# Problem: determination of optimal pricing for agri-environmental schemes

## Opportunity cost mechanism

To maximise the flow of benefits for society from environmental policy options that incentivise a land use change at minimum cost, the objective for the policymaker is to select the providers that can supply the most environmental benefits through an environmental scheme while only paying their opportunity cost, i.e. the lost revenues borne to enrol in the selected environmental scheme, subject to a budget constraint. This mechanism represents the best possible policy outcome, as no surpluses are passed through pricing to farmers; however, it is also highly unrealistic in practice, because it requires the policymaker to be able to obtain perfect knowledge of the opportunity costs for all farmers to provide individualised compensations. Moreover, individualised compensation may not be politically feasible.

This is a typical multiple-choice knapsack optimisation problem. In its most basic form, given classes of items characterised by a profit and weight, the multiple-choice knapsack problem aims to select one item for each class such that the sum of profits is maximised and the sum of weights does not exceed a set threshold value1.

We adapt this multiple-choice knapsack problem such that each class represents farmers, who can each enrol in any item , i.e. land management options. We define as a binary variable indicating whether a farmer *f* has enrolled in a scheme *o*. If there is uptake, each scheme generates a benefit for society . The policymaker aims to maximise benefits for society (Eq. 1):

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|  |  | Eq. 1 |

The cost to implement any of the measures contained in the agri-environmental schemes is specific to each farmer. We define this cost (the weight in the general formulation of the multiple-choice knapsack problem) as ; the policymaker pays farmers their cost. Given the available budget, we need to ensure that total spending does not exceed the budget *M* (Eq. 2):

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|  |  | Eq. 2 |

As each farmer can only enrol in one environmental scheme, we enforce the following constraint (Eq. 3):

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|  |  | Eq. 3 |

The cost includes the foregone farming income to take out of production the land enrolled into a scheme, the cost to implement the measures included in the scheme, plus an additional mark-up to cover private transaction costs2. The opportunity cost model is solved implementing the algorithm proposed by Pisinger 1.

## Flat rate mechanism

A flat rate payment mechanism offers all farmers a fixed price (which could target activities, environmental outcomes or ecosystem services), with some farmers able to participate if the price is above their costs and others not able to. The flat rates are designed with the overall policy objective in mind and are usually determined by considering a representative sample of farmers.

Such problem is a variation of the unit-demand envy free monopolistic pricing model3, a standard optimisation problem in which a number of consumers can buy from a set of items the one that maximises their surpluses, and the objective is to find the sets of prices maximising the expected profits for the seller (for a formulation of this problem see Fernandes, et al. 3).

Our version is a monopsonistic, unit-supply, envy-free declination, where a policymaker buys the uptake of agri-environmental schemes from a set of farmers whose aim is to maximise their surpluses. The objective is to find the set of prices to maximise public benefits from the farmers’ uptake of the schemes subject to a budget constraint for the policymakers.

### Flat rate for activity

Suppose a policymaker has designed a set of environmental land management options that any farmer can adopt upon receipt of a payment that equals a price per unit of land multiplied by the amount of land to be enrolled in the scheme. Each scheme, if adopted by a farmer *f*, would result on an amount of public benefit . A farmer would only adopt a scheme if the payment received exceeds the cost of implementing the measures required by the scheme. Given that the policymaker has a limited budget available , the problem is to find the set of optimal option prices promoting a pattern of uptake from farmers that maximises public benefits without overspending the available budget.

Thus, the objective function for the policymaker is to maximise public benefits, or (Eq. 1):

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|  |  | Eq. 1 |

As benefits are dependent on whether each farmer adopts one of the available environmental options, we define a binary choice variable that takes a value of one if an option *o* has been selected by farmer *f*, and zero otherwise. We defined the problem as a unit-supply one, as such we enforce the constraint (Eq.2):

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|  |  | Eq. 2 |

which ensures that each farmer can only enrol land in one environmental option.

