CS-E4850 Computer Vision
Exercise Round 12

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Spring Term Course 2020-2021

Exercise 1. Course feedback. (Worth of 1 bonus point in similar manner as other tasks.) Fill in the official course feedback through the link which you should receive by December 6 to your email. The feedback collection is anonymous. The system separately reports the emails of students, who have returned the feedback, but they are not associated with the answers.

Exercise 2. Epipolar geometry. (Pen & paper problem)
Let's assume that the camera projection matrices of two cameras are $\mathbf{P} = [\mathbf{I} \quad \mathbf{0}]$ and $\mathbf{P'} = [\mathbf{R} \quad \mathbf{t}]$, where $\mathbf{R}$ is a rotation matrix and $\mathbf{t} = (t_1, t_2, t_3)^\top$ describes the translation between the cameras. Hence, the cameras have identical internal parameters and the image points are given in the normalized image coordinates (the origin of the image coordinate frame is at the principal point and the focal length is 1).

The epipolar constraint is illustrated in Figure 1 below and it implies that if p and p' are corresponding image points then the vectors $\overrightarrow{Op}$, $\overrightarrow{O'p'}$ and $\overrightarrow{O'O}$ are coplanar, i.e.

$$\overrightarrow{O'p'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op}\right) = 0 \tag{1}$$

Let $\mathbf{x} = (x, y, 1)^\top$ and $\mathbf{x'} = (x', y', 1)^\top$ denote the homogeneous image coordinate vectors of p and p'. Show that the equation (1) can be written in the form

$$\mathbf{x'}^\top \mathbf{E} \mathbf{x} = 0, \tag{2}$$

where matrix $\mathbf{E}$ is the essential matrix $\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$ (as defined on slide 21 of Lecture 11).

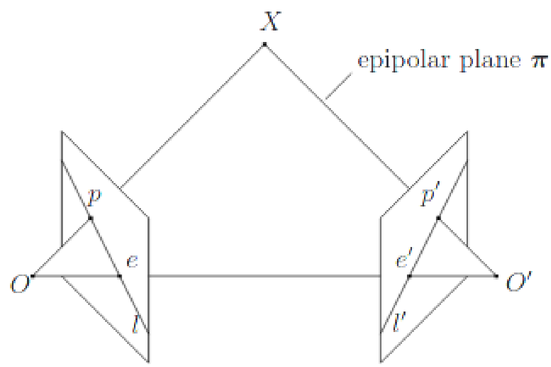


Figure 1: Epipolar geometry. Given a point p in the first image its corresponding point in the second image is constrained to lie on the line l' which is the epipolar line of p. Correspondingly, the line l is the epipolar line of p'. Points e and e' are the epipoles.

Solution.
We can write the first equation (1) as:

$$\overrightarrow{Op'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op}\right) = 0,$$

$$\mathbf{x}' \cdot (\mathbf{t} \times \mathbf{R}\mathbf{x}) = 0,$$
$$\mathbf{x}^{\top} \cdot [\mathbf{t}]_{\times} \mathbf{R}\mathbf{x} = 0.$$

And knowing that $\mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$ we obtain,

$$\mathbf{x}'^{\top}\mathbf{E}\mathbf{x} = 0.$$

□

Exercise 3. Stereo vision. (Pen & paper problem)
In Figure 2 below there is a picture of a typical stereo configuration, where two similar pinhole cameras are placed side by side. The focal length of the cameras is f and the distance between the camera centers is b. The point P is located in front of the cameras and its disparity d is the distance between the corresponding image points (i.e. $d = |x_l - x_r|$). The disparity depends only on the parameters b and f and the Z-coordinate of P.

a) Assume that $d = 1\ cm$, $b = 6\ cm$ and $f = 1\ cm$. Compute Z_P.

