CSIT 5410 Assignment2

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Written Part:

(a)

$A(1,3) \rightarrow E(4.5, 4.5)$

$B(1,1) \rightarrow F(2.5, 1.5)$

$C(3,1) \rightarrow G(5.5, 1.5)$

$D(3,3) \rightarrow H(7.5, 4.5)$.

which means:

$$\begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} \Rightarrow \begin{cases} 4.5 = a+3b \\ 4.5 = c+3d \end{cases} \quad ①$$

$$\begin{pmatrix} 2.5 \\ 1.5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} 2.5 = a+b \\ 1.5 = c+d \end{cases} \quad ②$$

$$\begin{pmatrix} 5.5 \\ 1.5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} 5.5 = 3a+b \\ 1.5 = 3c+d \end{cases} \quad ③$$

$$\begin{pmatrix} 7.5 \\ 4.5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} \Rightarrow \begin{cases} 7.5 = 3a+3b \\ 4.5 = 3c+3d \end{cases} \quad ④$$

Combining ① and ②, we get:

$$\begin{cases} a+3b = 4.5 \\ c+3d = 4.5 \\ a+b = 2.5 \\ c+d = 1.5 \end{cases} \Rightarrow \begin{cases} a = 1.5 \\ b = 1 \\ c = 0 \\ d = 1.5 \end{cases} \quad ⑤$$

we apply ⑤ into ③ and ④,

the equations still hold.

Therefore, the transformation $S = \begin{pmatrix} 1.5 & 1 \\ 0 & 1.5 \end{pmatrix}$.

(b) From (a), we know $S = \begin{pmatrix} 1.5 & 1 \\ 0 & 1.5 \end{pmatrix}$

so $S^{-1} = \frac{1}{1.5\times1.5 - 0} \begin{pmatrix} 1.5 & -1 \\ 0 & 1.5 \end{pmatrix} = \begin{pmatrix} 0.667 & -0.444 \\ 0 & 0.667 \end{pmatrix}$

we can get the coordinates of point J in the image I before transformation :

$j = S^{-1} J = \begin{pmatrix} 0.667 & -0.444 \\ 0 & 0.667 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 1.778 \\ 1.333 \end{pmatrix}$

And we also know that ;

For I: $2 = a + 2b + 2c + d$

For K: $5 = 2a + 2b + 4c + d$

For L: $9 = 2a + b + 2c + d$

For M: $15 = a + b + c + d$

$\Rightarrow \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 2 & 4 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 9 \\ 15 \end{pmatrix}$

Set matrix $A = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 2 & 4 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = A^{-1} \begin{pmatrix} 2 \\ 5 \\ 9 \\ 15 \end{pmatrix} = \begin{pmatrix} -15 \\ -22 \\ 9 \\ 43 \end{pmatrix}$

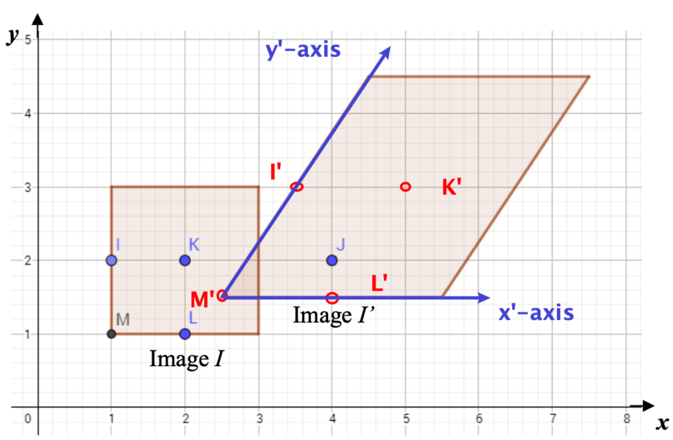
therefore, we get $g(x,y) = -15x - 22y + 9xy + 43$

So for $j: (1.778, 1.333)$,

$g(1.778, 1.333) = 8.334$.

Therefore, the intensity of point J is 8.334

In this problem, one method is map point J into original image I (the above one), and another method is map IKLM into image I' , and find the linear relations to get intensity value of point J:



Another Method:

If we project point I, k, L, M into Image $\hat{I}'$:

$$I' = SI = \begin{pmatrix} 1.5 & 1 \\ 0 & 1.5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 3 \end{pmatrix}$$

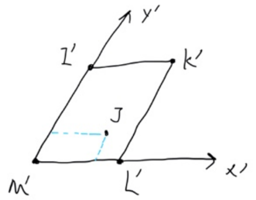
$$k' = Sk = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$L' = SL = \begin{pmatrix} 4 \\ 1.5 \end{pmatrix}$$

$$M' = SM = \begin{pmatrix} 2.5 \\ 1.5 \end{pmatrix}$$

But the linear property cannot apply to xy-coordinates now.

The linear property can still hold on the new x'y'-coordinates.



Set M' as origin, M'L' direction as x'-axis, M'I' direction as y'-axis: We can get the new coordinates of M', I', k', L' and J in x'y'-coordinates system.

M' (0, 0) L' (1.5, 0) J (1.167, 0.601)

I' (0, 1.803) k' (1.5, 1.803)

And for $g(x'y') = ax' + by' + cx'y' + d$:

$$\begin{pmatrix} 0 & 1.803 & 0 & 1 \\ 1.5 & 1.803 & 2.704 & 1 \\ 1.5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 9 \\ 15 \end{pmatrix}$$

Set matrix $B = \begin{pmatrix} 0 & 1.803 & 0 & 1 \\ 1.5 & 1.803 & 2.704 & 1 \\ 1.5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = B^{-1} \begin{pmatrix} 2 \\ 5 \\ 9 \\ 15 \end{pmatrix} = \begin{pmatrix} -4 \\ -7.211 \\ 3.328 \\ 15 \end{pmatrix}$$

therefore, $g(x'y') = -4x' - 7.211y' + 3.328x'y' + 15$

for point J $(1.167, 0.601)$:

$g(x'y') = 8.334$.

Therefore, the intensity value of point J is 8.334