1 Algorithms for Lecture 11

Algorithm 1.1

LATTICE ALGORITHM FOR PREDICTION ERRORS		Number of operations/ equation			Number of operations/ Step
		$\star$	/	+	
1. Initialize at $(N=0)$					
For $n = 0, \ldots, n_{max}-1$					
$\alpha_n(0) = 0$					
$\sigma_0(0) = \sigma_0^b(0) = 0$					
2. Recursions in time					$\mathcal{O}(n_{max}^2)$
For $N = 1, 2, \ldots, N_{max}$					
2.1 Initialize recursions in order					
$e_0^f(N) = \varepsilon_0^f(N) = y(N)$					
$e_0^b(N) = \varepsilon_0^b(N) = y(N-1)$					
$\sigma_0(N) = \sigma_0(N-1) + \varepsilon_0^{f2}(N)$	(1)	1	0	1	
$\sigma_0^b(N) = \sigma_0(N-1)$	(2)				
2.2 Recursions in order:					$\mathcal{O}(n_{max})$
For $n = 0, \ldots, n_{max}-1$					
$\alpha_n(N) = \alpha_n(N-1) + e_n^f(N)\varepsilon_n^b(N)$	(3)	1	0	1	
$K_{n+1}^f(N) = -\alpha_n(N)/\sigma_n^b(N)$ $(\star)$	(4)	0	1	0	
$K_{n+1}^b(N) = -\alpha_n(N)/\sigma_n(N)$ $(\star)$	(5)	0	1	0	
$\sigma_{n+1}(N) = \sigma_n(N)(1 - K_{n+1}^f(N)K_{n+1}^b(N))$	(6)	2	0	1	
$\sigma_{n+1}^b(N) = \sigma_n^b(N-1)(1 - K_{n+1}^f(N-1)K_{n+1}^b(N-1))$	(7)	1	0	0	
$e_{n+1}^f(N) = e_n^f(N) + K_{n+1}^f(N-1)e_n^b(N)$	(8)	1	0	1	
$e_{n+1}^b(N) = e_n^b(N-1) + K_{n+1}^b(N-2)e_n^f(N-1)$	(9)	1	0	1	
$\varepsilon_{n+1}^f(N) = \varepsilon_n^f(N) + K_{n+1}^f(N)\varepsilon_n^b(N)$	(10)	1	0	1	
$\varepsilon_{n+1}^b(N) = \varepsilon_n^b(N-1) + K_{n+1}^b(N-1)\varepsilon_n^f(N-1)$	(11)	1	0	1	
$(\star)$: If $\sigma_n^b(N) = 0$ assign $K_{n+1}^f(N) = 0$					
If $\sigma_n(N) = 0$ assign $K_{n+1}^b(N) = 0$					
Total: $8n_{max}$ multiplications; $2n_{max}$ divisions;$6n_{max}$ additions					

Extension of Algorithm 1.1

OPTIONAL SECTION FOR ORDER RECURSION OF PREDICTORS	Number of operations/ equation		
	$\star$	$/$	$+$
2.3 If the predictors are desired at time N			
Do:			
2.3.1 Initialize order recursions:			
$\theta_1(N) = -\alpha_0(N)/\sigma_0^b(N)$		1	
$\theta_1^b(N) = -\alpha_0(N-1)/\sigma_0(N-1)$		1	
$K_1^*(N) = -\varepsilon_0^b(N)/\sigma_0^b(N)$		1	
2.3.2 Order recursions:			
For $n = 1, \ldots, n_{max} - 1$			
$\theta_{n+1}(N) = \begin{bmatrix} \theta_n(N) \\ 0 \end{bmatrix} - \frac{\alpha_n(N)}{\sigma_n^b(N)} \begin{bmatrix} \theta_n^b(N) \\ 1 \end{bmatrix}$	n	1	n
$\theta_{n+1}^b(N) = \begin{bmatrix} 0 \\ \theta_n^b(N) \end{bmatrix} - \frac{\alpha_n(N-1)}{\sigma_n(N-1)} \begin{bmatrix} 1 \\ \theta_n(N) \end{bmatrix} - - (e_n^b(N) - \frac{\alpha_n(N-1)}{\sigma_n(N-1)} e_n^f(N)) \begin{bmatrix} 0 \\ K_n^*(N) \end{bmatrix}$	$2n+1$	1	$2n+1$
$K_{n+1}^*(N) = \begin{bmatrix} K_n^*(N) \\ 0 \end{bmatrix} - \frac{\varepsilon_n^b(N)}{\sigma_n^b(N)} \begin{bmatrix} \theta_n^b(N) \\ 1 \end{bmatrix}$	n	1	n
Total: $\mathcal{O}(n_{max}^2)$ multiplications; $\mathcal{O}(n_{max})$ divisions;$\mathcal{O}(n_{max}^2)$additions			