Solution.
Knowing that

$$\frac{x_l}{f} = \frac{x'_l}{Z_P} \longrightarrow x'_l = \frac{x_l Z_P}{f},$$

$$\frac{x_r}{f} = \frac{x'_r}{Z_P} \longrightarrow x'_r = \frac{x_r Z_P}{f},$$

$$d = |x_l - x_r|$$

we can write

$$x'_l - x'_r = b,$$

$$\frac{x_l Z_P}{f} - \frac{x_r Z_P}{f} = b,$$
$$\frac{Z_P}{f}(x_l - x_r) = b,$$
$$\frac{Z_P}{f}d = b.$$

Therefore

$$Z_p = \frac{bf}{d} = \frac{6 \cdot 1}{1} = 6\ cm$$

b) Assume that the smallest measurable disparity is 1 pixel and the pixel width is 0.01 mm. What is the range of Z-coordinates for those points for which the disparity is below 1 pixel?

Solution.
Having that

$$\frac{Z_P}{f}d = b$$

We want to obtain $d \leq 0.01\ mm$ hence,

$$d = \frac{bf}{Z_P} \leq 0.01\ mm$$

$$Z_P \geq \frac{bf}{0.01\ mm} = \frac{6\ cm \cdot 1\ cm}{0.01\ mm}$$
$$Z_P \geq 60m$$

c) In the configuration illustrated in Figure 2 the camera matrices are $\mathbf{P_l} = [\mathbf{I} \quad \mathbf{0}]$ and $\mathbf{P_r} = [\mathbf{I} \quad \mathbf{t}]$, where $\mathbf{I}$ is the identity matrix and $\mathbf{t} = (-6, 0, 0)^\top$. The point Q has coordinates $(3, 0, 3)$. Compute the image of Q on the image plane of the camera on the left and the corresponding epipolar line on the image plane of the camera on the right.

Solution.
Having

$$\mathbf{P_l} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{P_r} = \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$Q = (3, 0, 3),$$

and knowing that

$$\mathbf{x'}^\top \mathbf{E}\mathbf{x} = 0,$$
$$\mathbf{x} = \mathbf{P_l}Q,$$
$$\mathbf{x'} = \mathbf{P_r}Q,$$

the image of Q on the image plane of the camera on the left is

$$\mathbf{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix},$$

and the one of the camera on the right is

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix}.$$

Hence,

$$\mathbf{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix}.$$

Therefore the corresponding epipolar line on the image plane of the camera on the right is

$$\mathbf{E}^\top \mathbf{x}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -6 \\ 0 & 6 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -18 \\ 0 \end{bmatrix}.$$

And the one of the camera on the left would be

$$\mathbf{E}\mathbf{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}.$$

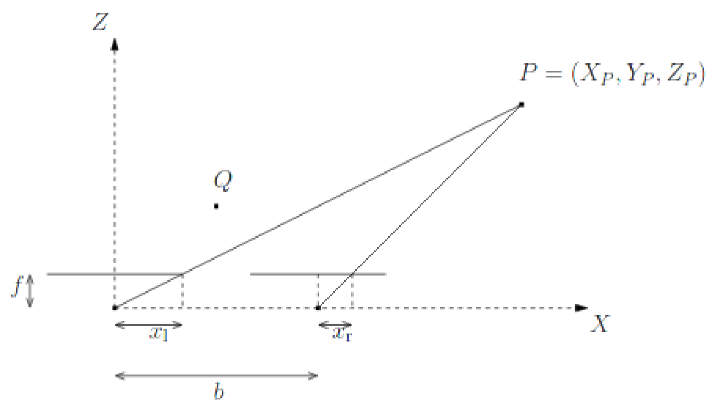


Figure 2: Top view of a stereo configuration where two pinhole cameras are placed side by side

exercise12

December 2, 2020

[1]:
```
# This cell is used for creating a button that hides/unhides code cells to␣
 ↪quickly look only the results.
# Works only with Jupyter Notebooks.

from IPython.display import HTML

HTML('''<script>
code_show=true;
function code_toggle() {
if (code_show){
$('div.input').hide();
} else {
$('div.input').show();
}
code_show = !code_show
}
$( document ).ready(code_toggle);
</script>
<form action="javascript:code_toggle()"><input type="submit" value="Click here␣
 ↪to toggle on/off the raw code."></form>''')
```

