Luce
Gurobi Optimization

Melanie Zeilinger
ETH Zurich

Anirudha Majumdar
Princeton University

Conclusion

Machine Learning tools
can help us
formulate optimization problems

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We should think
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⚙️ tune *train* → ✅ validate *certify* → 🚀 deploy

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@b_stellato bstellato@princeton.edu stellato.io