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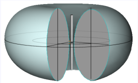
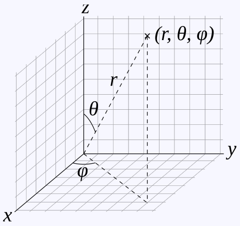
Far field of a very short current element (Hertzian dipole) orientated in $\boldsymbol{e}_z$-direction:

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{jk}{4\pi} Ih \frac{e^{-jkr}}{r} \sin\theta \boldsymbol{e}_\theta .$$



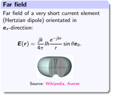
Source: Wikipedia, Averse

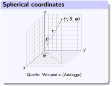
Spherical coordinates



Quelle: Wikipedia (Andeggs)

Herzscher Dipol



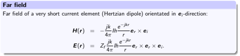


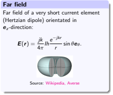
Far field

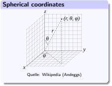
Far field of a very short current element (Hertzian dipole) orientated in $\boldsymbol{e}_i$-direction:

$$\begin{aligned}\boldsymbol{H}(\boldsymbol{r}) &= -\frac{jk}{4\pi} lh \frac{e^{-jkr}}{r} \boldsymbol{e}_r \times \boldsymbol{e}_i \\ \boldsymbol{E}(\boldsymbol{r}) &= Z_f \frac{jk}{4\pi} lh \frac{e^{-jkr}}{r} \boldsymbol{e}_r \times \boldsymbol{e}_r \times \boldsymbol{e}_i.\end{aligned}$$

Herzscher Dipol







Useful equations

$$\begin{aligned}
\boldsymbol{e}_r \times \boldsymbol{e}_\vartheta &= \boldsymbol{e}_\varphi \\
\boldsymbol{e}_\vartheta \times \boldsymbol{e}_\varphi &= \boldsymbol{e}_r \\
\boldsymbol{e}_r \times \boldsymbol{e}_\varphi &= -\boldsymbol{e}_\vartheta
\end{aligned}$$

and thus

$$\begin{aligned}
\boldsymbol{e}_r \times (A_r \boldsymbol{e}_r + A_\vartheta \boldsymbol{e}_\vartheta + A_\varphi \boldsymbol{e}_\varphi) &= (A_\vartheta \boldsymbol{e}_\varphi - A_\varphi \boldsymbol{e}_\vartheta) \\
\boldsymbol{e}_r \times \boldsymbol{e}_r \times (A_r \boldsymbol{e}_r + A_\vartheta \boldsymbol{e}_\vartheta + A_\varphi \boldsymbol{e}_\varphi) &= (-A_\vartheta \boldsymbol{e}_\vartheta - A_\varphi \boldsymbol{e}_\varphi) \,.
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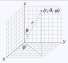
Source: Wikipedia, Averse

Spherical coordinates



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Useful equations

Consequently,

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) &= Z_f \frac{jk}{4\pi} Ih \frac{e^{-jkr}}{r}\boldsymbol{e}_r \times \boldsymbol{e}_r \times \boldsymbol{e}_i \\ &= -Z_f \frac{jk}{4\pi} Ih \frac{e^{-jkr}}{r}\left(\boldsymbol{e}_{i,\vartheta} + \boldsymbol{e}_{i,\varphi}\right).\end{aligned}$$

Herzscher Dipol

Useful equations

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Useful equations

Vector fields can be converted from cartesian to spherical by making use of (see Wikipedia as well):

$$\begin{aligned}A_r &= A_x \sin\vartheta\cos\varphi + A_y \sin\vartheta\sin\varphi + A_z\cos\vartheta \\ A_\vartheta &= A_x \cos\vartheta\cos\varphi + A_y \cos\vartheta\sin\varphi - A_z\sin\vartheta \\ A_\varphi &= -A_x\sin\varphi + A_y\cos\varphi\end{aligned}$$

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Fields of current distributions

Radiation in Free Space

Fields of Electrical Currents

The following considers electrically ideally conductive bodies in otherwise free space. The field radiated by currents[a] on the surface A of these bodies is given by[b]:

$$\boldsymbol{E}(\boldsymbol{r}, t) = -\boldsymbol{\nabla}\phi(\boldsymbol{r}, t) - \frac{\partial}{\partial t}\boldsymbol{A}(\boldsymbol{r}, t).$$

If the currents are known, the fields can be determined. However, determining the currents is generally not trivial.

