Multi-dimensional Regression Model: Pandemic and Bike Rents in London

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- Data Dictionary
- Data Exploration
- Model Building

```
install.packages("tidyverse")
install.packages("emmeans")
install.packages("Hmisc")
install.packages("car")
install.packages("RColorBrewer")
install.packages("gridExtra")
install.packages("ggpubr")


library(tidyverse)
library(emmeans) # for emmeans() and pairs()
library(Hmisc) # for correlation functions
library(car) # for vif()
library(RColorBrewer)
library(gridExtra)
library(ggpubr)
```

```
# Read in the datafile "London_COVID_bikes.csv"
data <- read_csv("London_COVID_bikes.csv")
```

```
## Rows: 4812 Columns: 15
## ── Column specification ────────────────────────────────────────────────────────
## Delimiter: ","
## chr   (2): day, month
## dbl  (12): Hires, schools_closed, pubs_closed, shops_closed, eating_places_c...
## date  (1): date
##
## ℹ Use `spec()` to retrieve the full column specification for this data.
## ℹ Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

Data Dictionary

Variable (Restriction Policy)	Description
School closures	Complete closures only
Pub closures	Excluding pubs that serve food
Shop closures	Non-essential shops only
Eating Places closures	Including pubs that serve food
Stay at home orders	When people are ordered to stay at homes, started on 2020-03-23
Household mixing indoors banned	Household mixing rules have been imposed to prevent people who do not live together from meeting.
Working from home encouraged (wfh)	When working from home is encouraged, first advised on 2020-03-17

Variable (Restriction Policy)	Description
Rule of 6 indoors	When people were prohibited from meeting more than six people socially, first announced on 2020-09-14 and implemented on 2020-09-22
10pm curfew on hospitality (curfew)	All hospitality venues must shut at 10pm
Eat Out to Help Out scheme	From 3 to 31 August, 2020, get a 50% discount when you eat in at restaurants that are registered with the Eat Out to Help Out Scheme

Data Exploration

We first start the exploration of data by checking its structure, summary, and whether there is any missing values contained.

```
# Check the structure and summary of the data
str(data)
```

```
## spc_tbl_ [4,812 × 15] (S3: spec_tbl_df/tbl_df/tbl/data.frame)
##  $ date                           : Date[1:4812], format: "2010-07-30" "2010-07-31" ...
##  $ Hires                          : num [1:4812] 6897 5564 4303 6642 7966 ...
##  $ schools_closed                 : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ pubs_closed                    : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ shops_closed                   : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ eating_places_closed           : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ stay_at_home                   : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ household_mixing_indoors_banned: num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ wfh                            : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ rule_of_6_indoors              : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ curfew                         : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ eat_out_to_help_out            : num [1:4812] 0 0 0 0 0 0 0 0 0 0 ...
##  $ day                            : chr [1:4812] "Fri" "Sat" "Sun" "Mon" ...
##  $ month                          : chr [1:4812] "Jul" "Jul" "Aug" "Aug" ...
##  $ year                           : num [1:4812] 2010 2010 2010 2010 2010 2010 2010 2010 2010 2010 ...
##  - attr(*, "spec")=
##   .. cols(
##   ..   date = col_date(format = ""),
##   ..   Hires = col_double(),
##   ..   schools_closed = col_double(),
##   ..   pubs_closed = col_double(),
##   ..   shops_closed = col_double(),
##   ..   eating_places_closed = col_double(),
##   ..   stay_at_home = col_double(),
##   ..   household_mixing_indoors_banned = col_double(),
##   ..   wfh = col_double(),
##   ..   rule_of_6_indoors = col_double(),
##   ..   curfew = col_double(),
##   ..   eat_out_to_help_out = col_double(),
##   ..   day = col_character(),
##   ..   month = col_character(),
##   ..   year = col_double()
##   .. )
##  - attr(*, "problems")=<externalptr>
```

```
summary(data)
```

```
##       date                Hires       schools_closed     pubs_closed
##  Min.   :2010-07-30   Min.   :    0   Min.   :0.00000   Min.   :0.00000
##  1st Qu.:2013-11-13   1st Qu.:19776   1st Qu.:0.00000   1st Qu.:0.00000
##  Median :2017-02-28   Median :26356   Median :0.00000   Median :0.00000
##  Mean   :2017-02-28   Mean   :26607   Mean   :0.02743   Mean   :0.05175
##  3rd Qu.:2020-06-15   3rd Qu.:33481   3rd Qu.:0.00000   3rd Qu.:0.00000
##  Max.   :2023-09-30   Max.   :73094   Max.   :1.00000   Max.   :1.00000
##   shops_closed     eating_places_closed  stay_at_home
##  Min.   :0.00000   Min.   :0.00000      Min.   :0.00000
##  1st Qu.:0.00000   1st Qu.:0.00000      1st Qu.:0.00000
##  Median :0.00000   Median :0.00000      Median :0.00000
##  Mean   :0.04634   Mean   :0.05175      Mean   :0.03616
##  3rd Qu.:0.00000   3rd Qu.:0.00000      3rd Qu.:0.00000
##  Max.   :1.00000   Max.   :1.00000      Max.   :1.00000
##  household_mixing_indoors_banned      wfh         rule_of_6_indoors
##  Min.   :0.00000                 Min.   :0.0000   Min.   :0.00000
##  1st Qu.:0.00000                 1st Qu.:0.0000   1st Qu.:0.00000
##  Median :0.00000                 Median :0.0000   Median :0.00000
##  Mean   :0.06525                 Mean   :0.2273   Mean   :0.01995
##  3rd Qu.:0.00000                 3rd Qu.:0.0000   3rd Qu.:0.00000
##  Max.   :1.00000                 Max.   :1.0000   Max.   :1.00000
##      curfew        eat_out_to_help_out     day               month
##  Min.   :0.00000   Min.   :0.000000    Length:4812        Length:4812
##  1st Qu.:0.00000   1st Qu.:0.000000    Class :character   Class :character
##  Median :0.00000   Median :0.000000    Mode  :character   Mode  :character
##  Mean   :0.01164   Mean   :0.005819
##  3rd Qu.:0.00000   3rd Qu.:0.000000
##  Max.   :1.00000   Max.   :1.000000
##       year
##  Min.   :2010
##  1st Qu.:2013
##  Median :2017
##  Mean   :2017
##  3rd Qu.:2020
##  Max.   :2023
```

```
summarise_all(data, ~ sum(is.na(.))) # no missing value contained
```

```
## # A tibble: 1 × 15
##    date Hires schools_closed pubs_closed shops_closed eating_places_closed
##   <int> <int>          <int>       <int>        <int>                <int>
## 1     0     0              0           0            0                    0
## # ℹ 9 more variables: stay_at_home <int>,
## #   household_mixing_indoors_banned <int>, wfh <int>, rule_of_6_indoors <int>,
## #   curfew <int>, eat_out_to_help_out <int>, day <int>, month <int>, year <int>
```

The data structure indicates that most of the variables are numerical data except variables “day” and “month”, which are characters. The earliest record in this data is on 2010-07-30, and the last record is on 2023-09-30, as we can learn from the summary. There is no missing value contained in the data.

Then, we convert time variables (day, month, year) into factors with appropriate levels.

```
# Check for unique month inputs and see if there's upper and lower case inconsistency issue (e.g. Jul and jul w
ould be identified as two distinct inputs).
unique(data$month)
```

```
##  [1] "Jul" "Aug" "Sep" "Oct" "Nov" "Dec" "Jan" "Feb" "Mar" "Apr" "May" "Jun"
```

```
# Check for unique day inputs and see if there's upper and lower case inconsistency issue.
unique(data$day)
```

```
## [1] "Fri" "Sat" "Sun" "Mon" "Tue" "Wed" "Thu"
```

```
# Convert "month" variable into factor with appropriate levels
data <- data %>% mutate(month=factor(month, levels=c("Jan", "Feb", "Mar", "Apr", "May", "Jun", "Jul", "Aug", "S
ep", "Oct", "Nov", "Dec")))

# Convert "day" variable into factor with appropriate levels
data <- data %>% mutate(day=factor(day, levels=c("Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun")))

# Convert "year" variable into factors; since years are in numeric form so we do not have to re-specify the lev
els.
data$year <- as.factor(data$year)

# Check the data summary again and see if the data type of these three time variables are changed successfully.
summary(data)
```

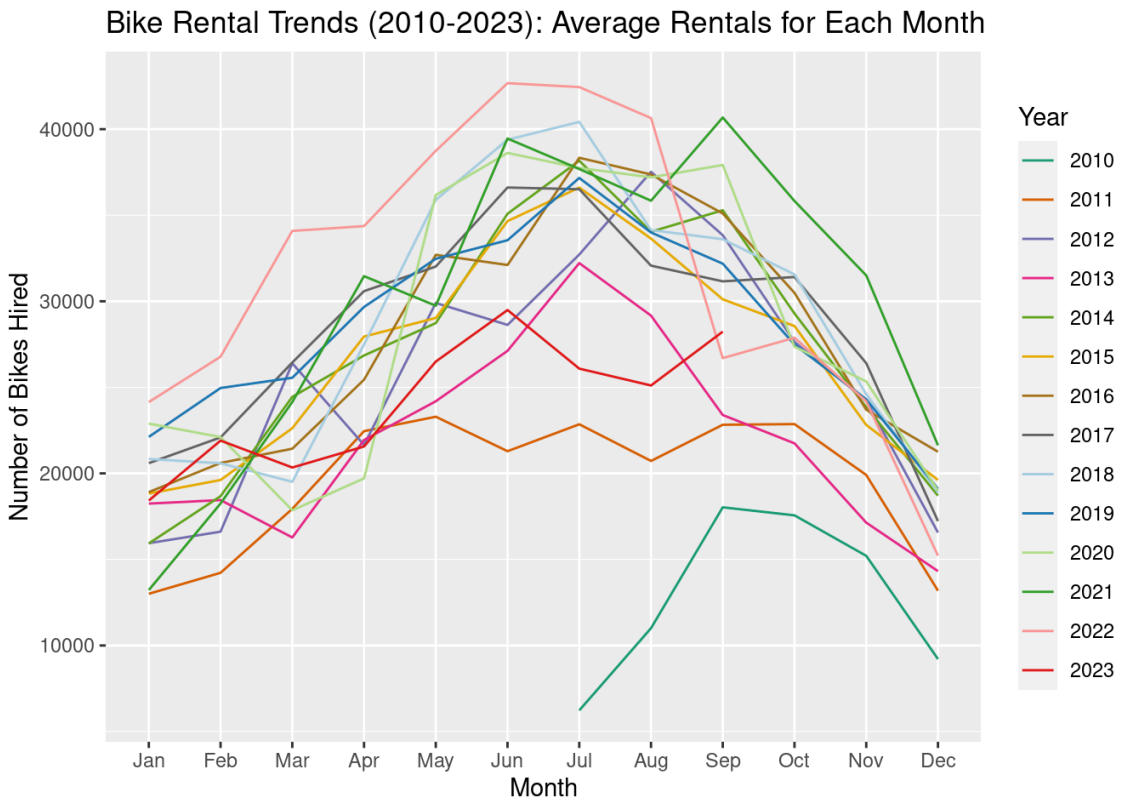
```
##       date                Hires       schools_closed     pubs_closed
##  Min.   :2010-07-30   Min.   :    0   Min.   :0.00000   Min.   :0.00000
##  1st Qu.:2013-11-13   1st Qu.:19776   1st Qu.:0.00000   1st Qu.:0.00000
##  Median :2017-02-28   Median :26356   Median :0.00000   Median :0.00000
##  Mean   :2017-02-28   Mean   :26607   Mean   :0.02743   Mean   :0.05175
##  3rd Qu.:2020-06-15   3rd Qu.:33481   3rd Qu.:0.00000   3rd Qu.:0.00000
##  Max.   :2023-09-30   Max.   :73094   Max.   :1.00000   Max.   :1.00000
##
##   shops_closed     eating_places_closed  stay_at_home
##  Min.   :0.00000   Min.   :0.00000      Min.   :0.00000
##  1st Qu.:0.00000   1st Qu.:0.00000      1st Qu.:0.00000
##  Median :0.00000   Median :0.00000      Median :0.00000
##  Mean   :0.04634   Mean   :0.05175      Mean   :0.03616
##  3rd Qu.:0.00000   3rd Qu.:0.00000      3rd Qu.:0.00000
##  Max.   :1.00000   Max.   :1.00000      Max.   :1.00000
##
##  household_mixing_indoors_banned      wfh         rule_of_6_indoors
##  Min.   :0.00000                 Min.   :0.0000   Min.   :0.00000
##  1st Qu.:0.00000                 1st Qu.:0.0000   1st Qu.:0.00000
##  Median :0.00000                 Median :0.0000   Median :0.00000
##  Mean   :0.06525                 Mean   :0.2273   Mean   :0.01995
##  3rd Qu.:0.00000                 3rd Qu.:0.0000   3rd Qu.:0.00000
##  Max.   :1.00000                 Max.   :1.0000   Max.   :1.00000
##
##      curfew        eat_out_to_help_out  day          month           year
##  Min.   :0.00000   Min.   :0.000000    Mon:688   Aug    : 434   2012   : 366
##  1st Qu.:0.00000   1st Qu.:0.000000    Tue:687   Sep    : 420   2016   : 366
##  Median :0.00000   Median :0.000000    Wed:687   Jul    : 405   2020   : 366
##  Mean   :0.01164   Mean   :0.005819    Thu:687   Dec    : 404   2021   : 366
##  3rd Qu.:0.00000   3rd Qu.:0.000000    Fri:688   Jan    : 403   2011   : 365
##  Max.   :1.00000   Max.   :1.000000    Sat:688   Mar    : 403   2013   : 365
##                                        Sun:687   (Other):2343   (Other):2618
```

After checking the data types, we move on to see the relationship between month and bike rents across 2010-2023.

```
# Set colors to be used in the following visualizations
mycolors = c(brewer.pal(name="Dark2", n = 8), brewer.pal(name="Paired", n = 6))

# Calculate the monthly average bike hires data to be used in the visualization
monthly_avg <- data %>%
  group_by(year, month) %>%
  summarise(monthly_avg_hires = mean(Hires))
```

```
## `summarise()` has grouped output by 'year'. You can override using the
## `.groups` argument.
```

```
# Plot the relationship between month and monthly average bike hires, with line color representing different ye
ars
ggplot(monthly_avg, aes(x=month, y=monthly_avg_hires, col=year, group=year)) +
  geom_line() +
  labs(title="Bike Rental Trends (2010-2023): Average Rentals for Each Month", x="Month", y="Number of Bikes Hi
red", col="Year")+
  scale_color_manual(values = mycolors)
```



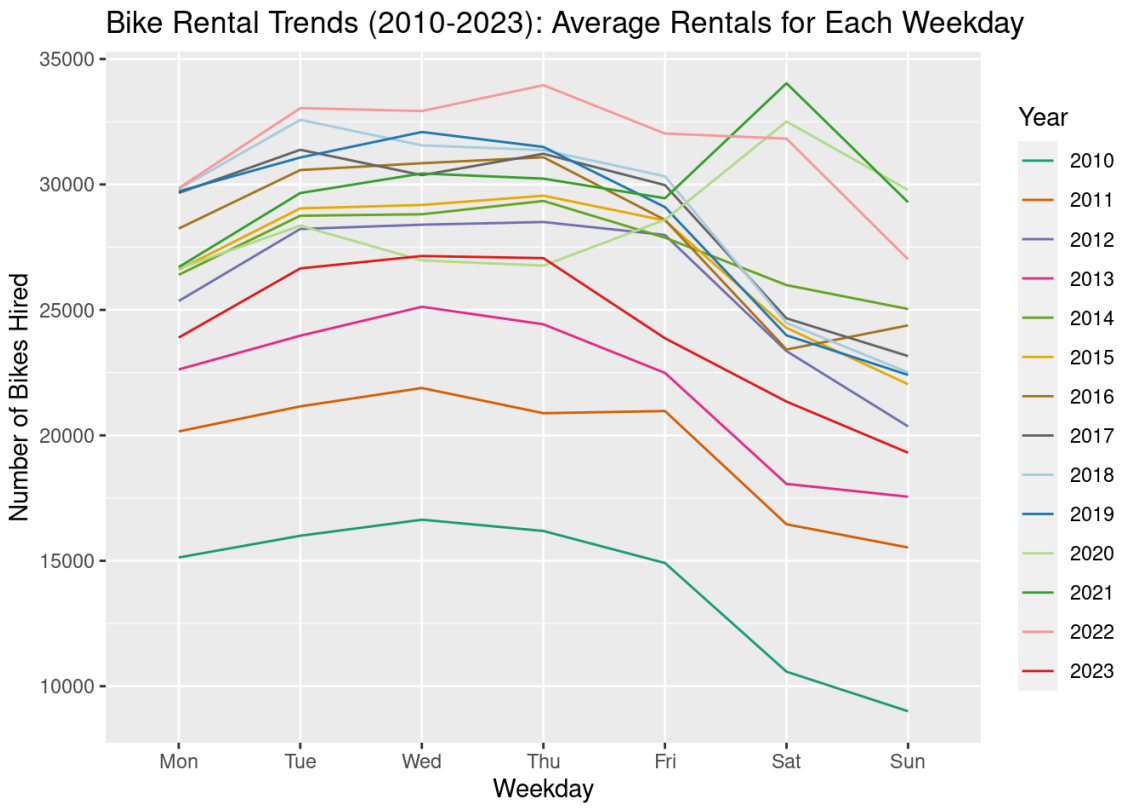
Overall looking, the number of bikes hired seems to increase year by year; in each year, bike rents reach a peak usually in the middle of the year, and such figure is low at the beginning and ending of the year.

Then we can also review the relationship between weekday and bike rents across 2010-2023.

