

Wave propagation

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Fields of current distributions

2.4 Fields of current distributions

2.4.1 Hertzian dipole

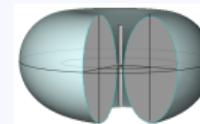
2.4.2 General solution

2.4.3 Far field of antennas

Far field

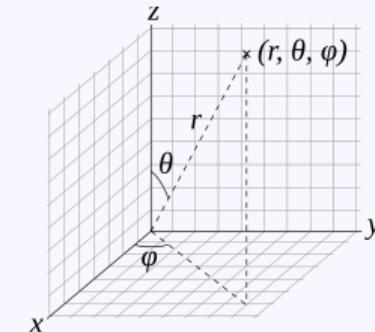
Far field of a very short current element (Hertzian dipole) orientated in \mathbf{e}_z -direction:

$$\mathbf{E}(\mathbf{r}) = \frac{jk}{4\pi} I h \frac{e^{-jkr}}{r} \sin \theta \mathbf{e}_\theta.$$

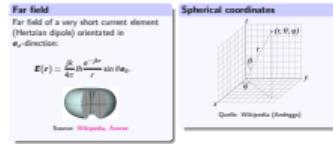


Source: [Wikipedia](#), Averse

Spherical coordinates



Quelle: Wikipedia (Andeggs)



Far field

Far field of a very short current element (Hertzian dipole) orientated in \mathbf{e}_i -direction:

$$\begin{aligned}\mathbf{H}(\mathbf{r}) &= -\frac{jk}{4\pi} lh \frac{e^{-jkr}}{r} \mathbf{e}_r \times \mathbf{e}_i \\ \mathbf{E}(\mathbf{r}) &= Z_f \frac{jk}{4\pi} lh \frac{e^{-jkr}}{r} \mathbf{e}_r \times \mathbf{e}_r \times \mathbf{e}_i.\end{aligned}$$

Far fieldFar field of a very short current element (Hertzian dipole) oriented in \mathbf{e}_r -direction:

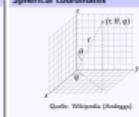
$$\begin{aligned} \mathbf{H}(r) &= -\frac{\mu_0}{4\pi r} \frac{e^{jkr}}{r} \mathbf{e}_r \times \mathbf{e}_t \\ \mathbf{E}(r) &= Z_0 \frac{\mu_0}{4\pi r} \frac{e^{jkr}}{r} \mathbf{e}_r \times \mathbf{e}_t \times \mathbf{e}_z. \end{aligned}$$

Far fieldFar field of a very short current element (Hertzian dipole) oriented in \mathbf{e}_ϑ -direction:

$$\mathbf{E}(r) = \frac{\mu_0}{4\pi} \frac{e^{jkr}}{r} \ln(\lambda r),$$



Source: Wikipedia, Author

Spherical coordinates

Quelle: Wikipedia (Autoren)

Useful equations

$$\mathbf{e}_r \times \mathbf{e}_\vartheta = \mathbf{e}_\varphi$$

$$\mathbf{e}_\vartheta \times \mathbf{e}_\varphi = \mathbf{e}_r$$

$$\mathbf{e}_r \times \mathbf{e}_\varphi = -\mathbf{e}_\vartheta$$

and thus

$$\begin{aligned} \mathbf{e}_r \times (A_r \mathbf{e}_r + A_\vartheta \mathbf{e}_\vartheta + A_\varphi \mathbf{e}_\varphi) &= (A_\vartheta \mathbf{e}_\varphi - A_\varphi \mathbf{e}_\vartheta) \\ \mathbf{e}_r \times \mathbf{e}_r \times (A_r \mathbf{e}_r + A_\vartheta \mathbf{e}_\vartheta + A_\varphi \mathbf{e}_\varphi) &= (-A_\vartheta \mathbf{e}_\vartheta - A_\varphi \mathbf{e}_\varphi). \end{aligned}$$

