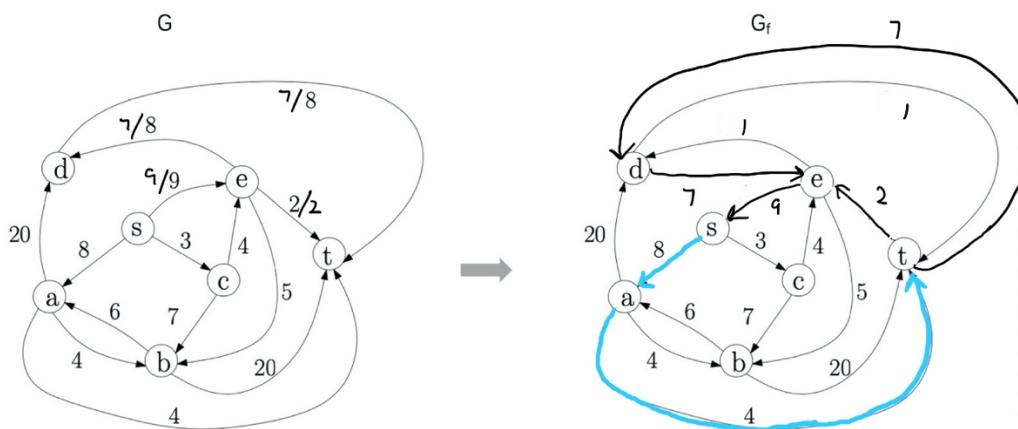
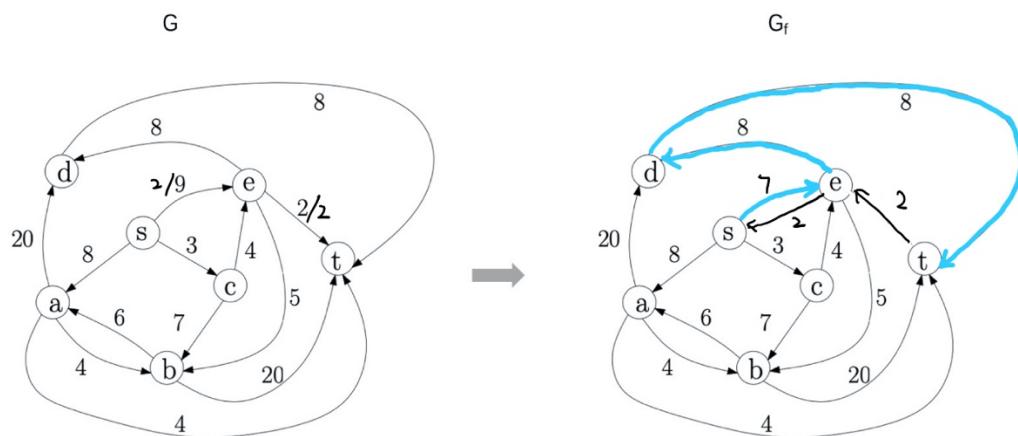
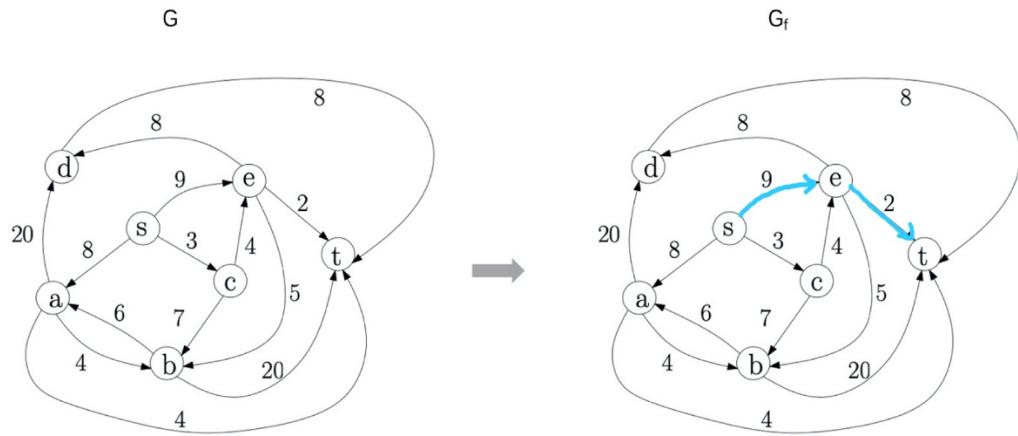