[1]: <IPython.core.display.HTML object>

[2]:
```
# Description:
#   Exercise12 notebook.
#
# Copyright (C) 2018 Santiago Cortes, Juha Ylioinas
#
# This software is distributed under the GNU General Public
# Licence (version 2 or later); please refer to the file
# Licence.txt, included with the software, for details.

# Preparations
import os
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

```python
# Select data directory
if os.path.isdir('/coursedata'):
    # JupyterHub
    course_data_dir = '/coursedata'
elif os.path.isdir('../../../coursedata'):
    # Local installation
    course_data_dir = '../../../coursedata'
else:
    # Docker
    course_data_dir = '/home/jovyan/work/coursedata/'

print('The data directory is %s' % course_data_dir)
data_dir = os.path.join(course_data_dir, 'exercise-12-data')
print('Data stored in %s' % data_dir)
```

```
The data directory is /coursedata
Data stored in /coursedata/exercise-12-data
```

Fill your name and student number below.

0.0.1 Name: Alex Herrero Pons

0.0.2 Student number: 918697

1 CS-E4850 Computer Vision Exercise Round 12

The problems should be solved before the exercise session and solutions returned via MyCourses. Upload to MyCourses both: this Jupyter Notebook (.ipynb) file containing your solutions to the programming tasks and the exported pdf version of this Notebook file. If there are both programming and pen & paper tasks kindly combine the two pdf files (your scanned/LaTeX solutions and the exported Notebook) into a single pdf and submit that with the Notebook (.ipynb) file. Note that (1) you are not supposed to change anything in the utils.py and (2) you should be sure that everything that you need to implement should work with the pictures specified by the assignments of this exercise round.

1.0.1 Make sure to complete the pen and paper exercices in the PDF attached.

1.1 Fundamental matrix estimation.

a) Implement the eight-point algorithm as explained on slide 28 of Lecture 11. Note the skeleton function and follow the input output structure

b) Implement the normalized eight-point algorithm as explained on slide 31 of Lecture 11 (Algorithm 11.1. in Hartley & Zisserman).

The epipolar lines obtained with both F-matrix estimates should be close to those visualized by the example script.

```
[3]: def estimateF(x1,x2):
         # Return the fundamental matrix F (3 by 3), based on two sets of␣
      ↪homogeneous 2D points x1 and x2.
         # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
      ↪points.
         # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.

         x1_0 = x1[0,:]
         x1_1 = x1[1,:]
         x2_0 = x2[0,:]
         x2_1 = x2[1,:]

         M = np.array([
         x1_0*x2_0, x1_0*x2_1, x1_0,
         x1_1*x2_0, x1_1*x2_1, x1_1,
              x2_0,      x2_1
         ])
         M = np.vstack((M,np.ones((1,11))))

         u,s,v = np.linalg.svd(M.T)
         a_min = np.argmin(s)
         F = np.reshape(v[a_min],(3,3))

         return F

     def compute_t(x):
         O = [np.average(x[0]),np.average(x[1])]
         t = np.array([x[0,:],x[1,:],x[2,:]])

         sum=0
         for i in range(len(t[0])):
             sum += t[0,i]**2+t[1,i]**2

         s = np.sqrt(2/(sum/len(t[0])))
         #T = np.array([[s,0.,0.],[0.,s,0],[0.,0.,1.]])@np.array([[1.,0.,O[0]],[0.,1.
      ↪,O[1]],[0.,0.,1.]])
         #t = T@np.array(x)
         T = np.dot(np.array([[s,0.,0.],[0.,s,0],[0.,0.,1.]]),np.array([[1.,0.
      ↪,O[0]],[0.,1.,O[1]],[0.,0.,1.]]))
         t = np.dot(T,np.array(x))