└ Wave propagation

└ Fields of current distributions

Useful equations

$$\begin{aligned}\mathbf{e}_r \times \mathbf{e}_\theta &= \mathbf{e}_\varphi \\ \mathbf{e}_\theta \times \mathbf{e}_r &= \mathbf{e}_\varphi \\ \mathbf{e}_r \times \mathbf{e}_\varphi &= -\mathbf{e}_\theta\end{aligned}$$

and thus

$$\begin{aligned}\mathbf{e}_r \times (\mathbf{A}_r \mathbf{e}_r + \mathbf{A}_\theta \mathbf{e}_\theta + \mathbf{A}_\varphi \mathbf{e}_\varphi) &= (\mathbf{A}_\theta \mathbf{e}_\varphi - \mathbf{A}_\varphi \mathbf{e}_\theta) \\ \mathbf{e}_\theta \times (\mathbf{A}_r \mathbf{e}_r + \mathbf{A}_\theta \mathbf{e}_\theta + \mathbf{A}_\varphi \mathbf{e}_\varphi) &= (-\mathbf{A}_\varphi \mathbf{e}_r - \mathbf{A}_r \mathbf{e}_\varphi)\end{aligned}$$

Useful equations

Consequently,

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &= Z_f \frac{jk}{4\pi} I h \frac{e^{-jkr}}{r} \mathbf{e}_r \times \mathbf{e}_r \times \mathbf{e}_i \\ &= -Z_f \frac{jk}{4\pi} I h \frac{e^{-jkr}}{r} (\mathbf{e}_{i,\vartheta} + \mathbf{e}_{i,\varphi}).\end{aligned}$$

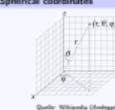
Far fieldFar field of a very short current element (Hertzian dipole) oriented in \mathbf{e}_r -direction:

$$\begin{aligned}\mathbf{H}(\mathbf{r}) &= -\frac{\mu_0}{4\pi} \frac{e^{-jkr}}{r} \mathbf{e}_r \times \mathbf{e}_i \\ \mathbf{E}(\mathbf{r}) &= Z_f \frac{\mu_0}{4\pi} \frac{e^{-jkr}}{r^2} \mathbf{e}_r \times \mathbf{e}_i \times \mathbf{e}_i.\end{aligned}$$

Far fieldFar field of a very short current element (Hertzian dipole) oriented in \mathbf{e}_r -direction:

$$\mathbf{E}(\mathbf{r}) = \frac{\mu_0}{4\pi} I h \frac{e^{-jkr}}{r^2} \sin(\theta) \mathbf{e}_r$$

Source: Wikipedia, Author

Spherical coordinates

Quelle: Wikipedia (Autoren)

Useful equations

Consequently,

$$\begin{aligned} \mathbf{E}(r) &= Z_0 \frac{\mu_0}{4\pi} \frac{e^{jkr}}{r} \mathbf{a}_r \times \mathbf{a}_\theta \times \mathbf{a}_\varphi \\ &= -Z_0 \frac{\mu_0}{4\pi} \frac{e^{jkr}}{r} (\mathbf{a}_{r,\theta} + \mathbf{a}_{r,\varphi}). \end{aligned}$$

Useful equations

Vector fields can be converted from cartesian to spherical by making use of (see [Wikipedia](#) as well):

$$\begin{aligned} \mathbf{a}_r \times \mathbf{a}_\theta &= \mathbf{a}_\varphi \\ \mathbf{a}_r \times \mathbf{a}_\varphi &= \mathbf{a}_\theta \\ \mathbf{a}_\theta \times \mathbf{a}_\varphi &= -\mathbf{a}_r \end{aligned}$$

and thus

$$\begin{aligned} \mathbf{a}_r \times (\mathbf{A}_x \mathbf{a}_r + \mathbf{A}_\theta \mathbf{a}_\theta + \mathbf{A}_\varphi \mathbf{a}_\varphi) &= (\mathbf{A}_\theta \mathbf{a}_\theta - \mathbf{A}_\varphi \mathbf{a}_\varphi) \\ \mathbf{a}_r \times \mathbf{a}_r \times (\mathbf{A}_x \mathbf{a}_r + \mathbf{A}_\theta \mathbf{a}_\theta + \mathbf{A}_\varphi \mathbf{a}_\varphi) &= (-\mathbf{A}_x \mathbf{a}_r - \mathbf{A}_\varphi \mathbf{a}_\varphi). \end{aligned}$$