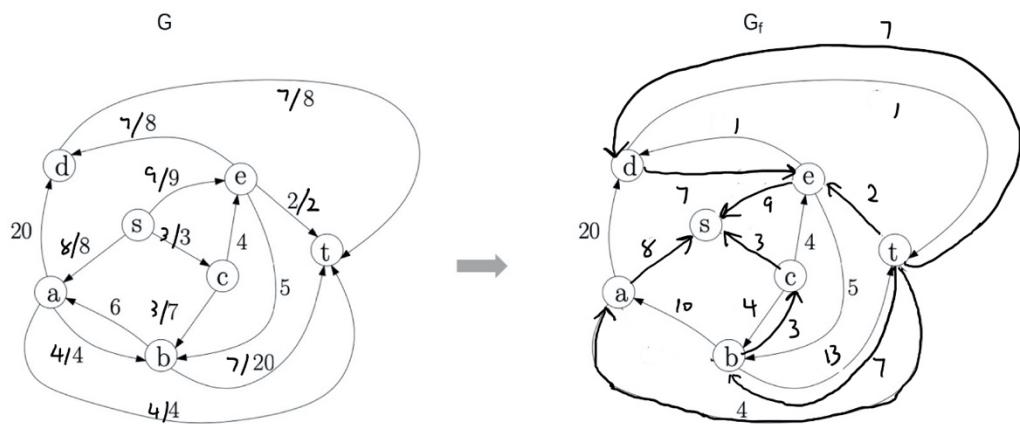
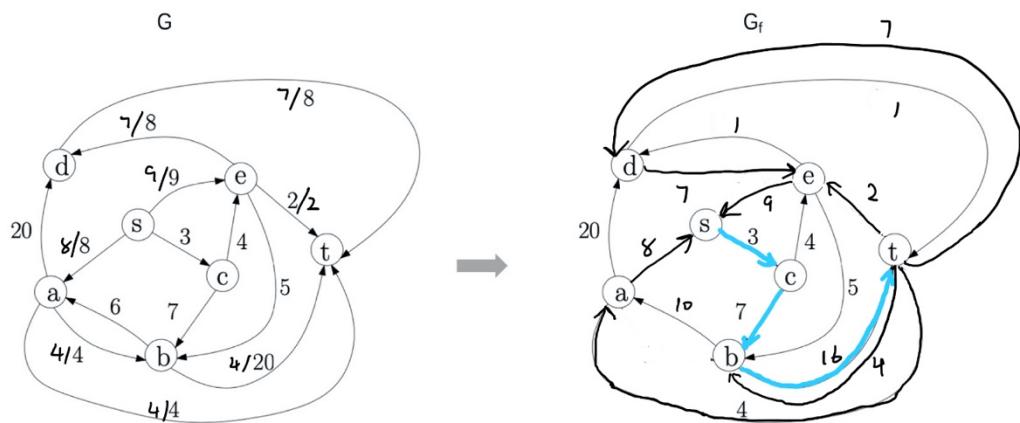
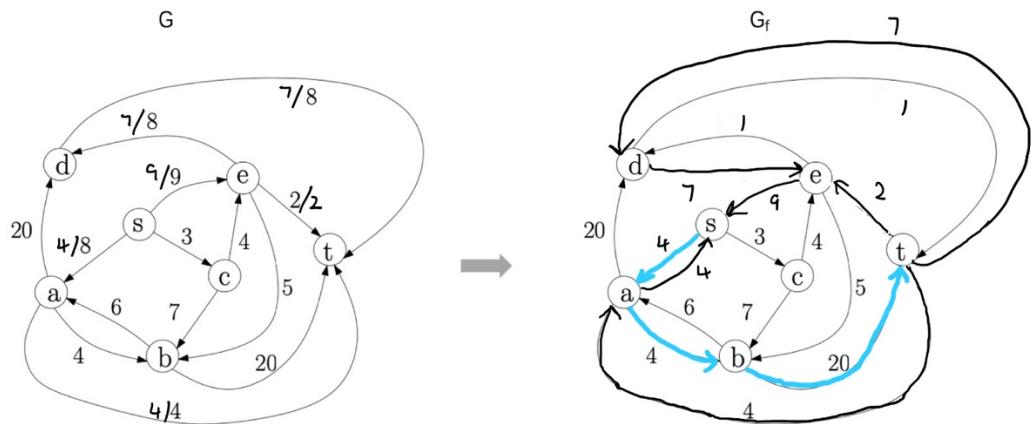
CSIT 5500 Advanced Algorithm

LIN Jialiang

Q1:

Step by step, showing the graph G with flow values in the left, and the corresponding residual graph G_f with augmenting path (blue line) in the right:





\Downarrow
 no more augmenting path
 in the residual graph.

