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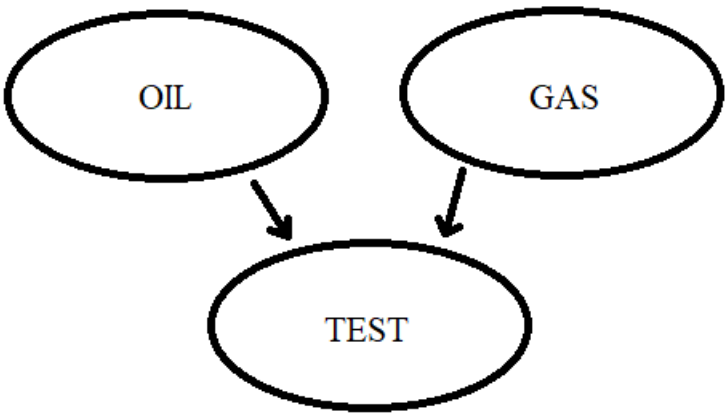
CS 161: Fundamentals of Artificial Intelligence
Assignment 6 – March 22, 2018

1a.




| Oil | P(Oil) |
|---|---|
| T | 0.5 |
| F | 0.5 |

| Gas | P(Gas) |
|---|---|
| T | 0.2 |
| F | 0.8 |

| Test | Oil | Gas | P(Test \| Oil, Gas) |
|---|---|---|---|
| T | F | F | 0.1 |
| T | F | T | 0.3 |
| T | T | F | 0.9 |
| T | T | T | 0 |

1b. $P(Oil|Test) = \frac{P(Test|Oil)*P(Oil)}{P(Test)} = \frac{0.9*0.5}{P(Test)}$ where $P(Test) = \sum_i P(Test, \{Oil, \sim Oil\})$.

Hence, $P(Test) = P(Test, Oil) + P(Test, \sim Oil)$
$= P(Test|Oil) * P(Oil) + P(Test|\sim Oil) * P(\sim Oil)$
$= 0.9 * 0.5 + 0.4 * 0.5$
$= 0.65$

This yields $P(Oil|Test) = \frac{P(Test|Oil)*P(Oil)}{P(Test)} = \frac{0.9*0.5}{0.65} = 0.69$

2a. $P(A,B,C,D,E,F,G,H)$
$= P(A) * P(B) * P(C|A) * P(D|A,B) * P(E|B) * P(F|C,D) * P(G|F) * P(H|F,E)$

2b. $P(E,F,G,H) = P(E|B) * P(F|C,D) * P(G,F) * P(H|F,E)$

2c. $P(a,\sim b,c,d,\sim e,f,\sim g,h)$
$= 0.2 * 0.3 * P(c|a) * 0.6 * 0.1 * P(f|c,d) * P(\sim g|f) * P(h|f,\sim e)$
$= 0.0036 * P(c|a) * P(f|c,d) * P(\sim g|f) * p(h|f,\sim e)$

2d. By definition of independence,
$P(a,\sim b) = P(a) * P(b) = 0.2 * 0.3 = 0.06$

Because node $e$ is independent of node $a$ due to parent $b$:
$P(\sim e|a) = \frac{P(\sim e,a)}{P(a)} = \frac{P(\sim e)*P(a)}{P(a)} = P(\sim e)$

Now, note that node $e$ is dependent on its parent $b$. By Law of Total Probabilities, partitioning on $b$, we have: $P(\sim e) = P(\sim e,b) + P(\sim e,\sim b)$.

By conditioning, $P(\sim e) = (\sim e,b) * P(b) + P(\sim e,\sim b) * P(\sim b)$
$= 0.9 * 0.7 + 0.1 * 0.3$
$= P(\sim e|a) = 0.66$

2e. Markovian assumptions:

- $I(A,\emptyset,BE)$
- $I(B,\emptyset,AC)$
- $I(C,A,DBE)$
- $I(D,AB,CE)$
- $I(E,B,ACDFG)$
- $I(F,CD,ABE)$
- $I(G,F,ABCDEH)$
- $I(H,EF,ABCDG)$

2f. Markov blanket for $D$: $\{A,B,C,F\}$

2g.

| $A$ | $B$ | $D$ | $P(D\|AB)$ |
|---|---|---|---|
| F | F | F | .5 |
| F | F | T | .5 |
| F | T | F | .6 |
| F | T | T | .4 |
| T | F | F | .1 |
| T | F | T | .9 |
| T | T | F | .8 |
| T | T | T | .2 |

| $B$ | $E$ | $P(E\|B)$ |
|---|---|---|
| F | F | .1 |
| F | T | .9 |
| T | F | .9 |
| T | T | .1 |

| $A$ | $B$ | $D$ | $E$ | $P(D\|AB) * P(E\|B)$ |
|---|---|---|---|---|
| F | F | F | F | .05 |
| F | F | F | T | .45 |
| F | F | T | F | .05 |
| F | F | T | T | .45 |
| F | T | F | F | .54 |
| F | T | F | T | .06 |
| F | T | T | F | .36 |
| F | T | T | T | .04 |
| T | F | F | F | .01 |
| T | F | F | T | .09 |
| T | F | T | F | .09 |
| T | F | T | T | .81 |
| T | T | F | F | .72 |
| T | T | F | T | .08 |
| T | T | T | F | .18 |
| T | T | T | T | .02 |

2h. Assign $x(A, B, D, E) = P(D|A, B) * P(E|B)$

$x(A, B, E) = x(A, B, d, E) + x(A, B, \sim d, E)$

| $A$ | $B$ | $E$ | $x(A, B, E)$ |
|---|---|---|---|
| T | T | T | .1 |
| T | T | F | .9 |
| T | F | T | .9 |
| T | F | F | .1 |
| F | T | T | .1 |
| F | T | F | .9 |
| F | F | T | .9 |
| F | F | F | .1 |