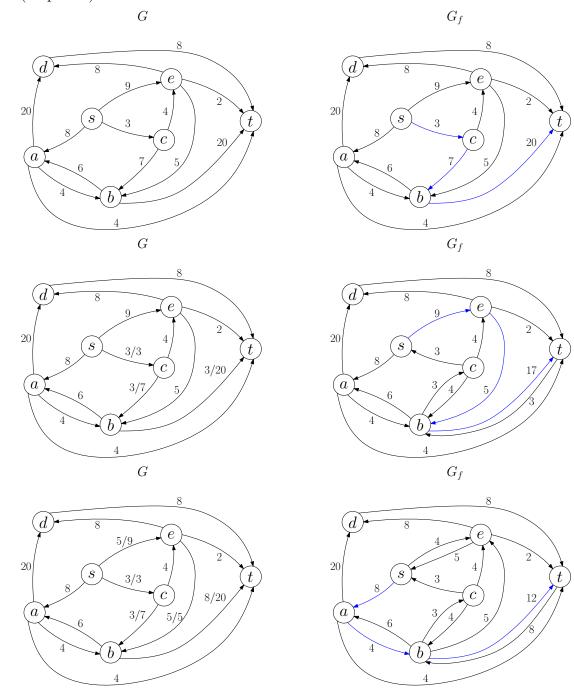
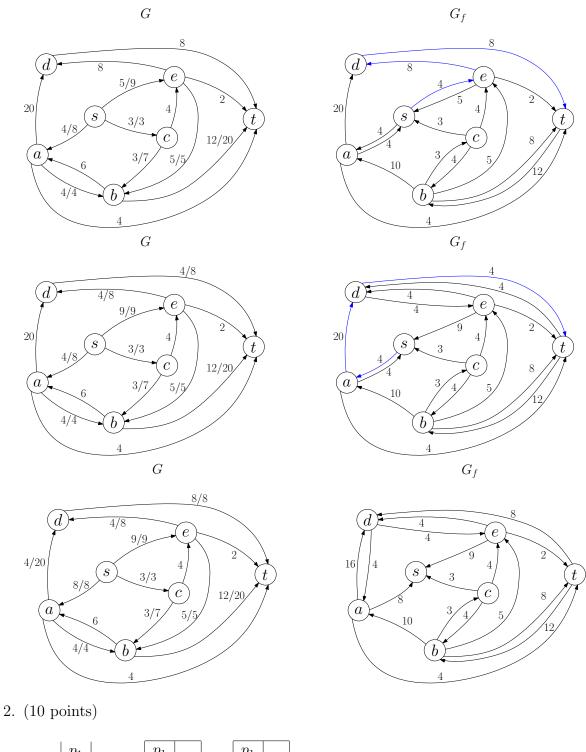
CSIT 5500 Advanced Algorithms 2020 Spring Semester

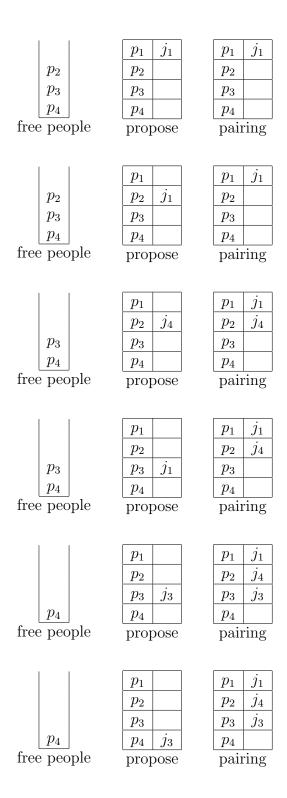
Written Assignment 4 solution

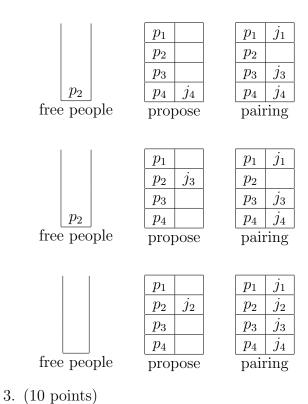
1. (10 points)





$\mid p_1 \mid$	p_1	p_1
$\mid p_2 \mid$	p_2	p_2
$\mid p_3 \mid$	p_3	p_3
$\mid p_4 \mid$	p_4	p_4
free people	propose	pairing





