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**Abstract**

1.5 Time series analysis is an important method and applied research field based on stochastic process theory and mathematical statistics. According to its statistical characteristics, it can be divided into stationary sequence and non-stationary sequence. The ARMA model is the most commonly used model for fitting stationary time series and has a wide range of applications in the financial field.

1.5 It is well known that for the estimation of parameters of the ARMA model, the most widely used maximum likelihood (ML) estimation and least squares (LS) estimation are currently used. For ML estimation and LS estimation, a better estimation effect can be obtained only under the condition that the error variance of the model exists. But financial data usually has a heavy-tailed nature. The variance of these data may be infinite. In this case, using ML estimation and LS estimation is not robust enough. In view of this, scholars have proposed to replace ML estimates and LS estimates with least squares (LAD) or M-estimators, where LAD estimates are a special form of M-estimation. Although the M-estimation does not need to assume that the error variance exists and has stronger robustness than the ML estimation and the LS estimation, it essentially gives the abnormal point the same weight as the normal point, which cannot effectively reduce the extreme value and the influence of the high leverage point, so that the asymptotic normality cannot be guaranteed in some cases.

1.5 In view of the problems existing in the above estimation methods, based on the M-estimation, different weights are given according to the difference of the data, and the SM-estimation of the ARMA model is proposed. The SM-estimation can not only effectively reduce the influence of the anomaly point, but also obtain the consistency and asymptotic normality of the estimated parameters under the condition that the error variance is infinite. Since we do not know whether the model has conditional heteroscedasticity in parameter estimation, we need to use SM-estimation to estimate the parameters of the heavy-tailed ARMA model under homoskedasticity and heteroscedasticity respectively, and allow the error variance to be infinite. Under the conditions, the global consistency and asymptotic normality of SM-estimation are proved. The data of the heavy-tailed homoscedastic ARMA model and the heavy-tailed heteroscedastic ARMA model are simulated. The SM-estimator, LS estimation and LAD estimation are compared under the condition that the error obeys several heavy-tailed distributions and standard normal distributions and there are abnormal points. The effect was found that the SM-estimated MSE was minimal and robust. Finally, the SM-estimation is used to study the daily yield of the Shenzhen Stock Exchange Day, and it is found that the SM-estimated MAE and MSE are both smaller than the LS estimate and the LAD estimate.

1.8 [0pt]

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1. ??LS???LAD???????????ARMA????SM-???????????????????????????????????????????????????????

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Box-Jenkins????????????????????????5????

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????([[201]](#Xf03f3f2febc46f3fa832d98251b0c98f64bc19b))????????? ????????????? ?LS??

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M-??????????????Huber(1973)?????????????????????????????????????([[201]](#Xf03f3f2febc46f3fa832d98251b0c98f64bc19b))????????? ?????????M-????????

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M-?????LSE?????????????????????????????????LS?????M-???????????????????????????????M-??????????????????????????????????????????????M-????????????????????SM-???

### SM-??

????([[201]](#Xf03f3f2febc46f3fa832d98251b0c98f64bc19b))????????? ????? Wang? Hu (2017)???????????M-??(SM-??)?SM-????????

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   ???? ????([[301]](#X87d41d9c35c57ef9e4aba799bacefac312149a4))?????????????? ????????

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? ? ??????????([[307]](#Xe5ffa6599c837509139132ef41c7856f33a50f7)) ?([[308]](#Xe0f83cc51276227de3cfebca941faace8aaa317))?????

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?????([[309]](#Xd2efc1c05342a60c2198a5e96773a237008956b))?([[310]](#Xf53d4aa0b9131f18f84130767ee5b1dcbcb63be))???????

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????([[311]](#Xd6d91089c7e219a7f4e86e3c2d57f936d367689)) ?([[312]](#X93c168323147d1135503939396cac628dc194c5))???

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?? ?? ? ? ??? ?([[302]](#Xd0613ba91fbab0c5af2827e308e487e267d28a0)) ???? ???????

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  ??? 3.1-3.5 ????????????????????

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???([[313]](#X7b31c942deab39118e64423efe46b9c31e669f7))???????????????

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?????????????([[315]](#X6b9b6ccd0440bc448ae4b0267c316b751bcf826))?????????? ????????

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?????????????([[316]](#X1c69212880f2e985e1dedf869c2483ece723d68)) ??Chebyshev????????

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?([[317]](#X44a286f486d11990238c4ae59a9b4f8b7a9edf4))?([[318]](#X54a31bf7fa827bce90915439a7a5f3aa0f2fa47))???

