CSIT 5500 Advanced Algorithms

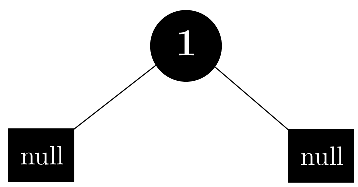
2020 Spring Semester

Written Assignment 1 suggestd solution

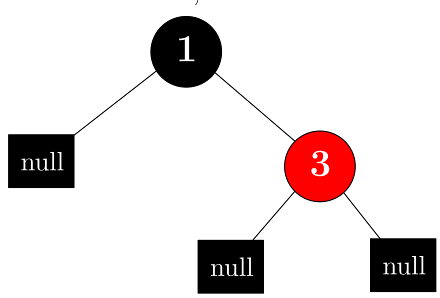
1. (10 points)

   (a) (7 points)

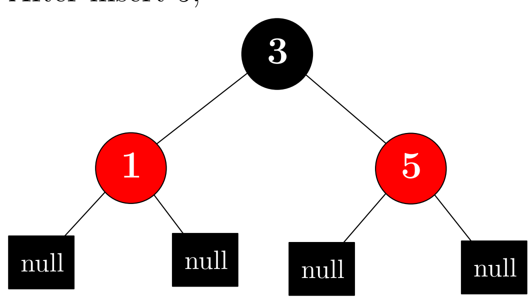
   After inserting 1,



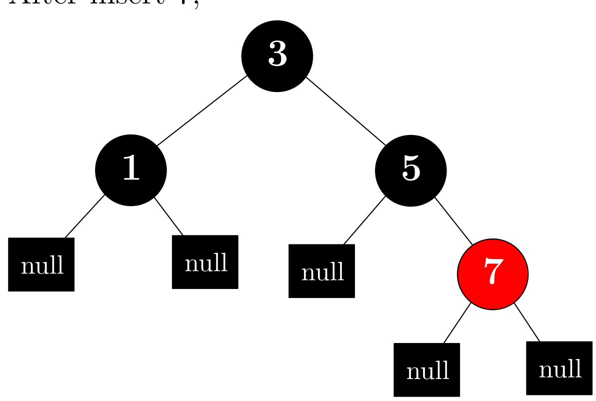
