CS-E5740 Complex Networks, Answers to exercise set 6

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Problem 1

a)

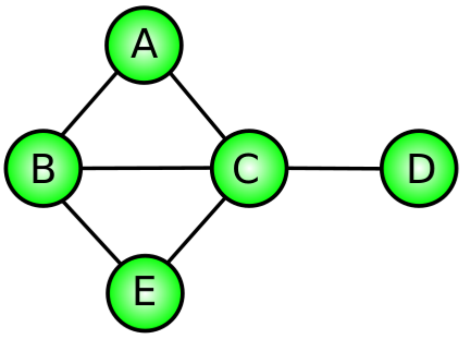

Figure 1: A small undirected network.

i) Betweenness centrality of node **C**.

$$bc(i) = \frac{1}{(N-1)(N-2)} \sum_{s \neq i} \sum_{t \neq i} \frac{\sigma_{sit}}{\sigma_{st}}$$

$$\sigma_{\mathbf{ACB}} = 0; \quad \sigma_{\mathbf{AB}} = 1; \qquad \sigma_{\mathbf{BCA}} = 0; \quad \sigma_{\mathbf{BA}} = 1$$

$$\sigma_{\mathbf{ACD}} = 1; \quad \sigma_{\mathbf{AD}} = 1; \qquad \sigma_{\mathbf{BCD}} = 1; \quad \sigma_{\mathbf{BD}} = 1$$

$$\sigma_{\mathbf{ACE}} = 1; \quad \sigma_{\mathbf{AE}} = 2; \qquad \sigma_{\mathbf{BCE}} = 0; \quad \sigma_{\mathbf{BE}} = 1$$

$$\sigma_{\mathbf{DCB}} = 1; \quad \sigma_{\mathbf{DB}} = 1; \qquad \sigma_{\mathbf{ECB}} = 0; \quad \sigma_{\mathbf{EB}} = 1$$

$$\sigma_{\mathbf{DCA}} = 1; \quad \sigma_{\mathbf{DA}} = 1; \qquad \sigma_{\mathbf{ECD}} = 1; \quad \sigma_{\mathbf{ED}} = 1$$

$$\sigma_{\mathbf{DCE}} = 1; \quad \sigma_{\mathbf{DE}} = 1; \qquad \sigma_{\mathbf{ECA}} = 1; \quad \sigma_{\mathbf{EA}} = 2$$

$$bc(\mathbf{C}) = \frac{1}{4\cdot 3}\sum_{s\neq \mathbf{C}}\sum_{t\neq \mathbf{C}}\frac{\sigma_{s\mathbf{C}t}}{\sigma_{st}}$$

$$= \frac{1}{12}\left(\frac{\sigma_{\mathbf{ACB}}}{\sigma_{\mathbf{AB}}} + \frac{\sigma_{\mathbf{ACD}}}{\sigma_{\mathbf{AD}}} + \frac{\sigma_{\mathbf{ACE}}}{\sigma_{\mathbf{AE}}} + \frac{\sigma_{\mathbf{BCA}}}{\sigma_{\mathbf{BA}}} + \frac{\sigma_{\mathbf{BCD}}}{\sigma_{\mathbf{BD}}} + \frac{\sigma_{\mathbf{BCE}}}{\sigma_{\mathbf{BE}}} + \right.$$

$$\left. + \frac{\sigma_{\mathbf{DCB}}}{\sigma_{\mathbf{DB}}} + \frac{\sigma_{\mathbf{DCA}}}{\sigma_{\mathbf{DA}}} + \frac{\sigma_{\mathbf{DCE}}}{\sigma_{\mathbf{DE}}} + \frac{\sigma_{\mathbf{ECB}}}{\sigma_{\mathbf{EB}}} + \frac{\sigma_{\mathbf{ECD}}}{\sigma_{\mathbf{ED}}} + \frac{\sigma_{\mathbf{ECA}}}{\sigma_{\mathbf{EA}}}\right) =$$

$$= \frac{1}{12}\left(\frac{0}{1} + \frac{1}{1} + \frac{1}{2} + \frac{0}{1} + \frac{1}{1} + \frac{0}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{0}{1} + \frac{1}{1} + \frac{1}{2}\right) = \frac{7}{12} = 0.5833$$

ii) Closeness centrality of node **C**.

$$C(i) = \frac{N-1}{\sum_{v\neq i} d(i,v)}$$

$$d(\mathbf{C},\mathbf{A}) = 1;\quad d(\mathbf{C},\mathbf{B}) = 1;\quad d(\mathbf{C},\mathbf{D}) = 1;\quad d(\mathbf{C},\mathbf{E}) = 1$$

$$C(\mathbf{C}) = \frac{4}{\sum_{v\neq \mathbf{C}} d(\mathbf{C},v)} = \frac{4}{d(\mathbf{C},\mathbf{A}) + d(\mathbf{C},\mathbf{B}) + d(\mathbf{C},\mathbf{D}) + d(\mathbf{C},\mathbf{E})} =$$

$$= \frac{4}{1+1+1+1} = \frac{4}{4} = 1$$

iii) K-shell centrality of all nodes.

