

## Wave propagation

2.1 Introduction

2.2 Maxwell's Equations

2.3 Electromagnetic waves

2.3.1 Wave equation

2.3.2 Plane waves

2.4 Fields of current distributions

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## Electromagnetic waves

### 2.3 Electromagnetic waves

#### 2.3.1 Wave equation

#### 2.3.2 Plane waves

**Source free area**

$$\operatorname{div} \mathbf{D} = 0$$

$$\operatorname{div} \mathbf{B} = 0$$

$$\operatorname{rot} \mathbf{E} = -j\omega \mathbf{B}$$

$$\operatorname{rot} \mathbf{H} = j\omega \mathbf{D}$$

Goal: Find equation which contains only electric or only magnetic fields.

Source free area

$$\begin{aligned}\operatorname{div} \mathbf{D} &= 0 \\ \operatorname{div} \mathbf{B} &= 0 \\ \operatorname{rot} \mathbf{E} &= -j\omega \mathbf{B} \\ \operatorname{rot} \mathbf{H} &= j\omega \mathbf{D}\end{aligned}$$

Goal: Find equation which contains only electric or only magnetic fields.

## Wave equation

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}$$

$$\nabla \times \mathbf{H} = j\omega \mathbf{D}$$

Subst. second eq. into first:

$$\frac{1}{j\omega\epsilon} \nabla \times \nabla \times \mathbf{H} = -j\omega\mu \mathbf{H}.$$

And thus:

$$\frac{1}{\omega^2\mu\epsilon} \nabla \times \nabla \times \mathbf{H} = \mathbf{H}.$$

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Goal: Find equation which contains only electric or only magnetic fields.

## Source free area

$$\frac{1}{\omega^2 \mu \epsilon} \nabla \times \nabla \times \mathbf{H} = \mathbf{H}$$

Using speed of light<sup>a</sup>

$$c = \frac{1}{\sqrt{\epsilon \mu}}$$

leads to

$$\frac{c^2}{\omega^2} \nabla \times \nabla \times \mathbf{H} = \mathbf{H}.$$

<sup>a</sup>in vacuum:  $c = 299\,792\,458 \text{ m s}^{-1}$

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**Wave equation**

$$\begin{aligned} \nabla \times \mathbf{E} &= -j\omega \mathbf{B} \\ \nabla \times \mathbf{H} &= j\omega \mathbf{D} \end{aligned}$$

Subst. second eq. into first:

$$\frac{1}{j\omega} \nabla \times \nabla \times \mathbf{H} = -j\omega \mathbf{H}$$

And thus:

$$\frac{1}{\omega^2 \mu^2} \nabla \times \nabla \times \mathbf{H} = \mathbf{H}$$

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$$\begin{aligned} \text{div } \mathbf{D} &= 0 \\ \text{div } \mathbf{B} &= 0 \\ \text{rot } \mathbf{E} &= -j\omega \mathbf{B} \\ \text{rot } \mathbf{H} &= j\omega \mathbf{D} \end{aligned}$$

Goal: Find equation which contains only electric or only magnetic fields.

## Source free area

With wavelength<sup>a</sup>  $\lambda$  and wavenumber

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

one gets:

$$\frac{c^2}{\omega^2} \nabla \times \nabla \times \mathbf{H} = \mathbf{H}$$

and thus one gets the **wave equation**:

$$\nabla \times \nabla \times \mathbf{H} = k^2 \mathbf{H}.$$

We are now looking for solutions.

$$^a \lambda = 0.3 \text{ m GHz}^{-1}. \text{ Example: } \lambda = 0.3 \text{ m @ 1 GHz}$$

## Electromagnetic waves

### 2.3 Electromagnetic waves

#### 2.3.1 Wave equation

#### 2.3.2 Plane waves

**Ansatz**

For the solution of the equation

$$\nabla \times \nabla \times \mathbf{H} = \mathbf{H}k^2$$

we are using the Ansatz

$$\mathbf{H} = \mathbf{e}_x e^{-jky}.$$

This fulfills the wave equation.



**Ansatz**

For the solution of the equation

$$\nabla \times \nabla \times \mathbf{H} = \mu_0 \mathbf{H}$$

we are using the Ansatz

$$\mathbf{H} = \mathbf{e}_x e^{-jky}$$

This fulfills the wave equation.

**Properties:**  $\mathbf{H} = \mathbf{e}_x e^{-jky}$



$$\mathbf{H}(\mathbf{r}, t) = \mathbf{e}_x \cos(\omega t - ky) :$$

- ▷ The magnetic field  $\mathbf{H}$  is perpendicular to the direction of propagation  $\mathbf{n} = \mathbf{e}_y$
- ▷ The corresponding electrical field is given by:

$$\mathbf{E} = \frac{1}{Z_f} \mathbf{n} \times \mathbf{H},$$

with

$$Z_f = \sqrt{\frac{\mu}{\epsilon}}$$

being the intrinsic **wave impedance** of the medium.

**Properties:**  $H = e_y e^{-jky}$

▷  $H(x, t) = e_y \cos(\omega t - ky)$

▷ The magnetic field  $H$  is perpendicular to the direction of propagation  $n = e_y$ .

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being the intrinsic **wave impedance** of the medium.

