~~7.1: 1, 5, 7, 15, 17, 25, 31, 37, 43, 45, 52~~

7.)

$$f(t) = \begin{cases} -1 & 0 \leq t < 1 \\ 1 & t > 1 \end{cases}$$

usando:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$\mathcal{L}\{f(t)\} = -\int_0^1 e^{-st}\, dt + \int_1^\infty e^{-st}\, dt$$

$$= \frac{e^{-st}}{s}\Big|_0^1 + \frac{e^{-st}}{s}\Big]_1^\infty$$

$$= \frac{e^{-s}}{s} - \frac{1}{s} + \frac{e^{-st}}{s} - \frac{e^{-s}}{s}$$

$$= \frac{2e^{-s}-1}{s}$$ //

5)

$$f(t) = \begin{cases} \text{sen}t & 0 \le t < \pi \\ 0 & t \geqslant \pi \end{cases}$$

$$L\{f(t)\} = \int_0^{\pi} e^{-st} \sin t \, dt + \int_{\pi}^{\infty} e^{-st} \cdot 0 \, dt$$

$$L\{f(t)\} = \int_0^{\pi} e^{-st} \sin t \, dt + \int_{\pi}^{\infty} 0 \, dt$$

$$L\{f(t)\} = \int_0^{\pi} e^{-st} \sin t \, dt$$

$$\int u \, dv = uv - \int v \, du$$

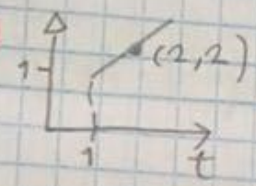
$u = \sin t$ $\qquad$ $v = e^{-st} dt$

$du = \cos t \, dt$ $\qquad$ $v = -\frac{1}{s} e^{-st}$

$$\int e^{-st} = -\frac{1}{s} e^{-st} \sin t + \int \frac{1}{s} e^{-st} \cos t \, dt$$

$$\frac{1 \pm e^{-s\pi}}{s^2+1}$$

7.)



$$f(t) = \begin{cases} 0 & 0 \le t < 1 \\ t & t \geq 1 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_1^{\infty} (t) e^{-st} dt$$

$$= \left(\frac{-t e^{-st}}{s} - \frac{e^{-st}}{s^2} \right) \Big|_1^{A}$$

$$= \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2}$$

$$2e^{-s}/s^2$$

15.) $f(t) = e^{-t} \operatorname{sen} t$

$$\int_0^{\infty} e^{-st} f(t)\, dt$$

$$\int_0^{\infty} e^{-t} \sin(t) e^{-st} dt$$

$$e^{ix} = \cos(x) + i \sin(x) \rightarrow e^{ix} - e^{-ix}$$

$$= 2i \sin(x) \rightarrow \sin(x) = \frac{1}{2i} (e^{ix} - e$$

$$\frac{1}{(s+1)^2+1}$$

5) $f(t) = t\cos t$

$$\int_0^{\infty} e^{-st} f(t)\,dt$$

$$\int_0^{\infty} t\cos(t)\, e^{-st}\,dt$$

$u = te^{-st} \rightarrow du = (1-st)e^{-st}dt$

$v = \sin(t) \rightarrow dv = \cos(t)dt$

$$t\sin(t)e^{-st}\Big|_0^{\infty} - \int_0^{\infty} \sin(t)\, e^{-st}dt +$$

$$\int_0^{\infty} st\sin(t)\, e^{-st}\,dt$$

$$= \frac{1}{s^2+1}$$

$$\int_0^{\infty} t\cos(t)\, e^{-st}dt = \frac{-1}{s^2+1} + \int_0^{\infty} st\sin(t)e^{-st}dt$$

$u = ste^{-st}$

$v = -\cos(t)$

$$\int_0^{\infty} t\cos(t)\, e^{-st}\,dt = \left(\frac{1}{s^2+1}\right)\left(\frac{s^2-1}{(s^2+1)^2} = \frac{s^2-1}{(s^2+1)}\right.$$

25.)

