Chapter 2-2

Modeling Concurrent Systems

Transition system

- State: the behavior of systems at a certain moment.
  - A state of a traffic light is the color of the light.
  - A state of a sequential program is the current values of variables and program counter.
  - A state of a synchronous hardware circuit the value of input bits and registers .

Transition systems

- Transition specify the evolution from one state to another.
  - Traffic light is a switch from one color to another.
  - Transition of a sequential program is the execution of a statement.
  - A transition of a synchronous hardware circuit is the change of registers and output bits on a new set of input.

Transition system

- Action: communication mechanisms between processes.
- Atomic propositions: formalize temporal characteristics.
  - X=10
  - X<200
  - There is more than a liter of fluid in the tank
  - There are no customers in the shop.

Transition system(TS)

- A *transition system TS* is a tuple (S, Act, $\rightarrow$, I, AP, L) where
  - S is a set of states,
  - Act is a set of actions,
  - $\rightarrow \subseteq S \times Act \times S$ is a transition relation,
  - $I \subseteq S$ is a set of initial states,
  - AP is a set of atomic propositions, and
  - $L : S \rightarrow 2^{AP}$ is a labeling function.
- *TS* is called *finite* if S, Act, and AP are finite.

TS(cont)

- $s \xrightarrow{\alpha} s'$ instead of $(s, \alpha, s') \in \rightarrow$
  - a transition originating from s is selected *nondeterministically*.
  - The action α is performed and the transition system evolves from state s into the state s'.
  - when the set of initial states consists of more than one state, the start state is selected nondeterministically.
  - Actions are for modeling communication mechanisms. Others can omit actions.

TS(cont)

- Label function $L(s) \in 2^{AP}$
  - A state *s* satisfies a propositional logic formula Φ if $L(s)$ makes the formula Φ true; that is:

$$s \models \Phi \text{ iff } L(s) \models \Phi.$$

  - AP is chosen depending on the characteristics of interest.

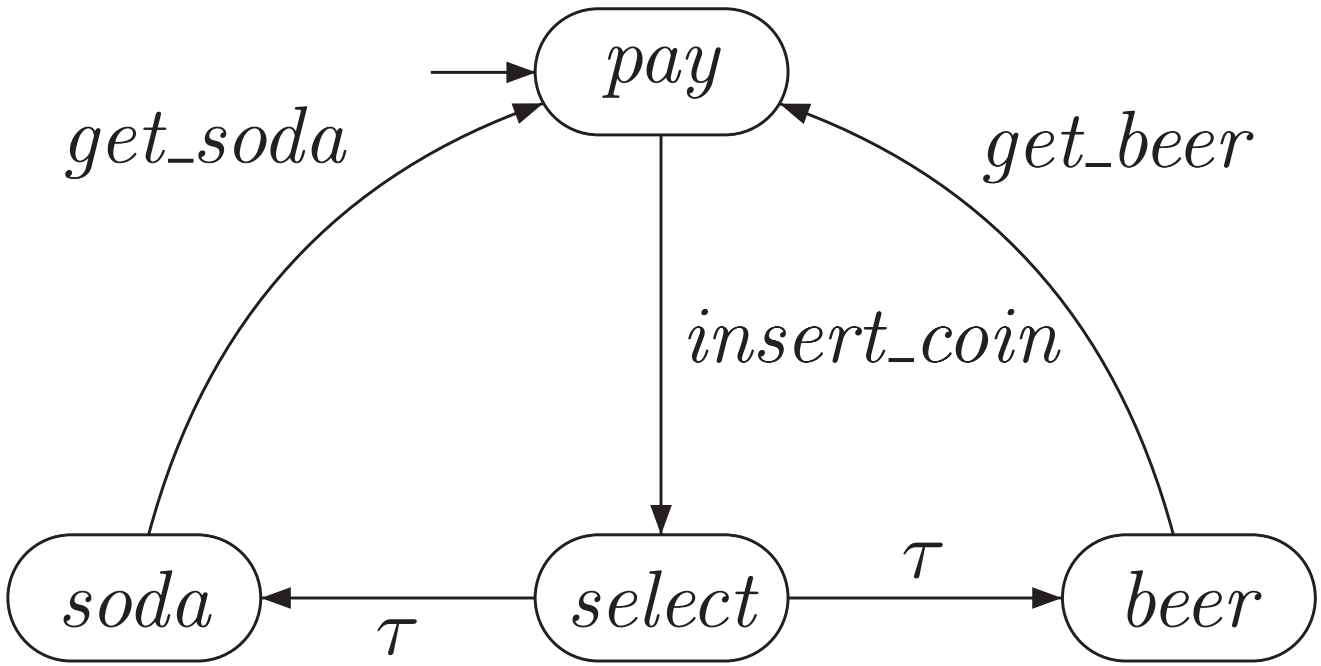
Satisfaction relation $|=$

- Evaluation for AP is function μ:AP$\rightarrow$\{0,1\}.
- Eval(AP) is the set of all evaluations for AP.
- Satisfaction relation $|=$ is a set of pair (μ,ϕ), indicating the evaluations μ for which a formula ϕ is true.
  - $\mu \;|= true$
  - $\mu \;|= a$ iff $\mu(a)=1$
  - $\mu \;|= \neg\, \phi$ iff $\mu \;|\neq \phi$
  - $\mu \;|= \phi \wedge \psi$ iff $\mu |= \phi \wedge \mu |= \psi$

Application of nondeterministic

- to model the parallel execution of independent activities by interleaving
- to model the conflict situations that arise if two processes aim to access a shared resource.
- To be so important for abstraction purposes, for underspecification
- to model the interface with an unknown or unpredictable environment.

Example: Beverage Vending



- S = { pay , select, soda, beer }
- I = { pay }
- Act = { insert_coin, get_soda, get_beer, τ }

Example(cont)

- After the insertion of a coin, the vending machine nondeterministically can choose to provide either beer or soda.
- $AP = \{ paid , drink \}$ *–property under consideration*
- Labeling function:

$$L(pay) = \varnothing,$$

$$L(soda) = L(beer) = \{ paid , drink \},$$

$$L(select) = \{ paid \}.$$

Actions and atomic propositions

- Actions are only necessary for modeling communication mechanisms
- The set of propositions AP is always chosen depending on the characteristics of interest.

Direct Successors

- Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system. For $s \in S$ and $\alpha \in Act$, the set of direct *α-successors* of s is defined as:

$$Post(s, \alpha) \quad = \quad \left\{ \, s' \in S \;\; \middle| \;\; s \xrightarrow{\alpha} s' \, \right\}$$

$$Post(s) \; = \; \bigcup_{\alpha \in Act} \; Post(s, \alpha)$$

Direct Predecessors

- The set of *α-predecessors* of *s* is defined by:

$$Pre(s,\alpha) \;=\; \left\{\, s' \in S \;\mid\; s' \xrightarrow{\alpha} s \,\right\}$$

$$Pre(s) \;=\; \bigcup_{\alpha\in Act} Pre(s,\alpha)$$

Direct Predecessors and Successors expanded to subset

$$Post(C, \alpha) = \bigcup_{s \in C} Post(s, \alpha), \quad Post(C) = \bigcup_{s \in C} Post(s)$$

$$Pre(C, \alpha) = \bigcup_{s \in C} Pre(s, \alpha), \quad Pre(C) = \bigcup_{s \in C} Pre(s)$$

Terminal State of a TS

- State s in TS is called *terminal* iff:
  - $Post(s) = \emptyset$.
- For a sequential computer program, terminal states represents the termination of the program.
- For parallel systems, terminal states are usually considered to be undesired

Deterministic Transition System

- Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system.
  - TS is called *action-deterministic* if for all states s and actions α

    $| I | \leq 1$ and $| Post(s, \alpha) | \leq 1$.
  - TS is called *AP-deterministic* if for all states s and $A \in 2^{AP}$

    $| I | \leq 1$ and

    $|Post(s) \cap \{s' \in S \mid L(s') = A\}| \leq 1$

Execution (also called run)

- Execution fragment
  - Let $TS = (S, Act, \to, I, AP, L)$. A *finite* execution fragment of TS:

$$\varrho \;=\; s_0\,\alpha_1\,s_1\,\alpha_2\ldots\alpha_n\,s_n$$

$$\text{such that } s_i \xrightarrow{\alpha_{i+1}} s_{i+1} \text{ for all } 0 \leqslant i < n$$

Where n≥0 is the length of the finite execution fragment.

Infinite execution fragment

- An *infinite* execution fragment ρ of *TS* :

$$\rho \;=\; s_0\,\alpha_1\,s_1\,\alpha_2\,s_2\,\alpha_3\ldots$$

$$\text{such that } s_i \xrightarrow{\alpha_{i+1}} s_{i+1} \text{ for all } 0 \leqslant i.$$

Maximal and Initial Execution Fragment

- A *maximal* execution fragment is either a finite execution fragment that ends in a terminal state, or an infinite execution fragment.
- An execution fragment is called *initial* if it starts in an initial state.

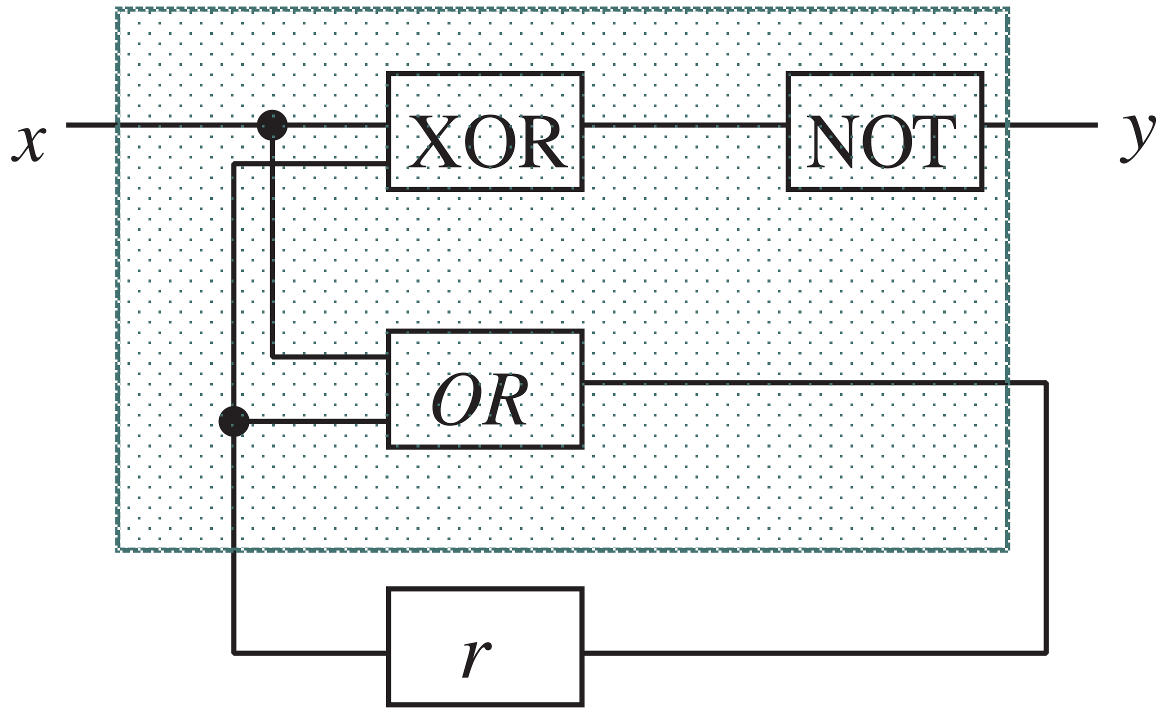
Execution and Reachable states

- An *execution* of *TS* is an initial, maximal execution fragment.
- Let $TS = (S, Act, \rightarrow, I, AP, L)$, a state $s \in S$ is called *reachable* in *TS* if there exists an initial, finite execution fragment

$$s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} \ldots \xrightarrow{\alpha_n} s_n = s$$

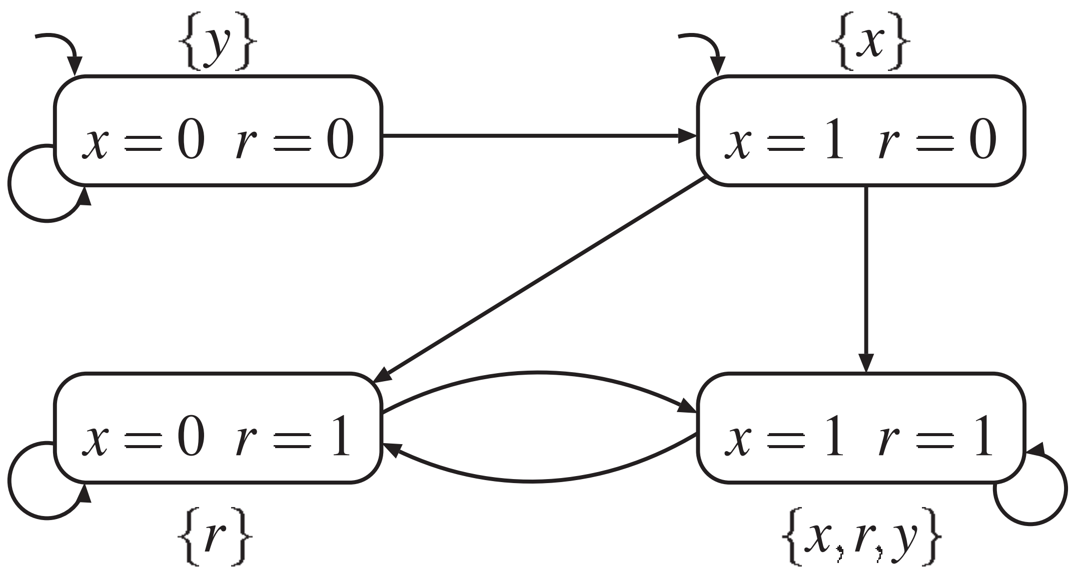
- *Reach*(*TS*) denotes the set of all reachable states in *TS*.

Sequential Hardware Circuits



$$\lambda_y = \neg(x \oplus r) \qquad\qquad \delta_r = x \vee r\ .$$

Sequential Hardware Circuits(con’t)



- state space: S = Eval(x, r)----the set of evaluation of x and r
- AP={x,y,r}
- Label function

Sequential Hardware Circuits(con’t)

- property: the output y is set infinitely often.
- AP’={x,y}
- r is invisible.