[a] In certain cases, such as horn antennas, magnetic currents are also considered
[b] See, for example, Blume, Theory of Electromagnetic Fields

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Potentials

$$\begin{aligned} \boldsymbol{A}(\boldsymbol{r},t) &= \frac{\mu}{4\pi}\int_A \frac{\boldsymbol{J}(\boldsymbol{r}',t')}{r}da', \\ \phi(\boldsymbol{r},t) &= \frac{1}{4\pi\varepsilon}\int_A \frac{\rho(\vec{r}',t')}{r}da', \end{aligned}$$

with

$$r = |\boldsymbol{r} - \boldsymbol{r}'|$$

and

$$t' = t - \frac{r}{c}.$$

Radiation in free space

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Fields of electrical currents

Thus, an integral representation for the electric field can be specified:

$$\boldsymbol{E}(\boldsymbol{r},t) = -\nabla\frac{1}{4\pi\varepsilon}\int_A \frac{\rho(\vec{r}',t')}{r}da' - \frac{\partial}{\partial t}\frac{\mu}{4\pi}\int_A \frac{\vec{J}(\vec{r}',t')}{r}da'.$$

The relationship between the electric current density $\vec{J}$ and the electric charge density ρ is described by the continuity equation:

$$\nabla\cdot\boldsymbol{J}(\boldsymbol{r}',t) + \frac{\partial}{\partial t}\rho(\boldsymbol{r}',t) = 0.$$

Radiation in free space

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Fields of electrical currents

Integral representation of the electric charge density:

$$\rho(\boldsymbol{r}\,',t) = -\int_{-\infty}^{t} \nabla\cdot\boldsymbol{J}(\boldsymbol{r}\,',t)dt.$$

And thus for $\boldsymbol{J}(\boldsymbol{r},t) = 0$ for $t < 0$:

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r},t) &= \nabla\frac{1}{4\pi\varepsilon}\int_A \frac{\int_0^t \nabla\cdot\boldsymbol{J}(\boldsymbol{r}\,',t')dt}{r}da'\\ &\quad -\frac{\partial}{\partial t}\frac{\mu}{4\pi}\int_A \frac{\boldsymbol{J}(\boldsymbol{r}\,',t')}{r}da'.\end{aligned}$$

Note that ∇ is applied to different coordinates (source (primed) and observation point (unprimed)).

Radiation in free space

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Field of electrical currents

Time harmonic case:

$$\boldsymbol{E}(\boldsymbol{r}) = \nabla \frac{1}{4\pi\varepsilon} \int_A \frac{\nabla \cdot \boldsymbol{J}(\boldsymbol{r}')}{j\omega} \frac{e^{-jkr}}{r} da' - \frac{j\omega\mu}{4\pi} \int_A \boldsymbol{J}(\boldsymbol{r}') \frac{e^{-jkr}}{r} da',$$

with

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Field of electrical currents

For large distances between antenna and observation point one can approximate

$$
\begin{aligned}
\boldsymbol{E}(\boldsymbol{r}) &= \nabla \frac{1}{4\pi\varepsilon} \int_A \frac{\nabla \cdot \boldsymbol{J}(\boldsymbol{r}\,')}{j\omega} \frac{e^{-jkr}}{r} da' - \frac{j\omega\mu}{4\pi} \int_A \boldsymbol{J}(\boldsymbol{r}\,') \frac{e^{-jkr}}{r} da' \\
&\approx \nabla \frac{1}{4\pi\varepsilon r_0} \int_A \frac{\nabla \cdot \boldsymbol{J}(\boldsymbol{r}\,')}{j\omega} e^{-jkr} da' - \frac{j\omega\mu}{4\pi r_0} \int_A \boldsymbol{J}(\boldsymbol{r}\,') e^{-jkr} da',
\end{aligned}
$$

with $r_0 \approx |\boldsymbol{r}|$ being the distance of the observation point to the center of gravity of the antenna.