$$\mathcal{L}\{(t+1)^3\} = \mathcal{L}\{t^3 + 3t^2 + 3t + 1\}$$

$$\mathcal{L}\{t^3\} + 3\mathcal{L}\{t^2\} + 3\mathcal{L}\{t\} + \mathcal{L}\{1\}$$

$$= \frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s} \; //$$

31.) $\mathcal{L}\{4t^2 - 5\sin 3t\} = 4\mathcal{L}\{t^2\} - 5\mathcal{L}\{\sin 3t\}$

$$= \frac{8}{s^3} - \frac{15}{s^2+9}$$

37.) $\sin 2t \cos 2t$

$$\frac{1}{2}\left[\sin(2t+2t) + \sin(2t-2t)\right]$$

$$\frac{1}{2}\left[\sin(4t) + 0\right]$$

$$\frac{1}{2}\mathcal{L}\{\sin(4t)\}$$

$$\frac{1}{2} \cdot \frac{4}{s^2+16}$$

$$\frac{2}{s^2+16} \; //$$

43) $f(t) = t^{-1/2}$

$$\mathcal{L}(t^{-1/2}) = \frac{\Gamma(-(1/2)+1)}{s^{(-1/2+1)}}$$

$$= \frac{\Gamma(1/2)}{s^{(1/2)}}$$

$$= \sqrt{\frac{\pi}{s}}$$

45.) $$\mathcal{L}(t^{3/2}) = \frac{\Gamma(3/2)+1}{s^{(3/2+1)}}$$

$$= \frac{(3/2)\Gamma(3/2)}{s^{(5/2)}}$$

$$= \frac{3}{4}\sqrt{\frac{\pi}{s^{5/2}}}$$

52)

$$f(t) = \begin{cases} t & 0 \leq t < 2 \\ t & 2 < t < 5 \\ & t > 5 \end{cases}$$

no existe!

7.2 ~~5~~, ~~7~~, ~~24~~, ~~27~~, ~~35~~, 37

5.) $\mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\}$

$\mathcal{L}^{-1}\left\{\frac{s^3+3s^2+3s+1}{s^4}\right\}$

$=\mathcal{L}^{-1}\left\{\frac{s^3}{s^4}+\frac{3s^2}{s^4}+\frac{3s}{s^4}+\frac{1}{s^4}\right\}$

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}+3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}+3\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}+$

$= 1+3t+\frac{3}{2}(t^2)+\frac{1}{6}(t^3)$

7.) $\mathcal{L}^{-1}\left\{\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s-2}\right\}$

$L^{-1}\left\{\frac{1}{s^2}\right\}-L^{-1}\left\{\frac{1}{s}\right\}+L^{-1}\left\{\frac{1}{s-2}\right\}$

$= t - 1e^{2t}$

$= t-1+e^{2t}$

24.) $\mathcal{L}^{-1}\left\{\frac{s^2+1}{s(s-1)(s+1)(s-2)}\right\}$

$$\frac{s^2+1}{s(s-1)(s+1)(s-2)} = \frac{a}{s} + \frac{b}{s-1} + \frac{c}{s+1} + \frac{d}{s-2}$$

$$\frac{s^2+1}{s(s-1)(s+1)(s-2)} = -\frac{1}{s-1} + \frac{1}{2s} - \frac{1}{3(s+1)} + \frac{5}{6(s-2)}$$

$$L^{-1}\left\{\frac{1}{s-1}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{3}L^{-1}\left\{\frac{1}{s-(-1)}\right\} + \frac{5}{6}L^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= -e^{t} + \frac{1}{2} - \frac{1}{3}e^{-t} + \frac{5}{6}e^{2t}$$

27.) $\mathcal{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\}$

$$\frac{2s-4}{(s^2+s)(s^2+1)} = \frac{2s-4}{s(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

$$s(s+1)(s^2+1)\left[\frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}\right.$$

$A + B + C = 0$
$A + C + D = 0$
$A + B + D = 2$
$A = -4$

$-4 + 3e^{-t} + \cos(t) + 3\sin(t)$

35.) $y'' + 5y' + 4y = 0$ ... $y(0) = 1$ $y'(0) = 0$

$L\{y'' + 5y + 4y\} = L\{0\}$

$Y(t) = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}$

$$L(y'') + L(y) = \sqrt{2}\,\mathcal{L}\{\sin\sqrt{2}t\}$$

37) $\dfrac{2}{(s^2+2)(s^2+1)} + \dfrac{10s}{s^2+1}$

$$= \frac{As+B}{s^2+2} + \frac{Cs+D}{s^2+1}$$

$$(As+B)(s^2+1) + (s^2+2)(Cs+D) = 2$$

$A + C = 0$

$B + D = 0$

$A + 2C = 0$

$B + 2D = 2$

$A = 0$

$B = -2$

$C = 0$

$D = 2$

$$Y(s) = \frac{-2}{s^2+2} + \frac{2}{s^2+1} + \frac{10s}{s^2+1}$$

$$\mathcal{L}^{-1}Y(s) = -\sqrt{2}\,\mathcal{L}^{-1}\left[\frac{\sqrt{2}}{s^2+2}\right]$$

$$y(t) = -\sqrt{2}\sin\sqrt{2}t + 2\sin t + 10\cos t$$

7.3

~~5~~, ~~7~~, ~~10~~, ~~15~~, ~~17~~, ~~20~~, ~~23~~, ~~26~~, ~~29~~, ~~33~~, ~~36~~,
~~39~~, ~~41~~, ~~47~~, ~~49~~, ~~54~~, ~~57~~, ~~60~~, ~~61~~, ~~62~~, ~~69~~
70, 73, 75, 77, 78