Sequential Hardware Circuits(con't)

- the general approach for sequential hardware circuits
  - state: $Eval(x_1,\ldots,x_n,r_1,\ldots,r_k)$
  - action: $\{\tau\}$
  - $AP=\{x_1,\ldots,x_n,y_1,\ldots,y_m,r_1,\ldots,r_k)$
  - transition:
    - $(a_1,\ldots,a_n,c_1,\ldots,c_k)\rightarrow(a'_1,\ldots,a'_n,c'_1,\ldots,c'_k)$

Data-Dependent Systems

- modeling conditional branching as a TS, the conditions of transitions could be omitted and could be replaced by nondeterminism;
  - if x%2=1 then x:=x+1; else x:=2x; if
- Result
  - very abstract transition system
  - a few relevant properties can be verified.
- New method
  - Conditional transition---program graph
  - Unfolded into a TS

Beverage Vending Machine

- counts the number of soda and beer bottles and returns inserted coins if the vending machine is empty.
- Location: start, select
- Conditional transition----- g:α

$$start \xhookrightarrow{true\ :\ coin} select$$

$$start \xhookrightarrow{true\ :\ refill} start$$

Beverage Vending Machine (cont)

- Conditional transition----- g:α
  - g is a Boolean condition (guard)
  - α is an action that is possible once g holds. If g does not hold, nothing to do.

$$select \xrightarrow{nsoda > 0 \ : \ sget} start$$

$$select \xrightarrow{nbeer > 0 \ : \ bget} start$$

$$select \xrightarrow{nsoda = 0 \wedge nbeer = 0: \ ret\_coin} start$$

Beverage Vending Machine (cont)

- coin: insert coin
- ret_coin:return coin
- Other actions

Action	Effect
$refill$	$nsoda := max;\ nbeer := max$
$sget$	$nsoda := nsoda - 1$
$bget$	$nbeer := nbeer - 1$

- Variables: $nsode$, $nbeer$

Typed variables

- *nsoda*, *nbeer* are typed variables: Var
- The domain of x: dom(x)
- The set of evaluation: Eval(Var)
- The set of Boolean condition over Var:Cond(Var)
- Propositional symbol: $x \in D$, where
  - $x=(x_1,x_2,\ldots x_n)$
  - $D=D_1 \times D_2 \times \ldots \times D_n$
  - Condition: propositional symbol
    - $-3<x-x'<=5$ and $x<=2*x'$

effect

- The effect of the actions is formalized by means of a mapping:
  - Effect:Act x Eval(Var)$\rightarrow$ Eval(Var)
  - How the evaluation η of variables is changed by performing an action.

Program Graph (PG)

- a digraph whose edges are labeled with conditions on these variables and actions.
- The effect of the actions is formalized by means of a mapping
  - $\text{Effect} : \text{Act} \times \text{Eval}(\text{Var}) \rightarrow \text{Eval}(\text{Var})$
  - α action $x := y+5$,
  - η evaluation $\eta(x) = 17$,$\eta(y) = -2$,
  - $\text{Effect}(\alpha,\eta)(x)=\eta(y) + 5 =-2 + 5 = 3$
  - $\text{Effect}(\alpha, \eta)(y) =\eta(y) = -2$

Program Graph (PG)

- A PG over set Var of typed variables is a tuple $(Loc, Act, Effect, \hookrightarrow, Loc_0, g_0)$
- Loc is a set of locations
- Act is a set of actions,
- $Effect : Act \times \mathrm{Eval(Var)} \rightarrow \mathrm{Eval(Var)}$
- $\hookrightarrow \subseteq Loc \times \mathrm{Cond(Var)} \times Act \times Loc$ is the conditional transition relation,
- $Loc_0 \subseteq Loc$ is a set of initial locations,
- $g_0 \in \mathrm{Cond(Var)}$ is the initial condition.

Beverage Vending Machine

- $Loc = \{start, select\}$
- $Loc_0 = \{start\}$ Var$=\{nsoda, nbeer\}$
- $Act = \{ bget, sget, coin, ret\_coin, refill \}$
- $g_0 = (nsoda = max \wedge nbeer = max)$
- Effect
  - Effect$(coin, \eta) = \eta$
  - Effect$(ret_coin, \eta) = \eta$
  - Effect$(sget, \eta) = \eta[nsoda := nsoda - 1]$
  - Effect$(bget, \eta) = \eta[nbeer := nbeer - 1]$
  - Effect$(refill, \eta) = [nsoda := max, nbeer := max]$

Translation from PG into TS

Definition 2.15. Transition System Semantics of a Program Graph

The transition system $TS(PG)$ of program graph

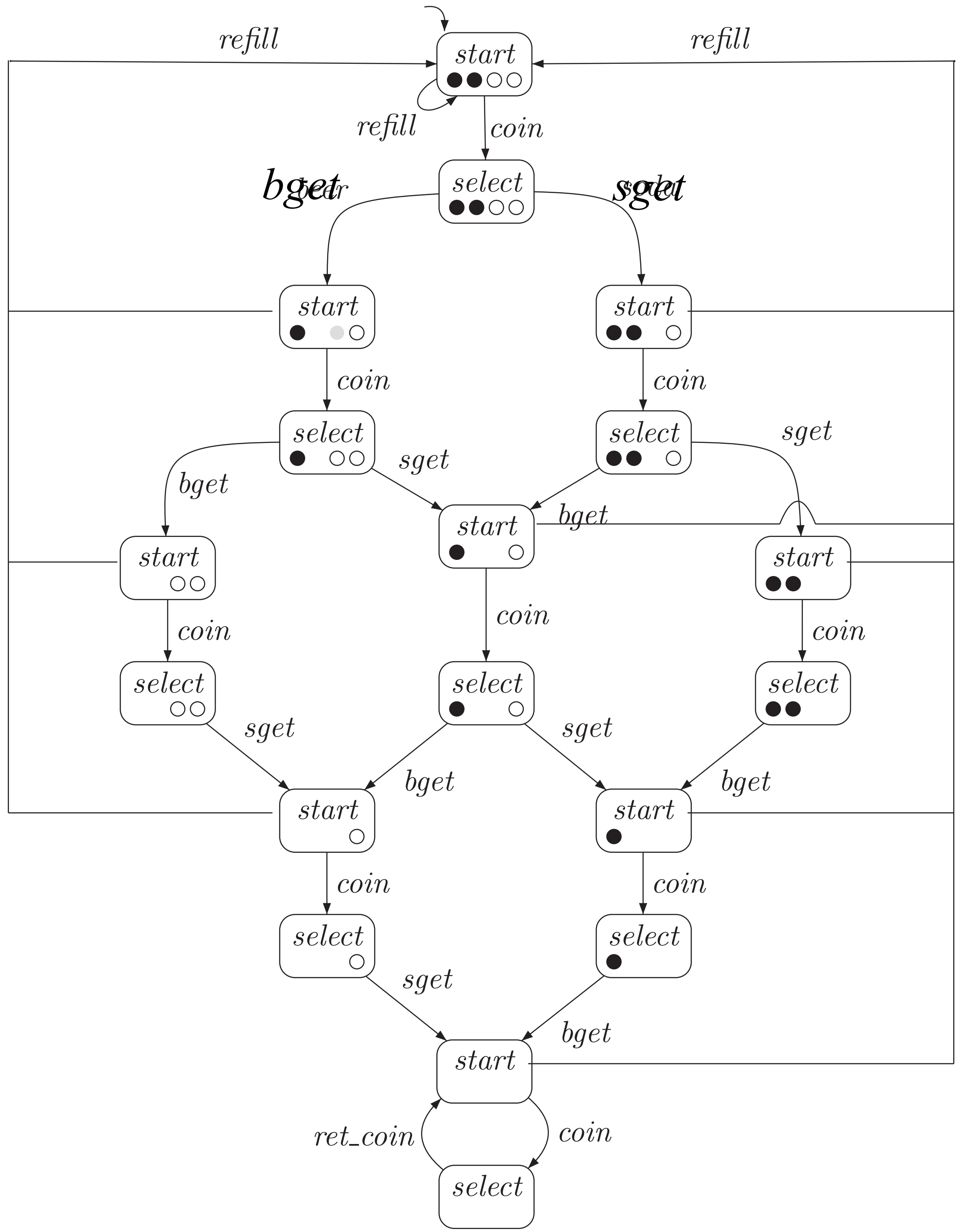
$$PG = (Loc, Act, Effect, \hookrightarrow, Loc_0, g_0)$$

over set Var of variables is the tuple $(S, Act, \longrightarrow, I, AP, L)$ where

- $S = Loc \times Eval(Var)$
- $\longrightarrow \subseteq S \times Act \times S$ is defined by the following rule (see remark below):

$$\frac{\ell \overset{g:\alpha}{\hookrightarrow} \ell' \quad \wedge \quad \eta \models g}{\langle \ell, \eta \rangle \xrightarrow{\alpha} \langle \ell', Effect(\alpha, \eta) \rangle}$$

- $I = \{\langle \ell, \eta \rangle \mid \ell \in Loc_0, \eta \models g_0\}$
- $AP = Loc \cup Cond(Var)$
- $L(\langle \ell, \eta \rangle) = \{\ell\} \cup \{g \in Cond(Var) \mid \eta \models g\}$.


refill
start
refill
refill
coin
bget
select
sget
start
start
coin
coin
select
select
bget
sget
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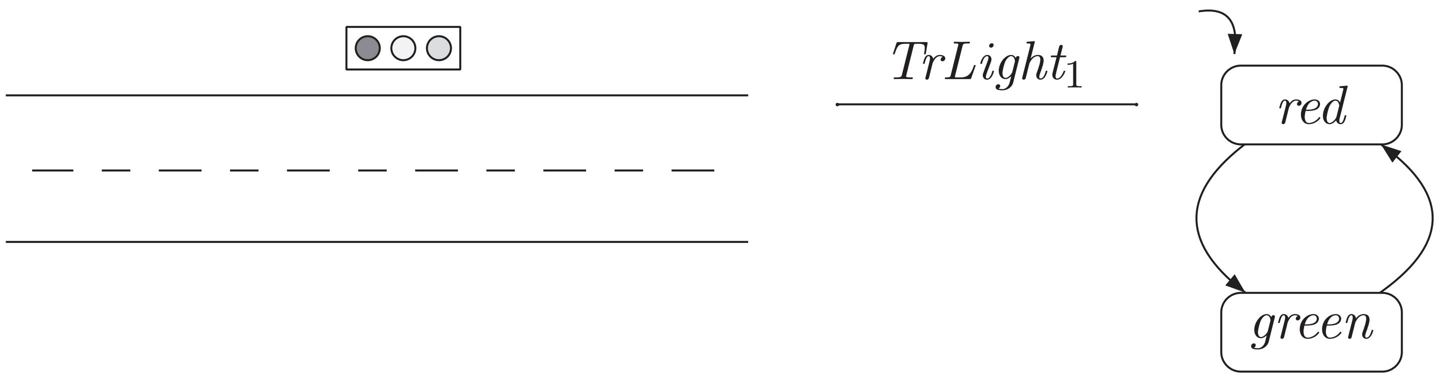
bget
start
start
coin
coin
coin
select
select
select
sget
sget
bget
bget
start
start
coin
coin
select
select
sget
bget
start
ret_coin
coin
select

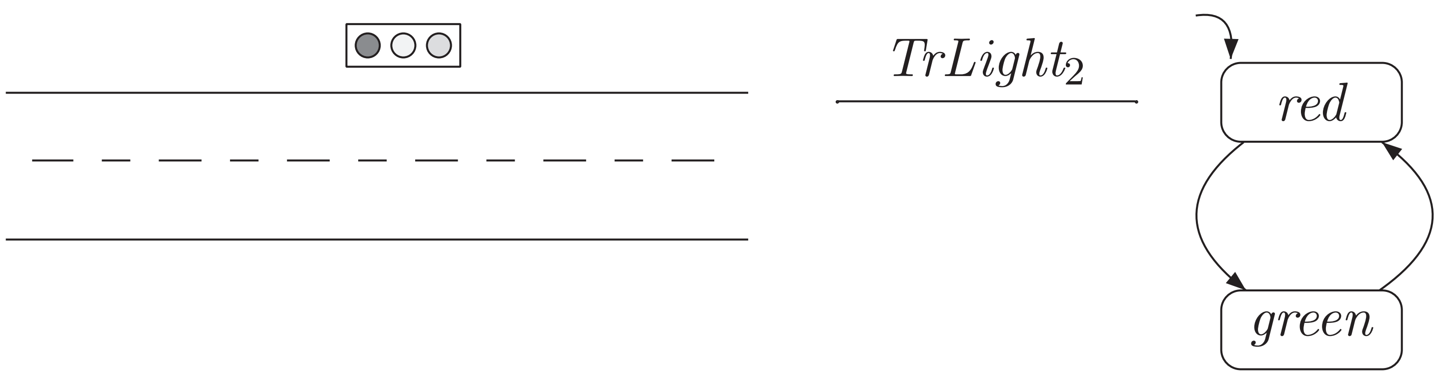
Parallelism and Communication

- define an operator, such that:
- $TS = TS_1 \mid\mid TS_2 \mid\mid \ \ldots \mid\mid TS_n$
- is a transition system that specifies the behavior of the parallel composition of transition systems TS_1 through TS_n.
- interleaving: the system is composed of a set of independent components.
  - example: two processes, P || Q
  - the order: PQPPQP..., or PPPQQQ, or PQPQPQ....