```
# Calculate the average bike hires data for each weekday to be used in the visualization
weekday_avg <- data %>%
  group_by(year, day) %>%
  summarise(day_avg_hires = mean(Hires))
```

```
## `summarise()` has grouped output by 'year'. You can override using the
## `.groups` argument.
```

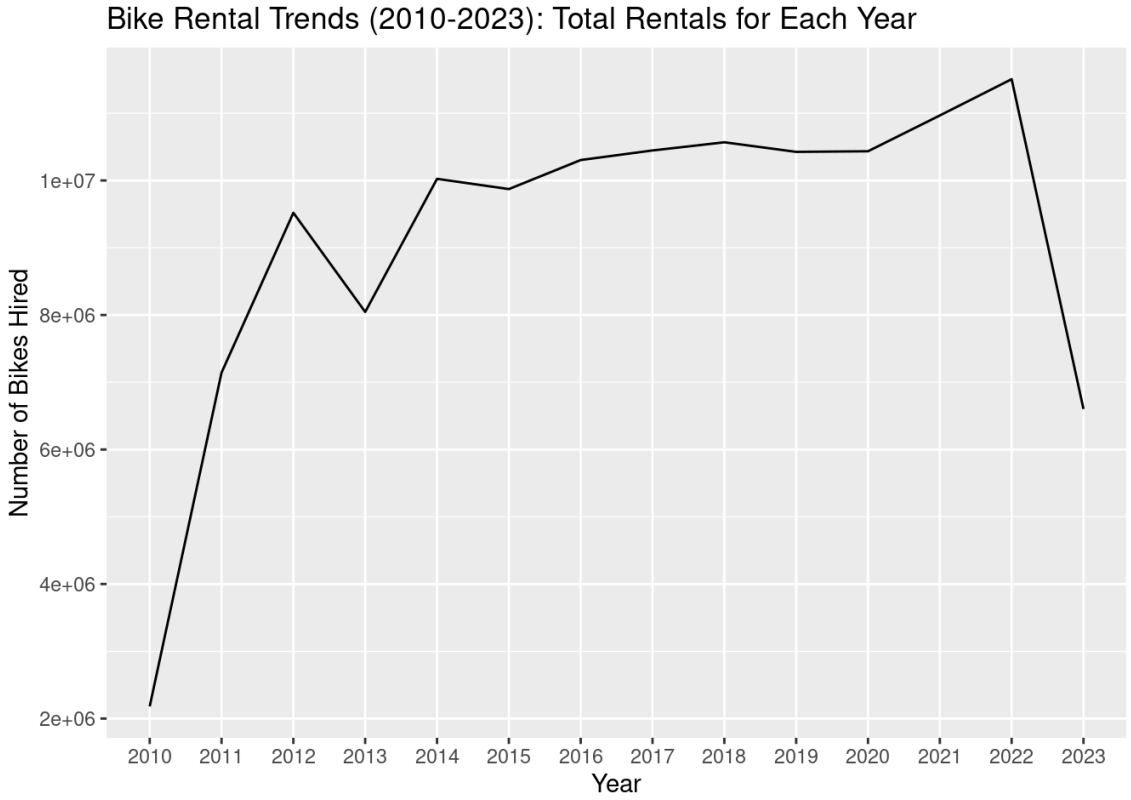
```
# Plot the relationship between weekday and daily average bike hires, with line color representing different ye
ars
ggplot(weekday_avg, aes(x=day, y=day_avg_hires, col=year, group=year)) +
  geom_line() +
  labs(title="Bike Rental Trends (2010-2023): Average Rentals for Each Weekday",x="Weekday", y="Number of Bikes
Hired", col="Year") +
  scale_color_manual(values = mycolors)
```



Again, we can see that the number of bikes hired seems to increase year by year; in each week, bike rents are lower on weekends (especially on Sunday) than on workdays.

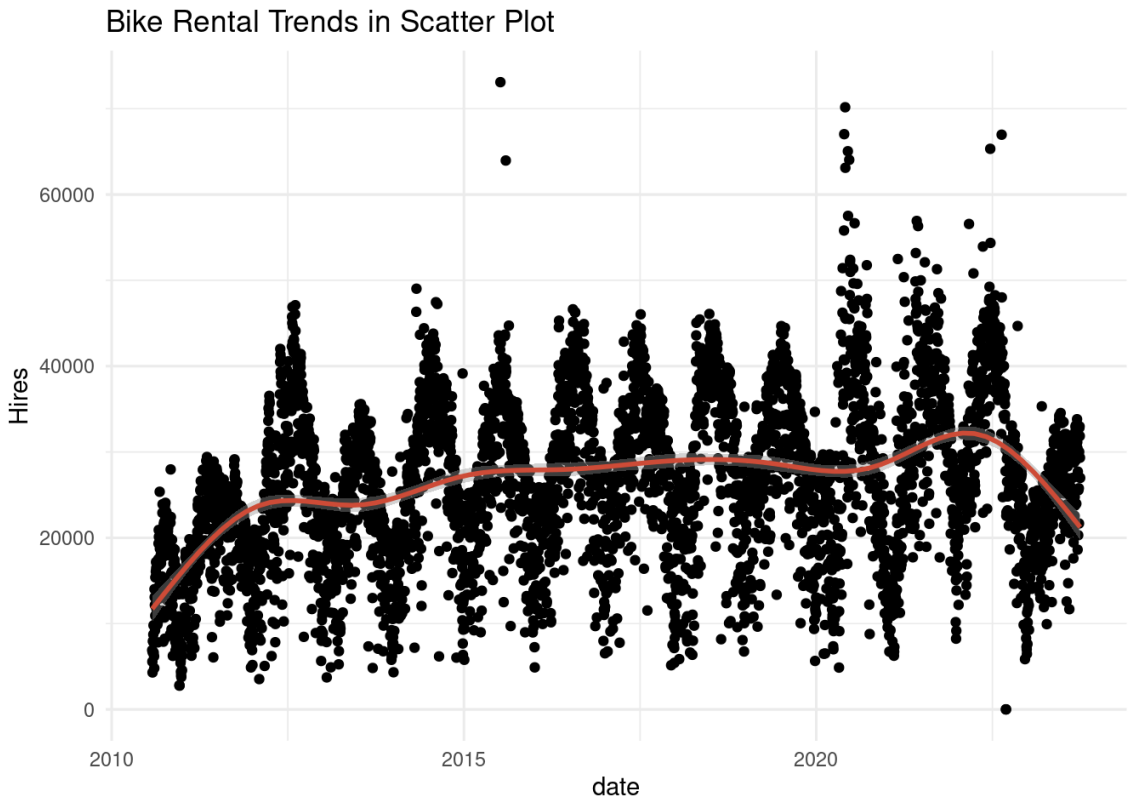
Moreover, we can use the following graph to review the overall bike rental trend across years:

```
# Calculate the yearly total bike rents
year_sum <- data %>%
  group_by(year) %>%
  summarise(total_hires = sum(Hires))

# Plot the summation out
ggplot(year_sum, aes(x=year, y=total_hires)) + geom_line(aes(group=1)) +
  labs(title="Bike Rental Trends (2010-2023): Total Rentals for Each Year",x="Year", y="Number of Bikes Hired")
```



```
# Plot the scatter graph as well to better understand the trend
ggplot(data, aes(y=Hires, x=date)) +
geom_jitter(width=0.1, height=0.1) + geom_smooth(color="tomato3")+
theme_minimal() + labels(title="Bike Rental Trends in Scatter Plot")
```

```
## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```



Bike rents increased at most of the years and reached the peak at 2022.

Since we are most interested in three variables- Rule of 6 indoors, Working from home, and Eat out to help out scheme- we can highlight them out on the scatter plot to better see their distribution.

```
# create new columns with some variables being turned into factor to better visualize them
wfh_factor <- as.factor(data$wfh)
ro6_factor <- as.factor(data$rule_of_6_indoors)
eatout_factor <- as.factor(data$eat_out_to_help_out)
```

We first count the number of 0 and 1 instances in these variables to get a rough idea about the implementation frequency of these policies.

```
# count the number of 0 and 1 instances
count(data, wfh)
```

```
## # A tibble: 2 × 2
##     wfh     n
##   <dbl> <int>
## 1     0  3718
## 2     1  1094
```

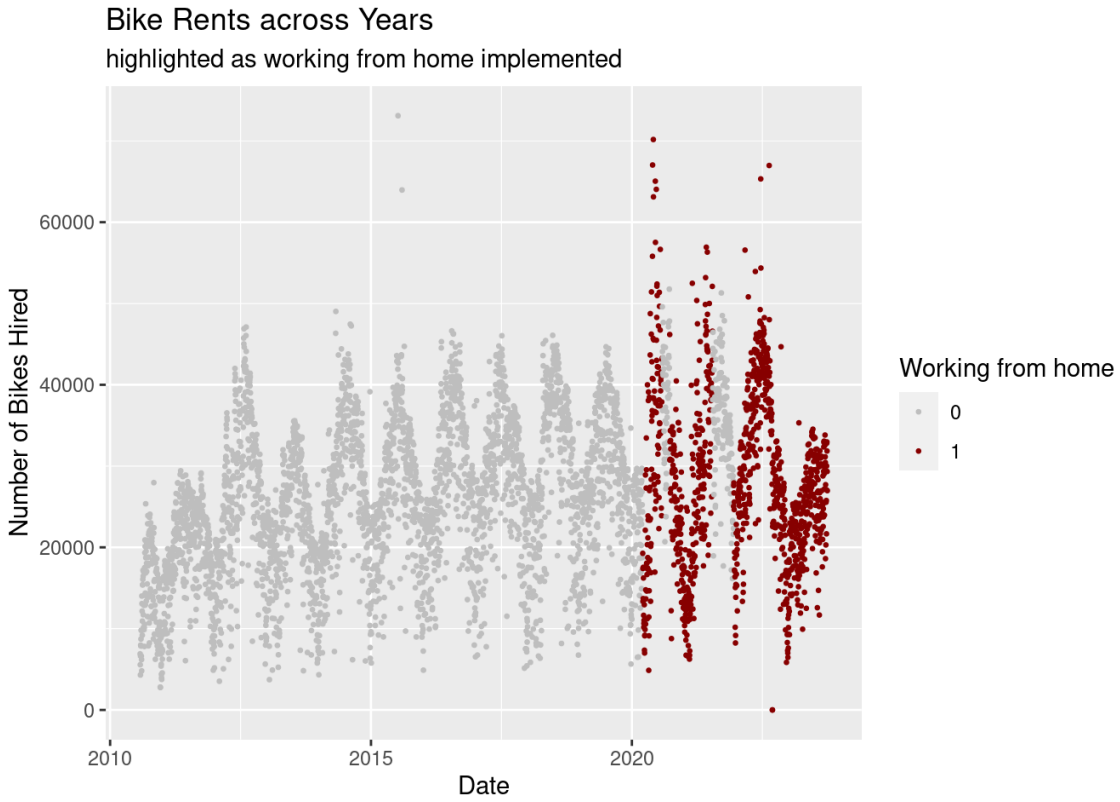
```
count(data, rule_of_6_indoors)
```

```
## # A tibble: 2 × 2
##   rule_of_6_indoors     n
##               <dbl> <int>
## 1                 0  4716
## 2                 1    96
```

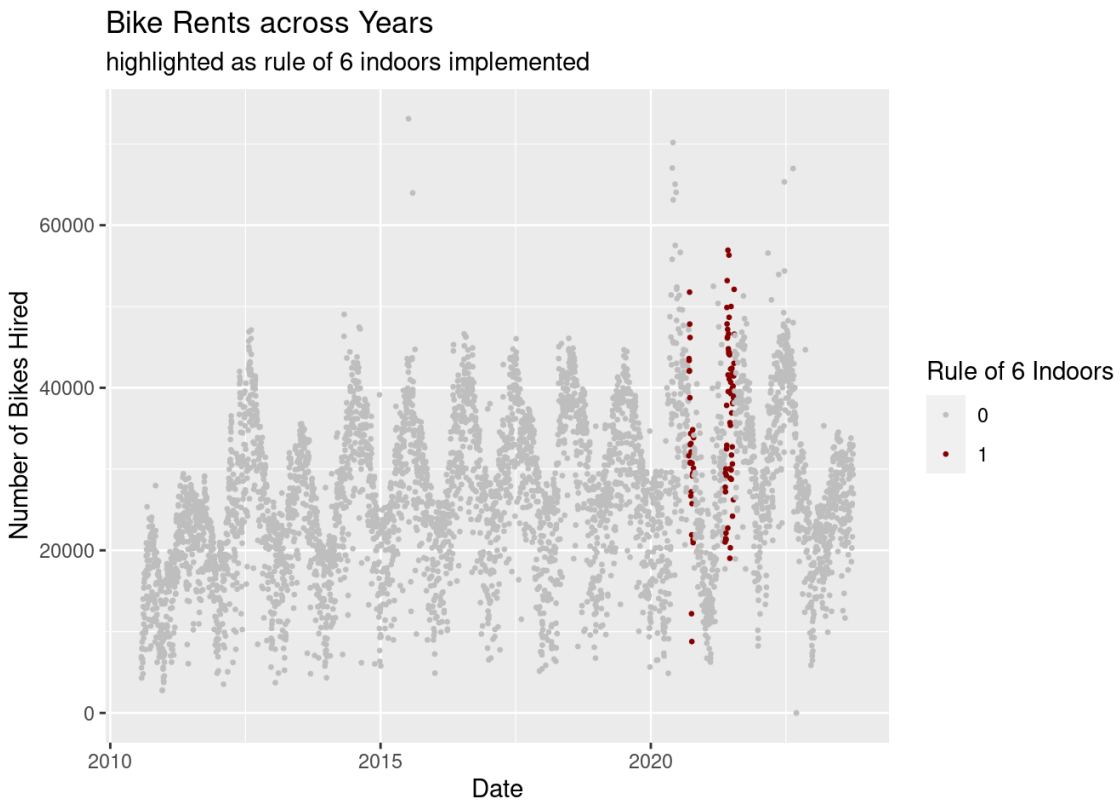
```
count(data, eat_out_to_help_out)
```

```
## # A tibble: 2 × 2
##   eat_out_to_help_out     n
##                 <dbl> <int>
## 1                   0  4784
## 2                   1    28
```

