

**CSIT 5500 Advanced Algorithms**  
**2020 Spring Semester**  
**Written Assignment 1**  
**Handed out: March 9**  
**Due: 21:00 on March 16**

**Please submit a soft copy via the canvas system by the due date and time shown above. Late assignments will not be graded.**

1. (10 points) This question is about red-black tree.
  - (a) (7 points) Starting from an initially empty red-black tree  $T$ , insert the numbers 1, 3, 5, 7, 9, 2, 4, 6, 8, 10 in this order into  $T$ . Show the tree  $T$  after inserting each number. You do not need to show other intermediate steps.
  - (b) (3 points) Starting from the final  $T$  that you obtained in (a) above containing the numbers 1 to 10, delete the numbers 7, 4, 1, in this order. Show the tree  $T$  after deleting each number. You do not need to show other intermediate steps.
2. (10 points) Let  $x_1, x_2, \dots, x_n$  be a list of  $n$  distinct input integers. We call the pair  $(i, j)$  an *inversion* if  $i < j$  and  $x_i > x_j$ . Give a divide-and-conquer algorithm that reports in  $O(n \log n)$  time the total number of inversions in the input list. Explain why your algorithm works and why it runs in  $O(n \log n)$  time.
3. (10 points) Prove that a complete binary tree of  $n$  nodes has  $O(\log n)$  height.