1-core = $\{A, B, C, D, E\}$
2-core = $\{A, B, C, E\}$
3-core = $\{\}$

1-shell = $\{D\}$
2-shell = $\{A, B, C, E\}$

b) See Figures below (2-**??**).

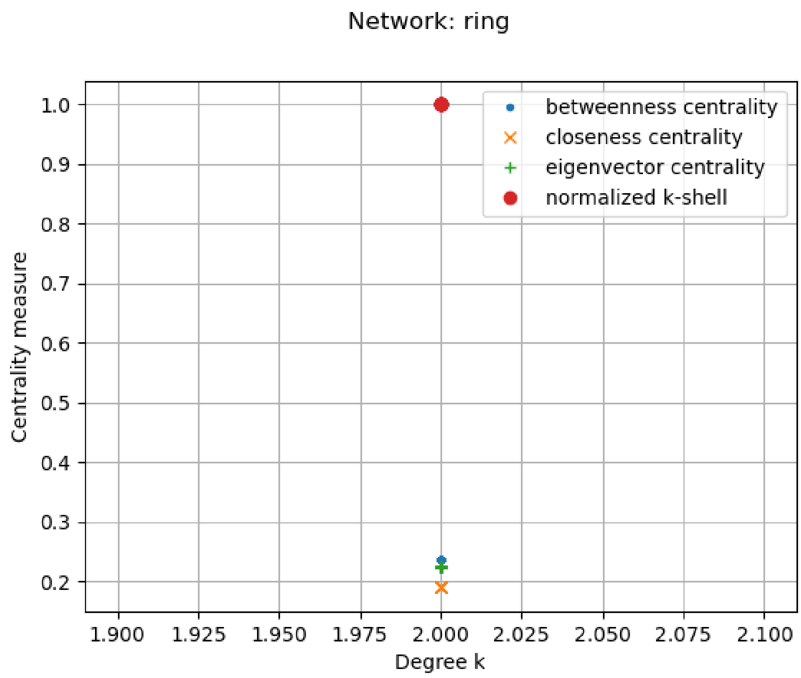

Figure 2: Scatter plot of the centrality measures of the ring network.

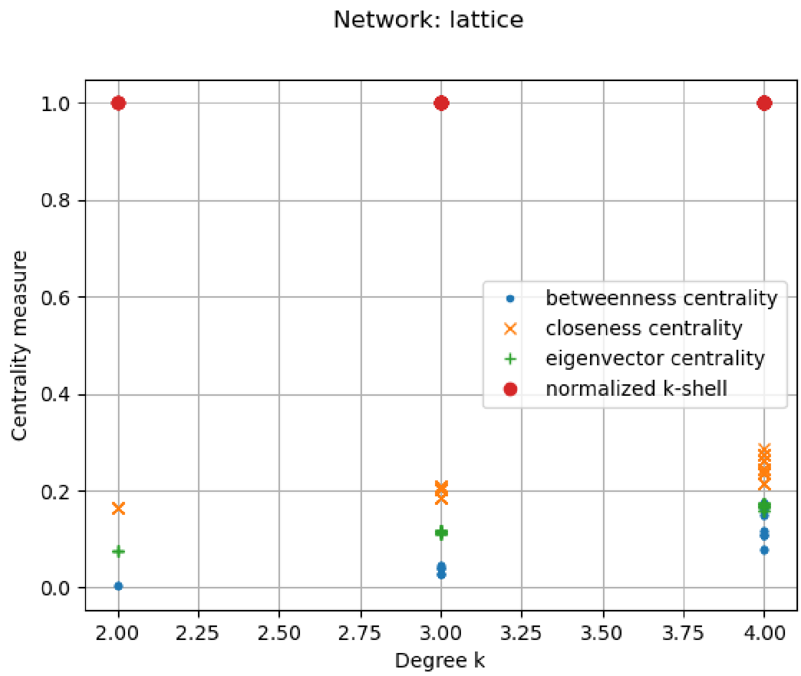

Figure 3: Scatter plot of the centrality measures of the lattice network.

