CS-E5740 Complex Networks,
Answers to exercise set 3

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Problem 1

a)
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The maximum degree is:  24
The total number of edges is:  200
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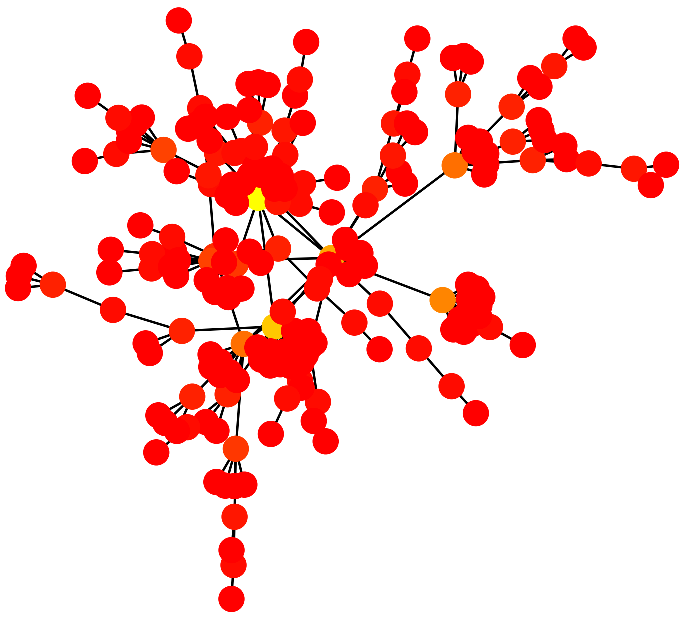

Figure 1: Barabási-Albert network with $N = 100$ and $m = 1$ (starting from a 3-clique seed network).

b)

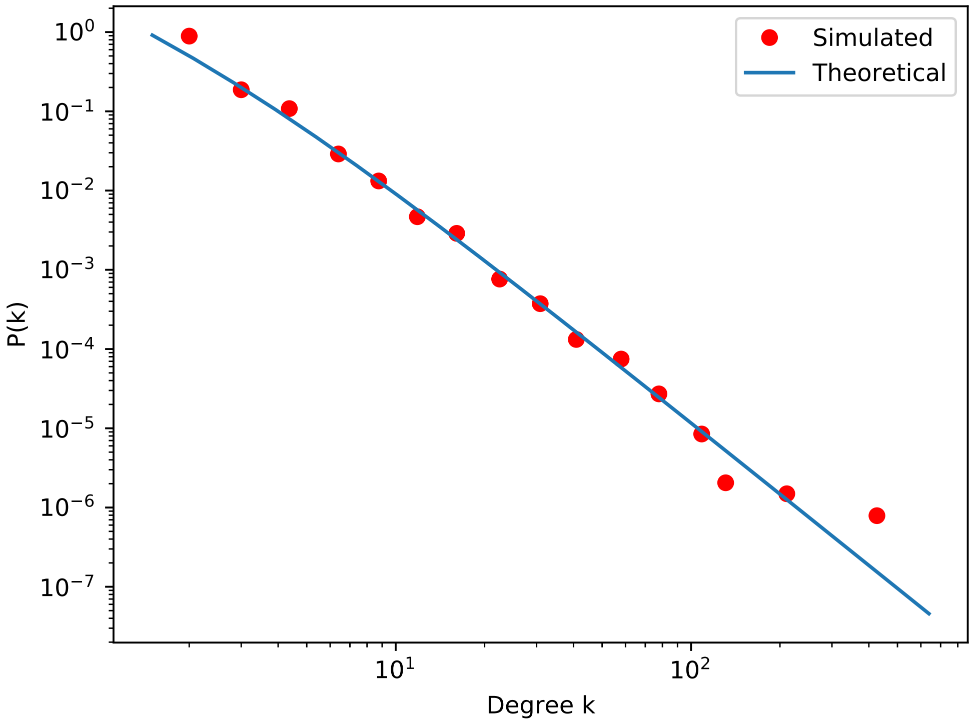

Figure 2: Logarithmically binned probability density function for degree, $P(k)$ for a BA network using parameters $N = 104$ with $m = 2$.