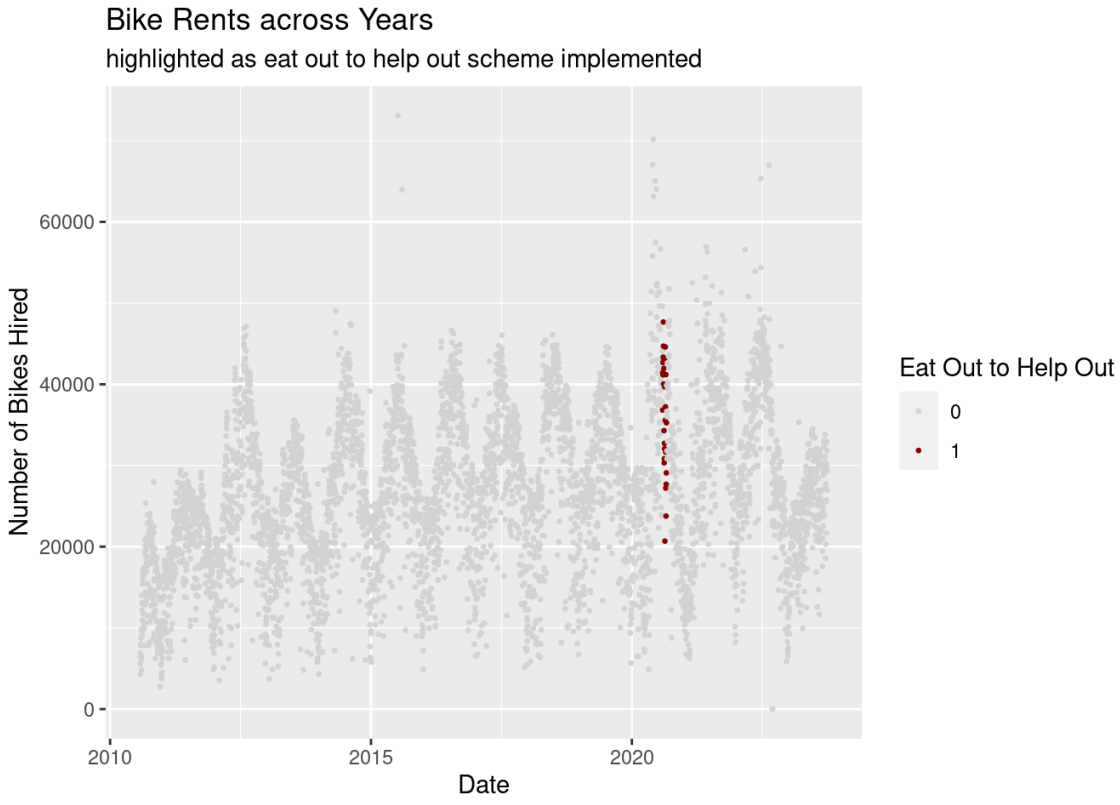
```
ggplot(data, aes(y=Hires, x=date, color=wfh_factor)) + geom_jitter(width=0.1, height=0.1, size=0.5) + scale_col
or_manual(values = c("gray", "darkred")) + labs(x="Date", y="Number of Bikes Hired", col="Working from home") +
labs(title="Bike Rents across Years", subtitle = "highlighted as working from home implemented")
```




```
ggplot(data, aes(y=Hires, x=date, color=ro6_factor)) + geom_jitter(width=0.1, height=0.1, size=0.5) + scale_col
or_manual(values = c("gray", "darkred")) + labs(x="Date", y="Number of Bikes Hired", col="Rule of 6 Indoors")+
labs(title="Bike Rents across Years", subtitle = "highlighted as rule of 6 indoors implemented")
```




```
ggplot(data, aes(y=Hires, x=date, color=eatout_factor)) + geom_jitter(width=0.1, height=0.1, size=0.5) + scale_
color_manual(values = c("lightgray", "darkred")) + labs(x="Date", y="Number of Bikes Hired", col="Eat Out to He
lp Out")+ labs(title="Bike Rents across Years", subtitle = "highlighted as eat out to help out scheme implement
ed")
```



As we can see, working from home policy had been implemented all the way from early 2020 to the end of 3rd quarter in 2023, with 1094 records (days) of implementation. Rule of 6 indoors was started being implemented in the late 2020 and stopped in the mid-2021, with only 96 days of implementation. Eat out to help out scheme started from 3 to 31 August, 2020, with 28 days of implementation.

Model Building

Regression model with time variables excluded

Before we start building the model, we first examine the pairwise correlations between each variable we have; if two variables are strongly correlated (either negatively or positively), our regression could be suffering from multicollinearity.

```
rcorr(as.matrix(select_if(data, is.numeric)))
```

```
##                                Hires schools_closed pubs_closed shops_closed
## Hires                           1.00          -0.09       -0.06        -0.08
## schools_closed                 -0.09           1.00        0.72         0.76
## pubs_closed                    -0.06           0.72        1.00         0.94
## shops_closed                   -0.08           0.76        0.94         1.00
## eating_places_closed           -0.06           0.72        1.00         0.94
## stay_at_home                   -0.15           0.72        0.83         0.88
## household_mixing_indoors_banned -0.04          0.63        0.87         0.83
## wfh                             0.08           0.31        0.43         0.41
## rule_of_6_indoors               0.13          -0.02       -0.03        -0.03
## curfew                          0.00          -0.02       -0.03        -0.02
## eat_out_to_help_out             0.08          -0.01       -0.02        -0.02
##                                eating_places_closed stay_at_home
## Hires                                         -0.06        -0.15
## schools_closed                                 0.72         0.72
## pubs_closed                                    1.00         0.83
## shops_closed                                   0.94         0.88
## eating_places_closed                           1.00         0.83
## stay_at_home                                   0.83         1.00
## household_mixing_indoors_banned                0.87         0.73
## wfh                                            0.43         0.36
## rule_of_6_indoors                             -0.03        -0.03
## curfew                                        -0.03        -0.02
## eat_out_to_help_out                           -0.02        -0.01
##                                household_mixing_indoors_banned   wfh
## Hires                                                 -0.04  0.08
## schools_closed                                         0.63  0.31
## pubs_closed                                            0.87  0.43
## shops_closed                                           0.83  0.41
## eating_places_closed                                   0.87  0.43
## stay_at_home                                           0.73  0.36
## household_mixing_indoors_banned                        1.00  0.49
## wfh                                                    0.49  1.00
## rule_of_6_indoors                                     -0.04  0.23
## curfew                                                 0.23  0.20
## eat_out_to_help_out                                   -0.02 -0.04
##                                rule_of_6_indoors curfew eat_out_to_help_out
## Hires                                       0.13   0.00                0.08
## schools_closed                             -0.02  -0.02               -0.01
## pubs_closed                                -0.03  -0.03               -0.02
## shops_closed                               -0.03  -0.02               -0.02
## eating_places_closed                       -0.03  -0.03               -0.02
## stay_at_home                               -0.03  -0.02               -0.01
## household_mixing_indoors_banned            -0.04   0.23               -0.02
## wfh                                         0.23   0.20               -0.04
## rule_of_6_indoors                           1.00   0.30               -0.01
## curfew                                      0.30   1.00               -0.01
## eat_out_to_help_out                        -0.01  -0.01                1.00
##
## n= 4812
##
##
## P
##                                Hires  schools_closed pubs_closed shops_closed
## Hires                                 0.0000         0.0000      0.0000
## schools_closed                 0.0000                 0.0000      0.0000
## pubs_closed                    0.0000 0.0000                      0.0000
## shops_closed                   0.0000 0.0000         0.0000
## eating_places_closed           0.0000 0.0000         0.0000      0.0000
## stay_at_home                   0.0000 0.0000         0.0000      0.0000
## household_mixing_indoors_banned 0.0032 0.0000        0.0000      0.0000
## wfh                            0.0000 0.0000         0.0000      0.0000
## rule_of_6_indoors              0.0000 0.0965         0.0208      0.0291
## curfew                         0.9347 0.2063         0.0787      0.0971
## eat_out_to_help_out            0.0000 0.3729         0.2152      0.2421
```

```
##                                eating_places_closed stay_at_home
## Hires                          0.0000               0.0000
## schools_closed                 0.0000               0.0000
## pubs_closed                    0.0000               0.0000
## shops_closed                   0.0000               0.0000
## eating_places_closed                                0.0000
## stay_at_home                   0.0000
## household_mixing_indoors_banned 0.0000              0.0000
## wfh                            0.0000               0.0000
## rule_of_6_indoors              0.0208               0.0553
## curfew                         0.0787               0.1449
## eat_out_to_help_out            0.2152               0.3041
##                                household_mixing_indoors_banned wfh
## Hires                          0.0032                          0.0000
## schools_closed                 0.0000                          0.0000
## pubs_closed                    0.0000                          0.0000
## shops_closed                   0.0000                          0.0000
## eating_places_closed           0.0000                          0.0000
## stay_at_home                   0.0000                          0.0000
## household_mixing_indoors_banned                                0.0000
## wfh                            0.0000
## rule_of_6_indoors              0.0089                          0.0000
## curfew                         0.0000                          0.0000
## eat_out_to_help_out            0.1609                          0.0040
##                                rule_of_6_indoors curfew eat_out_to_help_out
## Hires                          0.0000            0.9347 0.0000
## schools_closed                 0.0965            0.2063 0.3729
## pubs_closed                    0.0208            0.0787 0.2152
## shops_closed                   0.0291            0.0971 0.2421
## eating_places_closed           0.0208            0.0787 0.2152
## stay_at_home                   0.0553            0.1449 0.3041
## household_mixing_indoors_banned 0.0089           0.0000 0.1609
## wfh                            0.0000            0.0000 0.0040
## rule_of_6_indoors                                0.0000 0.4491
## curfew                         0.0000                   0.5648
## eat_out_to_help_out            0.4491            0.5648
```

Some predictors have an extremely high r value (above 0.85): shops_closed and pubs_closed (0.94), eating_places_closed and pubs_closed (1.00), eating_places_closed and shops_closed (0.94), household_mixing_indoors_banned and pubs_closed (0.87), stay_at_home and shops_closed (0.88), household_mixing_indoors_banned and eating_places_closed (0.87).

It is worth mentioning that if we check the data dictionary again, we can see that eating_places_closed and pubs_closed are nested; eating place closures has already included pub closures, and that's why their r value is 1. Thus, in order to prevent multicollinearity issues and isolate the individual effects of each variable, we retain the eating_places_closed variable and exclude the pubs_closed variable from our following model. As for other highly correlated variables, we will check their VIF score after building the model.

Here we are creating a model with variables that may have an effect on the dependent variable (number of bike rented). I am using all variables in the data to build the model because their occurrence can more or less affect the number of bike rented. For example, when working from home is encouraged, bike rents might decrease because the transportation need is weakened; this rationale can be extended to variables such as schools_closed, stay_at_home, and curfew. Also, since people might ride bikes to work on workdays, bike rents on working days might thus be higher than on weekends.

Since the variable "pubs closures" is already included in the variable "eating places closures", I am only using the latter for model building.

```
# Build the regression model with the above-mentioned variables used
m.hires <- lm(Hires~schools_closed+shops_closed+eating_places_closed+stay_at_home+household_mixing_indoors_bann
ed+curfew+eat_out_to_help_out+wfh+rule_of_6_indoors, data=data)

# Check the VIF score and modify the model if we have variables with VIF>=5
vif(m.hires)
```

```
##                  schools_closed                    shops_closed
##                        2.403903                       13.193574
##            eating_places_closed                    stay_at_home
##                       12.870333                        4.516650
## household_mixing_indoors_banned                          curfew
##                        6.337655                        1.578761
##             eat_out_to_help_out                             wfh
##                        1.001727                        1.434432
##               rule_of_6_indoors
##                        1.242229
```

Not surprisingly, VIF scores are high for the eating_places_closed, shops_closed, and household_mixing_indoors_banned scores, suggesting multicollinearity and reflecting the high pairwise correlation between eating_places_closed and shops_closed, and between household_mixing_indoors_banned and eating_places_closed.

Since compare to other two variables, we tend to use eating_places_closed as a predictor in our model, we choose to exclude household_mixing_indoors_banned and shops_closed from the model.

Before removing household_mixing_indoors_banned and shops_closed, we can use one-way ANOVA to test whether these variables have significant effects on bike rents.

```
# Conduct the ANOVA test
anova(m.hires)
```

```
## Analysis of Variance Table
##
## Response: Hires
##                                   Df     Sum Sq    Mean Sq  F value    Pr(>F)
## schools_closed                     1 3.7591e+09 3.7591e+09  42.9528 6.197e-11
## shops_closed                       1 1.5060e+08 1.5060e+08   1.7208    0.1897
## eating_places_closed               1 1.4210e+09 1.4210e+09  16.2365 5.677e-05
## stay_at_home                       1 1.0895e+10 1.0895e+10 124.4865 < 2.2e-16
## household_mixing_indoors_banned    1 2.1692e+08 2.1692e+08   2.4786    0.1155
## curfew                             1 8.3856e+07 8.3856e+07   0.9582    0.3277
## eat_out_to_help_out                1 2.6499e+09 2.6499e+09  30.2793 3.935e-08
## wfh                                1 6.0792e+09 6.0792e+09  69.4633 < 2.2e-16
## rule_of_6_indoors                  1 6.7856e+09 6.7856e+09  77.5354 < 2.2e-16
## Residuals                       4802 4.2026e+11 8.7517e+07
##
## schools_closed                  ***
## shops_closed
## eating_places_closed            ***
## stay_at_home                    ***
## household_mixing_indoors_banned
## curfew
## eat_out_to_help_out             ***
## wfh                             ***
## rule_of_6_indoors               ***
## Residuals
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

As wee can see from the ANOVA result, both of these two variables are not significant predictor upon bike rents. Thus, we can try to build the model again without these variable:

```
# Build the regression model again with the above-mentioned variables used (household_mixing_indoors_banned and
shops_closed excluded)
m.hires <- lm(Hires~schools_closed+eating_places_closed+stay_at_home+curfew+eat_out_to_help_out+wfh+rule_of_6_i
ndoors, data=data)

# Check the VIF score again
vif(m.hires)
```

```
##       schools_closed eating_places_closed         stay_at_home
##             2.292220             3.829955             3.543772
##               curfew  eat_out_to_help_out                  wfh
##             1.132010             1.001727             1.367402
##    rule_of_6_indoors
##             1.158946
```

This reduces all VIF scores to be less than 5.

Then, we can check the summary of this regression model and try interpret the coefficients of variables that we are interested in.

```
# Check the summary of the regression model
summary(m.hires)
```

```
##
## Call:
## lm(formula = Hires ~ schools_closed + eating_places_closed +
##     stay_at_home + curfew + eat_out_to_help_out + wfh + rule_of_6_indoors,
##     data = data)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -28541  -6576   -245   6675  46985
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)           26108.8      154.1 169.373  < 2e-16 ***
## schools_closed        -1462.2     1251.1  -1.169    0.243
## eating_places_closed   7657.4     1192.5   6.422 1.48e-10 ***
## stay_at_home         -16070.3     1361.0 -11.808  < 2e-16 ***
## curfew                -5395.7     1339.0  -4.030 5.67e-05 ***
## eat_out_to_help_out   10309.6     1776.1   5.805 6.87e-09 ***
## wfh                    2432.5      376.6   6.459 1.16e-10 ***
## rule_of_6_indoors      8685.4     1039.2   8.358  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9363 on 4804 degrees of freedom
## Multiple R-squared:  0.06889,    Adjusted R-squared:  0.06754
## F-statistic: 50.78 on 7 and 4804 DF,  p-value: < 2.2e-16
```

```
# View the coefficients of each variable and their respective confidence interval
cbind(coef(m.hires), confint(m.hires))
```

```
##                                  2.5 %      97.5 %
## (Intercept)           26108.819  25806.615  26411.0235
## schools_closed        -1462.211  -3914.932    990.5099
## eating_places_closed   7657.421   5319.651   9995.1911
## stay_at_home         -16070.323 -18738.550 -13402.0962
## curfew                -5395.671  -8020.736  -2770.6059
## eat_out_to_help_out   10309.609   6827.600  13791.6186
## wfh                    2432.475   1694.206   3170.7440
## rule_of_6_indoors      8685.396   6648.171  10722.6205
```

The three variables that we are interested in are eat_out_to_help_out, wfh, and rule_of_6_indoors. When estimating the effect of these three variables in the same regression model without considering the effect of time variables, we find that when controlling for other variables, the practice of Eat Out to Help Out scheme (eat_out_to_help_out=1 compare to eat_out_to_help_out=0) predicts 10309.6 additional rentals (t(4804) = 5.805, p<0.001, 95% CI [6827.600, 13791.6186]); the practice of Working from home encouraged (wfh=1 compare to wfh=0) predicts 2432.5 additional rentals (t(4804) = 6.459, p<0.001, 95% CI [1694.206, 3170.7440]); the practice of Rule of 6 indoors (rule_of_6_indoors=1 compare to rule_of_6_indoors=0) predicts 8685.4 additional rentals (t(4804) = 8.358, p<0.001, 95% CI [6648.171, 10722.6205]). As we can learn from their small p-values and positive confidence intervals, these three variables are significant predictors.

In the following part, we try to obtain estimated marginal means for different combinations of levels of these three variables and get their confidence intervals as well.