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?? ?? ???([[317]](#X44a286f486d11990238c4ae59a9b4f8b7a9edf4)) ?([[318]](#X54a31bf7fa827bce90915439a7a5f3aa0f2fa47)) ???????

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??????????????????????????([[320]](#Xfdec83a2662ffe53af456402cbaeafa380b15b4))?([[321]](#Xf6955d227a320c7f1f6c7da2a6d96a851a8118f))????([[314]](#Xe21fce62b88ee824118ee6f3d791d78a748f9a5))?????? 3.4???

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?([[324]](#X14127d1002e9938e2e1ee54cd92fd8195a248b0))?([[325]](#X551b2d552b0795735b70411d13d6b5ad82d6b8c))????????????

?????([[322]](#X1110df80ca4086e306c4c52ab485a35cf761acc))?([[323]](#Xb4dd52770e258826c4174c36202b18f649e262f))????(i)????

(ii) ????

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??Markov???????????????

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????([[326]](#X296abf737c7b57969d073b5dfa47c041e22782f))?([[327]](#X76e5a3a32038d0ba2d7e8fac5f6a5249a68e749))?????????

???????? (ii)??????

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(i) ???3.1??????3.4?3.5??([[305]](#Xa3d6127374af09c22015bf3ede3ac00a36e3ec6))???

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?([[329]](#Xd396f9047754b91e68d992920b521280cd9d351))???? 3.3?????

??(i)????

(ii) ??????3.1????????

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?([[328]](#X547f6312291149b2a3269420b59747949760a0d))?([[330]](#X609bb8ab05a0d13db9eb1d0d3b1383d1703c17d))????

?????([[326]](#X296abf737c7b57969d073b5dfa47c041e22782f))????? ?????

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???3.3?Ling(2007)???A.4???????

????? ??????????Minkowski?????

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?([[331]](#X28097ad29ab61bfec58d9b4de53bcdec687872e))?([[332]](#Xf2afd226e3384e34d9833fe09cd123db498754c))??? ????? Durrett (2005) ? Kronecker?????????3.6 ???

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????([[334]](#Xffb8e85796e61b713c68833d9f84ef0958681aa))?([[336]](#Xc882d5c6a1aa240b2672dce0ffb03360abcaca5))?([[337]](#X588f59014485510aac3bd9fdc2695c7b2a0d559))???(i)????

(ii) ?? ??([[331]](#X28097ad29ab61bfec58d9b4de53bcdec687872e))????????3.3???3.1(ii)??Minkowski????Hlder?????????

??(ii)????

(iii) ?????

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??([[339]](#Xc41101fe24a8f80c8cb51781f9e9ecf8c7a4d39))?([[340]](#Xe6bf6c89ba8a8b8b189f85975b0fab42bdc6d4a))????????????????

?Ling(2007)???A.4??????????????????([[335]](#X728a26498a7441a03a375eb0280b65c24a8cbd9))???????

?????????([[341]](#Xda4dabfaeb4a44681c9777c85db39140e3e12e6))??????(iii)????

(iv) ?Ling(2007)???A.4?????

??([[342]](#X415505dca69be631ca5d391b3ccd2b44b52d017))??????(i)?(iii)??????(iv)????

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(ii) ????([[305]](#Xa3d6127374af09c22015bf3ede3ac00a36e3ec6)) ?????

?(i)???????3.7?????

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?3.1-3.4??????????????????????????????????200?400???????????????5%??????????1000 ??LS ???LAD ???????????bias??????MSE??????????SLAD??? ???S-Huber???

??3.1-3.4??????????????? ???  ?????MSE?????LS???LAD ??????????bias??????0?? ??????????????????????????SM-?????????????????????????????200 ???400 ?? ??? ?????MSE?????LS???LAD????????MSE???????????????SM-????????????????????3.2?t(2)????3.4??????????3.1?????3.3 ????????????3.2? ?3.4 ??3.1? ?3.3 ????????????? ??? ????????????????

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   ???? ????([[401]](#X3b4f91cf8f3dcf5b300797302b9953cc8333368))?????????????? ????????

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   ? ????????????????? (i) ?? (ii) ??????????????????? (iii) ???????????????????

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   ?? 4.1 ???([[401]](#X3b4f91cf8f3dcf5b300797302b9953cc8333368)) ????????????????4.2 ?????????????????ARCH????????? ??3.2??? SM-?????????????????????????????????2.2.3???????????????????????4.4 ????????????????????????????4.5????? ??????????????????????????????????

