8005530 Adaptive Signal Processing - Spring 2004

3 CU

Lectures: TB 214, Tuesday 10:00-12:00

Exercises: Sun class, Wednesdays 10.00 - 12.00, TC407

Lecturer:

Ioan Tabus

office: TF 414,

e-mail tabus@cs.tut.fi

Contents of the course:

Basic adaptive signal processing methods

Linear adaptive filters

Supervised training

Requirements:

Project work: Exercises and programs for algorithm implementation

Final examination

Text book: Simon Haykin, Adaptive Filter Theory

Prentice Hall International, 2002

0	**Background and preview**	**10**	**Kalman Filters**
1	**Stationary Processes and Models**	**11**	**Square Root Adaptive Filters**
2	**Wiener Filters**	**12**	**Order Recursive Adaptive Filters**
3	**Linear Prediction**	**13**	**Finite Precision Effects**
4	**Method of Steepest Descent**	**14**	**Tracking of Time Varying Systems**
5	**Least-Mean-Square Adaptive Filters**	**15**	**Adaptive Filters using Infinite-Duration Impulse Response Structures**
6	**Normalized Least-Mean-Square Adaptive Filters**	**16**	**Blind Deconvolution**
7	**Frequency-Domain Adaptive Filters**	**17**	**Back-Propagation Learning**
8	**Method of Least Squares**		**Epilogue**
9	**Recursive Least-Squares Algorithm**		

1. Introduction to Adaptive Filtering

1.1 Example: Adaptive noise cancelling

- Found in many applications:

  Cancelling 50 Hz interference in electrocardiography (Widrow, 1975);

  Reduction of acoustic noise in speech (cockpit of a military aircraft: 10-15 dB reduction);

- Two measured inputs, $d(n)$ and $v_1(n)$:

  - $d(n)$ comes from a primary sensor: $d(n) = s(n) + v_0(n)$

    where $s(n)$ is the information bearing signal;

    $v_0(n)$ is the corrupting noise:

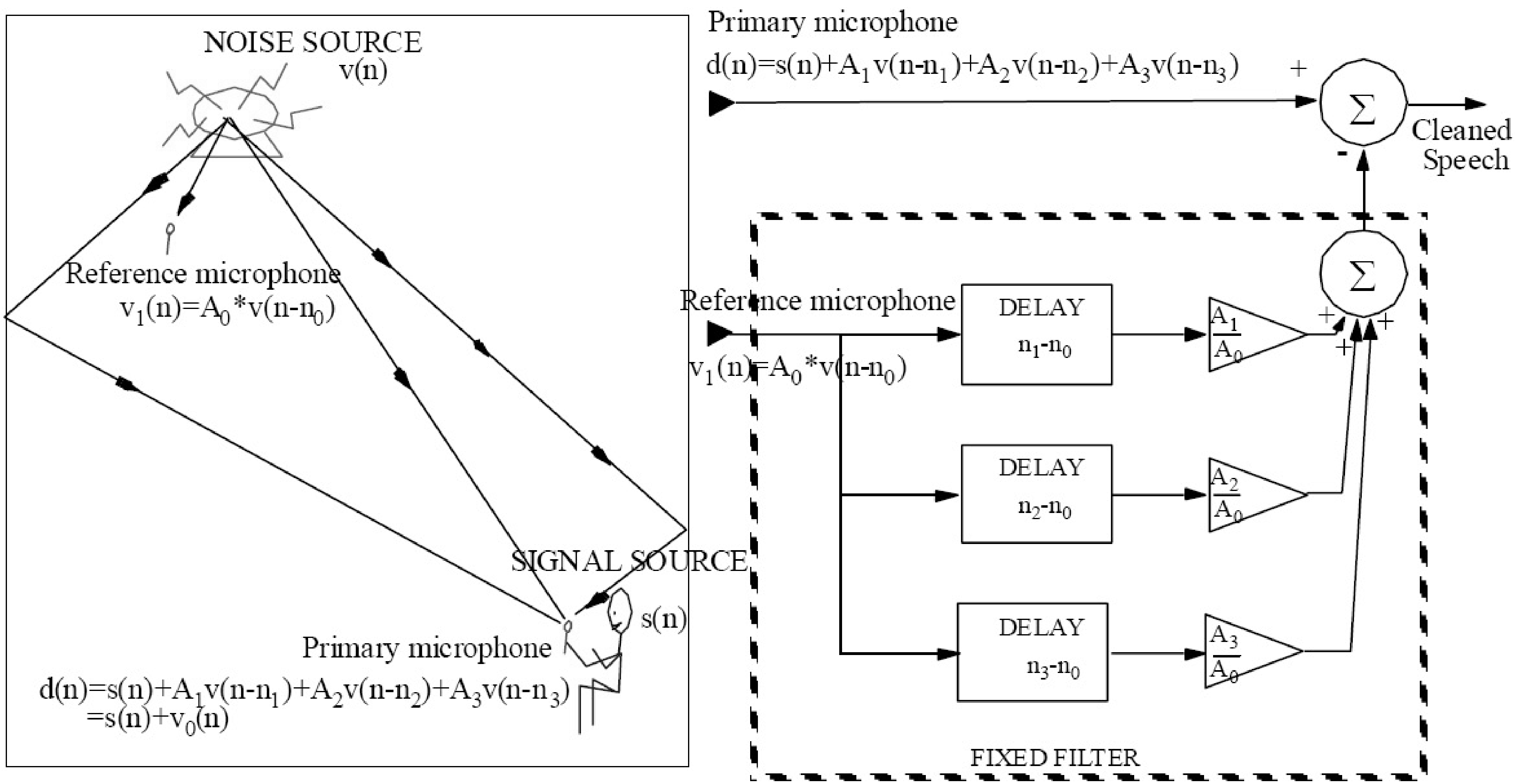
  - $v_1(n)$ comes from a reference sensor:

- Hypothesis:

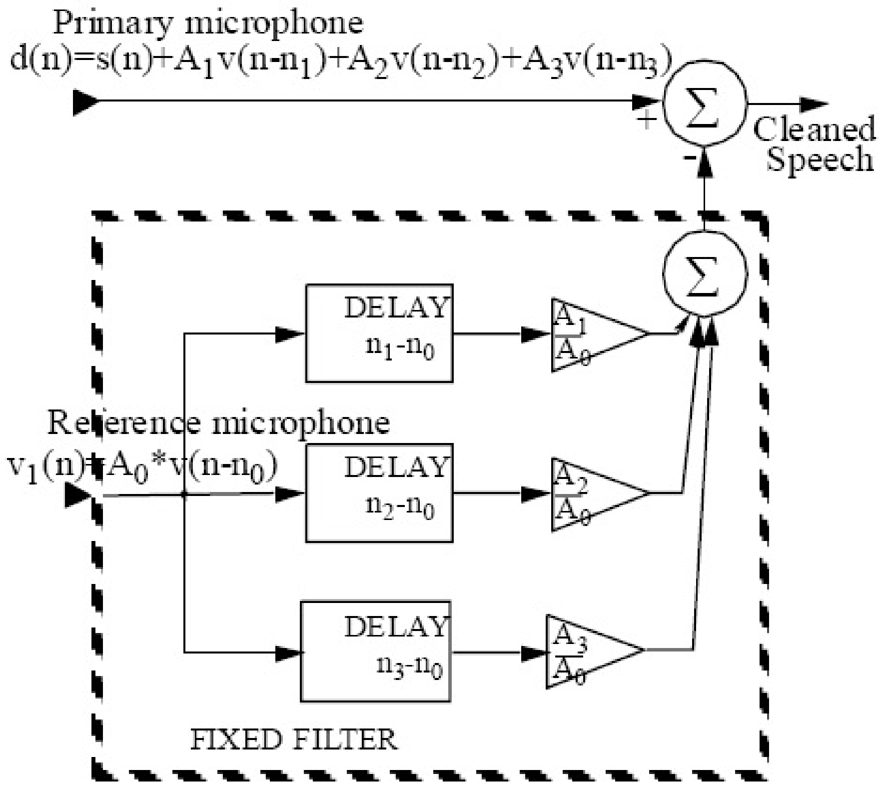
  * The ideal signal $s(n)$ is not correlated with the noise sources $v_0(n)$ and $v_1(n)$;

