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论文题目 **SC-FDMA Frequency Domain MMSE Equalizer**

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Abstract

Based on frequency domain oversampling and the Bayesian Gauss-Markov theorem, we propose a novel frequency domain oversampling MMSE equalization receiver for single carrier frequency division multiple access system. As shown in this dissertation, frequency domain oversampling MMSE equalizer greatly improves the BER performance in highly frequency selective channel by extracting frequency diversity. It is also robust against severe carrier frequency offset. Numerical results show that frequency domain oversampling scheme outperforms conventional scheme in SC-FDMA system with or without carrier frequency offset in highly frequency selective channel due to the highly extraction of frequency diversity. As number of channel path drops down to 1, oversampling scheme degenerates to conventional scheme due to there is no frequency diversity. In addition, it is shown that oversampling factor could be no larger than (N-Q+1)/N, where N is the subcarrier number and Q is the number of channel path components.

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1. INTRODUCTION

1.1 Background

Single carrier frequency division multiple access, known as SC-FDMA, which combines single carrier features and OFDMA properties, is a technique that has an equivalent performance and signal processing complexity as those of an OFDMA system. Comparing with OFDMA in the uplink communication, SC-FDMA shows better performance at the mobile terminal in terms of transmit power efficiency and terminal costs because of lower PAPR (Peak to Average Power Ratio). Meanwhile, frequency domain equalization (FDE), which is similar to the OFDMA system, also attracts researchers' attentions. SC-FDMA with FED in the receiver, which transfers complex channel equalization into simple point to point division, manages to deal with frequency selective fading channel effectively.

However, SC-FDMA, which is known as Pre-DFT OFDMA system, suffers greatly from carrier frequency offset (CFO) due to that CFO destroys the orthogonality among subcarriers, which is caused by oscillator instability, Doppler effect and imperfect carrier synchronization. Inter carrier interference (ICI), which is introduced by non-orthogonality among subcarriers, strongly decreases the system performance. Therefore carrier frequency offset compensation is critical for SC-FDMA system. In [1], the authors take a different approach, aiming at an OFDM receiver that is inherently robust against frequency offset. They mention that carrier frequency offset in multicarrier system and timing jitter in signal-carrier system is a dual problem, where timing jitter is traditionally solved via time domain oversampling techniques. Therefore frequency domain oversampling technique should be expected to be robust against carrier frequency offset. The results in [1] confirm the expectation and show that frequency domain oversampling techniques in OFDMA system extracts frequency diversity from a highly frequency selective channel. Therefore, frequency domain oversampling should also be expected to be robust against the CFO in SC-FDMA system.

1.2 Objectives

The objective of this project is to explore the performance improvement of frequency domain oversampling equalization in SC-FDMA system in several situations, by showing the bit error ratio (BER) performance.

1.3 Organization of Report

This report contains 7 chapters. Chapter 1 introduces the background, purpose and organization of this project. Chapter 2 is about the literature review of SC-FDMA system and of which talks about the frequency domain oversampling in OFDMA system. Chapter 3 is about the details overview of SC-FDMA system, including comparison with OFDMA system. Chapter 4 gives the Mathematic model of SC-FDMA with frequency domain oversampling MMSE equalizer (FDO-MMSE equalizer) and comparison with conventional MMSE equalizer. Chapter 5 shows the BER performance comparison between SC-FDMA system and OFDMA system considering conventional MMSE, FDO-MMSE, the effect of oversampling factor and oversampling gain in different fading channel. Chapter 6 gives the conclusion and recommendation of this report. Chapter 7 lists the references used in this project. Appendix A shows the Bayesian Gauss-Markov Theorem.

2. LITERATURE REVIEW

This chapter mainly discusses why SC-FDMA is so promising, and why frequency domain oversampling (FDO) should be expected in performance improvement of SC-FDMA system.

Orthogonal frequency division multiplexing (OFDM) could effectively cope with frequency selective channel by using cyclic prefix (CP) in the transmitter and frequency domain point to point equalization in the receiver due to the magic effect of cyclic prefix shown in [2], [3]. As presented in [4], single carrier frequency division multiple access (SC-FDMA) system also effectively cope with frequency selective channel in similar signal processing complexity. As shown in [5], SC-FDMA takes the advantage of simplicity of frequency domain equalization and lower power-to-average power ratio (PAPR) of signal carrier modulation. It has similar performance and signal process complexity as those of OFDM system. SC-FDMA has drawn a lot of attentions; especially in the uplink communications where lower PAPR strongly benefits the mobile terminal in terms of transmit power efficiency and manufacturing cost. [6] shows that as the wide variety of data service and the requirements for transparent operation across different technologies and quality of services constraints introduces a lot of challenges in the design of new generation systems, air interfaces will be reconfigurable, based on frequency domain transmission and multiple access scheme, where a generalized multi-carrier approach such as SC-FDMA and OFDMA is believed to a suitable choice for the modulation and multiple access in order to provide flexibility and adaptability. In [7], [8], [9], the authors claim that OFDM is very sensitive to carrier frequency offset due to the inter-carrier interference. At the same time, in [4], the authors claim that SC-FDMA also suffers from carrier frequency offset comparing with OFDMA system. Therefore, carrier frequency offset compensation is critical for both SC-FDMA system and OFDMA system. As an alternative novel equalization method, in [1], the author provides a novel equalization method named fractionally spaced frequency-domain minimum mean-square error (FSFD-MMSE), which is for OFDM system with zero postfix (ZP). It is all based on the fact that the dual problem of carrier frequency offset in frequency domain is timing jitter in time domain. It is shown that timing jitter is traditionally solved by time domain oversampling techniques in [10]. The basic idea is that the receiver is firstly projected onto a higher dimensional space via frequency-domain oversampling and then an optical linear receiver in the minimum mean square error sense(MMSE) is derived based on the Bayesian Gauss-Markov theorem. Hence, according to the similarity between OFMDA system and SC-FDMA system, this oversampling method is already proved to be robust against carrier frequency offset in DS-CDMA system [11] and multicarrier CDMA system [12]. Therefore, frequency domain oversampling MMSE equalization should be expected to be robust against carrier frequency offset in SC-FDMA system.

3. OVERVIEW OF SC-FDMA SYSTEM

3.1 Basic SC-FDMA Architectures and Mathematic Model

SC-FDMA system is an alternative scheme to OFDMA system. These two schemes are widely used in modern wireless communication systems, which have been adopted as the uplink and downlink multiple access schemes in 3GPP Long Term Evolution (LET), or Evolved Universal Terrestrial Radio Access (Evolved UTRA). Actually, SC-FDMA could be treated as Pre-DFT OFDMA, where a pre-coding Discrete Fourier Transform (DFT) method is utilized in OFDMA. The total architectures and mathematic model of SC-FDMA will be discussed briefly in the following sections.

3.1.1 Structure of SC-FDMA Systems

The basic structure of SC-FDMA system is shown is Figure 3.1.1. As shown in Figure 3.1, the only difference between OFDMA system and SC-FDMA system is the deep gray part, which is removed in the OFDMA case. In the other words, SC-FDMA is performed a pre M-point Discrete Fourier Transform (DFT) in the transmitter and an M-point Inverse Discrete Fourier Transform (IDFT) before detection in the receiver.

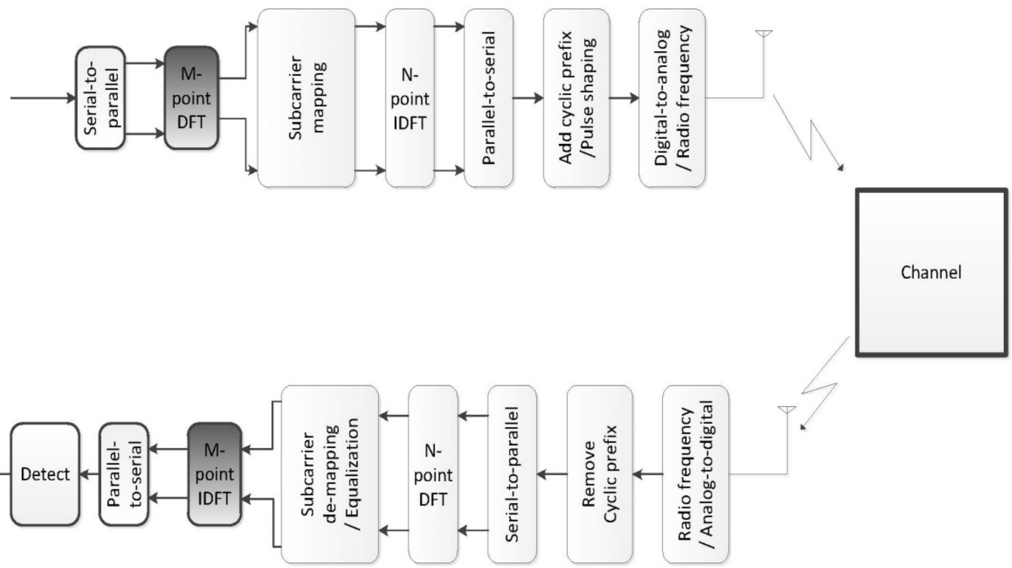

Figure 3.1.1: Structure of SC-FDMA System

In the transmitter, the input binary signal is firstly converted into complex modulated signals, e.g. QPSK or 16QAM signals. Then the totally N modulated signals $\{x_n\}$ are converted from serial sequences to parallel sequences. Each M modulated signal $\{x_m\}$ are grouped and performed M-point DFT operation to generate the frequency domain representation $\{X_m\}$. Then it maps each group frequency domain representation signal into one of the totally N orthogonal subcarriers. If $M = N/Q$, the system could handle Q simultaneous group to transmit without inter-interference, where Q is the bandwidth expansion factor of the symbol sequence. After subcarrier mapping, the result is the set $\tilde{X}_l$ $(l = 0,1,2 \dots, N$-1) where different subcarrier mapping schemes (usually interleaved subcarrier mapping or localized subcarrier mapping, shown in 3.2.1) lead to different symbol structure. As Q groups M point-DFT $\{X_m\}$ are mapped in N subcarriers, the entirely N subcarriers are performed N-point IDFT operation. Following a transformation from parallel sequence to serial sequence, cyclic prefix (CP) is inserted in order to prevent inter block interference (IBI) in multipath channel, where generally CP is a copy of the last part of transmit symbol. Interestingly, CP also makes the frequency domain equalization much easier, where point-to-point equalization is enough to recover the original signal. Pulse shaping is also performed after CP operation, which is used to suppress useless out band energy.After digital to analog conversion, the transmit symbol with CP are converted to radio frequency and transmitted by suitable antenna.

In the receiver, the opposite steps are performed. Firstly the receiver signals are down-converted to baseband and performed analog to digital conversion. Baseband signal convert from serial sequences to parallel sequences after remove CP. N point DFT transforms the receive signal into frequency domain. After subcarrier de-mapping, point-to-point equalization is performed in frequency domain. In every M point group, an extra M-point IDFT is performed and detection is made after parallel to serial conversion.

3.1.2 Mathematic Model of SC-FDMA System

As shown in Figure 3.1, the mathematic model of SC-FDMA system follows the procedure to present another view of SC-FMDA system.