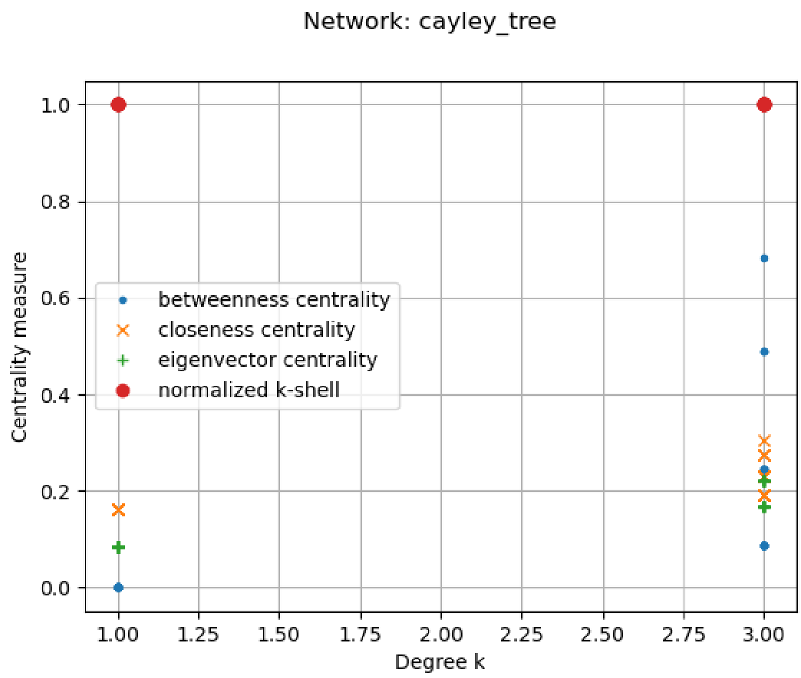

Figure 4: Scatter plot of the centrality measures of the cayley tree network.

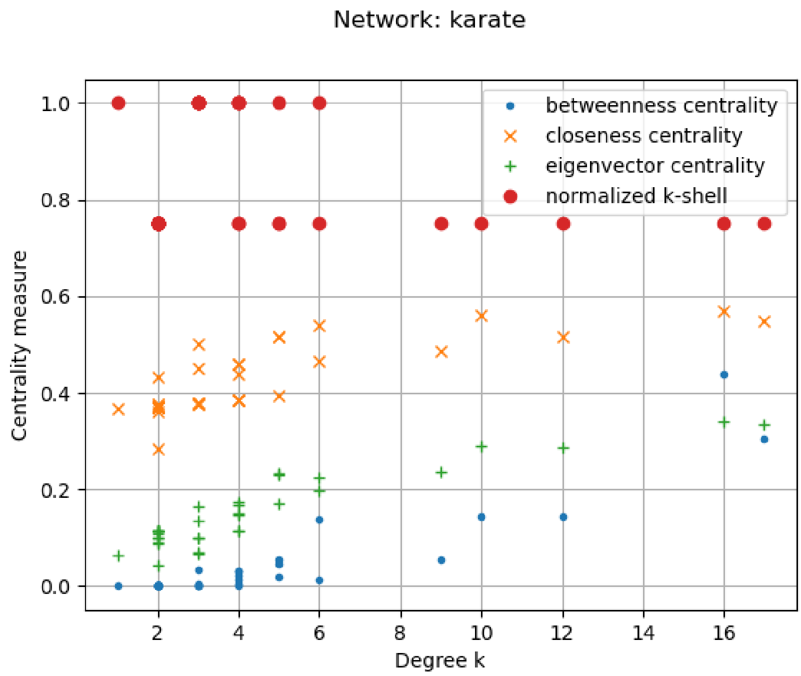

Figure 5: Scatter plot of the centrality measures of the karate network.

c) See Figures below (6-9).