For an option to be desirable, it must generate some level of surplus, i.e., the price received to enrol in the option needs at the very least to cover the costs to implement the measures required by the option. Accordingly, we define a farmer’s surplus as a set of variables such that (Eq.3):

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|  |  | Eq. 3 |

where is the quantity of land in hectares that the farmer owns, and is the price paid per hectare to enrol in option *o*. represents the total cost incurred by farmer *f* to implement the option *o*. This ensures that the surplus of a farmer is the highest surplus generated by any available option and represents the envy-free characterisation of the problem: a farmer only selects the option that generates the highest surplus, i.e., is envy-free relative to all other options at the set prices. To set an upper bound to the surplus that can be gained by a farmer, we enforce the following constraint (Eq. 4):

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|  |  | Eq. 4 |

Following the Big-M method, we here define a large number , as the product where , , and . In other words, is the product between the largest cost per hectare of all schemes across all farmers and the largest area to be converted across all options for each farmer; this ensures that, for each farmer, is at least as large as the largest potential cost that the costliest farmer could face if enrolling in any available scheme. If , i.e. farmer *f* selects the scheme *o*, the big-M term of the equation drops; due to the fact that only one scheme can be selected by any farmer (Eq. 2), Eq. 4 thus becomes the same as Eq.3, with the only difference being the sign, forcing to be equal to for the selected scheme. When , two alternative cases arise: 1) farmer *f* selects one of the ; 2) farmer *f* does not select any scheme. In both cases, is smaller or equal than ( or 0 respectively, where for one farmer, and lower for all the other farmers due to the way in which is defined); therefore, a farmer’s surplus can potentially reach a maximum of for the most costly farmer who selects one of the schemes , and for the farmers who do not select any scheme. What we want is for farmers to gain a surplus of when a scheme is selected, and zero otherwise; however, there is the potential, from Eq. 3 and 4, that a farmer could obtain a surplus even when not enrolled in any scheme. To prevent this, we set the following constraint (Eq. 5):

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|  |  | Eq. 5 |

Whereis a large number defined here as . This equation ensures that is lower or equal to 0 when . It is worth noting that the formulation of the constraints defining surplus in Equations 3, 4 and 5 have been specified such that the optimisation problem remains linear.

As the policymaker has only a limited budget, we need to ensure that spendings are lower or equal to the available budget. As farmers will only select a scheme if this allows to at least cover the costs and generate some surplus (assuming that a farmer would not change practices if only the costs are covered by the payment to enrol in a scheme), we set the following constraints (Eq. 6 and Eq. 7):

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|  |  | Eq. 6 |
|  |  | Eq. 7 |

Where is the total budget available for the policymaker, and is a multiplication factor defining the minimum surplus that a farmer is willing to receive in order to enrol in a scheme. As an example, with farmer would need to obtain a monetary surplus of at least £1 in order to enrol in scheme ; with , the minimum surplus is £2 (note that is a binary variable).

Finally, we limit the maximum price per hectare that a policymaker is willing to pay for a scheme to be equal to the cost per hectare of the most expensive farmer for that scheme (Eq. 8):

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|  |  | Eq. 8 |

Non-negativity constraints also apply (Eq. 9 and Eq. 10):

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|  |  | Eq. 9 |
|  |  | Eq. 10 |

As in the opportunity cost pricing scheme, the cost includes the foregone farming income to take out of production the land enrolled into a scheme, the cost to implement the measures included in the scheme, plus an additional mark-up to cover private transaction costs2.

### Flat rate for environmental outcomes

For this scenario, suppose that a policymaker a policymaker has designed a set of environmental land management schemes that any farmer can adopt. Unlike in the previous case, the monetary compensation that a farmer would is based on the environmental outcome that the land management required by the scheme would produce, for example a flat rate per tonne of CO2 sequestered, per any additional species of flora/fauna reintroduced on the landscape, per microgram of dissolved nitrogen sequestered from waterways. As such, a farmer would enrol in a scheme only if the flat rates for the unit of environmental outcome of interest multiplied by the quantity of environmental outcome generated by the new land management produce a surplus for the farmer, compared with the alternative case in which no scheme is adopted. The objective of the policymaker is to find the optimal flat rate for each environmental outcome in order to maximise benefits for society without overspending the available budget.

