SC-FDMA Oversampling MMSE Equalizer

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Outline

- Background

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- Introduction to **Oversampling MMSE Equalizer SC-FDMA System**

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- Simulation Results

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- SC-FDMA has been adopted as the uplink multiple access scheme in 3GPP Long Term Evolution (LTE), or Evolved UTRA
- SC-FDMA effectively copes with frequency-selective fading channels by using simple frequency-domain equalization

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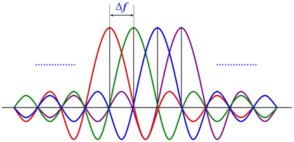

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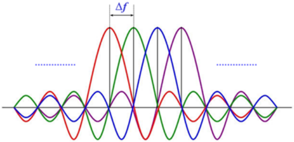

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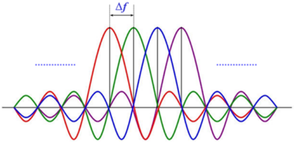

- Inter-carrier Interference is introduced

- Frequency offset estimation and compensation is critical

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- It is clear by noting that a dual problem of frequency offset for multicarrier systems is timing jitter for signal-carrier systems, which is traditionally solved via time-domain-oversampling

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- Oversampling MMSE Equalization may be also an effective way to combat the frequency offset
- It is clear by noting that a dual problem of frequency offset for multicarrier systems is timing jitter for signal-carrier systems, which is traditionally solved via time-domain-oversampling
- Frequency oversampling should be expected to be robust against frequency offset

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Comparison between Conventional MMSE and Oversampling MMSE

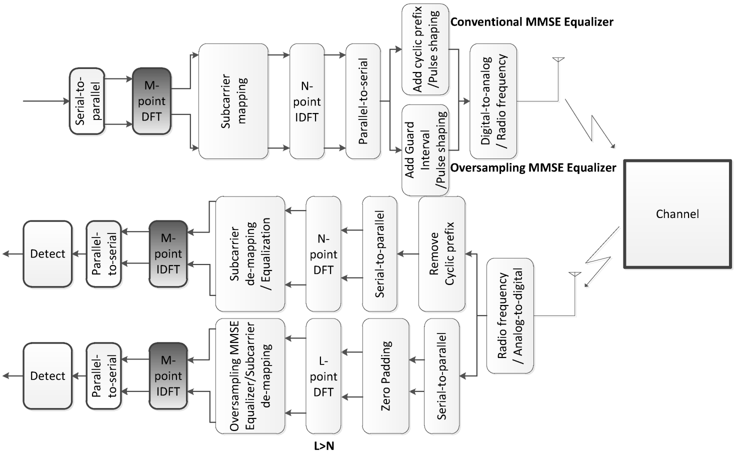

Fig: Comparison between Conventional MMSE Equalizer and Oversampling MMSE equalizer

Mathmatical Model-Transmitter

Mathmatical Model-Transmitter

- DFT operation

$$X^i = F_M d^i$$

where

$$d^i = [d_1^i, d_2^i, ..., d_M^i]^T$$

$$[F_M]_{p,q} = (1/\sqrt{M})e^{-j2\pi(pq/M)}$$

$$\mathrm{i = 1, 2, ..., Q, Q = N/M}$$

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- Subcarrier Mapping

$$\overline{X}^i_{N\times 1} = M_T^i X^i_{M\times 1}$$

where M_T^i is subcarrier mapping matrix

Mathmatical Model-Transmitter

- IDFT operation

$$x = F_N^{-1} \sum_{i=1}^{Q} M_T^i X^i$$

Mathmatical Model-Transmitter

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$$x = F_N^{-1} \sum_{i=1}^{Q} M_T^i X^i$$