Concurrency and Interleaving

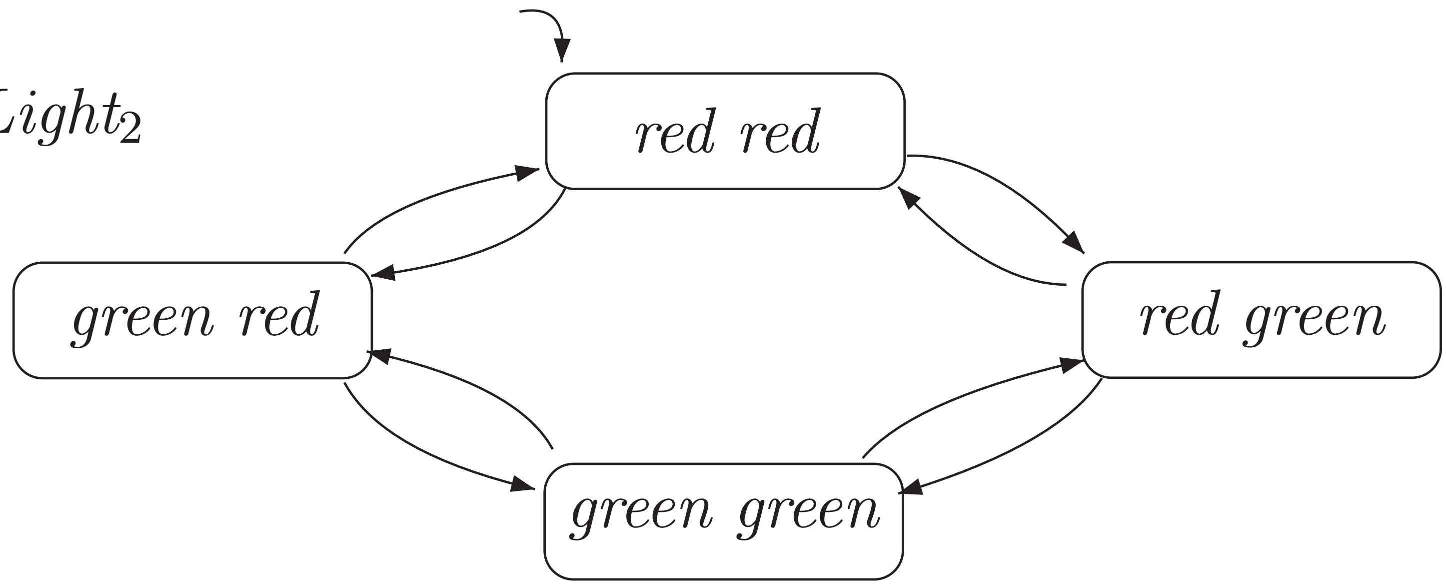
- Two Independent Traffic Lights----the transition systems of two traffic lights are nonintersecting (i.e., parallel) roads.





Two Independent Traffic Lights(cont)

$TrLight_1 \; ||| \; TrLight_2$



where ||| denotes the interleaving operator.

Interleaving of Transition Systems

- Let $TS_i = (S_i, Act_i, \rightarrow_i, I_i, AP_i, L_i)$ i=1, 2,

$TS_1 \mid\mid\mid TS_2$

$= (S_1 \times S_2, Act_1 \cup Act_2, \rightarrow, I_1 \times I_2, AP_1 \cup AP_2, L)$,

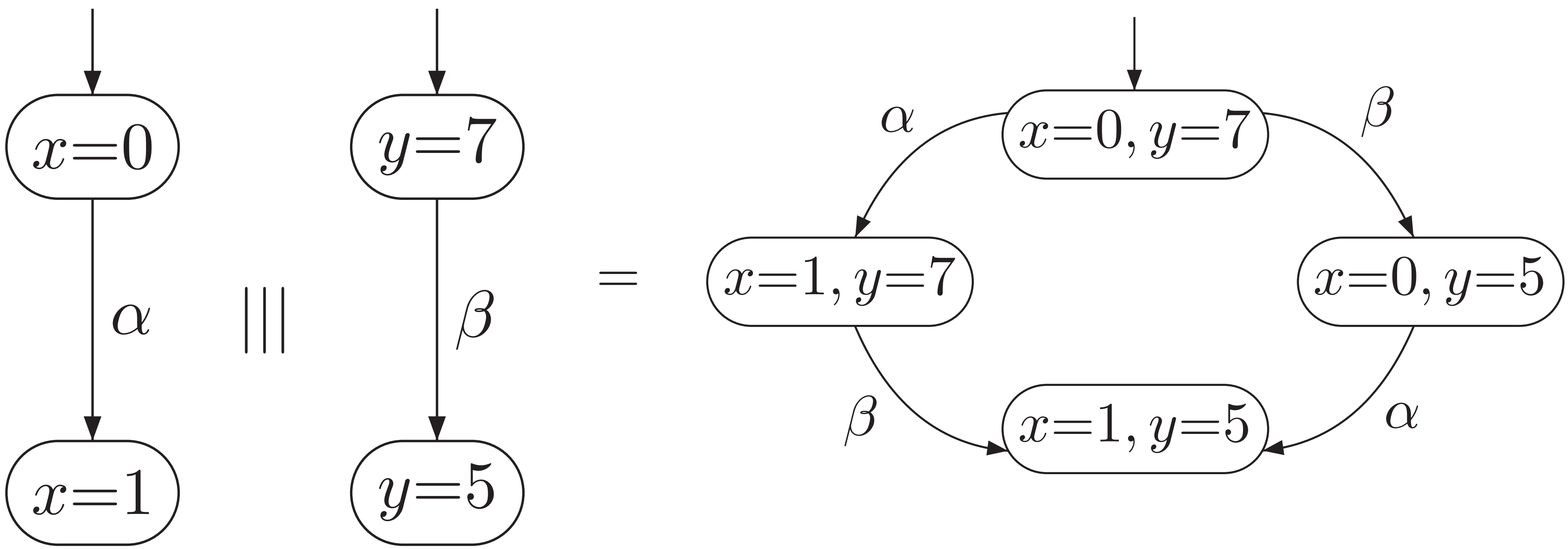
where $\rightarrow$

$$\frac{s_1 \xrightarrow{\alpha}_1 s_1'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle} \quad \text{and} \quad \frac{s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle}$$

- the labeling function is defined by $L(<s_1, s_2>) = L(s_1) \cup L(s_2)$.

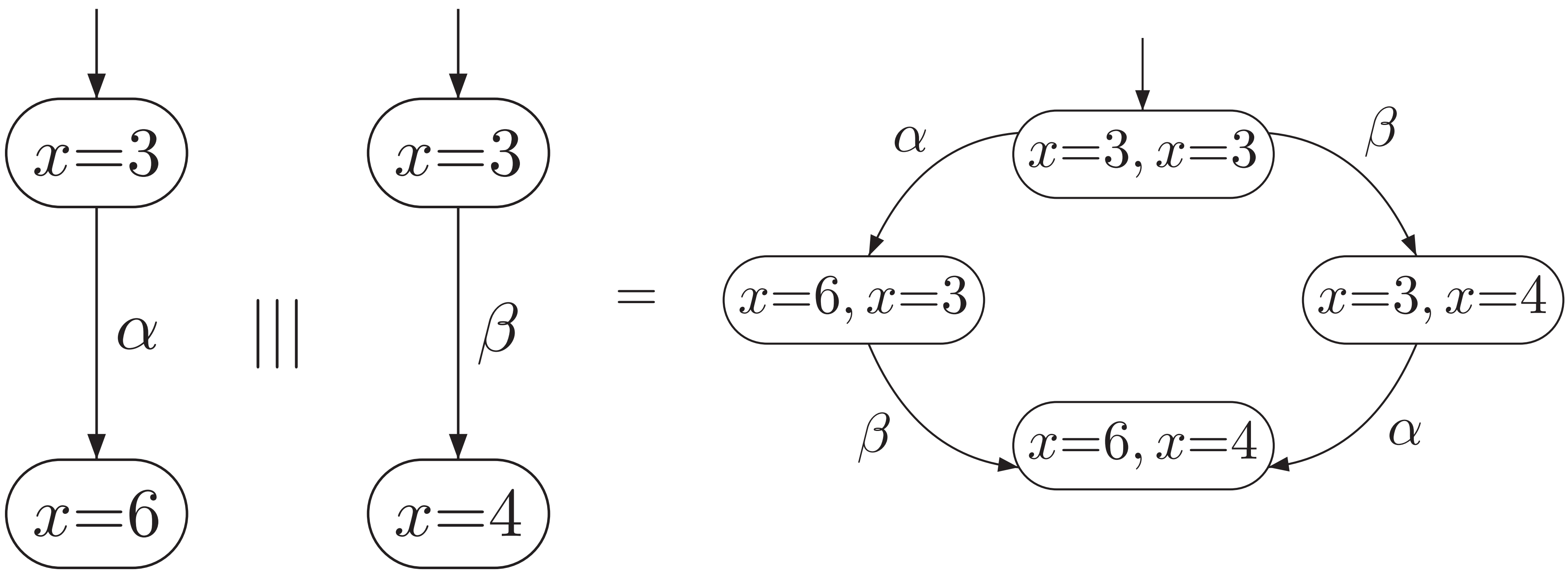
concurrent execution of independent activities.

$\underbrace{x := x + 1}_{=\alpha} \;|||\; \underbrace{y := y - 2}_{=\beta}.$ initially x = 0 and y = 7,



Communication via Shared variables

- Example: $\underbrace{x := 2 \cdot x}_{\text{action } \alpha} \;|||\; \underbrace{x := x + 1}_{\text{action } \beta}$
- Initial x=3
- TS(PG1) ||| TS(PG2)



problem

- the actions α and β access the shared variable x and therefore are competing.
- Solution: an interleaving operator will be defined on the level of program graphs
- The underlying transition system of the resulting program graph PG1 ||| PG2, i.e., TS(PG1 |||PG2) faithfully describes a parallel system whose components communicate via shared variables.

Interleaving of Program Graphs

Let $PG_i = (Loc_i, Act_i, \mathit{Effect}_i, \hookrightarrow_i, Loc_{0,i}, g_{0,i})$, for i=1, 2 be two program graphs over the variables Var_i. Program graph $PG_1 \,|||\, PG_2$ over $Var_1 \cup Var_2$ is defined by

$$PG_1 \,|||\, PG_2 \;=\; (Loc_1 \times Loc_2, Act_1 \uplus Act_2, \mathit{Effect}, \hookrightarrow, Loc_{0,1} \times Loc_{0,2}, g_{0,1} \wedge g_{0,2})$$

where $\hookrightarrow$ is defined by the rules:

$$\frac{\ell_1 \overset{g:\alpha}{\hookrightarrow}_1 \ell_1'}{\langle \ell_1, \ell_2 \rangle \overset{g:\alpha}{\hookrightarrow} \langle \ell_1', \ell_2 \rangle} \quad \text{and} \quad \frac{\ell_2 \overset{g:\alpha}{\hookrightarrow}_2 \ell_2'}{\langle \ell_1, \ell_2 \rangle \overset{g:\alpha}{\hookrightarrow} \langle \ell_1, \ell_2' \rangle}$$

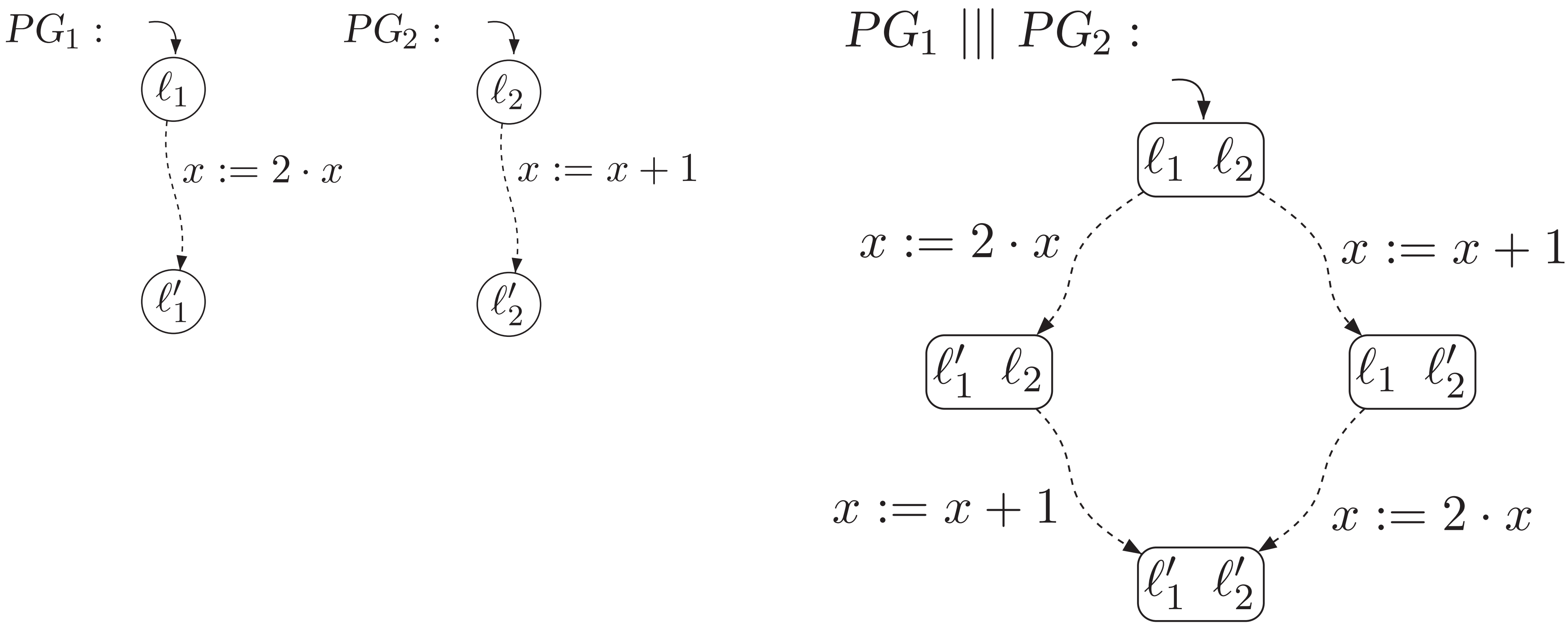
and $\mathit{Effect}(\alpha, \eta) \;=\; \mathit{Effect}_i(\alpha, \eta)$ if $\alpha \in Act_i$. ■

Interleaving of Program Graphs

- $Var_1 \cap Var_2 \neq \varnothing$------shared variables (sometimes called "global"), critical
- Var_1/Var_2 -----local variables of PG_1. noncritical
- Var_2/Var_1 -----local variables of PG_2. noncritical

