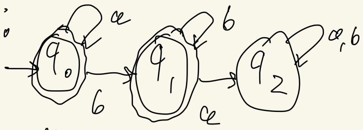
$A^{(1)}$:



$L_k = L(A^{(k)})$

$L(A^{(1)}) = L_1 := \{a^m b^n \mid m,n \ge 0\}$

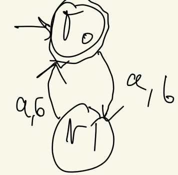
$\underbrace{aaaa\ldots a}_{m \ge 0}\underbrace{bbb\ldots b}_{n \ge 0}$

$a^0 := \varepsilon$

$a^{n+1} := aa^n$

inductive definition

$A^{(2)}$:



ε ✓
ab ✓
abab
abbb
abaaab

$L_2 = \{w \mid |w| \text{ even}\}$

$= \{w \mid |w| \equiv 0 \pmod 2\}$

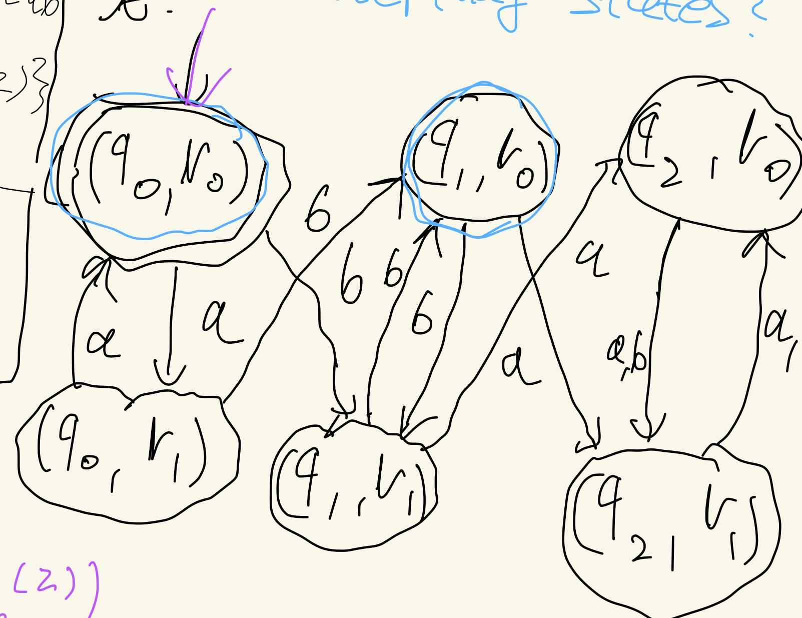
$= \{w \mid |w| \% 2 = 0\}$

Q: How to get an A such that $L(A) = L_1 \cap L_2$?

Parallel computations

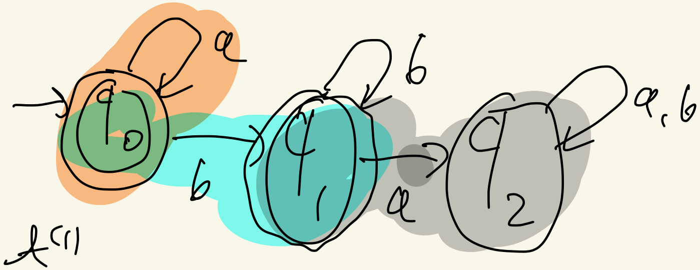
⇝ Product automaton

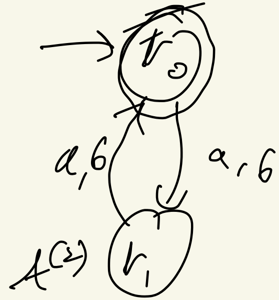
A: accepting states?