$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	(a) Table A	(b) Frequ	enc	ies	tab	les
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	a 7		1	2	3	4
Estimated frequencies 0 0 0 0 0 0 0 0 0		_	0	0	0	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$[II[a] \mid I]$	Estimated frequencies	0	0	0	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		a	1	2	3	4
Estimated frequencies $0 0 0 1 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0$ $0 0 0 0 $		True frequencies	0	0	1	0
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$A[a] \mid A \mid A$	Estimated frequencies	0	0	1	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	7 7 1	a	1	2	3	4
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		True frequencies	1	0	1	0
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$A[a] \mid 1 \mid 1 \mid 1$	Estimated frequencies	1	0	1	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	7 9 1	\overline{a}	1	2	3	4
Estimated frequencies 2 0 1 0 0 0 0 0 0 0 0		True frequencies	2	0	1	0
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$A[a] \mid 1 \mid 1 \mid 2$	Estimated frequencies	2	0	1	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		\overline{a}	1	2	3	4
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		True frequencies	2	0	1	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$A[a] \mid 1 \mid 1 \mid 2 \mid 1$		2	0	1	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		a	1	2	3	4
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		True frequencies	2	1	1	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$A[a] \mid 1$		1	0	0	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		_	1	2	3	4
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		True frequencies				
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$A[a] \mid 1 \mid 1$	_	1	0	0	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		a	1	2	3	4
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		True frequencies	2	1		0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$A[a] \mid 1 \mid 1 \mid 1$		1	0	0	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			1	2	3	4
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		True frequencies	2			0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$A[a] \mid 1 \mid 1 \mid 1 \mid 1$		1	0	1	0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		a	1	2	3	4
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$A[a] \mid 1 \mid 1 \mid 2 \mid 1$		1	0	1	0
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			1	2	3	4
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$A[a] \mid 1$	_				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		_	1	2	3	
Estimated frequencies $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$ $\begin{vmatrix} a & 5 & 1 \\ 4 & 2 & 1 \end{vmatrix}$ True frequencies $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 0 \end{vmatrix}$		**				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$A[a] \mid 1 \mid 1$	_				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$						
		**				
Estimated frequencies 1 0 0 0	$[A[a] \mid 2 \mid 1]$	Estimated frequencies	1	0	0	0

6 7 8

6 7

 $0 \mid 2$

0 1

6 7

 $0 \mid 0$

6 7

0 2

6 7

 $0 \mid 1$

 $6 \mid 7$

0 1

6 7

$\begin{bmatrix} a & 5 & 1 & 4 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$A[a] \ 2 \ 1 \ 1$	True frequencies	3	1	2	1	4	0	2	1
$A[u] \mid Z \mid I \mid I$	Estimated frequencies	1	0	0	1	2	0	0	0
$\begin{bmatrix} a & 5 & 1 & 4 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	3	1	2	2	4	0	2	1
$A[a] \mid Z \mid 1 \mid Z$	Estimated frequencies	1	0	0	2	2	0	0	0
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	3	2	2	2	4	0	2	1
$A[a] \mid Z \mid 1 \mid Z \mid 1$	Estimated frequencies	1	1	0	2	2	0	0	0
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	2	2	2	4	0	2	1
$A[a] \mid Z \mid Z \mid Z \mid 1$	Estimated frequencies	2	1	0	2	2	0	0	0
$\begin{bmatrix} a & 5 & 1 & 4 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	2	2	2	4	1	2	1
$A[a] \mid A \mid A \mid A$	Estimated frequencies	1	0	0	1	1	0	0	0
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	2	2	2	4	1	2	2
$A[a] \mid 1 \mid 1 \mid 1 \mid 1$	Estimated frequencies	1	0	0	1	1	0	0	1
	a	1	2	3	4	5	6	7	8
$\begin{bmatrix} a \\ A[a] \end{bmatrix}$	True frequencies	4	2	2	2	4	1	3	2
A[a]	Estimated frequencies	0	0	0	0	0	0	0	0
$\begin{bmatrix} a & 6 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$\begin{bmatrix} a & 6 \\ A[a] & 1 \end{bmatrix}$	True frequencies	4	2	2	2	4	2	3	2
$A[a] \mid 1$	Estimated frequencies	0	0	0	0	0	1	0	0
$\begin{bmatrix} a & 6 & 8 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	2	2	2	4	2	3	3
$[A[a] \mid 1 \mid 1]$	Estimated frequencies	0	0	0	0	0	1	0	1
$\begin{bmatrix} a & 6 & 8 & 7 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	2	2	2	4	2	4	3
$A[a] \mid 1 \mid 1 \mid 1$	Estimated frequencies	0	0	0	0	0	1	1	1
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	2	2	2	5	2	4	3
$A[a] \mid 1 \mid 1 \mid 1 \mid 1$	Estimated frequencies	0	0	0	0	1	1	1	1
$\begin{bmatrix} a & 6 & 8 & 7 & 5 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
	True frequencies	4	2	2	2	5	2	5	3
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Estimated frequencies	0	0	0	0	1	1	2	1
_ a _ 7	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	3	2	2	5	2	5	3
A[a] 1	Estimated frequencies	0	0	0	0	0	0	1	0