$$A_r = A_x \sin \vartheta \cos \varphi + A_y \sin \vartheta \sin \varphi + A_z \cos \vartheta$$

$$A_\theta = A_x \cos \vartheta \cos \varphi + A_y \cos \vartheta \sin \varphi - A_z \sin \vartheta$$

$$A_\varphi = -A_x \sin \varphi + A_y \cos \varphi$$

Far fieldFar field of a very short current element (Hertzian dipole) oriented in \mathbf{a}_r -direction:

$$\begin{aligned} \mathbf{H}(r) &= -\frac{\mu_0}{4\pi} \frac{g^2 e}{r} \mathbf{a}_r \times \mathbf{a}_\theta \\ \mathbf{E}(r) &= Z_0 \frac{\mu_0}{4\pi} \frac{g^2 e}{r} \mathbf{a}_r \times \mathbf{a}_\theta \times \mathbf{a}_\varphi. \end{aligned}$$

Fields of current distributions

2.4 Fields of current distributions

2.4.1 Hertzian dipole

2.4.2 General solution

2.4.3 Far field of antennas

Fields of Electrical Currents

The following considers electrically ideally conductive bodies in otherwise free space. The field radiated by currents^a on the surface A of these bodies is given by^b:

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\phi(\mathbf{r}, t) - \frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t).$$

If the currents are known, the fields can be determined. However, determining the currents is generally not trivial.

^aIn certain cases, such as horn antennas, magnetic currents are also considered

^bSee, for example, Blume, Theory of Electromagnetic Fields

Fields of Electrical Currents

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^aIn certain cases, such as horn antennas, magnetic currents are also considered.

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Potentials

$$\begin{aligned}\mathbf{A}(\mathbf{r}, t) &= \frac{\mu}{4\pi} \int_A \frac{\mathbf{J}(\mathbf{r}', t')}{r} da', \\ \phi(\mathbf{r}, t) &= \frac{1}{4\pi\epsilon} \int_A \frac{\rho(\mathbf{r}', t')}{r} da',\end{aligned}$$

with

$$r = |\mathbf{r} - \mathbf{r}'|$$

and

$$t' = t - \frac{r}{c}.$$

Potentials

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu}{4\pi r} \int_A \frac{\mathbf{J}(\mathbf{r}', t')}{r'} d\mathbf{a}'$$

$$\phi(\mathbf{r}, t) = \frac{1}{4\pi r} \int_A \frac{\rho(\mathbf{r}', t')}{r'} d\mathbf{a}'$$

with

$$r = |\mathbf{r} - \mathbf{r}'|$$

and

$$t' = t - \frac{r}{c}$$

Fields of electrical currents

Thus, an integral representation for the electric field can be specified:

$$\mathbf{E}(\mathbf{r}, t) = -\nabla \frac{1}{4\pi\epsilon} \int_A \frac{\rho(\mathbf{r}', t')}{r} d\mathbf{a}' - \frac{\partial}{\partial t} \frac{\mu}{4\pi} \int_A \frac{\mathbf{J}(\mathbf{r}', t')}{r} d\mathbf{a}'.$$

The relationship between the electric current density \vec{J} and the electric charge density ρ is described by the continuity equation:

$$\nabla \cdot \mathbf{J}(\mathbf{r}', t) + \frac{\partial}{\partial t} \rho(\mathbf{r}', t) = 0.$$

Fields of Electrical Currents

The following considers electrically ideally conductive bodies in otherwise free space. The field radiated by currents* on the surface A of these bodies is given by^{**}:

$$\mathbf{E}(\mathbf{r}, t) = -\nabla \phi(\mathbf{r}, t) - \frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t).$$

If the currents are known, the fields can be determined. However, determining the currents is generally not trivial.

* In certain cases, such as here, aperiodic, magnetic currents are also considered.

^{**}See, for example, Blau, Theory of Electromagnetic Fields.

