Field and Service Robotics - Homework 4

Luciana Ercolanese - P38000197

a.y. 2023-2024

Exercise 1

Buoyancy is a lift force that is generated when a body is floating in a fluid. It is a hydrostatic effect because it does not depend on the relative movement between the body and the fluid. In other words, it is a restoring force, e.g. a force that is active also when the body is stationary, like gravity. This effect is not negligible in underwater robotics, since the density of water is comparable to that of the robot. This is not true for aerial robotics, because the density of air is much lower than that of the robot. Buoyancy can be computed as

$$b = \rho \Delta \|\bar{g}\|, \tag{1}$$

where Δ is the robot's volume, and $\bar{g} = [0,\ 0, -g]^T$ is the gravity vector. As seen in the formula, buoyancy is directly proportional to the density of water ρ. When modelling buoyancy, it is often coupled with gravity in a cumulative term that represents restoring forces. However, they are applied in different points of the body: gravity is applied at the center of mass, while buoyancy at the center of buoyancy. Notice how, usually, the body frame is placed at the CoM and the dynamic model is referred to this frame. Therefore, when designing the mechanics of a UUV, it is convenient to make these two points coincide, or at least align them along the vertical axis. This way the buoyancy effect will generate a force, but not a torque since the direction of the force would be parallel to the arm, as seen in Fig. 1. This issue would be most noticeable in a large robot: as seen in Eq. 1, buoyancy is proportional to the volume of the robot. This means large thrusts are needed to compensate for the forces and torques generated by the restoring forces.

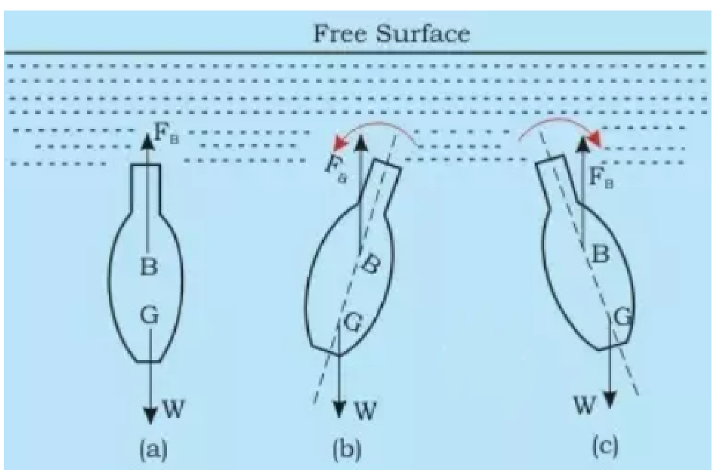


Figure 1: Effects of the placement of the centre of mass and centre of buoyancy on a UUV. F_b indicates buoyancy and W is gravity.

Exercise 2

a) **False.** The added mass effect represents the reaction forces generated when the fluid surrounding the body is accelerated by the movement of the body itself. Therefore, it is not a load that increases the total mass of the robot. The effect is shown in the dynamic model of the UUV by means of an additional inertial contribution, as well as a Coriolis and centripetal term. The additional inertia term is modelled in the matrix M_A, that depends on the geometry of the UUV and therefore isn't necessarily positive definite. It is shown that, under the assumptions of ideal fluid, low-speed regime and absence of ocean current, M_A is symmetric and positive-definite.

b) **True.** The added mass effect is generated by the fluid surrounding the body being accelerated by the robot itself. However, in wheeled, aerial and legged robotics it is not considered since the density of air is much lower than that of the robot. In underwater robotics, instead, the density of water is much higher, and comparable to that of the robot, so that the effect is not negligible.

c) **True.** The damping effect is generated by dissipative forces acting on the body because the viscosity of the fluid. Since these forces act on velocities, instead of accelerations like the added mass effect, they are helpful in proving the stability of the system. When considering a Lyapunov candidate in terms of kinetic energy, these dissipative terms make the time derivative of the Lyapunov candidate more negative.

d) **False.** The ocean current is conventionally expressed as a twist vector, that has only linear components (e.g. is irrotational), and is constant if expressed in the world frame. The world frame is more convenient to model it because the current would be highly nonlinear if expressed in the body frame. It is included in the UUV dynamic model by means of an additional term, that is a function of the relative velocity of the body and of the current itself. This relative velocity is expressed in the body frame, so we can report the ocean current back to the body frame by multiplying it by R_b^T (rotation matrix of the body frame).

Exercise 3

Analysis of gaits

We completed and used the provided software to compare the available gaits for the quadruped: trot, bound, pacing, gallop, trot run and crawl. The desired trajectory has a phase with a constant acceleration $a_d = 1m/s^2$ and a subsequent phase at constant velocity $v_d = 0.5m/s$. The bound gait is an exception that will be discussed later. The simulations of all gaits with the standard values can be seen in Fig. 13.

Trot is a gait that alternates a stance phase and swing phases with the two diagonal feet moving together. This generates a moderate oscillation around the x-axis (roll). The stance phase makes this gait very stable, especially compared to trot run. The ground reaction forces have moderate magnitude and asymptotically approximate a square wave with a smooth transient. The velocity along the x-axis has an oscillatory behavior, whose mean value asymptotically reaches the constant velocity reference.