```
# Obtain the estimated mean value by using emmeans()
m.hires.emm <- emmeans(m.hires, ~eat_out_to_help_out+wfh+rule_of_6_indoors)

# Check the summary of the emmeans() and the mean values and confidence interval
summary(m.hires.emm)
```

```
##  eat_out_to_help_out wfh rule_of_6_indoors emmean   SE   df lower.CL upper.CL
##                    0   0                 0  18473  850 4804    16808    20139
##                    1   0                 0  28783 1957 4804    24947    32619
##                    0   1                 0  20906  785 4804    19367    22444
##                    1   1                 0  31216 1942 4804    27409    35022
##                    0   0                 1  27159 1238 4804    24733    29585
##                    1   0                 1  37468 2155 4804    33244    41693
##                    0   1                 1  29591 1119 4804    27398    31785
##                    1   1                 1  39901 2100 4804    35784    44018
##
## Results are averaged over the levels of: schools_closed, eating_places_closed, stay_at_home, curfew
## Confidence level used: 0.95
```

The mean bike rents when these three variables equal to 0 (not practiced) is 18473 with 95% CI [16808-20139]; the mean bike rents when only eat_out_to_help_out equals to 1 (practiced) and the other two variables equal to 0 (not practiced) is 28783 with 95% CI [24947-32619], which is indeed an increase compare to the first original situation; the mean bike rents when only wfh equals to 1 (practiced) and the other two variables equal to 0 (not practiced) is 20906 with 95% CI [19367-22444], which is also an increase compare to the first situation; the mean bike rents when only rule_of_6_indoors equals to 1 (practiced) and the other two variables equal to 0 (not practiced) is 27159 with 95% CI [24733-29585], which is as well an increase compare to the first situation.

We can also check the mean number of bike rents for each of these three variables without controlling the other two variables:

```
# Eat out to help out scheme
# Obtain the estimated mean value by using emmeans()
m.hires.eat.emm <- emmeans(m.hires, ~eat_out_to_help_out)

# Check the summary of the emmeans() and the mean values and confidence interval
summary(m.hires.eat.emm)
```

```
##  eat_out_to_help_out emmean   SE   df lower.CL upper.CL
##                    0  24032  851 4804    22363    25702
##                    1  34342 1964 4804    30492    38192
##
## Results are averaged over the levels of: schools_closed, eating_places_closed, stay_at_home, curfew, wfh, ru
le_of_6_indoors
## Confidence level used: 0.95
```

The mean bike rents when eat_out_to_help_out equals to 0 (not practiced) is 23180 with 95% CI [21424-24937]; the mean bike rents when eat_out_to_help_out equals to 1 (practiced) is 33492 with 95% CI [29605-37378], which is indeed an increase compare to the first situation.

```
# Working from home
# Obtain the estimated mean value by using emmeans()
m.hires.wfh.emm <- emmeans(m.hires, ~wfh)

# Check the summary of the emmeans() and the mean values and confidence interval
summary(m.hires.wfh.emm)
```

```
##  wfh emmean   SE   df lower.CL upper.CL
##    0  27971 1274 4804    25474    30468
##    1  30403 1206 4804    28040    32767
##
## Results are averaged over the levels of: schools_closed, eating_places_closed, stay_at_home, curfew, eat_out
_to_help_out, rule_of_6_indoors
## Confidence level used: 0.95
```

The mean bike rents when wfh equals to 0 (not practiced) is 27237 with 95% CI [24697-29776]; the mean bike rents when wfh equals to 1 (practiced) is 29436 with 95% CI [26991-31880], which is indeed an increase compare to the former situation.

```
# Rule of 6 Indoors
# Obtain the estimated mean value by using emmeans()
m.hires.ro6.emm <- emmeans(m.hires, ~rule_of_6_indoors)

# Check the summary of the emmeans() and the mean values and confidence interval of each combination
summary(m.hires.ro6.emm)
```

```
##  rule_of_6_indoors emmean   SE   df lower.CL upper.CL
##                  0  24844 1187 4804    22517    27172
##                  1  33530 1461 4804    30665    36395
##
## Results are averaged over the levels of: schools_closed, eating_places_closed, stay_at_home, curfew, eat_out
_to_help_out, wfh
## Confidence level used: 0.95
```

The mean bike rents when rule_of_6_indoors equals to 0 (not practiced) is 23603 with 95% CI [21137-26070]; the mean bike rents when wfh equals to 1 (practiced) is 33069 with 95% CI [30192-35945], which is indeed an increase compare to the former situation.

However, there is a big mistake exists in the analysis above- the effect of time variables was not considered when building the model. As we can see in the visualization with title “Bike Rental Trends (2010-2023): Total Rentals for Each Year”, it is obvious that bike rents increased almost yearly overall, and reached a peak on 2022. We all know that before the pandemic, non of these policies was implemented (denoted as 0), so with the overall upward bike rental trend and the fact that in early years non of the policies has even occurred, the comparison between average bike rents when the three variables (eat_out_to_help_out, wfh, and rule_of_6_indoors) equal to 0 and when one of them is implemented is flawed and the latter will always have more bike rents. The former situation (all 3 variables equal to 0) is most likely happened in early years, while the latter situations (one of the 3 variables equal to 1) happened only after the outbreak of the pandemic.

Thus, we should build a regression model with time variables included to assess the effect of these policies upon bike rents correctly.

Regression model with time variables considered

```
# Build the regression model with variables used in the final m.hires model and time variables added (year, mon
th, day)
m.hires.time <- lm(Hires~schools_closed+eating_places_closed+stay_at_home+curfew+eat_out_to_help_out+wfh+rule_o
f_6_indoors+year+month+day, data=data)

# Check the VIF score and modify the model if we have variables with GVIF^(1/(2*Df)) above 1.6
vif(m.hires.time)
```

```
##                          GVIF Df GVIF^(1/(2*Df))
## schools_closed        2.415286  1        1.554119
## eating_places_closed  6.508757  1        2.551227
## stay_at_home          3.814975  1        1.953196
## curfew                1.695970  1        1.302294
## eat_out_to_help_out   1.242613  1        1.114725
## wfh                  12.171102  1        3.488711
## rule_of_6_indoors     1.679369  1        1.295905
## year                 24.574817 13        1.131046
## month                 1.514675 11        1.019052
## day                   1.000502  6        1.000042
```

Ramzi(2024) mentions that when using adjusted generalized standard error inflation factor (aGSIF), we must take the square-root of our rules of thumb for what is a large value – aGSIF values above $\sqrt{2.5}$ (1.6) may be of concern, and values above $\sqrt{10}$ (3.2) are indicative of a more serious problem.

After checking the VIF score, we can see that eating_places_closed and wfh have fairly high aGSIF, 2.55 and 3.49 respectively. Since wfh is one of the three variables that we are interested in, we first try removing eating_places_closed variable from the model and see if this helps reduce aGSIF score.

Before removing eating_places_closed variable, we can use one-way ANOVA to test whether the variable eating_places_closed has a significant effect on bike rents.

```
anova(m.hires.time)
```

```
## Analysis of Variance Table
##
## Response: Hires
##                         Df     Sum Sq    Mean Sq  F value    Pr(>F)
## schools_closed           1 3.7591e+09 3.7591e+09  95.7799 < 2.2e-16 ***
## eating_places_closed     1 5.1065e+07 5.1065e+07   1.3011    0.2541
## stay_at_home             1 1.2222e+10 1.2222e+10 311.4077 < 2.2e-16 ***
## curfew                   1 3.8956e+04 3.8956e+04   0.0010    0.9749
## eat_out_to_help_out      1 2.6396e+09 2.6396e+09  67.2545 3.038e-16 ***
## wfh                      1 6.3647e+09 6.3647e+09 162.1708 < 2.2e-16 ***
## rule_of_6_indoors        1 6.1240e+09 6.1240e+09 156.0368 < 2.2e-16 ***
## year                    13 7.0192e+10 5.3994e+09 137.5734 < 2.2e-16 ***
## month                   11 1.4155e+11 1.2868e+10 327.8647 < 2.2e-16 ***
## day                      6 2.2032e+10 3.6721e+09  93.5628 < 2.2e-16 ***
## Residuals             4774 1.8737e+11 3.9247e+07
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

As wee can see from the ANOVA result, bike rents do not differ significantly with and without the closure of eating places, F(1,4774)=1.3011, p=.2541. Thus, we try to build the model again without eating_places_closed variable:

```
# Build the regression model again without eating_places_closed variable
m.hires.time <- lm(Hires~schools_closed+stay_at_home+curfew+eat_out_to_help_out+wfh+rule_of_6_indoors+year+mont
h+day, data=data)

# Check the VIF score again
vif(m.hires.time)
```

```
##                         GVIF Df GVIF^(1/(2*Df))
## schools_closed       2.261822  1        1.503935
## stay_at_home         2.784857  1        1.668789
## curfew               1.493394  1        1.222045
## eat_out_to_help_out  1.235195  1        1.111393
## wfh                  8.879793  1        2.979898
## rule_of_6_indoors    1.497901  1        1.223888
## year                14.910494 13        1.109518
## month                1.472587 11        1.017747
## day                  1.000420  6        1.000035
```

Most of the aGSIF scores for each variable are decreased after the modification; though the aGSIF score for wfh is still high, we consider it might exhibit characteristics as a combination of other variables or has a relationship with multiple predictors. Since work from home is the variable we are interested in and its aGSIF score is now below 3.2, we will keep this model as it is and move on to analyze the results.

```
# Check the summary of the regression model
summary(m.hires.time)
```

```
##
## Call:
## lm(formula = Hires ~ schools_closed + stay_at_home + curfew +
##     eat_out_to_help_out + wfh + rule_of_6_indoors + year + month +
##     day, data = data)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -33775  -3435    543   3672  39642
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)           5883.6      652.0   9.024  < 2e-16 ***
## schools_closed       -1261.6      834.0  -1.513 0.130405
## stay_at_home         -6298.0      809.7  -7.779 8.93e-15 ***
## curfew               -2184.2     1032.1  -2.116 0.034374 *
## eat_out_to_help_out   -637.1     1323.5  -0.481 0.630278
## wfh                  -1892.9      644.0  -2.939 0.003306 **
## rule_of_6_indoors      283.3      792.8   0.357 0.720840
## year2011              5424.8      612.6   8.856  < 2e-16 ***
## year2012             11888.9      612.4  19.414  < 2e-16 ***
## year2013              7888.2      612.6  12.877  < 2e-16 ***
## year2014             13308.4      612.6  21.725  < 2e-16 ***
## year2015             12891.7      612.6  21.044  < 2e-16 ***
## year2016             14023.7      612.4  22.900  < 2e-16 ***
## year2017             14481.2      612.6  23.639  < 2e-16 ***
## year2018             14804.0      612.6  24.166  < 2e-16 ***
## year2019             14407.3      612.6  23.519  < 2e-16 ***
## year2020             17692.8      717.9  24.644  < 2e-16 ***
## year2021             18626.4      681.7  27.323  < 2e-16 ***
## year2022             19272.3      877.4  21.964  < 2e-16 ***
## year2023             10756.3      903.0  11.912  < 2e-16 ***
## monthFeb              1650.3      453.4   3.640 0.000276 ***
## monthMar              4227.1      444.2   9.516  < 2e-16 ***
## monthApr              7703.2      449.0  17.156  < 2e-16 ***
## monthMay             11816.6      448.9  26.322  < 2e-16 ***
## monthJun             14570.6      455.8  31.970  < 2e-16 ***
## monthJul             16043.0      448.0  35.807  < 2e-16 ***
## monthAug             13137.5      445.9  29.466  < 2e-16 ***
## monthSep             12222.4      443.0  27.591  < 2e-16 ***
## monthOct              9241.0      453.1  20.396  < 2e-16 ***
## monthNov              5144.8      452.3  11.374  < 2e-16 ***
## monthDec             -1021.4      450.4  -2.268 0.023393 *
## dayTue                2185.6      338.9   6.449 1.24e-10 ***
## dayWed                2255.9      338.9   6.657 3.12e-11 ***
## dayThu                2292.0      338.9   6.763 1.52e-11 ***
## dayFri                1061.0      338.8   3.132 0.001748 **
## daySat               -1710.5      338.8  -5.049 4.61e-07 ***
## daySun               -3692.3      338.9 -10.895  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6283 on 4775 degrees of freedom
## Multiple R-squared:  0.5832, Adjusted R-squared:  0.5801
## F-statistic: 185.6 on 36 and 4775 DF,  p-value: < 2.2e-16
```

```
# View the coefficients of each variable and their respective confidence interval
cbind(coef(m.hires.time), confint(m.hires.time))
```

```
##                                  2.5 %     97.5 %
## (Intercept)          5883.6315  4605.3643  7161.8988
## schools_closed      -1261.6452 -2896.6678   373.3773
## stay_at_home        -6298.0160 -7885.3412 -4710.6907
## curfew              -2184.1754 -4207.5519  -160.7990
## eat_out_to_help_out  -637.1107 -3231.8737  1957.6522
## wfh                 -1892.8773 -3155.4096  -630.3449
## rule_of_6_indoors     283.3146 -1270.9448  1837.5740
## year2011             5424.8498  4223.8867  6625.8129
## year2012            11888.9399 10688.3710 13089.5088
## year2013             7888.1755  6687.2121  9089.1389
## year2014            13308.3609 12107.3974 14509.3244
## year2015            12891.6649 11690.7013 14092.6285
## year2016            14023.6700 12823.1039 15224.2361
## year2017            14481.2245 13280.2620 15682.1870
## year2018            14803.9744 13603.0112 16004.9377
## year2019            14407.3426 13206.3792 15608.3060
## year2020            17692.7505 16285.2633 19100.2377
## year2021            18626.3641 17289.9042 19962.8241
## year2022            19272.3106 17552.1054 20992.5159
## year2023            10756.2687  8985.9950 12526.5424
## monthFeb             1650.3215   761.3756  2539.2674
## monthMar             4227.1010  3356.2079  5097.9941
## monthApr             7703.1910  6822.9506  8583.4315
## monthMay            11816.5865 10936.4831 12696.6900
## monthJun            14570.5578 13677.0698 15464.0458
## monthJul            16043.0088 15164.6506 16921.3671
## monthAug            13137.4827 12263.3958 14011.5695
## monthSep            12222.3847 11353.9299 13090.8396
## monthOct             9240.9726  8352.7511 10129.1941
## monthNov             5144.8352  4258.0568  6031.6137
## monthDec            -1021.4031 -1904.4286  -138.3775
## dayTue               2185.6147  1521.2074  2850.0220
## dayWed               2255.9372  1591.5287  2920.3456
## dayThu               2291.9746  1627.5608  2956.3883
## dayFri               1060.9883   396.8224  1725.1543
## daySat              -1710.5043 -2374.6684 -1046.3401
## daySun              -3692.2883 -4356.6949 -3027.8816
```

We can find that when controlling for other variables, the practice of Eat Out to Help Out scheme (eat_out_to_help_out=1 compare to eat_out_to_help_out=0) predicts 637.1 less rentals (t(4775) = -0.481, p=0.630278, 95% CI [-3231.8737, 1957.6522]); the practice of Working from home encouraged (wfh=1 compare to wfh=0) predicts 1892.9 less rentals (t(4775) = -2.939, p<0.01, 95% CI [-3155.4096, -630.3449]); the practice of Rule of 6 indoors (rule_of_6_indoors=1 compare to rule_of_6_indoors=0) predicts 283.3 additional rentals (t(4775) = 0.357, p=0.72, 95% CI [-1270.9448, 1837.5740]).

We can learn from their p-values that after adding the time variables in the regression model, the effect of eat_out_to_help_out and rule_of_6_indoors upon bike rents are no more significant; there is a significant bike rents decrease (-1892.9) when wfh is implemented, with t(4775)=-2.939 and p<0.01.

As for time variables, all three time variables (year, month, and day) have significant effects upon bike rents (p-value < 0.05). We will further analyze the effect of time variables with some visualizations.

