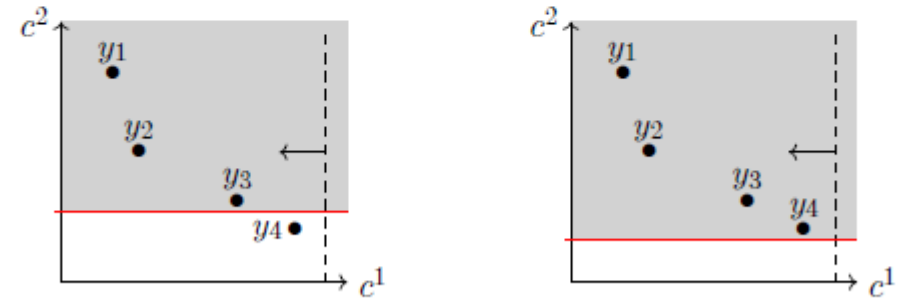
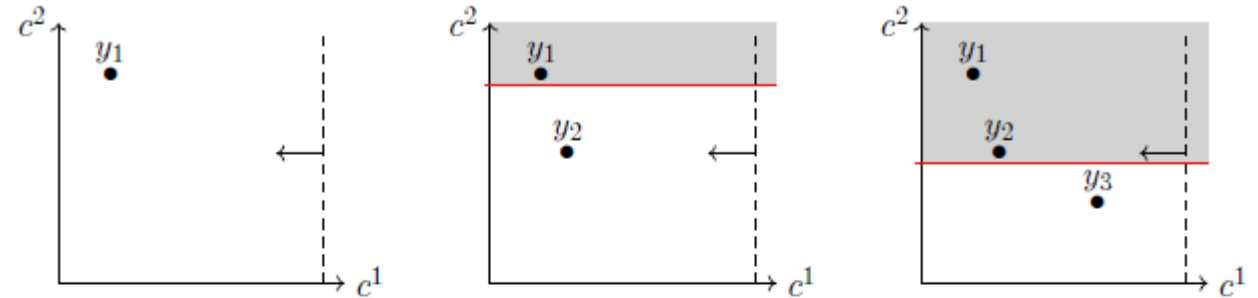


MOO method

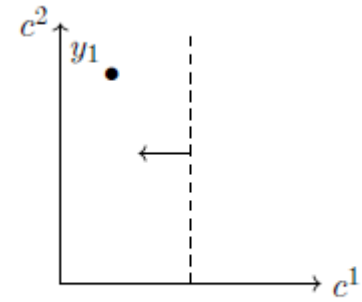
# Simple limit (ex of FINE)

- Only one objective function
- Second one defined as a variable
- Constraint on the variable
- Declining at every step

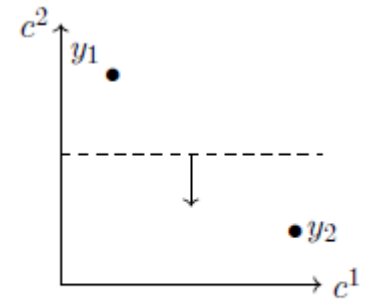


# Sigma-constraint (ex of code from scratch)

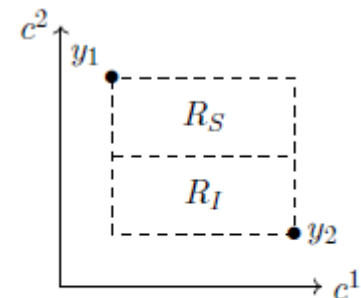
- Definition of functions A and B
- Deactivation of function A, activation of function B
- **Optimization**: max of function A registered
- Activation of function A, deactivation of function B
- **Optimization**: min of function A registered
- Deactivation of function A, activation of function B
- **Optimization** with constraint of function A between min and max
- Plot of the Pareto front



(a) Initialisation : recherche du point  $c^1$ -extrême  $y_1$ .



(b) Initialisation : recherche du point  $c^2$ -extrême  $y_2$ .



(c) Itération 1 : division de la région de recherche définie par  $y_1$  et  $y_2$ .