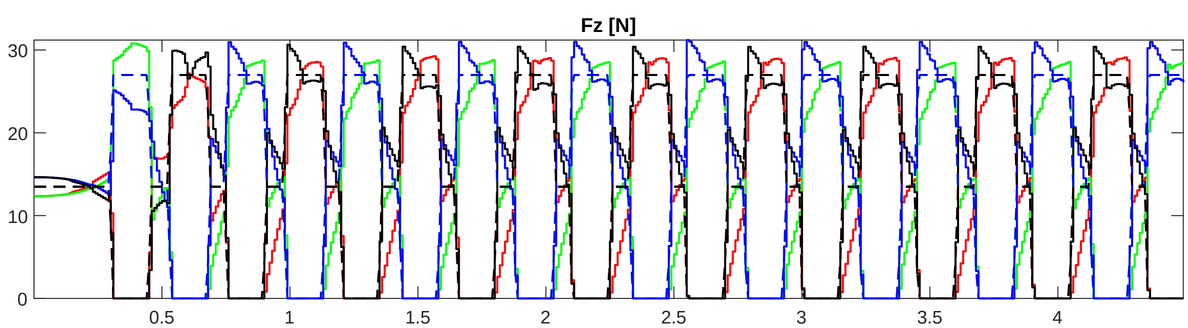
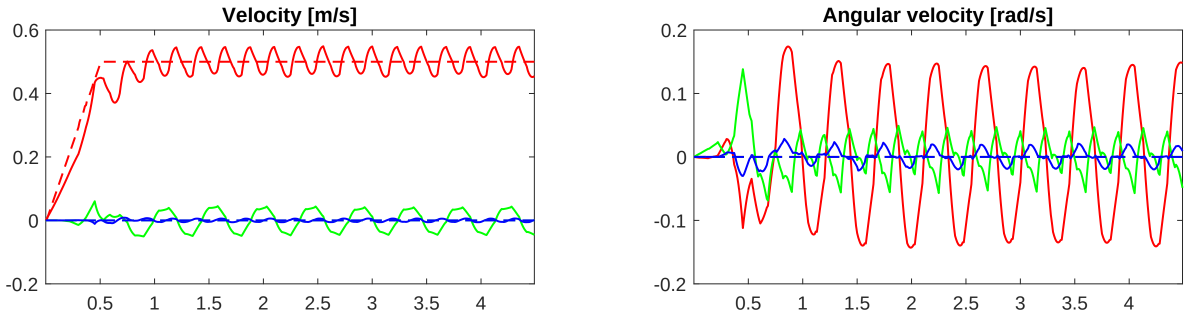


Figure 2: Trot: force plot



(a) Linear velocity (b) Angular velocity

Figure 3: Trot: velocity plot

Bound is a gait where the fore legs are coordinated, and the hind legs as well. In between these swing phases, where two feet are in contact with the ground, there is a third swing phase without any stance feet. This implies that this gait needs to be dynamically balanced. Furthermore, the ground reaction forces are much higher in magnitude compared to the other gaits, and their increase is more impulsive when a foot switches from swing to stance. The alternating contact of fore and hind legs generates a strong pitch motion, as well as a noticeable vertical motion (see blue trace in Fig. 5a). Also note that this gait is most suitable at higher velocities, therefore the trajectory tracks a constant reference $v_d = 2m/s$ by default.

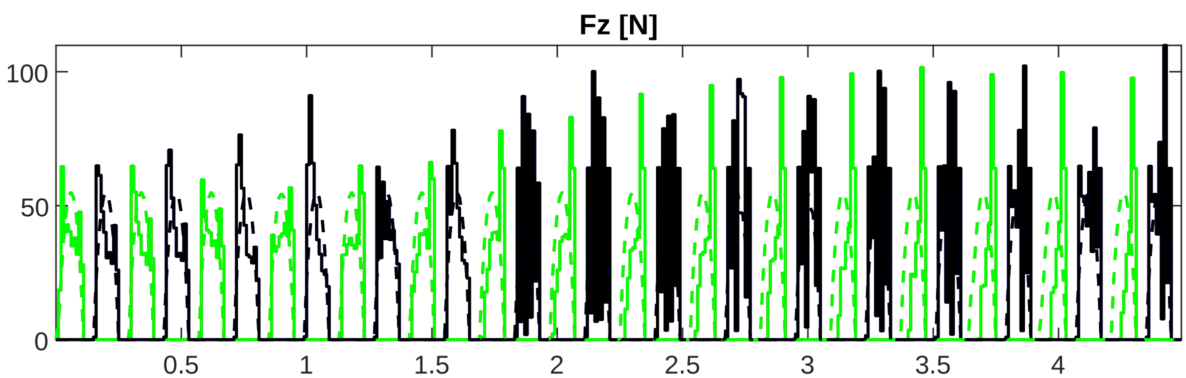
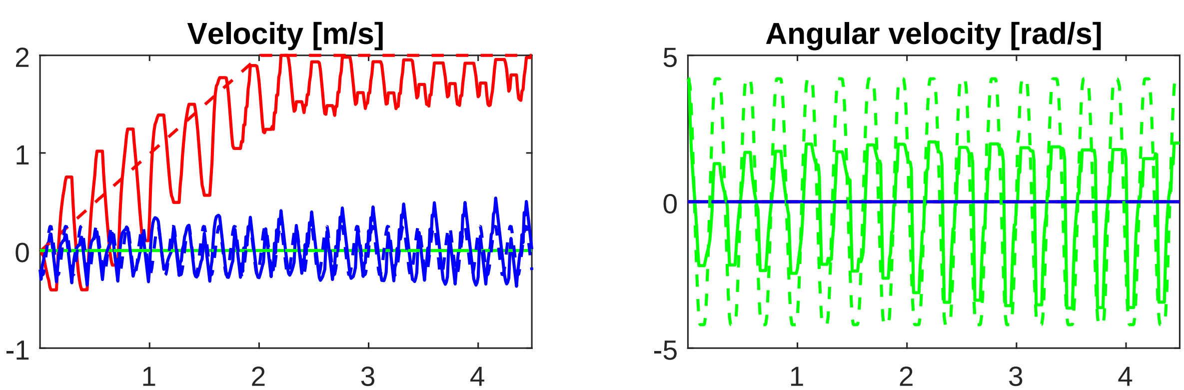