In the following part, we again try to obtain estimated marginal means for different combinations of levels of variables we are interested in and time variables, and get their confidence intervals as well.

```
# In case that the rows of requested reference grid would exceed the limit of 10000
emm_options(rg.limit = 100000)
options(max.print = 3000)

# Obtain the estimated mean value by using emmeans()
m.hires.time.emm <- emmeans(m.hires.time, ~eat_out_to_help_out+wfh+rule_of_6_indoors+year+month+day)

# Check the summary of the emmeans() and the mean values and confidence interval of each combination
summary(m.hires.time.emm)
```

```
##  eat_out_to_help_out wfh rule_of_6_indoors year month day   emmean   SE   df
##                    0   0                 0 2010 Jan   Mon  1011.71  972 4775
##                    1   0                 0 2010 Jan   Mon   374.60 1686 4775
##                    0   1                 0 2010 Jan   Mon  -881.16  956 4775
##                    1   1                 0 2010 Jan   Mon -1518.27 1772 4775
##                    0   0                 1 2010 Jan   Mon  1295.03 1279 4775
##                    1   0                 1 2010 Jan   Mon   657.92 1862 4775
##                    0   1                 1 2010 Jan   Mon  -597.85 1129 4775
##                    1   1                 1 2010 Jan   Mon -1234.96 1853 4775
##                    0   0                 0 2011 Jan   Mon  6436.56  866 4775
##                    1   0                 0 2011 Jan   Mon  5799.45 1621 4775
##                    0   1                 0 2011 Jan   Mon  4543.69  860 4775
##                    1   1                 0 2011 Jan   Mon  3906.57 1716 4775
##                    0   0                 1 2011 Jan   Mon  6719.88 1202 4775
##                    1   0                 1 2011 Jan   Mon  6082.77 1805 4775
##                    0   1                 1 2011 Jan   Mon  4827.00 1051 4775
##                    1   1                 1 2011 Jan   Mon  4189.89 1801 4775
##                    0   0                 0 2012 Jan   Mon 12900.65  866 4775
##                    1   0                 0 2012 Jan   Mon 12263.54 1621 4775
##                    0   1                 0 2012 Jan   Mon 11007.78  860 4775
##                    1   1                 0 2012 Jan   Mon 10370.67 1716 4775
##                    0   0                 1 2012 Jan   Mon 13183.97 1202 4775
##                    1   0                 1 2012 Jan   Mon 12546.86 1805 4775
##                    0   1                 1 2012 Jan   Mon 11291.09 1051 4775
##                    1   1                 1 2012 Jan   Mon 10653.98 1801 4775
##                    0   0                 0 2013 Jan   Mon  8899.89  866 4775
##                    1   0                 0 2013 Jan   Mon  8262.78 1621 4775
##                    0   1                 0 2013 Jan   Mon  7007.01  860 4775
##                    1   1                 0 2013 Jan   Mon  6369.90 1716 4775
##                    0   0                 1 2013 Jan   Mon  9183.20 1202 4775
##                    1   0                 1 2013 Jan   Mon  8546.09 1805 4775
##                    0   1                 1 2013 Jan   Mon  7290.33 1051 4775
##                    1   1                 1 2013 Jan   Mon  6653.22 1801 4775
##                    0   0                 0 2014 Jan   Mon 14320.07  866 4775
##                    1   0                 0 2014 Jan   Mon 13682.96 1621 4775
##                    0   1                 0 2014 Jan   Mon 12427.20  860 4775
##                    1   1                 0 2014 Jan   Mon 11790.09 1716 4775
##                    0   0                 1 2014 Jan   Mon 14603.39 1202 4775
##                    1   0                 1 2014 Jan   Mon 13966.28 1805 4775
##                    0   1                 1 2014 Jan   Mon 12710.51 1051 4775
##                    1   1                 1 2014 Jan   Mon 12073.40 1801 4775
##                    0   0                 0 2015 Jan   Mon 13903.38  866 4775
##                    1   0                 0 2015 Jan   Mon 13266.27 1621 4775
##                    0   1                 0 2015 Jan   Mon 12010.50  860 4775
##                    1   1                 0 2015 Jan   Mon 11373.39 1716 4775
##                    0   0                 1 2015 Jan   Mon 14186.69 1202 4775
##                    1   0                 1 2015 Jan   Mon 13549.58 1805 4775
##                    0   1                 1 2015 Jan   Mon 12293.82 1051 4775
##                    1   1                 1 2015 Jan   Mon 11656.70 1801 4775
##                    0   0                 0 2016 Jan   Mon 15035.38  866 4775
##                    1   0                 0 2016 Jan   Mon 14398.27 1621 4775
##                    0   1                 0 2016 Jan   Mon 13142.51  860 4775
##                    1   1                 0 2016 Jan   Mon 12505.40 1716 4775
##                    0   0                 1 2016 Jan   Mon 15318.70 1202 4775
##                    1   0                 1 2016 Jan   Mon 14681.59 1805 4775
##                    0   1                 1 2016 Jan   Mon 13425.82 1051 4775
##                    1   1                 1 2016 Jan   Mon 12788.71 1801 4775
##                    0   0                 0 2017 Jan   Mon 15492.94  866 4775
##                    1   0                 0 2017 Jan   Mon 14855.83 1621 4775
##                    0   1                 0 2017 Jan   Mon 13600.06  860 4775
##                    1   1                 0 2017 Jan   Mon 12962.95 1716 4775
##                    0   0                 1 2017 Jan   Mon 15776.25 1202 4775
##                    1   0                 1 2017 Jan   Mon 15139.14 1805 4775
##                    0   1                 1 2017 Jan   Mon 13883.38 1051 4775
##                    1   1                 1 2017 Jan   Mon 13246.26 1801 4775
```

```
##                   0   0                 0 2018 Jan   Mon 15815.69  866 4775
##                   1   0                 0 2018 Jan   Mon 15178.58 1621 4775
##                   0   1                 0 2018 Jan   Mon 13922.81  860 4775
##                   1   1                 0 2018 Jan   Mon 13285.70 1716 4775
##                   0   0                 1 2018 Jan   Mon 16099.00 1202 4775
##                   1   0                 1 2018 Jan   Mon 15461.89 1805 4775
##                   0   1                 1 2018 Jan   Mon 14206.13 1051 4775
##                   1   1                 1 2018 Jan   Mon 13569.01 1801 4775
##                   0   0                 0 2019 Jan   Mon 15419.06  866 4775
##                   1   0                 0 2019 Jan   Mon 14781.95 1621 4775
##                   0   1                 0 2019 Jan   Mon 13526.18  860 4775
##                   1   1                 0 2019 Jan   Mon 12889.07 1716 4775
##                   0   0                 1 2019 Jan   Mon 15702.37 1202 4775
##                   1   0                 1 2019 Jan   Mon 15065.26 1805 4775
##                   0   1                 1 2019 Jan   Mon 13809.49 1051 4775
##                   1   1                 1 2019 Jan   Mon 13172.38 1801 4775
##                   0   0                 0 2020 Jan   Mon 18704.46  890 4775
##                   1   0                 0 2020 Jan   Mon 18067.35 1472 4775
##                   0   1                 0 2020 Jan   Mon 16811.59  653 4775
##                   1   1                 0 2020 Jan   Mon 16174.48 1459 4775
##                   0   0                 1 2020 Jan   Mon 18987.78 1244 4775
##                   1   0                 1 2020 Jan   Mon 18350.67 1689 4775
##                   0   1                 1 2020 Jan   Mon 17094.90  922 4775
##                   1   1                 1 2020 Jan   Mon 16457.79 1576 4775
##                   0   0                 0 2021 Jan   Mon 19638.08  965 4775
##                   1   0                 0 2021 Jan   Mon 19000.97 1620 4775
##                   0   1                 0 2021 Jan   Mon 17745.20  778 4775
##                   1   1                 0 2021 Jan   Mon 17108.09 1621 4775
##                   0   0                 1 2021 Jan   Mon 19921.39 1234 4775
##                   1   0                 1 2021 Jan   Mon 19284.28 1775 4775
##                   0   1                 1 2021 Jan   Mon 18028.51  931 4775
##                   1   1                 1 2021 Jan   Mon 17391.40 1680 4775
##                   0   0                 0 2022 Jan   Mon 20284.02 1261 4775
##                   1   0                 0 2022 Jan   Mon 19646.91 1772 4775
##                   0   1                 0 2022 Jan   Mon 18391.15  866 4775
##                   1   1                 0 2022 Jan   Mon 17754.04 1621 4775
##                   0   0                 1 2022 Jan   Mon 20567.34 1617 4775
##                   1   0                 1 2022 Jan   Mon 19930.23 2025 4775
##                   0   1                 1 2022 Jan   Mon 18674.46 1202 4775
##                   1   1                 1 2022 Jan   Mon 18037.35 1805 4775
##                   0   0                 0 2023 Jan   Mon 11767.98 1269 4775
##                   1   0                 0 2023 Jan   Mon 11130.87 1780 4775
##                   0   1                 0 2023 Jan   Mon  9875.10  880 4775
##                   1   1                 0 2023 Jan   Mon  9237.99 1630 4775
##                   0   0                 1 2023 Jan   Mon 12051.30 1626 4775
##                   1   0                 1 2023 Jan   Mon 11414.19 2033 4775
##                   0   1                 1 2023 Jan   Mon 10158.42 1215 4775
##                   1   1                 1 2023 Jan   Mon  9521.31 1815 4775
##                   0   0                 0 2010 Feb   Mon  2662.03  976 4775
##                   1   0                 0 2010 Feb   Mon  2024.92 1688 4775
##                   0   1                 0 2010 Feb   Mon   769.16  961 4775
##                   1   1                 0 2010 Feb   Mon   132.05 1775 4775
##                   0   0                 1 2010 Feb   Mon  2945.35 1281 4775
##                   1   0                 1 2010 Feb   Mon  2308.24 1863 4775
##                   0   1                 1 2010 Feb   Mon  1052.47 1132 4775
##                   1   1                 1 2010 Feb   Mon   415.36 1855 4775
##                   0   0                 0 2011 Feb   Mon  8086.88  870 4775
##                   1   0                 0 2011 Feb   Mon  7449.77 1623 4775
##                   0   1                 0 2011 Feb   Mon  6194.01  865 4775
##                   1   1                 0 2011 Feb   Mon  5556.90 1719 4775
##                   0   0                 1 2011 Feb   Mon  8370.20 1204 4775
##                   1   0                 1 2011 Feb   Mon  7733.09 1806 4775
##                   0   1                 1 2011 Feb   Mon  6477.32 1054 4775
##                   1   1                 1 2011 Feb   Mon  5840.21 1804 4775
##                   0   0                 0 2012 Feb   Mon 14550.97  869 4775
##                   1   0                 0 2012 Feb   Mon 13913.86 1623 4775
```

```
##                 0   1                 0 2012 Feb   Mon 12658.10  864 4775
##                 1   1                 0 2012 Feb   Mon 12020.99 1719 4775
##                 0   0                 1 2012 Feb   Mon 14834.29 1204 4775
##                 1   0                 1 2012 Feb   Mon 14197.18 1806 4775
##                 0   1                 1 2012 Feb   Mon 12941.41 1054 4775
##                 1   1                 1 2012 Feb   Mon 12304.30 1803 4775
##                 0   0                 0 2013 Feb   Mon 10550.21  870 4775
##                 1   0                 0 2013 Feb   Mon  9913.10 1623 4775
##                 0   1                 0 2013 Feb   Mon  8657.33  865 4775
##                 1   1                 0 2013 Feb   Mon  8020.22 1719 4775
##                 0   0                 1 2013 Feb   Mon 10833.52 1204 4775
##                 1   0                 1 2013 Feb   Mon 10196.41 1806 4775
##                 0   1                 1 2013 Feb   Mon  8940.65 1054 4775
##                 1   1                 1 2013 Feb   Mon  8303.54 1804 4775
##                 0   0                 0 2014 Feb   Mon 15970.40  870 4775
##                 1   0                 0 2014 Feb   Mon 15333.28 1623 4775
##                 0   1                 0 2014 Feb   Mon 14077.52  865 4775
##                 1   1                 0 2014 Feb   Mon 13440.41 1719 4775
##                 0   0                 1 2014 Feb   Mon 16253.71 1204 4775
##                 1   0                 1 2014 Feb   Mon 15616.60 1806 4775
##                 0   1                 1 2014 Feb   Mon 14360.83 1054 4775
##                 1   1                 1 2014 Feb   Mon 13723.72 1804 4775
##                 0   0                 0 2015 Feb   Mon 15553.70  870 4775
##                 1   0                 0 2015 Feb   Mon 14916.59 1623 4775
##                 0   1                 0 2015 Feb   Mon 13660.82  865 4775
##                 1   1                 0 2015 Feb   Mon 13023.71 1719 4775
##                 0   0                 1 2015 Feb   Mon 15837.01 1204 4775
##                 1   0                 1 2015 Feb   Mon 15199.90 1806 4775
##                 0   1                 1 2015 Feb   Mon 13944.14 1054 4775
##                 1   1                 1 2015 Feb   Mon 13307.03 1804 4775
##                 0   0                 0 2016 Feb   Mon 16685.70  869 4775
##                 1   0                 0 2016 Feb   Mon 16048.59 1623 4775
##                 0   1                 0 2016 Feb   Mon 14792.83  865 4775
##                 1   1                 0 2016 Feb   Mon 14155.72 1719 4775
##                 0   0                 1 2016 Feb   Mon 16969.02 1204 4775
##                 1   0                 1 2016 Feb   Mon 16331.91 1806 4775
##                 0   1                 1 2016 Feb   Mon 15076.14 1054 4775
##                 1   1                 1 2016 Feb   Mon 14439.03 1803 4775
##                 0   0                 0 2017 Feb   Mon 17143.26  870 4775
##                 1   0                 0 2017 Feb   Mon 16506.15 1623 4775
##                 0   1                 0 2017 Feb   Mon 15250.38  865 4775
##                 1   1                 0 2017 Feb   Mon 14613.27 1719 4775
##                 0   0                 1 2017 Feb   Mon 17426.57 1204 4775
##                 1   0                 1 2017 Feb   Mon 16789.46 1806 4775
##                 0   1                 1 2017 Feb   Mon 15533.70 1054 4775
##                 1   1                 1 2017 Feb   Mon 14896.59 1804 4775
##                 0   0                 0 2018 Feb   Mon 17466.01  870 4775
##                 1   0                 0 2018 Feb   Mon 16828.90 1623 4775
##                 0   1                 0 2018 Feb   Mon 15573.13  865 4775
##                 1   1                 0 2018 Feb   Mon 14936.02 1719 4775
##                 0   0                 1 2018 Feb   Mon 17749.32 1204 4775
##                 1   0                 1 2018 Feb   Mon 17112.21 1806 4775
##                 0   1                 1 2018 Feb   Mon 15856.45 1054 4775
##                 1   1                 1 2018 Feb   Mon 15219.34 1803 4775
##                 0   0                 0 2019 Feb   Mon 17069.38  870 4775
##                 1   0                 0 2019 Feb   Mon 16432.27 1623 4775
##                 0   1                 0 2019 Feb   Mon 15176.50  865 4775
##                 1   1                 0 2019 Feb   Mon 14539.39 1719 4775
##                 0   0                 1 2019 Feb   Mon 17352.69 1204 4775
##                 1   0                 1 2019 Feb   Mon 16715.58 1806 4775
##                 0   1                 1 2019 Feb   Mon 15459.81 1054 4775
##                 1   1                 1 2019 Feb   Mon 14822.70 1804 4775
##                 0   0                 0 2020 Feb   Mon 20354.79  894 4775
##                 1   0                 0 2020 Feb   Mon 19717.67 1474 4775
##                 0   1                 0 2020 Feb   Mon 18461.91  659 4775
##                 1   1                 0 2020 Feb   Mon 17824.80 1462 4775
```

```
##                   0   0                1 2020 Feb   Mon 20638.10 1245 4775
##                   1   0                1 2020 Feb   Mon 20000.99 1691 4775
##                   0   1                1 2020 Feb   Mon 18745.22  926 4775
##                   1   1                1 2020 Feb   Mon 18108.11 1579 4775
##                   0   0                0 2021 Feb   Mon 21288.40  968 4775
##                   1   0                0 2021 Feb   Mon 20651.29 1622 4775
##                   0   1                0 2021 Feb   Mon 19395.52  783 4775
##                   1   1                0 2021 Feb   Mon 18758.41 1624 4775
##                   0   0                1 2021 Feb   Mon 21571.71 1236 4775
##                   1   0                1 2021 Feb   Mon 20934.60 1777 4775
##                   0   1                1 2021 Feb   Mon 19678.84  934 4775
##                   1   1                1 2021 Feb   Mon 19041.73 1682 4775
##                   0   0                0 2022 Feb   Mon 21934.35 1263 4775
##                   1   0                0 2022 Feb   Mon 21297.23 1774 4775
##                   0   1                0 2022 Feb   Mon 20041.47  870 4775
##                   1   1                0 2022 Feb   Mon 19404.36 1623 4775
##                   0   0                1 2022 Feb   Mon 22217.66 1618 4775
##                   1   0                1 2022 Feb   Mon 21580.55 2026 4775
##                   0   1                1 2022 Feb   Mon 20324.78 1204 4775
##                   1   1                1 2022 Feb   Mon 19687.67 1806 4775
##                   0   0                0 2023 Feb   Mon 13418.30 1271 4775
##                   1   0                0 2023 Feb   Mon 12781.19 1781 4775
##                   0   1                0 2023 Feb   Mon 11525.43  884 4775
##                   1   1                0 2023 Feb   Mon 10888.32 1633 4775
##                   0   0                1 2023 Feb   Mon 13701.62 1627 4775
##                   1   0                1 2023 Feb   Mon 13064.51 2034 4775
##                   0   1                1 2023 Feb   Mon 11808.74 1217 4775
##                   1   1                1 2023 Feb   Mon 11171.63 1816 4775
##                   0   0                0 2010 Mar   Mon  5238.81  984 4775
##                   1   0                0 2010 Mar   Mon  4601.70 1689 4775
##                   0   1                0 2010 Mar   Mon  3345.94  953 4775
##                   1   1                0 2010 Mar   Mon  2708.83 1767 4775
##                   0   0                1 2010 Mar   Mon  5522.13 1292 4775
##                   1   0                1 2010 Mar   Mon  4885.02 1868 4775
##                   0   1                1 2010 Mar   Mon  3629.25 1131 4775
##                   1   1                1 2010 Mar   Mon  2992.14 1851 4775
##                   0   0                0 2011 Mar   Mon 10663.66  879 4775
##                   1   0                0 2011 Mar   Mon 10026.55 1625 4775
##                   0   1                0 2011 Mar   Mon  8770.79  856 4775
##                   1   1                0 2011 Mar   Mon  8133.68 1711 4775
##                   0   0                1 2011 Mar   Mon 10946.98 1217 4775
##                   1   0                1 2011 Mar   Mon 10309.87 1811 4775
##                   0   1                1 2011 Mar   Mon  9054.10 1053 4775
##                   1   1                1 2011 Mar   Mon  8416.99 1800 4775
##                   0   0                0 2012 Mar   Mon 17127.75  879 4775
##                   1   0                0 2012 Mar   Mon 16490.64 1625 4775
##                   0   1                0 2012 Mar   Mon 15234.88  855 4775
##                   1   1                0 2012 Mar   Mon 14597.77 1711 4775
##                   0   0                1 2012 Mar   Mon 17411.07 1216 4775
##                   1   0                1 2012 Mar   Mon 16773.96 1811 4775
##                   0   1                1 2012 Mar   Mon 15518.19 1053 4775
##                   1   1                1 2012 Mar   Mon 14881.08 1799 4775
##                   0   0                0 2013 Mar   Mon 13126.99  879 4775
##                   1   0                0 2013 Mar   Mon 12489.88 1625 4775
##                   0   1                0 2013 Mar   Mon 11234.11  856 4775
##                   1   1                0 2013 Mar   Mon 10597.00 1711 4775
##                   0   0                1 2013 Mar   Mon 13410.30 1217 4775
##                   1   0                1 2013 Mar   Mon 12773.19 1811 4775
##                   0   1                1 2013 Mar   Mon 11517.43 1053 4775
##                   1   1                1 2013 Mar   Mon 10880.32 1800 4775
##                   0   0                0 2014 Mar   Mon 18547.18  879 4775
##                   1   0                0 2014 Mar   Mon 17910.06 1625 4775
##                   0   1                0 2014 Mar   Mon 16654.30  856 4775
##                   1   1                0 2014 Mar   Mon 16017.19 1711 4775
##                   0   0                1 2014 Mar   Mon 18830.49 1217 4775
##                   1   0                1 2014 Mar   Mon 18193.38 1811 4775
```

```
##                   0   1                1 2014 Mar   Mon 16937.61 1053 4775
##                   1   1                1 2014 Mar   Mon 16300.50 1800 4775
##                   0   0                0 2015 Mar   Mon 18130.48  879 4775
##                   1   0                0 2015 Mar   Mon 17493.37 1625 4775
##                   0   1                0 2015 Mar   Mon 16237.60  856 4775
##                   1   1                0 2015 Mar   Mon 15600.49 1711 4775
##                   0   0                1 2015 Mar   Mon 18413.79 1217 4775
##                   1   0                1 2015 Mar   Mon 17776.68 1811 4775
##                   0   1                1 2015 Mar   Mon 16520.92 1053 4775
##                   1   1                1 2015 Mar   Mon 15883.81 1800 4775
##  lower.CL upper.CL
##   -894.46   2917.9
##  -2930.77   3680.0
##  -2756.34    994.0
##  -4992.21   1955.7
##  -1211.54   3801.6
##  -2991.78   4307.6
##  -2810.68   1615.0
##  -4867.32   2397.4
##   4738.93   8134.2
##   2621.14   8977.8
##   2857.88   6229.5
##    541.82   7271.3
##   4363.24   9076.5
##   2544.56   9621.0
##   2766.74   6887.3
##    658.63   7721.1
##  11203.52  14597.8
##   9085.55  15441.5
##   9322.58  12693.0
##   7006.26  13735.1
##  10827.69  15540.2
##   9008.94  16084.8
##   9231.32  13350.9
##   7123.05  14184.9
##   7202.25  10597.5
##   5084.47  11441.1
##   5321.20   8692.8
##   3005.14   9734.7
##   6826.56  11539.9
##   5007.89  12084.3
##   5230.06   9350.6
##   3121.95  10184.5
##  12622.44  16017.7
##  10504.66  16861.3
##  10741.39  14113.0
##   8425.33  15154.9
##  12246.75  16960.0
##  10428.08  17504.5
##  10650.25  14770.8
##   8542.14  15604.7
##  12205.75  15601.0
##  10087.96  16444.6
##  10324.69  13696.3
##   8008.63  14738.1
##  11830.05  16543.3
##  10011.38  17087.8
##  10233.55  14354.1
##   8125.45  15188.0
##  13337.90  16732.9
##  11220.09  17576.5
##  11456.95  14828.1
##   9140.80  15870.0
##  12962.16  17675.2
##  11143.50  18219.7
##  11365.76  15485.9
```

```
##   9257.61  16319.8
##  13795.30  17190.6
##  11677.52  18034.1
##  11914.25  15285.9
##   9598.19  16327.7
##  13419.61  18132.9
##  11600.94  18677.3
##  11823.11  15943.6
##   9715.01  16777.5
##  14118.41  17513.0
##  12000.46  18356.7
##  12237.36  15608.3
##   9921.12  16650.3
##  13742.62  18455.4
##  11923.86  18999.9
##  12146.15  16266.1
##  10037.93  17100.1
##  13721.42  17116.7
##  11603.64  17960.2
##  11840.37  15212.0
##   9524.31  16253.8
##  13345.73  18059.0
##  11527.06  18603.5
##  11749.23  15869.8
##   9641.12  16703.6
##  16958.83  20450.1
##  15181.96  20952.8
##  15531.27  18091.9
##  13314.85  19034.1
##  16549.51  21426.0
##  15038.74  21662.6
##  15287.12  18902.7
##  13367.20  19548.4
##  17746.08  21530.1
##  15824.34  22177.6
##  16220.68  19269.7
##  13930.76  20285.4
##  17501.79  22341.0
##  15804.16  22764.4
##  16203.70  19853.3
##  14097.98  20684.8
##  17812.19  22755.9
##  16172.77  23121.1
##  16693.51  20088.8
##  14575.73  20932.3
##  17397.04  23737.6
##  15960.85  23899.6
##  16317.82  21031.1
##  14499.15  21575.5
##   9279.44  14256.5
##   7642.03  14619.7
##   8150.01  11600.2
##   6041.88  12434.1
##   8863.98  15238.6
##   7428.76  15399.6
##   7776.61  12540.2
##   5963.55  13079.1
##    749.25   4574.8
##  -1284.79   5334.6
##  -1115.06   2653.4
##  -3347.27   3611.4
##    434.81   5455.9
##  -1344.66   5961.1
##  -1166.82   3271.8
##  -3221.41   4052.1
##   6381.87   9791.9
```

```
##   4266.98  10632.6
##   4498.19   7889.8
##   2186.60   8927.2
##   6009.36  10731.0
##   4191.60  11274.6
##   4410.15   8544.5
##   2304.43   9376.0
##  12847.13  16254.8
##  10731.74  17096.0
##  10963.56  14352.6
##   8651.38  15390.6
##  12474.29  17194.3
##  10656.30  17738.1
##  10875.28  15007.5
##   8769.18  15839.4
##   8845.20  12255.2
##   6730.30  13095.9
##   6961.52  10353.1
##   4649.93  11390.5
##   8472.69  13194.4
##   6654.93  13737.9
##   6873.47  11007.8
##   4767.76  11839.3
##  14265.38  17675.4
##  12150.49  18516.1
##  12381.70  15773.3
##  10070.11  16810.7
##  13892.88  18614.5
##  12075.12  19158.1
##  12293.66  16428.0
##  10187.94  17259.5
##  13848.69  17258.7
##  11733.79  18099.4
##  11965.01  15356.6
##   9653.42  16394.0
##  13476.18  18197.8
##  11658.42  18741.4
##  11876.96  16011.3
##   9771.25  16842.8
##  14981.51  18389.9
##  12866.28  19230.9
##  13097.93  16487.7
##  10785.93  17525.5
##  14608.77  19329.3
##  12790.86  19873.0
##  13009.72  17142.6
##  10903.73  17974.3
##  15438.24  18848.3
##  13323.35  19689.0
##  13554.57  16946.2
##  11242.98  17983.6
##  15065.74  19787.4
##  13247.98  20331.0
##  13466.53  17600.9
##  11360.81  18432.4
##  15761.35  19170.7
##  13646.29  20011.5
##  13877.68  17268.6
##  11565.91  18306.1
##  15388.74  20109.9
##  13570.90  20653.5
##  13789.57  17923.3
##  11683.73  18754.9
##  15364.36  18774.4
##  13249.47  19615.1
##  13480.68  16872.3
```

```
##  11169.09  17909.7
##  14991.86  19713.5
##  13174.10  20257.1
##  13392.64  17527.0
##  11286.92  18358.5
##  18603.11  22106.5
##  16828.03  22607.3
##  17169.99  19753.8
##  14959.36  20690.2
##  18196.59  23079.6
##  16686.16  23315.8
##  16930.67  20559.8
##  15013.00  21203.2
##  19390.32  23186.5
##  17470.50  23832.1
##  17860.61  20930.4
##  15575.54  21941.3
##  19148.45  23995.0
##  17451.44  24417.8
##  17846.79  21510.9
##  15743.76  22339.7
##  19459.20  24409.5
##  17820.24  24774.2
##  18336.45  21746.5
##  16221.56  22587.2
##  19045.61  25389.7
##  17609.35  25551.8
##  17963.95  22685.6
##  16146.19  23229.2
##  10926.52  15910.1
##   9289.54  16272.8
##   9793.14  13257.7
##   7687.78  14088.9
##  10512.61  16890.6
##   9077.29  17051.7
##   9422.83  14194.6
##   7610.65  14732.6
##   3309.80   7167.8
##   1289.64   7913.8
##   1478.33   5213.6
##   -754.79   6172.4
##   2988.70   8055.6
##   1222.73   8547.3
##   1412.03   5846.5
##   -636.93   6621.2
##   8940.42  12386.9
##   6841.28  13211.8
##   7093.40  10448.2
##   4779.57  11487.8
##   8561.79  13332.2
##   6758.68  13861.1
##   6989.12  11119.1
##   4889.11  11944.9
##  15405.01  18850.5
##  13305.69  19675.6
##  13558.10  16911.7
##  11244.02  17951.5
##  15026.23  19795.9
##  13223.05  20324.9
##  13453.70  17582.7
##  11353.53  18408.6
##  11403.75  14850.2
##   9304.61  15675.1
##   9556.72  12911.5
##   7242.90  13951.1
##  11025.11  15795.5
```

```
##   9222.00  16324.4
##   9452.44  13582.4
##   7352.43  14408.2
##  16823.94  20270.4
##  14724.80  21095.3
##  14976.91  18331.7
##  12663.09  19371.3
##  16445.30  21215.7
##  14642.19  21744.6
##  14872.63  19002.6
##  12772.62  19828.4
##  16407.24  19853.7
##  14308.10  20678.6
##  14560.21  17915.0
##  12246.39  18954.6
##  16028.60  20799.0
##  14225.49  21327.9
##  14455.93  18585.9
##  12355.92  19411.7
##  [ reached getOption("max.print") -- omitted 9136 rows ]
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew
## Confidence level used: 0.95
```