Algorithm 1.2

LATTICE ALGORITHM FOR PREDICTION ERRORS AND PREDICTORS		Num. op. $\star$	$/$	$+$
1. Initialize at time $(N=0)$				
For $n=0,\dots,n_{max}-1$ $\quad \alpha_n(0)=0$				
$\sigma_0(0)=\sigma_0^b(0)=0$				
2. Time recursions				
For $N=1,2,\dots,N_{max}$				
2.1 Initialize order recursions				
$\varepsilon_0^f(N)=y(N);\quad \varepsilon_0^b(N)=y(N-1)$				
$\sigma_0(N)=\sigma_0(N-1)+\varepsilon_0^{f2}(N);\quad \sigma_0^b(N)=\sigma_0(N-1)$		1	0	1
$\gamma_0^0(N)=1$				
2.2 Recursions in order:				
ntru $n=0,\dots,n_{max}-1$				
$\alpha_n(N)=\alpha_n(N-1)+\varepsilon_n^f(N)\varepsilon_n^b(N)/\gamma_n^0(N)$	(1)	1	1	1
$K_{n+1}^f(N)=-\alpha_n(N)/\sigma_n^b(N)\quad(\star)$	(2)	0	1	0
$K_{n+1}^b(N)=-\alpha_n(N)/\sigma_n(N)\quad(\star)$	(3)	0	1	0
$\sigma_{n+1}(N)=\sigma_n(N)(1-K_{n+1}^f(N)K_{n+1}^b(N))$	(4)	2	0	1
$\sigma_{n+1}^b(N)=\sigma_n^b(N-1)(1-K_{n+1}^f(N-1)K_{n+1}^b(N-1))$	(5)	1	0	0
$\varepsilon_{n+1}^f(N)=\varepsilon_n^f(N)+K_{n+1}^f(N)\varepsilon_n^b(N)$	(6)	1	0	1
$\varepsilon_{n+1}^b(N)=\varepsilon_n^b(N-1)+K_{n+1}^b(N-1)\varepsilon_n^f(N-1)$	(7)	1	0	1
$\gamma_{n+1}^0(N)=\gamma_n^0(N)-\varepsilon_n^{b2}(N)/\sigma_n^b(N)\quad\star$	(8)	1	0	1
2.3 If the predictors are needed at time N				
Then:				
2.3.1 Initialize order recursions:				
$\theta_1(N)=K_1^f(N);\quad \theta_1^b(N)=K_1^b(N-1)$				
$K_1^*(N)=-\varepsilon_0^b(N)/\sigma_0^b(N)$	(9)		1	
2.3.2 Recursions in order:				
For $n=1,\dots,n_{max}-1$				
$\theta_{n+1}(N)=\begin{bmatrix}\theta_n(N)\\0\end{bmatrix}+K_{n+1}^f(N)\begin{bmatrix}\theta_n^b(N)\\1\end{bmatrix}$	(10)	n	0	n
$\theta_{n+1}^b(N)=\begin{bmatrix}0\\\theta_n^b(N)\end{bmatrix}+K_{n+1}^b(N-1)\begin{bmatrix}1\\\theta_n(N)\end{bmatrix}-$				
$-\frac{(\varepsilon_n^b(N)+K_{n+1}^b(N-1)\varepsilon_n^f(N))}{\gamma_n^0(N)}\begin{bmatrix}0\\K_n^*(N)\end{bmatrix}$	(11)	$2n+1$	1	$2n+1$
$K_{n+1}^*(N)=\begin{bmatrix}K_n^*(N)\\0\end{bmatrix}-\frac{\varepsilon_n^b(N)}{\sigma_n^b(N)}\begin{bmatrix}\theta_n^b(N)\\1\end{bmatrix}$	(12)	n	1	n
$(\star)$: If any of $\sigma_n(N),\sigma_n^b(N)$ sau $\gamma_n^0(N)$ are zero, set the corresponding division results to 0 0.				