Therefore, the maximum flow value is: $9+8+3=20$

Q2:

Initial:

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2

P1
P2
P3
P4

P1	
P2	
P3	
P4	

P1	
P2	
P3	
P4	

free person

propose

pairing

Propose-and-Reject Algorithm:

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2

P1
P2
P3
P4

P1	j1
P2	
P3	
P4	

P1	
P2	
P3	
P4	

free person

propose

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2

P2
P3
P4

P1	j1
P2	
P3	
P4	

P1	j1
P2	
P3	
P4	

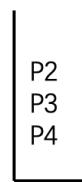
free person

propose

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



P1	
P2	j1
P3	
P4	

free person

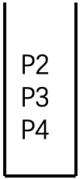
propose

P1	j1
P2	
P3	
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



P1	
P2	j4
P3	
P4	

free person

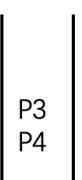
propose

P1	j1
P2	
P3	
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



P1	
P2	j4
P3	
P4	

free person

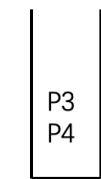
propose

P1	j1
P2	j4
P3	
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	
P3	j1
P4	

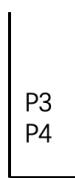
propose

P1	j1
P2	j4
P3	
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	
P3	j3
P4	

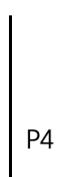
propose

P1	j1
P2	j4
P3	
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	
P3	j3
P4	

propose

P1	j1
P2	j4
P3	j3
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	
P3	
P4	j3

propose

P1	j1
P2	j4
P3	j3
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	
P3	
P4	j4

propose

P1	j1
P2	j4
P3	j3
P4	

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	
P3	
P4	j4

propose

P1	j1
P2	
P3	j3
P4	j4

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	j3
P3	
P4	

propose

P1	j1
P2	
P3	j3
P4	j4

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	j2
P3	
P4	

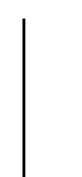
propose

P1	j1
P2	
P3	j3
P4	j4

pairing

People	Jobs			
p_1	j_1	j_2	j_4	j_3
p_2	j_1	j_4	j_3	j_2
p_3	j_1	j_3	j_4	j_2
p_4	j_3	j_4	j_2	j_1

Jobs	People			
j_1	p_4	p_1	p_2	p_3
j_2	p_4	p_1	p_3	p_2
j_3	p_3	p_1	p_2	p_4
j_4	p_1	p_3	p_4	p_2



free person

P1	
P2	j2
P3	
P4	

propose

P1	j1
P2	j2
P3	j3
P4	j4

pairing

So the final pairing is: (p1,j1) (p2,j2) (p3,j3) (p4,j4)

Q3:

According the element stream:

7, 3, 1, 1, 5, 2, 7, 5, 3, 5, 8, 1, 5, 4, 4, 2, 1, 6, 8, 7, 6, 8, 7, 5, 7, 2, 5, 4, 5, 8, 4, 5, 1, 1, 5, 2, 7, 1, 2, 1, 3, 4

see element: 7

Table A

7	1

Estimated Frequency:

1	
2	
3	
4	
5	
6	
7	1
8	

True Frequency:

1	
2	
3	
4	
5	
6	
7	1
8	

see element: 3

Table A

7	1
3	1

Estimated Frequency:

1	
2	
3	1
4	
5	
6	
7	1
8	

True Frequency:

1	
2	
3	1
4	
5	
6	
7	1
8	

see element: 1

Table A

7	1
3	1
1	1

Estimated Frequency:

1	1
2	
3	1
4	
5	
6	
7	1
8	

True Frequency:

1	1
2	
3	1
4	
5	
6	
7	1
8	

see element: 1

Table A

7	1
3	1
1	2

Estimated Frequency:

1	2
2	
3	1
4	
5	
6	
7	1
8	

True Frequency:

1	2
2	
3	1
4	
5	
6	
7	1
8	

see element: 5

Table A

7	1
3	1
1	2
5	1

Estimated Frequency:

1	2
2	
3	1
4	
5	1
6	
7	1
8	

True Frequency:

1	2
2	
3	1
4	
5	1
6	
7	1
8	

see element: 2

Table A

1	1

Estimated Frequency:

1	1
2	
3	
4	
5	
6	
7	
8	

True Frequency:

1	2
2	1
3	1
4	
5	1
6	
7	1
8	

see element: 7

Table A

7	1
1	1

Estimated Frequency:

1	1
2	
3	
4	
5	
6	
7	1
8	

True Frequency:

1	2
2	1
3	1
4	
5	1
6	
7	2
8	

see element: **5**

Table A

7	1
5	1
1	1

Estimated Frequency:

1	1
2	
3	
4	
5	1
6	
7	1
8	

True Frequency:

1	2
2	1
3	1
4	
5	2
6	
7	2
8	

see element: **3**

Table A

7	1
5	1
1	1
3	1

Estimated Frequency:

1	1
2	
3	1
4	
5	1
6	
7	1
8	

True Frequency:

1	2
2	1
3	2
4	
5	2
6	
7	2
8	

see element: **5**

Table A

7	1
5	2
1	1
3	1

Estimated Frequency:

1	1
2	
3	1
4	
5	2
6	
7	1
8	

True Frequency:

1	2
2	1
3	2
4	
5	3
6	
7	2
8	

see element: **8**

Table A

5	1

Estimated Frequency:

1	
2	
3	
4	
5	1
6	
7	
8	

True Frequency:

1	2
2	1
3	2
4	
5	3
6	
7	2
8	1

see element: 1

Table A

1	1
5	1

Estimated Frequency:

1	1
2	
3	
4	
5	1
6	
7	
8	

True Frequency:

1	3
2	1
3	2
4	
5	3
6	
7	2
8	1

see element: 5

Table A

1	1
5	2

Estimated Frequency:

1	1
2	
3	
4	
5	2
6	
7	
8	

True Frequency:

1	3
2	1
3	2
4	
5	4
6	
7	2
8	1

see element: 4

Table A

1	1
5	2
4	1

Estimated Frequency:

1	1
2	
3	
4	1
5	2
6	
7	
8	

True Frequency:

1	3
2	1
3	2
4	1
5	4
6	
7	2
8	1

see element: 4

Table A

1	1
5	2
4	2

Estimated Frequency:

1	1
2	
3	
4	2
5	2
6	
7	
8	

True Frequency:

1	3
2	1
3	2
4	2
5	4
6	
7	2
8	1

see element: **2**

Table A

1	1
5	2
4	2
2	1

Estimated Frequency:

1	1
2	1
3	
4	2
5	2
6	
7	
8	

True Frequency:

1	3
2	2
3	2
4	2
5	4
6	
7	2
8	1

see element: **1**

Table A

1	2
5	2
4	2
2	1

Estimated Frequency:

1	2
2	1
3	
4	2
5	2
6	
7	
8	

True Frequency:

1	4
2	2
3	2
4	2
5	4
6	
7	2
8	1

see element: **6**

Table A

1	1
5	1
4	1

Estimated Frequency:

1	1
2	
3	
4	1
5	1
6	
7	
8	

True Frequency:

1	4
2	2
3	2
4	2
5	4
6	1
7	2
8	1

see element: **8**

Table A

1	1
5	1
4	1
8	1

Estimated Frequency:

1	1
2	
3	
4	1
5	1
6	
7	
8	1

True Frequency:

1	4
2	2
3	2
4	2
5	4
6	1
7	2
8	2

see element: **7**

Table A

Estimated Frequency:

1	
2	
3	
4	
5	
6	
7	
8	

True Frequency:

1	4
2	2
3	2
4	2
5	4
6	1
7	3
8	2

see element: **6**

Table A

6	1

Estimated Frequency:

1	
2	
3	
4	
5	
6	1
7	
8	

True Frequency:

1	4
2	2
3	2
4	2
5	4
6	2
7	3
8	2

see element: **8**

Table A

6	1
8	1

Estimated Frequency:

1	
2	
3	
4	
5	
6	1
7	
8	1

True Frequency:

1	4
2	2
3	2
4	2
5	4
6	2
7	3
8	3

see element: **7**

Table A

6	1
8	1
7	1

Estimated Frequency:

1	
2	
3	
4	
5	
6	1
7	1
8	1

True Frequency:

1	4
2	2
3	2
4	2
5	4
6	2
7	4
8	3

see element: 5

Table A

6	1
8	1
7	1
5	1

Estimated Frequency:

1	
2	
3	
4	
5	1
6	1
7	1
8	1

True Frequency:

1	4
2	2
3	2
4	2
5	5
6	2
7	4
8	3

see element: 7

Table A

6	1
8	1
7	2
5	1

Estimated Frequency:

1	
2	
3	
4	
5	1
6	1
7	2
8	1

True Frequency:

1	4
2	2
3	2
4	2
5	5
6	2
7	5
8	3

see element: 2

Table A

7	1

Estimated Frequency:

1	
2	
3	
4	
5	
6	
7	1
8	

True Frequency:

1	4
2	3
3	2
4	2
5	5
6	2
7	5
8	3

see element: 5

Table A

5	1
7	1

Estimated Frequency:

1	
2	
3	
4	
5	1
6	
7	1
8	

True Frequency:

1	4
2	3
3	2
4	2
5	6
6	2
7	5
8	3

see element: 4

Table A

5	1
4	1
7	1

Estimated Frequency:

1	
2	
3	
4	1
5	1
6	
7	1
8	

True Frequency:

1	4
2	3
3	2
4	3
5	6
6	2
7	5
8	3

see element: 5

Table A

5	2
4	1
7	1

Estimated Frequency:

1	
2	
3	
4	1
5	2
6	
7	1
8	

True Frequency:

1	4
2	3
3	2
4	3
5	7
6	2
7	5
8	3

see element: 8

Table A

5	2
4	1
7	1
8	1

Estimated Frequency:

1	
2	
3	
4	1
5	2
6	
7	1
8	1

True Frequency:

1	4
2	3
3	2
4	3
5	7
6	2
7	5
8	4

see element: 4

Table A

5	2
4	2
7	1
8	1

Estimated Frequency:

1	
2	
3	
4	2
5	2
6	
7	1
8	1

True Frequency:

1	4
2	3
3	2
4	4
5	7
6	2
7	5
8	4

see element: 5

Table A

5	3
4	2
7	1
8	1

Estimated Frequency:

1	
2	
3	
4	2
5	3
6	
7	1
8	1

True Frequency:

1	4
2	3
3	2
4	4
5	8
6	2
7	5
8	4

see element: 1

Table A

5	2
4	1

Estimated Frequency:

1	
2	
3	
4	1
5	2
6	
7	
8	

True Frequency:

1	5
2	3
3	2
4	4
5	8
6	2
7	5
8	4

see element: 1

Table A

5	2
4	1
1	1

Estimated Frequency:

1	1
2	
3	
4	1
5	2
6	
7	
8	

True Frequency:

1	6
2	3
3	2
4	4
5	8
6	2
7	5
8	4

see element: 5

Table A

5	3
4	1
1	1

Estimated Frequency:

1	1
2	
3	
4	1
5	3
6	
7	
8	

True Frequency:

1	6
2	3
3	2
4	4
5	9
6	2
7	5
8	4

see element: 2

Table A

5	3
4	1
1	1
2	1

Estimated Frequency:

1	1
2	1
3	
4	1
5	3
6	
7	
8	

True Frequency:

1	6
2	4
3	2
4	4
5	9
6	2
7	5
8	4

see element: 7

Table A

5	2

Estimated Frequency:

1	
2	
3	
4	
5	2
6	
7	
8	

True Frequency:

1	6
2	4
3	2
4	4
5	9
6	2
7	6
8	4

see element: 1

Table A

5	2
1	1

Estimated Frequency:

1	1
2	
3	
4	
5	2
6	
7	
8	

True Frequency:

1	7
2	4
3	2
4	4
5	9
6	2
7	6
8	4

see element: 2

Table A

5	2
1	1
2	1

Estimated Frequency:

1	1
2	1
3	
4	
5	2
6	
7	
8	

True Frequency:

1	7
2	5
3	2
4	4
5	9
6	2
7	6
8	4

see element: 1

Table A

5	2
1	2
2	1

Estimated Frequency:

1	2
2	1
3	
4	
5	2
6	
7	
8	

True Frequency:

1	8
2	5
3	2
4	4
5	9
6	2
7	6
8	4

see element: 3

Table A

5	2
1	2
2	1
3	1

Estimated Frequency:

1	2
2	1
3	1
4	
5	2
6	
7	
8	

True Frequency:

1	8
2	5
3	3
4	4
5	9
6	2
7	6
8	4

see element: 4

Table A

5	1
1	1

Estimated Frequency:

1	1
2	
3	
4	
5	1
6	
7	
8	

True Frequency:

1	8
2	5
3	3
4	5
5	9
6	2
7	6
8	4