- Add Guard Interval

$$x = P_{GI} F_N^{-1} \sum_{i=1}^{Q} M_T^i X^i$$

where

$$P_{GI} = [I_N^T, 0_{N_g \times N}^T]^T$$

Mathmatical Model-Channel Model

- Channel Response

$$h = \sum_{k=0}^{K-1} h_k \delta(\tau - \tau_k)$$

where

$$\sum_{k=0}^{K-1} \mathrm{E}\left[|h_k|^2 \right] = 1$$

Mathmatical Model-Channel and Receiver

- Receiver Signal after Zeros Padding

$$\widetilde{r} = D(\varepsilon)\, P_{zp} \widetilde{H} x + n$$

where

$$P_{zp} = \left[I_P^T 0_{(L-P)\times P}^T \right]^T$$

$$D(\varepsilon) = diag\left\{ 1, e^{-j\frac{2\pi}{N}\varepsilon}, ..., e^{-j\frac{2\pi}{N}(L-1)\varepsilon} \right\}$$

and $\widetilde{H}$ is a $P \times P$ lower triangular Toeplitz matrix with the first column $[h_0, h_1, ..., h_{K-1}, 0, ..., 0]^T$ and ε is normalized by subcarrier spacing 1/T

Mathmatical Model-Receiver

- Channel Matrix Transformation

$$\widetilde{H}_{zp} = P_{zp}\widetilde{H}P_{GI}$$

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By defining

$$F_{L\times L} \triangleq \left\{ \frac{1}{\sqrt{L}} \exp\left(-j\frac{2\pi}{L}np\right)\right\}_{L\times L}$$

$$F_{L\times N} \triangleq \left\{ \frac{1}{\sqrt{L}} \exp\left(-j\frac{2\pi}{L}lp\right)\right\}_{L\times N}$$

$$\widetilde{H}_{zp} = F^{H}_{L\times L}HF_{L\times N}$$

where $H = diag\{H_0, H_1, ..., H_{L-1}\}$ and S = L/N, which is the oversampling factor

$$H_l \triangleq H\left(\frac{l}{ST}\right) = \sum_{k=0}^{K-1} h_k \exp(-j2\pi\left[\frac{l}{S}\right]\frac{\tau_q}{T}), l = 0, 1, ..., L-1.$$

Mathmatical Model-Receiver

- Receive Signal after L-point DFT

$$
\begin{aligned}
y &= F_{L\times L} D(\varepsilon) P_{zp} \widetilde{H} P_{GI} F_{N\times N}^{H} \sum_{i=1}^{Q} M_T^i X^i + F_{L\times L} P_{zp} n \\
&= F_{L\times L} D(\varepsilon) F_{L\times L}^{H} H F_{L\times N} F_{N\times N}^{H} \sum_{i=1}^{Q} M_T^i X^i + F_{L\times P} n \\
&= F_{L\times L} D(\varepsilon) F_{L\times L}^{H} H \Omega \Psi + \eta
\end{aligned}
$$

where

$$\Omega = F_{L\times N} F_{N\times N}^{H}, \Psi = \sum_{i=1}^{Q} M_T^i X^i, \eta = F_{L\times P} n$$

$$C_\eta = E\left[\eta \eta^H\right] = \sigma_n^2 \Gamma, \qquad \Gamma = F_{L\times P} F_{L\times P}^{H}$$

Mathmatical Model - Receiver

- Oversampling MMSE Receiver

$$\begin{aligned} w^H &= E\left[dy^H\right] \cdot E\left[yy^H\right]^\dagger \\ &= \left(\widehat{H}\Omega\right)^H \left\{\widehat{H}\Omega\Omega^H\widehat{H}^H + \frac{\sigma_n^2}{\sigma_s^2}\Gamma\right\}^\dagger \end{aligned}$$

  Based on Bayesian Gauss-Markov theorem

- Estimation

$$\widehat{\Psi} = w^H y$$

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- N-point IDFT operation

$$d^i = F^H_{N\times N}\widehat{X^i}$$

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Simulation outline

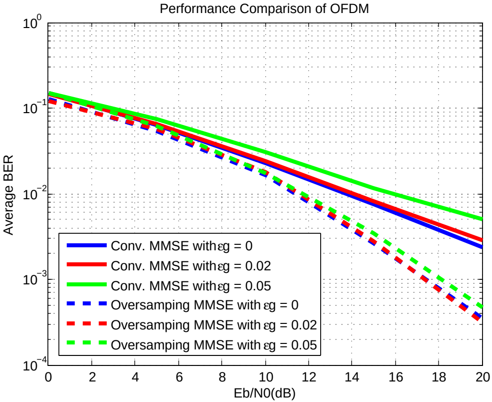
- Simulation Setup
- Reconsctrction of OFDM Oversampling Equalizer based on [1]
- BER Performance Comparison between SC-FDMA and OFDM without CFO
- BER Performance Comparison between SC-FDMA and OFDM with CFO
- BER Performance of SC-FDMA under different CFO
- Effect of Oversampling Factor
- Oversampling Gain Discussion
- One Path Rayleigh Channel with Different CFO