5) $\mathcal{L}\{t(e^t+e^{2t})^2\} = L\{t(e^{2t}+2e^{3t}+e^{4t})\}$

$L\{te^{2t}\} + L\{2te^{3t}\} + L\{te^{4t}\}$

$\frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^2}$

7) $L\{e^t \sin 3t\}$

$= \sin L\{\sin 3t\}$

$= \frac{3}{(s-1)^2+9}$

10.) $\mathcal{L}\{e^{3t}(9-4t+10\sin(\frac{t}{2})$

$9L\{e^{3t}\} - 4L\{te^{3t}\} + 10L\{e^{3t}\sin(\frac{t}{2})\}$

$\frac{9}{s-3} - \frac{4}{(s-3)^2} + \frac{4}{(s-3)^2+\frac{1}{4}}$

15.) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+1}\right.$

$$\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1} - \frac{2}{(s+2)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+1}\right\}$$

$$= e^{-2t}\cos(t) - 2e^{-2t}\sin(t)$$

17.) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s}\right\}$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1-1}{(s+1)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\lim_{s \to s-(-1)} \frac{s}{s^2} - \lim_{s \to s-(-1)} \frac{1}{s^2}\right\}$$

$$= e^{-t} - te^{-t}$$

20.) $\mathcal{L}^{-1}\left\{\frac{(s+1)^2}{(s+2)^4}\right\} = \frac{a}{s+2} + \frac{b}{(s+2)^2} + \frac{c}{(s+2)^3}$

$$+ \frac{d}{(s+2)^4}$$

$$\frac{(s+1)^2}{(s+2)^4} = \frac{1}{(s+2)^2} - \frac{2}{(s+2)^3} + \frac{1}{(s+2)^4}$$

$te^{-2t} - t^2e^{-2t} + \frac{1}{3!}t^3e^{-2t}$

23.) $f(t) = e^{-t}(1+2t)$

$(s^2+2s+1)\,\mathcal{L}\{y\} = (s+3)$

$\mathcal{L}\{y\} = \frac{s+3}{(s+1)^2} = \frac{(s+1)+2}{(s+1)^2}$

$= \frac{1}{s+1} + \frac{2}{(s+1)^2}$

26.) $y(t) = \frac{3}{4} + \frac{9}{8}t + \frac{3}{4}t^2 + \frac{1}{4}t^3 - \frac{3}{4}e^{2t} + \frac{3}{8}te^{2t}$
$+ e^{2t} - 2te^{2t}$

$\mathcal{L}\{y'' - 4y' + 4y\} = \mathcal{L}\{t^3\}$

29.) $y'' - y' = e^t\cos(t)$

$\mathcal{L}\{y'' - y'\} = \mathcal{L}\{e^t\cos(t)\}$

$s^2F(s) - sf(0) - f'(0) - sF(s) - f(0) = \frac{s-1}{(s-1)^2+1}$

$F(s) = \frac{1}{s(s^2-2s+2)}$

$y = \frac{1}{2} - \frac{1}{2}e^t\cos(t) + \frac{1}{2}e^t\sin(t)$

fracciones parciales

$= \frac{A}{s} + \frac{Bs+C}{s^2-2s+2}$

33)

$$\mathcal{L}^{-1}\{x(s)\} = -\frac{3}{2}L^{-1}\left\{\frac{s+7/2}{(s+7/2)^2+15/4}\right\}$$

$$-\frac{21}{4}L^{-1}\left\{\frac{1}{(s+7/2)^2+15/4}\right\}$$

$$x(t) = -\frac{3}{2}L^{-1}\left\{\frac{s}{s^2+\frac{15}{4}}\right\}\cdot e^{-7t/2} - \frac{21}{4}L^{-1}\left\{\frac{1}{s^2+\frac{15}{4}}\right\}$$

$$\cdot e^{-7t/2}$$

$$x(t) = -\frac{3}{2}\cos\left(\frac{\sqrt{15}}{2}t\right)e^{-7t/2} - \frac{7\sqrt{15}}{10}\sin\left(\frac{\sqrt{15}}{2}t\right)e^{-7t/2}$$

