CS-E5740 Complex Networks, Answers to exercise set 4

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Problem 1

a)

$$n_d = \langle q \rangle \, n_{d-1}$$
$$n_{d-1} = \langle q \rangle \, n_{d-2}$$
$$n_{d-2} = \langle q \rangle \, n_{d-3}$$
$$\dots$$
$$n_0 = 1$$

So we obtain that

$$n_d = \langle q \rangle \, n_{d-1} = \langle q \rangle^d$$

In the other hand we know that $\langle q \rangle = \frac{\langle k^2 \rangle}{\langle k \rangle} - 1$, and that the mean and the variance are equal ($\langle k \rangle = \langle k^2 \rangle - \langle k \rangle^2$) since the degree distribution of an ER network is a Poisson distribution when $N \to \infty$ and $\langle k \rangle$ is constant.

So we have that,

$$\left\langle k^2 \right\rangle = \langle k \rangle^2 + \langle k \rangle \, ,$$

$$\langle q \rangle = \frac{\langle k^2 \rangle}{\langle k \rangle} - 1 = \frac{\langle k \rangle^2 + \langle k \rangle}{\langle k \rangle} - 1 = \langle k \rangle + 1 - 1 = \langle k \rangle \, .$$

Hence,

$$n_d = \langle k \rangle \, n_{d-1} = \langle k \rangle^d \, .$$

Now we can easily observe that a giant component appears when $\langle k \rangle > 1$.

b) In the first plot of the following figures (1-6) we can observe the results of this section.