The summary can be interpreted as follows: When all 3 variables equal to 0 and on Mondays in January, 2010, the average bike rents is 1011.71 with 95% CI [-894.46-2917.9]. Other conditions are same, when eat_out_to_help_out equals to 1, the average bike rents is 374.60 with 95% CI [-2930.77-3680.0]. The latter CI is way larger than the first one because in our dataset, there was no such policy implemented (or existed) at that time, and the CI is so wide due to such uncertainty about a situation that did not exist.

It is not easy to see the overall trend with so many variables controlled, so we then only focus on time variables first:

```
# Year

# Obtain the estimated mean value by using emmeans()
m.hires.year.emm <- emmeans(m.hires.time, ~year)

# Check the summary of the emmeans() and the mean values and confidence interval
summary(m.hires.year.emm)
```

```
##  year emmean   SE   df lower.CL upper.CL
##  2010   8125 1160 4775     5850    10399
##  2011  13549 1082 4775    11428    15671
##  2012  20014 1082 4775    17892    22135
##  2013  16013 1082 4775    13891    18135
##  2014  21433 1082 4775    19311    23555
##  2015  21016 1082 4775    18895    23138
##  2016  22148 1082 4775    20027    24270
##  2017  22606 1082 4775    20484    24728
##  2018  22929 1082 4775    20807    25050
##  2019  22532 1082 4775    20410    24654
##  2020  25817  908 4775    24038    27597
##  2021  26751 1029 4775    24734    28768
##  2022  27397 1276 4775    24896    29898
##  2023  18881 1289 4775    16353    21408
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, eat_out_to_help_out, wfh, rul
e_of_6_indoors, month, day
## Confidence level used: 0.95
```

The mean bike rents is lowest on 2010, with a value of 18620 95% CI [16026, 21215]; the mean bike rents is highest on 2022, with a value of 41909 95% CI [39681, 44138].

```
# Month

# Obtain the estimated mean value by using emmeans()
m.hires.month.emm <- emmeans(m.hires.time, ~month)

# Check the summary of the emmeans() and the mean values and confidence interval
summary(m.hires.month.emm)
```

```
##  month emmean   SE   df lower.CL upper.CL
##  Jan    12763 1066 4775    10674    14853
##  Feb    14414 1069 4775    12318    16509
##  Mar    16990 1072 4775    14888    19092
##  Apr    20467 1073 4775    18363    22570
##  May    24580 1073 4775    22476    26684
##  Jun    27334 1085 4775    25207    29460
##  Jul    28806 1087 4775    26676    30937
##  Aug    25901 1025 4775    23891    27910
##  Sep    24986 1065 4775    22897    27075
##  Oct    22004 1044 4775    19957    24052
##  Nov    17908 1076 4775    15798    20018
##  Dec    11742 1073 4775     9637    13846
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, eat_out_to_help_out, wfh, rul
e_of_6_indoors, year, day
## Confidence level used: 0.95
```

The mean bike rents is lowest in December, with a value of 11742 95% CI [9637, 13846]; the mean bike rents is highest in July, with a value of 28806 95% CI [26676, 30937].

```
# Weekday

# Obtain the estimated mean value by using emmeans()
m.hires.day.emm <- emmeans(m.hires.time, ~day)

# Check the summary of the emmeans() and the mean values and confidence interval
summary(m.hires.day.emm)
```

```
##  day emmean   SE   df lower.CL upper.CL
##  Mon  20316 1047 4775    18264    22368
##  Tue  22502 1046 4775    20450    24553
##  Wed  22572 1047 4775    20520    24625
##  Thu  22608 1047 4775    20556    24660
##  Fri  21377 1046 4775    19326    23428
##  Sat  18606 1046 4775    16554    20657
##  Sun  16624 1046 4775    14573    18675
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, eat_out_to_help_out, wfh, rul
e_of_6_indoors, year, month
## Confidence level used: 0.95
```

The mean bike rents is lowest on Sumday, with a value of 16624 95% CI [14573, 18675]; the mean bike rents is highest on Thursday, with a value of 22608 95% CI [20556, 24660].