In the transmitter, M-point input signals of complex amplitude symbols are performed

DFT operation

$$\boldsymbol{X}^i = \boldsymbol{F}_M \boldsymbol{d}^i \tag{3-1}$$

where

$$\boldsymbol{d}^i = \left[d_1^i, d_1^i, \dots \dots, d_1^i\right]^T \tag{3-2}$$

$$[\boldsymbol{F}_M]_{p,q} = (\frac{1}{\sqrt{M}})e^{-j2\pi(\frac{pq}{M})} \tag{3-3}$$

$$i = 1,2, \dots, Q, Q = N/M \tag{3-4}$$

As all Q group signals are done, totally N (=M * Q) signals are mapped into N subcarriers as follows:

$$\overline{\boldsymbol{X}}_{N\times 1}^i = \boldsymbol{M}_T^i \overline{\boldsymbol{X}}_{M\times 1}^i \tag{3-5}$$

where

$\boldsymbol{M}_T^i$ is subcarrier mapping matrix

Following operation is the N point IDFT, which converts the frequency domain signals back into the time domain.

$$\boldsymbol{x} = \boldsymbol{F}_N^{-1} \sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i \tag{3-6}$$

While the whole symbol is generated, CP is inserted to provide guard time, which could not only prevent inter block interference but also benefit the complexity of signal processing in the receiver. The length of CP is determined by the maximum delay of the channel, which is longer than the maximum delay spread of channel.

$$\boldsymbol{x}_{CP} = \boldsymbol{P}_{add}\boldsymbol{x} = \boldsymbol{P}_{add}\boldsymbol{F}_N^{-1} \sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i \tag{3-7}$$

where

$$\boldsymbol{P}_{add} = [\boldsymbol{C}, \boldsymbol{I}_N^{\boldsymbol{T}}]^T \tag{3-8}$$

$$\boldsymbol{C} = \left[\boldsymbol{0}_{N_C\times(M-N_C)}, \boldsymbol{I}_{N_C}\right] \tag{3-9}$$

$$P = N_C + N \tag{3-10}$$

The channel model is quasi-static frequency selective fading channel, where we assume that it is flat fading in each subcarrier. The impulse response of the channel is modeled as

$$h = \sum_{k=0}^{K-1} h_k \delta(\tau - \tau_k) \tag{3-11}$$

where

$$\sum_{k=0}^{K-1} E[|h_k|^2] = 1 \tag{3-12}$$

In the receiver, the receive signal with carrier frequency offset is shown as

$$\tilde{\boldsymbol{r}} = \widetilde{\boldsymbol{D}}(\varepsilon)\widetilde{\boldsymbol{H}}\boldsymbol{x}_{CP} + \boldsymbol{n}_{CP} \tag{3-13}$$

where

$$[\widetilde{\boldsymbol{D}}(\varepsilon)]_{n,n} = e^{j2\pi\varepsilon n/M}, m = 0, \dots\dots, N + N_C - 1 \tag{3-14}$$

$$\boldsymbol{D}(\varepsilon) = \boldsymbol{diag}\left\{1, e^{-j\frac{2\pi}{N}\epsilon}, \dots, e^{-j\frac{2\pi}{N}(P-1)\epsilon}\right\} \tag{3-15}$$

and $\widetilde{\boldsymbol{H}}$ is a $P \times P$ lower triangular Toeplitz matrix with the first column $[h_0, h_1, \dots, h_{K-1}, 0, \dots, 0]^T$ and ε is normalized by subcarrier spacing 1/T.

The receive signal removes the CP at the very first to prevent inter block interference as

$$\boldsymbol{r} = \boldsymbol{P}_{rem}\tilde{\boldsymbol{r}} = \boldsymbol{D}(\varepsilon)\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{3-16}$$

where

$$\boldsymbol{P}_{rem} = [\boldsymbol{0}_{(N \times N_C)}, \boldsymbol{I}_N] \tag{3-17}$$

$$\boldsymbol{D}(\varepsilon) = diag\left\{1, e^{-j\frac{2\pi}{N}\epsilon}, \dots, e^{-j\frac{2\pi}{N}(P-1)\epsilon}\right\} \tag{3-18}$$

and **H** is the $N \times N$ lower triangular Toeplitz matrix with the first column $[h_0, h_1, \dots, h_{K-1}, 0, \dots, 0]^T$.

After the N-point DFT operation in the receiver, the channel matrix is transformed to a diagonal matrix, which is the most attractive part of SC-FDMA and OFDMA system. The channel equalization could be performed by point-to point division. Based on feature of Toeplitz matrix, the channel matrix could be denoted as $\boldsymbol{H} = \boldsymbol{F}_N^{-1}\Lambda\boldsymbol{F}_N$. So the signal after N-point DFT operation is as follows:

$$\boldsymbol{R} = \boldsymbol{F}_N\boldsymbol{r} = \boldsymbol{F}_N\boldsymbol{D}(\varepsilon)\boldsymbol{F}_N^{-1}\boldsymbol{\Lambda}\boldsymbol{F}_N\boldsymbol{x} + \boldsymbol{F}_N\boldsymbol{n} = \boldsymbol{\Omega_{cir}}\boldsymbol{\Lambda}\sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i\boldsymbol{X}^i + \boldsymbol{F}_N\boldsymbol{n} \tag{3-19}$$

where

$$\boldsymbol{\Omega_{cir}} = \boldsymbol{F}_N\boldsymbol{D}(\varepsilon)\boldsymbol{F}_N^{-1} \tag{3-20}$$

Λ is an $N \times N$ diagonal matrix containing the DFT of the circulant sequence of $\mathbf{H}$.
If there is no carrier frequency offset presented, the formula is simplified as

$$\boldsymbol{R} = \boldsymbol{\Lambda} \sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i + \boldsymbol{F}_N \boldsymbol{n} \tag{3-21}$$

As we assume that the receiver is blind to the carrier frequency offset, so the receiver formula could perform frequency domain equalization no matter carrier frequency offset is existed or not.

$$\boldsymbol{R}_{equ} = \boldsymbol{W}_{\boldsymbol{equ}} \boldsymbol{\Lambda} \sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i + \boldsymbol{W}_{\boldsymbol{equ}} \boldsymbol{F}_N \boldsymbol{n} \tag{3-22}$$

where $\boldsymbol{W}_{\boldsymbol{equ}}$ is the M by M diagonal equalization matrix.
After equalization, each group signals are de-mapped by the subcarrier de-mapping matrix. Here we use the first user as an example, i.e. i = 1. As the subcarrier de-mapping matrix of first user is orthogonal to any other users, all the data from all other users are cancelled by the subcarrier de-mapping matrix.

$$\widetilde{\boldsymbol{X}}_{M\times 1}^1 = \boldsymbol{M}^1 \boldsymbol{R}_{equ} = \boldsymbol{M}^1 \boldsymbol{W}_{\boldsymbol{equ}} \boldsymbol{\Lambda} \boldsymbol{M}_T^1 \boldsymbol{X}^1 + \boldsymbol{M}^1 \boldsymbol{W}_{\boldsymbol{equ}} \boldsymbol{F}_N \boldsymbol{n} \tag{3-23}$$

Finally, an M-point IDFT is performed to get the final estimated data as

$$\widetilde{\boldsymbol{d}}^1 = \boldsymbol{F}_M^{-1} \widetilde{\boldsymbol{X}}_{M\times 1}^1 \tag{3-24}$$

3.2 Key Techniques in conventional SC-FDMA

There are two key techniques in SC-FDMA. Firstly, there are two different subcarrier mapping schemes which is described in 3.2.1. Secondly, the most important reason why SC-FDMA is so popular is the simplicity of the receiver equalization, which transfers the complex equalization into point-to-point division by the use of CP.

3.2.1 Subcarrier Mapping

There are mainly two types of subcarrier mapping schemes, which are distributed subcarrier mapping and localized subcarrier mapping. In distributed mapping scheme, M point outputs of the pre M-point DFT operation in each group are mapped over the entire unused subcarriers. One special case of distributed subcarrier mapping is interleaved subcarrier mapping, where M point outputs are mapped with equidistance. In localized subcarrier mapping scheme, the M point outputs are allocated in consecutive subcarriers. From diversity point of view, distributed subcarrier mapping could get more frequency diversity than the localized mode. With the channel-dependent scheduling, localized subcarrier mapping scheme could extract frequency gain. An example of different subcarrier mapping schemes is shown in Figure 2.2, where IFDMA donates interleaved subcarrier mapping, DFDMA donates conventional distributed subcarrier mapping and LFDMA is localized subcarrier mapping.

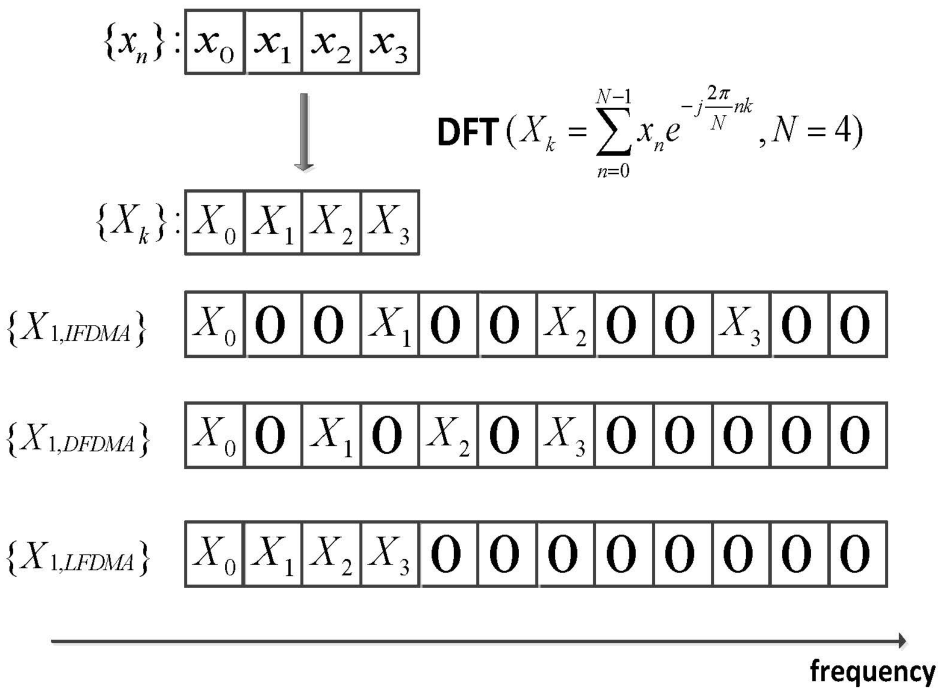

Figure 3.2.1: An example of different subcarrier mapping schemes for N=4, Q=3 and M = 12

3.2.2 Conventional Equalization

Two popular linear equalization schemes are introduced in this section. Zero Forcing (ZF) is the simplest equalization scheme, which simply divides the channel frequency response subcarrier by subcarrier. Another well-known equalization scheme is Minimum Mean Square Error (MMSE), which refers to estimation in a Bayesian setting with quadratic cost function. In order to show the difference between these two schemes in SC-FDMA system, we just base on the mathematic model in 3.1.2. The mathematic model before equalization is shown as follows:

$$\boldsymbol{R} = \boldsymbol{\Lambda} \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i + \boldsymbol{F}_N \boldsymbol{n} \tag{3-25}$$

The purpose of equalization is to remove the frequency selective channel effect**,** which is already transformed into point-to-point effect $\boldsymbol{\Lambda}$ by using CP.