Figure 4: Bound: force plot



(a) Linear velocity (b) Angular velocity

Figure 5: Bound: velocity plot

Pacing is a gait where the lateral legs are coordinated in two alternating swing phases. This generates a strong periodic roll, and a large periodic lateral motion (see green trace in Fig. 18c).

However, the oscillations of velocity are small in magnitude and the GRFs are moderate as well, similarly to trot.

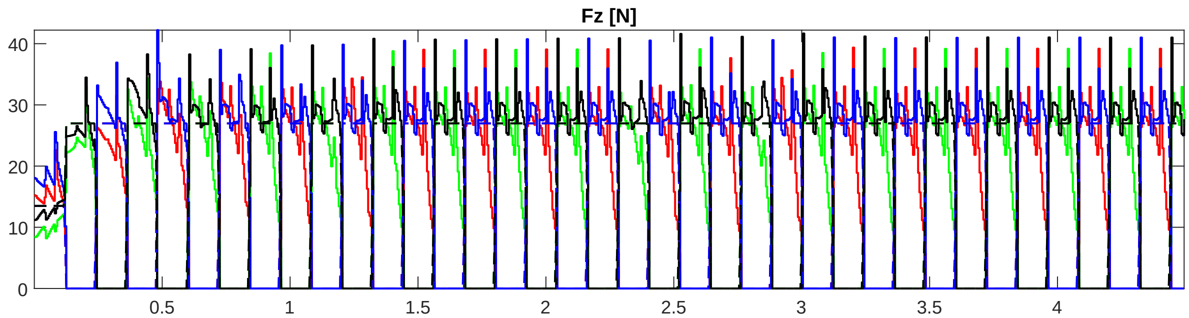


Figure 6: Pacing: force plot

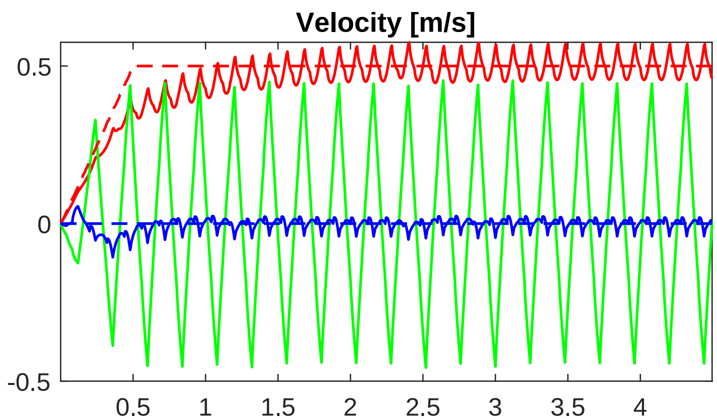


Figure 7: Pacing: linear velocity plot

Gallop is a highly dynamic gait, where only one foot (or none) is in contact with the ground. The spikes in the GRFs are pretty high in magnitude, similarly to bound. The dynamic switching between the feet that touch the ground generates high oscillations in the orientation of the robot, around all three axes. The velocity along x show the largest oscillation around the reference value of all the examined gaits.