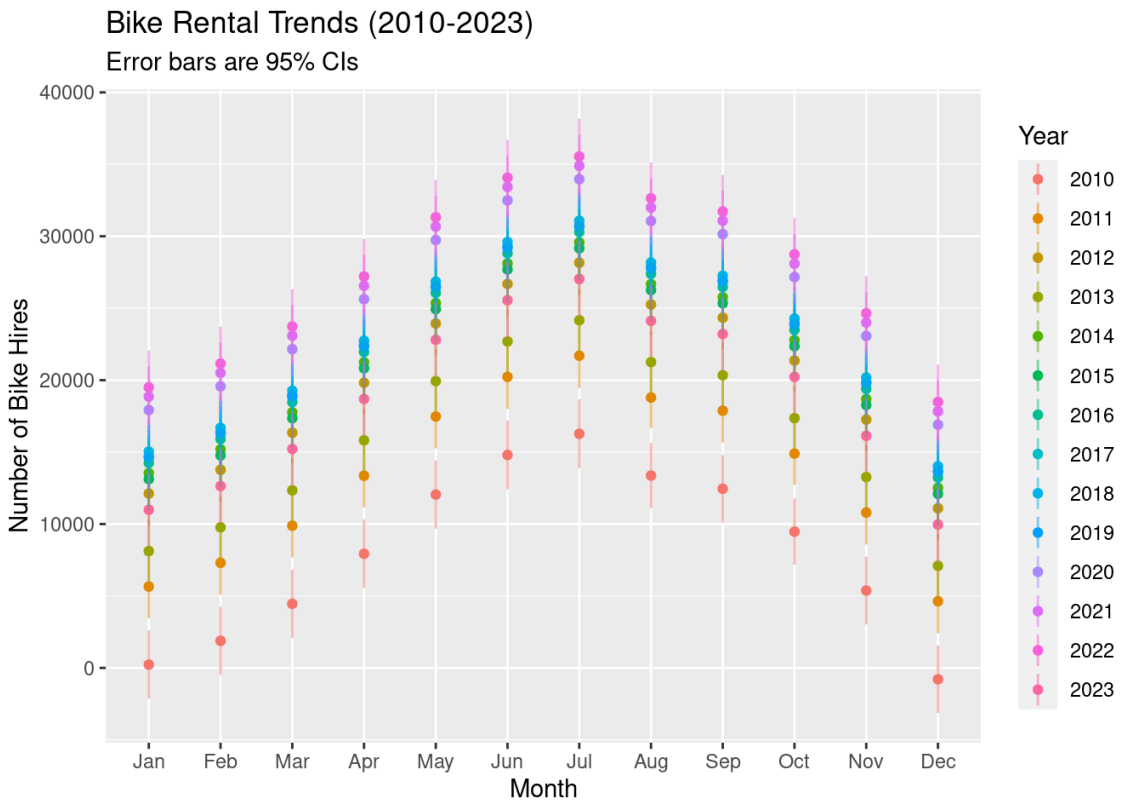
```
# Year+Month

# Obtain the estimated mean value by using emmeans()
m.hires.year.month.emm <- emmeans(m.hires.time, ~year+month)

# Check the summary of the emmeans() and the mean values and confidence interval of each combination
summary(m.hires.year.month.emm)
```

```
##  year month emmean   SE   df lower.CL upper.CL
##  2010 Jan      230 1207 4775    -2137     2597
##  2011 Jan     5655 1125 4775     3449     7860
##  2012 Jan    12119 1125 4775     9914    14324
##  2013 Jan     8118 1125 4775     5913    10324
##  2014 Jan    13538 1125 4775    11333    15744
##  2015 Jan    13122 1125 4775    10916    15327
##  2016 Jan    14254 1125 4775    12048    16459
##  2017 Jan    14711 1125 4775    12506    16917
##  2018 Jan    15034 1125 4775    12829    17239
##  2019 Jan    14637 1125 4775    12432    16843
##  2020 Jan    17923  953 4775    16055    19791
##  2021 Jan    18856 1071 4775    16758    20955
##  2022 Jan    19502 1301 4775    16952    22053
##  2023 Jan    10986 1312 4775     8414    13559
##  2010 Feb     1880 1210 4775     -492     4253
##  2011 Feb     7305 1128 4775     5093     9517
##  2012 Feb    13769 1128 4775    11558    15981
##  2013 Feb     9769 1128 4775     7557    11981
##  2014 Feb    15189 1128 4775    12977    17401
##  2015 Feb    14772 1128 4775    12560    16984
##  2016 Feb    15904 1128 4775    13693    18115
##  2017 Feb    16362 1128 4775    14150    18574
##  2018 Feb    16684 1128 4775    14472    18896
##  2019 Feb    16288 1128 4775    14076    18500
##  2020 Feb    19573  956 4775    17698    21448
##  2021 Feb    20507 1074 4775    18402    22612
##  2022 Feb    21153 1303 4775    18598    23707
##  2023 Feb    12637 1314 4775    10060    15213
##  2010 Mar     4457 1210 4775     2084     6830
##  2011 Mar     9882 1128 4775     7670    12094
##  2012 Mar    16346 1128 4775    14134    18558
##  2013 Mar    12345 1128 4775    10133    14558
##  2014 Mar    17766 1128 4775    15553    19978
##  2015 Mar    17349 1128 4775    15137    19561
##  2016 Mar    18481 1128 4775    16269    20693
##  2017 Mar    18938 1128 4775    16726    21151
##  2018 Mar    19261 1128 4775    17049    21473
##  2019 Mar    18864 1128 4775    16652    21077
##  2020 Mar    22150  963 4775    20261    24039
##  2021 Mar    23084 1080 4775    20967    25200
##  2022 Mar    23729 1316 4775    21150    26309
##  2023 Mar    15213 1327 4775    12613    17814
##  2010 Apr     7933 1209 4775     5563    10304
##  2011 Apr    13358 1126 4775    11150    15566
##  2012 Apr    19822 1126 4775    17614    22030
##  2013 Apr    15821 1126 4775    13613    18029
##  2014 Apr    21242 1126 4775    19034    23450
##  2015 Apr    20825 1126 4775    18617    23033
##  2016 Apr    21957 1126 4775    19749    24165
##  2017 Apr    22414 1126 4775    20206    24622
##  2018 Apr    22737 1126 4775    20529    24945
##  2019 Apr    22341 1126 4775    20133    24549
##  2020 Apr    25626  969 4775    23727    27525
##  2021 Apr    26560 1083 4775    24436    28683
##  2022 Apr    27206 1326 4775    24606    29805
##  2023 Apr    18690 1337 4775    16069    21310
##  2010 May    12047 1209 4775     9676    14417
##  2011 May    17471 1126 4775    15264    19679
##  2012 May    23936 1126 4775    21728    26143
##  2013 May    19935 1126 4775    17727    22142
##  2014 May    25355 1126 4775    23148    27562
##  2015 May    24938 1126 4775    22731    27146
##  2016 May    26070 1126 4775    23863    28277
##  2017 May    26528 1126 4775    24321    28735
```

```
##  2018 May    26851 1126 4775    24643    29058
##  2019 May    26454 1126 4775    24247    28661
##  2020 May    29739  968 4775    27842    31637
##  2021 May    30673 1085 4775    28547    32799
##  2022 May    31319 1329 4775    28714    33924
##  2023 May    22803 1339 4775    20177    25428
##  2010 Jun    14801 1219 4775    12412    17189
##  2011 Jun    20225 1136 4775    17998    22453
##  2012 Jun    26690 1136 4775    24462    28917
##  2013 Jun    22689 1136 4775    20461    24916
##  2014 Jun    28109 1136 4775    25881    30337
##  2015 Jun    27692 1136 4775    25465    29920
##  2016 Jun    28824 1136 4775    26597    31052
##  2017 Jun    29282 1136 4775    27054    31509
##  2018 Jun    29605 1136 4775    27377    31832
##  2019 Jun    29208 1136 4775    26980    31436
##  2020 Jun    32493  978 4775    30576    34410
##  2021 Jun    33427 1097 4775    31276    35578
##  2022 Jun    34073 1341 4775    31444    36702
##  2023 Jun    25557 1351 4775    22908    28206
##  2010 Jul    16273 1220 4775    13880    18666
##  2011 Jul    21698 1140 4775    19463    23933
##  2012 Jul    28162 1140 4775    25927    30397
##  2013 Jul    24161 1140 4775    21926    26396
##  2014 Jul    29581 1140 4775    27347    31816
##  2015 Jul    29165 1140 4775    26930    31400
##  2016 Jul    30297 1140 4775    28062    32531
##  2017 Jul    30754 1140 4775    28519    32989
##  2018 Jul    31077 1140 4775    28842    33312
##  2019 Jul    30680 1140 4775    28446    32915
##  2020 Jul    33966  977 4775    32050    35881
##  2021 Jul    34899 1095 4775    32753    37046
##  2022 Jul    35545 1338 4775    32923    38168
##  2023 Jul    27029 1349 4775    24386    29673
##  2010 Aug    13368 1155 4775    11104    15631
##  2011 Aug    18792 1088 4775    16659    20925
##  2012 Aug    25256 1088 4775    23124    27389
##  2013 Aug    21256 1088 4775    19123    23389
##  2014 Aug    26676 1088 4775    24543    28809
##  2015 Aug    26259 1088 4775    24126    28392
##  2016 Aug    27391 1088 4775    25258    29524
##  2017 Aug    27849 1088 4775    25716    29982
##  2018 Aug    28172 1088 4775    26039    30304
##  2019 Aug    27775 1088 4775    25642    29908
##  2020 Aug    31060  913 4775    29271    32850
##  2021 Aug    31994 1026 4775    29982    34006
##  2022 Aug    32640 1269 4775    30151    35129
##  2023 Aug    24124 1281 4775    21612    26636
##  2010 Sep    12452 1193 4775    10113    14792
##  2011 Sep    17877 1128 4775    15665    20089
##  2012 Sep    24341 1128 4775    22129    26553
##  2013 Sep    20341 1128 4775    18128    22553
##  2014 Sep    25761 1128 4775    23549    27973
##  2015 Sep    25344 1128 4775    23132    27556
##  2016 Sep    26476 1128 4775    24264    28688
##  2017 Sep    26934 1128 4775    24721    29146
##  2018 Sep    27256 1128 4775    25044    29469
##  2019 Sep    26860 1128 4775    24648    29072
##  2020 Sep    30145  948 4775    28287    32003
##  2021 Sep    31079 1067 4775    28986    33171
##  2022 Sep    31725 1296 4775    29185    34265
##  2023 Sep    23209 1307 4775    20646    25771
##  2010 Oct     9471 1170 4775     7177    11765
##  2011 Oct    14896 1105 4775    12729    17063
##  2012 Oct    21360 1105 4775    19193    23527
##  2013 Oct    17359 1105 4775    15192    19526
```

```
##  2014 Oct    22779 1105 4775    20612    24947
##  2015 Oct    22363 1105 4775    20196    24530
##  2016 Oct    23495 1105 4775    21328    25662
##  2017 Oct    23952 1105 4775    21785    26119
##  2018 Oct    24275 1105 4775    22108    26442
##  2019 Oct    23878 1105 4775    21711    26046
##  2020 Oct    27164  935 4775    25330    28997
##  2021 Oct    28097 1045 4775    26048    30147
##  2022 Oct    28743 1284 4775    26227    31260
##  2023 Oct    20227 1305 4775    17670    22785
##  2010 Nov     5375 1199 4775     3025     7725
##  2011 Nov    10800 1135 4775     8574    13025
##  2012 Nov    17264 1135 4775    15039    19489
##  2013 Nov    13263 1135 4775    11038    15488
##  2014 Nov    18683 1135 4775    16458    20909
##  2015 Nov    18267 1135 4775    16041    20492
##  2016 Nov    19399 1135 4775    17173    21624
##  2017 Nov    19856 1135 4775    17631    22082
##  2018 Nov    20179 1135 4775    17953    22404
##  2019 Nov    19782 1135 4775    17557    22008
##  2020 Nov    23068  965 4775    21176    24959
##  2021 Nov    24001 1080 4775    21884    26118
##  2022 Nov    24647 1313 4775    22072    27222
##  2023 Nov    16131 1333 4775    13517    18745
##  2010 Dec     -791 1193 4775    -3130     1547
##  2011 Dec     4634 1129 4775     2421     6846
##  2012 Dec    11098 1129 4775     8885    13310
##  2013 Dec     7097 1129 4775     4884     9310
##  2014 Dec    12517 1129 4775    10304    14730
##  2015 Dec    12100 1129 4775     9888    14313
##  2016 Dec    13232 1129 4775    11020    15445
##  2017 Dec    13690 1129 4775    11477    15903
##  2018 Dec    14013 1129 4775    11800    16225
##  2019 Dec    13616 1129 4775    11403    15829
##  2020 Dec    16901  968 4775    15003    18799
##  2021 Dec    17835 1078 4775    15722    19948
##  2022 Dec    18481 1324 4775    15885    21077
##  2023 Dec     9965 1344 4775     7330    12600
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, eat_out_to_help_out, wfh, rul
e_of_6_indoors, day
## Confidence level used: 0.95
```

```
# Visualize the data and review the trend of bike rents
ggplot(summary(m.hires.year.month.emm), aes(x=month, y=emmean, ymin=lower.CL, ymax=upper.CL, col=year)) + geom_
point() + geom_linerange(alpha=0.5) + labs(x="Month", y="Number of Bike Hires", col="Year", title="Bike Rental
Trends (2010-2023)",subtitle="Error bars are 95% CIs")
```



As we can see from this graph, bike rents are higher during the middle of the years and lower at the beginning and ending of the years. Take 2020 for example, the mean bike rents is 17923 with 95% CI[16055-19791] in January, and this figure increases all the way to 2020 July, with an average 33966 bike rents 95% CI[32050-35881]. Then, this figure decrease month by month to the end of the year, 2020 December, with an average bike rents of 16901 95% CI[15003-18799]. This trend can be seen in every year.