For zero forcing, it just simply use the inverse matrix of $\boldsymbol{\Lambda}$ to cancel the channel effect, which is shown as

$$\boldsymbol{R}_{ZF} = \boldsymbol{\Lambda}^{-1} \boldsymbol{\Lambda} \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i + \boldsymbol{\Lambda}^{-1} \boldsymbol{F}_N \boldsymbol{n} = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i + \boldsymbol{\Lambda}^{-1} \boldsymbol{F}_N \boldsymbol{n} \tag{3-26}$$

As shown in the formula, zero forcing perfectly removes the effect of frequency selective channel and keeps the orthogonality among different subcarriers. But due to the inverse matrix of the second term, i.e. the noise term, the noise will be enlarged to infinite if there is a deep fading in one subcarrier, i.e. there is the very small value in the diagonal matrix $\boldsymbol{\Lambda}$. Hence, zero forcing seems to be a good scheme because it keeps the orthogonality, which do not introduce inter carrier interference. But the enlargement of noise is a quite big problem because zero forcing method works poor when there is a deep fading in the channel.

For minimum mean square error, the formula after equalization is presented as

$$\boldsymbol{R}_{MMSE} = \boldsymbol{W}^{-1} \boldsymbol{\Lambda}^* (\boldsymbol{\Lambda} \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i + \boldsymbol{F}_N \boldsymbol{n}) \tag{3-27}$$

where

$$\boldsymbol{W} = diag(|H_1|^2 + \frac{1}{SNR}, \dots, |H_N|^2 + \frac{1}{SNR}) \tag{3-28}$$

As shown in the formula, MMSE scheme does not keep the orthogonality among subcarriers. But it keeps a roughly noise power in the denominator if signal power is normalized. Because the $\frac{1}{SNR}$ term, it won't enlarge any noise if there is a deep fading. Hence MMSE combats the deep fading while it introduces a little bit of inter carrier interference.

4. FREQUENCY DOMAIN OVERSAMPLING MMSE EQUALIZER IN SC-FDMA SYSTEM

This chapter introduces the frequency domain oversampling MMSE equalizer (FDO-MMSE equalizer), including the block diagram and mathematic model.

4.1 Structure of SC-FDMA Systems with frequency domain oversampling MMSE equalizer

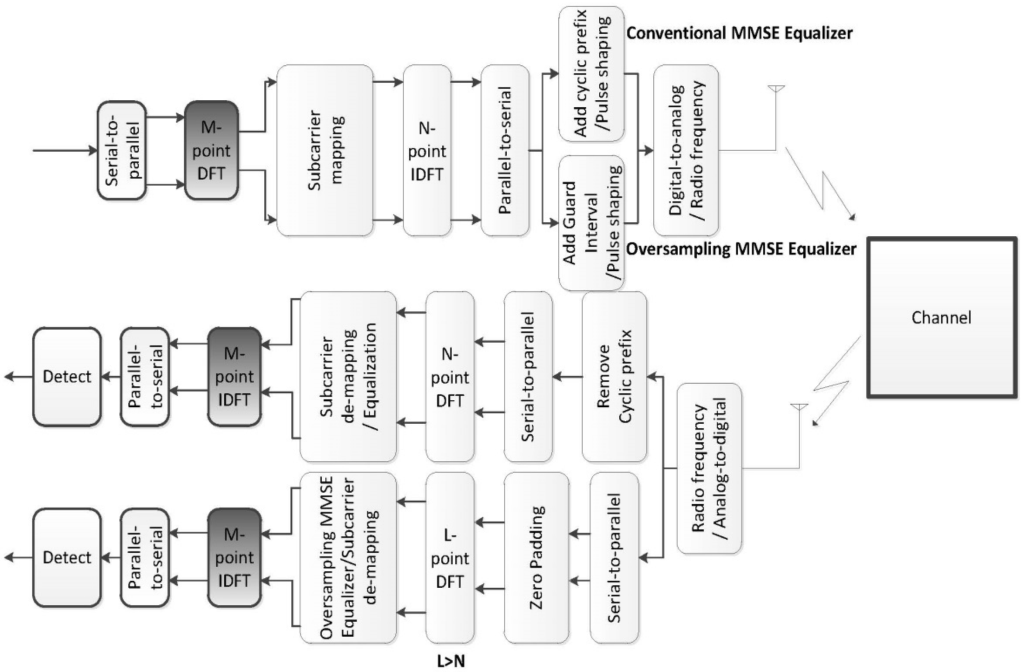

Figure 4.1: Comparison between conventional SC-FDMA and FDO-MMSE SC-FDMA

The block diagram comparison between conventional MMSE equalizer and FDO-MMSE equalizer of SC-FDMA system is depicted in Figure 4.1.1. As shown in Chapter 3, the block diagram of conventional MMSE equalizer follows the upper diagram shown in Figure 3.1, where FDO-MMSE equalizer is shown in the lower diagram.

In the transmitter part, the only difference between these two schemes is that frequency domain oversampling equalizer scheme adds zero postfix instead of cyclic prefix which is widely used in conventional MMSE equalizer scheme. Same as cyclic prefix, zero postfix is the guard interval where certain numbers of zeros are added right behind the SC-FDMA symbol.

In the receiver, instead of removing CP in the conventional MMSE equalization scheme, FDO-MMSE equalization scheme adds more zeros at the end of the receive symbol and performs a L-point FFT operation where L equals the sum of receive symbol length and zero padding length. So the receive signal is not only converted to frequency domain but also oversampled in the frequency domain by the L-point FFT. Hence, based on the Bayesian Gauss-Markov theorem, an optical linear MMSE receiver is derived and performed in the frequency domain right after L-point FFT. Finally the estimated frequency domain data is de-mapped into the correct order and an M-point DFT is performed to get the estimated data.

4.2 Mathematic Model of SC-FDMA System

Based on Figure 4.1.1, the mathematic model of SC-FDMA system with FDO-MMSE equalizer is presented as follows where most operation keeps the same as conventional MMSE.

In the transmitter, M-point input signals of complex amplitude symbols are performed DFT operation

$$\boldsymbol{X}^i = \boldsymbol{F}_M \boldsymbol{d}^i \tag{4-1}$$

where

$$\boldsymbol{d}^i = \left[d_1^i, d_1^i, \dots \dots, d_1^i\right]^T \tag{4-2}$$

$$[\boldsymbol{F}_M]_{p,q} = \left(\frac{1}{\sqrt{M}}\right)e^{-j2\pi\left(\frac{pq}{M}\right)} \tag{4-3}$$

$$i = 1,2, \dots, Q, Q = N/M \tag{4-4}$$

As all Q group signals are done, totally N (=M * Q) signals are mapped into N subcarriers as follows:

$$\overline{\boldsymbol{X}}_{N\times 1}^i = \boldsymbol{M}_T^i \overline{\boldsymbol{X}}_{M\times 1}^i \tag{4-5}$$

where

$\boldsymbol{M}_T^i$ is subcarrier mapping matrix

Following operation is the N point IDFT, which converts the frequency domain signals back into time domain.

$$\boldsymbol{x} = \boldsymbol{F}_N^{-1} \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i \tag{4-6}$$

While the whole symbol is generated, instead of CP, FDO-MMSE equalization scheme add zeros at the end of transmit symbol named guard interval which is inserted to provide guard time. The zero padding could not transmit the channel matrix into Toeplitz matrix. But zero padding aims at preventing inter-block interference. The length of zero padding is determined by the maximum delay of the channel, which is longer than the maximum delay spread of channel.

$$\boldsymbol{x}_{GI} = \boldsymbol{P}_{GI}\boldsymbol{x} = \boldsymbol{P}_{GI}\boldsymbol{F}_N^{-1} \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{Q}} \boldsymbol{M}_T^i \boldsymbol{X}^i \tag{4-7}$$

where

$$\boldsymbol{P}_{GI} = [\boldsymbol{I}_{\boldsymbol{N}}^{\boldsymbol{T}}, \boldsymbol{0}_{\boldsymbol{N_g}\times\boldsymbol{N}}^{\boldsymbol{T}}]^T \tag{4-8}$$

The channel model is quasi-static frequency selective fading channel, where we assume that it is flat fading in every subcarrier. The impulse response of the channel is modeled as

$$h = \sum_{k=0}^{K-1} h_k \delta(\tau - \tau_k) \tag{4-9}$$

where

$$\sum_{k=0}^{K-1} E[|h_k|^2] = 1 \tag{4-10}$$

In the receiver, the receive signal with carrier frequency offset is shown as

$$\tilde{\boldsymbol{r}} = \widetilde{\boldsymbol{D}}(\varepsilon)\boldsymbol{P}_{ZP}\widetilde{\boldsymbol{H}}\boldsymbol{x}_{GI} + \boldsymbol{n}_{ZP} \tag{4-11}$$

where

$$\boldsymbol{P}_{ZP} = [I_P^T, 0_{(L-P)\times P}^T]^{\boldsymbol{T}} \tag{4-12}$$

$$\widetilde{\boldsymbol{D}}(\varepsilon) = diag\left\{1, e^{-j\frac{2\pi}{N}\epsilon}, \dots, e^{-j\frac{2\pi}{N}(L-1)\epsilon}\right\} \tag{4-13}$$

and $\widetilde{\boldsymbol{H}}$ is a $P\times P$ lower triangular Toeplitz matrix with the first column $[h_0, h_1, \dots, h_{K-1}, 0, \dots, 0]^T$ and ε is normalized by subcarrier spacing 1/T.