Fields of electrical currents

Thus, an integral representation for the electric field can be specified:

$$\mathbf{E}(\mathbf{r}, t) = -\nabla \frac{1}{4\pi\epsilon_0} \int_A \frac{J(\mathbf{r}', t')}{r'} d\mathbf{a}' - \frac{\partial}{\partial t} \frac{\mu}{4\pi} \int_A \frac{\hat{J}(\mathbf{r}', t')}{r'} d\mathbf{a}'.$$

The relationship between the electric current density J and the electric charge density ρ is described by the continuity equation:

$$\nabla \cdot \mathbf{J}(\mathbf{r}', t) + \frac{\partial}{\partial t} \rho(\mathbf{r}', t) = 0.$$

Potentials

$$\begin{aligned}\mathbf{A}(\mathbf{r}, t) &= \frac{\mu}{4\pi} \int_A \frac{J(\mathbf{r}', t')}{r} d\mathbf{a}', \\ \phi(\mathbf{r}, t) &= \frac{1}{4\pi\epsilon_0} \int_A \frac{\rho(\mathbf{r}', t')}{r} d\mathbf{a}',\end{aligned}$$

with

$$r = |\mathbf{r} - \mathbf{r}'|$$

and

$$t' = t - \frac{r}{c}.$$

Fields of Electrical CurrentsThe following considers electrically ideally conductive bodies in otherwise free space. The field radiated by currents* on the surface A of these bodies is given by^b:

$$\mathbf{E}(\mathbf{r}, t) = -\nabla \phi(\mathbf{r}, t) - \frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t).$$

If the currents are known, the fields can be determined. However, determining the currents is generally not trivial.

^aIn certain cases, such as horn antennas, magnetic currents are also considered.^bSee, for example, Blasius, Theory of Electromagnetic Fields.

Fields of electrical currents

Integral representation of the electric charge density:

$$\rho(\mathbf{r}', t) = - \int_{-\infty}^t \nabla \cdot \mathbf{J}(\mathbf{r}', t') dt'.$$

And thus for $\mathbf{J}(\mathbf{r}, t) = 0$ for $t < 0$:

$$\begin{aligned}\mathbf{E}(\mathbf{r}, t) &= \nabla \frac{1}{4\pi\epsilon_0} \int_A \frac{\int_0^t \nabla \cdot \mathbf{J}(\mathbf{r}', t') dt'}{r} d\mathbf{a}' \\ &\quad - \frac{\partial}{\partial t} \frac{\mu}{4\pi} \int_A \frac{\mathbf{J}(\mathbf{r}', t')}{r} d\mathbf{a}'.\end{aligned}$$

Note that ∇ is applied to different coordinates (source (primed) and observation point (unprimed)).

Fields of electrical currents

Integral representation of the electric charge density:

$$\rho(r', t) = - \int_{-\infty}^t \nabla \cdot \mathbf{J}(r', t') dt'$$

And thus for $\mathbf{J}(r, t) = 0$ for $t < 0$:

$$\begin{aligned} \mathbf{E}(r, t) &= -\nabla \frac{1}{4\pi\epsilon_0} \int_A \frac{\nabla \cdot \mathbf{J}(r', t')}{r'} da' \\ &\quad - \frac{\partial}{\partial t} \frac{\mu}{4\pi} \int_A \frac{\mathbf{J}(r', t')}{r'} da'. \end{aligned}$$

Note that ∇ is applied to different coordinates [source (primed) and observation point (unprimed)].**Fields of electrical currents**

Thus, an integral representation for the electric field can be specified:

$$\mathbf{E}(r, t) = -\nabla \frac{1}{4\pi\epsilon_0} \int_A \frac{\mathbf{J}(r', t')}{r'} da' - \frac{\partial}{\partial t} \frac{\mu}{4\pi} \int_A \frac{\mathbf{J}(r', t')}{r'} da'.$$

The relationship between the electric current density \mathbf{j} and the electric charge density ρ is described by the continuity equation:

$$\nabla \cdot \mathbf{J}(r', t) + \frac{\partial}{\partial t} \rho(r', t) = 0.$$