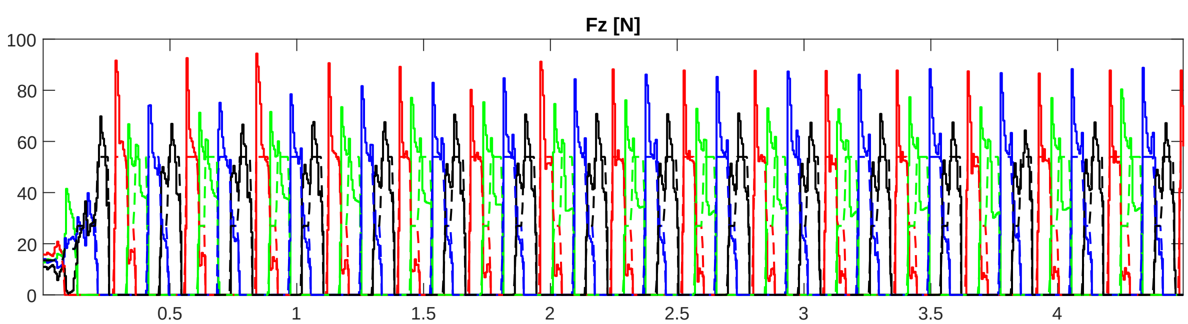


Figure 8: Gallop: force plot

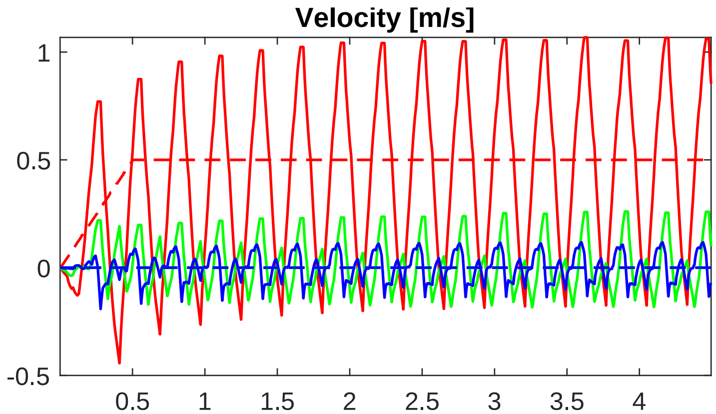


Figure 9: Gallop: linear velocity plot

Trot run is a variation of the trot without a stance phase. Instantly, the feet on one diagonal switch from stance to swing, and vice-versa for the other diagonal. Therefore a moderate yaw oscillation is observed. Being more dynamic than trot, it shows a more accentuated periodic vertical motion (blue trace in Fig. 11a) The GRFs are much higher in magnitude than the trot, and they show a more impulsive time evolution. However, this is the gait where the velocity reference is tracked most closely, with negligible periodic oscillations.

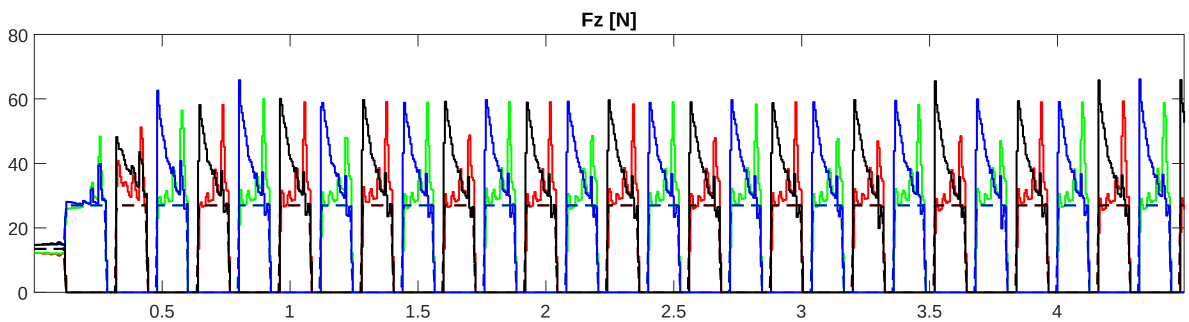
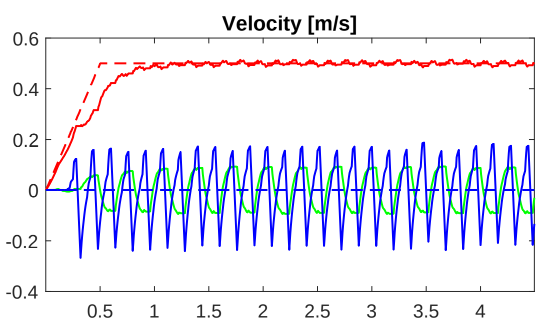
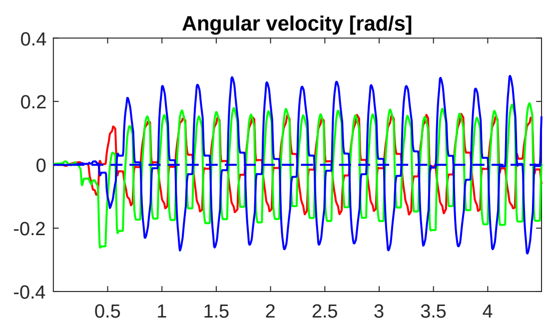


Figure 10: Trot run: force plot



(a) Linear velocity



(b) Angular velocity

Figure 11: Trot run: velocity plot

Crawl is a statically balanced gait, where 3 feet are in contact with the ground the whole time. Therefore the GRFs are very low, similarly to trot. A small pitch motion is observed in Fig. 12. However, at low speeds, the velocity reference is tracked closely.