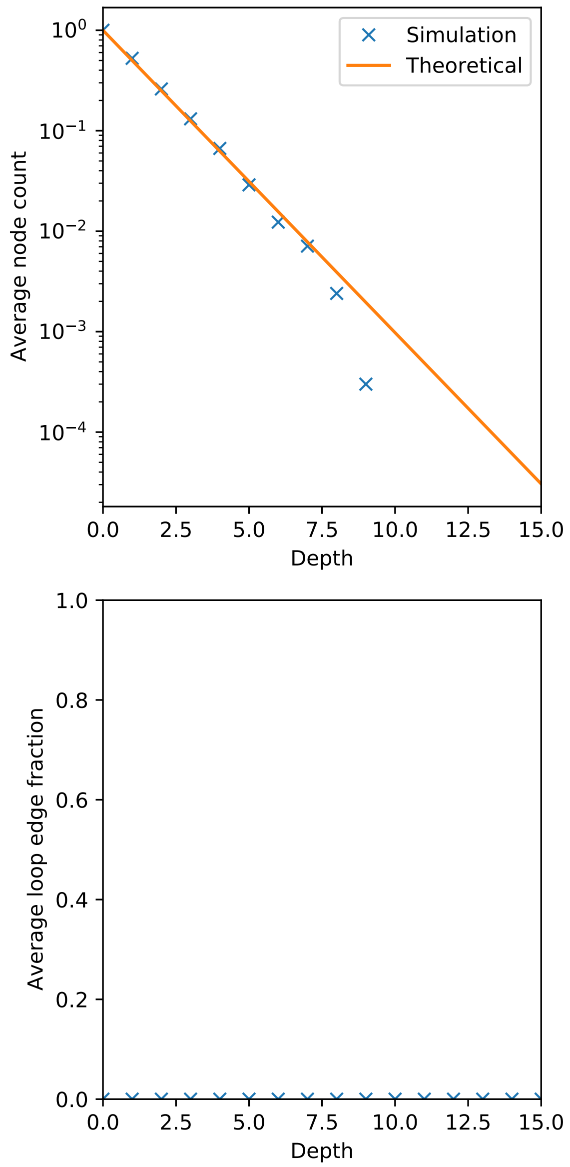

Figure 1: Average degree = 0.5; Net size = 10k

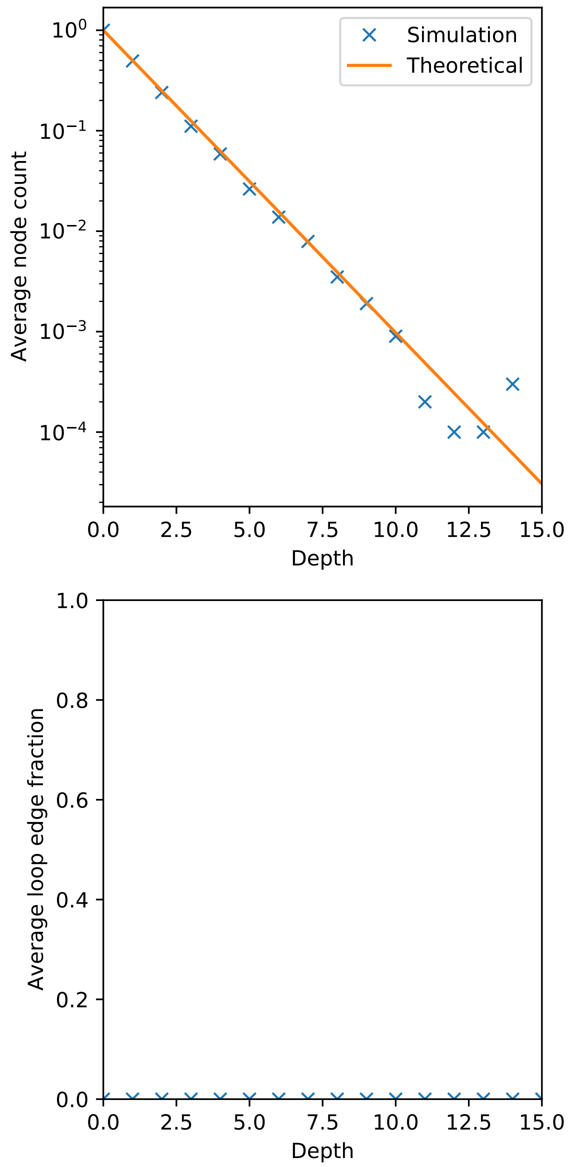

Figure 2: Average degree = 0.5; Net size = 100k

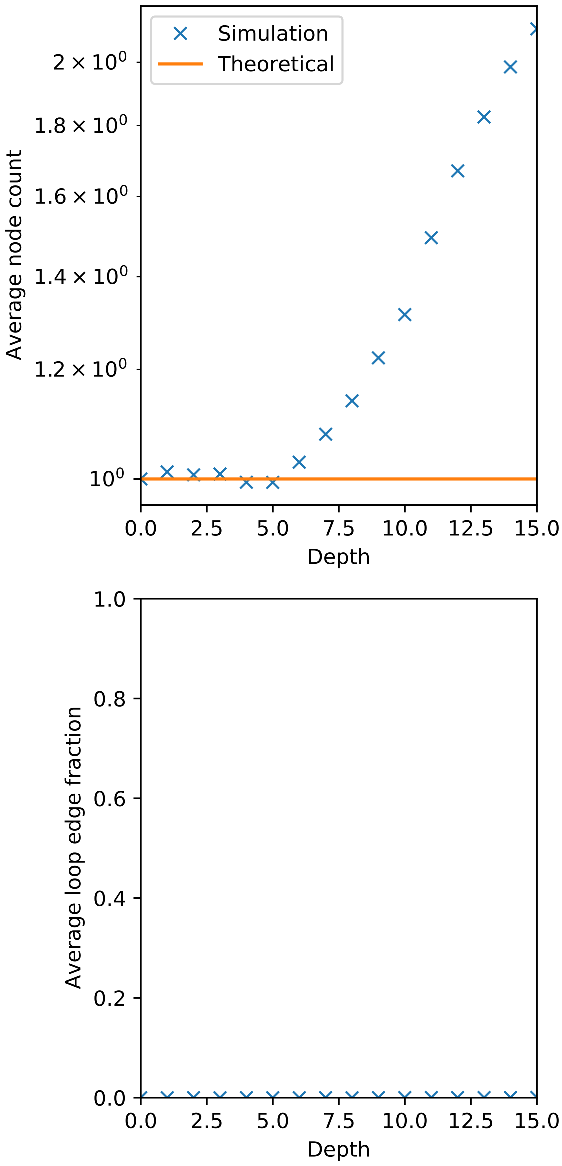

