CS-E5740 Complex Networks,
Answers to exercise set 2

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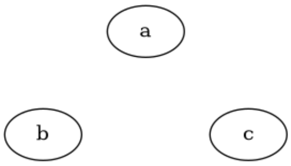
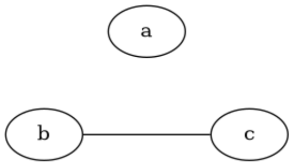
Problem 1

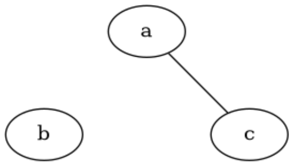
$$G(N=3,p)$$



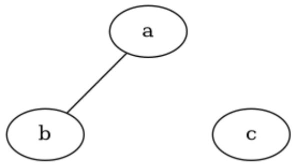
(G_1)
$\pi_1 = (1-p)^3$
$\mathrm{k}(\mathrm{G}_1) = 0$
$\mathrm{c}(\mathrm{G}_1) = 0$
$\mathrm{d}^*(G_1) = 0$



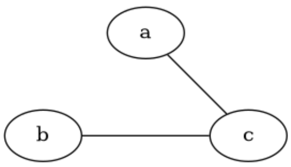
(G_2)
$\pi_2 = p(1-p)^2$
$\mathrm{k}(\mathrm{G}_2) = \frac{2}{3}$
$\mathrm{c}(\mathrm{G}_2) = 0$
$\mathrm{d}^*(G_2) = 1$



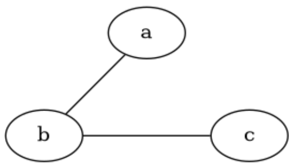
(G_3)
$\pi_3 = p(1-p)^2$
$\mathrm{k}(\mathrm{G}_3) = \frac{2}{3}$
$\mathrm{c}(\mathrm{G}_3) = 0$
$\mathrm{d}^*(G_3) = 1$



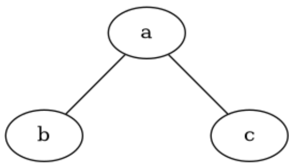
(G_4)
$\pi_4 = p(1-p)^2$
$\mathrm{k}(\mathrm{G}_4) = \frac{2}{3}$
$\mathrm{c}(\mathrm{G}_4) = 0$
$\mathrm{d}^*(G_4) = 1$



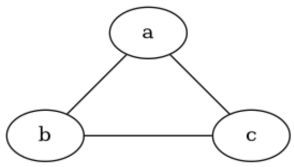
(G_5)
$\pi_5 = p^2(1-p)$
$\mathrm{k}(\mathrm{G}_5) = \frac{4}{3}$
$\mathrm{c}(\mathrm{G}_5) = 0$
$\mathrm{d}^*(G_5) = 2$



(G_6)
$\pi_6 = p^2(1-p)$
$\mathrm{k}(\mathrm{G}_6) = \frac{4}{3}$
$\mathrm{c}(\mathrm{G}_6) = 0$
$\mathrm{d}^*(G_6) = 2$



(G_7)
$\pi_7 = p^2(1-p)$
$\mathrm{k}(\mathrm{G}_7) = \frac{4}{3}$
$\mathrm{c}(\mathrm{G}_7) = 0$
$\mathrm{d}^*(G_7) = 2$



(G_8)
$\pi_8 = p^3$
$\mathrm{k}(\mathrm{G}_8) = 2$
$\mathrm{c}(\mathrm{G}_8) = 1$
$\mathrm{d}^*(G_8) = 1$