  $$Es(n)v_0(n-k) = 0, \quad Es(n)v_1(n-k) = 0, \quad \text{for all } k$$

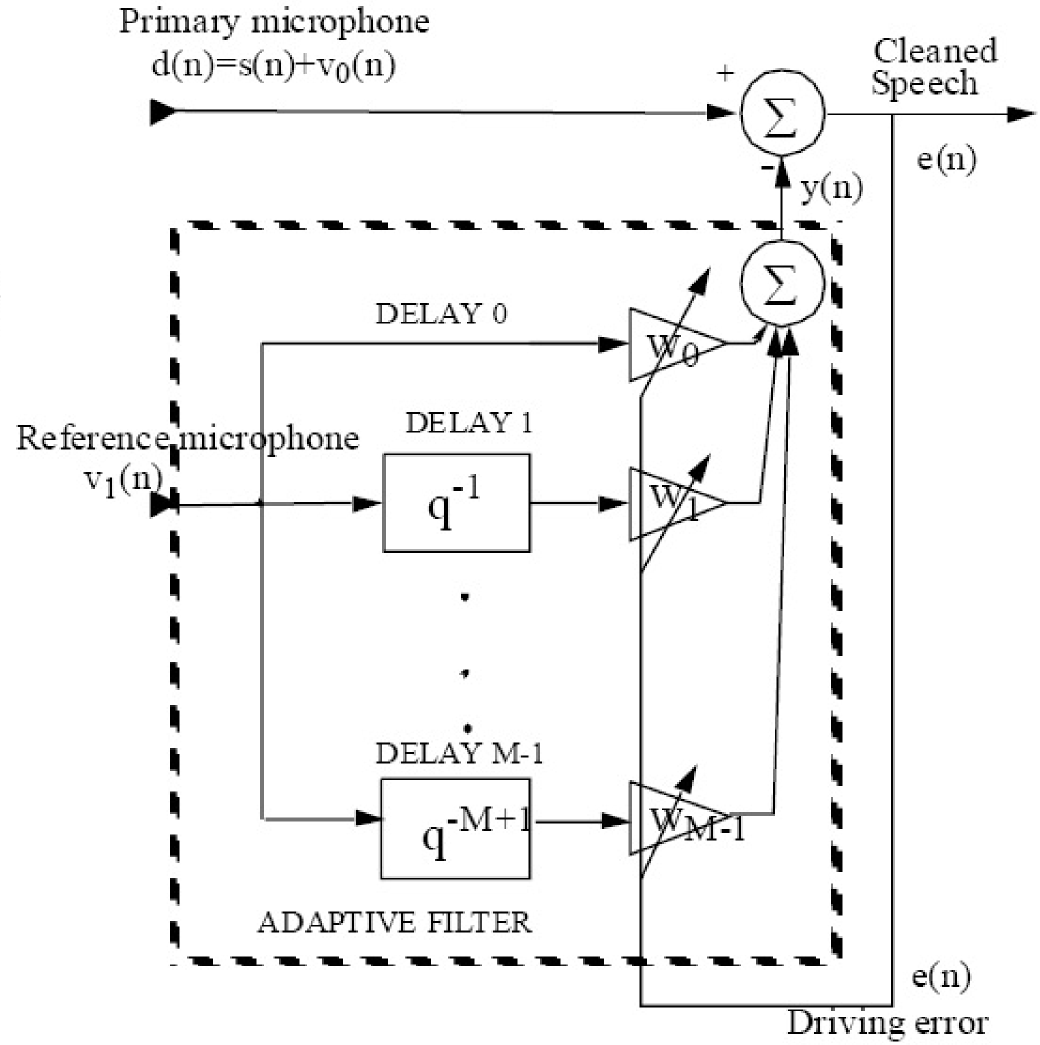
  * The reference noise $v_1(n)$ and the noise $v_0(n)$ are **correlated**, with unknown crosscorrelation $p(k)$,
    $Ev_0(n)v_1(n-k) = p(k)$



NOISE CANCELLATION WITH A FIXED FILTER



FIXED FILTER NOISE CANCELLING



ADAPTIVE NOISE CANCELLING

- Description of adaptive filtering operations, at any time instant, n:

  * The reference noise $v_1(n)$ is processed by an adaptive filter, with time varying parameters $w_0(n), w_1(n), \ldots, w_{M-1}(n)$, to produce the output signal

$$y(n) = \sum_{k=0}^{M-1} w_k(n) v_1(n-k)$$

.