Figure 3: Average degree = 1; Net size = 10k

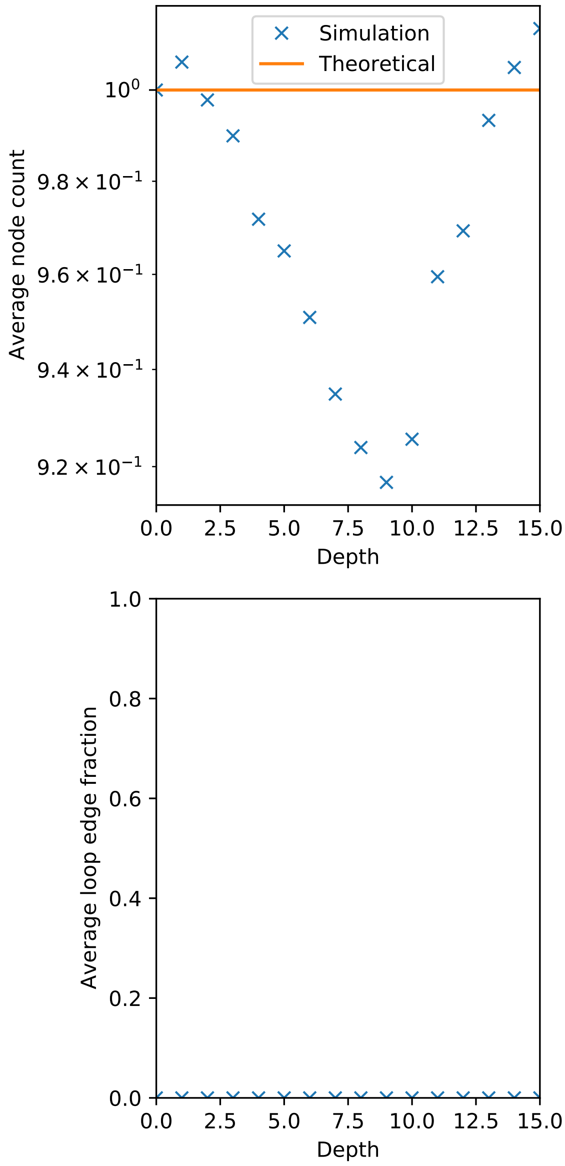

Figure 4: Average degree = 1; Net size = 100k

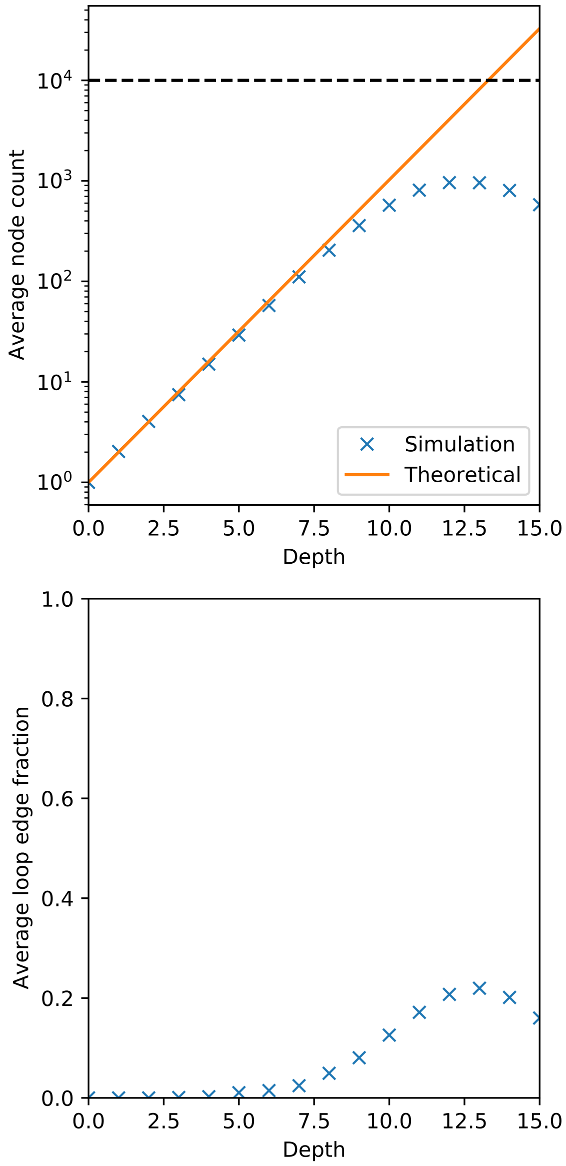

Figure 5: Average degree = 2; Net size = 10k