Field of electrical currents

Time harmonic case:

$$\mathbf{E}(r) = -\nabla \frac{1}{4\pi\epsilon_0} \int_A \frac{\nabla \cdot \mathbf{J}(r') e^{-jkr}}{j\omega r} da' - \frac{j\omega\mu}{4\pi} \int_A \mathbf{J}(r') \frac{e^{-jkr}}{r} da',$$

with

$$r = |\mathbf{r} - \mathbf{r}'|.$$

Potentials

$$\begin{aligned} \mathbf{A}(r, t) &= \frac{\mu}{4\pi} \int_A \frac{\mathbf{J}(r', t')}{r'} da', \\ \phi(r, t) &= \frac{1}{4\pi\epsilon_0} \int_A \frac{\rho(r', t')}{r'} da', \end{aligned}$$

with

$$r = |\mathbf{r} - \mathbf{r}'|$$

and

$$t' = t - \frac{r}{c}.$$

Fields of current distributions

2.4 Fields of current distributions

2.4.1 Hertzian dipole

2.4.2 General solution

2.4.3 Far field of antennas

Field of electrical currents

For large distances between antenna and observation point one can approximate

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &= \nabla \frac{1}{4\pi\varepsilon} \int_A \frac{\nabla \cdot \mathbf{J}(\mathbf{r}')}{j\omega} \frac{e^{-jkr}}{r} d\mathbf{a}' - \frac{j\omega\mu}{4\pi} \int_A \mathbf{J}(\mathbf{r}') \frac{e^{-jkr}}{r} d\mathbf{a}' \\ &\approx \nabla \frac{1}{4\pi\varepsilon r_0} \int_A \frac{\nabla \cdot \mathbf{J}(\mathbf{r}')}{j\omega} e^{-jkr} d\mathbf{a}' - \frac{j\omega\mu}{4\pi r_0} \int_A \mathbf{J}(\mathbf{r}') e^{-jkr} d\mathbf{a}',\end{aligned}$$

with $r_0 \approx |\mathbf{r}|$ being the distance of the observation point to the center of gravity of the antenna.

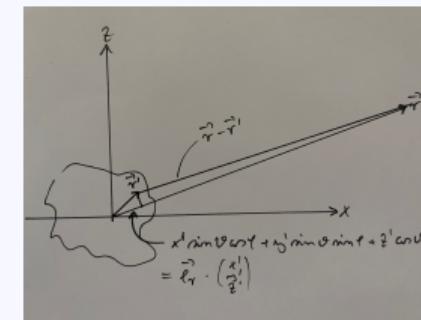
Field of electrical currents

For large distances between antenna and observation point one can approximate

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \nabla \frac{1}{4\pi\epsilon_0} \int_A \frac{\nabla \cdot \mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} e^{-jk|\mathbf{r}'|} d\mathbf{r}' - \frac{jk\mu_0}{4\pi\epsilon_0} \int_A \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}'|}}{|\mathbf{r}'|} d\mathbf{r}' \\ &\approx \frac{1}{4\pi\epsilon_0\eta_0} \int_A \frac{\nabla \cdot \mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} e^{-jk|\mathbf{r}'|} d\mathbf{r}' - \frac{jk\mu_0}{4\pi\epsilon_0\eta_0} \int_A \mathbf{J}(\mathbf{r}') e^{-jk|\mathbf{r}'|} d\mathbf{r}', \end{aligned}$$

with $\eta_0 \approx |\mathbf{r}|$ being the distance of the observation point to the center of gravity of the antenna.**Field of electrical currents**The phase term can be approximated as follows^a:

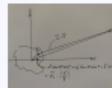
$$e^{-jk|\mathbf{r}-\mathbf{r}'|} \approx e^{-jk|\mathbf{r}|} e^{+jk(x' \sin \vartheta \cos \varphi + y' \sin \vartheta \sin \varphi + z' \cos \vartheta)}$$



$$^a \mathbf{e}_r = \sin \theta \cos \varphi \mathbf{e}_x + \sin \theta \sin \varphi \mathbf{e}_y + \cos \theta \mathbf{e}_z$$

Field of electrical currentsThe phase term can be approximated as follows^a:

$$e^{-jk(r-r')} \approx e^{-jk(r-r')}\left(1 + jk(r-r')\sin\vartheta\sin\varphi\right)$$



$$\mathbf{J}_r = \sin\vartheta\sin\varphi\mathbf{J}_\theta + \sin\vartheta\cos\varphi\mathbf{J}_\phi + \cos\vartheta\mathbf{J}_r$$