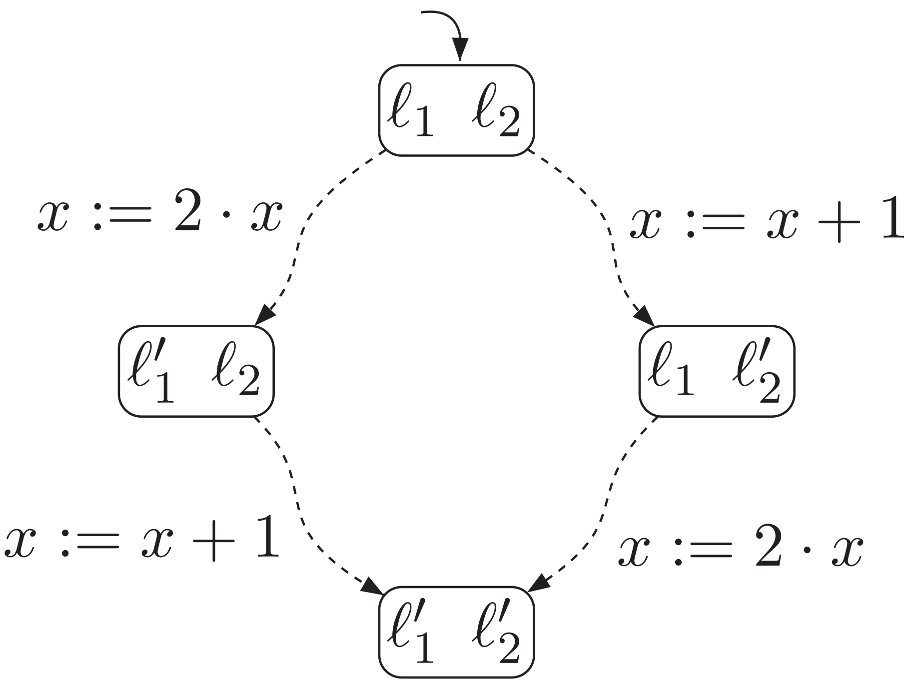
example

- x=x+1 ||| x:=2·x

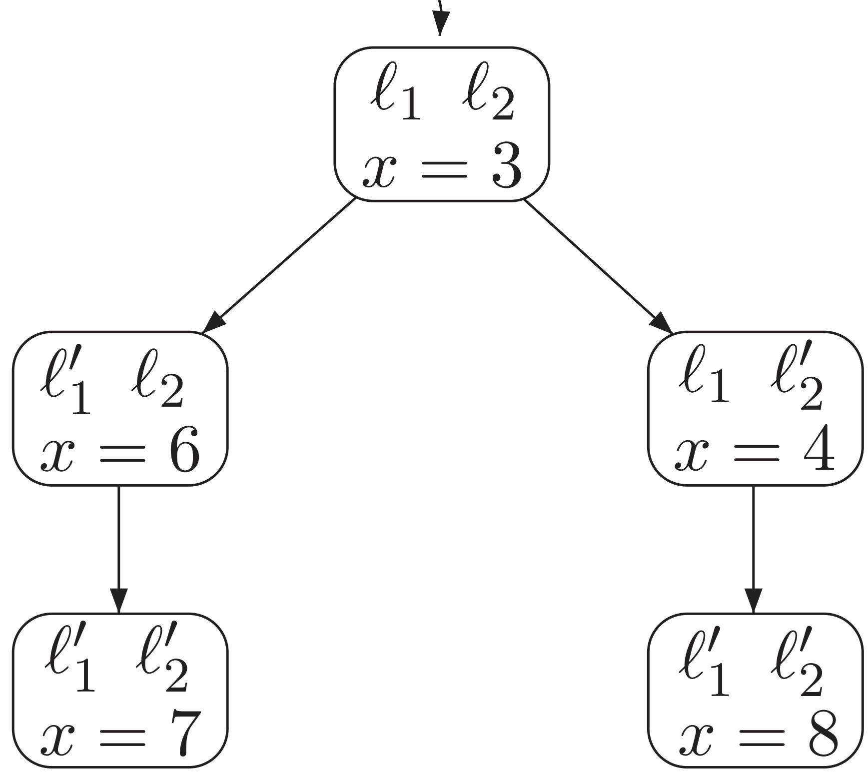


TS of example

$PG_1 \mid\mid\mid PG_2$:



$TS(PG_1 \mid\mid\mid PG_2)$



nondeterminism in a state of this transition system

- An “internal” nondeterministic choice within program graph PG_1 or PG_2,
- The interleaving of noncritical actions of PG_1 and PG_2, or
- The resolution of a contention between critical actions of PG_1 and PG_2 (concurrency).
  - The value of the shared variables depends on the order of executing Critical actions of PG_1 and PG_2.

Atomicity

- an action α with its effect being described by the statement sequence.
- declared *atomic* , surrounded by brackets<...>
- $x{:=}x+1; y=2x+1;$ if $x<12$ then $z{:=}(x-2)^2*y$;
  - $\mathrm{Effect}(\alpha,\eta)(x)= \eta(x)+1$
  - $\mathrm{Effect}(\alpha, \eta)(y)=2(\eta(x)+1)+1$
  - $\mathrm{Effect}(\alpha, \eta)(z)=\begin{cases} (\eta(x)+1-\eta(z))^2*2(\eta(x)+1)+1 & \text{if } \eta(x)+1<12 \\ \eta(z) & \text{otherwise} \end{cases}$

Mutual Exclusion with Semaphores

- share the binary semaphore y.
- y=0 indicates that the semaphore—the lock to get access to the critical section—is currently possessed by one of the processes.
- y=1, the semaphore is free.

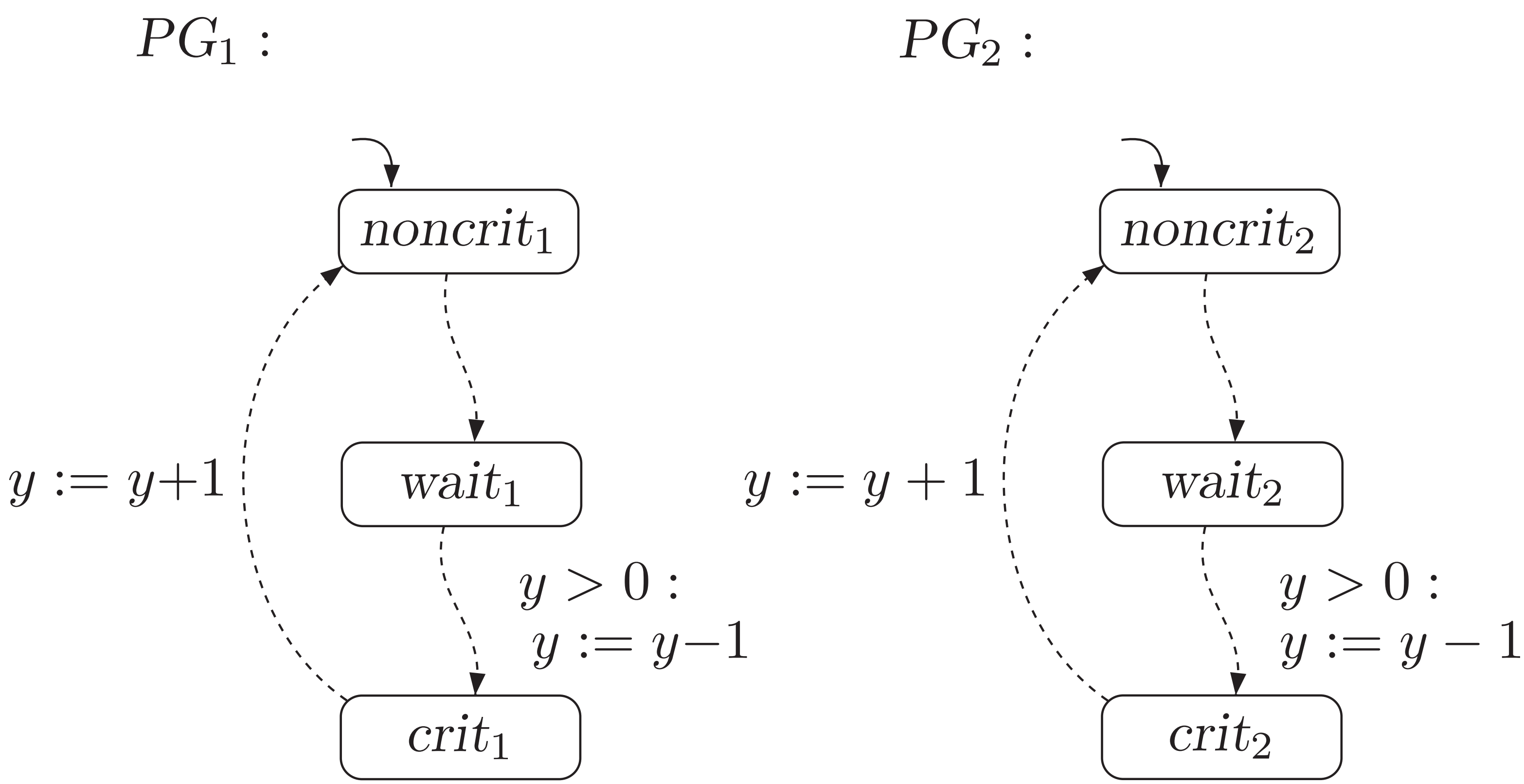
Mutual Exclusion with Semaphores (cont)

- Consider two simplified processes P_i, i=1,2 of the form:

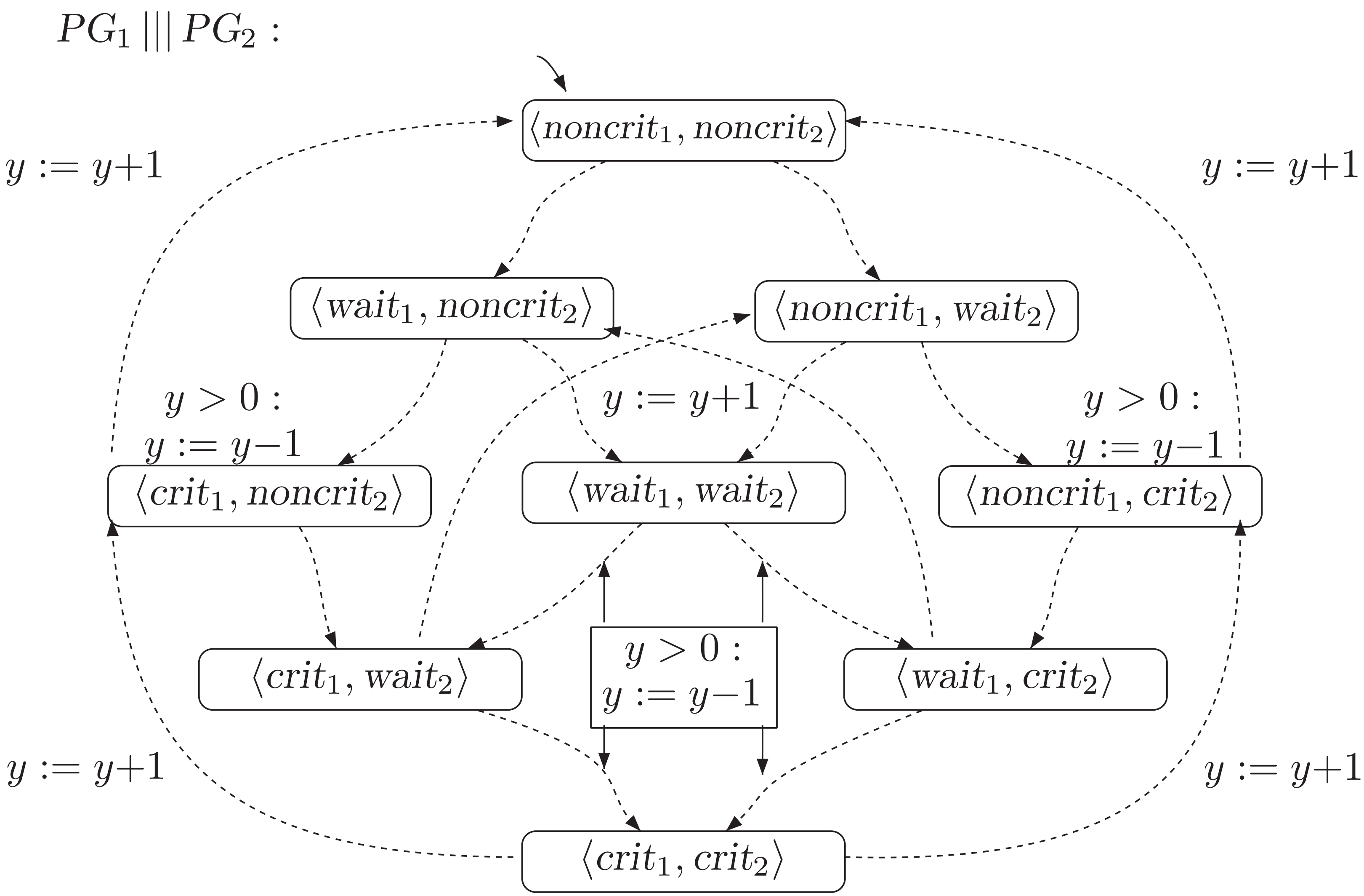
$$
\begin{array}{lll}
P_i & \textbf{loop forever} & \\
 & \vdots & \text{(* noncritical actions *)} \\
 & \textit{request} & \\
 & \text{critical section} & \\
 & \textit{release} & \\
 & \vdots & \text{(* noncritical actions *)} \\
 & \textbf{end loop} &
\end{array}
$$

Mutual Exclusion with Semaphores(cont)