         return t

     def estimateFnorm(x1,x2):
         # Return the fundamental matrix F (3 by 3), based on two sets of␣
      ↪homogeneous 2D points x1 and x2.
```

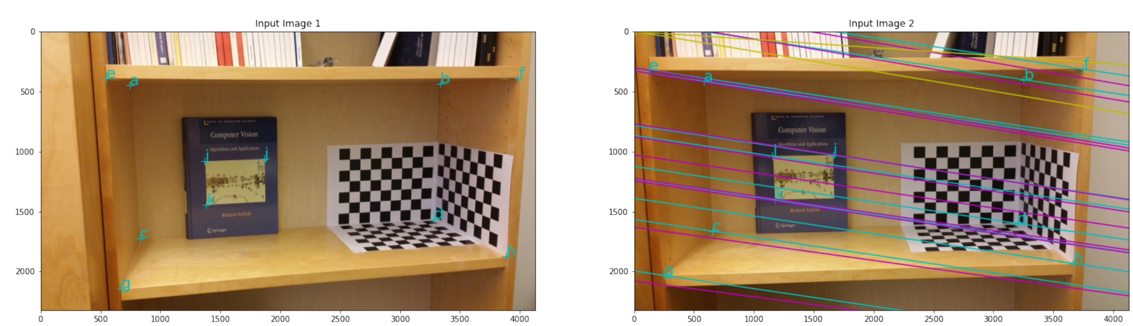
```
    # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
↪points.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix based␣
↪on normalized homogeneous points.
    t1 = compute_t(x1)
    t2 = compute_t(x2)

    F = estimateF(t1,t2)

    return F

def vgg_F_from_P(P1,P2):
    # Return the fundamental matrix F (3 by 3), based on two camera parameter␣
↪arrays.
    # Input: P1, P2 numpy ndarray (3 by 4) containing intrinsic and extrinsic␣
↪parameters.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.
    X=[]
    Y=[]
    X.append(P1[[1,2],:])
    X.append(P1[[2,0],:])
    X.append(P1[[0,1],:])
    Y.append(P2[[1,2],:])
    Y.append(P2[[2,0],:])
    Y.append(P2[[0,1],:])
    F=np.zeros([3,3])

    for i in range(3):
        for j in range(3):
            M=np.concatenate([X[j],Y[i]])
            F[i,j]=np.linalg.det(M)
    return F
```

[4]:

```
# Point locations
x1 = 1.0e+03*np.array([0.7435,3.3315,0.8275,3.2835,0.5475,3.9875,0.6715,3.
↪8835,1.3715,1.8675,1.3835])
y1 = 1.0e+03*np.array([0.4455,0.4335,1.7215,1.5615,0.3895,0.3895,2.1415,1.
↪8735,1.0775,1.0575,1.4415])
x2 = 1.0e+03*np.array([0.5835,3.2515,0.6515,3.1995,0.1275,3.7475,0.2475,3.
↪6635,1.1555,1.6595,1.1755])
y2 = 1.0e+03*np.array([0.4135,0.4015,1.6655,1.5975,0.3215,0.3135,2.0295,1.
↪9335,1.0335,1.0255,1.3975])

# Camera parameters
P1= np.row_stack([[-0.001162918366053,0.000102986385133,-0.000344703214391,0.
↪995200644722518],\
```

```
                 [-0.000019974831639,0.001106889654747,-0.000150591916681,0.
↪097841118173777],\
                 [-0.000000053632777,0.000000044849673,-0.000000270734766,0.
↪000249501614496]])

P2= np.row_stack([[-0.001272880601540, 0.000093061493378,-0.000574486218854, 0.
↪996457618133488],\
                 [-0.000002971652037, 0.001271207503106,-0.000200323351541, 0.
↪084074548573989],\
                 [-0.000000020226464, 0.000000043518811,-0.000000316928290, 0.
↪000265554210072]])

# Make homogenous representations of points
pts1=np.row_stack([x1,y1,np.ones_like(x1)])
pts2=np.row_stack([x2,y2,np.ones_like(x2)])

# Read images
im1 = cv2.imread(data_dir+'/im1.jpg')
im2 = cv2.imread(data_dir+'/im2.jpg')

im1 = cv2.cvtColor(im1, cv2.COLOR_BGR2RGB)
im2 = cv2.cvtColor(im2, cv2.COLOR_BGR2RGB)