  * The error signal is computed as $e(n) = d(n) - y(n)$.

  * The parameters of the filters are modified in an adaptive manner. For example, using the LMS algorithm (the simplest adaptive algorithm)

$$w_k(n+1) = w_k(n) + \mu v_1(n-k) e(n) \qquad (LMS)$$

where μ is the adaptation constant.

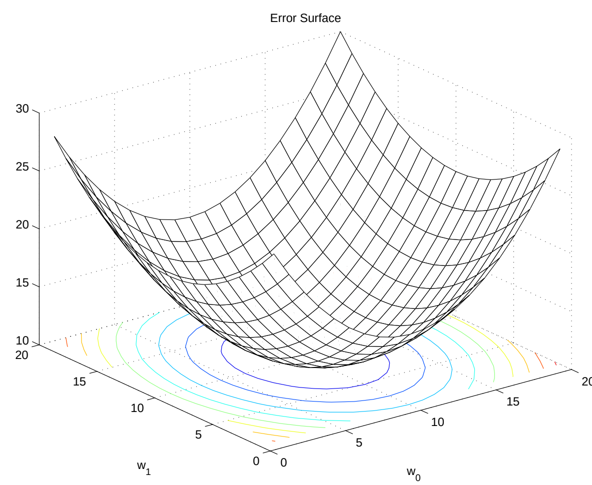
- Rationale of the method:

  * $e(n) = d(n) - y(n) = s(n) + v_0(n) - y(n)$
  * $Ee^2(n) = Es^2(n) + E(v_0(n) - y(n))^2$ (follows from hypothesis: Exercise)
  * $Ee^2(n)$ depends on the parameters $w_0(n), w_1(n), \ldots, w_{M-1}(n)$
  * The algorithm in equation (LMS) modifies $w_0(n), w_1(n), \ldots, w_{M-1}(n)$ such that $Ee^2(n)$ is minimized
  * Since $Es^2(n)$ does not depend on the parameters $\{w_k(n)\}$, the algorithm (LMS) minimizes $E(v_0(n) - y(n))^2$, thus statistically $v_0(n)$ will be close to $y(n)$ and therefore $e(n) \approx s(n)$, ($e(n)$ will be close to $s(n)$).
  * Sketch of proof for Equation (LMS)
    · $e^2(n) = (d(n) - y(n))^2 = (d(n) - w_0 v_1(n) - w_1 v_1(n-1) - \ldots w_{M-1} v_1(n - M + 1))^2$

    · The square error surface

    $$e^2(n) = F(w_0, \ldots, w_{M-1})$$

    is a paraboloid.



    · The gradient of square error is $\nabla_{w_k} e^2(n) = \frac{de^2(n)}{dw_k} = -2e(n)v_1(n-k)$

