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CHE 2215

HW 7

HW7_textsolutions

① $NaNO_3 = 90$ kg
$H_2O = 110$ kg

Pure H_2O → $\dot{m}_{in}$ → [$NaNO_3$ / H_2O] → solution → $\dot{m}_{out}$

over time $C_{NaNO_3 + H_2O}$ decreases

a) $\dot{m}_{accumulation} = \dot{m}_{in} - \dot{m}_{out}$

$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$ * $\dot{m}_{in} = \dot{m}_{out}$

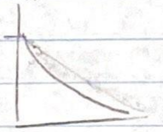
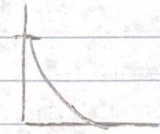
$\boxed{\frac{dm}{dt} = 0}$

b) $x(t, \dot{m})$ = mass fraction of $NaNO_3$ in tank

$\boxed{\frac{d(m_{tot} \cdot X_{tank})}{dt} = (-\dot{m}_{out})(X_{NaNO_3 \, out})}$ * $x(t, \dot{m}) = $ kg $NaNO_3$ / kg tank

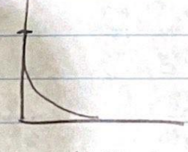
* initial condition
$t = 0$, $m_{total} = 200$ kg

c) ① $\dot{m}_N = 50$ kg/min   ② $\dot{m}_N = 100$ kg/min

x(kg)    x(kg) 

time   time

③ $\dot{m}_N = 200$ kg/min

x(kg) 

time

* Integrating (b) will result in exponential decreasing curve. $\frac{d(X_{tank})}{X_{NaNO_3}} = \frac{-\dot{m}_{out}}{m_{tank}} \rightarrow \ln|X_{NaNO_3}| = \frac{-\dot{m}_{out}}{m_{tank}} t$

$\boxed{X_{NaNO_3} = e^{-\dot{m}_{out}/m_{tank} \cdot t}}$

d) $\frac{d(m_{tot} \cdot X_{tank})}{dt} = -\dot{m}_{outlet} X_{NaNO_3}$

$(200) \frac{dX_{tank}}{dt} = -\dot{m}_{outlet} X_{NaNO_3}$ @ $t=0$, $m_{tank} = 200$

$\frac{dX_{tank}}{X_{NaNO_3}} = -\frac{1}{200} \dot{m}_{outlet}\, dt$

$\ln|X_{NaNO_3}| = -t/200\, \dot{m}_{outlet} + C$

$X_{NaNO_3} = e^{-t/200\, \dot{m}_{outlet} + C}$

$= e^{C} e^{-t/200\, \dot{m}_{outlet}}$

$X_{NaNO_3} = Ce^{-t/200\, \dot{m}_{outlet}}$ @ $t=0$, $X_{NaOH} = 90/200 = 0.45$