Before the operation moves on, let's first take a look at the channel matrix transformation here.

$$\widetilde{\boldsymbol{H}}_{ZP} = \boldsymbol{P}_{ZP}\widetilde{\boldsymbol{H}}P_{GI} \tag{4-14}$$

By defining

$$\boldsymbol{F}_{L\times L} \triangleq \left\{\frac{1}{\sqrt{L}} \exp\left(-j\frac{2\pi}{L}np\right)\right\}_{L\times L} \tag{4-15}$$

$$\boldsymbol{F}_{L\times N} \triangleq \left\{\frac{1}{\sqrt{L}} \exp\left(-j\frac{2\pi}{L}np\right)\right\}_{L\times N} \tag{4-16}$$

Therefore based on the feature of Teoplitz matrix, the channel matrix which combines the effect of ZP insertion, wireless propagation channel, and zero-padding operations $\widetilde{\boldsymbol{H}}_{ZP}$ could be expressed as

$$\widetilde{\boldsymbol{H}}_{ZP} = \boldsymbol{F}_{L\times L}^{H}\boldsymbol{H}\boldsymbol{F}_{L\times N} \tag{4-17}$$

where $\boldsymbol{H} = diag\{H_0, H_1, \dots, H_{L-1}\}$ and S = L/N, which is the oversampling factor and

$$H_l \triangleq H\left(\frac{l}{ST}\right) = \sum_{k=0}^{K-1} h_k \exp\left(-j2\pi\left[\frac{l}{S}\right]\frac{\tau_q}{T}\right), l = 0,1,\dots,L-1 \tag{4-18}$$

After the derivation of $\widetilde{\boldsymbol{H}}_{ZP}$, the receiver signal after L-point DFT is shown as bellow:

$$\begin{aligned}
\boldsymbol{y} &= \boldsymbol{F}_{L\times L}\tilde{\boldsymbol{r}} \qquad (4\text{-}19)\\
&= \boldsymbol{F}_{L\times L}\boldsymbol{D}(\varepsilon)\boldsymbol{P}_{ZP}\widetilde{\boldsymbol{H}}P_{GI}\boldsymbol{F}_{N\times N}^{H}\sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}}\boldsymbol{M}_{T}^{i}\boldsymbol{X}^{i} + \boldsymbol{F}_{L\times L}\boldsymbol{P}_{ZP}\boldsymbol{n}\\
&= \boldsymbol{F}_{L\times L}\boldsymbol{D}(\varepsilon)\boldsymbol{F}_{L\times L}^{H}\boldsymbol{H}\boldsymbol{F}_{L\times N}\boldsymbol{F}_{N\times N}^{H}\sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}}\boldsymbol{M}_{T}^{i}\boldsymbol{X}^{i} + \boldsymbol{F}_{L\times P}\boldsymbol{n}\\
&= \boldsymbol{F}_{L\times L}\boldsymbol{D}(\varepsilon)\boldsymbol{F}_{L\times L}^{H}\boldsymbol{H}\boldsymbol{\Omega}\boldsymbol{\Psi} + \boldsymbol{\eta}
\end{aligned}$$

where

$$\boldsymbol{\Omega} = \boldsymbol{F}_{L\times N}\boldsymbol{F}_{N\times N}^{H}, \boldsymbol{\Psi} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{Q}}\boldsymbol{M}_{T}^{i}\boldsymbol{X}^{i}\,,\ \ \boldsymbol{\eta} = \boldsymbol{F}_{L\times P}\boldsymbol{n} \tag{4-20}$$

$$\mathrm{C}_{\boldsymbol{\eta}} = \mathrm{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{H}] = \sigma_n^2\boldsymbol{\Gamma},\ \ \boldsymbol{\Gamma} = \boldsymbol{F}_{L\times P}\boldsymbol{F}_{L\times P}^{H} \tag{4-21}$$

Based on Bayesian Gauss-Markov theorem which is shown in Appendix A, the oversampling MMSE receiver is presented as

$$\begin{aligned}
\boldsymbol{\omega}^{H} &= E[d\boldsymbol{y}^{H}]\cdot E[\boldsymbol{y}\boldsymbol{y}^{H}]^{\dagger} \qquad (4\text{-}22)\\
&= (\widetilde{\mathbf{H}}\boldsymbol{\Omega})^{H}\left\{\widetilde{\mathbf{H}}\boldsymbol{\Omega}\boldsymbol{\Omega}^{H}\widetilde{\mathbf{H}}^{H} + \frac{\sigma_n^2}{\sigma_s^2}\boldsymbol{\Gamma}\right\}
\end{aligned}$$

The estimated frequency domain data $\widetilde{\boldsymbol{\Psi}}$ equals to

$$\widetilde{\boldsymbol{\Psi}} = \omega^{H}\boldsymbol{y} \tag{4-23}$$

Then de-mapping process is adopted as

$$\widetilde{\mathbf{X}^{\iota}} = \boldsymbol{M}^{i}\widetilde{\boldsymbol{\Psi}} \tag{4-24}$$

Finally, N-point IDFT process is operated

$$\mathbf{d}^{i} = \boldsymbol{F}_{N\times N}^{H}\widetilde{\mathbf{X}^{\iota}} \tag{4-25}$$

5. BER PERFORMANCE DISCUSSION

In this chapter, both conventional MMSE equalizer and frequency domain oversampling MMSE equalizer (FDO-MMSE equalizer) are processed to compare the performance improvement of FDO-MMSE equalizer over conventional MMSE equalizer. As shown in chapter 3, conventional MMSE equalizer is performed by adding cyclic prefix and point to point equalization. On the other hand, FDO-MMSE equalizer adds guard interval instead of cyclic prefix. Based on this big difference, this chapter will show the BER performance comparison in variety situation including different CFO and channel path number.

5.1 Simulation

5.1.1 Simulation System Set Up

Total Subcarriers	64
Pre-DFT Subcarriers Number	16
Modulation Scheme	QPSK
Equalization	Conventional MMSE and oversampling MMSE
Mapping scheme	Interleaved
Cyclic Prefix Number	16
Channel Model	16 Paths equal power Rayleigh Channel
Normalized Carrier Frequency Offset	[0,0.02,0.05,0.1]
Coding	uncoded
Iteration Number	100000000
Number of Stop Iteration	10000

Table 5.1.1: Simulation System Parameters

The system parameters are shown in Figure 5.1.1. The number of total subcarriers of

SC-FDMA system is 64, while pre-DFT subcarrier number is 16, which means the system manages to sustain 4 groups simultaneously transmitting without any inter-carrier interference. For illustration, the system modulation scheme is QPSK and the mapping scheme is interleaved. For comparison, conventional MMSE equalizer and FDO-MMSE equalizer are both adopted to explore the BER improvement. The simulation channel is 16 paths Rayleigh channel with equal power in each path. Hence, the length of cyclic prefix is 16, which copes with the multipath effect and perfectly prevents inter block interference. For carrier frequency offset, the values shown in Figure 5.1.1 are normalized by subcarrier space. In order to show the benefit of FDO-MMSE equalizer, this system is uncoded. The iteration number is the numbers of simulation loop and number of stop iteration means the iteration stops once the error number reaches it.

5.1.2 BER Performance Comparison between SC-FDMA and OFDM without CFO

This section presents the BER performance comparison between SC-FDMA system and OFDM system without CFO, where both conventional MMSE equalizer and oversampling MMSE equalizer are simulated.

In Figure 5.1.2, EbN0 is the energy per bit to noise power spectral density ratio. It is clear that BER performance of SC-FDMA system is always better than OFDM due to the correlated data caused by pre-DFT operation gives the benefit of frequency diversity, where OFDM is uncoded system. For FDO-MMSE OFDMA system, the system gets the frequency diversity from the oversampling operation, which shows better performance than conventional MMSE OFDM system, which does not have any frequency diversity. The green line is FDO-MMSE scheme, which shows greatly BER performance improvement than conventional MMSE equalizer in high SNR. At lower SNR, the reason why the BER performance for four schemes has no big difference is that noise dominates this area, as multi-path effect dominates the high SNR area.

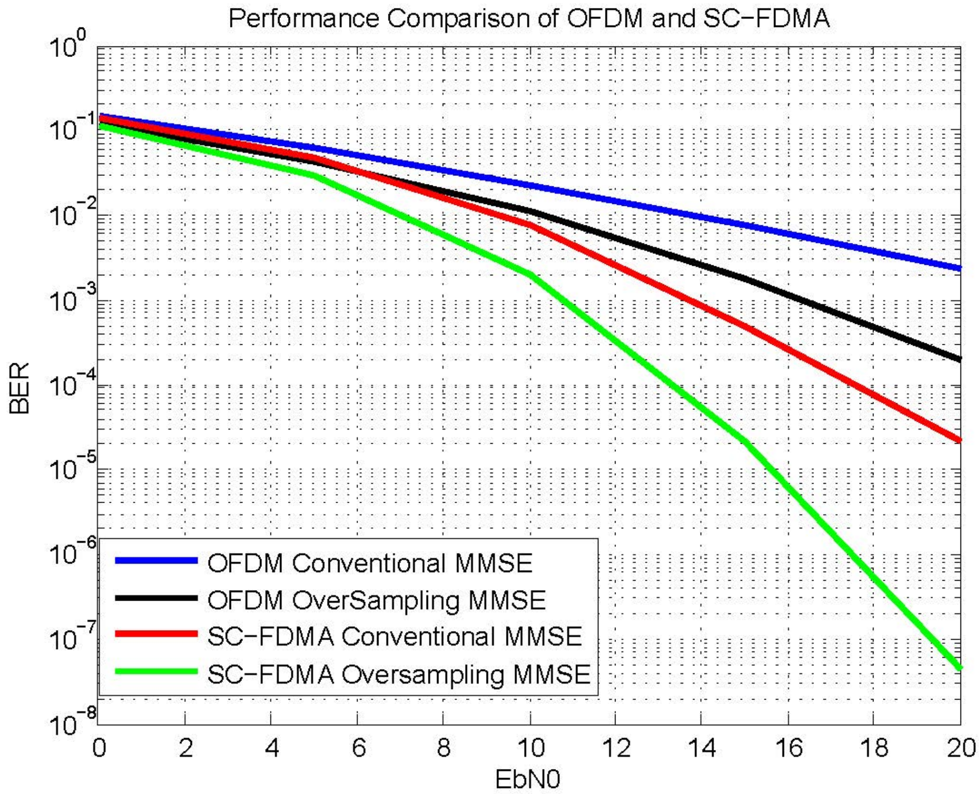

Figure 5.1.2: Performance Comparison of OFDM and SC-FDMA without CFO

5.1.3 BER Performance Comparison between SC-FDMA and OFDM with CFO = 0.02

In Figure 5.1.3, the BER performance comparison with CFO = 0.02 is illustrated. Compared with Figure 5.1.2, there is not obvious difference of these two figures. As normalized carrier frequency offset is so small, the non-orthogonolity caused by carrier frequency offset effects a little, where only a little inter-carrier interference is introduced. As talked in 5.1.2, green line shows the best performance due to the frequency diversity.

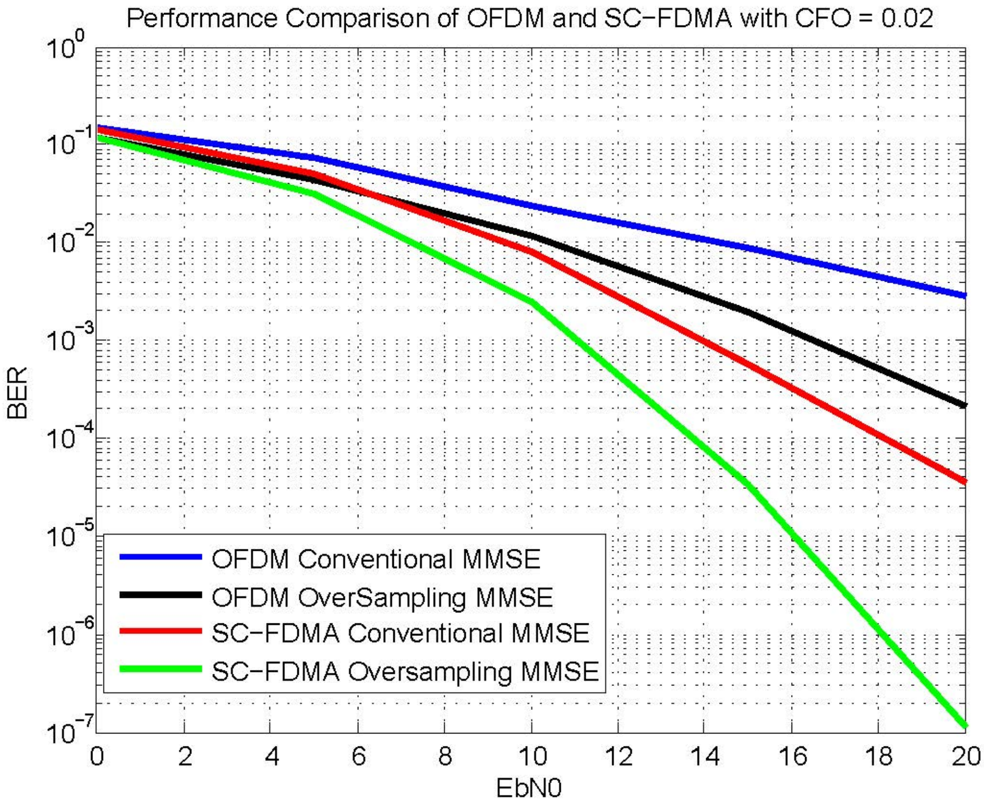

Figure 5.1.3: Performance Comparison of OFDM and SC-FDMA with CFO = 0.02

5.1.4 BER Performance Comparison between SC-FDMA and OFDM with CFO = 0.05

As the CFO increases, the carrier frequency offset causes more subcarriers non-orthononality, where more inter-subcarrier interference is introduced. In Figure 5.1.4, the blue line and red line which presents the OFDM system and SC-FDMA system with conventional MMSE equalizer respectively shows that the BER performance meets a BER floor due to severe carrier frequency offset. While FDO-MMSE equalizer is adopted, the BER floor disappears. So oversampling scheme helps the system to be robust against carrier frequency offset.