$q_0 = (q_0^{(1)}, q_0^{(2)})$

$F = F^{(1)} \times F^{(2)}$





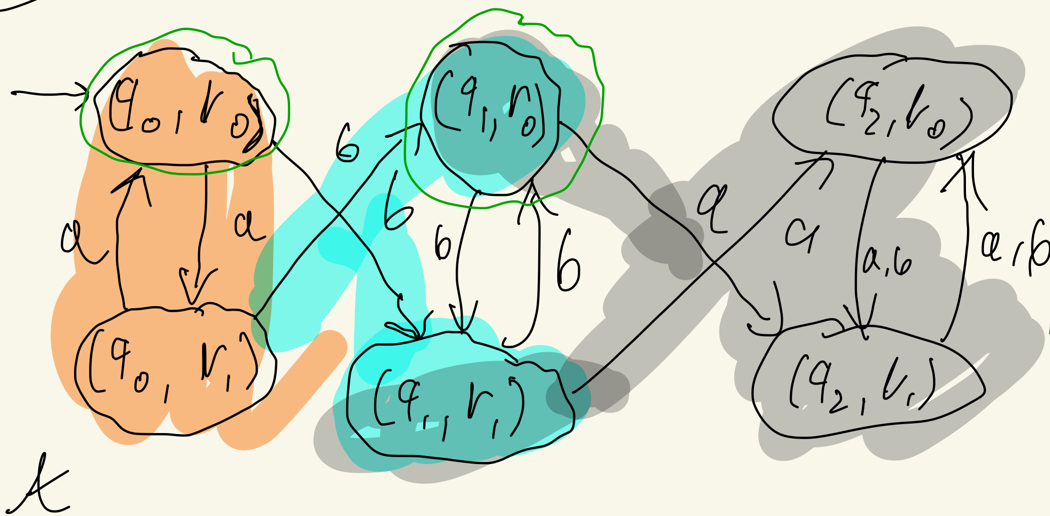
Parallel computation

Product automaton: $Q := Q^{(1)} \times Q^{(2)}$

Idea: new states (q, r)

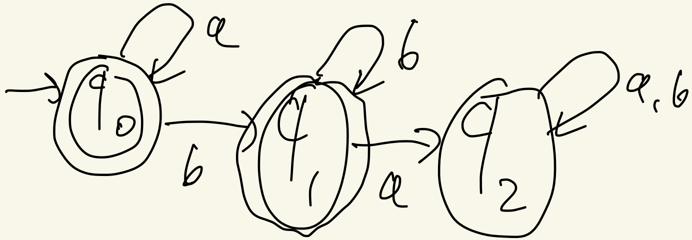
wanted:

$L(A) = L(A^{(1)}) \cap L(A^{(2)})$



$\boxed{F = F^{(0)} \times F^{(1)}}$

$F = \{(q, r) \mid q \in F^{(0)}, r \in F^{(1)}\}$
$= \{(q_0, r_0), (q_1, r_0)\}$



L_1: any word w/o ba

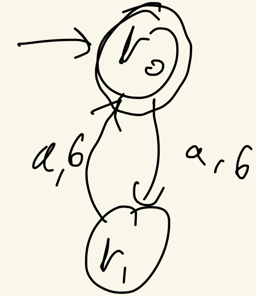
$L_1 := \{a^n b^m \mid m,n \geq 0\}$

$\underbrace{aaa\ldots a}_{n \geq 0}\underbrace{bb\ldots b}_{m \geq 0}$

$w = a^n b^m$

$a^2 = aa$

$$\boxed{\begin{array}{l} a^0 := \varepsilon \\ a^{n+1} := a a^n \end{array}}$$



aa ✓

ab ✓

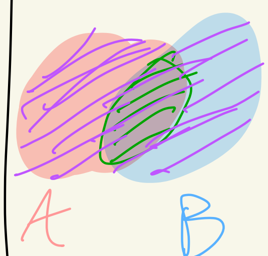
a ✗

aba ✗

...

$L_2 := \{w \mid |w| \text{ even}\}$

ab in both ⇝ ab $\in L_1 \cap L_2$



$A \cap B$ intersection

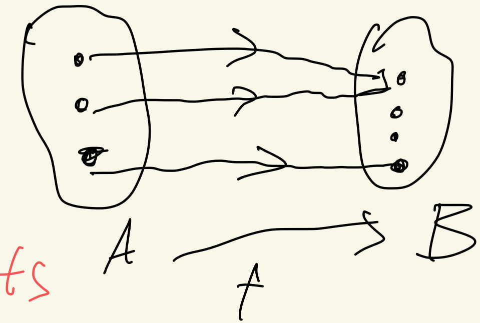
$A \cup B$ union

Can we find an α s.t. $L(\alpha) = L_1 \cup L_2$?