# Labels
labels = ['a','b','c','d','e','f','g','h','i','j','k']

# Create figure
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(25,25))
ax = axes.ravel()
ax[0].imshow(im1)
ax[0].plot(x1, y1, 'c+', markersize=10)

# Put labels
for i in range(len(x1)):
    ax[0].annotate(labels[i], (x1[i], y1[i]), color='c', fontsize=20)
ax[0].set_title("Input Image 1")
ax[1].imshow(im2)
ax[1].plot(x2, y2, 'c+', markersize=10)
for i in range(len(x2)):
    ax[1].annotate(labels[i], (x2[i], y2[i]), color='c', fontsize=20)
ax[1].set_title("Input Image 2")

# Get ground truth fundamental matrix
F=vgg_F_from_P(P1,P2)

# Create lines
```

```
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)

#  Plot lines
px=np.array([0,np.shape(im2)[1]])
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'c-');

# Get fundamental matrix and draw epipolar lines
F=estimateF(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'m-');

# Get fundamental matrix from normalized algorithm and draw epipolar lines
F=estimateFnorm(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'y-');

ax[1].axes.set_xlim([0,np.shape(im2)[1]])
ax[1].axes.set_ylim([np.shape(im2)[0],0])
plt.show()
```

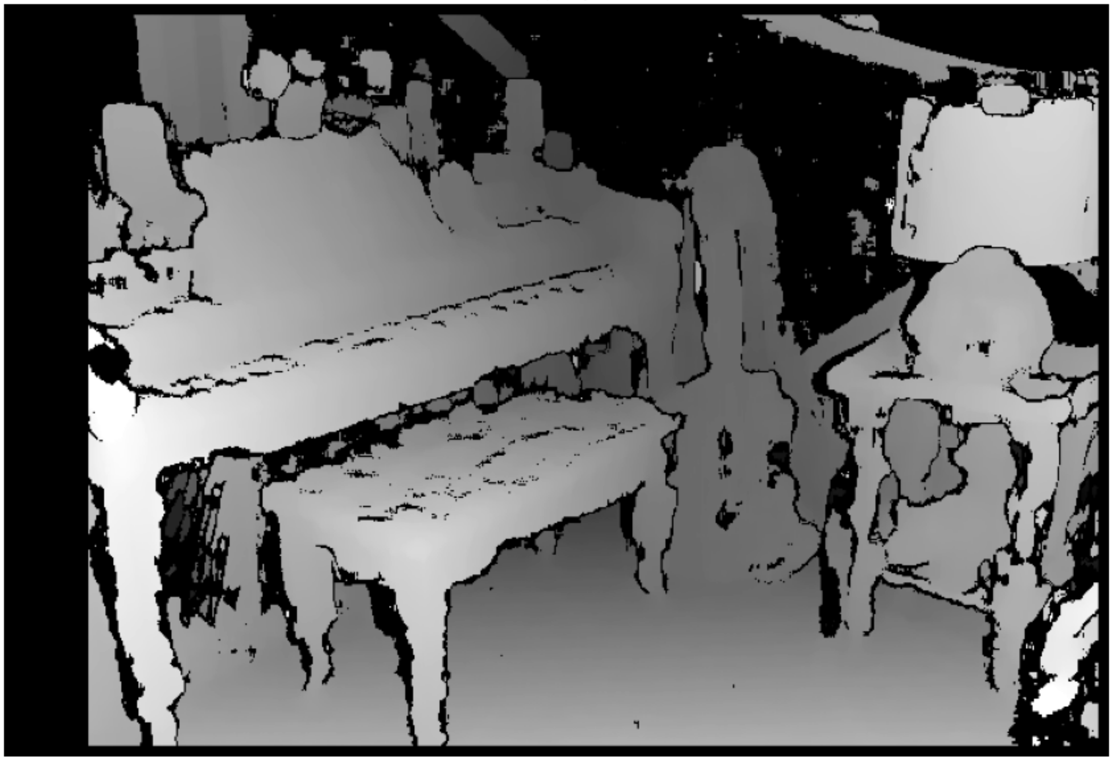


1.2 Demo. Stereo disparity computation. (Just a demo, no points given)

Run and study the opencv stereo disparity and depth estimation.

[5]:
```
# Import images
sc=0.25