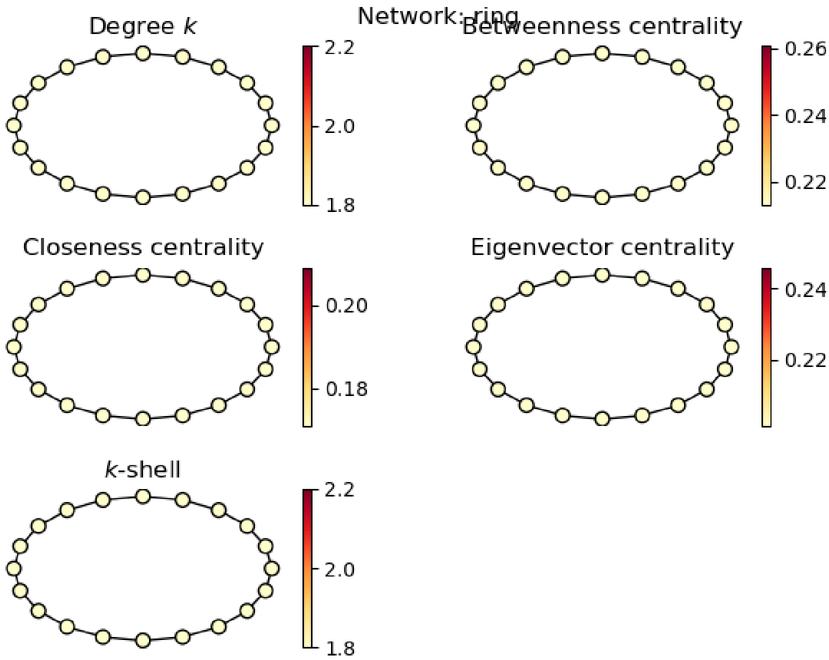

Figure 6: Visualization of the centrality measures of the ring network.

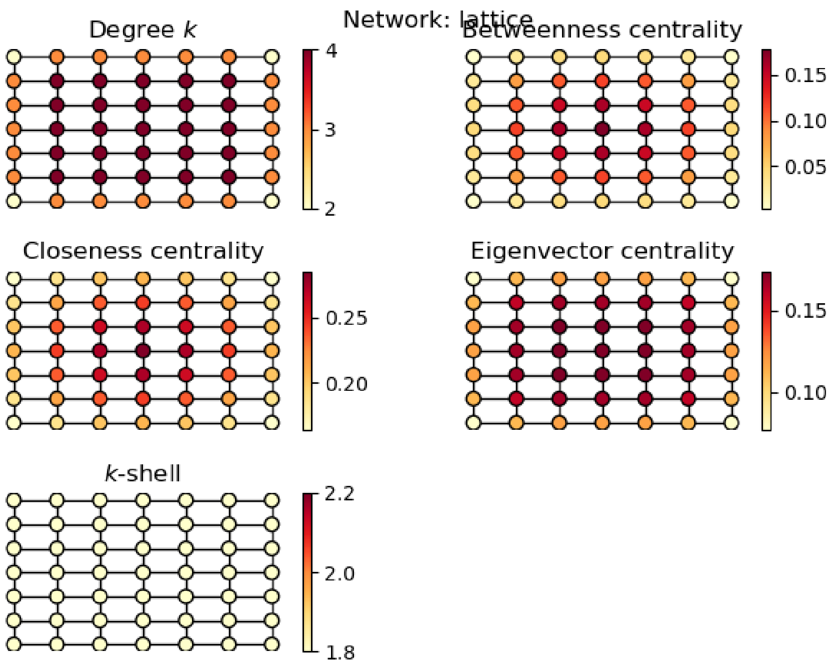

Figure 7: Visualization of the centrality measures of the lattice network.

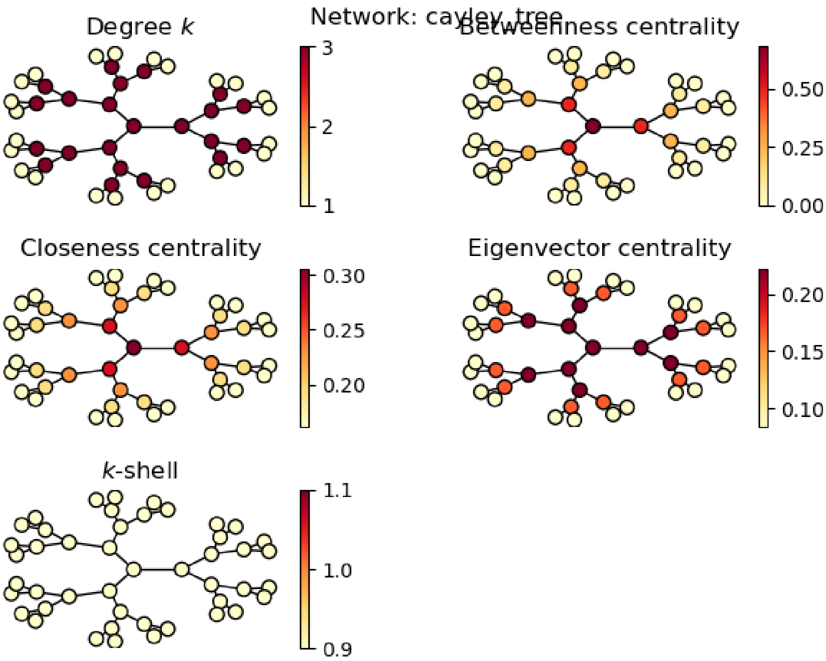

Figure 8: Visualization of the centrality measures of the cayley tree network.