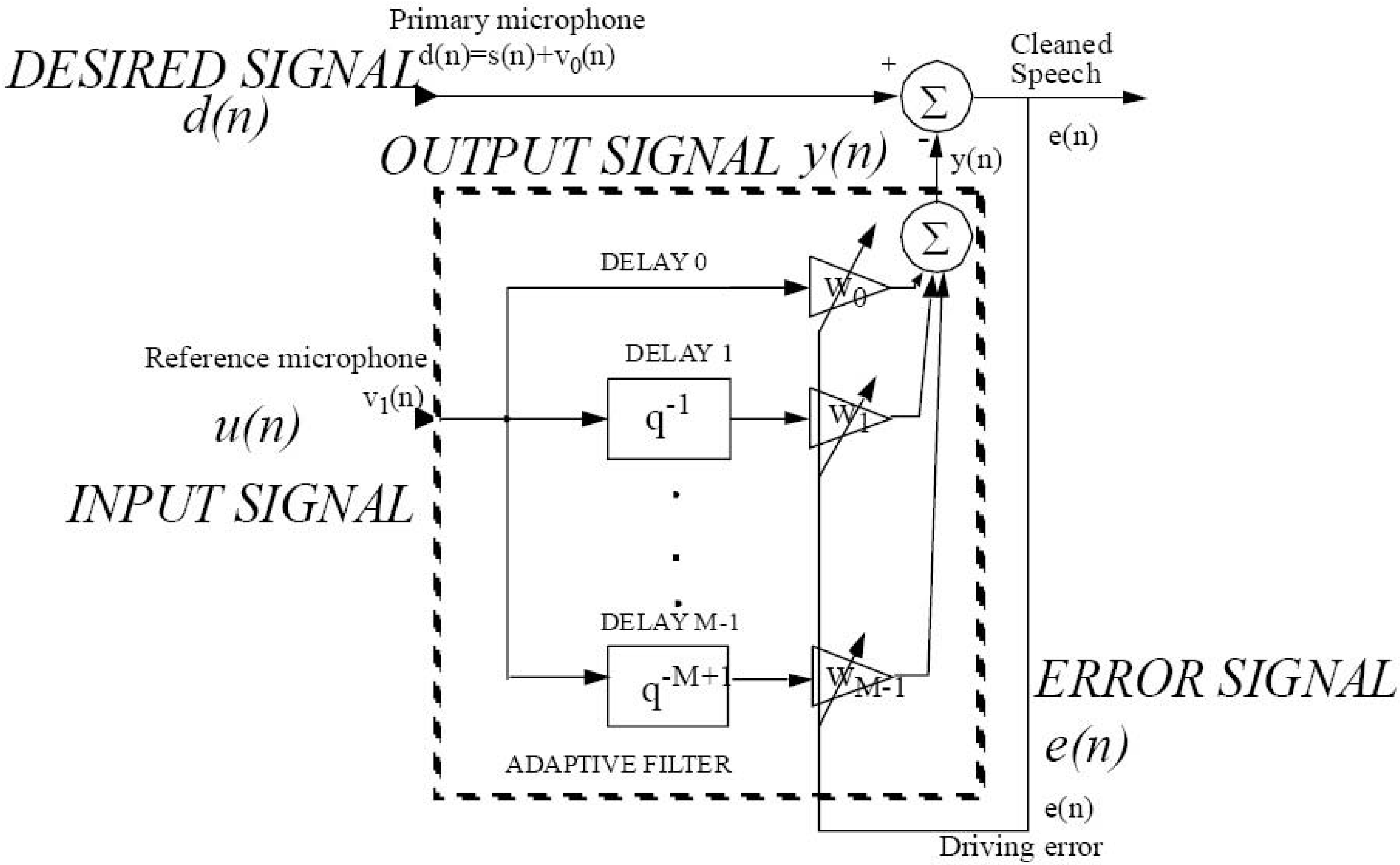
- The method of gradient descent minimization: $w_k(n+1) = w_k(n) - \mu \nabla_{w_k} e^2(n) = w_k(n) + \mu v_1(n-k)e(n)$