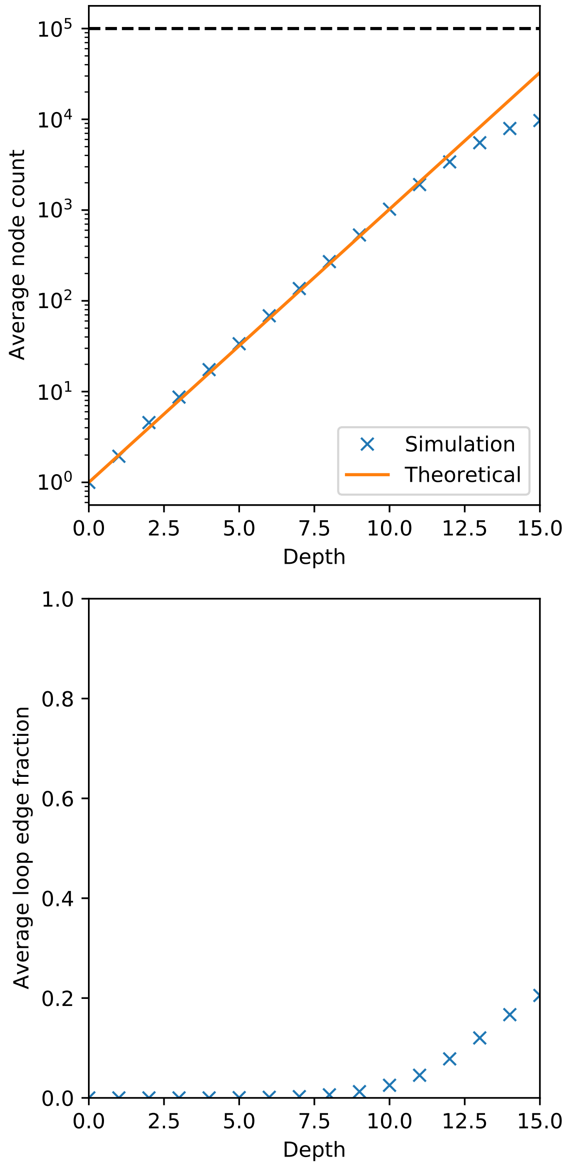

Figure 6: Average degree = 2; Net size = 100k

c) The results of this section are represented in the second plot of figures above (1-6).

d) The results of this section are shown in Figure 7.