We have that:

$$\langle k \rangle = \sum_i \pi_i k(G_i) = 3(p(1-p)^2 \cdot 2/3) + 3(p^2(1-p) \cdot 4/3) + (p^3 \cdot 2) = 2p(1-p)^2 + 4p^2(1-p) + p^3 =$$

$$= 2p(1-2p+p^2) + 4p^2 - 4p^3 + p^3 = 2p - 4p^2 + 2p^3 + 4p^2 - 4p^3 + 2p^3 = 2p$$

$$\langle c \rangle = \sum_i \pi_i c(G_i) = p^3$$

$$\langle d^* \rangle = \sum_i \pi_i d^*(G_i) = 3(1 \cdot p(1-p)^2) + 3(2 \cdot p^2(1-p)) + p^3 = 3p(1-p)^2 + 6p^2(1-p) + p^3 =$$

$$= 3p(1-2p+p^2) + 6p^2 - 6p^3 + p^3 = 3p - 6p^2 + 3p^3 + 6p^2 - 6p^3 + p^3 = 3p - 2p^3$$

Proof ($p = 1/3$):

$\langle \mathbf{k} \rangle = \mathbf{2p} = 2/3$.

$\langle \mathbf{c} \rangle = \mathbf{p^3} = 1/27$.

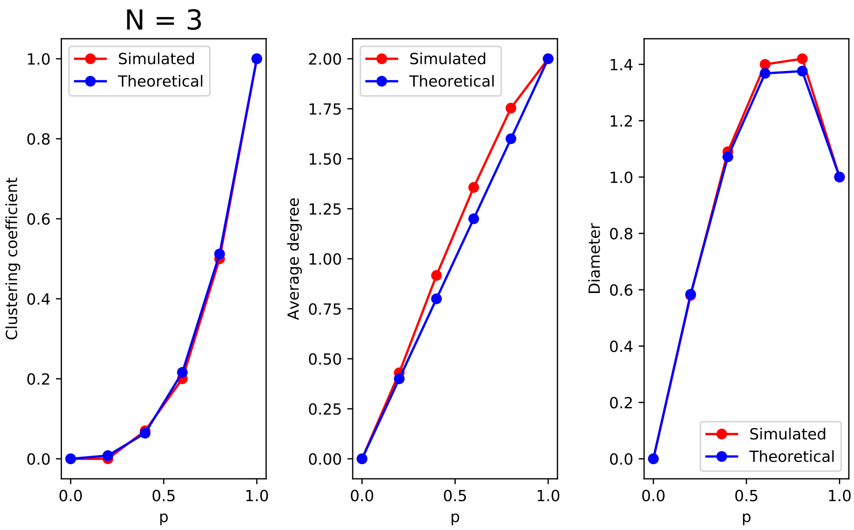
$\langle \mathbf{d}^* \rangle = \mathbf{3p - 2p^3} = 25/27$.

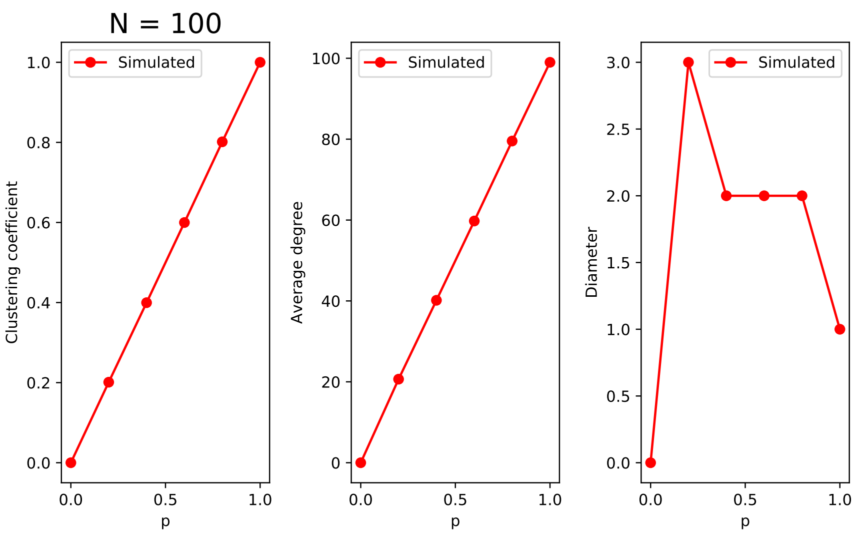
Problem 2

a) As said in the exercise 1f of the ES1. The clustering coefficient of a node in an undirected graph is given by the proportion of links between the vertices within its neighbourhood divided by the number of links that could possibly exist between them. So the average clustering coefficient $\langle c \rangle$ can also be seen as the probability that exists a link between two nodes in the same neighbourhood.