* Checking for effectiveness of Equation (LMS) in reducing the errors

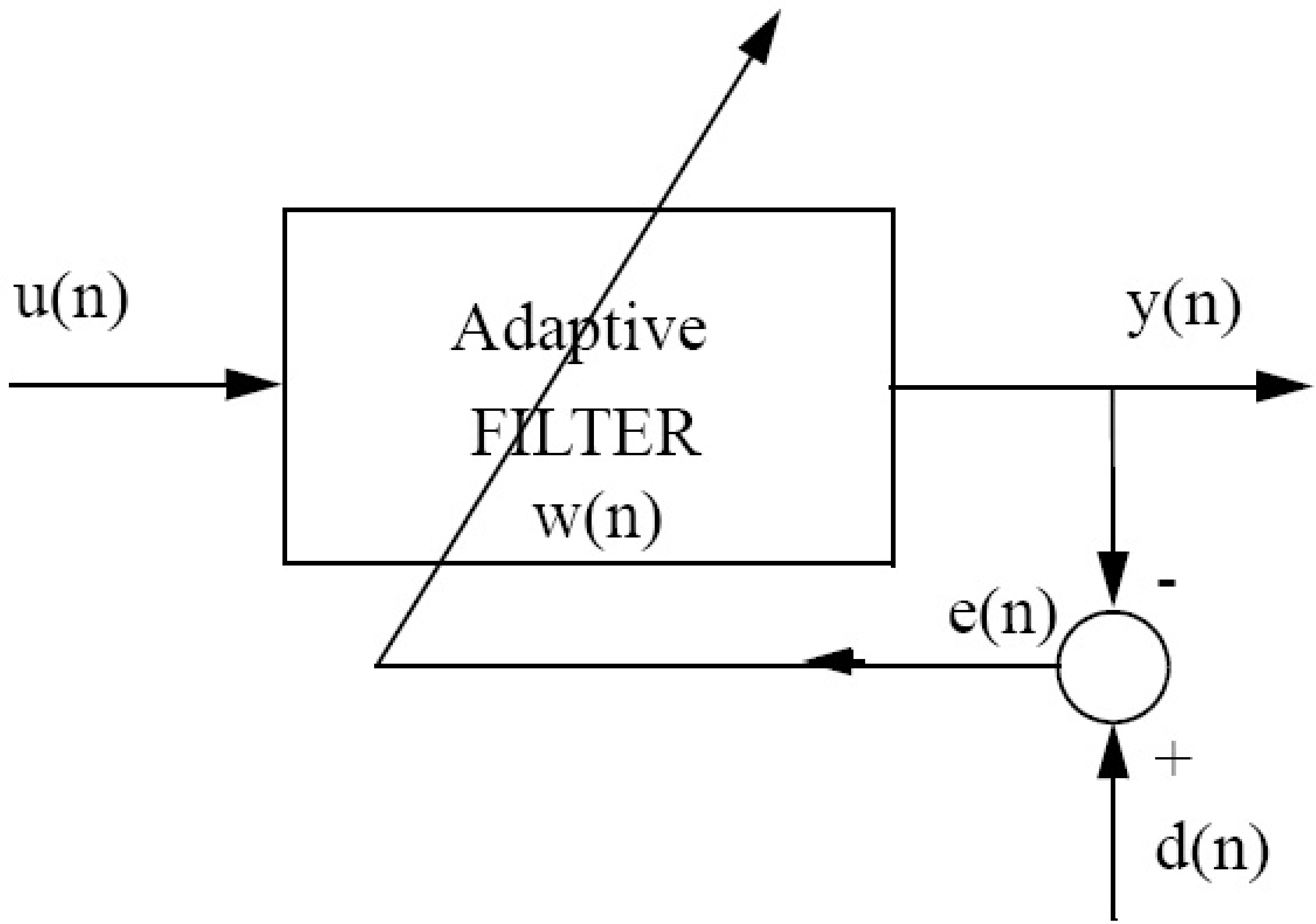
$$
\begin{aligned}
\varepsilon(n) &= d(n) - \sum_{k=0}^{M-1} w_k(n+1) v_1(n-k) \\
&= d(n) - \sum_{k=0}^{M-1} (w_k(n) + \mu v_1(n-k)e(n)) v_1(n-k) \\
&= d(n) - \sum_{k=0}^{M-1} w_k(n) v_1(n-k) - e(n)\mu \sum_{k=0}^{M-1} v_1^2(n-k) \\
&= e(n) - e(n)\mu \sum_{k=0}^{M-1} v_1^2(n-k) \\
&= e(n)(1 - \mu \sum_{k=0}^{M-1} v_1^2(n-k))
\end{aligned}
$$

In order to reduce the error by using the new parameters, $w(n+1)$

$$
\begin{aligned}
|\varepsilon(n)| &< |e(n)| \\
0 < \mu &< \frac{2}{\sum_{k=0}^{M-1} v_1^2(n-k)}
\end{aligned}
$$