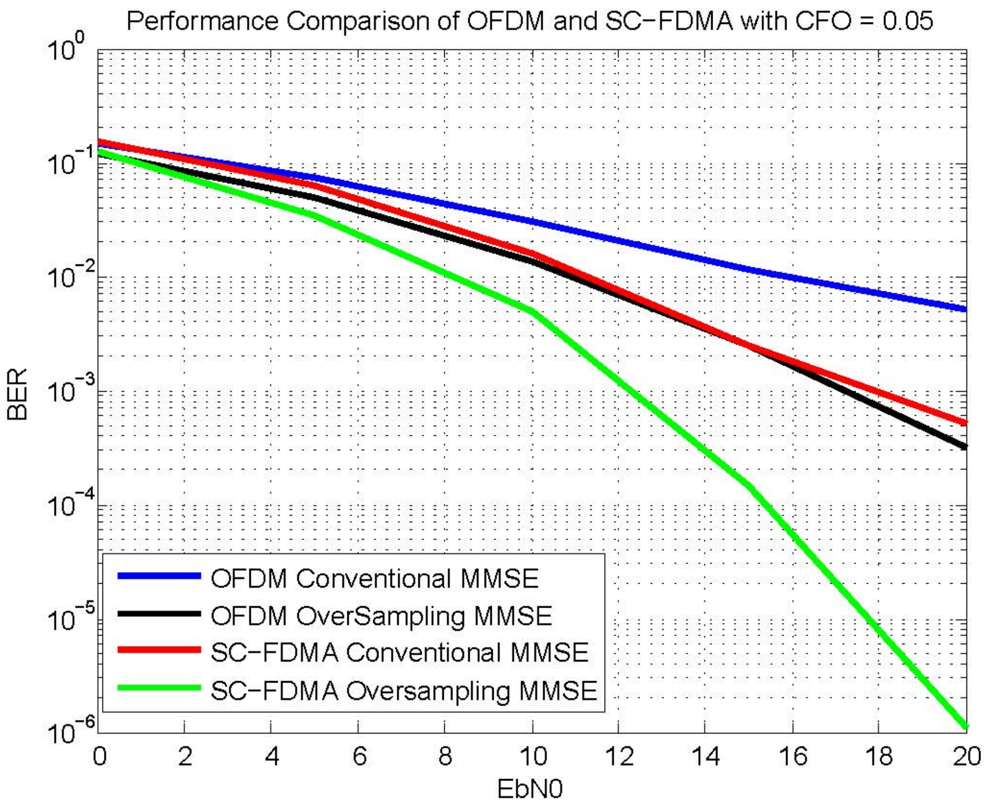

Figure 5.1.4: Performance Comparison of OFDM and SC-FDMA with CFO = 0.05

5.1.5 BER Performance Comparison between SC-FDMA and OFDM with CFO = 0.1

As CFO = 0.1, which means the carrier frequency offset is extremely severe, the BER performances of conventional MMSE OFDMA system and conventional MMSE SC-FDMA system are nearly same no matter what value SNR is. This is because severe CFO mainly dominates the BER performance. So both the noise dominated part and multipath dominated part are mainly affected by CFO. Even the frequency diversity of conventional MMSE SC-FDMA is useless. But as shown in Figure 5.1.3, FDO-MMSE equalizer makes both OFDMA system and SC-FDMA system to be robust against severe CFO. They all outperform the conventional MMSE equalizer scheme, where SC-FDMA is the best scenario as SNR increases.

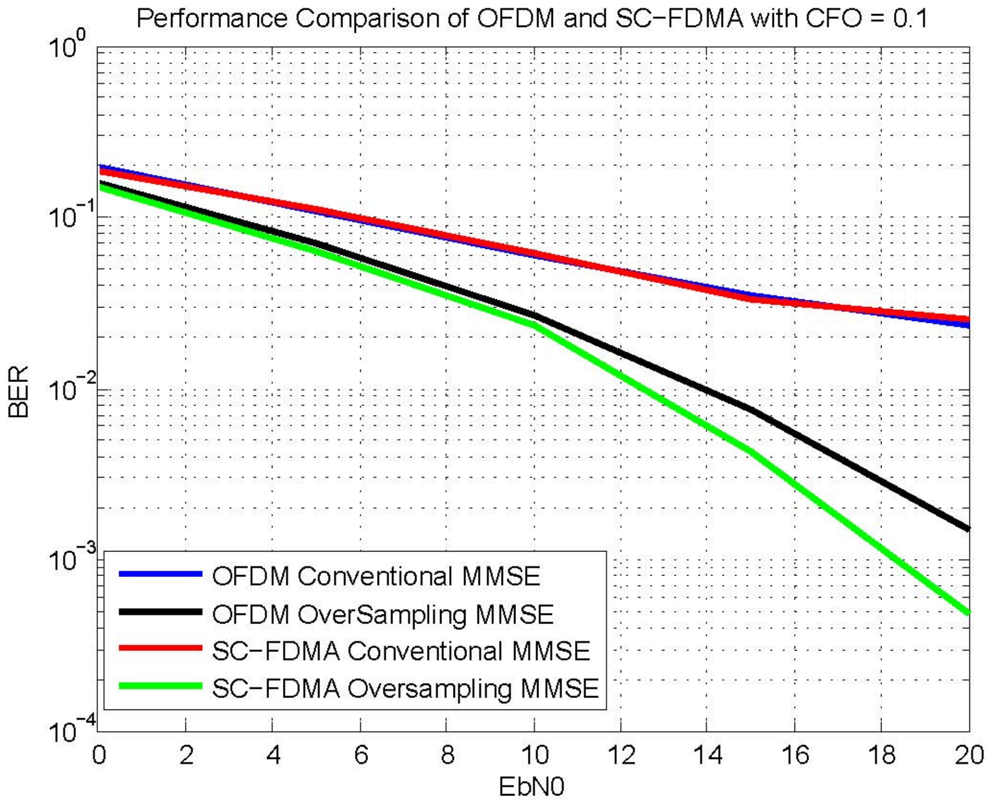

Figure 5.1.5: Performance Comparison of OFDM and SC-FDMA with CFO = 0.1

5.1.6 BER Performance of SC-FDMA system with different CFO

This section shows all the BER performance for different CFO presented in one figure. In Figure 5.1.6, FDO-MMSE equalizer always shows best performance no matter what value CFO presents. As SC-FDMA system with conventional MMSE equalizer meets the BER floor and is extremely destroyed by severe CFO, i.e. CFO = 0.05 and CFO = 0.1, SC-FDMA system with FDO-MMSE equalizer shows great robustness against severe CFO.

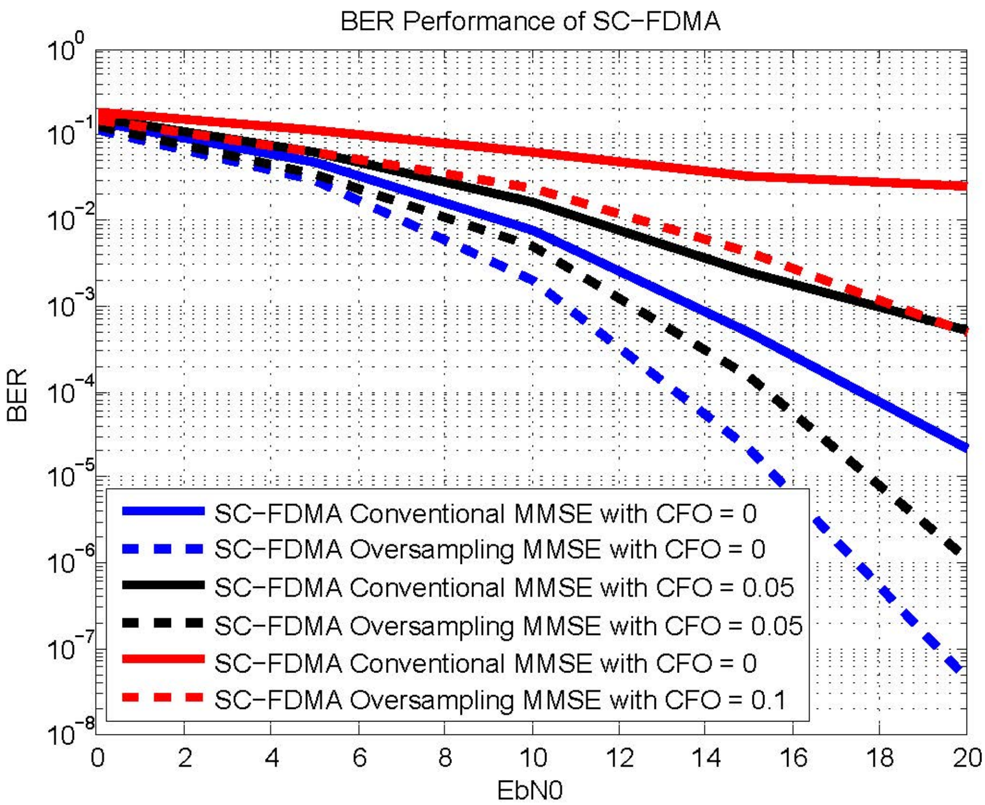

Figure 5.1.6: BER Performance of SC-FDMA with different CFO

5.1.7 Discussion

As shown in this chapter, FDO-MMSE equalizer does not only work in OFDM system, which is proved in [3], but also benefits SC-FDMA system.

