

# HW4 Text Solutions

1)

$$\begin{aligned} 8.2) \quad a) \quad [A] &= 3 \times 2 & [D] &= 2 \times 4 & [G] &= 1 \times 3 \\ [B] &= 3 \times 3 & [E] &= 3 \times 3 \\ [C] &= 3 \times 1 & [F] &= 2 \times 3 \end{aligned}$$

b) Square: B and E  
column: C  
Row: G

$$\begin{aligned} c) \quad a_{12} &= 7 & d_{32} &= \text{not possible} & f_{12} &= 0 \\ b_{23} &= 7 & e_{22} &= 2 & g_{12} &= 6 \end{aligned}$$

$$d) \quad ① [E] + [B] = \begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}$$

$$② [A] + [F] = \text{not possible}$$

$$③ [B] - [E] = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -6 & 0 & 4 \\ -2 & 0 & -2 \end{bmatrix}$$

$$④ 7[B] = 7 \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 28 & 21 & 49 \\ 7 & 14 & 49 \\ 14 & 0 & 28 \end{bmatrix}$$

$$⑤ [C]^T = [3 \ 6 \ 1]$$

$$⑥ [E] \times [B] = \begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 28 & 13 & 74 \\ 36 & 21 & 71 \\ 28 & 12 & 52 \end{bmatrix}$$

$$⑦ [B] \times [A] = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 54 & 76 \\ 41 & 53 \\ 28 & 38 \end{bmatrix}$$

$$⑧ [D]^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}$$

$$(9) [A] \times [C] = (3 \times 2)(3 \times 1) = \text{not possible}$$

$$(10) [I] \times [B] = [B]$$

$$(11) [E]^T \times [E] = \begin{bmatrix} 1 & 7 & 4 \\ 5 & 2 & 0 \\ 8 & 3 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 66 & 19 & 53 \\ 19 & 29 & 46 \\ 53 & 46 & 109 \end{bmatrix}$$

$$(12) [C]^T \times [C] = \begin{bmatrix} 3 & 6 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} = [46]$$

2a)



$$\begin{aligned}
 2) a) \quad & 2x_1 + x_2 - x_3 = -1 \\
 & 4x_1 + 2x_3 = -2 \\
 & 8x_1 + x_2 = -3
 \end{aligned}$$

$$\begin{aligned}
 & \begin{array}{c} * A_2/A_1 = 2 \\ \downarrow A_3/A_1 = 4 \end{array} \\
 & \left[ \begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 4 & 0 & 2 & -2 \\ 8 & 1 & 0 & -3 \end{array} \right] \xrightarrow{A_2 - 2A_1} \left[ \begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -2 & 4 & 0 \\ 8 & 1 & 0 & -3 \end{array} \right] \xrightarrow{A_3 - 4A_1} \left[ \begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -2 & 4 & 0 \\ 0 & -3 & 4 & 1 \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \xrightarrow{A_3/A_2 = 3/2} \\
 & \left[ \begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -2 & 4 & 0 \\ 0 & -3 & 4 & 1 \end{array} \right] \xrightarrow{A_3 - \frac{3}{2}A_2} \left[ \begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -2 & 4 & 0 \\ 0 & 0 & -2 & 1 \end{array} \right] \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & \left[ \begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -2 & 4 & 0 \\ 0 & 0 & -2 & 1 \end{array} \right] \quad \begin{aligned} 2x_1 + x_2 - x_3 &= -1 \\ -2x_2 + 4x_3 &= 0 \\ -2x_3 &= 1 \end{aligned} \rightarrow \boxed{\begin{aligned} x_1 &= -1/4 \\ x_2 &= -1 \\ x_3 &= -1/2 \end{aligned}}
 \end{aligned}$$

$$b) \quad \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & 2 \\ 8 & 1 & 0 \end{bmatrix} = A = [U][L]$$

$$[U] = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & 2 \\ 8 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 4 \\ 0 & 0 & -2 \end{bmatrix} \checkmark$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ +2 & 1 & 0 \\ +4 & +\frac{1}{2} & 1 \end{bmatrix} \checkmark$$

\*using Annotations from part A

$$[L][U]x = b$$

$$\text{let } [U]x = d$$

$$[L]d = b$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 2 & 1 & 0 & -2 \\ 4 & 3/2 & 1 & -3 \end{array} \right] \rightarrow \begin{array}{l} d_1 = -1 \\ d_2 = 0 \\ d_3 = 1 \end{array} \rightarrow \vec{d} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$d = [U]\vec{x}$$

$$\left[ \begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -2 & 4 & 0 \\ 0 & 0 & -2 & 1 \end{array} \right] \rightarrow \begin{array}{l} x_1 = -1/4 \\ x_2 = -1 \\ x_3 = -1/2 \end{array}$$

2f)

Problem 1: The diagonals are 0, so when factors are calculated for forward elimination, it is not possible because it would try to divide by 0.

$$0 + 2x_2 + 3x_3 = 10$$

$$x_1 + 0 + 5x_3 = 20$$

$$7x_1 + 7x_2 + 12x_3 = 30$$

Problem 2: The diagonals are almost 0, so the factors would become infinity. So it is not possible to calculate the factor.

$$0.00000001x_1 + 2x_2 + 3x_3 = 10$$

$$x_1 + 0 + 5x_3 = 20$$

$$7x_1 + 7x_2 + 12x_3 = 30$$