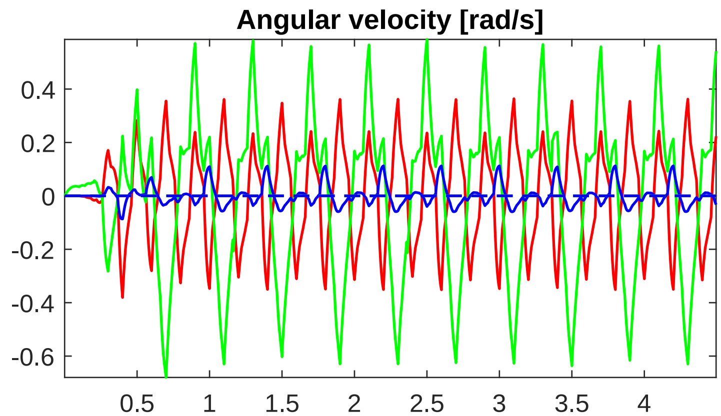
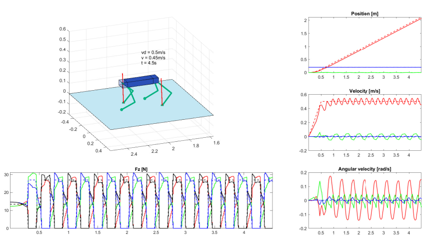
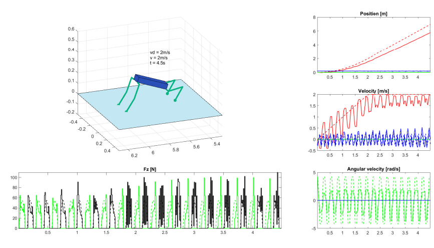


Figure 12: Crawl: angular velocity plot

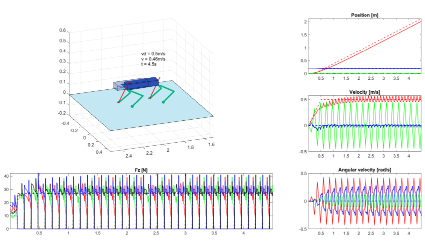
Note: in the dynamically balanced gaits and for some other extreme conditions (very high mass and very low friction), it was observed that the legs can cross the ground level and go under it. Supposedly, this is due to the simulation environment that does not include a ground collision check for the legs. In a real situation, there would be a collision with the ground and the robot would fall.



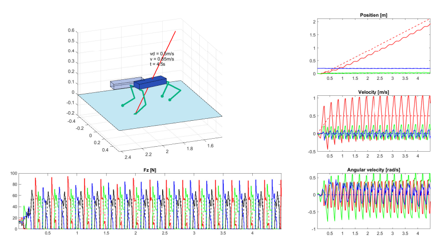
(a) Trot



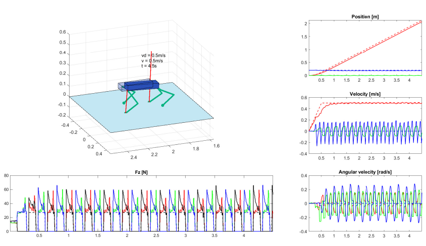
(b) Bound



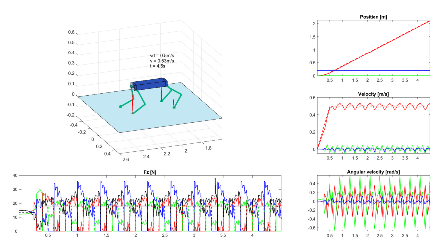
(c) Pacing



(d) Gallop



(e) Trot run

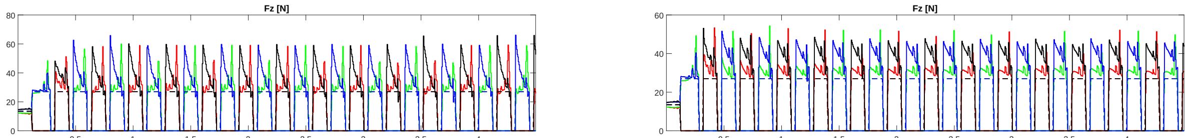


(f) Crawl

Figure 13: Overview of different gaits

Decrease in friction

We conducted simulations with a strong decrease in the friction coefficient, from the standard value $\mu = 1$ to $\mu = 0.1$. Friction is needed to develop the ground reaction forces, so that the foot sticks to the ground and is able to push against it to propel the motion. As of the Coulomb friction model, the GRFs need to stay within the friction cone, whose radius is proportional to the friction coefficient: in other words, the smaller the friction coefficient, the easier it is to slip. The simulations show that the magnitude of the GRFs are greatly reduced, and they drop to 0 more abruptly when a foot switches from stance to swing: a closeup for the trot run can be seen in Fig. 14.

