6) $\frac{dy}{dt} = \frac{1}{3} - t + 3y$ $y(0) = 1$, $t = 0.1,\ 0.2,\ 0.3$

$\Delta t = 0.1$

a) $f(y,t) = \frac{1}{3} - t + 3y$

* at $t = 0.1$

$\begin{aligned}
y(0.1) &= y(0) + f(y(0), 0)(0.1) \\
&= 1 + f(1,0)(0.1) \\
&= 1 + (\frac{1}{3} - 0 + 3)(0.1)
\end{aligned}$

$y(0.1) = \frac{4}{3}$

* at $t = 0.2$

$\begin{aligned}
y(0.2) &= y(0.1) + f(y(0.1), 0.1)(0.1) \\
&= \frac{4}{3} + f(\frac{4}{3}, 0.1)(0.1) \\
&= \frac{4}{3} + (\frac{1}{3} - 0.1 + 4)(0.1)
\end{aligned}$

$y(0.2) = 1.7567$

* at $t = 0.3$

$\begin{aligned}
y(0.3) &= y(0.2) + f(y(0.2), 0.2)(0.1) \\
&= 1.7567 + f(1.7567, 0.2)(0.1) \\
&= 1.7567 + (\frac{1}{3} - 0.2 + 3(1.7567))(0.1)
\end{aligned}$

$y(0.3) = 2.297$

Euler's

$f(y,t) = \frac{1}{3} - t + 3y$

$t = 0.1, 0.2, 0.3$

$y(0) = 1$

① $f(y(0), 0) = f(1,0) = \frac{1}{3} - 0 + 3 = \frac{10}{3}$

$y(0.1) = y(0) + f(y(0),1)(0.1) = 1 + (\frac{10}{3})(0.1) = \frac{4}{3}$

$f(y(0.1), 0.1) = \frac{1}{3} - 0.1 + 3(\frac{4}{3}) = 4.233$

$y(0.1) = y(0) + \frac{1}{2}(0.1)(4.233 + \frac{4}{3}) = \boxed{1.378}$

② $f(y(0.1), 0.1) = \frac{1}{3} - 0.1 + 3(1.378) = 4.367$

$y(0.2) = y(0.1) + 0.1(4.367) = 1.8347$

$f(y(0.2), 0.2) = \frac{1}{3} - 0.2 + 3(1.8347) = 5.637$

$y(0.2) = y(0.1) + (0.1)(\frac{1}{2})(4.367 + 5.637) = \boxed{1.88}$

③ $f(y(0.2), 0.2) = \frac{1}{3} - 0.2 + 3(1.88) = 5.76$

$y(0.3) = y(0.2) + 0.1(5.76) = 2.46$

$f(y(0.3), 0.3) = \frac{1}{3} - 0.3 + 3(2.46) = 7.4$

$y(0.3) = y(0.2) + (0.1)(\frac{1}{2})(7.4 + 5.76) = \boxed{2.54}$

Analytical solution: $y(t) = e^{3t} + \frac{t}{3}$

$y(0.1) = 1.38$

$y(0.2) = 1.88$

$y(0.3) = 2.56$