   After insert 3,



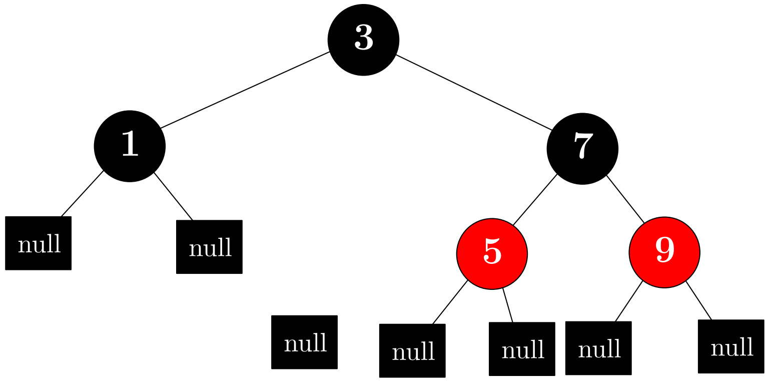
   After insert 5,



   After insert 7,



After insert 9,



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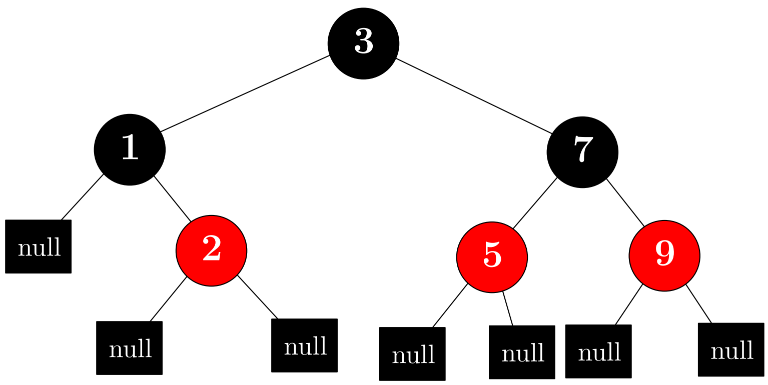
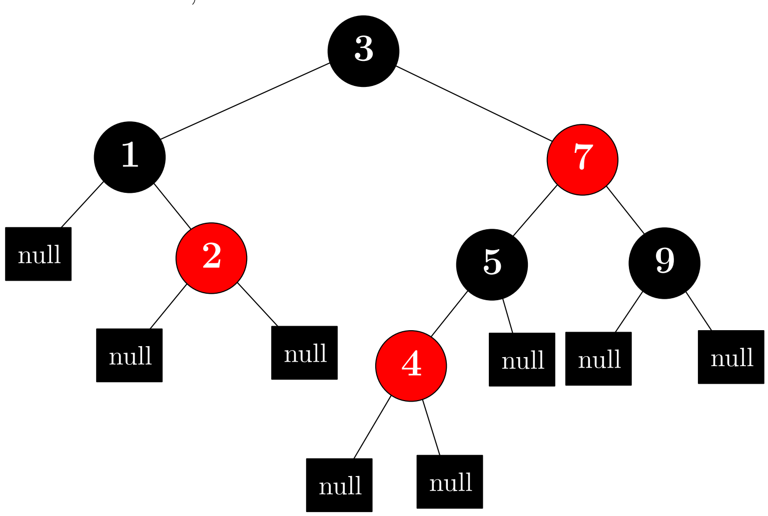
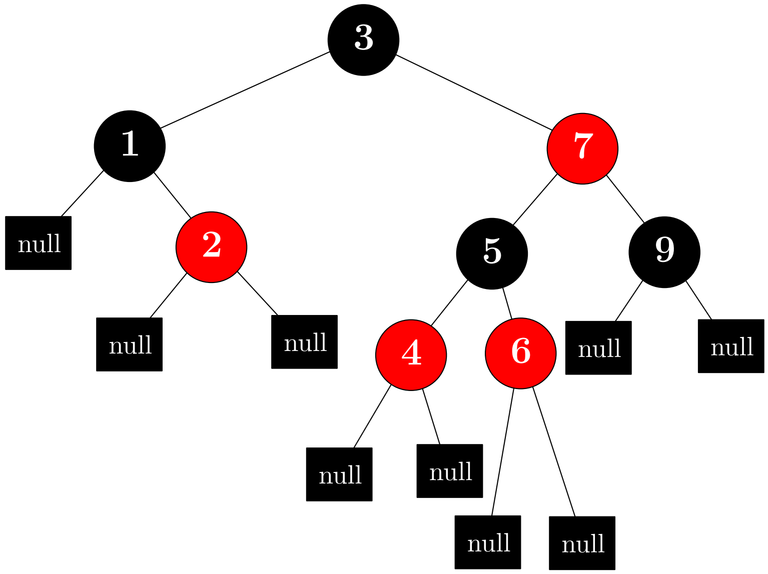