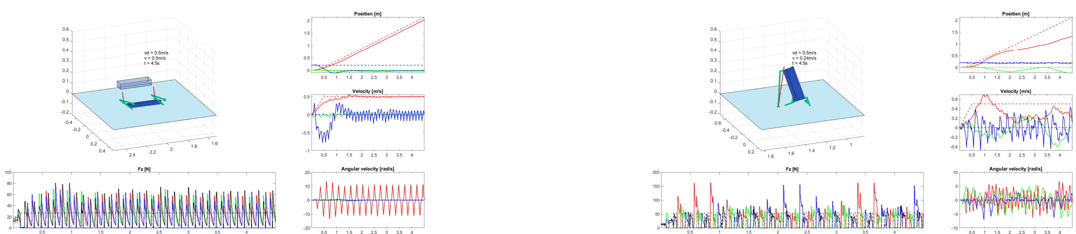


(a) $\mu = 1$ (b) $\mu = 0.1$

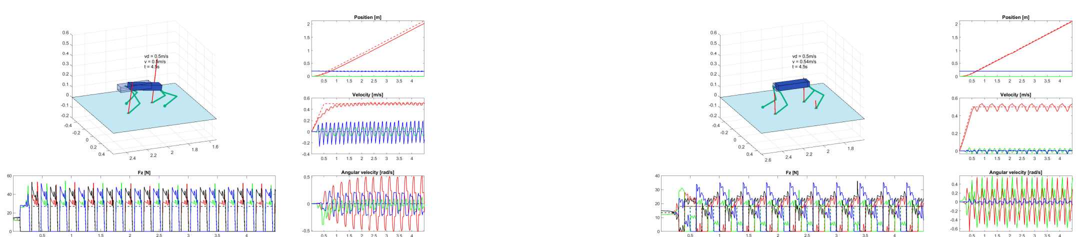
Figure 14: Comparison of force plots with different friction (trot run)

In the simulations, we observed that the more dynamic gaits, that rely on high values of the GRFs, fail on a slippery ground. This is intuitive, because in the dynamically balanced gaits the zero moment point does not always fall inside of the support polygon, so the GRFs are needed in order to keep the robot standing. As an example, see the attached videos *gait2_mu01.mp4* and *gait3_mu01.mp4*. On the contrary, crawl performs almost perfectly in this harsh condition because of its exceptional stability. In trot and trot run, the desired velocity is reached after a longer transient. The simulation results are shown in Fig. 15.

(a) Trot (b) Bound



(c) Pacing (d) Gallop

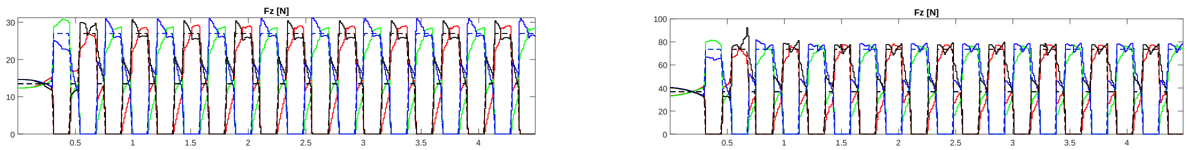


(e) Trot run (f) Crawl

Figure 15: Overview of gaits with lower friction

Increase in mass

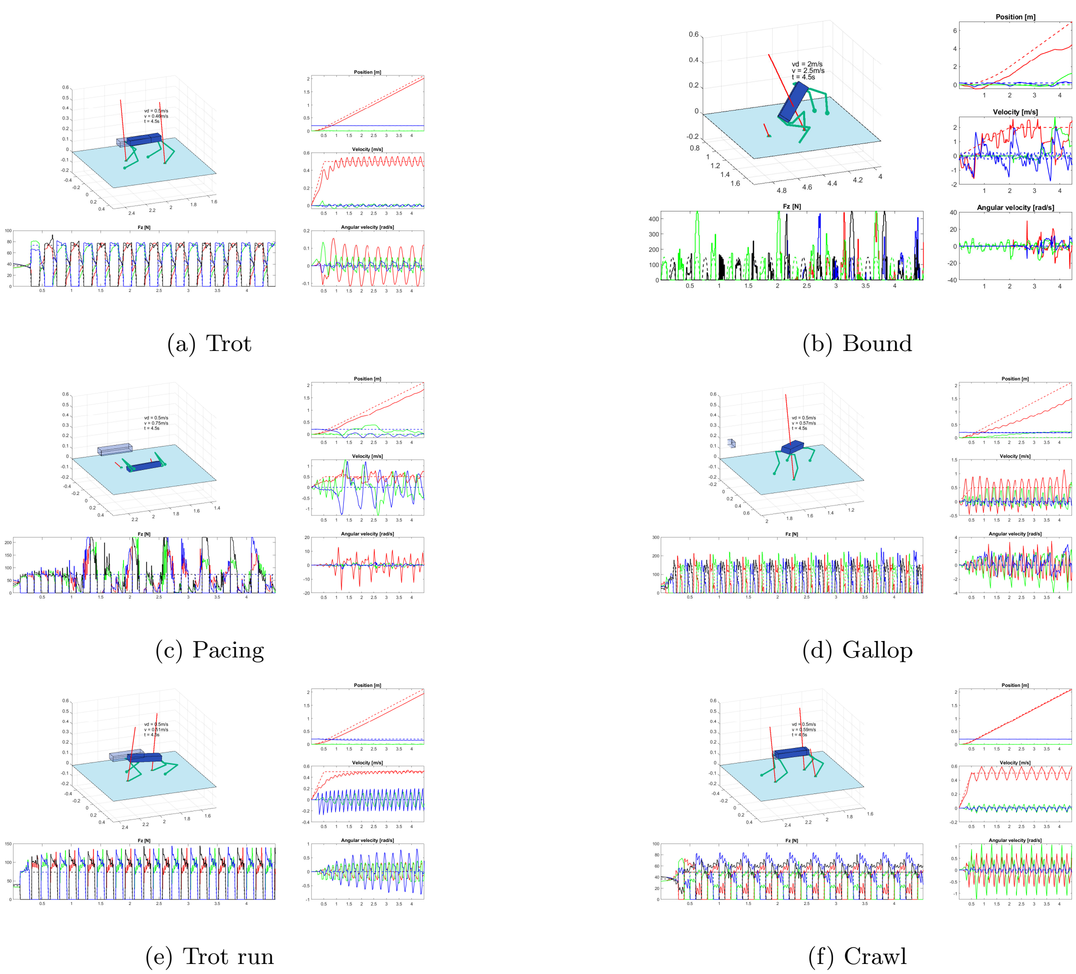
Some simulations were conducted to highlight the influence of a larger mass on the gaits, as seen in Fig. 17. The mass was modified from the standard value $m = 5.5kg$ to $m = 15kg$. We were expecting a significant increase in the magnitude of the GRFs: as an example, see the comparison in Fig. 16.