?????????????

?????????????4.4(i)?????

??????

??????? ???????????????? ????????????? ? ??? ?? ??????? ?????????????([[403]](#X980dc86c1e9ea324ec99f8b15a2e9a92d0a4d1e)) ????

? ? ???????? ????????????

?????????SM-?????????

  ???? 4.1-4.5 ?????

?????4.1??????????SM-???????????

  ???? 4.1-4.5 ????? (i) , (ii) ???

## ????

?????????4.1-4.2???????????????4.1????????????????????

  ? ??? 4.3 ????????? 4.1?4.2 ???????? ????????????? (i) ? (ii) ? (iii) ?

  ????Ling(2007)????A.1????

  ??? 4.1-4.4 ???? 4.1????????

?? ?? ? ???? ????

????????? ????????([[402]](#Xe63673cbc075e2d21cef6bde4c591c263380bee)) ?????

???

????

???????????????????([[404]](#X35a9fc52bb556c79f8fa540df587a2bf465b940))???

?????????

???????????????????????????????????

??????

??????????????????3.3(i)??????????4.1????

?????????4.2????????????????????

  ???4.1-4.5???????

  ???????????

??

????????4.4? ??4.1(ii)??Minkowski?????

?Ling(2007)???A.4?????

????([[405]](#Xee51d9582ef3d3b56ec2fc25b77fc147d8563e5))??????

????????4.4-4.5? ??4.1?Minkowski?????Hlder?????

?([[405]](#Xee51d9582ef3d3b56ec2fc25b77fc147d8563e5))-([[407]](#X6de89674d9f1d7968a70dad4f41b844965b4fdc))????

???Kronecker?????????4.2???

  ???4.1-4.5???????

  ??????

??

?Ling(2007)???A.4???4.1???

????([[405]](#Xee51d9582ef3d3b56ec2fc25b77fc147d8563e5))?????

?Ling(2007)???A.4??????

????([[407]](#X6de89674d9f1d7968a70dad4f41b844965b4fdc))??????

????????Kronecker?????????4.3???

??????????????????????4.2????

  ??????4.4????4.1???

???????????????? ???????????????????Chow?Teicher(1978)???1?????????

??????????????????

??([[408]](#Xeba4d5d3ffb8fac7fe5ce87ac1eb2f75c4cd1a2))?([[409]](#X352d0d8278c176fa61d82326d7e51dabd2a032e))??????????

????4.3?????????

?????([[410]](#X29dc1daf9fb9d5e75d687dd9e0740e1c72796c3))?([[411]](#X3fdc3407ccf68718bfb9aaddefa7cc0e40529db))???????????

??????4.1??

??????????????4.1???4.2 ??([[412]](#Xe9b99384f21dc46400b5347a3eb341f6daeb334)) ???

?([[404]](#X35a9fc52bb556c79f8fa540df587a2bf465b940))????????0?

?????Kolmogorov(1929)??????????

??4.2(i)?????????????([[413]](#X715aa02de07dc08f6197a5850b92d7640766663))??????????4.2(ii)???

## ????

???????????????????ARMA?????????? ???????SM- ?? ???????????????????????

????

?????????(4.15)(?GJR(1,1)??)???(4.16)(????GARCH??)???

??(4.14)?(4.15)?????????????????????? ?????(4.14)?(4.15)???????????? ??????(4.14)?(4.15)? ??????  (0.1,0.2,0.8,0.0) ? (0.1,0.1,0.8,0.3)??????????????(4.14)?(4.15)?? ??? (0.1,0.2,0.8,0.1)? (0.1,0.1,0.9,0.2)?????2.2.3??([[204]](#Xcc641954099c249e0e4ef0402da3fd0364d95f0))?([[205]](#Xf1cd7c3fb68ae7c679f8c33966610670d32ff1e))????????? ? 90% ?????????????R?????

?4.1-4.3??????????????????????????????????200?400???????????????5%??????????1000 ??LS ???LAD ???????????bias??????MSE??????????SLAD??? ??????Huber ???

??4.1-4.3????????????????? ?????MSE?????LS???LAD ??????????bias??????0?? ??????????????????????????SM-?????????????????????????????200 ???400?? ??? ?????MSE?????LS???LAD????????MSE???????????????SM-????????

## ????

????????????????????????????????????????????ARMA???SM-??????????LS???LAD?????????R????SM-???????????????????????LS???LAD?????????

????????2016?2?19??2019?5?31????????801??????????????????????????????????800????????