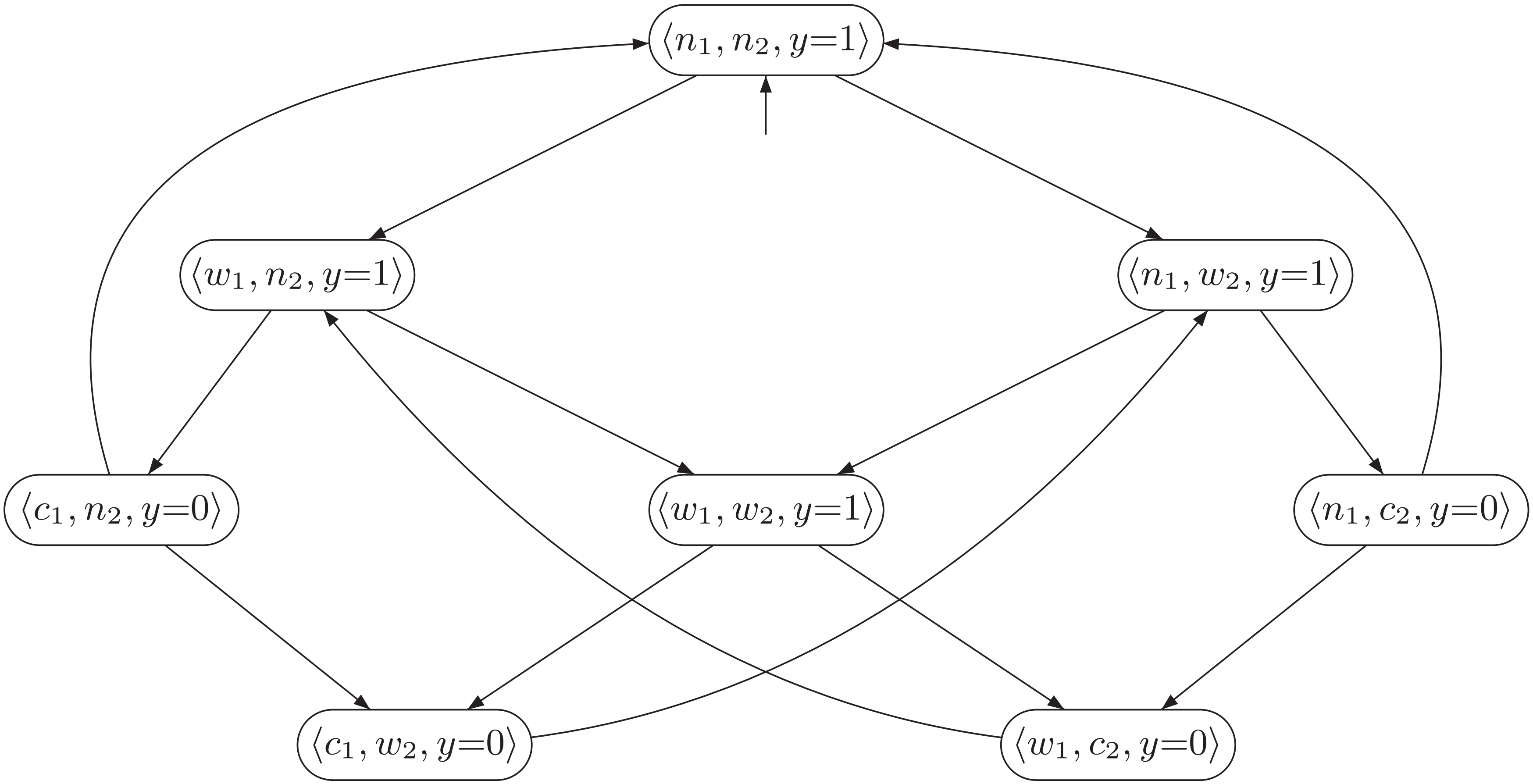
- PG_1 and PG_2.



Mutual Exclusion with Semaphores(cont)



Translation from PG into TS



Peterson's Mutual exclusion

- Shared variables b_1,b_2 and x.
- Initial b_1=b_2=false

P_1 **loop forever**

…

<b_1 := true; x := 2;>

wait until ($x = 1 \vee \neg b_2$)

do critical section **od**

b_1 := false

…

end loop

P_2 **loop forever**

…

<b_2 := true; x := 1;>

wait until ($x = 2 \vee \neg b_1$)

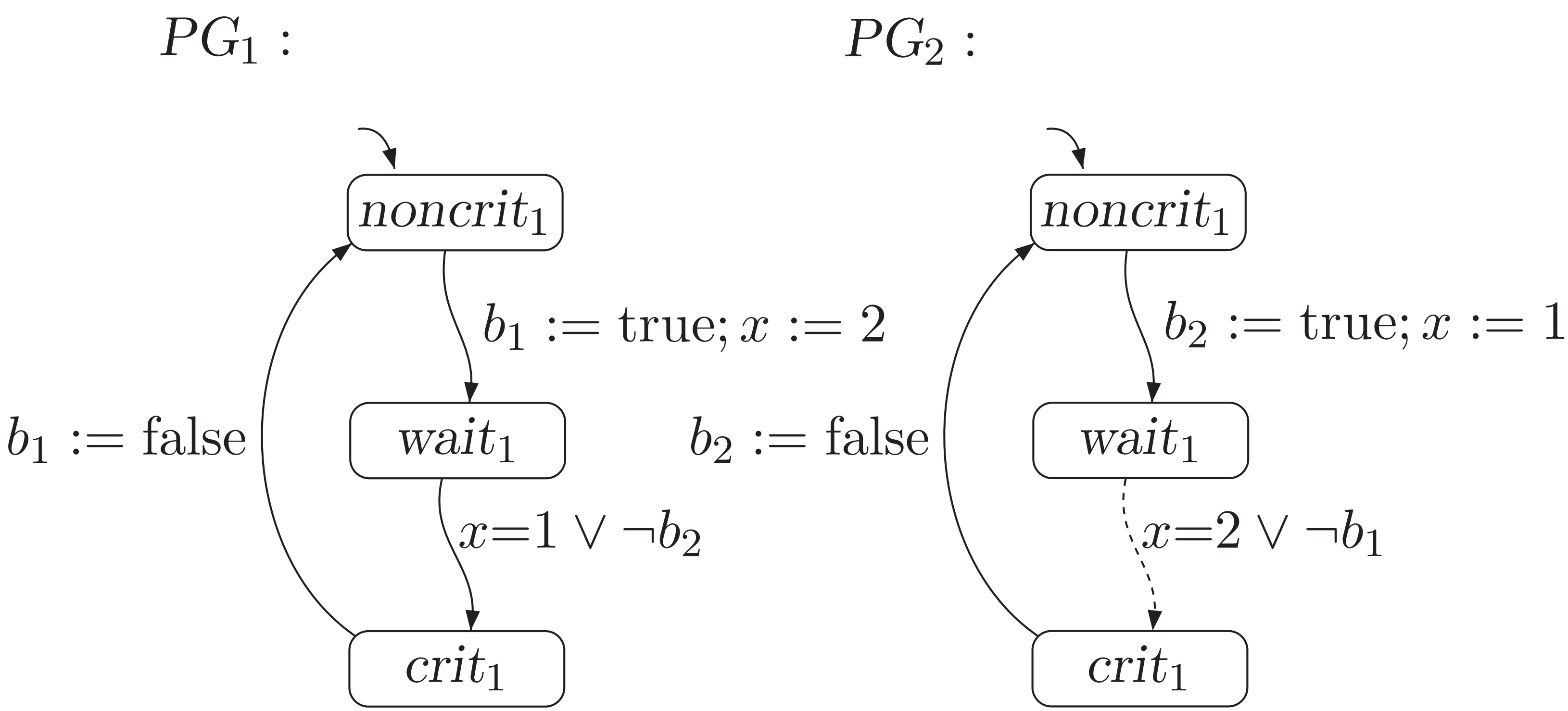
do critical section **od**

b_2 := false

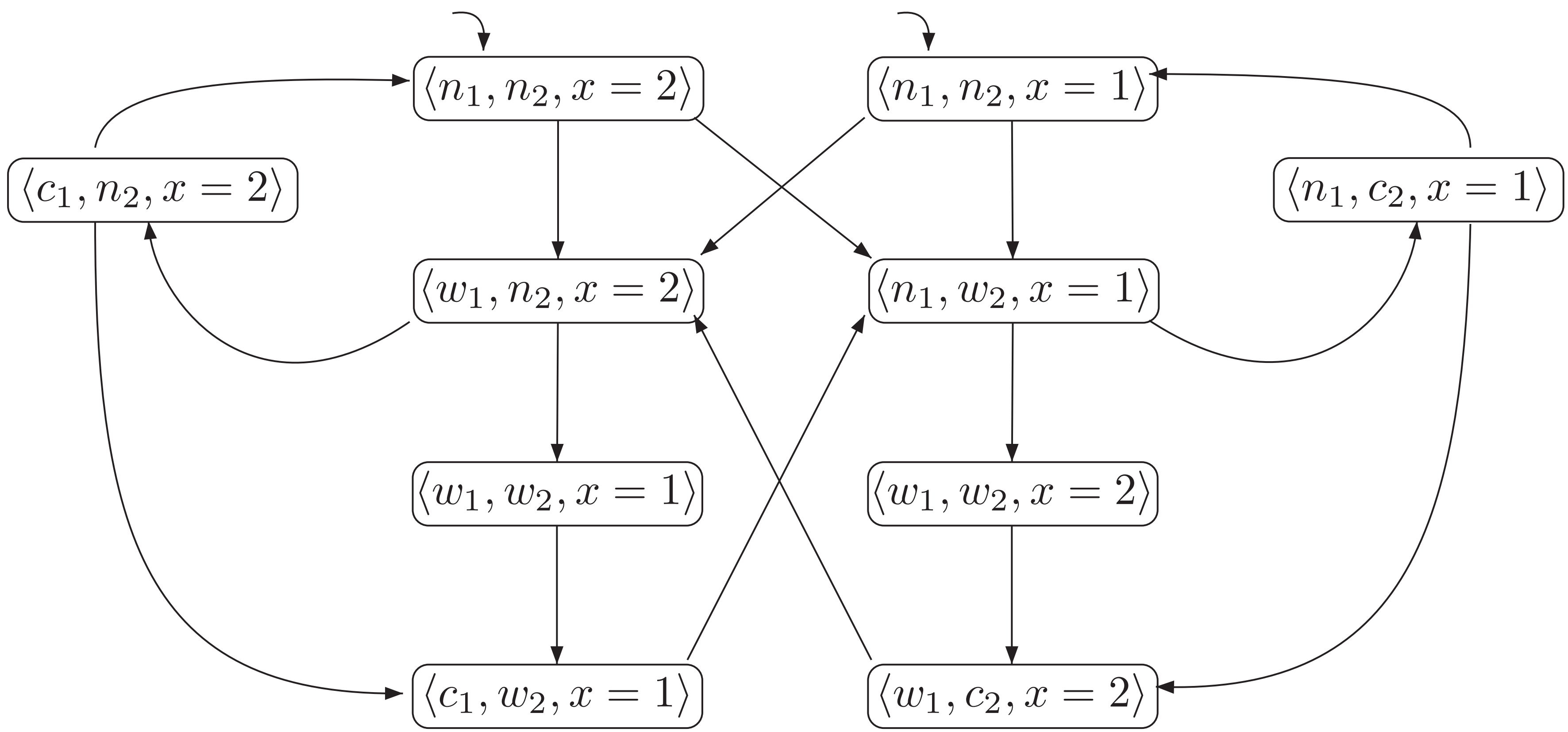
…

end loop

PG of these processes



TS of PG_1 ||| PG_2.



$\langle n_1, n_2, x = 2\rangle$
$\langle n_1, n_2, x = 1\rangle$
$\langle c_1, n_2, x = 2\rangle$
$\langle n_1, c_2, x = 1\rangle$
$\langle w_1, n_2, x = 2\rangle$
$\langle n_1, w_2, x = 1\rangle$
$\langle w_1, w_2, x = 1\rangle$
$\langle w_1, w_2, x = 2\rangle$
$\langle c_1, w_2, x = 1\rangle$
$\langle w_1, c_2, x = 2\rangle$

questions?

- if b_i:=true;x:=...in this order but in a nonatomic way, can this algorithm satisfy the mutual exclusion property?
  - » Yes
- if x:=...; b_i:=true, can this algorithm satisfy the mutual exclusion property?
  - » No.

handshaking

- Interleaving: completely autonomously, no communicative
- Shared-variable: communicates by shared variable, asynchronous
- Handshaking: synchronous fashion----they are both participating in this interaction at the same time.
  - Exchanged information can: integer, complex data structure.
  - Abstract: only communication (also called synchronization) actions are considered that represent the occurrence of a handshake
- Channel: first-in,first-out buffers that may contain messages.

Handshake (cont')

- H: handshake actions is distinguished with $\tau \notin H$
- Only if both participating processes are ready to execute the same handshake action, can message passing take place.
- actions in Act/ H are independent

Definition of handshaking
(Synchronous Message Passing)

Let $TS_i = (S_i, Act_i, \rightarrow_i, I_i, AP_i, L_i)$, i=1, 2 TSs, and $H \subseteq Act_1 \cap Act_2$ with $\tau \notin H$.

$TS_1 ||_H TS_2$ is defined :

$$TS_1 ||_H TS_2 = (S_1 \times S_2, Act_1 \cup Act_2, \rightarrow, I_1 \times I_2, AP_1 \cup AP_2, L)$$

where $L(s_1, s_2) = L_1(s_1) \cup L_2(s_2)$,

$\rightarrow$ is defined:

Definition of handshaking(cont')

(Synchronous Message Passing)

- interleaving for $\alpha \notin H$:

$$\frac{s_1 \xrightarrow{\alpha}_1 s_1'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle} \qquad \frac{s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle}$$

- handshaking for $\alpha \in H$:

$$\frac{s_1 \xrightarrow{\alpha}_1 s_1' \quad \wedge \quad s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2' \rangle}$$

$TS_1 \,||\, TS_2 = TS_1 \,||_H\, TS_2$ for $H = Act_1 \cap Act_2$

- **Empty Set of Handshake Actions**
  - $TS_1 \,||_{\varnothing}\, TS_2 = TS_1 \,|||\, TS_2$.
- Handshaking is commutative
  - $TS_1 \,||_H\, TS_2 = TS_2 \,||_H\, TS_1$
- if $H \subseteq Act_1 \cap \ldots \cap Act_n$,
  - $TS = TS_1 \,||_H\, TS_2 \,||_H \ldots ||_H\, TS_n$, TS is associative

processes communicate in a pairwise

- $TS_1 \| \ldots \| TS_n$ denote the parallel
- TS_i and TS_j $(0 < i \neq j \leq n)$ synchronize , $H_{i,j} = Act_i \cap Act_j$ such that $H_{i,j} \cap Act_k = \varnothing$ for $k \notin \{i, j\}$. $\tau \notin H_{i,j}$.