(a) $m = 5.5kg$ (b) $m = 15kg$

Figure 16: Comparison of force plots with different mass (trot)

We observed that the gaits where the robot oscillates less can withstand an increase in mass with minor issues: the GRFs and the oscillations of the velocity profile increase, but the reference velocity is reached (see trot, trot run and crawl). The other gaits, instead, either lose balance (pacing and bound), or cannot reach the desired velocity (gallop).



(a) Trot (b) Bound

(c) Pacing (d) Gallop

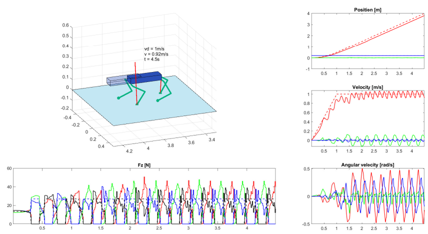
(e) Trot run (f) Crawl

Figure 17: Overview of gaits with a much higher mass

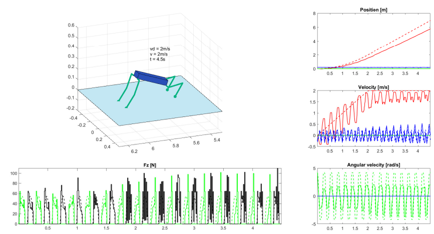
It should be noticed that the only parameter that was modified for these simulations is mass, while inertia was kept at the standard value. Physically, this would mean that the additional mass is concentrated in the center of mass. In reality, a proportional increase in inertia would most likely occur. In the simulations where the inertia is increased proportionally to the mass, we observed similar effects to the simple increase in mass. One notable difference is a decrease in the oscillatory motion of the robot, when the inertia is increased rather than kept constant.

Increase in reference velocity

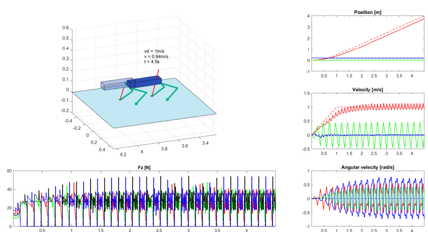
Finally, we examined the effects of an increase in the desired velocity, from $v_d = 0.5m/s$ to $v_d = 1m/s$. We observed a slight increase in the magnitude and steepness of the ground reaction forces. As a result, the legged robot oscillates more in its motion. Naturally, the transient to reach the reference velocity increases slightly. All the simulations are shown in Fig. 18.



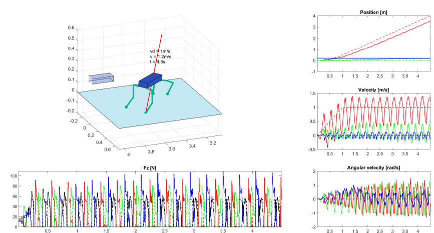
(a) Trot



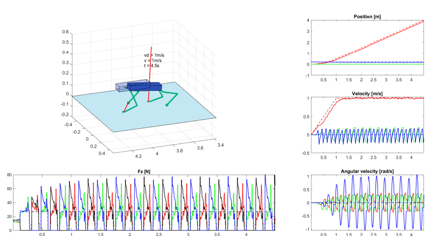
(b) Bound



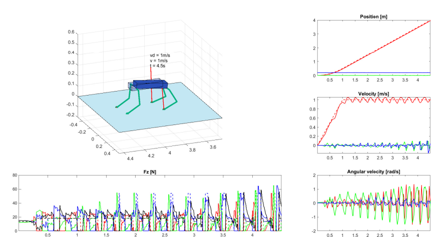
(c) Pacing



(d) Gallop



(e) Trot run



(f) Crawl

Figure 18: Overview of gaits with a reference velocity $v_d = 1m/s$

Exercise 4

a) Consider the leg in Fig. 19. The foot is in stance configuration (e.g. is fixed on the ground), and both the leg and foot are massless, so that the mass is concentrated in m. If the ankle joint is not actuated, the whole system behaves like an inverted pendulum. This implies that the point $\theta^* = \frac{\pi}{2}$ is an unstable equilibrium. In other words, the leg at configuration $\theta = \frac{\pi}{2} + \epsilon$ would fall, since there would be no torque to counteract gravity, namely

$$\tau_g = mgl \sin(\hat{\theta}),$$

where the variable transformation $\hat{\theta} = \frac{\pi}{2} - \theta$ was employed. This allows us to write the inverted pendulum equation

$$ml^2 \ddot{\hat{\theta}} = \tau_g.$$

Using linearization, it is easily proved that the equilibrium $\hat{\theta}^* = 0$ is unstable, and therefore also $\theta^* = \frac{\pi}{2}$. This would be true also if we considered linear stiffness and damping terms for the leg.