The formulation of this pricing problem closely follows the one for optimally pricing activities listed above. As in the previous case, the objective function for the policymaker is to maximise public benefits, or (Eq. 1):

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|  |  | Eq. 11 |

Equally, we enforce the unit supply constraint in the same way (Eq. 12):

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|  |  | Eq. 12 |

Here we need to define a set of environmental outcomes which are of interest for the policymaker and are generated in different amounts by each farmer enrolling in each environmental scheme. For a farmer to enrol in an environmental scheme, the surplus generated by the payment for the environmental outcomes produced must exceed the cost of the implementation of the scheme; as in the previous formulation, farmers aim to maximise their surplus (Eq. 13 and 14):

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|  |  | Eq. 13 |
|  |  | Eq. 14 |

where is the amount of environmental outcome generated by the uptake of the scheme, and is the price paid per unit of envioronmental outcome *e*. represents the total cost incurred by farmer *f* to implement the scheme *o*, including foregone agrciultural income, costs to implement the measures in the scheme and a 15% markup to cover transaction costs. In this case, is defined as (Eq. 15):

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|  | Eq. 15 |

Where , and and is the quantity of environmental outcome *e* produced by farmer *f* from the adoption of scheme *o*. In other words,  is the product between the maximum quantity of each environmental outcome across all options that each farmer would generate and the cost per unit of each environmental outcome from the costliest farmer; these total costs for each environmental outcomes are then summed across all environmental outcomes to obtain a value representing our big-M, i.e. a very large cost, larger than any cost any farmer would incur.

Once again, we now need to set up a constraint that forces the surplus to be zero if no environmental scheme is selected by a farmer: this mirrors Eq.5 in the previous formulation (Eq. 16):

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|  |  | Eq. 16 |

Where . Equations 17 and 18 are the same as Eq. 6 and 7 and constrain public expenditures to be lower or equal to the budget available for the policymaker, and ensure that a farmer would enrol in a scheme if the surplus gained is above a small threshold , to avoid the case in which a farmer is indifferent to both the status quo or the enrollment in a scheme:

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|  |  | Eq. 17 |
|  |  | Eq. 18 |

Similarly to Eq. 8, we set an upper bound for the price per unit of outcome *e* to be at most equal to the highest cost per unit of environmental outcome for the costliest environmental quantity for the costliest farmer (Eq. 19):

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|  |  | Eq. 19 |

Finally, we enforce the usual non negativity constraints (Eq. 20 and 21):

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|  |  | Eq. 20 |
|  |  | Eq. 21 |

### Flat rate for ecosystem services.

This pricing model closely mirrors the flat rate for environmental outcome pricing model, with the only difference being that policymaker wishes to spend a predetermined budget by rewarding farmers for the ecosystem service values that they deliver following the implementation of an environmental scheme; farmers are thus paid a flat rate per pound of improvement in a range of ecosystem services, rather than a flat rate per unit of environmental outcome delivered. As a consequence, in this formulation we define as the set of ecosystem services of interest, as the price paid per pound of improvement in ecosystem service *e*, and as the quantity of improvement (in value terms) for ecosystem service *e* from the adoption of scheme *o* by farmer *f.* With the exception of these differences in definitions, the mathematical formulation is the same as described in Equations 11 to 21.

1 Pisinger, D. A minimal algorithm for the multiple-choice knapsack problem. *European Journal of Operational Research* **83**, 394-410 (1995).

2 Mettepenningen, E., Verspecht, A. & Van Huylenbroeck, G. Measuring private transaction costs of European agri-environmental schemes. *Journal of Environmental Planning and Management* **52**, 649-667 (2009).

3 Fernandes, C. G., Ferreira, C. E., Franco, A. J. & Schouery, R. C. The envy-free pricing problem, unit-demand markets and connections with the network pricing problem. *Discrete Optimization* **22**, 141-161 (2016).