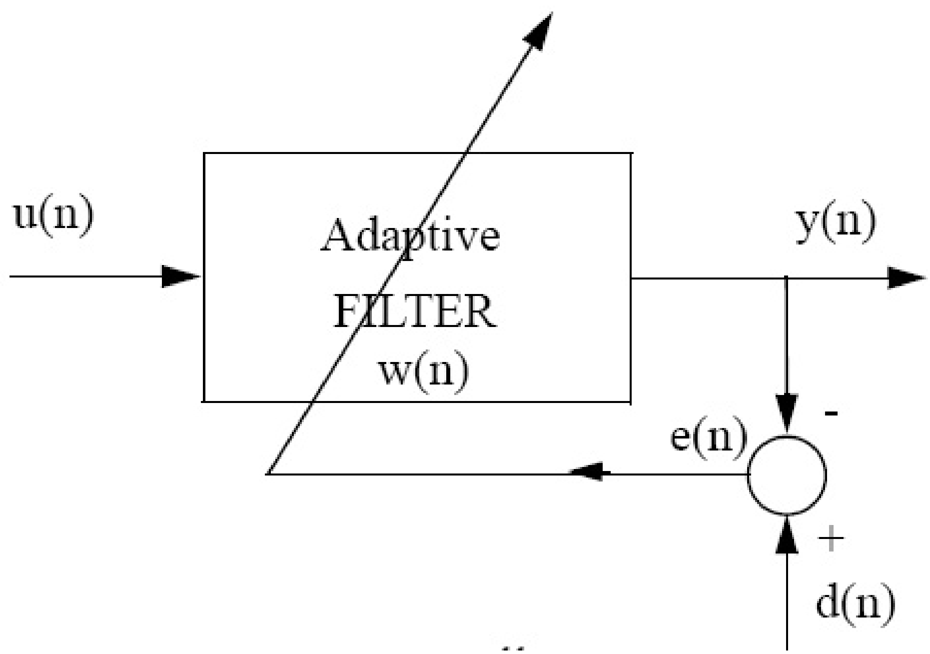
ADAPTIVE NOISE CANCELLING

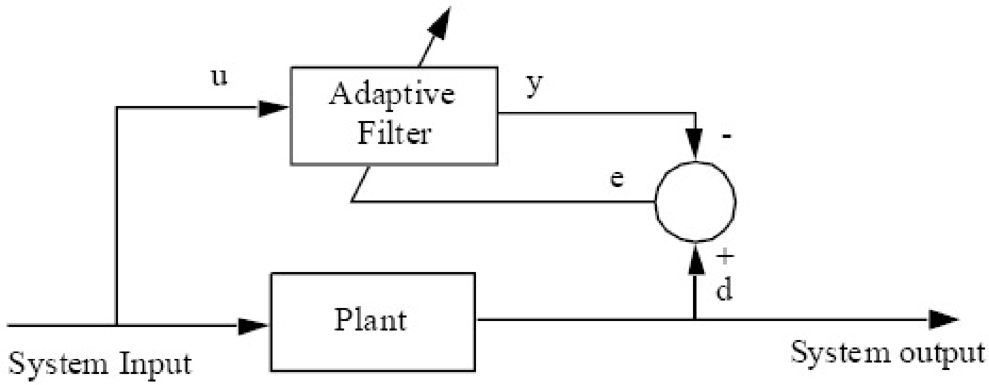

u(n)
Adaptive
FILTER
w(n)
y(n)
-
e(n)
+
d(n)

Applications using adaptive filters

We have to define: who is the input signal into the filter u(t)
what type of filter structure is used to compute y(t)
who is the desired (ideal) signal d(t) (against which we compare y(t))

Seldom the filter output, y(t), is useful by itself.
We may need only the parameters w(n) or the error signal e(n)

