CSIT 5500 Advanced Algorithms

2020 Spring Semester

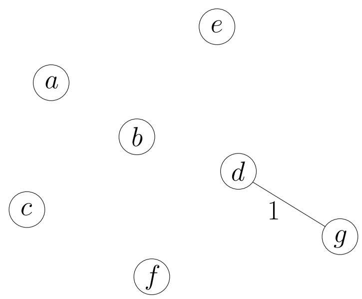
Written Assignment 3 solution

1. (10 points)

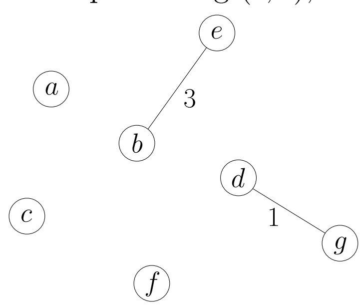
   (a) (d, g), (b, e), (a, b), (a, e), (d, f), (f, h), (g, h), (b, c), (c, f), (e, g).

   (b)

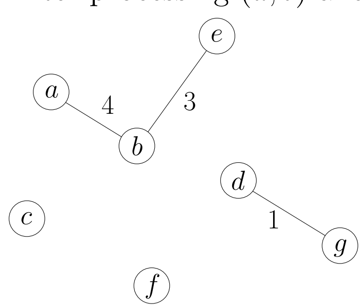
   After processing (d, g),



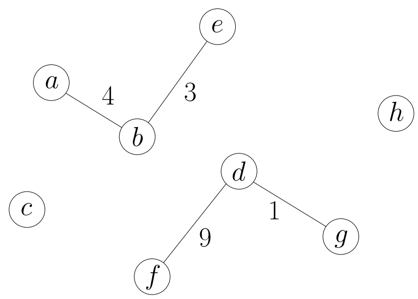
   After processing (b, e),



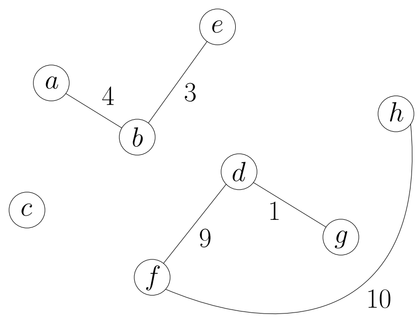
   After processing (a, b) and (a, e),



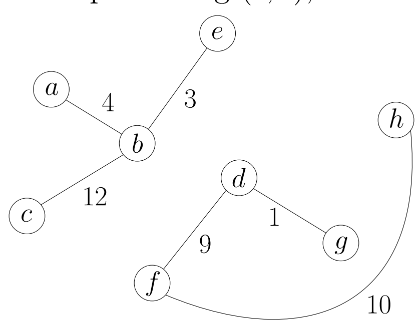
   After processing (d, f),



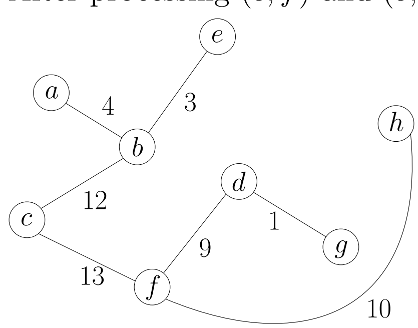
After processing (f, h) and (g, h),



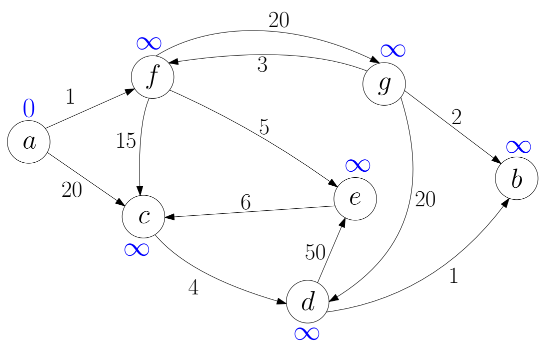
After processing (b, c),



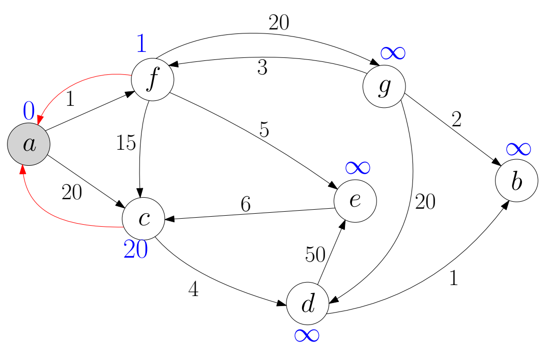
After processing (c, f) and (e, g),



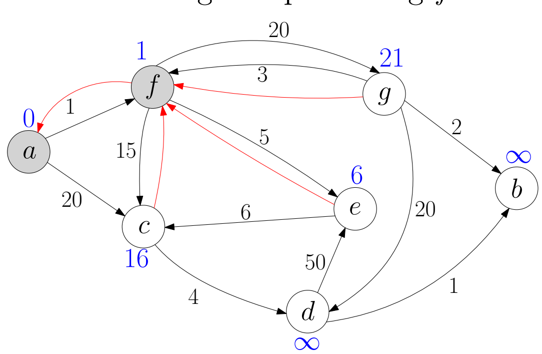
2. (10 points)
   Before removing a from Q,