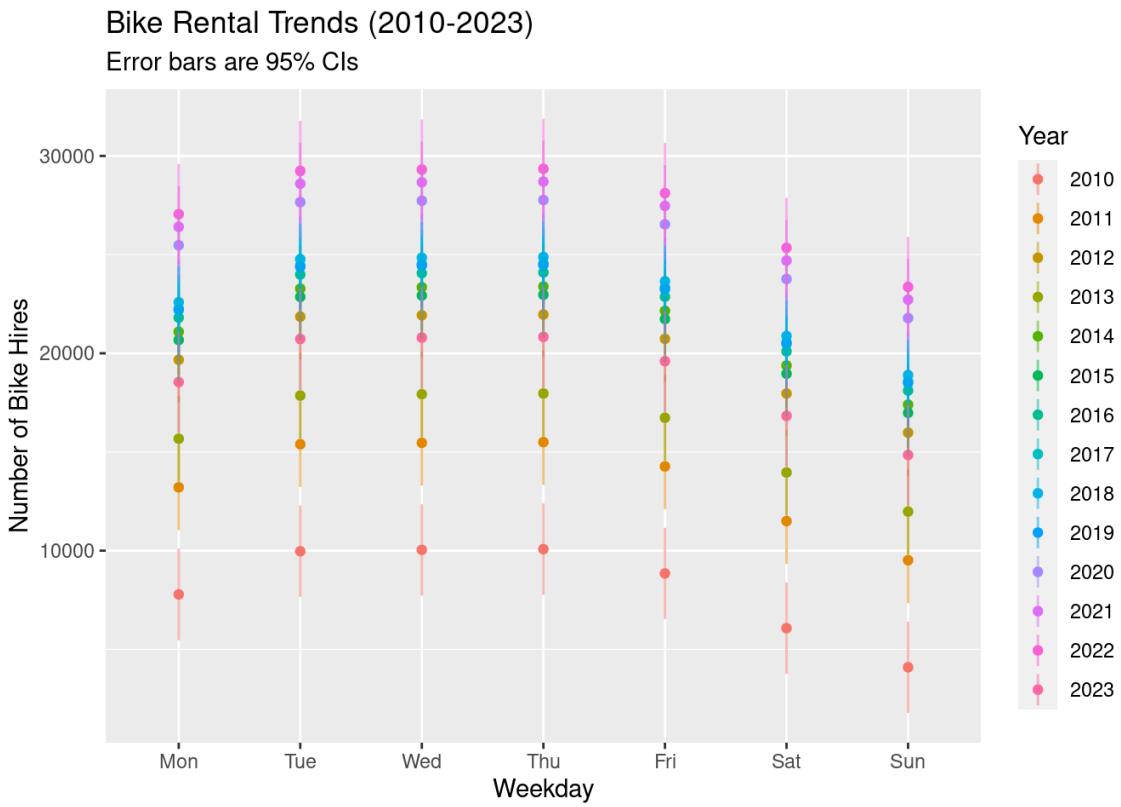
As for yearly trends, take January for example, the mean value increased from 2010 January(230 bike rents 95% CI[-2137-2597]) to 2012 January(12119 bike rents 95% CI[9914-14324]), then decreased in 2023 January(8118 bike rents with 95% CI[5913-10324]), increased in 2014 January and decreased again in 2015 January, then increased all the way to 2018 January, decreased a little bit in 2019 January and increased again and reached the peak at 2022 January(19502 bike rents 95% CI[16952-22053]). Such trend can be seen in every month throughout these years.

```
# Year+Weekday

# Obtain the estimated mean value by using emmeans()
m.hires.year.day.emm <- emmeans(m.hires.time, ~year+day)

# Check the summary of the emmeans() and the mean values and confidence interval of each combination
summary(m.hires.year.day.emm)
```

```
##  year day emmean   SE   df lower.CL upper.CL
##  2010 Mon   7783 1181 4775     5467    10099
##  2011 Mon  13208 1105 4775    11041    15374
##  2012 Mon  19672 1105 4775    17506    21838
##  2013 Mon  15671 1105 4775    13504    17838
##  2014 Mon  21091 1105 4775    18925    23258
##  2015 Mon  20675 1105 4775    18508    22841
##  2016 Mon  21807 1105 4775    19640    23973
##  2017 Mon  22264 1105 4775    20098    24431
##  2018 Mon  22587 1105 4775    20421    24753
##  2019 Mon  22190 1105 4775    20024    24357
##  2020 Mon  25476  935 4775    23644    27308
##  2021 Mon  26409 1053 4775    24346    28473
##  2022 Mon  27055 1295 4775    24517    29593
##  2023 Mon  18539 1308 4775    15975    21104
##  2010 Tue   9969 1181 4775     7653    12284
##  2011 Tue  15393 1105 4775    13228    17559
##  2012 Tue  21858 1104 4775    19692    24023
##  2013 Tue  17857 1104 4775    15692    20022
##  2014 Tue  23277 1105 4775    21111    25442
##  2015 Tue  22860 1105 4775    20695    25026
##  2016 Tue  23992 1104 4775    21827    26158
##  2017 Tue  24450 1105 4775    22284    26615
##  2018 Tue  24773 1105 4775    22607    26938
##  2019 Tue  24376 1104 4775    22211    26541
##  2020 Tue  27661  935 4775    25829    29494
##  2021 Tue  28595 1053 4775    26531    30659
##  2022 Tue  29241 1295 4775    26702    31780
##  2023 Tue  20725 1308 4775    18160    23290
##  2010 Wed  10039 1181 4775     7723    12355
##  2011 Wed  15464 1105 4775    13297    17630
##  2012 Wed  21928 1105 4775    19761    24094
##  2013 Wed  17927 1105 4775    15760    20094
##  2014 Wed  23347 1105 4775    21181    25514
##  2015 Wed  22931 1105 4775    20764    25097
##  2016 Wed  24063 1105 4775    21896    26229
##  2017 Wed  24520 1105 4775    22354    26687
##  2018 Wed  24843 1105 4775    22676    27009
##  2019 Wed  24446 1105 4775    22280    26613
##  2020 Wed  27732  935 4775    25899    29564
##  2021 Wed  28665 1053 4775    26601    30730
##  2022 Wed  29311 1296 4775    26771    31851
##  2023 Wed  20795 1309 4775    18229    23361
##  2010 Thu  10075 1181 4775     7759    12391
##  2011 Thu  15500 1105 4775    13334    17666
##  2012 Thu  21964 1105 4775    19798    24130
##  2013 Thu  17963 1105 4775    15797    20129
##  2014 Thu  23383 1105 4775    21217    25549
##  2015 Thu  22967 1105 4775    20801    25132
##  2016 Thu  24099 1105 4775    21933    26264
##  2017 Thu  24556 1105 4775    22390    26722
##  2018 Thu  24879 1105 4775    22713    27045
##  2019 Thu  24482 1105 4775    22316    26648
##  2020 Thu  27768  935 4775    25936    29600
##  2021 Thu  28701 1053 4775    26637    30765
##  2022 Thu  29347 1295 4775    26808    31886
##  2023 Thu  20831 1308 4775    18266    23396
##  2010 Fri   8844 1181 4775     6530    11158
##  2011 Fri  14269 1105 4775    12103    16434
##  2012 Fri  20733 1104 4775    18568    22898
##  2013 Fri  16732 1105 4775    14567    18898
##  2014 Fri  22152 1105 4775    19987    24318
##  2015 Fri  21736 1105 4775    19570    23901
##  2016 Fri  22868 1104 4775    20703    25032
##  2017 Fri  23325 1105 4775    21160    25491
```

```
##  2018 Fri  23648 1105 4775    21483    25813
##  2019 Fri  23251 1105 4775    21086    25417
##  2020 Fri  26537  934 4775    24705    28368
##  2021 Fri  27470 1052 4775    25407    29533
##  2022 Fri  28116 1295 4775    25578    30654
##  2023 Fri  19600 1308 4775    17036    22165
##  2010 Sat   6072 1181 4775     3757     8388
##  2011 Sat  11497 1105 4775     9332    13663
##  2012 Sat  17961 1105 4775    15796    20127
##  2013 Sat  13961 1105 4775    11795    16126
##  2014 Sat  19381 1105 4775    17215    21547
##  2015 Sat  18964 1105 4775    16798    21130
##  2016 Sat  20096 1104 4775    17931    22261
##  2017 Sat  20554 1105 4775    18388    22720
##  2018 Sat  20876 1105 4775    18711    23042
##  2019 Sat  20480 1105 4775    18314    22646
##  2020 Sat  23765  934 4775    21934    25597
##  2021 Sat  24699 1052 4775    22636    26762
##  2022 Sat  25345 1294 4775    22807    27882
##  2023 Sat  16829 1308 4775    14265    19393
##  2010 Sun   4091 1181 4775     1776     6406
##  2011 Sun   9516 1105 4775     7350    11681
##  2012 Sun  15980 1104 4775    13815    18144
##  2013 Sun  11979 1105 4775     9813    14144
##  2014 Sun  17399 1105 4775    15234    19564
##  2015 Sun  16982 1105 4775    14817    19148
##  2016 Sun  18114 1104 4775    15949    20280
##  2017 Sun  18572 1104 4775    16407    20737
##  2018 Sun  18895 1105 4775    16729    21060
##  2019 Sun  18498 1105 4775    16333    20663
##  2020 Sun  21783  934 4775    19952    23614
##  2021 Sun  22717 1052 4775    20654    24780
##  2022 Sun  23363 1294 4775    20826    25900
##  2023 Sun  14847 1307 4775    12284    17410
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, eat_out_to_help_out, wfh, rul
e_of_6_indoors, month
## Confidence level used: 0.95
```

```
# Visualize the data and review the trend of bike rents
ggplot(summary(m.hires.year.day.emm), aes(x=day, y=emmean, ymin=lower.CL, ymax=upper.CL, col=year)) + geom_poin
t() + geom_linerange(alpha=0.5) + labs(x="Weekday", y="Number of Bike Hires", col="Year", title="Bike Rental Tr
ends (2010-2023)",subtitle="Error bars are 95% CIs")
```



As we can see from this graph, bike rents are lower on weekends, which also reflects the negative coefficients of Saturday and Sunday in our regression model. Take 2023 for example, the mean bike rents is 18539 with 95% CI[15975-21104] on Monday, and this figure increases all the way to Thursday, with an average 20831 bike rents 95% CI[18266-23396]. Then, this figure decreases to the end of the week, Sunday, with an average bike rents of 14847 95% CI[12284-17410]. This trend can be seen in every year.

We then move on to assess the effect of the three policies we are interested in with and without time variables added:

```
# Working from home
# Obtain the estimated mean value by using emmeans()
m.hires.wfh.time.emm <- emmeans(m.hires.time, ~wfh)

# Check the summary of the emmeans() and the mean values
summary(m.hires.wfh.time.emm)
```

```
##  wfh emmean   SE   df lower.CL upper.CL
##    0  21604 1120 4775    19409    23800
##    1  19711 1022 4775    17707    21716
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, eat_out_to_help_out, rule_of_
6_indoors, year, month, day
## Confidence level used: 0.95
```

The mean bike rents when wfh equals to 0 (not practiced) is 21604 with 95% CI [19409-23800]; the mean bike rents when wfh equals to 1 (practiced) is 19711 with 95% CI [17707-21716], which is a decrease compare to the former situation.

```
# Rule of 6 Indoors
# Obtain the estimated mean value by using emmeans()
m.hires.ro6.time.emm <- emmeans(m.hires.time, ~rule_of_6_indoors)

# Check the summary of the emmeans() and the mean values
summary(m.hires.ro6.time.emm)
```

```
##  rule_of_6_indoors emmean   SE   df lower.CL upper.CL
##                  0  20516  966 4775    18623    22409
##                  1  20800 1214 4775    18419    23180
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, eat_out_to_help_out, wfh, yea
r, month, day
## Confidence level used: 0.95
```

The mean bike rents when rule_of_6_indoors equals to 0 (not practiced) is 20516 with 95% CI [18623-22409]; the mean bike rents when rule_of_6_indoors equals to 1 (practiced) is 20800 with 95% CI [18419-23180], which is a slight increase compare to the former situation.

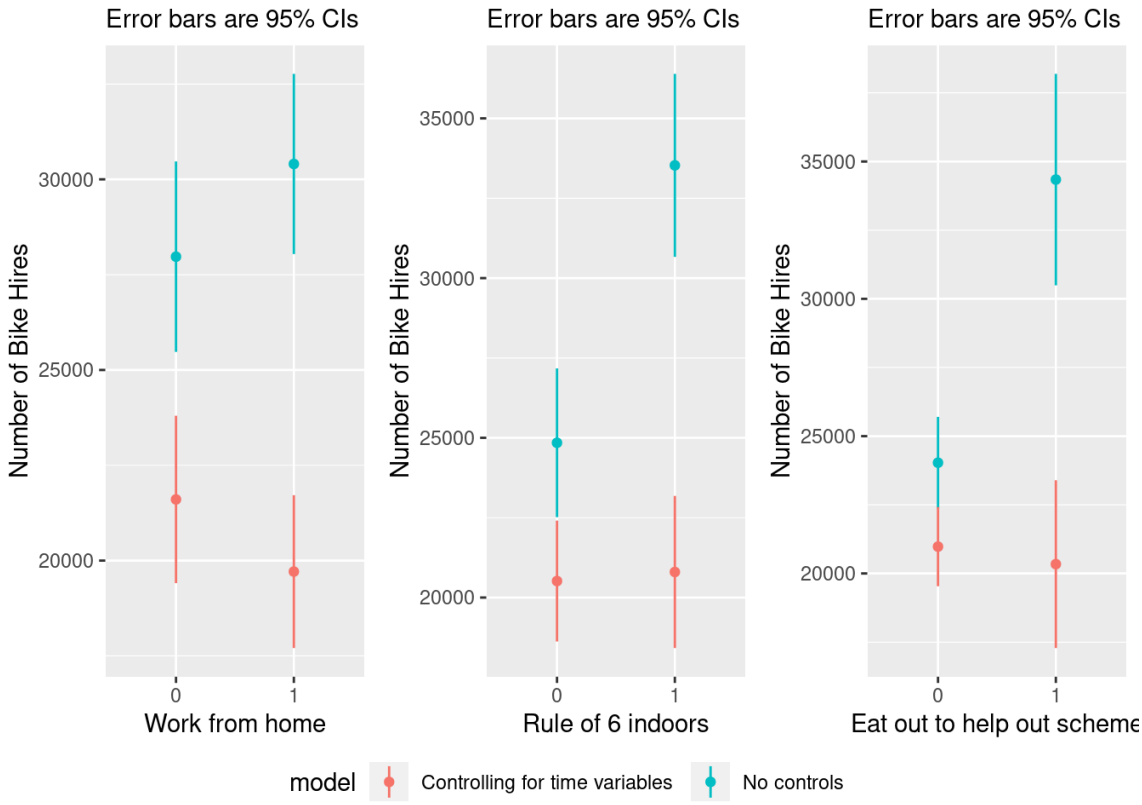
```
# Eat out to help out scheme
# Obtain the estimated mean value by using emmeans()
m.hires.eat.time.emm <- emmeans(m.hires.time, ~eat_out_to_help_out)

# Check the summary of the emmeans() and the mean values
summary(m.hires.eat.time.emm)
```

```
##  eat_out_to_help_out emmean   SE   df lower.CL upper.CL
##                    0  20976  735 4775    19536    22417
##                    1  20339 1558 4775    17285    23394
##
## Results are averaged over the levels of: schools_closed, stay_at_home, curfew, wfh, rule_of_6_indoors, year,
month, day
## Confidence level used: 0.95
```

The mean bike rents when eat_out_to_help_out equals to 0 (not practiced) is 20976 with 95% CI [19536-22417]; the mean bike rents when rule_of_6_indoors equals to 1 (practiced) is 20339 with 95% CI [17285-23394], which is a slight decrease compare to the former situation.

We can compare the effect of the three policies with and without time variables added by using the visualization below:

```
# Combine two emmeans (with and without time variables) for each of the three variables
both.wfh.emms <- bind_rows(list(data.frame(m.hires.wfh.emm, model="No controls"), data.frame(m.hires.wfh.time.e
mm, model="Controlling for time variables")))

both.wfh.emms$wfh <- as.factor(both.wfh.emms$wfh)

both.ro6.emms <- bind_rows(list(data.frame(m.hires.ro6.emm, model="No controls"), data.frame(m.hires.ro6.time.e
mm, model="Controlling for time variables")))

both.ro6.emms$rule_of_6_indoors <- as.factor(both.ro6.emms$rule_of_6_indoors)

both.eat.emms <- bind_rows(list(data.frame(m.hires.eat.emm, model="No controls"), data.frame(m.hires.eat.time.e
mm, model="Controlling for time variables")))

both.eat.emms$eat_out_to_help_out <- as.factor(both.eat.emms$eat_out_to_help_out)


# Plot them together and compare the mean bike rents
ggarrange(ggplot(both.wfh.emms, aes(x=wfh, y=emmean, ymin=lower.CL, ymax=upper.CL, col=model)) + geom_point() +
geom_linerange() + labs(x="Work from home", y="Number of Bike Hires", subtitle="Error bars are 95% CIs")  + sca
le_y_continuous(breaks = seq(15000, 40000, 5000)),
  ggplot(both.ro6.emms, aes(x=rule_of_6_indoors, y=emmean, ymin=lower.CL, ymax=upper.CL, col=model)) + geom_poi
nt() + geom_linerange() + labs(x="Rule of 6 indoors", y="Number of Bike Hires", subtitle="Error bars are 95% CI
s")+ scale_y_continuous(breaks = seq(15000, 40000, 5000)),
  ggplot(both.eat.emms, aes(x=eat_out_to_help_out, y=emmean, ymin=lower.CL, ymax=upper.CL, col=model)) + geom_p
oint() + geom_linerange() + labs(x="Eat out to help out scheme", y="Number of Bike Hires", subtitle="Error bars
are 95% CIs")+ scale_y_continuous(breaks = seq(15000, 40000, 5000)), ncol=3, nrow=1, common.legend = TRUE, lege
nd="bottom")
```



As we can see from this chart, after controlling time variables, the mean bike rents after the implementation of these policies are closer to the mean value before implementation, which is more reasonable and meaningful in terms of analyzing the effect of these variables. For example, with time variables controlled, the implementation of working from home will decrease the number of bike rents.

Finally, we can use a two-way ANOVA test to check if the regression model is significantly improved after introducing time variables.

```
anova(m.hires, m.hires.time)
```

```
## Analysis of Variance Table
##
## Model 1: Hires ~ schools_closed + eating_places_closed + stay_at_home +
##     curfew + eat_out_to_help_out + wfh + rule_of_6_indoors
## Model 2: Hires ~ schools_closed + stay_at_home + curfew + eat_out_to_help_out +
##     wfh + rule_of_6_indoors + year + month + day
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1   4804 4.2114e+11
## 2   4775 1.8851e+11 29 2.3262e+11 203.18 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The ANOVA result shows that after adding time variables, the overall model fit is significantly improved ($F(29,4775) = 203.18$, $p < 0.001$).