- for $\alpha \in Act_i \setminus (\bigcup\limits_{\substack{0<j\leqslant n \\ i\neq j}} H_{i,j})$ and $0 < i \leqslant n$:

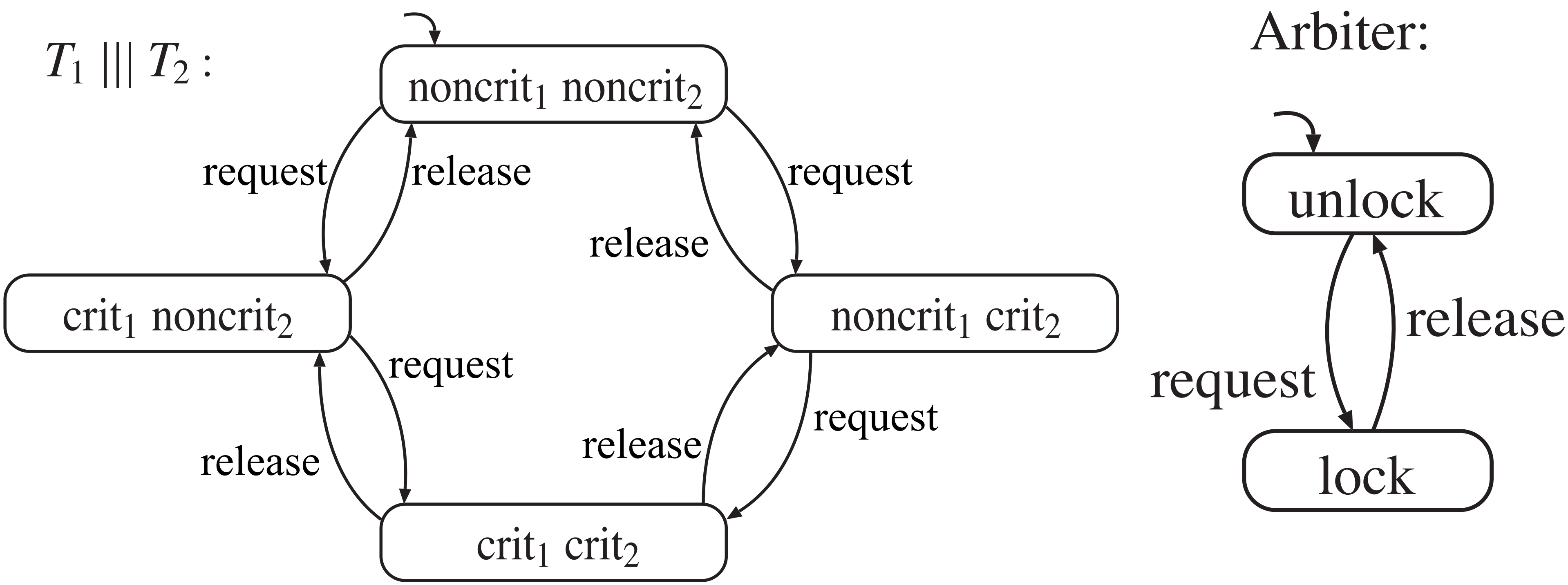
$$\frac{s_i \xrightarrow{\alpha}_i s_i'}{\langle s_1, \ldots, s_i, \ldots, s_n \rangle \xrightarrow{\alpha} \langle s_1, \ldots, s_i', \ldots s_n \rangle}$$

- for $\alpha \in H_{i,j}$ and $0 < i < j \leqslant n$:

$$\frac{s_i \xrightarrow{\alpha}_i s_i' \quad \wedge \quad s_j \xrightarrow{\alpha}_j s_j'}{\langle s_1, \ldots, s_i, \ldots, s_j, \ldots, s_n \rangle \xrightarrow{\alpha} \langle s_1, \ldots, s_i', \ldots, s_j', \ldots, s_n \rangle}$$

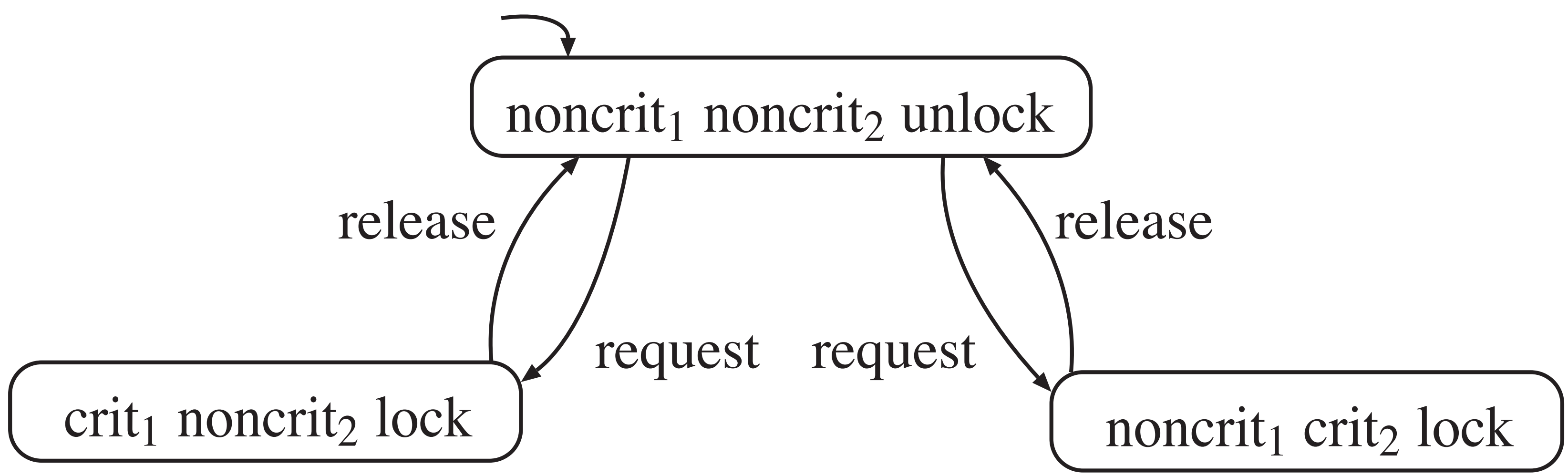
Example: Mutual exclusion by handshaking

- Add a arbiter
  - $TS_{Arb} = (TS_1 \mid\mid\mid TS_2) \mid\mid Arbiter$



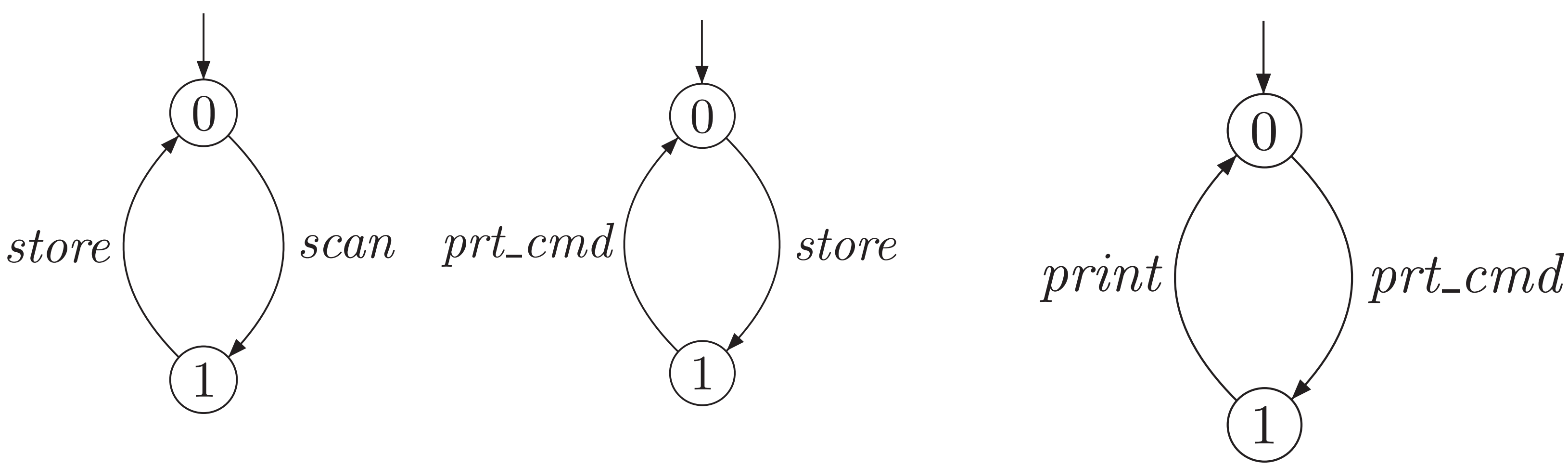
Example: Mutual exclusion by handshaking(cont')

$(T_1 \,|||\, T_2) \,||\, \text{Arbiter}:$



Example: booking system

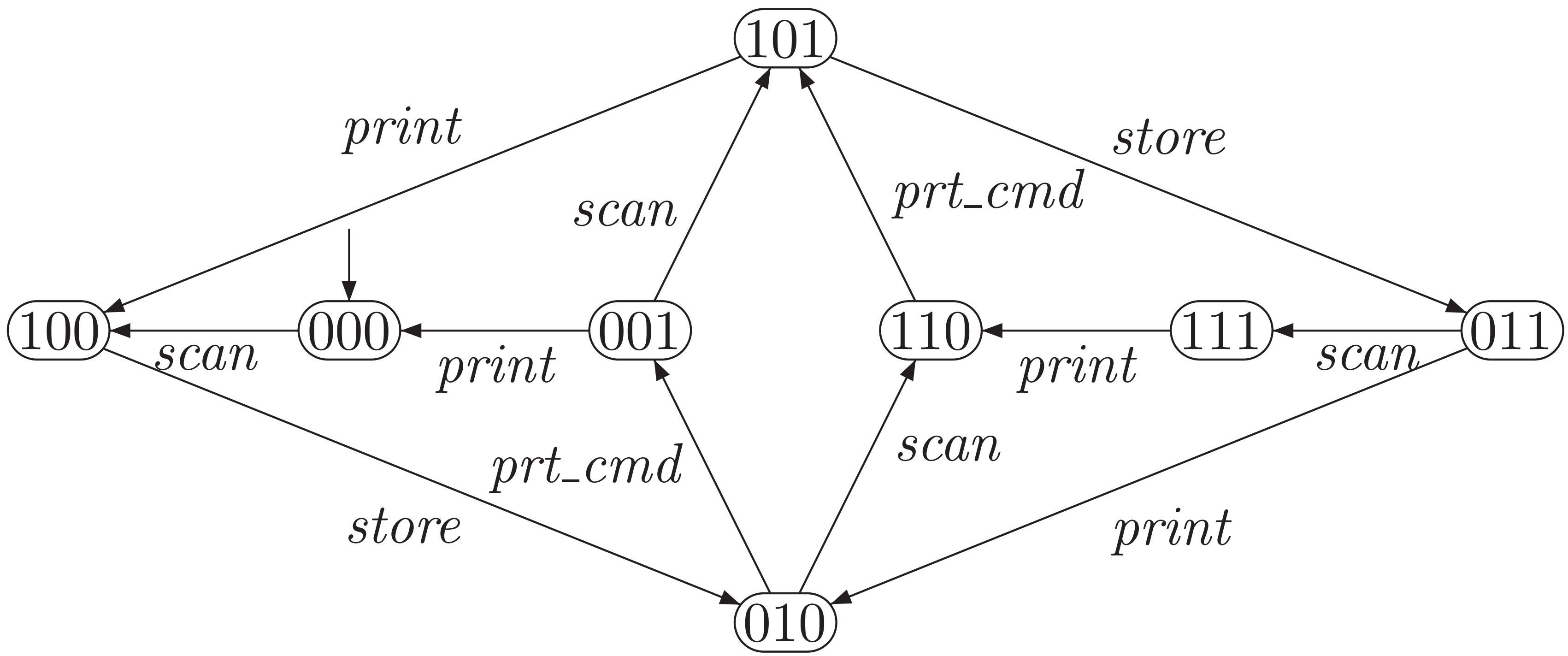
- Three processes:
  - The bar code reader :BCR
  - The actual booking program :BP
  - The printer: Printer



BCR BP Printer

Example: booking system(cont')

- The complete system is given:
  - BCR || BP || Printer

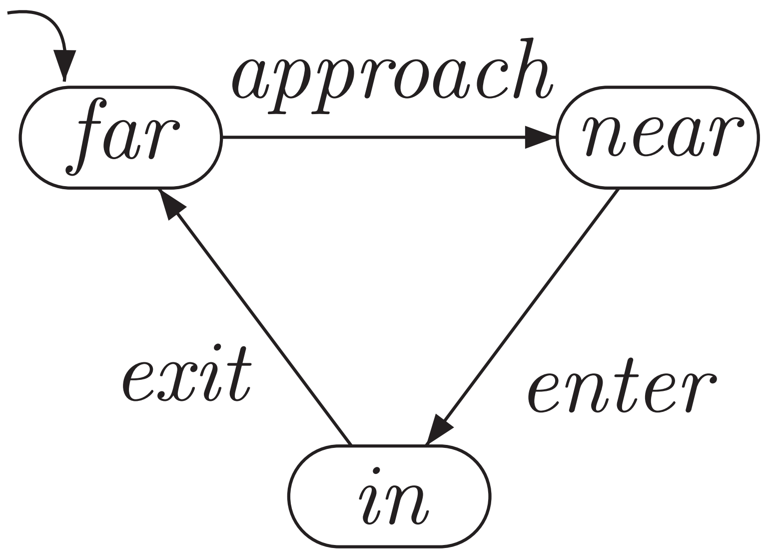


Example railroad crossing

- Requirement:
  - a train is approaching closes the gates, and only opens these gates after the train crossed the road.
  - The gates are always closed when the train is crossing the road.
- Three components: Train, Gate and Controller
  - Train || Gate || Controller

Example railroad crossing(cont')