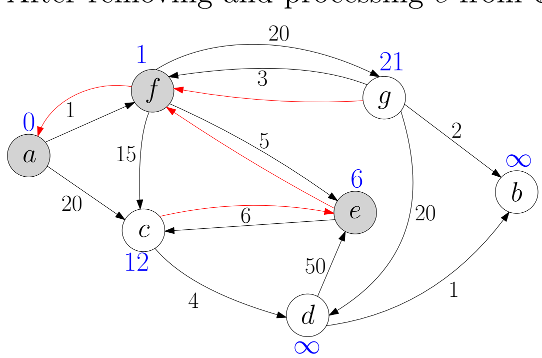
After removing and processing a from Q,



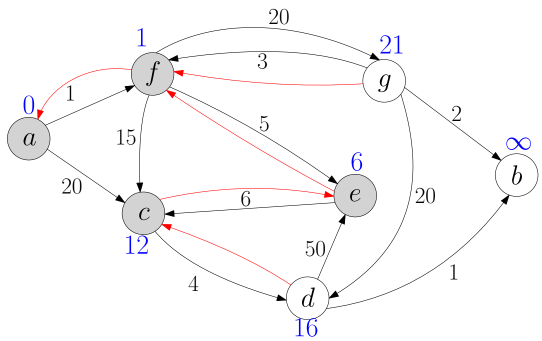
After removing and processing f from Q,



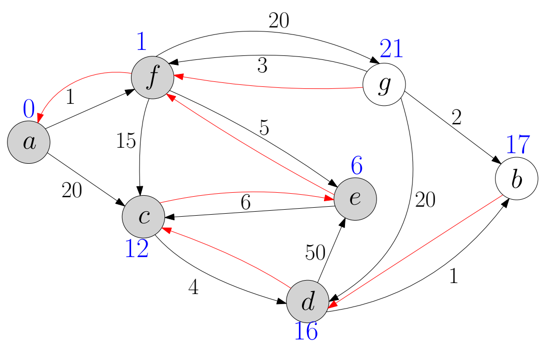
After removing and processing e from Q,



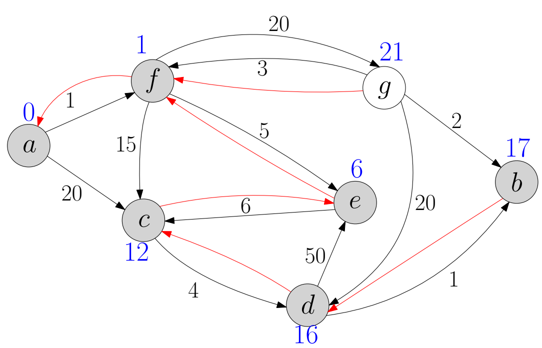
After removing and processing c from Q,



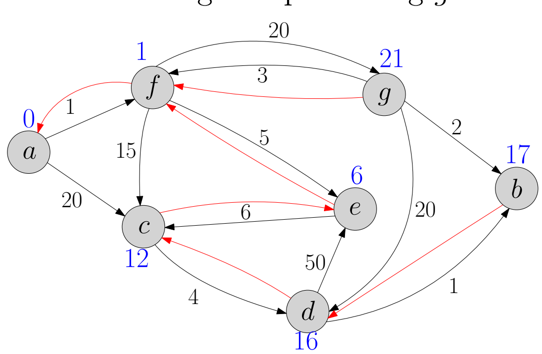
After removing and processing d from Q,



After removing and processing b from Q,



After removing and processing g from Q,



3. (10 points) For each connected component C of G, compute the minimum spanning tree T_C of C by Prim's or Kruskal's algorithm. We then obtain a

forest F, each tree of F is the minimum spanning tree of the corresponding connected component of G. This forest represents the minimum width paths for every pair of vertices in G. The running time is $O(m \log n)$.

Correctness: For any pair of distinct vertices u and v in G, if there is no path from u to v in F, then there is no path from u to v in G because u and v belong to different connected components of G. Suppose u and v belong to the same connected component C in G, there is a unique path P from u to v in T_C. Let $w(P)$ denote the width of P. Suppose there is another path P' in C such that $w(P') < w(P)$. Let e denote the maximum weight edge in P. The edge e does not belong to P' because $w(P') < w(P)$. There exists a cycle C' in G such that C' contains only edges in $P \cup P'$ and C' contains e. The edge e is the maximum weight edge in C'. If e is removed from T_C, T_C is split into two connected components C_1 and C_2. There is an edge $e' \neq e$ and e' in $C' \setminus T_C$ such that one endpoint of e' in C_1 and the other endpoint in C_2. Therefore, replacing e by e' results in a spanning tree with weight smaller than T_C, a contradiction.