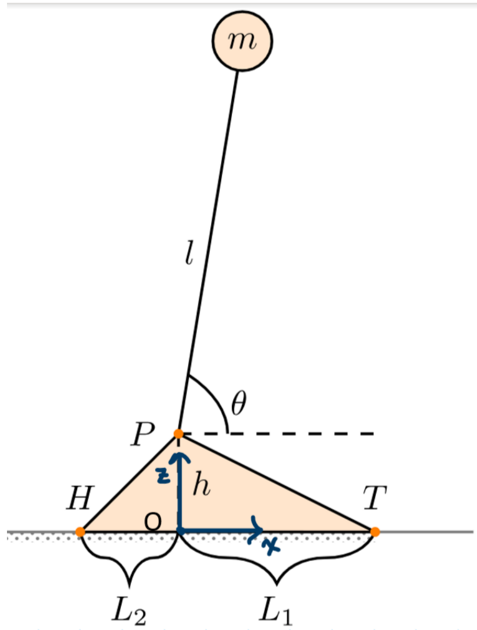


Figure 19: Depiction of the leg and the assumed reference frame.

b) Consider again the leg in Fig. 19: as in the case above, the ankle joint is not actuated. Therefore, no torque is employed to counteract for gravity. In the 2D case, the x-coordinate of the ZMP on the ground can be computed as:

$$p_z^x = p_c^x - \frac{p_c^z}{\ddot{p}_c^z - g_0^z}(\ddot{p}_c^x - g_0^x) - \frac{1}{m(\ddot{p}_c^z - g_0^z)}\dot{L}^y, \tag{2}$$

where

- $p_c = [p_c^x,\ p_c^z]^T = [l\cos(\theta),\ h + l\sin(\theta)]^T$ is the position of the centre of mass;
- $g_0 = [g_0^x,\ g_0^z]^T = [0,\ -g]^T$ is the gravity vector on flat ground;
- $\dot{L}^y = I^y\dot{\omega}^y = m(l^2+h^2)\ddot{\theta}$ is the time variation of the angular momentum about the y axis. The inertia of the leg was computed by using the Huygens-Steiner theorem, assuming that the leg and foot are massless.

From these definitions, the ZMP is computed as:

$$p_z^x = l\cos(\theta) + \frac{h + l\sin(\theta)}{l\cos(\theta)\ddot{\theta} - l\sin(\theta)\dot{\theta}^2 + g}(l\sin(\theta)\ddot{\theta} + l\cos(\theta)\dot{\theta}^2) - \frac{1}{m(l\cos(\theta)\ddot{\theta} - l\sin(\theta)\dot{\theta}^2 + g)}(m(l^2+h^2)\ddot{\theta}) \tag{3}$$

$$= l\cos(\theta) + \frac{(h + l\sin(\theta))(l\sin(\theta)\ddot{\theta} + l\cos(\theta)\dot{\theta}^2) - (l^2+h^2)\ddot{\theta}}{l\cos(\theta)\ddot{\theta} - l\sin(\theta)\dot{\theta}^2 + g} \tag{4}$$

The position of the ZMP is, thus, a function of the angle θ and its time derivatives, and of the geometric parameters l and h. This is intuitive since they characterize the distance of the centre of mass from the ground.

c) The condition so that the leg does not fall is that the centre of pressure (or zero moment point) p_z lies within the support polygon, that is the convex hull of the contact points of the stance feet. In this 2-dimensional case with only one leg, this implies that its x-coordinate p_z^x lies within the line connecting the points H and T:

$$-L_2 \leq p_z^x \leq L_1, \tag{5}$$

assuming the frame of reference of centre O displayed in Fig. 19. Assume that the ankle joint is actuated, and the torque provided can perfectly counteract the torque due to gravity. This implies that the posture of the leg is a fixed point, e.g. $\ddot{p}_c = \dot{L} = 0$, where $\ddot{p}_c$ is the acceleration of the centre of mass and $\dot{L}$ the time derivative of the angular momentum. In the case of fixed point, the formula for the ZMP reads:

$p_z^x = p_c^x - \frac{p_c^z}{g_0^z} g_0^x$.

Since the gravity vector $g_0 = [0, \ -g]^T$ for the 2D case of the foot standing on a flat surface, the ZMP coincides with the projection of the centre of mass onto the ground: $p_z^x = p_c^x = lcos(\theta)$. Substituting into Eq. 5, we obtain:

$$\arccos(\frac{L_1}{l}) \leq \theta \leq \arccos(-\frac{L_2}{l}) \tag{6}$$

That is the condition for the static balance of the leg in a fixed point configuration.