# Sigma-constraint: different options

- while loop
  - while the minimum value that we know has not been reached
- Official sigma-constraint method
  - Calculate a fixed number of value that will be used as constraints
  - $step = \frac{Max - Min}{\alpha}$  then go through  $list(range(Min, Max, step)) + Max$
  - Conventional method: simple  $\leq$  sign
  - Augmented method:
    - $Objectivef = costs + \delta * s$  to minimize
    - $emissions - s = e$  new constraint
- (difference is relevant for GLPK, not for gurobi)

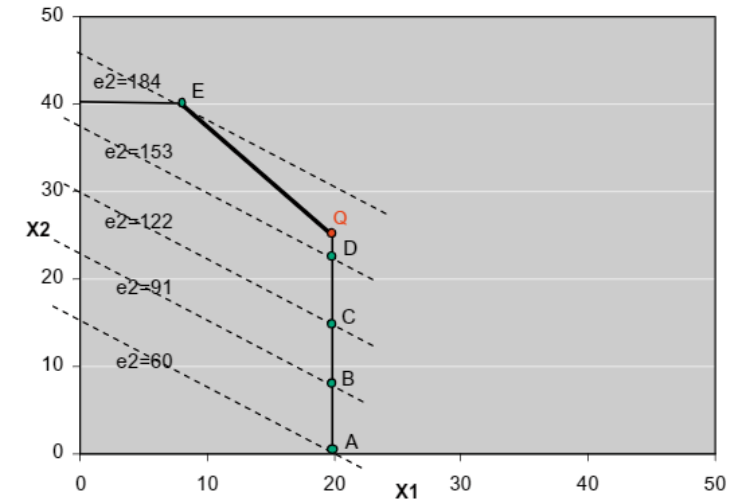


Fig. 2. Results of the conventional  $\epsilon$ -constraint method.

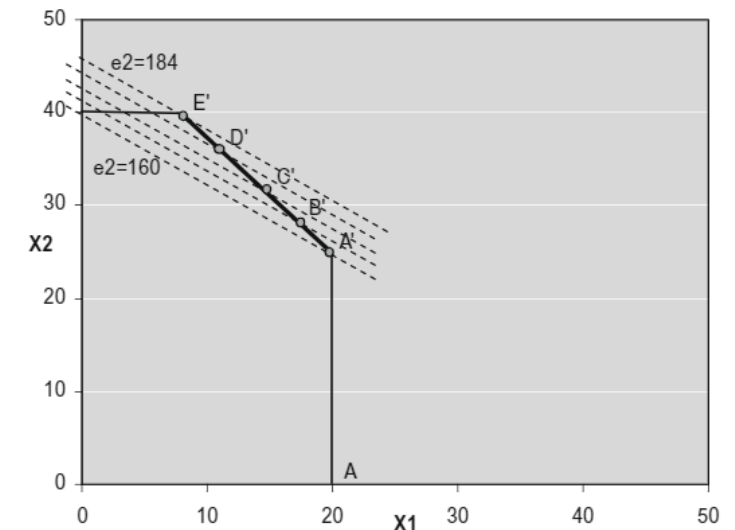
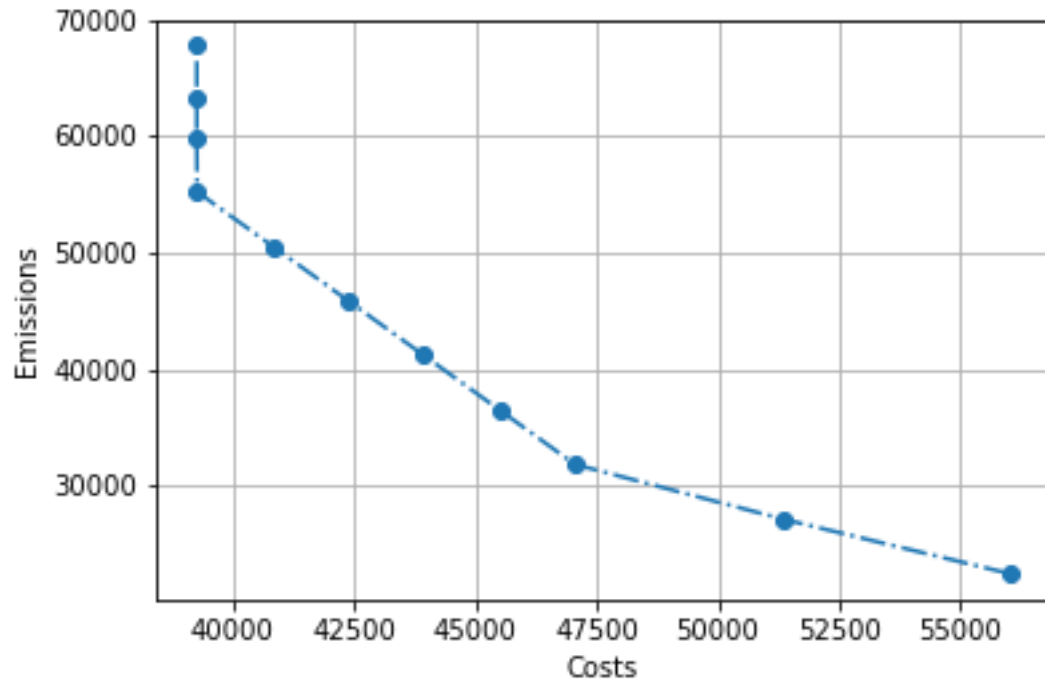


