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Discrete Time & Applications

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Filters

Study goals

- Classify basic types of filters
- Name limitations
- Transform from continuous time to discrete time and vice versa

Study goals
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- Name limitations
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Definition

Referring to LTI systems, the most common meaning of filter is a device that removes/amplifies/attenuates parts of the spectrum. An example is shown below:

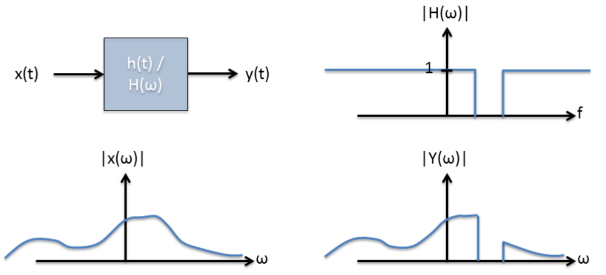

Figure 25: Example filter

Applications

Definition

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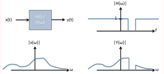

Figure 25: Example filter

Study goals

- Classify basic types of filters
- Name limitations
- Transform from continuous time to discrete time and vice versa

Examples

Figure 26: Equalizer

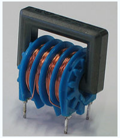

Figure 27: Choke

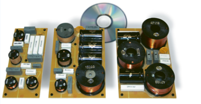

Figure 28: Audio crossover

Figure 29: Waveguide filter

Figure 30: AC line filter

Note: All pictures from Wikipedia

Classification

Examples


Figure 26: Equalizer


Figure 28: Audio crossover


Figure 30: AC line filter


Figure 27: Choke


Figure 29: Waveguide filter

Note: All pictures from Wikipedia

Definition

Referring to LTI systems, the most common meaning of filter is a device that removes/amplifies/attenuates parts of the spectrum. An example is shown below:

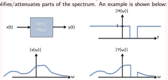

Figure 25: Example filter

Study goals

- ▷ Classify basic types of filters
- ▷ Name limitations
- ▷ Transform from continuous time to discrete time and vice versa

Examples

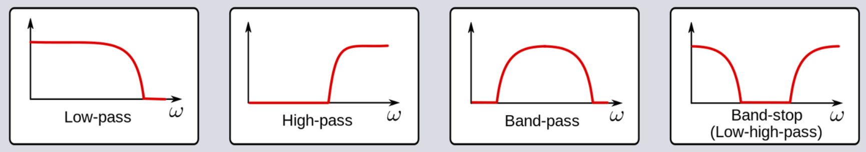

Figure 31: Filter types

Other types:

- ▷ Notch filter: Band-stop filter with a narrow stopband
- ▷ Comb filter: Multiple equally spaced passbands

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Filters

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Filter properties

Ideal low-pass

Transfer function

$$H(\omega) \quad = \quad \begin{cases} 1 & \text{for } |\omega| < 0.5\Delta\omega \\ 0 & \text{else} \end{cases}$$

Impulse response

$$\begin{aligned} h(t) \quad &= \quad \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega \\ &= \quad \frac{\sin \pi t}{\pi t} t \end{aligned}$$

Properties

This is a non-causal filter with unlimited impulse response

Filter properties

Phase delay

Properties

Phase is the integral of the frequency. Example:

$$\omega(t) = \omega_0 + \Delta\omega t,$$

then

$$\varphi(t) = \omega_0 t + 0.5\Delta\omega t^2.$$

Definition

The phase delay is the time delay of the phase:

$$\tau_{ph}(\omega) = -\frac{\varphi(\omega)}{\omega} = -\frac{\arg\{H(\omega)\}}{\omega}$$