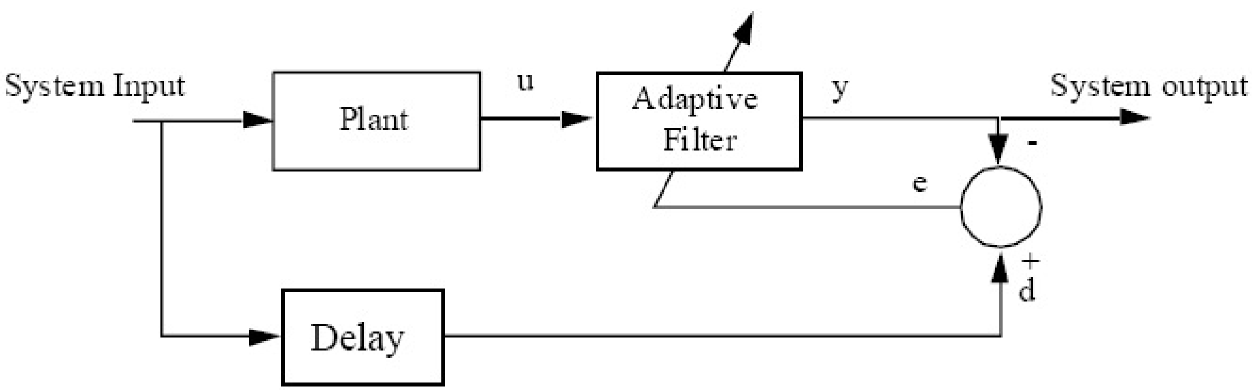




IDENTIFICATION

SYSTEM
IDENTIFICATION

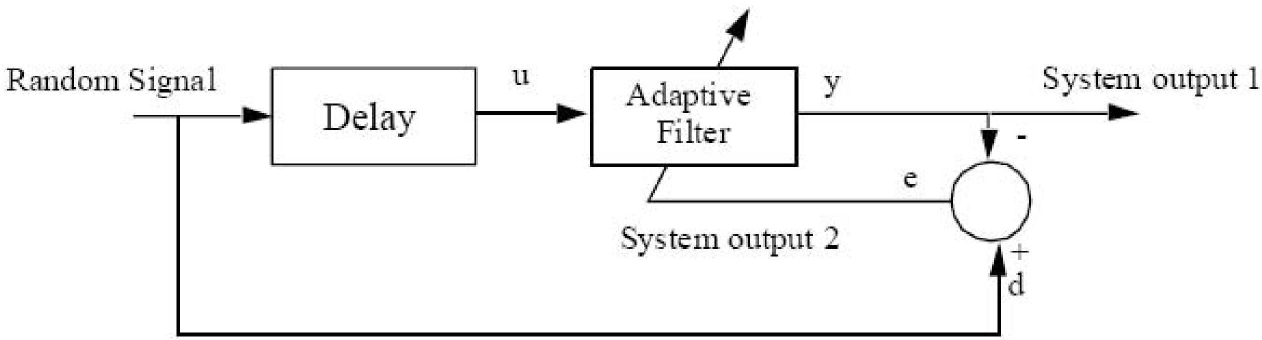
LAYERED EARTH
MODELLING



INVERSE MODELLING

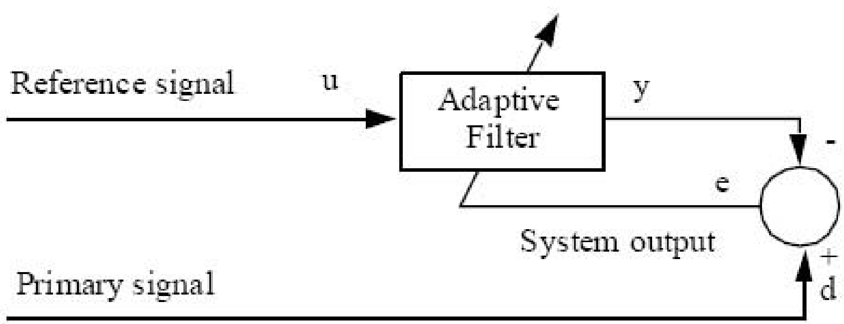
PREDICTIVE
DECONVOLUTION

ADAPTIVE
EQUALIZATION



PREDICTION

- LINEAR PREDICTIVE CODING
- ADPCM
- AUTOREGRESSIVE SPECTRUM ANALYSIS
- SIGNAL DETECTION



INTERFERENCE CANCELLING

- ADAPTIVE NOISE CANCELLING
- ECHO CANCELLATION
- RADAR POLARIMETRY
- ADAPTIVE BEAMFORMING