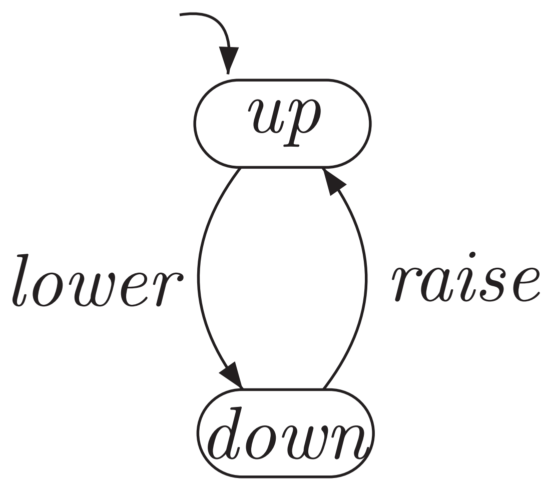
- Train: has the same direction



Train

Example railroad crossing(cont')

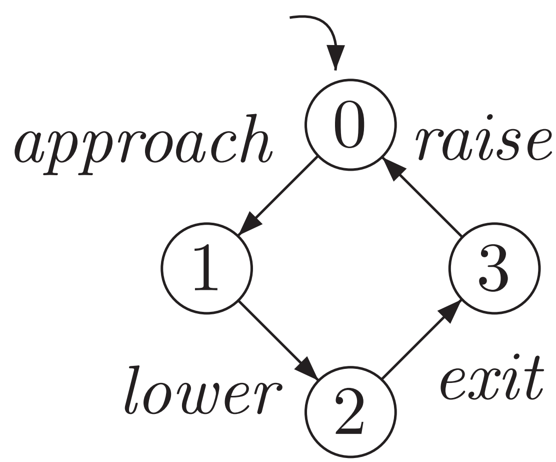
- Gate



Gate

Example railroad crossing(cont')

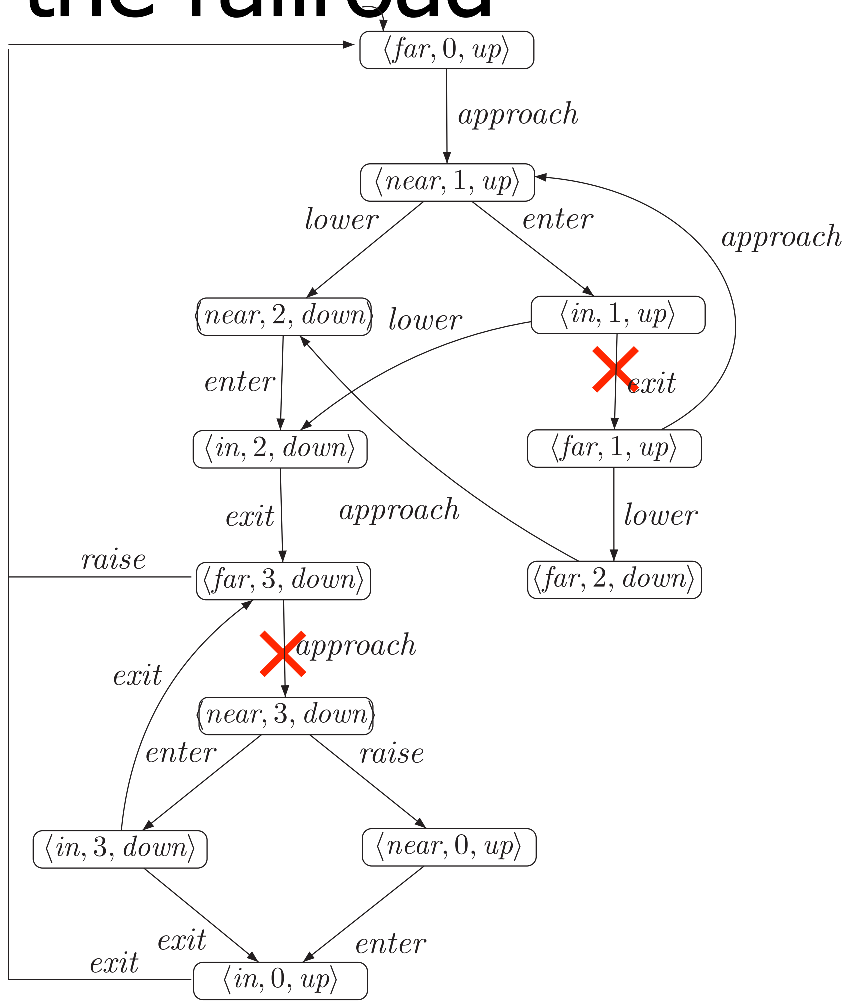
- Controller: The state changes stand for handshaking with the trains and the Gate (the Controller causes the gate to close or to open, respectively).



Controller

Example railroad crossing(cont')

- TS for the railroad



Channel systems

- Channels: first-in, first-out buffers that may contain messages.
- Constraint: channel systems that are closed.
- PG_i represents a processes from P_1 to P_n.
  - Transition:
    - conditional
    - communicative action’s transition.
      - c!v transmit the value v along channel c,
      - c?x receive a message via channel c and assign it to variable x.

Type

- Comm $=\{c!v, c?x \mid c \in$ Chan, $v \in$ dom(c), $x \in$ Var with dom(x) $\supseteq$ dom(c) $\}$
  - Where Chan is a finite set of channels with typical element c.

Capacity

- Channel has a (finite or infinite) capacity cap(c) $\in 2^{\mathbb{N}} \cup \{\infty\}$
  - cap(c)= ∞
  - cap(c)=0: handshaking (simultaneous transmission) plus the exchange of some data.
  - cap(c)>0:sending and reading of the same message take place at different moments. This is called asynchronous message passing.

Channel system (CS)

A program graph over $(Var, Chan)$ is a tuple

$$PG = (Loc, Act, Effect, \hookrightarrow, Loc_0, g_0)$$

$$\hookrightarrow \quad \subseteq \quad Loc \times (Cond(Var) \times (Act \cup Comm) \times Loc.$$

A channel system CS over (*Var*, *Chan*) consists of program graphs PG_i over (Var_i, *Chan*) (for $1 \leq i \leq n$) with $Var = \bigcup_{1 \leq i \leq n} Var_i$. We denote $CS = [PG_1 \mid \ldots \mid PG_n]$.

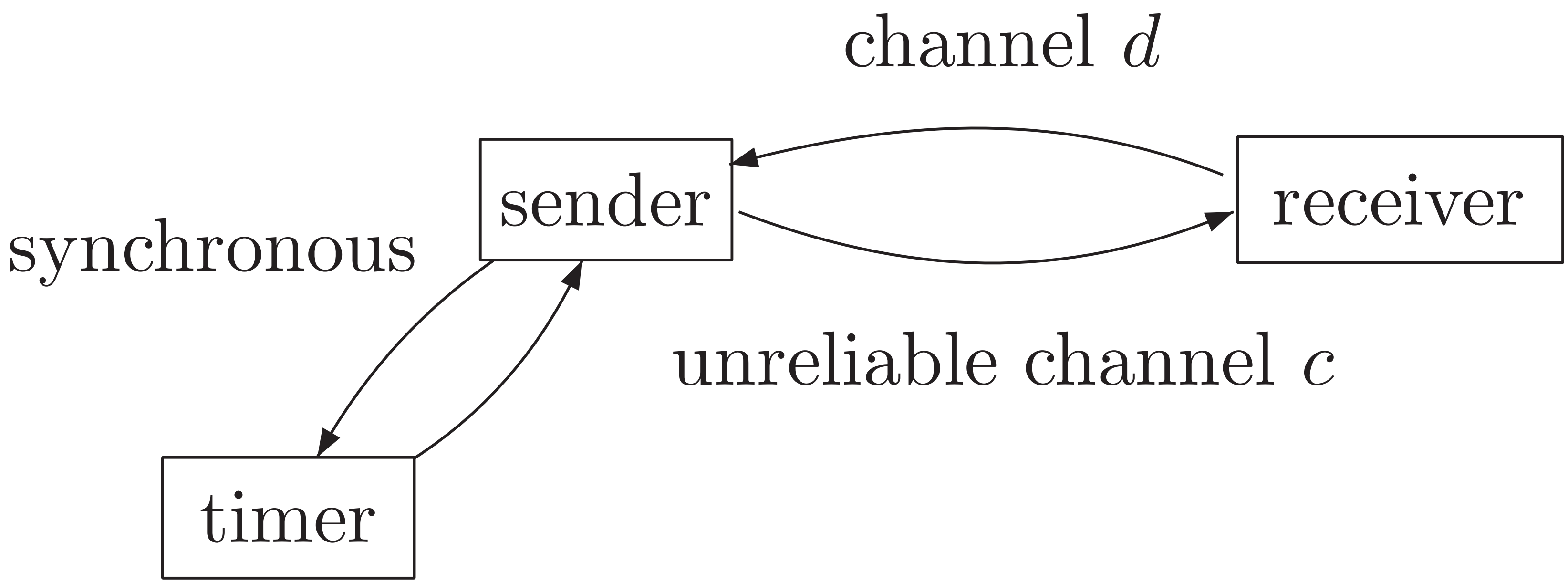
Transition of CS

- Conditional transition: $\ell \overset{g:\alpha}{\hookrightarrow} \ell'$
- Conditional transition with communication actions
  - $\ell \overset{g:c!v}{\hookrightarrow} \ell'$:for sending v along c
  - $\ell \overset{g:c?x}{\hookrightarrow} \ell'$:for receiving a message along c
    - Handshaking :cap(c)=0
    - Asynchronous message passing: cap(c)≠0
      - Send a message to a channel iff c is not full
      - receiving a message from a channel iff c is not

Transition of CS(cont)

	executable if ...	effect
$c!v$	c is not "full"	$Enqueue(c, v)$
$c?x$	c is not empty	$\langle x := Front(c)\,;\; Dequeue(c)\rangle;$

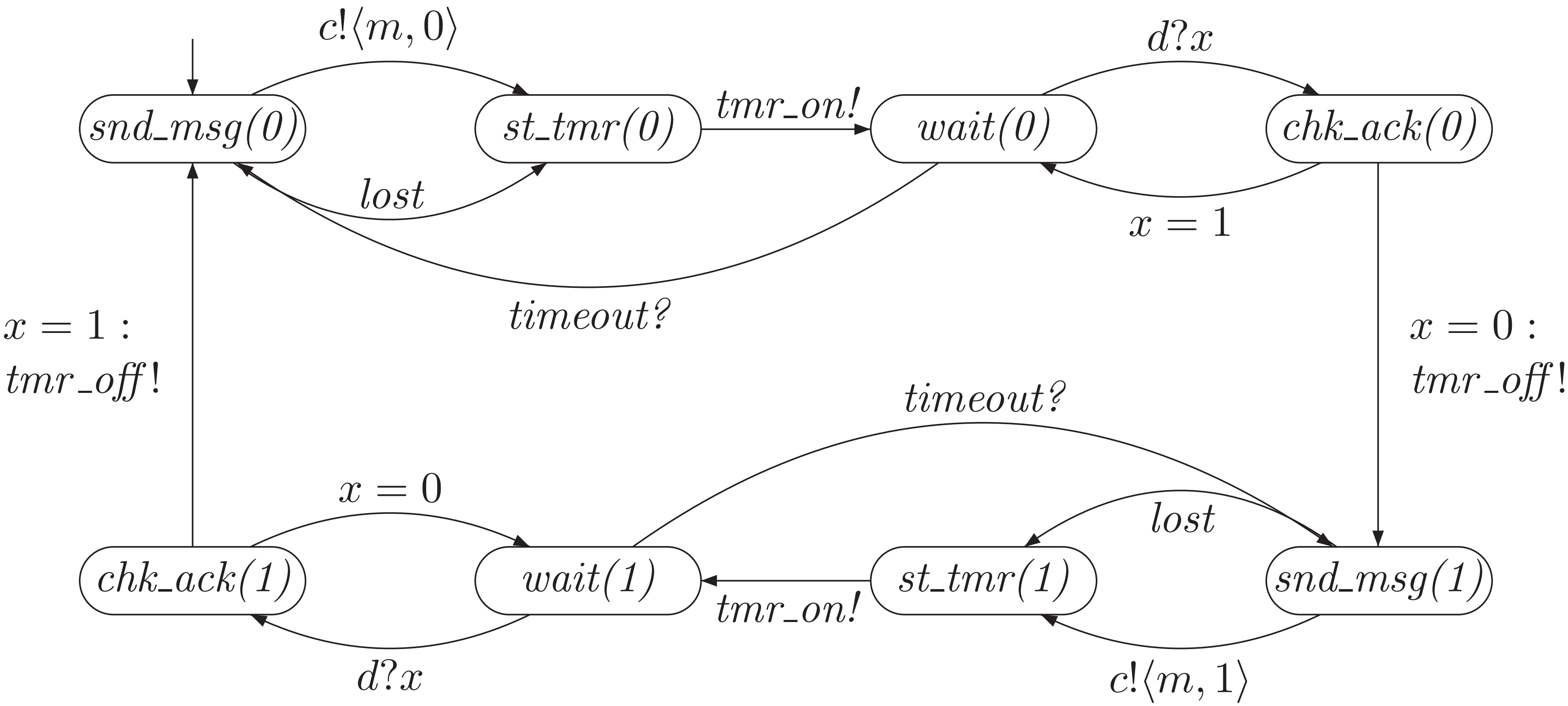
Example: Alternating Bit Protocol



Example: Alternating Bit Protocol (ABP) cont

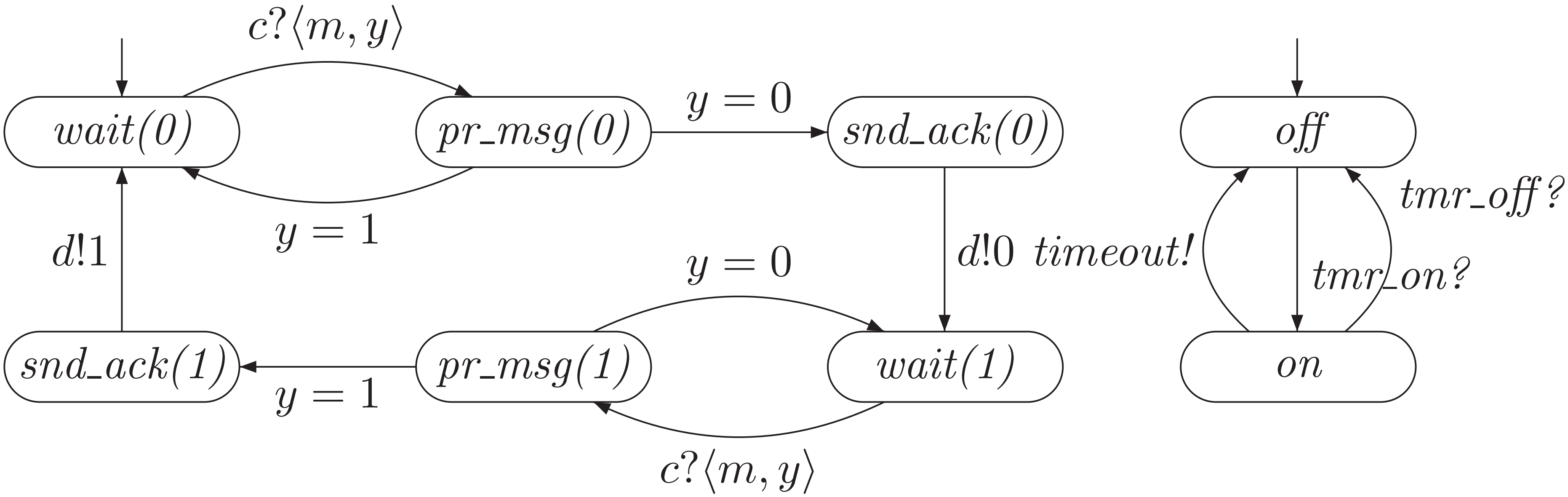
- S sends the successive messages m_0,m_1, . . . together with control bits b_0, b_1, . . . over channel c to R.
  - $<m_0, 0>$, $<m_1, 1>$, $<m_2, 0>$, $<m_3, 1>$, . . .