Problem 2

a) We have that in the BA model, the probability Π_i that a new edge attaches to a *particular* vertex of degree k_i equals:

$$\Pi_i = \frac{k_i}{\sum_{j=1}^{N} k_j}.$$

The sum of degree on N vertices can be expressed as:

$$\sum_{j=1}^{N} k_j = 2mN,$$

Then we can derive that the probability $\Pi(k)$ that a new edge attaches to *any* vertex of degree k in a network of N vertices as follows.

$$\Pi(k) = \Pi_i n_{k,N} = \frac{k}{\sum_{j=1}^{N} k_j} n_{k,N} = \frac{k n_{k,N}}{2mN} = \frac{kNp_{k,N}}{2mN} = \frac{kp_{k,N}}{2m}.$$

b) This happens because after a long time all the non new nodes will already have $k > m$ degree, hence, the only nodes with degree $k = m$ will be the new nodes.

Equations for the *net change* of the number of vertices of degree k as the network grows in size from N to $N+1$:

$$(N+1)p_{k,N+1} - Np_{k,N} = n_k^+ - n_k^- = \begin{cases} \frac{1}{2}(k-1)p_{k-1,N} - \frac{1}{2}kp_{k,N} & k > m \\ 1 - \frac{1}{2}kp_{k,N} & k = m \end{cases}$$

$$= \begin{cases} \frac{1}{2}(k-1)p_{k-1,N} - \frac{1}{2}kp_{k,N} & k > m \\ 1 - \frac{1}{2}mp_{m,N} & k = m \end{cases}$$

c)

$$p_k = (N+1)p_k - Np_k = (N+1)p_{k,N+1} - Np_{k,N} =$$
$$= \frac{1}{2}(k-1)p_{k-1,N} - \frac{1}{2}kp_{k,N} = \frac{1}{2}(k-1)p_{k-1} - \frac{1}{2}kp_k$$
$$p_k = \frac{k-1}{k+2}p_{k-1}$$

$$p_m = 1 - \frac{1}{2}mp_{m,N} = 1 - \frac{1}{2}mp_m$$
$$p_m = \frac{2}{m+2}$$

d)

$$p_{m+1} = \frac{(m+1)-1}{(m+1)+2}p_{(m+1)-1} = \frac{m}{m+3}p_m = \frac{2m}{(m+3)(m+2)}$$

$$p_{m+2} = \frac{(m+2)-1}{(m+2)+2}p_{(m+2)-1} = \frac{m+1}{m+4}p_{m+1} = \frac{(m+1)2m}{(m+4)(m+3)(m+2)}$$

$$p_{m+3} = \frac{(m+3)-1}{(m+3)+2}p_{(m+3)-1} = \frac{m+2}{m+5}p_{m+2} = \frac{(m+2)(m+1)2m}{(m+5)(m+4)(m+3)(m+2)} =$$
$$= \frac{(m+1)2m}{(m+5)(m+4)(m+3)}$$

$$p_{m+4} = \frac{(m+4)-1}{(m+4)+2} p_{(m+4)-1} = \frac{m+3}{m+6} p_{m+3} = \frac{(m+3)(m+1)2m}{(m+6)(m+5)(m+4)(m+3)} =$$

$$= \frac{(m+1)2m}{(m+6)(m+5)(m+4)}$$

$$p_{m+5} = \frac{(m+5)-1}{(m+5)+2} p_{(m+5)-1} = \frac{m+4}{m+7} p_{m+4} = \frac{(m+4)(m+1)2m}{(m+7)(m+6)(m+5)(m+4)} =$$

$$= \frac{(m+1)2m}{(m+7)(m+6)(m+5)}$$

$$p_k = \frac{(m+1)2m}{(k+2)(k+1)k}$$

$$P(k) = 2m(m+1)/[k(k+1)(k+1)]$$

$\square$