This is equivalent to the time a signal with fixed frequency f needs to pass the system:

$$x(t) = \sin(2\pi f_0 t) \rightarrow y(t) \sim \sin(2\pi f_0(t - \tau_{ph}))$$

Filter properties

Group delay

Definition

Assuming a signal $a(t)$ with a small bandwidth and

$$x(t) = a(t)\cos(\omega_0 t),$$

then one can derive the output of an LTI system as follows:

$$y(t) = |H(\omega_0)|\, a(t-\tau_g)\cos\left(\omega_0(t-\tau_{ph})\right).$$

The group delay is the time delay of the amplitude envelope:

$$\tau_g(\omega) = -\frac{\mathrm{d}}{\mathrm{d}\omega}\varphi(\omega)$$

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Filter properties

Cascading

Example

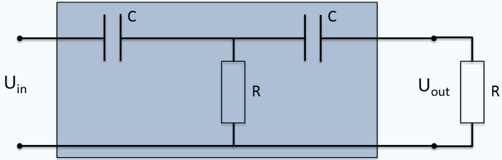


Figure 32: high-pass

Properties

The transfer function of this system **is not** given by $H_1(\omega)^2$

Example

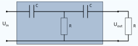


Figure 32: high-pass

Properties

The transfer function of this system **is not** given by $H_1(\omega)^2$

Example

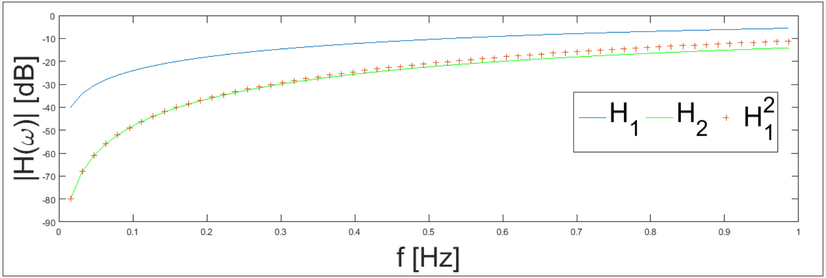


Figure 33: high-pass (R $= 1\text{k}\Omega$, C $= 100\ \mu$F)

Cascading

Example

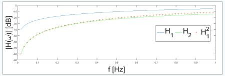


Figure 33: high-pass ($R = 1k\Omega$, $C = 100\ \mu F$)

Example

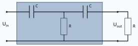


Figure 32: high-pass

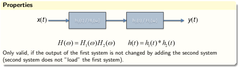
Properties

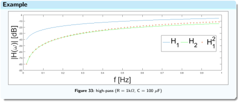
The transfer function of this system **is not** given by $H_1(\omega)^2$

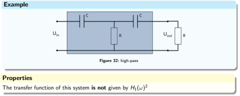
Properties

$$H(\omega) = H_1(\omega) H_2(\omega) \qquad h(t) = h_1(t) * h_2(t)$$

Only valid, if the output of the first system is not changed by adding the second system (second system does not "load" the first system).







Example

Using a buffer amplifier:

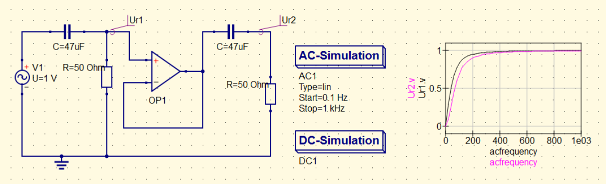


Figure 34: Cascading using a unity gain buffer amplifier

Note: QUCS[a] model can be found on ILIAS

[a] http://qucs.sourceforge.net/

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Filters

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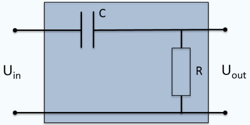
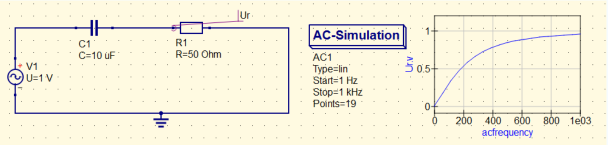
Filter structures

Example



Simulation with QUCS[a]:



[a] http://qucs.sourceforge.net/

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Filter structures

A simple band-pass

Example

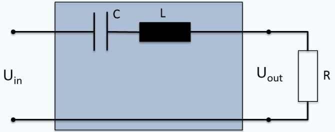


Figure 35: band-pass

$$H(\omega) = \frac{j\omega RC}{1 + j\omega RC + \omega^2 LC}$$

Filter structures

Butterworth filter

Properties

- ▷ a low pass filter that shows no ripples in the pass band
- ▷ also known as maximally flat amplitude filter
- ▷ normalized frequency response for a filter with order n:

$$|H(\omega)| = \sqrt{\frac{1}{1+\omega^{2n}}}$$

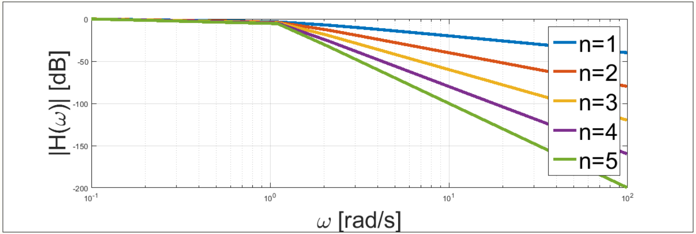
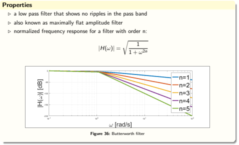


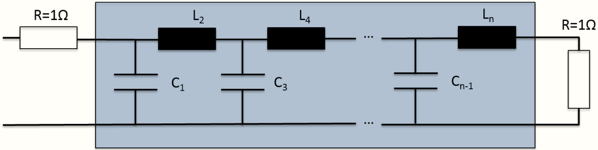
Figure 36: Butterworth filter

Butterworth filter



Properties

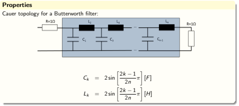
Cauer topology for a Butterworth filter:

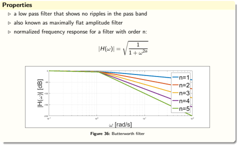


$$C_k = 2\sin\left[\frac{2k-1}{2n}\pi\right]\,[F]$$

$$L_k = 2\sin\left[\frac{2k-1}{2n}\pi\right]\,[H]$$

Butterworth filter





Design process

Consider a lowpass filter with

- ▷ Passband: $1 - \delta_p \leq |H(\omega)| \leq 1 + \delta_p$ for $|\omega| \leq \omega_p$
- ▷ Stopband $|H(\omega)| \leq \delta_s$ for $\omega > \omega_s$

Steps:

1. Determine order N
2. Determine transfer function
3. Map frequencies

Design process

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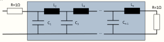

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- a low pass filter that shows no ripples in the pass band
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- normalized frequency response for a filter with order n:

$$|H(\omega)| = \sqrt{\frac{1}{1+\omega^{2n}}}$$

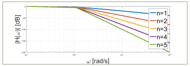

Figure 36: Butterworth filter

Design process

Steps:

1. Determine order N

$$N = \frac{1}{2}\frac{\ln(G_p/G_s)}{\ln(\omega_p/\omega_s)},$$

   with $G_p = \frac{1}{(1-\delta_p)^2} - 1,\ G_s = \frac{1}{(\delta_s)^2} - 1$

2. Determine transfer function
3. Map frequencies

Butterworth filter

Design process
Steps:
1. Determine order N
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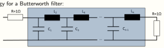

$$C_k = 2\sin\left[\frac{2k-1}{2n}\pi\right] [F]$$
$$L_k = 2\sin\left[\frac{2k-1}{2n}\pi\right] [H]$$

Design process

Steps:

1. Determine order N
2. Determine transfer function $H(s) = \frac{1}{D(s)}$, with

Order	Denominator $D(s)$
1	$(s+1)$
2	$(s^2 + 1.414214s + 1)$
3	$(s+1)(s^2 + s + 1)$
4	$(s^2 + 0.765367s + 1)(s^2 + 1.847759s + 1)$
5	$(s+1)(s^2 + 0.618034s + 1)(s^2 + 1.618034s + 1)$
6	$s^2 + 0.517638s + 1)(s^2 + 1.414214s + 1)(s^2 + 1.931852s + 1)$
7	$(s+1)(s^2 + 0.445042s + 1)(s^2 + 1.246980s + 1)(s^2 + 1.801938s + 1)$