Example: Alternating Bit Protocol



Program graph of ABP sender S.

Example: Alternating Bit Protocol



Program graph of ABP receiver R

Timer.

Example: Alternating Bit Protocol (ABP) cont

- The complete system :
  - Chan={ c, d, tmr_on, tmr_off , timeout } and Var = { x, y,m_i }
  - ABP = [S | Timer | R]

TS of a channel system(cont')

- Term:
  - $\xi(c) = v_1\, v_2\, .\ .\ .\ v_k$ (where cap(c)≥k), len($\xi(c)$) = k
  - $\xi[c := v_1 \ldots v_k](c') = \begin{cases} \xi(c') & \text{if } c \neq c' \\ v_1 \ldots v_k & \text{if } c = c'. \end{cases}$
  - $\xi_0(c)$ =ε for any channel c. Let len(ε) = 0.
- Actions: consists of actions $\alpha \in Act_i$ of component PG_i and the distinguished symbol τ representing all communication actions in which data is exchanged.

TS of a channel system

- TS(CS)
  - States of TS(CS):
    - $<l_1, \ldots, l_n, \eta, \xi>$
    - η keeps track of the current values of the variables, and
    - ξ records the current content of the various channels
    - Initial $l_i \in Loc_{0,i}$ must be initial and variable evaluation η must satisfy the initial condition $g_{0,i}$.
    - every channel is initially assumed to be empty, denoted ε.

TS of a channel system(cont')

Let $CS = [PG_1 \mid \ldots \mid PG_n]$ be a channel system over $(Chan, Var)$ with

$$PG_i = (Loc_i, Act_i, Effect_i, \hookrightarrow_i, Loc_{0,i}, g_{0,i}), \qquad \text{for } 0 < i \leqslant n.$$

The transition system of CS, denoted $TS(CS)$, is the tuple $(S, Act, \rightarrow, I, AP, L)$ where:

- $S \;=\; (Loc_1 \times \ldots \times Loc_n) \times Eval(Var) \times Eval(Chan)$,
- $Act \;=\; \biguplus_{0<i\leqslant n} Act_i \;\uplus\; \{\tau\}$,
- $\rightarrow$ is defined by the rules of Figure 2.20 (page 61),
- $I \;=\; \left\{ \langle \ell_1, \ldots, \ell_n, \eta, \xi_0 \rangle \;\middle|\; \forall 0 < i \leqslant n.\ (\ell_i \in Loc_{0,i} \;\wedge\; \eta \models g_{0,i}) \right\}$,
- $AP \;=\biguplus_{0<i\leqslant n} Loc_i \;\uplus\; Cond(Var)$,
- $L(\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle) \;=\; \{\ell_1, \ldots, \ell_n\} \;\cup\; \{ g \in Cond(Var) \mid \eta \models g \}$.

TS of a channel system(cont')

- interleaving for $\alpha \in Act_i$:

$$\frac{\ell_i \overset{g:\alpha}{\hookrightarrow} \ell_i' \quad \wedge \quad \eta \models g}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\alpha} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta', \xi \rangle}$$

  where $\eta' = \mathit{Effect}(\alpha, \eta)$.

- asynchronous message passing for $c \in \mathit{Chan}$, $\mathit{cap}(c) > 0$:

  – receive a value along channel c and assign it to variable x:

$$\frac{\ell_i \overset{g:c?x}{\hookrightarrow} \ell_i' \ \wedge\ \eta \models g \ \wedge\ \mathit{len}(\xi(c)) = k > 0 \ \wedge\ \xi(c) \ = \ v_1 \ldots v_k}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta', \xi' \rangle}$$

  where $\eta' = \eta[x := v_1]$ and $\xi' = \xi[c := v_2 \ldots v_k]$.

  – transmit value $v \in \mathit{dom}(c)$ over channel c:

$$\frac{\ell_i \overset{g:c!v}{\hookrightarrow} \ell_i' \ \wedge\ \eta \models g \ \wedge\ \mathit{len}(\xi(c)) = k < \mathit{cap}(c) \ \wedge\ \xi(c) \ = \ v_1 \ldots v_k}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta, \xi' \rangle}$$

  where $\xi' = \xi[c := v_1\, v_2 \ldots v_k\, v]$.

- synchronous message passing over $c \in \mathit{Chan}$, $\mathit{cap}(c) = 0$:

$$\frac{\ell_i \overset{g_1:c?x}{\hookrightarrow} \ell_i' \ \wedge\ \eta \models g_1 \ \wedge\ \eta \models g_2 \ \wedge\ \ell_j \overset{g_2:c!v}{\hookrightarrow} \ell_j' \ \wedge\ i \neq j}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_j, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_j', \ldots, \ell_n, \eta', \xi \rangle}$$

  where $\eta' = \eta[x := v]$.

NanoPromela

- the core input language for the SPIN
- support
  - communication by shared variables
  - message passing along either synchronous or buffer FIFO-channels
- the formal semantics can be provided by means of a *channel system*.
- channel system can be unfolded into transition system.
- no use action name,but specify the effect of action

syntax of nanoPromela

$$
\begin{array}{lll}
\mathsf{stmt} & ::= & \texttt{skip} \;\mid\; x := \mathsf{expr} \;\mid\; c?x \;\mid\; c!\mathsf{expr} \;\mid \\
& & \mathsf{stmt}_1 \,;\, \mathsf{stmt}_2 \;\mid\; \texttt{atomic}\{\mathsf{assignments}\} \;\mid \\
& & \mathbf{if} \quad :: g_1 \Rightarrow \mathsf{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \mathsf{stmt}_n \quad \mathbf{fi} \quad \mid \\
& & \mathbf{do} \quad :: g_1 \Rightarrow \mathsf{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \mathsf{stmt}_n \quad \mathbf{do}
\end{array}
$$

peterson's mutual exclusion algorithm

$$
\begin{array}{llll}
\textbf{do} & :: & \text{true} \Rightarrow & \texttt{skip}; \\
 & & & \texttt{atomic}\{b_1 := \text{true}; x := 2\}; \\
 & & & \textbf{if} \;\; :: \; (x = 1) \vee \neg b_2 \;\; \Rightarrow \;\; \mathit{crit}_1 := \text{true} \;\; \textbf{fi} \\
 & & & \texttt{atomic}\{\mathit{crit}_1 := \text{false}; b_1 := \text{false}\} \\
\textbf{od} & & &
\end{array}
$$

Vending Machine

$$
\begin{array}{lll}
\textbf{do} & :: & \text{true} \Rightarrow \\
& & \quad \texttt{skip}; \\
& & \quad \begin{array}{lllll}
\textbf{if} & :: & nsoda > 0 & \Rightarrow & nsoda := nsoda - 1 \\
& :: & nbeer > 0 & \Rightarrow & nbeer := nbeer - 1 \\
& :: & nsoda = nbeer = 0 & \Rightarrow & \texttt{skip}
\end{array} \\
& & \quad \textbf{fi} \\
& :: & \text{true} \Rightarrow \texttt{atomic}\{nbeer := max; nsoda := max\} \\
\textbf{od} & &
\end{array}
$$

Synchronous Parallelism

Let $TS_i = (S_i, Act, \rightarrow_i, I_i, AP_i, L_i)$, i=1, 2,

be transition systems with the same set of actions Act.

$Act \times Act \rightarrow Act, \quad (\alpha, \beta) \rightarrow \alpha * \beta$

synchronous product $TS_1 \otimes TS_2$ is given by:

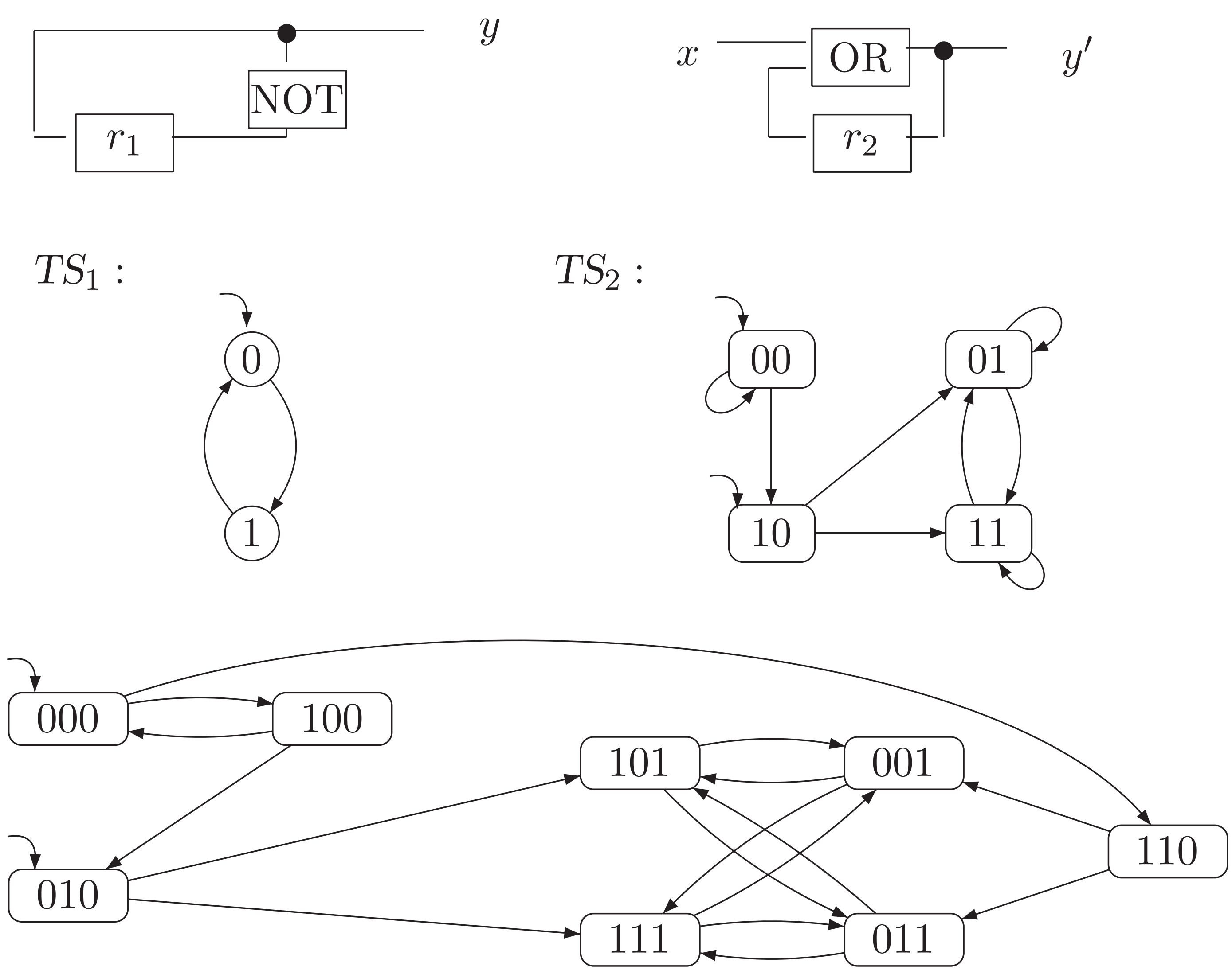
$$TS_1 \otimes TS_2 = (S_1 \times S_2, Act, \rightarrow, I_1 \times I_2, AP_1 \cup AP_2, L),$$

$$\frac{s_1 \xrightarrow{\alpha}_1 s_1' \quad \wedge \quad s_2 \xrightarrow{\beta}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha * \beta} \langle s_1', s_2' \rangle}$$

$$L(\langle s_1, s_2 \rangle) = L_1(s_1) \cup L_2(s_2).$$

example

- Synchronous Product of Two Circuits



the State-Space Explosion Problem

- Program Graph Representation

$$|\,Loc\,| \cdot \prod_{x \in Var} |\,dom(x)\,| \,.$$

- Channel System

$$\prod_{i=1}^{n} \left(|\,Loc_i\,| \cdot \prod_{x \in Var_i} |\,dom(x)\,| \right) \cdot \prod_{c \in Chan} |\,dom(c)\,|^{cp(c)}.$$

summary

- TS
- PG
- parallel,concurrent
- shared variable system
- handshake
- channel system
- transition to TS