36.) $R\frac{dq}{dt} + \frac{q}{C}$ $\quad E(t) = E_0 e^{-kt}$

$$RsQ(s) + \frac{Q(s)}{C} = \frac{E_0}{s+k}$$

$$v(t) = \frac{e^{-t/RC}}{k-1/RC} + \frac{e^{-kt}}{\frac{1}{RC}-k}$$

$$\frac{E_0}{R}e^{-t/RC} = q(t)$$

39.)

$$\mathcal{L}\{tU(t-2)\} = \mathcal{L}\{(t-2+2)U(t-2)\}$$

$$= \mathcal{L}\{(t-2)U(t-2)\} + 2\mathcal{L}\{U(t-2)\}$$

$$Y(s) = \frac{1}{s^2}e^{-2s} + \frac{2}{1}e^{-2s}$$

$$Y(s) = \frac{1}{s^2}e^{-2s} + \frac{2}{s}e^{-2s}$$

41.) $\mathcal{L}\{\cos(2t)U(t-\pi)\} = \mathcal{L}\{\cos 2(t-\pi)+\ldots$ $(t-\pi)$

$$\cos(x-2\pi) = \cos(x)$$

$$\mathcal{L}(\cos \omega t) = \frac{s}{s^2+\omega^2}$$

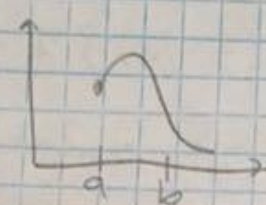
$$Y(s) = \frac{s}{s^2+4}e^{-s\pi}$$

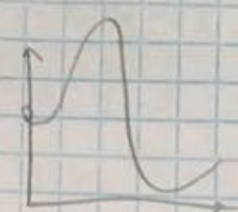
47.) $\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$

$$\frac{1}{s(s+1)} = \frac{a}{s} + \frac{b}{s+1} = \frac{1}{s} - \frac{1}{s+1}$$

$$y(t) = U(t-1) - e^{-(t-1)}U(t-1)$$

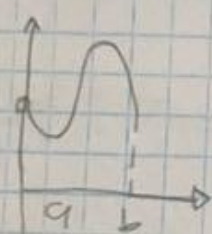
49.) c

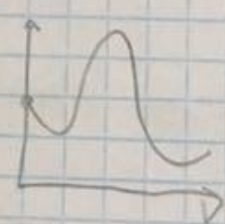




$f(t)\,u(t-a)$

54.)





$f(t) - f(t)\,u(t-b)$

57.) $e^{-s}\,\mathcal{L}\{t^2+2t+1\}$

$$e^{-s}\left(\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}\right)$$

$$f(t)=\begin{cases}0 & 0\le t<1\\ t^2 & t\geq 1\end{cases}$$

60) $f(t) = \begin{cases} \sin t & 0 \le t < 2\pi \\ 0 & t \geq 2\pi \end{cases}$

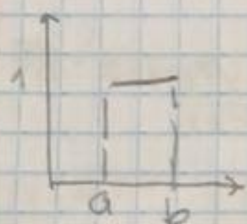
$\sin(t) - \sin(t)\,U(t-2\pi)$

$y(t) = \sin(t) - \sin(t-2\pi)\,U(t-2\pi)$

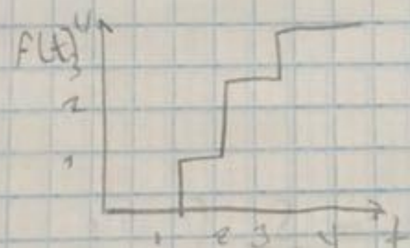
$Y(s) = L\{\sin(t)\} - L\{\sin(t-2\pi)U(t-2\pi)\}$

$Y(s) = \frac{1}{s^2+1} - \frac{1}{s^2+1}e^{-2\pi s}$

61.) $y(t) = U(t-a) - U(t-b)$



$Y(s) = \frac{e^{-as}}{s} - \frac{e^{-bs}}{s}$

62) 

$Y(s) = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \frac{e^{-4s}}{s} + \frac{e^{-5s}}{s}$

$y(t) = \frac{1}{s}\left[\frac{e^{s}}{1-e^{s}}\right]$

65.)