**Ansatz**

For the solution of the equation

$$\nabla \times \nabla \times H = H \omega^2$$

we are using the Ansatz

$$H = e_y e^{-jky}$$

This fulfills the wave equation.

## Properties

In free space, the corresponding electrical field is given by:

$$E = \frac{1}{Z_f} n \times H = \frac{1}{\pi 120 \Omega} e_z e^{-jky}.$$

**Property**

In free space, the corresponding electrical field is given by:

$$\mathbf{E} = \frac{1}{Z_0} \mathbf{a} \times \mathbf{H} = \frac{1}{120 \Omega} \mathbf{a}_y e^{-jky} e^{j\omega t}$$

**Property:**  $\mathbf{H} = \mathbf{a}_y e^{-jky} e^{j\omega t}$ 

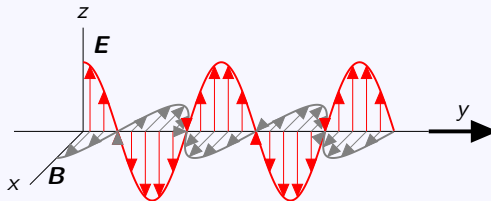
$$\mathbf{H}(x, t) = \mathbf{a}_y \cos(\omega t - ky)$$

- ▷ The magnetic field  $\mathbf{H}$  is perpendicular to the direction of propagation  $\mathbf{a} = \mathbf{a}_y$ .
- ▷ The corresponding electrical field is given by:

$$\mathbf{E} = \frac{1}{Z_0} \mathbf{a} \times \mathbf{H},$$

with

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}$$

being the intrinsic **wave impedance** of the medium.**Fields for  $t = 0$** based upon a blog entry of [Henri Menke](#)

The direction of the electric field is called **polarisation**. Here the wave is linear polarized. Other polarizations are elliptical and circular.

**Ansatz**

For the solution of the equation

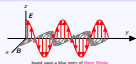
$$\nabla \times \nabla \times \mathbf{H} = -\mu \nabla^2 \mathbf{H}$$

we are using the Ansatz

$$\mathbf{H} = \mathbf{a}_y e^{-jky} e^{j\omega t}$$

This fulfills the wave equation.

Fields for  $t = 0$



The direction of the electric field is called **polarization**. Here the wave is linear polarized. Other polarizations are elliptic and circular.

Properties

In free space, the corresponding electrical field is given by:

$$\mathbf{E} = \frac{1}{Z_0} \mathbf{a} \times \mathbf{H} = \frac{1}{120\pi} \mathbf{a}_x e^{j(\omega t - ky)}$$

Properties:  $\mathbf{H} = \mathbf{a}_x e^{j\omega t}$

$$\mathbf{H}(x, t) = \mathbf{a}_x \cos(\omega t - ky)$$

- ▷ The magnetic field  $\mathbf{H}$  is perpendicular to the direction of propagation  $\mathbf{a} = \mathbf{a}_x$ .
- ▷ The corresponding electrical field is given by:

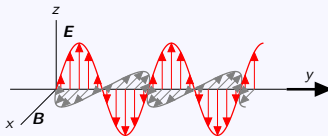
$$\mathbf{E} = \frac{1}{Z_0} \mathbf{a} \times \mathbf{H}$$

with

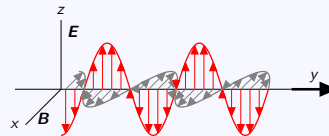
$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}$$

being the intrinsic **wave impedance** of the medium.

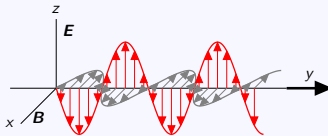
Fields for  $t = \frac{T}{4}$



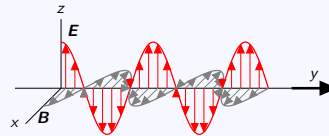
Fields for  $t = 2\frac{T}{4}$

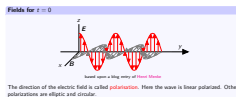
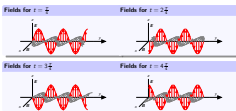


Fields for  $t = 3\frac{T}{4}$



Fields for  $t = 4\frac{T}{4}$





## Proof of correctness

It follows

$$\begin{aligned}
 \frac{1}{k^2} \nabla \times \nabla \times \mathbf{H} &= \frac{1}{k^2} \nabla \times \left[ \left( \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \right) \times \mathbf{e}_x e^{-jky} \right] \\
 &= \frac{1}{k^2} \nabla \times \left[ \mathbf{e}_z \left( -\frac{\partial}{\partial y} e^{-ky} \right) \right] \\
 &= -\frac{1}{k^2} \left( \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \right) \times \left[ \mathbf{e}_z k e^{-jky} \right] \\
 &= \frac{1}{k} \mathbf{e}_x \frac{\partial}{\partial y} e^{-ky} = \mathbf{e}_x e^{-jky} = \mathbf{H}
 \end{aligned}$$

### Property

In free space, the corresponding electrical field is given by:

$$\mathbf{E} = \frac{1}{\epsilon_0} \nabla \times \mathbf{H} = \frac{1}{\epsilon_0 \omega} \nabla \times \frac{\partial \mathbf{H}}{\partial t}$$