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Wave propagation

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Wave equation

Source free area

$$
\begin{aligned}
\operatorname{div} \boldsymbol{D} &= 0 \\
\operatorname{div} \boldsymbol{B} &= 0 \\
\operatorname{rot} \boldsymbol{E} &= -j\omega \boldsymbol{B} \\
\operatorname{rot} \boldsymbol{H} &= j\omega \boldsymbol{D}
\end{aligned}
$$

Goal: Find equation which contains only electric or only magnetic fields.

Source free area

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Wave equation

$$\begin{aligned} \nabla \times \boldsymbol{E} &= -j\omega \boldsymbol{B} \\ \nabla \times \boldsymbol{H} &= j\omega \boldsymbol{D} \end{aligned}$$

Subst. second eq. into first:

$$\frac{1}{j\omega\epsilon} \nabla \times \nabla \times \boldsymbol{H} = -j\omega\mu \boldsymbol{H}.$$

And thus:

$$\frac{1}{\omega^2\mu\epsilon} \nabla \times \nabla \times \boldsymbol{H} = \boldsymbol{H}.$$

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$$\frac{1}{\omega^2\mu\epsilon}\nabla \times \nabla \times \boldsymbol{H} = \boldsymbol{H}$$

Using speed of light[a]

$$c = \frac{1}{\sqrt{\epsilon\mu}}$$

leads to

$$\frac{c^2}{\omega^2}\nabla \times \nabla \times \boldsymbol{H} = \boldsymbol{H}.$$

[a] in vacuum: $c = 299\,792\,458\,\mathrm{m\,s^{-1}}$

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Source free area

With wavelength[a] λ and wavenumber

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

one gets:

$$\frac{c^2}{\omega^2}\nabla\times\nabla\times\boldsymbol{H} = \boldsymbol{H}$$

and thus one gets the wave equation:

$$\boxed{\nabla\times\nabla\times\boldsymbol{H} = k^2\boldsymbol{H}.}$$

We are now looking for solutions.

[a] $\lambda = 0.3\,\mathrm{m\,GHz^{-1}}$. Example: $\lambda = 0.3\,\mathrm{m}$ @ $1\,\mathrm{GHz}$

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Electromagnetic waves

Example

Ansatz

For the solution of the equation

$$\nabla \times \nabla \times \boldsymbol{H} \quad = \quad \boldsymbol{H} k^2$$

we are using the Ansatz

$$\boldsymbol{H} = \boldsymbol{e}_x e^{-jky}.$$

This fulfills the wave equation.

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▷

$$\boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{e}_x \cos(\omega t - ky) :$$

▷ The magnetic field $\boldsymbol{H}$ is perpendicular to the direction of propagation $\boldsymbol{n} = \boldsymbol{e}_y$

▷ The corresponding electrical field is given by:

$$\boldsymbol{E} = \frac{1}{Z_f} \boldsymbol{n} \times \boldsymbol{H},$$

with

$$Z_f = \sqrt{\frac{\mu}{\varepsilon}}$$

being the intrinsic wave impedance of the medium.

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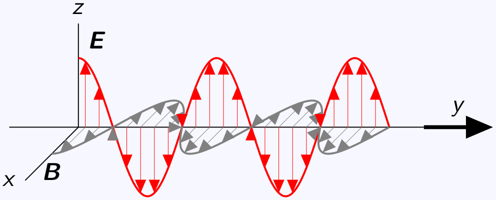
Fields for $t = 0$



based upon a blog entry of Henri Menke

The direction of the electric field is called polarisation. Here the wave is linear polarized. Other polarizations are elliptic and circular.

Example

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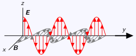

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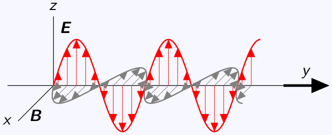
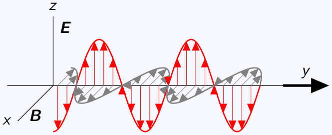
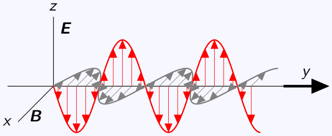


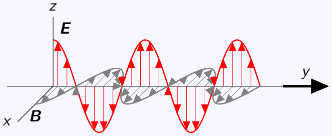
Fields for $t = 2\frac{T}{4}$



Fields for $t = 3\frac{T}{4}$

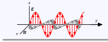


Fields for $t = 4\frac{T}{4}$

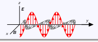


Example

Fields for $t = \frac{T}{4}$



Fields for $t = 2\frac{T}{4}$



Fields for $t = 3\frac{T}{4}$

Fields for $t = 4\frac{T}{4}$

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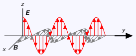

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In free space, the corresponding electrical field is given by:

$$\boldsymbol{E} = \frac{1}{Z_f}\boldsymbol{n} \times \boldsymbol{H} = \frac{1}{\pi 120\,\Omega}\boldsymbol{e}_z e^{-jky}.$$

Proof of correctness

It follows

$$\begin{aligned}
\frac{1}{k^2}\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{H} &= \frac{1}{k^2}\boldsymbol{\nabla} \times \left[\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \boldsymbol{e}_x e^{-jky}\right] \\
&= \frac{1}{k^2}\boldsymbol{\nabla} \times \left[\boldsymbol{e}_z\left(-\frac{\partial}{\partial y}e^{-ky}\right)\right] \\
&= -\frac{1}{k^2}\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \left[\boldsymbol{e}_z k e^{-jky}\right] \\
&= \frac{1}{k}\boldsymbol{e}_x \frac{\partial}{\partial y}e^{-ky} = \boldsymbol{e}_x e^{-jky} = \boldsymbol{H}
\end{aligned}$$