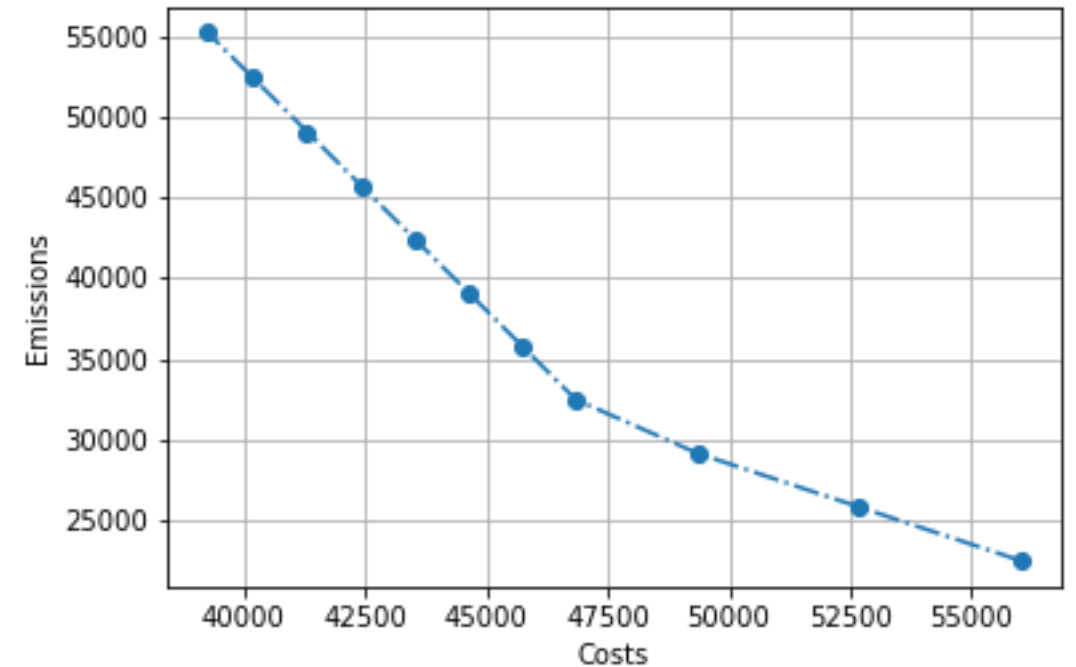
Fig. 3. Results of the  $\epsilon$ -constraint method from model using lex-optimization in the payoff table.

# Augmented method of sigma-constraint

Conventional method with GLPK :



Augmented method with GLPK,  
Both methods with Gurobi :



# References

- [minimize - Multi-objective optimization example Pyomo - Stack Overflow](#)
- [Effective implementation of the  \$\varepsilon\$ -constraint method in Multi-Objective Mathematical Programming problems | Elsevier Enhanced Reader](#)
- [Méthodes exactes pour les problèmes combinatoires bi-objectif : Application sur les problèmes de tournées de véhicules - TEL - Thèses en ligne \(archives-ouvertes.fr\)](#)