Simulation Setup

- Simulation System Parameters

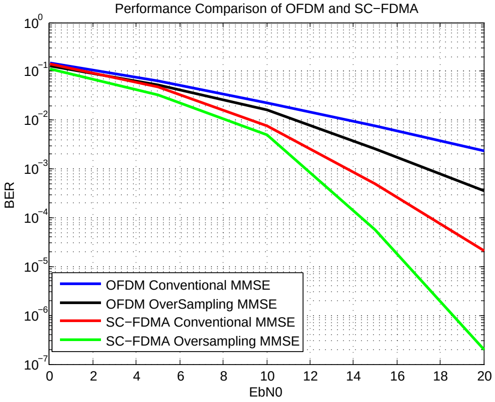
Total Subcarriers	64
Pre-DFT subcarriers number	16
Modulation	QPSK
Equalization	Conventional and Oversampling MMSE
Mapping Scheme	Interleaved
CP number	16
Iteration Number	1000000
Channel	16path Rayleigh channel
Carrier Frequency Offset(CFO)	[0,0.02,0.05,0.1]

Reconsctruction of OFDM Oversampling Equalizer based on [1]



- 1. Shi, Q., Liu, L., Member, S., and Guan, Y. L. (2010). Fractionally Spaced Frequency-Domain MMSE Receiver for OFDM Systems, 59(9), 4400-4407.

BER Performance Comparison between SC-FDMA and OFDM without CFO



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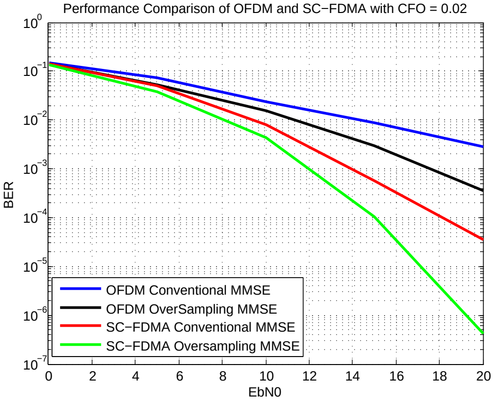

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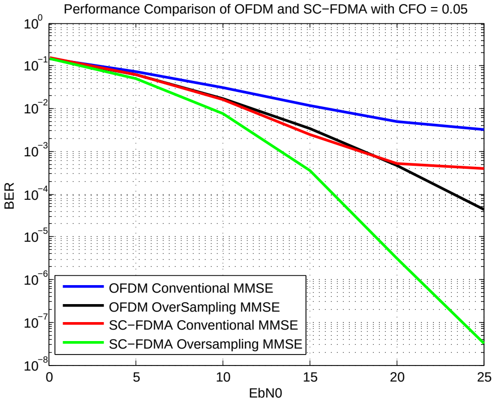

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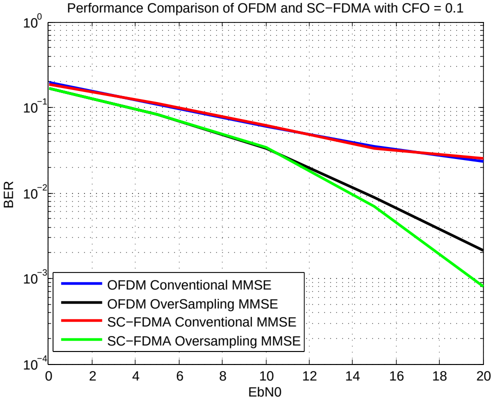
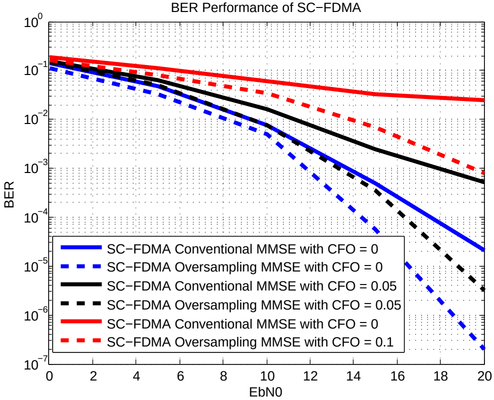
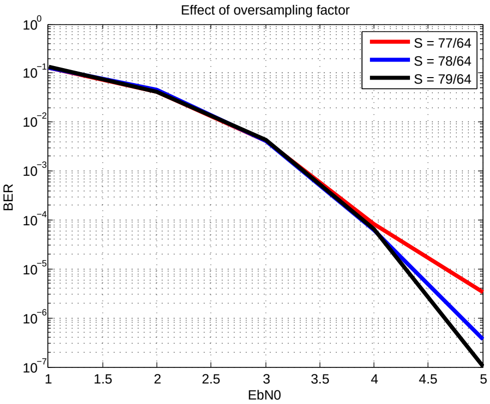