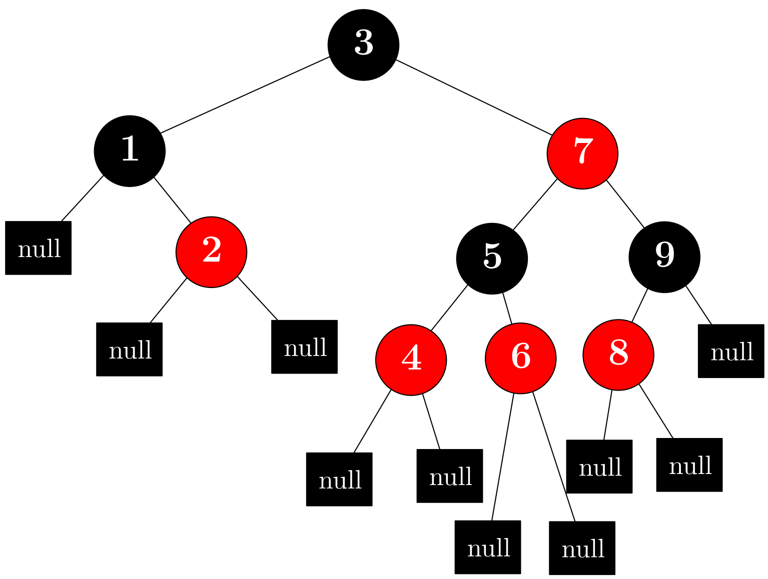
After insert 4,



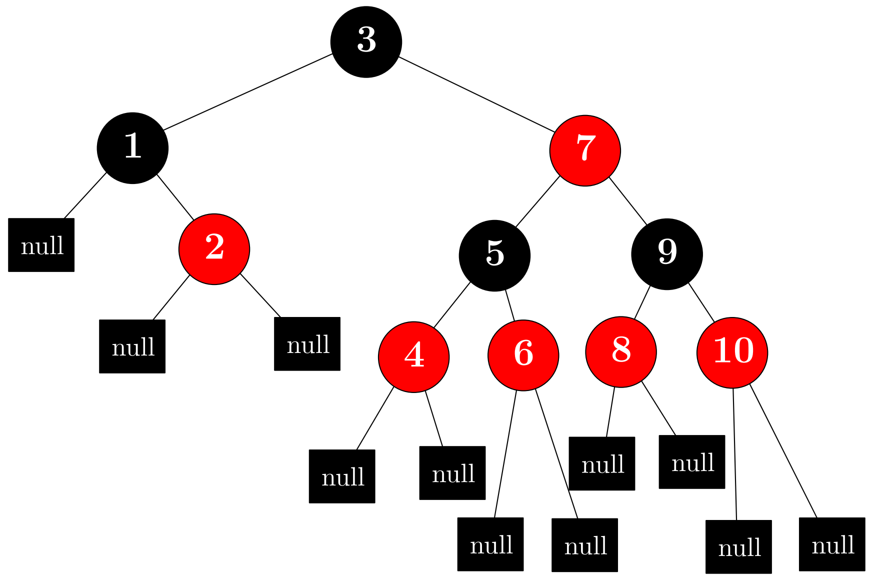
After insert 6,



After insert 8,



After insert 10,



(b) (3 points)
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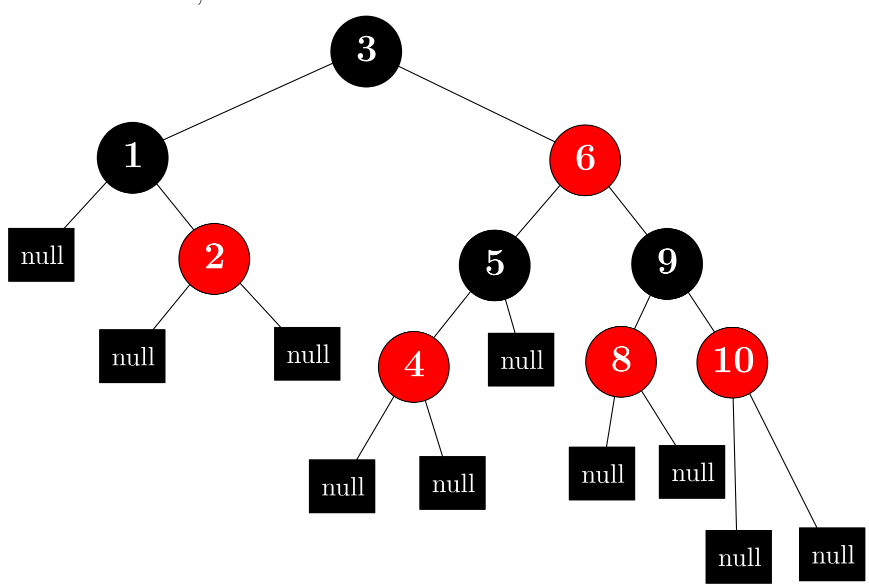

