CSIT 5500 Advanced Algorithm

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Q1:

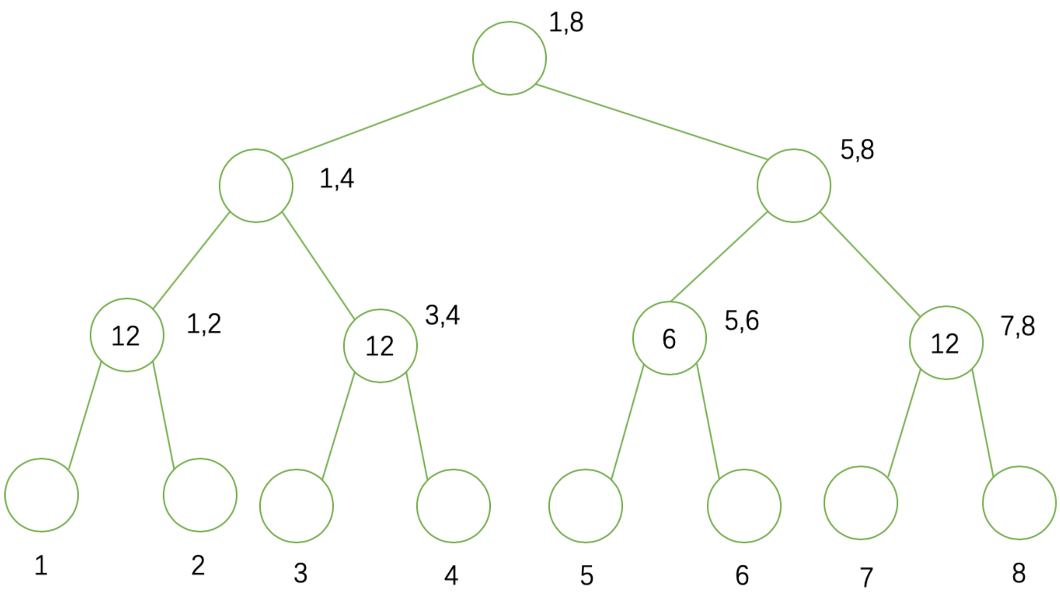
Final array C:

13	14	15
13	15	14

Estimation of each element:

element	0	1	2	3	4	5	6	7	8	9
count	14	13	15	14	13	15	14	13	15	14

Q2:



Estimate the 20-th number:

node range	1,2	3,4	5,6	7,8
count	12	12	6	12

Scan from left to right:

12<20,

12+12>20, then output 4.

4 is the estimation of 20-th number in the non-decreasing sorted list.

Q3:

The algorithm is as the following:

Algorithm :

1. pick an arbitrary point $P_0(x_0, y_0) \in D$.
2. For $t := 1, 2, \cdots$

$$z_t := \arg\min_{z \in D} \langle \nabla f(P_t), z \rangle$$

if stopping condition is satisfied :

return P_t

else :

$$\alpha_t := \arg\min_{\alpha \in [0,1]} f(P_t + \alpha(z_t - P_t))$$

3. $P_{t+1} := P_t + \alpha_t (z_t - P_t)$

The detailed specification of the above algorithm and explanation:

Since f(x) = x^2 + y^2, to avoid the confusion, we use p_t represent x_t and α represent ρ:

① $\nabla f_x = 2x$

$\nabla f_y = 2y$

$\langle \nabla f(P_t), z \rangle = 2x_t z_x + 2y_t z_y$

② since domain D is convex,

$\arg\min_{z \in D} \langle \nabla f(P_t), z \rangle$ can be obtained by emurating

all points in $P = \{P_1, P_2 \cdots P_n\}$ $(n \geq 3)$.

③ $-\langle \nabla f(P_t), Z_t - P_t \rangle = \langle \nabla f(P_t), P_t - Z_t \rangle$

$$= 2x_t(x_t - z_x) + 2y_t(y_t - z_y)$$

$$= 2(x_t^2 + y_t^2 - x_t z_x - y_t z_y)$$

stopping condition is :

$$-\langle \nabla f(P_t), Z_t - P_t \rangle \le \frac{\varepsilon}{1+\varepsilon} f(P_t)$$

using this stopping condition can guarantee $(1+\varepsilon)$-approximation.

Proof :

According to Lemma 2,

$$f(P_t) - f^* \le -\langle \nabla f(P_t), Z_t - P_t \rangle$$

$$\therefore f(P_t) - f^* \le \frac{\varepsilon}{1+\varepsilon} f(P_t)$$

$$\therefore \frac{1}{1+\varepsilon} f(P_t) \le f^*$$

$$\therefore f(P_t) \le (1+\varepsilon) f^*$$

therefore, satisfying $(1+\varepsilon)$-approximation.

④ $f(p_t + \alpha(z_t - p_t))$

$$= f((1-\alpha)p_t + \alpha z_t)$$

$$= ((1-\alpha)x_t + \alpha z_x)^2 + ((1-\alpha)y_t + \alpha z_y)^2$$

$$= \left[(z_x - x_t)^2 + (z_y - y_t)^2\right]\alpha^2 + 2(x_t z_x - x_t^2 + y_t z_y - y_t^2)\alpha$$

$$+ x_t^2 + y_t^2$$

we can see it is a convex function. So the min value would appear in the extreme points or boundary points.

Therefore, we can emurate these points, (here, just 3 pts) to get best α_t.

⑤ $\|\nabla f(x) - \nabla f(y)\| \leq L\|x-y\|$

$\therefore \|(2x_1, 2x_2) - 2(y_1, y_2)\| \leq L\|(x_1 - y_1, x_2 - y_2)\|$

$\therefore L \geq 2$

and we want to minimize $\dfrac{\alpha^2 L\|z_t - p_t\|^2}{2}$

$L = 2.$