???????????????????4.1??4.4?

???????

???4.1????????????????????????????????????????????????????????????????

?????????????

| ?? | ?? | ??? | ??? | ??? | ??? | ?? | ?? | JB??? |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 800 | -0.0002 | -0.0001 | 0.0544 | -0.0786 | 0.0139 | -0.5569 | 4.0400 | 590.3197(0) |

???4.4?????????????????????????????????0??????????????????????3?????????????????JB?????????p???? 0????????????

??ADF??????????????????????DF??-8.632???0.01???????0.05???????????????????

???????Hill????????????

????????????????4.2????????????????????1?2???????????????????([[204]](#Xcc641954099c249e0e4ef0402da3fd0364d95f0))???SM-???????

????Hill?

??ARMA?????????????ARMA?????????ACF??PACF???????????

????ACF??PACF?

?4.3?????10???????????3????????3????????????2???????ACF???20??23??PACF???14???2?????????????????????????????????????????????? ARMA (0,1)? ARMA (0,2)? ARMA (0,3)? ARMA (1,0)? ARMA (1,1)? ARMA (1,2)? ARMA (1,3)? ARMA (2,0)? ARMA (2,1)? ARMA (2,2)? ARMA (2,3)? ARMA (3,0)? ARMA (3,1)? ARMA (3,2)? ARMA  (3,3)?????([[202]](#Xe7b95c5614637fdcde70eb7f2d109134c95c6bf))??AIC????????????ARMA???????LS???LAD???SLAD???S-Huber???????ARMA ????????ARMA(2,1)???

????????????? ARMA (2,1)?????????Box-Ljung??????????????????????4.5?????p???????????????????ARMA(2,1) ????????????

Box-Ljung??

|  | LS | LAD | SLAD | S-Huber |
| --- | --- | --- | --- | --- |
| Q(6) | 3.1792 | 4.9676 | 4.9276 | 3.6344 |
| p-value | 0.786 | 0.548 | 0.5531 | 0.726 |
| Q(12) | 8.1198 | 10.3 | 10.302 | 8.8337 |
| p-value | 0.7757 | 0.5897 | 0.5895 | 0.7171 |

?????4.4??QQ???????????????????????

??QQ?

????????R???ArchTest?????ARMA?????????6?12????????????4.6???????????????????????ARMA ?????????????

ArchTest??

|  | LS | LAD | SLAD | S-Huber |
| --- | --- | --- | --- | --- |
|  | 27.473 | 27.916 | 27.92 | 28.397 |
| p-value | 1.18e-04 | 9.74e-05 | 9.73e-05 | 7.91e-05 |
|  | 28.231 | 29.797 | 29.836 | 29.777 |
| p-value | 5.12e-03 | 3.00e-03 | 2.96e-03 | 3.02e-03 |

??????????ARMA(2,1)???????????????????????????????????GARCH(1,1) ????????GARCH(1,1)???????????????????????????????????ARMA(2,1)-GARCH(1,1)????800??????????????????????(MSE)???????(MAE)?????4.7?

???????????

|  | LS | LAD | SLAD | S-Huber |
| --- | --- | --- | --- | --- |
| MSE | 0.000442 | 0.000399 | 0.000379 | 0.000368 |
| MAE | 0.015286 | 0.014525 | 0.014151 | 0.013932 |

??????????S-Huber?????ARMA(2,1)-GARCH(1,1)??????????LS?????MSE???16.73%?MAE???8.85%??LAD?????MSE ???7.71%?MAE???4.08%???SLAD????????????????SM-???????????????S-Huber??????????ARMA(2,1)-GARCH(1,1)????

???????????

# ?????

## ??

?Box?Jenkins(1970)??????ARMA??????????????????????????????????????????????????????LS???LAD??????????????????????????????????????????????LS??????????LAD ???????????????????????????SM-???????????????????????????

?1????ARMA???????????????????LS???LAD???SM-????????????ARMA???SM-???????????ARMA???SM-???

?2?????????????ARMA????????????????????????????SM-?????????????

?3??????????????????SM-???LAD???LS?????????????????????????????????????????????????SM-??????????

?4?????????????????SM-???LAD????LS???????ARMA??????????GARCH??????????SM-??????????????????

## ??

?1???????ARMA???SM-?????????????????????????????????????????????????????????????SM-??????

?2????SM-?????????????SLAD???S-Huber???????????SM-????????????????Hampel ???????????????????????????SM- ?????????

?3???????????????????????????????SM-?????ARMA?????????????

1.2

99

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