DFA $\rightsquigarrow$ NFA

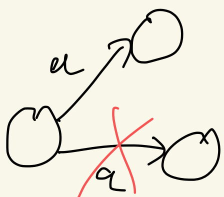
$$\delta : Q \times \Sigma \longrightarrow Q$$

(state, symbol) $\longmapsto$ new state



determinism:

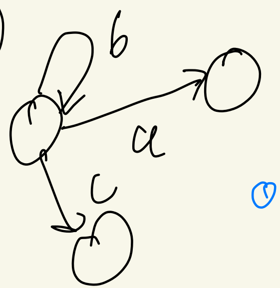
(1)



predictable

∘∘ for all q, a exists at most one outcome

(2)

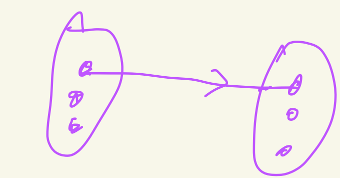


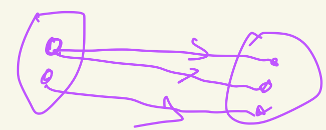
totality

∘∘ for all q, a exists at least one

for all q, a exists exactly one outcome

disallowed





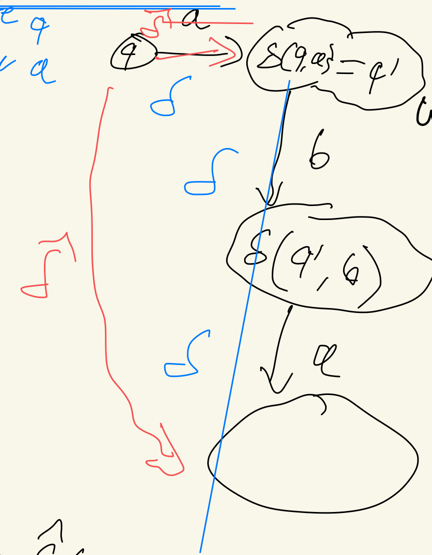
$$Q \times \Sigma \xrightarrow{\delta} Q$$

$i \downarrow$ $\qquad \nearrow \hat{\delta}$

$$Q \times \Sigma^*$$

in: state q
letter a

in: state q
word w



$w = aba$

$x = ab$

$a = a$

$$i(q,a) = (q,a) \qquad \hat{\delta}(q,a) = \hat{\delta}(i(q,a)) = \delta(q,a)$$

For every $w \in \Sigma^*$:

base:

$w = \varepsilon$

step:

$w = xa,\ x \in \Sigma^*,\ a \in \Sigma$

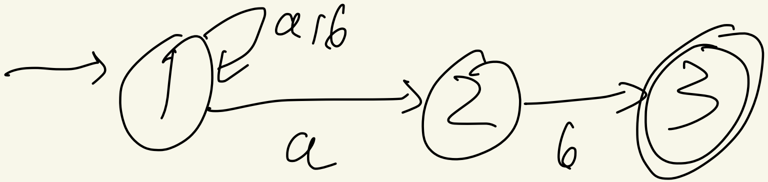
Inductive def. of $\hat{\delta} : Q \times \Sigma^* \to Q$

$$\hat{\delta}(q,w) = \begin{cases} q & \text{if } w = \varepsilon \\ \delta(\hat{\delta}(q,x), a) & \text{if } w = xa \end{cases}$$

For every $n \in \mathbb{N}$:

base: $n = 0$

step: $n = k+1,\ k \in \mathbb{N}$

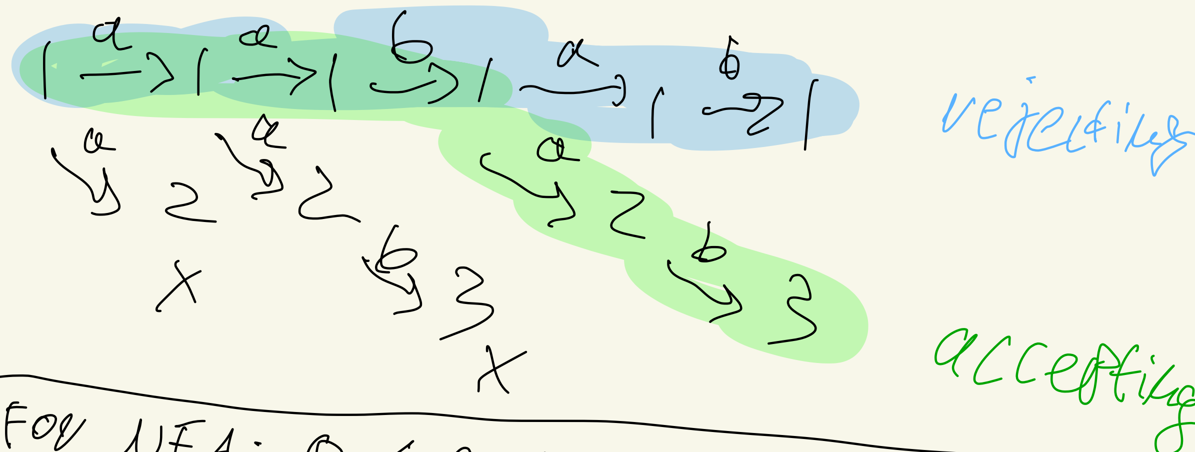


w <u>accepted</u> if some computation accepts.

$w = aabab$

Not def.:

- $1 \xrightarrow{a} 1$, $1 \xrightarrow{a} 2$
- $2 \xrightarrow{a} X$
- no trans. out of 3

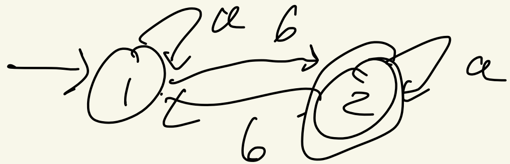


$\delta(1,a) = \{1,2\}$

$\delta(2,a) = \{\ \} = \emptyset$

For NFA: Q, Σ, q_0, F (just as DFA)

$\delta : Q \times \Sigma \to \mathcal{P}(Q)$ <u>Power set</u>



"guessing" for NFA.

DFA:

$\delta: Q \times \Sigma \to Q$

$\delta(1,b) = 2$

$\delta(q,a) = q'$

NFA:

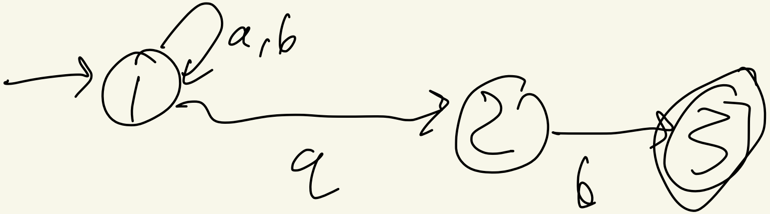
$\delta': Q \times \Sigma \to \mathcal{P}(Q)$

$\delta'(1,b) = \{2\}$

$\delta'(q,a) = \{q'\}$

Is the converse true?

Every DFA can be regarded as NFA!

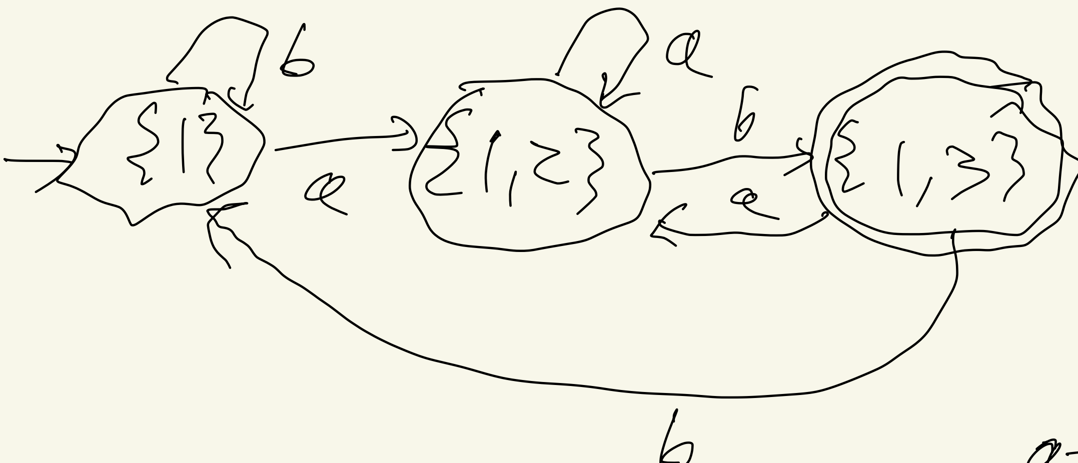


NFA

Power set construction

new DFA: states $\mathcal{P}(Q)$

determinization



States S are accepting if they contain at least one accepting state