Field of electrical currents

For large distances between antenna and observation point one can approximate

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0|\mathbf{r}|} \int_A \frac{\nabla \cdot \mathbf{J}(x', y', z')}{j\omega} e^{+jk(x' \sin\vartheta \cos\varphi + y' \sin\vartheta \sin\varphi + z' \cos\vartheta)} dx' dy' dz' \\ &\approx \frac{j\omega\mu}{4\pi|\mathbf{r}|} e^{-jk|\mathbf{r}|} \int_A \mathbf{J}(x', y', z') e^{+jk(x' \sin\vartheta \cos\varphi + y' \sin\vartheta \sin\varphi + z' \cos\vartheta)} dx' dy' dz' \end{aligned}$$

$$\text{with } \eta_0 = |\mathbf{r}'| \text{ being the distance of the observation point to the center of gravity of the antenna.}$$

Field of electrical currents

Thus, one gets^a:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &\approx \nabla \frac{1}{4\pi\epsilon_0|\mathbf{r}|} e^{-jk|\mathbf{r}|} \int_A \frac{\nabla \cdot \mathbf{J}(x', y', z')}{j\omega} e^{+jk(x' \sin\vartheta \cos\varphi + y' \sin\vartheta \sin\varphi + z' \cos\vartheta)} dx' dy' dz' \\ &\quad - \frac{j\omega\mu}{4\pi|\mathbf{r}|} e^{-jk|\mathbf{r}|} \int_A \mathbf{J}(x', y', z') e^{+jk(x' \sin\vartheta \cos\varphi + y' \sin\vartheta \sin\varphi + z' \cos\vartheta)} dx' dy' dz' \\ &= -\frac{jk}{4\pi} \mathbf{E}_0(\vartheta, \varphi) \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|} \end{aligned}$$

^aDue to the gradient in spherical coordinates given by

$$\text{grad } f = \frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \vartheta} \mathbf{e}_\vartheta + \frac{1}{r \sin \vartheta} \frac{\partial f}{\partial \varphi} \mathbf{e}_\varphi$$

the first anti-derivative can be neglected.

Field of electrical currentsThus, one gets^a:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \nabla \frac{1}{4\pi r_0 |\mathbf{r}|} e^{-jkr_0} \int_A \frac{\nabla \cdot \mathbf{J}(x', y', z')}{|z'|} j(k' \sin \vartheta \cos \varphi' \sin \vartheta' \sin \varphi' + k' \sin \vartheta \cos \varphi' \sin \vartheta' \cos \varphi') dz' dy' dz' \\ &= -\frac{jk}{4\pi r_0} \int_A \mathbf{J}(x', y', z') e^{-j(k' \sin \vartheta \cos \varphi' \sin \vartheta' \sin \varphi' + k' \sin \vartheta \cos \varphi' \sin \vartheta' \cos \varphi')} dz' dy' dz' \\ &= -\frac{jk}{4\pi} \mathbf{E}_0(\vartheta, \varphi) \frac{e^{-jkr_0}}{|\mathbf{r}|} \end{aligned}$$

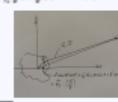
Due to the gradient in spherical coordinates follows by:

$$\text{grad } f = \frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \vartheta} \mathbf{e}_\vartheta + \frac{1}{r \sin \vartheta} \frac{\partial f}{\partial \varphi} \mathbf{e}_\varphi$$

the first anti-derivative can be neglected.