3. Map frequencies

Butterworth filter

Design process

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3. Map frequencies

Design process

Steps:

1. Determine order N

$$N = \frac{1}{2}\frac{\ln(G_p/G_s)}{\ln(\omega_p/\omega_s)},$$

   with $G_p = \frac{1}{(1-\delta_p)^2} - 1$, $G_s = \frac{1}{(\delta_s)^2} - 1$
2. Determine transfer function
3. Map frequencies

Design process

Consider a lowpass filter with

- Passband: $1-\delta_p \leq |H(\omega)| \leq 1+\delta_p$ for $|\omega| \leq \omega_p$
- Stopband $|H(\omega)| \leq \delta_s$ for $\omega > \omega_s$

Steps:

1. Determine order N
2. Determine transfer function
3. Map frequencies

Design process

Steps:

1. Determine order N
2. Determine transfer function
3. Map frequencies

$$H(s) = H(S)|_{S=s/\omega_c},$$

with

$$\omega_c = \frac{\omega_p}{(G_p)^{0.5/N}} \text{ (passband contraint) or (stopband constraint) } \omega_c = \frac{\omega_s}{(G_s)^{0.5/N}}$$

and the gain terms given $G_p = \frac{1}{\delta_p^2} - 1$ and $G_s = \frac{1}{\delta_s^2} - 1$, where δ_p and δ_s are the linear limits for the pass and stop band.

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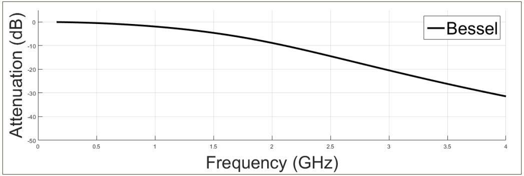
Filter structures

Bessel filter

Properties

- Maximally linear phase response and thus prevents the wave shape of filtered signals in the passband.
- Not as steep as Butterworth



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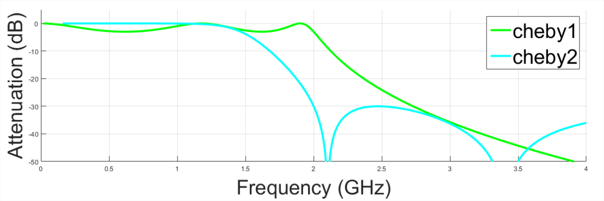
Filter structures

Tschebyscheff (Chebyshev) filter

Properties

- steeper roll-off and passband ripples (type 1) or stopband ripples (type 2).
- Digital implementation available in closed form[a]



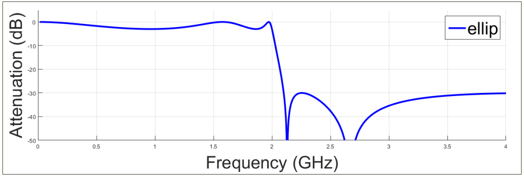
[a] See e.g. Xiao, Fast Design of IIR Digital Filters With a General Chebyshev Characteristic

Filter structures

Cauer filter (elliptical filter)

Properties

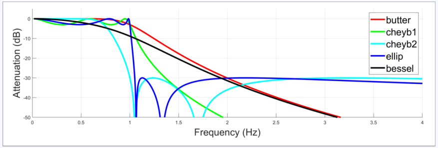
- very steep roll-off
- Ripples in both bands

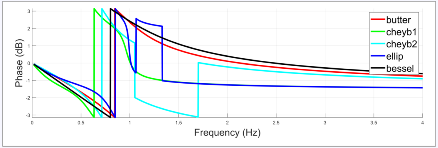


Filter structures

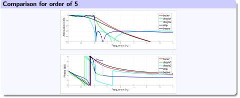
Comparision

Comparison for order of 5





Comparision



Properties

- ▷ Narrower transition band → more ripples

Type	**passband**	**stopband**
Butterworth	flat	flat
Bessel	flat	flat
Chebyshev 1	ripples	flat
Chebyshev 2	flat	ripples
Cauer	ripples	ripples

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Filters

Bandform transformation

Definition

The following bandform transformations can be used:

- Normalized lowpass to lowpass

$$j\omega' = \frac{j\omega}{j\omega_c}$$

- Lowpass to highpass

$$j\omega' = \frac{1}{j\omega}$$

- Lowpass to bandpass

$$j\omega' = \frac{1}{2\pi B}\left(1 + \frac{1}{j\omega}\right),$$

  with B being the bandwidth.