imgL = cv2.resize(cv2.imread(data_dir+'/im0.png',0), (0,0), fx=sc, fy=sc)
imgR = cv2.resize(cv2.imread(data_dir+'/im1.png',0), (0,0), fx=sc, fy=sc)
imgL_col = cv2.resize(cv2.imread(data_dir+'/im0.png'), (0,0), fx=sc, fy=sc)
imgR_col = cv2.resize(cv2.imread(data_dir+'/im1.png'), (0,0), fx=sc, fy=sc)

# Show images
plt.figure(figsize=[15,15])
plt.subplot(121)
plt.imshow(imgL_col[:,:,[2,1,0]])
plt.axis('off')
plt.subplot(122)
plt.imshow(imgR_col[:,:,[2,1,0]])
plt.axis('off')

# Compute disparity
stereo = cv2.StereoBM_create(numDisparities=16*3, blockSize=15)
disparity = stereo.compute(imgL,imgR)

# Show disparity
plt.figure(figsize=[15,15])
plt.imshow(disparity,'gray')
plt.axis('off')
plt.title('Disparity')
#ndistp=cv2.guidedFilter(imgL, disparity, 9, 4,0.1)

# Calibration data
baseline=17.8089 #cm
f_length=2826.171*sc #pixels
c_point=np.array([1415.97,965.806])*sc # pixels

# Get depth from disparity
point=np.zeros([np.count_nonzero(disparity>1),6])
ind=0
for i in range(np.shape(disparity)[0]):
    for j in range(np.shape(disparity)[1]):
        if disparity[i,j]>1:
            # Save point information into point cloud
            # [pixel_x,pixel_y,disparity,color]
            point[ind,0:3]=j,i,disparity[i,j]
            point[ind,3:6]=imgL_col[i,j]/255.0
            ind+=1
```

```
# Z=baseline*focal/disparity
# openCV disparity is (16*actual_disparity). This depends on the algorithm.
# It is in order to use signed shorts and keep good subpixel accuracy.
point[:,2]=baseline*f_length/(point[:,2]/16.0)
#X=Z*(pixel_u-center_u)/focal
point[:,0]=point[:,2]*(point[:,0]-c_point[0])/f_length
#Y=Z*(pixel_v-center_v)/focal
point[:,1]=-point[:,2]*(point[:,1]-c_point[1])/f_length

# Delete points on the far background
inl=(point[:,2]<2000)
point=point[inl,:]

plt.show()
```




```
[6]: def visualize_points(pts,R,img,f=1000,cp=[400,300]):
         #visualize colored points given a rotation matrix
         # rotate around the mean of the point cloud
         c=np.mean(point[:,0:3],0)
         #r_point=((point[:,0:3]-c)@R_y)+c
         r_point=np.dot((point[:,0:3]-c),R_y)+c

         #Project back to the same camera model
         K=np.float32([[f,0,cp[0]],[0,f,cp[1]],[0,0,1]])

         # Sort by depth (painter's algorithm)
         ind=np.argsort(r_point[:,2])
         r_point=r_point[np.flip(ind,0),:]

         #Project
         #uvk=K@r_point.T
         uvk=(np.dot(K,r_point.T))
         color=point[:,[5,4,3]]
         color=color[np.flip(ind,0),:]

         # Normalize homogeneous  coordinates
         uv=uvk[0:2,:]/(uvk[2,:])
```

```
    # Draw projected points
    plt.scatter(uv[0,:],uv[1,:],marker='.',s=10,c=color)
    plt.xlim([0,np.shape(imgL)[1]])
    plt.ylim([0,np.shape(imgL)[0]])
    plt.axis('off')



# Visualize points from two different angles
plt.figure(figsize=[30,15])
plt.subplot(121)
# Rotate around y axis to visualize
ang_y=-20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
 ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.subplot(122)
# Rotate around y axis to visualize
ang_y=20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
 ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.show()
```