$0.45 = Ce^{0}$

$C = 0.45$

$\boxed{X_{NaNO_3} = (0.45)e^{-t/200\, \dot{m}_{outlet}}}$

$\dot{m} = 100$ kg/min → 90% out

e) ① 90% $X_{NaNO_3} = 9\text{kg}/200\text{kg} = 0.045$

$X_{NaNO_3} = 0.45e^{-1/2 t}$

$0.045 = 0.45e^{-1/2 t}$

$0.1 = e^{-1/2 t}$

$\boxed{t_{90\%} = 4.61 \text{ min}}$

② 99%

$0.0045 = 0.45e^{-1/2 t}$

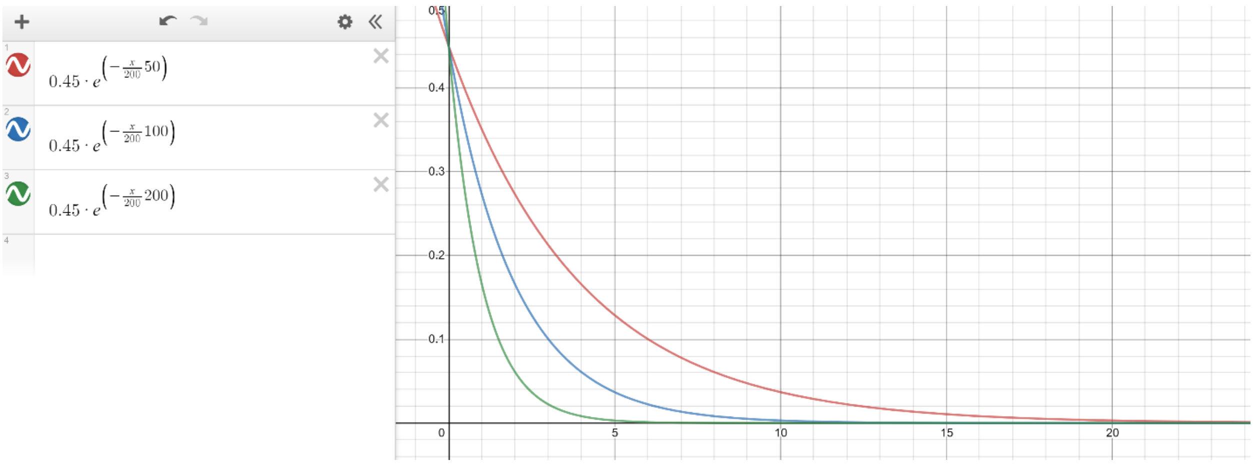
$\boxed{t = 9.21 \text{ min}}$

③ 99.9%

$0.001 = 0.45e^{-1/2 t}$

$\boxed{t = 13.82 \text{ min}}$

Q1D)


$0.45 \cdot e^{\left(-\frac{x}{200}50\right)}$
$0.45 \cdot e^{\left(-\frac{x}{200}100\right)}$
$0.45 \cdot e^{\left(-\frac{x}{200}200\right)}$
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20

2) $CO + Cl_2 \rightarrow COCl_2$

$R_P(mol/min) = \dfrac{8.75\, C_{CO} C_{Cl_2}}{(1 + 58.6\, C_{Cl_2} + 34.3\, C_{COCl_2})}$

~~A4~~ $T = 303.8\ K$ w/ 1g charcoal

let $A = CO$ $A + B \rightarrow C$

$B = Cl_2$

$C = COCl_2$

a) $V = 3L$ 40% mol Cl_2

$T = 303.8K$ 60% mol CO

$P = 1atm$

$PV = nRT$

→ $(1)(3) = n(0.0821)(303.8\ K)$ → $(n)(0.4) = 0.0723$ mol Cl_2 (B)

$n = 0.12043$ mol (total) $(n)(0.6) = 0.0481$ mol CO (A)

→ $P_A = X_A P$ $\left\langle \begin{array}{l} P_{Cl_2} = 0.4\ atm \\ P_{CO} = 0.6\ atm \end{array} \right.$

→ Intial condition/concentration

$PV = nRT$

$Cl_2 \rightarrow (0.4)(1) = (Cl_{2_0})(0.0821)(303.8)$

$\boxed{C_{Cl_2 0} = 0.01605\ mol/L}$

$CO \rightarrow (0.6)(1) = C_{O_0}(0.0821)(303.8)$

$\boxed{CO_0 = 0.02407\ mol/L}$

} initial concentrations

→

$\boxed{\begin{array}{l} C_{CO} = 0.02407 - C_P(t) \\ C_{Cl_2} = 0.01605 - C_P(t) \end{array}}$

2d) Using the trapezoidal rule from 0 – 0.012, the value calculated was 89.6293.

$T_{steam} =$

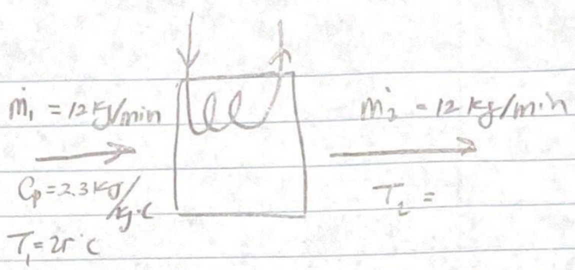
$P = 7.5$ bar

$T_{steam\,out} =$

$\swarrow 167.8\,^\circ C$

3) $\dot{Q} = UA(T_{steam} - T)$

$\dot{Q} = 11.5\ \text{kJ/min}\cdot{}^\circ C$



m_{Tank} at $T(0) = 760$ kg

$T(0) = 25\,^\circ C$

a) $T(0) = 25\,^\circ C$

$M C_v \, dT/dt = \dot{m} C_p (T_{in} - T_{out}) + UA(T_{steam} - T)$

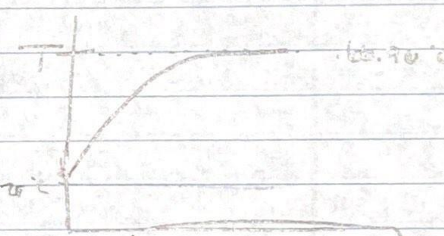
$dT/dt = \frac{(12)(2.3)}{(760)(2.3)} (25 - T_{out}) + \frac{11.5}{(760)(2.3)} (167.8 - T_{out})$

$$\boxed{dT/dt = 1.50 - 0.0224\,T}$$

b)