Field of electrical currentsThe phase term can be approximated as follows^a:

$$e^{j(k(r-r'))} \approx e^{-jkr'} e^{-jk(r-r') \cos \vartheta \cos \varphi' + (r-r') \sin \vartheta \sin \varphi' + k(r-r') \sin \vartheta \sin \varphi}$$

^a $\theta = \sin^{-1}(\sin \vartheta \cos \varphi + \sin \vartheta \sin \varphi \cos \theta)$, $\varphi = \tan^{-1}(\sin \vartheta \sin \varphi / \cos \vartheta)$ **Field of electrical currents**

For large distances between antenna and observation point one can approximate

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \nabla \frac{1}{4\pi r_0} \int_A \frac{\nabla \cdot \mathbf{J}(\mathbf{r}')}{|z'|} \frac{e^{-jkr'}}{r} dz' \\ &\approx -\frac{1}{4\pi r_0} \int_A \frac{\nabla \cdot \mathbf{J}(\mathbf{r}')}{|z'|} e^{-jkr'} dz' - \frac{jkr_0}{4\pi} \int_A \mathbf{J}(\mathbf{r}') \frac{e^{-jkr'}}{r^2} dz' \end{aligned}$$

with $r_0 = |\mathbf{r}'|$ being the distance of the observation point to the center of gravity of the antenna.**Field of electrical currents**

For large distances between antenna and observation point one can approximate

$$\mathbf{E}(\mathbf{r}) = -\frac{jk}{4\pi} \mathbf{E}_0(\vartheta, \varphi) \frac{e^{-jkr}}{|\mathbf{r}|},$$

where $\mathbf{E}_0(\vartheta, \varphi)$ contains anti-derivatives of the form

$$\int_A \mathbf{J}(x', y', z') e^{jk(x' \cos \vartheta \cos \varphi + y' \sin \vartheta \sin \varphi + z' \cos \vartheta)} dx' dy' dz'$$

Compare^a to **Fourier transform**: $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \mathcal{F}\{f(t)\}$.^aSee Klark, Antennen und Strahlungsfelder as well (available as ebook in library)

Field of electrical currents

For large distances between antenna and observation point one can approximate

$$\mathbf{E}(r) = -\frac{\rho}{4\pi} \mathbf{E}_d(\theta, \varphi) \frac{e^{jkr}}{|r|},$$

where $\mathbf{E}_d(\theta, \varphi)$ contains anti-derivatives of the form

$$\int J(x', y', z') j^k [x' \cos \theta \sin \varphi, y' \cos \theta \cos \varphi, z' \sin \theta] dx' dy' dz'$$

Compare* to Fourier transform: $F(w) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \mathcal{F}\{f(t)\}$.

*See Klark, Antennen und Strahlungsfelder as well (available as eBook in library)

Field of electrical currents

Thus, one gets*

$$\begin{aligned} \mathbf{E}(r) &\approx -\frac{1}{4\pi r^2} e^{-jkr} \int_{\text{ant}} \frac{\nabla \cdot J(x', y', z')}{j\omega} e^{-j[k(x' \cos \theta \sin \varphi + y' \cos \theta \cos \varphi + z' \sin \theta)]} dx' dy' dz' \\ &= -\frac{j\omega}{4\pi |r|^2} e^{-jkr} \int_{\text{ant}} J(x', y', z') j^k [x' \cos \theta \sin \varphi, y' \cos \theta \cos \varphi, z' \sin \theta] dx' dy' dz' \\ &= -\frac{\rho}{4\pi} \mathbf{E}_d(\theta, \varphi) \frac{e^{-jkr}}{|r|} \end{aligned}$$

*Due to the gradient in spherical coordinates given by

$$\operatorname{grad} f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \hat{\varphi}$$

the first anti-derivative can be neglected.

Summary

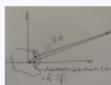
A more detailed examination is beyond the scope here. For further consideration, it is important to note:

- ▷ The use of potentials simplifies calculations (assumption: currents are known).
- ▷ An analysis of the time-harmonic case leads to the use of retarded potentials.
- ▷ The calculation of potentials leads to integrals of the type of Fourier transformation.
- ▷ Similarly to signals and systems, we will later examine exemplary current distributions.

Field of electrical currents

The phase term can be approximated as follows*.

$$e^{-j(kr - \tau)} \approx e^{-jkr} e^{-j\tau} [1 + jk(r \cos \theta \sin \varphi + r \cos \theta \cos \varphi + r \sin \theta)]$$

* $\mathbf{E}_d = \sin \theta \cos \varphi \mathbf{e}_x + \sin \theta \sin \varphi \mathbf{e}_y + \cos \theta \mathbf{e}_z$