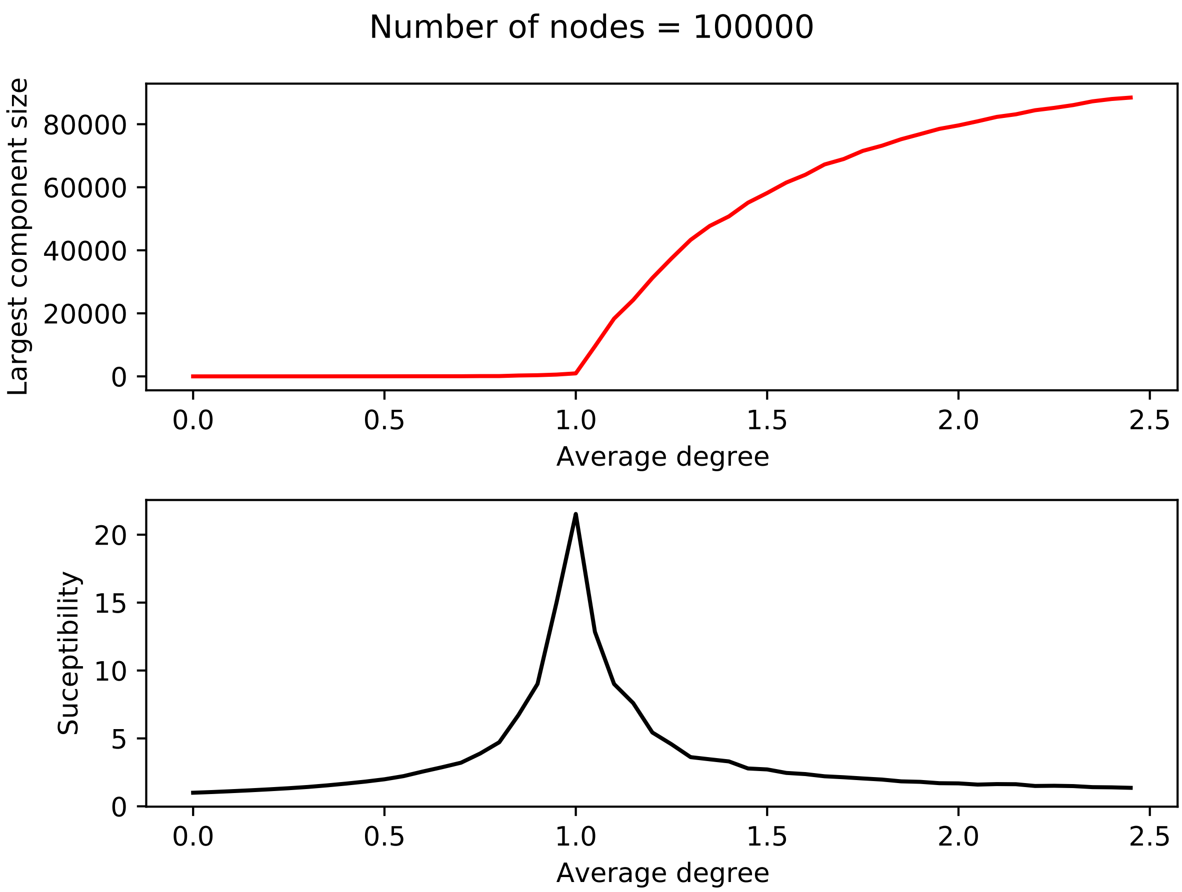

Figure 7: Average fraction of edges in function of d

We can easily observe how for $\langle k \rangle < 1$ the largest connected component is small (size being measured as number of nodes), and for $\langle k \rangle > 1$ it quickly reaches the network size.

e) Susceptibility is a large point where percolation transition happens, hence, when minor changes of the value of k will lead to big changes in the size of the largest component.

Also we can observe how when $k \approx 1$ there is a sudden change in both plot lines because it's when the percolation happens.

Problem 2

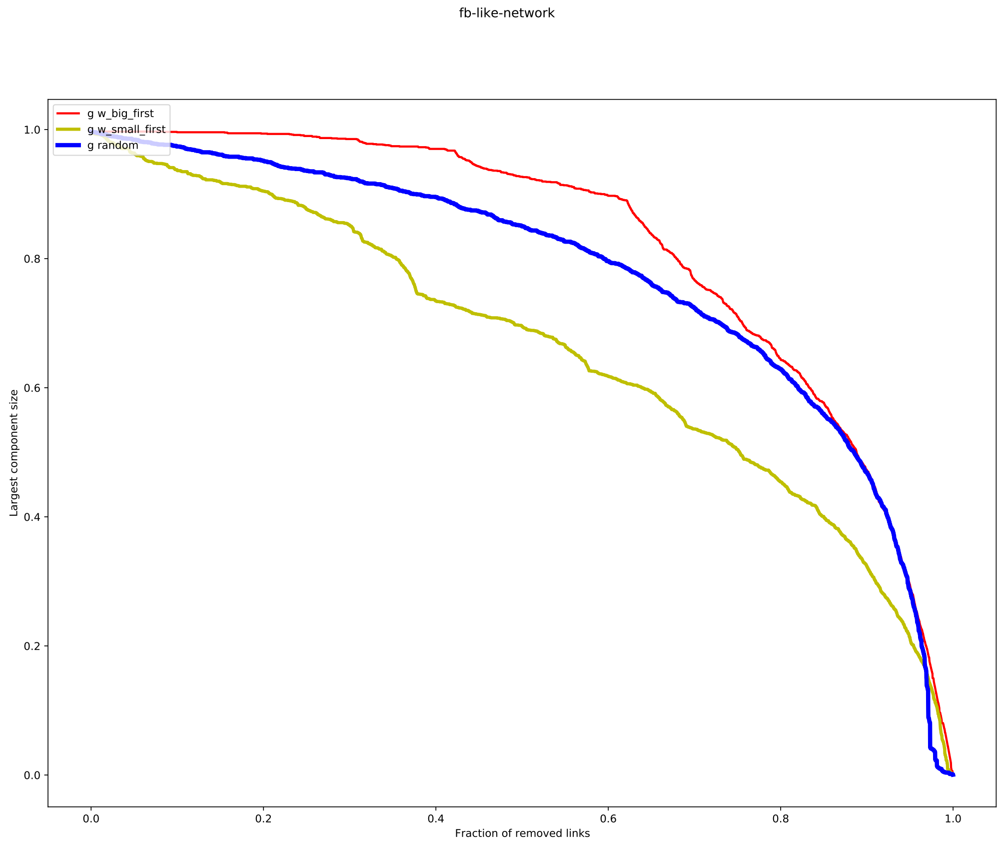

Figure 8: Caption

As we can observe in Figure 8, the small first (ascending link weight) is the most vulnerable. And in the other hand the big first (descending link weight) is the least vulnerable.