Radiation in free space

Field of electrical currents

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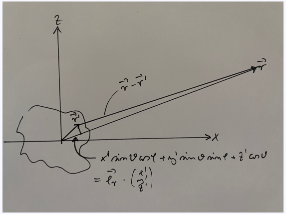
Field of electrical currents

The phase term can be approximated as follows[a]:

$$e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|} \approx e^{-jk|\boldsymbol{r}|} e^{+jk(x' \sin\vartheta \cos\varphi + y' \sin\vartheta \sin\varphi + z' \cos\vartheta)}$$



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Radiation in free space

Field of electrical currents

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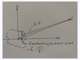

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Field of electrical currents

For large distances between antenna and observation point one can approximate

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) &= \nabla\frac{1}{4\pi\varepsilon}\int_A \frac{\nabla\cdot\boldsymbol{J}(\boldsymbol{r}')}{j\omega}\frac{e^{-jkr}}{r}da' - \frac{j\omega\mu}{4\pi}\int_A \boldsymbol{J}(\boldsymbol{r}')\frac{e^{-jkr}}{r}da' \\ &\approx \nabla\frac{1}{4\pi\varepsilon r_0}\int_A \frac{\nabla\cdot\boldsymbol{J}(\boldsymbol{r}')}{j\omega}e^{-jkr}da' - \frac{j\omega\mu}{4\pi r_0}\int_A \boldsymbol{J}(\boldsymbol{r}')e^{-jkr}da',\end{aligned}$$

with $r_0 \approx |\boldsymbol{r}|$ being the distance of the observation point to the center of gravity of the antenna.

Field of electrical currents

Thus, one gets[a]:

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) \approx\;& \nabla\frac{1}{4\pi\varepsilon\,|\boldsymbol{r}|}e^{-jk|\boldsymbol{r}|}\int_A \frac{\nabla\cdot\boldsymbol{J}(x',y',z')}{j\omega}e^{+jk(x'\sin\vartheta\cos\varphi+y'\sin\vartheta\sin\varphi+z'\cos\vartheta)}dx'dy'dz' \\ & -\frac{j\omega\mu}{4\pi\,|\boldsymbol{r}|}e^{-jk|\boldsymbol{r}|}\int_A \boldsymbol{J}(x',y',z')e^{+jk(x'\sin\vartheta\cos\varphi+y'\sin\vartheta\sin\varphi+z'\cos\vartheta)}dx'dy'dz' \\ =\;& -\frac{jk}{4\pi}\boldsymbol{E}_0(\vartheta,\varphi)\frac{e^{-jk|\boldsymbol{r}|}}{|\boldsymbol{r}|}\end{aligned}$$

[a] Due to the gradient in spherical coordinates given by

$$\mathrm{grad}\, f = \frac{\partial f}{\partial r}\boldsymbol{e}_r + \frac{1}{r}\frac{\partial f}{\partial\vartheta}\boldsymbol{e}_\vartheta + \frac{1}{r\sin\vartheta}\frac{\partial f}{\partial\varphi}\boldsymbol{e}_\varphi$$

the first anti-derivative can be neglected.

Radiation in free space

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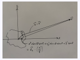

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Field of electrical currents

For large distances between antenna and observation point one can approximate

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Field of electrical currents

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$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{jk}{4\pi} \boldsymbol{E}_0(\vartheta,\varphi) \frac{e^{-jk|\boldsymbol{r}|}}{|\boldsymbol{r}|},$$

where $\boldsymbol{E}_0(\vartheta,\varphi)$ contains anti-derivatives of the form

$$\int_A \boldsymbol{J}(x',y',z') e^{jk(x'\cos\vartheta\cos\varphi+y'\sin\vartheta\sin\varphi+z'\cos\vartheta)} dx'dy'dz'$$

Compare[a] to Fourier transform: $F(\omega) = \int\limits_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \mathcal{F}\{f(t)\}$.

[a] See Klark, Antennen und Strahlungsfelder as well (available as ebook in library)

Radiation in free space

Field of electrical currents

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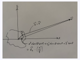
Field of electrical currents

The phase term can be approximated as follows[a]:

$$e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}\approx e^{-jk|\boldsymbol{r}|}e^{+jk(x'\sin\vartheta\cos\varphi+y'\sin\vartheta\sin\varphi+z'\cos\vartheta)}$$



[a] $\mathbf{e}_r=\sin\theta\cos\varphi\,\mathbf{e}_x+\sin\theta\sin\varphi\,\mathbf{e}_y+\cos\theta\,\mathbf{e}_z$

Summary

A more detailed examination is beyond the scope here. For further consideration, it is important to note:

- ▷ The use of potentials simplifies calculations (assumption: currents are known).
- ▷ An analysis of the time-harmonic case leads to the use of retarded potentials.
- ▷ The calculation of potentials leads to integrals of the type of Fourier transformation.
- ▷ Similarly to signals and systems, we will later examine exemplary current distributions.