$dT/dt = 0 = 1.50 - 0.0224\,T$

$\boxed{T = 66.96\,^\circ C}$ steady state



$\boxed{T(0) = 25\,^\circ C \text{ initial}}$

c) $dT/dt = 1.50 - 0.0224\,T$

$\frac{dT}{1.50 - 0.0224\,T} = dt$

$\frac{-1}{0.0224} \left(\frac{du}{u} \right) = dt$

$\frac{-1}{0.0224} (\ln|1.50 - 0.0224\,T|) = t + c$

$(\ln|1.5 - 0.0224\,T| = -0.0224\,t - 0.0224\,C$

$1.5 - 0.0224\,T = (e^{-0.0224t})(e^{-0.0224c})$

$\frac{1.5 - e^{-0.0224t}(k)}{0.0224} = T(t)$

$k = 0.94$

$\frac{1.5 - e^{-0.0224t}(0.94)}{0.0224} = T(t)$

$\boxed{T(40) = 49.83\,^\circ C}$

4) a) $\frac{dCA}{dt} + 2CA = 0.2$

(1) Separation Variable

$dCA = (0.2 - 2CA)dt$

$\frac{dCA}{0.2 - 2CA} = dt$

$-\frac{1}{2} \ln|0.2 - 2CA| = t + C$

$\ln|0.2 - 2CA| = -2(t + C)$

$0.2 - 2CA = e^{-2t} \cdot C$

$\left[\frac{0.2}{2} + e^{-2t}(C) = CA \right]$ $CA(0) = 0.5$

$C = 0.4$

$\boxed{CA = 0.4e^{-2t} + 0.1}$

(2) undetermined Coefficients

$\frac{dCA}{dt} + 2CA = 0.2$

Homogeneous: $\frac{dCA}{dt} + 2CA = 0$ Particular: $y_p = B$

$\frac{dCA}{CA} = -2dt$

$CA = e^{-2t} \rightarrow CA = Ae^{-2t}$

General: $CA = Ae^{-2t} + B$ with initial condition

$dCA/dt = -2Ae^{-2t}$ $A + B = 0.5$

→ Plugging into starting equation

$-2Ae^{-2t} + 2Ae^{-2t} + 2B = 0.2$

$B = 0.1 \quad A = 0.4$

$\boxed{CA = 0.4e^{-2t} + 0.1}$

5) $\frac{dC_{A}}{dt} = \frac{1}{\tau_1}(C_{A_0} - C_{A_1}) - k_1 C_{A_1}$

$\frac{dC_{B_1}}{dt} = -1/\tau_1 (C_{B_1}) + k_1 C_{A_1}$

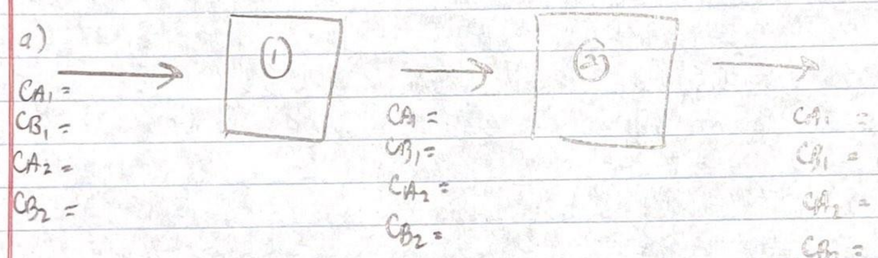
$dC_{A_2}/dt = \frac{1}{\tau_2} C_{A_1} - 1/\tau_2 C_{A_2} - k_2 C_{A_2}$

$dC_{B_2}/dt = 1/\tau_2 C_{B_1} - 1/\tau_2 C_{B_2} + k_2 C_{A_2}$

$\tau_1 = 10$ min
$\tau_2 = 7$ min
$k_1 = 0.15$ /min
$k_2 = 0.14$ /min
$C_{A_0} = 0.4$

* Total volumes of both reactors are in steady state.

a)



→ C_{A_1} : $\frac{dN_{A_1}}{dt} = N_{A_1 in} - N_{A_1 out} - N_{A_1 con}$

$\frac{dC_{A_1}}{dt}(V_{tank}) = C_{A_1 in} V_{in} - C_{A_1 out} V_{out} - k C_{A_1} V_{tank}$

$$\boxed{\frac{dC_{A_1}}{dt} = \frac{1}{\tau}(C_{A_0} - C_{A_1}) - k_1 C_{A_1}}$$

C_{B_1} : $\frac{dN_{B_1}}{dt} = \cancel{N_{B_1 in}}^{0} - N_{B_1 out} - \cancel{N_{B_1 con}}^{0} + N_{B_1 gen}$

$\frac{dC_{B_1}}{dt} V_{tank} = -C_{B_1} V_{in} + k C_{B_1 gen}$

$= -C_{B_1} V_{in} + k C_{A_1}$

* $C_{A_1 con} = C_{B_1 gen}$

$$\boxed{\frac{dC_B}{dt} = -\frac{1}{\tau}(C_{B_1}) + k_1 C_{A_1}}$$

C_{A_2} : $\frac{dN_{A_2}}{dt} = N_{A_2 in} - N_{A_2 out} + \cancel{N_{A_2 gen}}^{0} - N_{A_2 con}$

