Pyung Lee
PKL4FR
Project2
Text Solutions

1a)

$$* \quad \frac{dY_L}{dt} = \left(\frac{-kA}{V_L} - \frac{Q_A}{V_L}\right)Y_L + \frac{QY_{air}}{V_L}$$

$$* \quad \frac{dX_p}{dt} = \left(\frac{kA}{V_p}\right)X_p + \left(-\frac{k_{eq}kA}{V_p} - \frac{\sigma Q_B}{V_p}\right)X_p + \left(\sigma Q_B / V_p\right)X_p$$

$$* \quad \frac{dX_A}{dt} = \left(\frac{\sigma Q_B}{dt}\right)X_A - \left(\frac{Q_B}{V_A}\right)X_A + \left(\frac{(1-\sigma)Q_B}{V_A}\right)X_A$$

$$* \quad \frac{dX_V}{dt} = \left(\frac{Q_B}{V_V}\right)X_V + \left(\frac{-Q_B}{V_V}\right)X_V - M/V_V$$

1c)

$$* \quad \frac{dY_L}{dt} = \frac{-kA(Y_L - Y_3^*)}{V_L} - \frac{kA(Y_L - Y_2^*)}{V_L} - \frac{kA(Y_L - Y_1^*)}{V_L} - \frac{Q_A Y_L}{V_L} + \frac{Q_A Y_{air}}{V_L} - \frac{Y_L}{V_L}\left(\frac{dV_L}{dt}\right)$$

$$* \quad \frac{dX_{p_1}}{dt} = \frac{kA(Y_L - Y_1^*)}{V_p} + \frac{\sigma Q_B X_V}{V_p} - \frac{\sigma Q_B X_{p_1}}{V_p}$$

$$* \quad \frac{dX_{p_2}}{dt} = \frac{kA(Y_L - Y_2^*)}{V_p} + \frac{\sigma Q_B X_A}{V_p} - \frac{\sigma Q_B X_{p_2}}{V_p}$$

$$* \quad \frac{dX_{p_3}}{dt} = \frac{kA(Y_L - Y_3^*)}{V_p} + \frac{\sigma Q_B X_{p_2}}{V_p} - \frac{\sigma Q_B X_{p_3}}{V_p}$$

$$* \quad \frac{dX_A}{dt} = \frac{\sigma Q_B X_{p_3}}{V_A} + \frac{(1-\sigma)Q_B X_V}{V_A} - \frac{Q_B X_A}{V_A} \quad \checkmark$$

$$* \quad \frac{dX_V}{dt} = \frac{Q_B X_A}{V_V} - \frac{M}{V_V} - \frac{Q_B X_V}{V_V} \quad \checkmark$$

$$V_L = 2720 + 260\cos\left[2\pi\left(\frac{t}{\tau}\right)\right]$$

$$\frac{dV_L}{dt} = \left[-260 * \frac{2\pi}{\tau}\right]\sin\left(2\pi\left(\frac{t}{\tau}\right)\right)$$

$$V_{Lmax} = 2980 \text{ mL}$$

$$* \quad Y_j^*(X_{pj}) = [(0.0003 X_{pj})/(6.22 - X_{pj})] + [(11.2 - X_{pj})(0.055 X_{pj})]$$

1d)

1d

$$\frac{dY_L}{dt} = \frac{-kA(Y_L - Y_3^*)}{Y_L} - \frac{kA(Y_L - Y_2^*)}{Y_L} - \frac{kA(Y_L - Y_1^*)}{Y_L}$$

$$\frac{dX_{p1}}{dt} = \frac{kA(Y_L - Y_1^*)}{V_p} + \frac{\sigma Q_B X_v}{V_p} - \frac{\sigma Q_B X_{p1}}{V_p}$$

$$\frac{dX_{p2}}{dt} = \frac{kA(Y_L - Y_2^*)}{V_p} + \frac{\sigma Q_B X_{p1}}{V_p} - \frac{\sigma Q_B X_{p2}}{V_p}$$

$$\frac{dX_{p3}}{dt} = \frac{kA(Y_L - Y_3^*)}{V_p} + \frac{\sigma Q_B X_{p2}}{V_p} - \frac{\sigma Q_B X_{p3}}{V_p}$$

$$\frac{dX_A}{dt} = \frac{\sigma Q_B X_{p3}}{V_A} + \frac{(1-\sigma) Q_B X_v}{V_A} - \frac{Q_B X_A}{V_A}$$

$$\frac{dX_v}{dt} = \frac{Q_B X_A}{V_v} - \frac{M}{V_v} - \frac{Q_B X_v}{V_v}$$

2a) It took approximately 1 to 2 minutes to reach steady state.
2b) Since there is oscillation in breathing, there is an oscillation in the graph in the lung and pulmonary compartment lines. Steady state is still reached between 1 to 2 minutes. The oxygen concentration varies from high to low concentrations because there is an oscillation in breathing.
2c) To reach 65% of the initial value, it took about approximately 8.7 minutes.
2d) The graph has the oscillations trending downwards, as compared to the constant oscillatory lines in the graph of b. The Xv line trends negatively toward a concentration of 0 while the Xa concentration of appears to reach a steady state value.