As shown from Figure 5.1.2 to Figure 5.1.3, where there is no CFO effect or slight CFO influence, the BER performance relationship between these four scenarios are that SC-FDMA system with conventional MMSE equalizer outperforms FDO-MMSE OFDM system, while conventional MMSE OFDM system is the worst and SC-FDMA system with FDO-MMSE equalizer is the best one. The reason for this result is that OFDM system with conventional MMSE equalizer has no frequency diversity due to the orthogonality among subcarriers. Even if the system is strictly no inter-carrier interference, the system has no ability to cope with deep fading. As OFDM system with FDO-MMSE equalizer samples in non-orthogonal position in frequency domain, it introduces inter carrier interference but also correlates subcarriers with each other. Then FDO-MMSE equalizer makes use of the information to extract more frequency diversity to compensate inter-carrier interference and get a better performance. Comparing with conventional MMSE SC-FDMA system, the pre-DFT operation highly correlates the subcarriers among each other. Thus the system extracts frequency diversity. As shown in the figures above, conventional MMSE SC-FDMA system extracts more frequency diversity than OFDM system with FDO-MMSE equalizer. Finally, for the SC-FDMA system with FDO-MMSE equalizer, both pre-DFT operation and oversampling gives the frequency diversity, which shows the best BER performance.

System	Conventional OFDM	Conventional SC-FDMA	Oversampling OFDM	Oversampling SC-FDMA
Detection Method	Point to Point Detection	Point to Point Detection	Sequence Detection	Sequence Detection
Inter carrier Interference	N	Y	Y	Y
Orthogonality	Y	Y	N	N
Noise	Y	Y	Y	Y
Frequency Diversity	N	Y	Y	Y
Deep Fading Recoverability	N	Y	Y	Y

Table 5.1.7: Comparison between different systems

As shown from Figure 5.1.4 to Figure 5.1.5, where the CFO increases to the severe region, the BER performance result changes a little bit, where conventional MMSE SC-FDMA system has the same or even worse performance as compared with OFDM system with FDO-MMSE equalizer. This is because CFO dominates the system and these two scenarios shows similar performance as CFO breaks the orthogonality severely. But FDO-MMSE SC-FDMA still outperforms. This scenario extracts the most frequency diversity and has the ability to cope with deep fading and recover the subcarriers in deep fading. The main reasons are also shown in Table 5.1.7. In the form, Y means this effect presents and N equals to the phrase ‘not present’.

5.2 Oversampling Factor and Oversampling Gain in different Channel

This part mainly discusses the effect of oversampling factor and oversampling gain in different channel. For oversampling factor, the simulation result presents when the oversampling factor will affect the BER performance. For oversampling gain, this chapter will illustrate the effect of different channel paths for BER performance.

5.2.1 Oversampling Factor

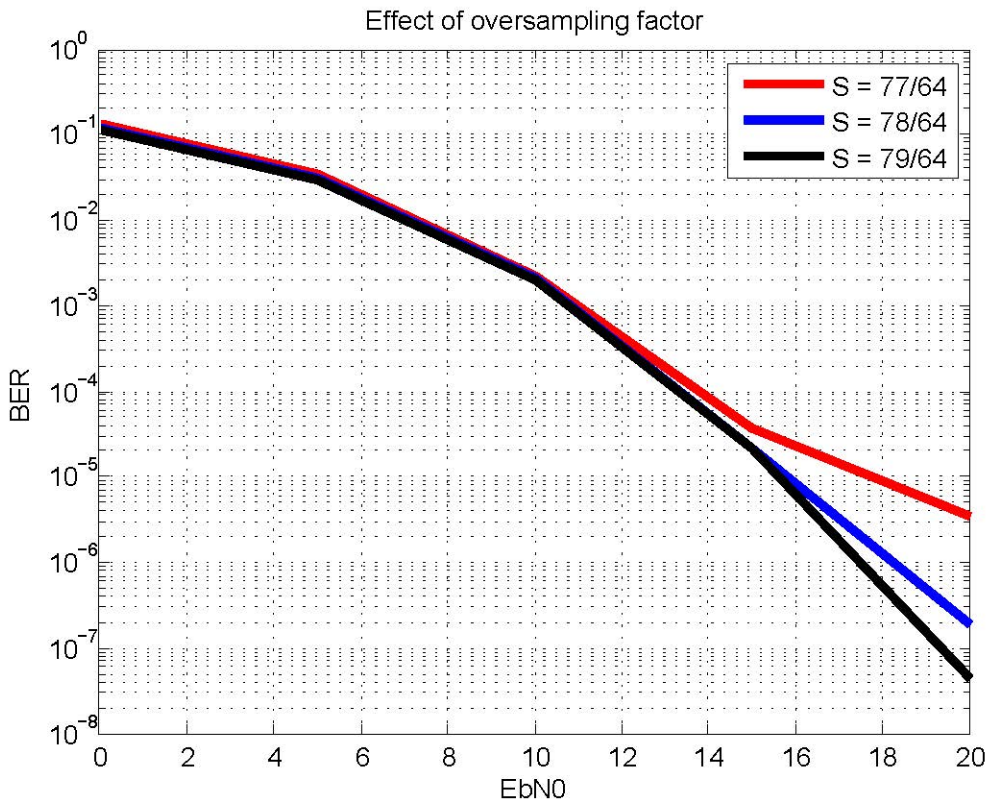

Figure 5.2.1: Effect of Oversampling Factor

In Figure 5.2.1, S is the oversampling factor which the BER performance keeps the same of different oversampling factor as SNR is low. In high SNR area, the BER performance degrades as the oversampling factor decrease from S = 79/64. This is because the path number of channel is 16, so as the oversampling factor remains in 79/80, the last symbol contains a little information of the signal from the last path. If oversampling factor decreases to 78/80, it loses the last information. As in high SNR, where multipath dominates the BER performance, this information is critical. Therefore, this results indicate that S = 1 + (Q-1/N) is the minimum oversampling factor that enables the frequency domain oversampling MMSE equalizer to achieve a near optimal performance.

5.2.2 Oversampling Gain in different path

As shown at the beginning of this chapter, FDO-MMSE equalizer mainly helps the system to be robust against carrier frequency offset. This section gives the simulation result to show how frequency domain oversampling MMSE equalizer gives the robustness to the system and why it outperforms the conventional MMSE equalizer.

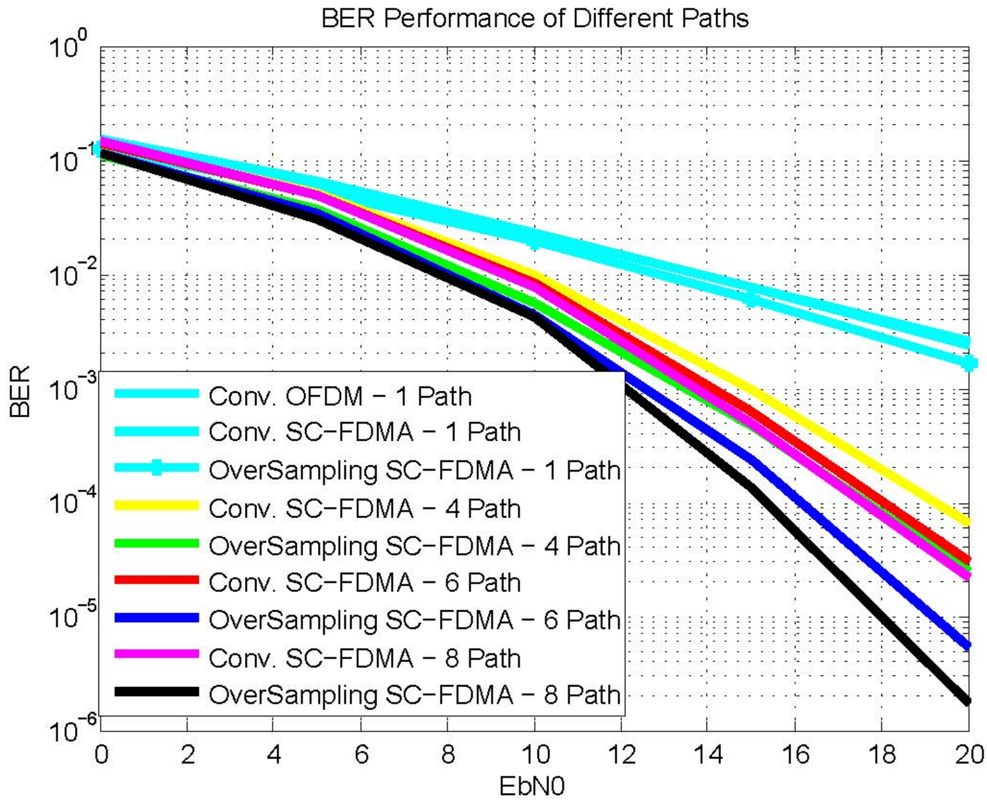

Figure 5.2.2: Oversampling Gain in different numbers of paths.

In Figure 5.2.2, it should be highlighted that the reason why the BER performance of

FDO-MMSE SC-FDMA is a little bit better than the conventional OFDM system and conventional SC-FDMA system is that the SNR of FDO-MMSE SC-FDMA is higher than the conventional one in same situation, where FDO-MMSE SC-FDMA uses ZP instead of CP. CP costs the system energy while ZP doesn'It is clear that the more paths you have the better performance you get. It seems to be paradoxical where multipath causes frequency selective channel, which severely influences the system with time domain equalization. But as sequence detection showing here, the transmit data are correlated with each other. So sequence detection could make use of frequency diversity if exists. For higher frequency selective channel, more frequency diversity could be used. Therefore, the more channel paths exist, better performance shows. As a result, FDO-MMSE equalizer makes use of highly frequency selective channel to achieve better BER performance by getting more frequency diversity. As channel path drops to 1, there is no frequency diversity which the system could make use of. So there is no BER performance improvement with FDO-MMSE equalizer in both SC-FDMA and OFDM system.

5.2.3 Oversampling gain in one path channel with and without CFO

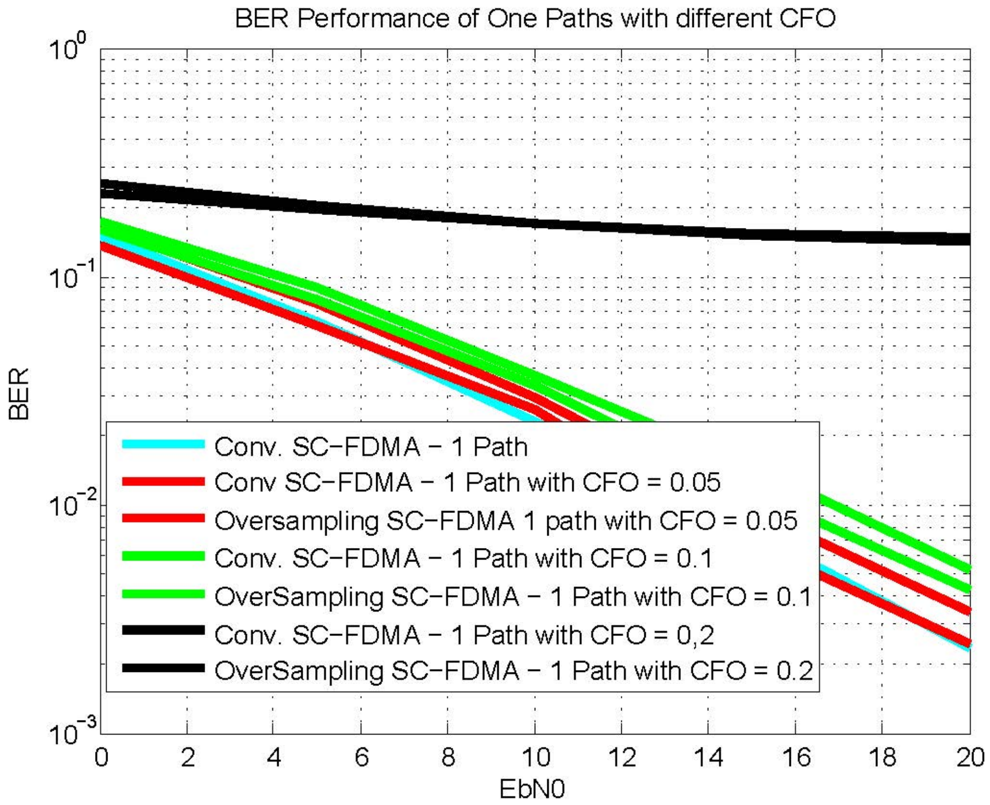

Figure 5.2.3: Oversampling Gain in one path with and without CFO.