$\frac{dC_{A_2}}{dt} V_{tank} = C_{A_1 in} \dot{V}_{in} - C_{A_2 out} \dot{V}_{out} - k_2 C_{A_2} V_{tank}$

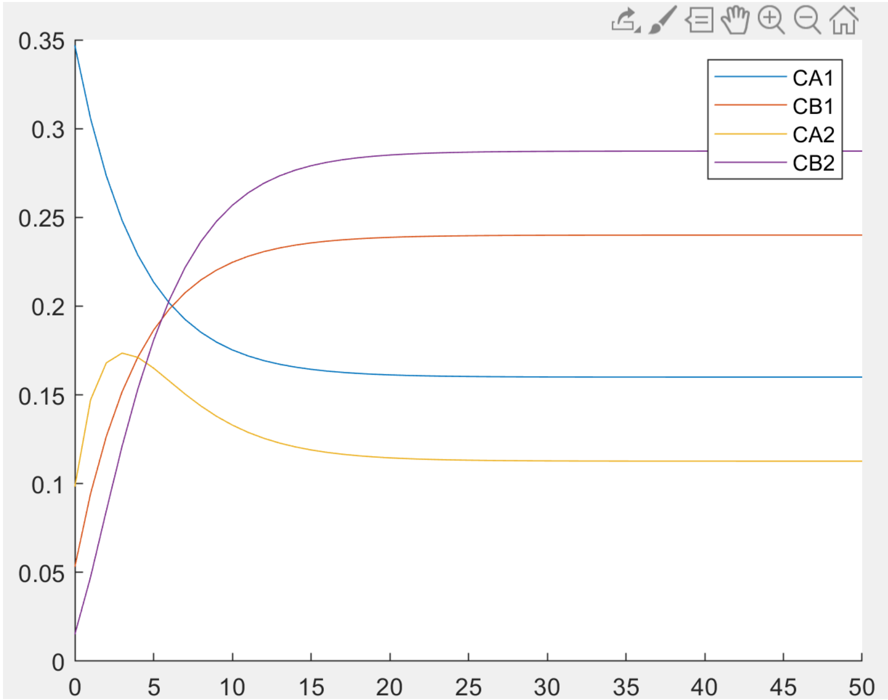
$$\boxed{\frac{dC_{A_2}}{dt} = \frac{1}{\tau_2} C_{A_1} - \frac{1}{\tau_2} C_{A_2} - k_2 C_{A_2}}$$

C_{B_2} : $\frac{dN_{B_2}}{dt} = N_{B_1 in} - N_{B_2 out} + N_{B_2 gen} - \cancel{N_{B_2 con}}^{0}$

$\frac{dC_{B_2}}{dt} V_{tank} = C_{B_1} \dot{V}_{in} - C_{B_2} \dot{V}_{out} + C_{B_2}(k_2) V_{tank}$

$$\boxed{\frac{dC_{B_2}}{dt} = \frac{1}{\tau_2} C_{B_1} - \frac{1}{\tau_2} C_{B_2} + k_2 C_{A_2}}$$

5C)



This graph was created using matlab. It shows that as the reaction continues onwards with time, both CA and CB reach an equilibrium concentration. CA1 decreases because it is being consumed continuously. CB1 and CB2 increased and reached their respective concentrations. CA2 increased initially and then decreased. This is because in the first reactor it was being generated then it was consumed in the second reactor.

The MATLAB Functions I used were:

EIG: calculated the eigenvalues and vectors for a matrix.

b) $\frac{dy}{dt} = \frac{1}{3} - t + 3y \quad y(0) = 1, \quad t = 0.1, 0.2, 0.3$

$\Delta t = 0.1$

a) $f(y,t) = \frac{1}{3} - t + 3y$

Euler's

* at $t = 0.1$

$y(0.1) = y(0) + f(y(0), 0)(0.1)$

$= 1 + f(1, 0)(0.1)$

$= 1 + (\frac{1}{3} - 0 + 3)(0.1)$

$y(0.1) = \frac{4}{3}$

* at $t = 0.2$

$y(0.2) = y(0.1) + f(y(0.1), 0.1)(0.1)$

$= \frac{4}{3} + f(\frac{4}{3}, 0.1)(0.1)$

$= \frac{4}{3} + (\frac{1}{3} - 0.1 + 4)(0.1)$

$y(0.2) = 1.7567$

* at $t = 0.3$

$y(0.3) = y(0.2) + f(y(0.2), 0.2)(0.1)$

$= 1.7567 + f(1.7567, 0.2)(0.1)$

$= 1.7567 + (\frac{1}{3} - 0.2 + 3(1.7567))(0.1)$

$y(0.3) = 2.297$