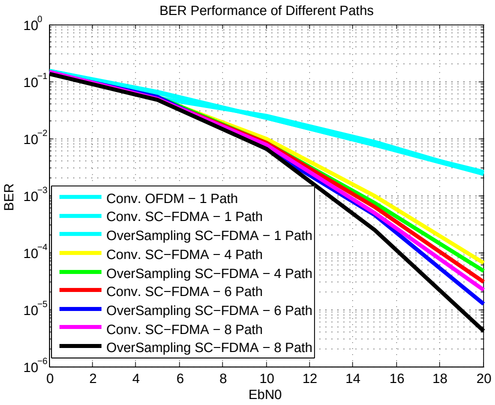
BER Performance of SC-FDMA under different CFO



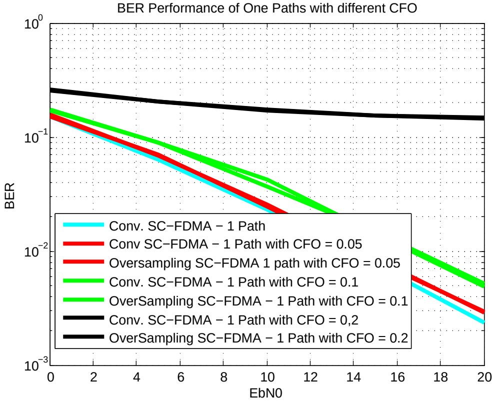
Effect of Oversampling Factor



Oversampling Gain



One Path Channel with Different CFO



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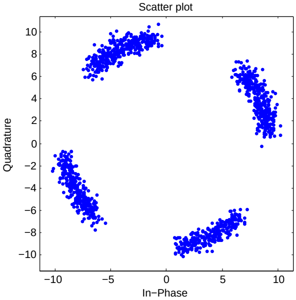

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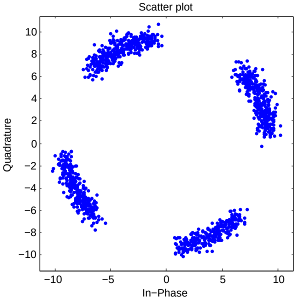

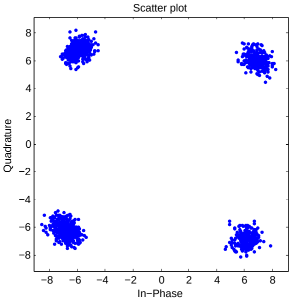

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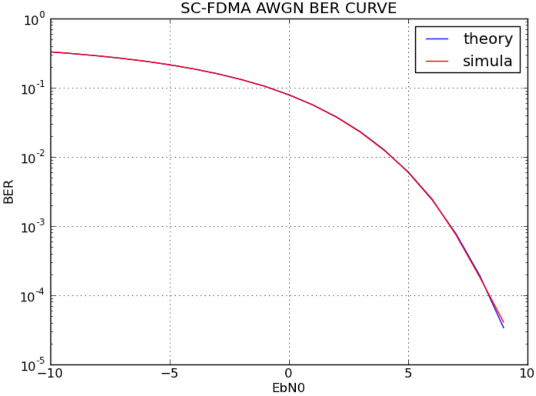

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