$L\{y' + 2y\} = L\{f(t)\} \quad \ldots \quad y(0) = 0$

$f(t) = t - t\,U(t-1)$

$y(t) = -\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}e^{-2t} - \frac{1}{4}U(t-1) - \frac{1}{2}(t-1) + \frac{1}{4}e^{-2(t-1)}U(t-1)$

70.) $y(t) = \frac{1}{3} + \frac{1}{6}e^{-3t} - \frac{1}{2}e^{-t} - [\frac{1}{3} + \frac{1}{6}e^{-3(t-2)}$

$- \frac{1}{2}e^{-(t-2)}]U(t-2) - [\frac{1}{3} + \frac{1}{6}e^{-3(t-4)}$

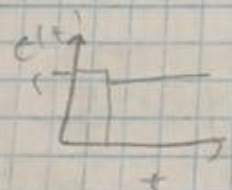
$- \frac{1}{2}e^{-(t-4)}]U(t-4) +$

$[\frac{1}{3} + \frac{1}{6}e^{-3(t-6)} - \frac{1}{2}e^{-(t-6)}]U(t-6)$

73.) $q(0)=0$

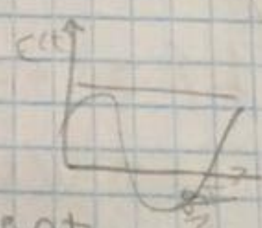
$R=25$

$C=0.08f$



$Q(s) = \frac{5}{s(Rs+\frac{1}{C})} = \frac{5}{R}\left(\frac{1}{s(s+1/RC)}\right)$

75.) $R + L\frac{di}{dt} = E(t)$

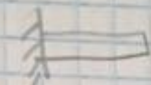


$i(t) = -\cos t + \frac{e^{-t}}{2} + \frac{\sin t}{2}$

$I(s) = e^{-\frac{3\pi}{2}} - 1/2 / s+1/0$

77.)

$$w(x) = \begin{cases} w_0 & 0 < x < L/2 \\ 0 & L/2 \le x < L \end{cases}$$



$$EI \frac{d^4 y}{dx^4} = w(x) \qquad y'(0) = 0$$

$$y''(0)$$

$$y'''(0) = 0$$

no recomto

78.) $EI \dfrac{d^4 y}{dx^4} = 0 \qquad \dfrac{2L}{3} < x < L$

$$y(x) = y\left(\tfrac{2}{3}L\right) + y'\left(\tfrac{2L}{3}\right) + y''\frac{\left(\frac{2L}{3}\right)^2}{2} + y'''\frac{\left(\frac{2L}{3}\right)}{6}$$

$$Y(s) = \frac{y\left(\frac{2L}{3}\right)}{s} + \frac{y'\left(\frac{2L}{3}\right)}{s^2} + \frac{y''\left(\frac{2L}{3}\right)}{s^3} + \frac{y'''\left(\frac{2L}{3}\right)}{s^4}$$

60) $f(t) = \begin{cases} \sin t & 0 \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$

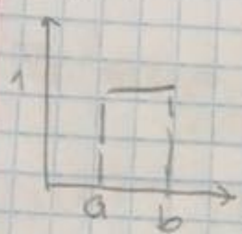
$\sin(t) - \sin(t)\,U(t-2\pi)$

$y(t) = \sin(t) - \sin(t-2\pi)\,U(t-2\pi)$

$Y(s) = L\{\sin(t)\} - L\{\sin(t-2\pi)U(t-2\pi)\}$

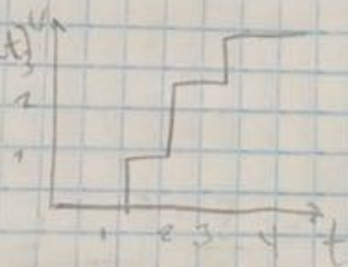
$Y(s) = \frac{1}{s^2+1} - \frac{1}{s^2+1}e^{-2\pi s}$

61) $y(t) = U(t-a) - U(t-b)$



$Y(s) = \frac{e^{-as}}{s} - \frac{e^{-bs}}{s}$

62)



$Y(s) = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \frac{e^{-4s}}{s} + \frac{e^{-5s}}{s}$

$y(t) = \frac{1}{s}\left[\frac{e^{s}}{1-e^{s}}\right]$

65)

$L\{y' + 2y\} = L\{f(t)\} \quad \dots \quad y(0)=0$

$f(t) = t - t\,U(t-1)$

$y(t) = -\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}e^{-2t} - \frac{1}{4}U(t-1) - \frac{1}{2}(t-$

$+ \frac{1}{4}e^{-2(t-1)}U(t-1)$