Comparing with Figure 5.2.3, the gap between conventional MMSE SC-FDMA and FDO-MMSE SC-FDMA is also due to the energy difference between CP and ZP. As shown in 5.2.2, FDO-MMSE equalizer extracts frequency diversity from highly frequency selective channel to improve the performance. Also shown in 5.1, FDO-MMSE equalizer shows the best performance no matter what value CFO is. So this section explore if FDO-MMSE equalizer also improves the BER performance through other aspects, besides frequency diversity from highly frequency selective channel. In Figure 5.2.3, the BER performance presents that FDO-MMSE equalizer has similar performance comparing with conventional MMSE equalizer in one path Rayleigh channel. As there is no frequency diversity in one path Rayleigh channel, it means FDO-MMSE equalizer is only able to make use of frequency diversity to improve BER performance. Once the frequency diversity doesn't exist, FDO-MMSE equalizer degenerates to conventional MMSE equalizer due to no multipath information could be used. Consequently, FDO-MMSE equalizer is robust against carrier frequency offset only through frequency diversity.

6. CONCLUSION & RECOMMENDATION

6.1 Conclusion

FDO-MMSE equalizer shows greatly performance improvement in SC-FDMA system not only in OFDM system, but also in SC-FDMA system. One nice feature of the proposed equalization scheme is that it can extract more frequency diversity from non-orthogonality among subcarrier, as well as the frequency diversity of pre-DFT operation. Numerical results show that FDO-MMSE equalizer outperforms the conventional MMSE receiver under highly frequency selective channel. It also shows that the oversampling factor can be reduced to a number given by $S_{min} = 1 + (Q - \frac{1}{N})$ to minimize the receiver complexity while achieve the optimal BER performance. However, without highly frequency diversity, FDO-MMSE equalizer shows no big improvement comparing with conventional MMSE equalizer, where FDO-MMSE equalizer degenerates to conventional MMSE equalizer as channel path is 1.

6.2 Future work and Recommendation

Conventional MMSE equalizer is formulized in [4]. Future work should focus on mathematic analysis of frequency domain oversampling MMSE equalizer. Moreover, frequency domain oversampling MMSE equalizer is expected to be adopted in USRP (Universal Software Radio Peripheral) to show the real world BER performance through constellation.

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Appendix A

Bayesian Gauss-Markov Theorem

If the data are the Bayesian linear model form

$$\mathbf{x} = \mathbf{H\theta} + \mathbf{w} \tag{1}$$

where $\mathbf{x}$ is an $\mathrm{N}\times 1$ data vector, $\mathbf{H}$ is a known $\mathrm{N}\times\mathrm{p}$ observation matrix, $\boldsymbol{\theta}$ is a $\mathrm{p}\times 1$ random vector of parameters whose realization is to be estimated and has mean $\mathrm{E}(\boldsymbol{\theta})$ and covariance matrix $\mathbf{C}_{\boldsymbol{\theta\theta}}$, and $\mathbf{w}$ is an $\mathrm{N}\times 1$ random vector with zero mean and covariance matrix $\mathbf{C}_{\boldsymbol{w}}$ and is uncorrelated with $\boldsymbol{\theta}$ (the joint PDF $\mathrm{p}(\mathbf{w},\boldsymbol{\theta})$ is otherwise arbitrary), then the LMMSE estimator of $\boldsymbol{\theta}$ is

$$\begin{aligned}\widehat{\boldsymbol{\theta}} &= \boldsymbol{E}(\boldsymbol{\theta}) + \mathbf{C}_{\boldsymbol{\theta\theta}}\boldsymbol{H}^{T}(\boldsymbol{H}\mathbf{C}_{\boldsymbol{\theta\theta}}\boldsymbol{H}^{T} + \mathbf{C}_{w})^{-1}\big(\mathbf{x} - \boldsymbol{HE}(\boldsymbol{\theta})\big) \\ &= \boldsymbol{E}(\boldsymbol{\theta}) + (\boldsymbol{C}_{\boldsymbol{\theta\theta}}^{-1} + \boldsymbol{H}^{T}\boldsymbol{C}_{\boldsymbol{w}}^{-1}\boldsymbol{H})^{-1}\boldsymbol{H}^{T}\boldsymbol{C}_{\boldsymbol{w}}^{-1}(\mathbf{x} - \boldsymbol{HE}(\boldsymbol{\theta}))\end{aligned} \tag{2}$$

The performance of the estimator is measured by the error $\boldsymbol{\varepsilon} = \boldsymbol{\theta} - \widehat{\boldsymbol{\theta}}$ whose mean is zero and whose covariance matrix is

$$\begin{aligned}\mathbf{C}_{\boldsymbol{\varepsilon}} &= E_{\boldsymbol{x},\boldsymbol{\theta}}(\boldsymbol{\varepsilon\varepsilon}^{T}) \\ &= \mathbf{C}_{\boldsymbol{\theta\theta}} - \mathbf{C}_{\boldsymbol{\theta\theta}}\boldsymbol{H}^{T}(\boldsymbol{H}\mathbf{C}_{\boldsymbol{\theta\theta}}\boldsymbol{H}^{T} + \mathbf{C}_{w})^{-1}\boldsymbol{H}\mathbf{C}_{\boldsymbol{\theta\theta}} \\ &= (\boldsymbol{C}_{\boldsymbol{\theta\theta}}^{-1} + \boldsymbol{H}^{T}\boldsymbol{C}_{\boldsymbol{w}}^{-1}\boldsymbol{H})^{-1}\end{aligned} \tag{3}$$

The error covariance matrix is also the minimum MSE matrix $\mathbf{M}_{\widehat{\boldsymbol{\theta}}}$ whose diagonal elements yield the minimum Bayesian MSE

$$\begin{aligned}[\boldsymbol{M}_{\widehat{\boldsymbol{\theta}}}]_{\boldsymbol{ii}} &= [\boldsymbol{C}_{\boldsymbol{\varepsilon}}]_{\boldsymbol{ii}} \\ &= Bmse(\widehat{\boldsymbol{\theta}}_{i}).\end{aligned} \tag{4}$$

These results are identical to those in Theorem 11.1 for the Bayesian linear model except that the error vector is not necessary Gaussian. An example of the determination of this estimator and its minimum Bayesian MSE has already been given in Section 10.6 . The Bayesian Gauss-Markov theorem states that within the class of linear estimator the one that minimizes the Bayesian MSE for each element of $\boldsymbol{\theta}$ is given by (12.26) or (12.27). It will not be optimal unless the conditional expectation $\mathrm{E}(\boldsymbol{\theta}|\boldsymbol{x})$ happens to be linear. Such was the case for the jointly Gaussian PDF. Although suboptimal the LMMSE estimator is in practice quite useful, being available in closed form and depending only on the means and covariance.

Literature Translation

Original Literature

Single Carrier Orthogonal Multiple Access Technique for Broadband Wireless Communications

Hyung G. Myung

1.1. Evolution of Cellular Wireless Communications

During the 1950s and 1960s, researchers at AT&T Bell Laboratories and companies around the world developed the idea of cellular radiotelephony. The concept of cellular wireless communications is to break the coverage zone into small cells and reuse the portions of the available radio spectrum. In 1979, the world's first cellular system was deployed by Nippon Telephone and Telegraph (NTT) in Japan and thus began the evolution of cellular wireless communications [1], [2]. The first generation of cellular wireless communication systems utilized analog communication techniques and its focus was on accommodating voice traffic. Frequency modulation (FM) and frequency division multiple access (FDMA) were the basis of the first generation systems. AMPS (Advanced Mobile Phone System) in US and ETACS (European Total Access Cellular System) in Europe were among the first generation systems. The second generation systems saw the advent of digital communication techniques which greatly improved spectrum efficiency. Also they vastly enhanced the voice quality and made possible the packet data transmission. The main multiple access schemes are time division multiple access (TDMA) and code division multiple access (CDMA). GSM (Global System for Mobile) which is based on TDMA and IS-95 which is based on CDMA are two most widely accepted second generation systems. In the mid-1980s, the concept for IMT-2000 (International Mobile Telecommunications-2000) was born at the ITU (International Telecommunication Union) as the third generation (3G) system for mobile communications [3]. Key objectives of IMT-2000 are to provide seamless global roaming and to provide seamless delivery of services over a number of media via higher data rate link. In 2000, a unanimous approval of the technical specifications for 3G system under the brand IMT-2000 was made and UMTS/WCDMA (Universal

Mobile Telecommunications System/Wideband CDMA) and cdma2000 are two prominent standards under IMT-2000 both of which are based on CDMA. IMT-2000 provides higher transmission rates; a minimum speed of 2 Mbps for stationary or walking users and 348 kbps in a moving vehicle whereas second generation systems only provide speeds ranging from 9.6 kbps to 28.8 kbps. Since the initial standardization, both WCDMA and cdma2000 have evolved into socalled "3.5G"; UMTS through HSD/UPA (High Speed Downlink/Uplink Packet Access) and cdma2000 through 1xEV-DO Rev A (1x Evolution Data-Optimized Revision A). Currently, 3rd Generation Partnership Project Long Term Evolution (3GPP LTE) is considered as the prominent path to the next generation of cellular system beyond 3G.

1.2. 3GPP Long Term Evolution

3GPP's work on the evolution of the 3G mobile system started with the Radio Access Network (RAN) Evolution workshop in November 2004 [4]. Operators, manufacturers, and research institutes presented more than 40 contributions with views and proposals on the evolution of the Universal Terrestrial Radio Access Network (UTRAN) which is the foundation for UMTS/WCDMA systems. They identified a set of high level requirements at the workshop; reduced cost per bit, increased service provisioning, flexibility of the use of existing and new frequency bands, simplified architecture and open interfaces, and allow for reasonable terminal power consumption. With the conclusions of this workshop and with broad support from 3GPP members, a feasibility study on the Universal Terrestrial Radio Access (UTRA) and UTRAN Long Term Evolution started in December 2004. The objective was to develop a framework for the evolution of the 3GPP radio access technology towards a high-data-rate, low-latency, and packet-optimized radio access technology. The study focused on means to support flexible transmission bandwidth of up to 20 MHz, introduction of new transmission schemes, advanced multi-antenna technologies, signaling optimization, identification of the optimum UTRAN network architecture, and functional split between RAN network nodes.

The first part of the study resulted in an agreement on the requirements for the Evolved UTRAN (E-UTRAN). Key aspects of the requirements are as follows [5].