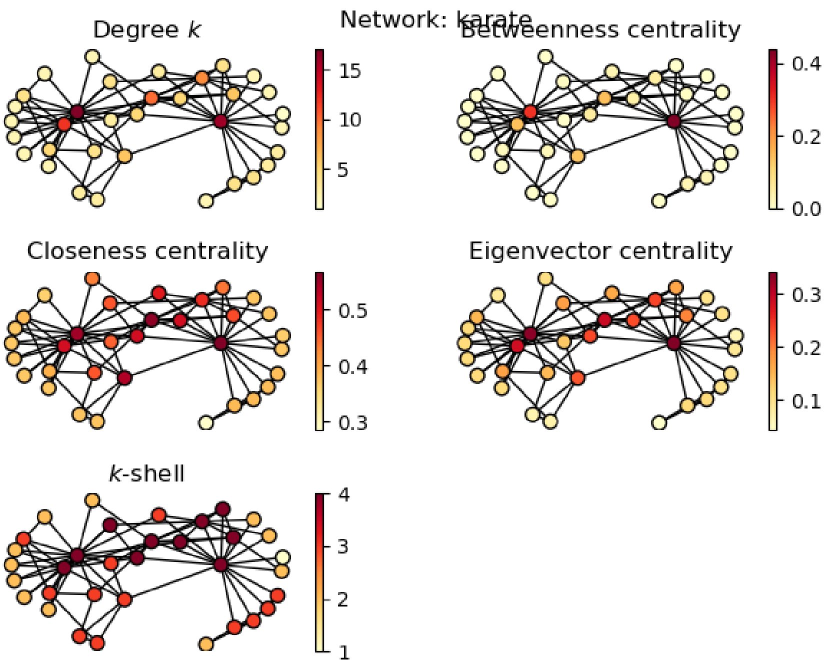

Figure 9: Visualization of the centrality measures of the karate network.

d) Because is near to nodes with very high degree so that it has very short distance to a lot of nodes.

Problem 2

Not done.

Problem 3

a)

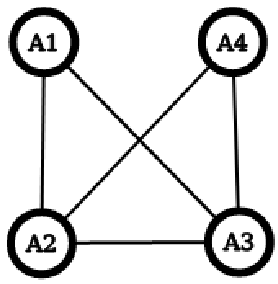

Figure 10: Unipartite projection of actors.

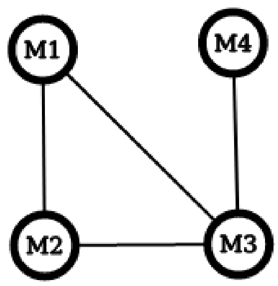

Figure 11: Unipartite projection of movies.

b) Counter example:

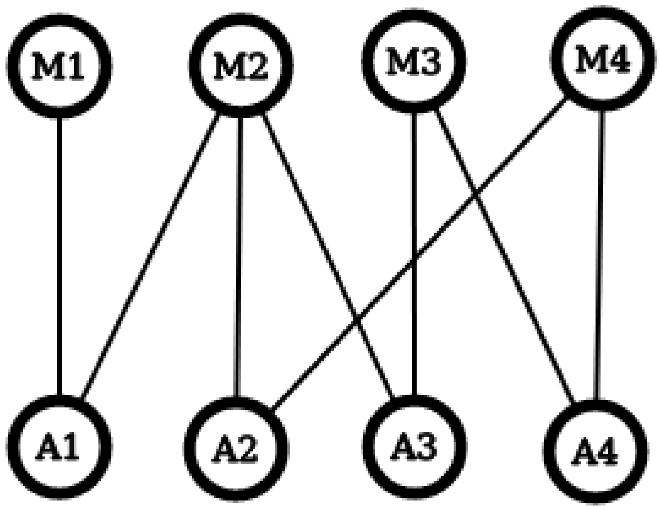

Figure 12: Bipartite network that can't be uniquely reconstructed from its two unipartite projections.