		1	0	0	4		0	-	
$\begin{bmatrix} a & 7 & 5 \end{bmatrix}$		1	2	3	4	5	6	7	8
A[a] 1 1	True frequencies	4	3	2	2	6	2	5	3
	Estimated frequencies	0	0	0	0	1	0	1	0
$\begin{bmatrix} a & 7 & 5 & 4 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$A[a] \ 1 \ 1 \ 1$	True frequencies	4	3	2	3	6	2	5	3
	Estimated frequencies	0	0	0	1	1	0	1	0
$\begin{bmatrix} a & 7 & 5 & 4 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$A[a] \ 1 \ 2 \ 1$	True frequencies	4	3	2	3	7	2	5	3
	Estimated frequencies	0	0	0	1	2	0	1	0
a 7 5 4 8	a	1	2	3	4	5	6	7	8
$A[a] \ 1 \ 2 \ 1 \ 1$	True frequencies	4	3	2	3	7	2	5	4
Estimated frequencies		0	0	0	1	2	0	1	1
a 7 5 4 8	a	1	2	3	4	5	6	7	8
$A[a] \ 1 \ 2 \ 2 \ 1$	True frequencies	4	3	2	4	7	2	5	4
	Estimated frequencies	0	0	0	2	2	0	1	1
a 7 5 4 8	a	1	2	3	4	5	6	7	8
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	4	3	2	4	8	2	5	4
	Estimated frequencies	0	0	0	2	3	0	1	1
$\begin{bmatrix} a & 5 & 4 \end{bmatrix}$	a	1	2	3	4	5	6	7	8
$\begin{array}{c cccc} a & 5 & 4 \\ \hline A[a] & 2 & 1 \\ \end{array}$	True frequencies	5	3	2	4	8	2	5	4
$A[a] \ Z \ I$	Estimated frequencies	0	0	0	1	2	0	0	0
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	a	1	2	3	4	5	6	7	8
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	True frequencies	6	3	2	4	8	2	5	4
$A[a] \mid Z \mid 1 \mid 1$	Estimated frequencies	1	0	0	1	2	0	0	0
	a	1	2	3	4	5	6	7	8
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	6	3	2	4	9	2	5	4
$A[a] \mid 3 \mid 1 \mid 1$	Estimated frequencies	1	0	0	1	3	0	0	0
	a	1	2	3	4	5	6	7	8
	True frequencies	6	4	2	4	9	2	5	4
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Estimated frequencies	1	1	0	1	3	0	0	0
	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c} a & 5 \\ \hline A[a] & 2 \\ \hline \end{array}$	True frequencies	6	4	2	4	9	2	6	4
$\lfloor A[a] \mid 2 \rfloor$	Estimated frequencies	0	0	0	0	2	0	0	0
	a	1	2	3	4	5	6	7	8
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	7	4	2	4	9	2	6	4
$[A[a] \mid 2 \mid 1]$	Estimated frequencies	1	0	0	0	2	0	0	0
a F 1 0	a	1	2	3	4	5	6	7	8
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	7	5	2	4	9	2	6	4
$A[a] \mid 2 \mid 1 \mid 1$	Estimated frequencies	1	1	0	0	2	0	0	0
F 1 0	\overline{a}	1	2	3	4	5	6	7	8
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	True frequencies	8	5	2	4	9	2	6	4
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Estimated frequencies	2	1	0	0	2	0	0	0

a	5	1	2	3
A[a]	2	2	1	1

a	5	1
A[a]	1	1

a	1	2	3	4	5	6	7	8
True frequencies	8	5	3	4	9	2	6	4
Estimated frequencies	2	1	1	0	2	0	0	0
a	1	2	3	4	5	6	7	8
	_	_	9	_	~		'	_
True frequencies	8	5	3	5	9	2	6	4