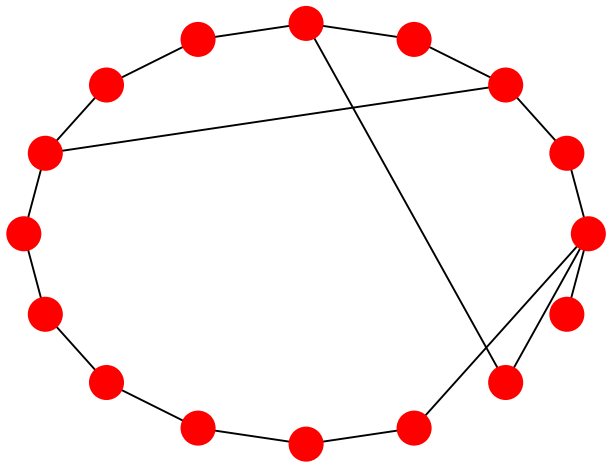
b) Since $\langle k \rangle$ is bounded $p(k)$ will follow a Poison distribution. Hence, the nodes will have a very low degree and therefore there will be a high number of connected components and a very low clustering coefficient. So, when $N \to \infty$, $\langle c \rangle \to 0$.

c) The resulting plots are shown below.

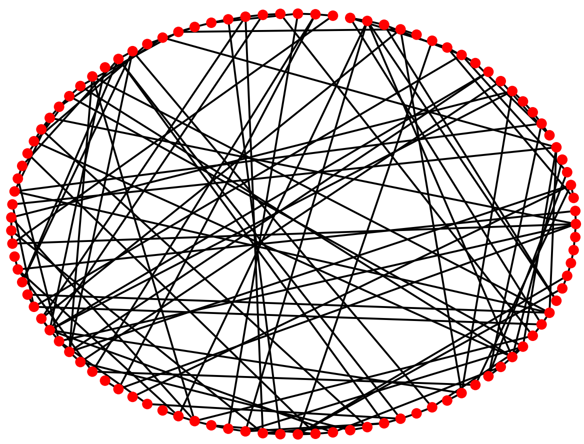


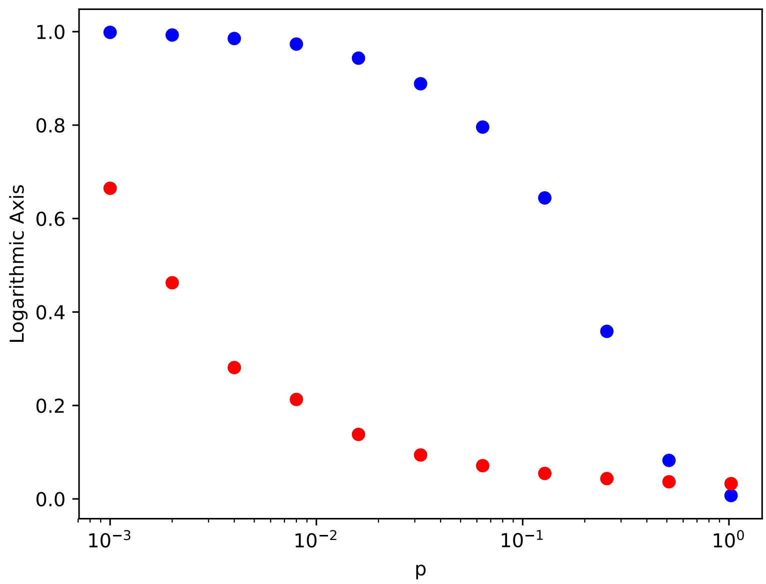


Problem 3



a)





b)