4) a) $\frac{dC_A}{dt} + 2C_A = 0.2$

(1) Separation Variable

$dC_A = (0.2 - 2C_A)dt$

$\frac{dC_A}{0.2 - 2C_A} = dt$

$-\frac{1}{2}\ln|0.2 - 2C_A| = t + C$

$\ln|0.2 - 2C_A| = -2(t + C)$

$0.2 - 2C_A = e^{-2t} \cdot C$

$\left[\frac{0.2}{2} + e^{-2t}(C) = C_A\right]$

$C_A(0) = 0.1$

$C = 0.4$

$\boxed{C_A = 0.4e^{-2t} + 0.1}$

(2) undetermined Coefficients

$\frac{dC_A}{dt} + 2C_A = 0.2$

Homogenous: $\frac{dC_A}{dt} + 2C_A = 0$ Particular: $y_p = B$

$\frac{dC_A}{C_A} = -2\,dt$

$C_A = e^{-2t} \rightarrow C_A = Ae^{-2t}$

General: $C_A = Ae^{-2t} + B$

$dC_A/dt = -2Ae^{-2t}$

with initial condition

$A + B = 0.5$

→ Plugging into starting equation

$-2Ae^{-2t} + 2Ae^{-2t} + 2B = 0.2$

$B = 0.1 \quad A = 0.4$

$\boxed{C_A = 0.4e^{-2t} + 0.1}$

b) $\frac{dC_A}{dt} + 2C_A = 3e^{-3t}$ $\quad C_A(0) = 0.5$

* Homogeneous: $\frac{dC_A}{dt} = -2C_A$

$\ln C_A = -2t$

$C_A = Ae^{-2t}$

* Particular: $y_p = Be^{-3t}$

* general: $C_A(t) = Ae^{-2t} + Be^{-3t}$

$\frac{dC_A}{dt} = -2Ae^{-2t} - 3Be^{-3t}$

* Initial condition: $0.5 = A + B$

Original Equation: $-2Ae^{-2t} - 3Be^{-3t} + 2Ae^{-2t} + 2Be^{-3t} = 3e^{-3t}$

$-Be^{-3t} = 3e^{-3t}$

$B = -3$

$A = 3.5$

$\boxed{C_A(t) = 7/2e^{-2t} - 3e^{-3t}}$

c) $\frac{dC_A}{dt} + 2C_A = \sin(2t) + \cos(2t)$ $\quad C_A(0) = 0.5$

* Homogen: $\frac{dC_A}{C_A} = -2dt$

$C_A(t) = Ae^{-2t}$

* particular: $y_p = A\sin(2t) + B\cos(2t)$

* general: $C_A(t) = Ae^{-2t} + B\sin(2t) + C\cos(2t)$

$dC_A/dt = -2Ae^{-2t} + 2B\cos(2t) - 2C\sin(2t)$

* initial condition

$0.5 = A + C$ ←

original:

$-2Ae^{-2t} + 2B\cos(2t) - 2C\sin(2t) + 2Ae^{-2t} + 2B\sin(2t) + 2C\cos(2t) =$

$\sin(2t) + \cos(2t)$

$0.5 = A + C$

$2B + 2C = 1$

$-2C + 2B = 1$

$A = 0.5$

$B = 1/2$

$C = 0$

$\rightarrow \boxed{C_A(t) = 1/2e^{-2t} + 1/2\sin(2t)}$