- Peak data rate: Instantaneous downlink peak data rate of 100 Mbps within a 20

MHz downlink spectrum allocation (5 bps/Hz) and instantaneous uplink peak data rate of 50 Mbps (2.5 bps/Hz) within a 20 MHz uplink spectrum allocation.

• Control-plane capacity: At least 200 users per cell should be supported in the active state for spectrum allocations up to 5 MHz.

• User-plane latency: Less than 5 ms in an unloaded condition (i.e. single user with single data stream) for small IP packet.

• Mobility: E-UTRAN should be optimized for low mobile speed from 0 to 15km/h. Higher mobile speeds between 15 and 120 km/h should be supported with high performance. Mobility across the cellular network shall be maintained at speeds from 120 to 350 km/h (or even up to 500 km/h depending on the frequency band).

• Coverage: Throughput, spectrum efficiency, and mobility targets should be met for 5 km cells and with a slight degradation for 30 km cells. Cells ranging up to 100 km should not be precluded.

• Enhanced multimedia broadcast multicast service (E-MBMS).

• Spectrum flexibility: E-UTRA shall operate in spectrum allocations of different sizes including 1.25 MHz, 1.6 MHz, 2.5 MHz, 5 MHz, 10 MHz, 15 MHz, and 20 MHz in both uplink and downlink.

• Architecture and migration: Packet-based single E-UTRAN architecture with provision to support systems supporting real-time and conversational class traffic and support for an end-to-end quality-of-service (QoS).

• Radio resource management: Enhanced support for end-to-end QoS, efficient support for transmission of higher layers, and support of load sharing and policy management across different radio access technologies. The wide set of options initially identified by the early LTE work was narrowed down in December 2005 to a working assumption that the downlink would use Orthogonal Frequency Division Multiple Access (OFDMA) and the uplink would use Single Carrier Frequency Division Multiple Access (SC-FDMA). Supported downlink data modulation schemes are QPSK, 16QAM, and 64QAM, and possible uplink data modulation schemes are p/2-shifted BPSK, QPSK, 8PSK and 16QAM. They agreed the use of Multiple Input Multiple Output (MIMO) scheme with possibly up to four antennas at the mobile side and four antennas at the base station. Re-using the expertise from the UTRAN, they agreed to the same channel coding type as UTRAN (turbo codes).

They agreed to a transmission time interval (TTI) of 1 ms to reduce signaling overhead and to improve efficiency. The study item phase ended in September 2006 and the LTE works are scheduled to conclude in early 2008 and produce a technical standard. More technical details on 3GPP LTE are at [6] and [7].

1.3. Single Carrier FDMA

Ever increasing demand for higher data rate is leading to utilization of wider transmission bandwidth. Broadband wireless mobile communications suffer from multipath frequency selective fading. For broadband multipath channels, conventional time domain equalizers are impractical for complexity reason. Orthogonal frequency division multiplexing (OFDM), which is a multicarrier communication technique, has become widely accepted primarily because of its robustness against frequency-selective fading channels which are common in broadband mobile wireless communications [8]. Orthogonal frequency division multiple access (OFDMA) is a multiple access scheme which is an extension of OFDM to accommodate multiple simultaneous users. OFDM/OFDMA technique is currently adopted in wireless LAN (IEEE 802.11a & 11g), WiMAX (IEEE 802.16), and 3GPP LTE downlink systems. Despite the benefits of OFDM and OFDMA, they suffer a number of drawbacks including: high peak-to-average power ratio (PAPR), a need for an adaptive or coded scheme to overcome spectral nulls in the channel, and high sensitivity to frequency offset. Single carrier frequency division multiple access (SC-FDMA) which utilizes single carrier modulation and frequency domain equalization is a technique that has similar performance and essentially the same overall complexity as those of OFDMA system [9]. One prominent advantage over OFDMA is that the SC-FDMA signal has lower PAPR because of its inherent single carrier structure. SC-FDMA has drawn great attention as an attractive alternative to OFDMA, especially in the uplink communications where lower PAPR greatly benefits the mobile terminal in terms of transmit power efficiency and manufacturing cost. SC-FDMA has two different subcarrier mapping schemes：distributed and localized. In distributed subcarrier mapping scheme, user’s data occupy a set of distributed subcarriers and we achieve frequency diversity. In localized subcarrier mapping scheme, user’s data inhabit a set of consecutive localized subcarriers and we achieve frequency-selective gain through channel-dependent scheduling (CDS).

SC-FDMA is currently a working assumption for the uplink multiple access scheme in 3GPP LTE [10].

Literature Translation

宽带通信的单载波频分多址技术

1.1 蜂窝无线网的革新

在 20 世纪 50 年代和 60 年代，在 AT&T 和贝尔实验室及全世界公司的研究者们产生了蜂窝无线电话的想法。无线蜂窝网的概念将一个覆盖区域打破为很多小区域并重复使用那部分可用频谱。在 1979 年，世界上第一个蜂窝网系统在日本 NTT 建成，因此开始了无线蜂窝通信的革新[1], [2]。第一代无线蜂窝通信系统使用了模拟通信技术并且它重点倾向于语音通信。频率调制和频分复用是第一代系统的基础。美国的 AMPS 和欧洲的 ETACS 是第一代通信系统。第二代通信系统看到了能极大提高频谱效率的数字通信的到来。并且他们极大的增加了语音通信质量并把包数据传输变为可能。主要的多址接入方案是时分复用和码分复用。基于时分复用的 GSM 和基于 CDMA 系统的 IS-95 是 2 个主要广泛被接受的第二代通信系统。在 20 世纪 80 年代中期，IMT-2000 提出的概念在 ITU 诞生，这个概念是移动通信的第三代系统。IMT-2000 的主要目标是提供无缝的全球漫游并在高速率链路上一系列媒体中提供无缝的服务。在 2000 年，在 IMT-2000 的一个 3G 系统的技术参数被一致同意并且 UMTS/WCDMA 和 cdma2000 是 2 个重要的 IMT-2000 下的均基于 CDMA 系统的标准。IMT-2000 提供高速率；一个最小传输 2Mbps 的静止速率或行走路人及 348kpsk 的移动汽车，然而第二代移动通信系统只能提供在 9.6kbps 到 28.8kbps 的速率。由于最初的标准，WCDMA 和 cdma2000 都被称为 3.5G；通过 HSD/UPA 的 UMTS 及通过 1Xev-DO Rev A 的 cdma2000，第三代合作项目的长期演进计划被考虑为通向下一代通信系统的重要通道。

1.2 第三代合作项目的长期演进计划

3GPP 在演变 3G 移动系统的工作开始于 2004 年 11 月的 RAN 演变工作站[4]。运营商，制造商和研究院提出了超出 40 个基于 UTRAN 演变的想法和方案。他们坚定了一系列高眼球的需求，减少每比特的花费，提高服务供应，使用已存在带快及新带宽的灵活性，简化架构和接口，允许合理的终端能耗。在 3GPP 成员的广泛支持及这个工作站的结论，一个在 UTRA 和

UTRAN 长期演变的灵活的研究开始于 2004 年 12 月。主要的目的是发展一个通过高速率，低成本，包最优化的 3GPP 无线接入技术需要被框架化。他们主要集中于支持超过 20MHz 的灵活性传输带宽，引入新的传输方案，先进的多天线技术，信号最优化，最优 UTRAN 网络的鉴别以及基本的 RAN 网络接口的功能切割。

第一个 E-UTRAN 演进的研究结果达成了一致。主要的要求如下[5]。

- 峰值数据速率：连续的下行 100Mbps 的峰值速率于 20MHz 的带宽中，50Mbps 上行峰值速率于 20MHz 的上行带宽中。
- 控制层面容量：最少每个蜂窝小区 200 个用户能够处于激活状态并分配 5MHz 带宽
- 潜在用户层面：少于 5ms 的非装载状态（也就是一个用户配置一个单数据流）对小 IP 包
- 移动性：E-UTRAN 应该被最优化于 0 到 15km/h 的低移动速率及 15 到 120km/h 的高速率高性能应该被支持。跨蜂窝网的移动性应该被维持在 120 到 350km/h（或者甚至根据带宽最高到 500km/h）
- 覆盖性：吞吐量，频谱效率和移动目标应该在半径 5km 小区和半径 30km 小区下被满足。半径大于 100km 的小区不应该被排除。
- 增强的多媒体广播服务
- 频谱效率：E-UTRA 应该在不同的频谱分配下操作，其中在上下行中包括.25 MHz, 1.6 MHz, 2.5 MHz, 5 MHz, 10 MHz, 15 MHz, and 20 MHz。
- 结构和扩展：基于包的单 E-UTRAN 结构支持系统实时并且会话级的通信兵支持终端对终端的服务质量。
- 无线资源管理：增强终端对终端的服务质量，有效的支持高层传输，支持跨层的负载分享和管理策略技术。广泛的选择开始定于早起的 LTE 工作中并被局限于 2005 年 12 月，其中上行使用 OFDM 而下行使用 SC-FDMA。支持的下行数据调制方案包括 QPSK, 16QAM, and 64QAM 以及可能的上行传输方案包括 p/2-shifted BPSK, QPSK, 8PSK and 16QAM。他们同意使用堕入多出技术，最多可能在移动设备上使用 4 个天线，基站处使用 4 个天线。从 UTRAN 中复用专业技术，他们同意相同的信道编码如 Turbo 码。他们同样同意一个可以降低信号开销及提高效率的传输间隔 1ms。他们的研究停止与 2006 年 9 月并且 LTE 工作被安排在 2008 年早期并提出了一个技术标准。更多的 3GPP LTE 技术细节在[6]和[7]中。

1.3 单载波频分多址接入

每一次对高数据速率提升的要求都会促进一个宽带传输的使用。宽带无线网通信受制于多径频率选择性衰落。对于宽带多径信道，传统的时域均衡由于复杂度的原因已经不太现实。正交频分多址技术是一个多载波通信技术，由于它对于频率选择性信道的坚韧性而被广泛接受，并且被广泛的使用在宽带无线通信系统中[8]。正交频分多址接入作为一个多址接入方

案，是对 OFDM 系统的扩展来保证多用户同时传输。OFDM/OFDMA 技术是如今被无线 LAN 和 WiMAX 及 3GPP LTE 下行系统采用的技术。尽管有 OFDM 和 OFDMA 系统的优势，它们同样受制于一系列的缺点：高的峰均功率比，对自适应或编码方案来解决频率零点的需求以及对频率偏移的高度敏感。单载波频分多址接入使用了单载波调制和频率域均衡，它是一个和 OFDMA 系统有着相同性能和信号处理复杂度的技术[9]。相对于 OFDMA 系统，它的一个重要的优点就是低的峰均功率比，这个是因为它继承了单载波的结构。SC-FDMA 已经吸引了很多注意，被作为 OFDMA 系统外的另一个选择，特别是需要低峰均功率比来提高传输能量效率和制造价格的移动终端。SC-FDMA 有 2 个不同的子载波分配方案：分布式和集中式。在分布式子载波分配方案中，用户的数据占用一系列分离的子载波并有频率分级。在集中式分配方案中，用户的数据占用连续的本地载波并通过信道自适应调度来实现频率选择性增益。SC-FDMA 已经是 3GPP LTE 上行多址接入方案的一个工作假设。