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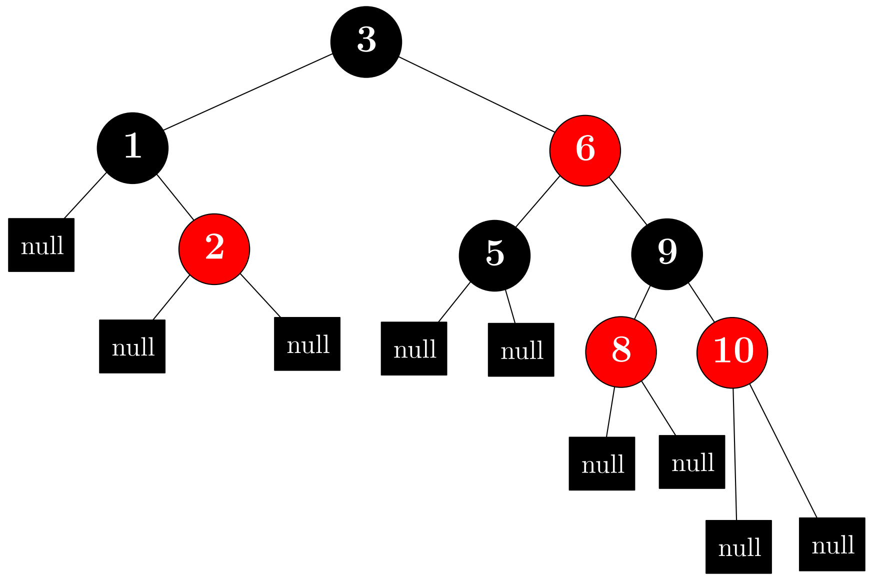
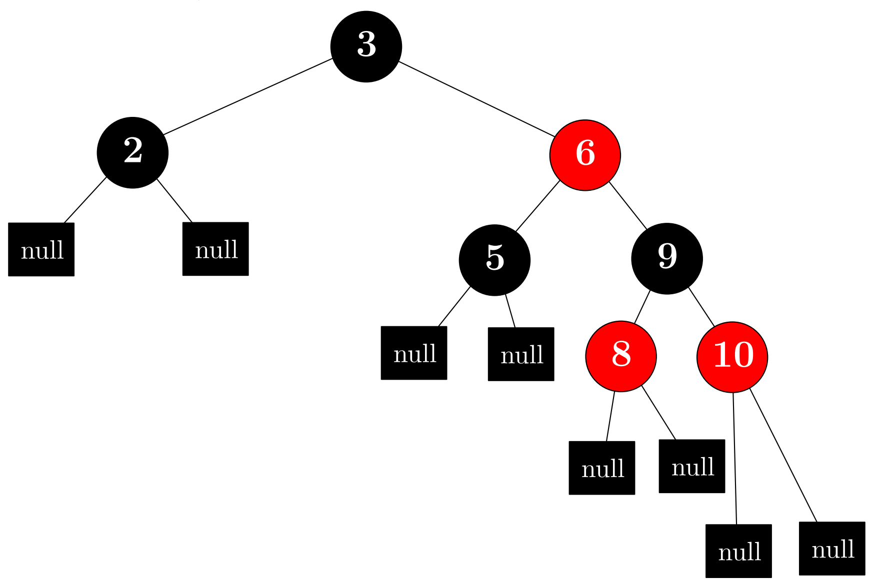


After delete 1,



2. (10 points) Set a counter c for counting the number of inversion. The initial value of c is 0. We perform merge sort for sorting the n integers in ascending order. During the merge operation for merging a left sublist L and a right sublist R, there are two indices k and l for comparing $L[k]$ and $R[l]$. If $L[k] > R[l]$, then we increase c by $|L| - k + 1$. This is because $L[j] > R[l]$ for $k \leq j \leq |L|$, so each such $L[j]$ forms an inversion with $R[l]$. There are exactly $|L| - k + 1$ such $L[j]$'s. Morever, since $R[l]$ will be moved into the merged list, $R[l]$ will not be counted in any inversion in the future. So there is no danger of overcounting. The running time for each update of c is $O(1)$, which is absorted by the comparison of x_i and x_j. So, the total running time is $O(n \log n)$.

3. (10 points) Recall that every level of a complete binary tree is full except possibly the bottommost level. So, for a complete binary tree of n nodes with h height, the number of nodes from level 0 to level $h-1$ is $2^0 + 2^1 + \cdots + 2^{h-1} = 2^h - 1$ which is less than n. So, we have $2^h \leq n+1$, and thus $h \leq \log_2(n+1)$. Therefore, the height is $O(\log n)$.