Filters

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Digital filters

Overview

Ways to design digital filters

- Analog prototypes
  - Direct discretization
  - Impulse invariance method
  - Bilinear transformation
- Window method

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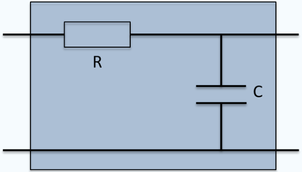
Digital filters

Direct discretization

Example

Consider the low-pass shown below:

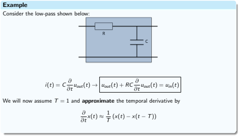


$$i(t) = C\frac{\partial}{\partial t}u_{out}(t) \rightarrow \boxed{u_{out}(t) + RC\frac{\partial}{\partial t}u_{out}(t) = u_{in}(t)}$$

We will now assume $T = 1$ and **approximate** the temporal derivative by

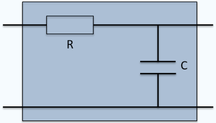
$$\frac{\partial}{\partial t}x(t) \approx \frac{1}{T}\left(x(t) - x(t-T)\right)$$

Direct discretization


Example
Consider the low-pass shown below:

$i(t) = C\frac{\partial}{\partial t}u_{out}(t) \rightarrow \boxed{u_{out}(t) + RC\frac{\partial}{\partial t}u_{out}(t) = u_{in}(t)}$
We will now assume $T = 1$ and **approximate** the temporal derivative by
$\frac{\partial}{\partial t}x(t) \approx \frac{1}{T}(x(t) - x(t-T))$

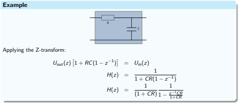
Example

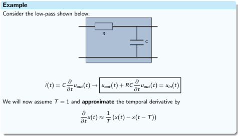


Applying the Z-transform:

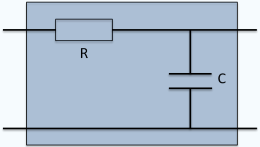
$$
\begin{aligned}
U_{out}(z)\left[1 + RC(1 - z^{-1})\right] &= U_{in}(z) \\
H(z) &= \frac{1}{1 + CR(1 - z^{-1})} \\
H(z) &= \frac{1}{(1 + CR)}\frac{1}{1 - \frac{z^{-1}CR}{1+CR}}
\end{aligned}
$$

Direct discretization





Example



- ▷ Transfer function

$$\boxed{H(z) = \frac{1}{(1+CR)}\frac{1}{1-z^{-1}\frac{CR}{1+CR}}}$$

- ▷ Compare this with standard form

$$H_d(z) = \frac{a_0 z^0 + a_1 z^{-1} + \ldots}{1 + b_1 z^{-1} + \ldots}$$

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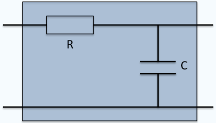
Digital filters

Solution in frequency domain

Example



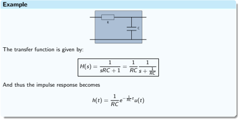
The transfer function is given by:

$$\boxed{H(s) = \frac{1}{sRC+1} = \frac{1}{RC}\frac{1}{s+\frac{1}{RC}}}$$

And thus the impulse response becomes

$$h(t) = \frac{1}{RC}e^{-\frac{1}{RC}t}u(t)$$

Impulse invariance method



Definition

Application of impulse invariance method: Derive sampled impulse response

$$h_n = Th_a(nT)$$

and the spectrum of the sampled impulse response:

$$H_d(\omega) = \sum_{k=-\infty}^{k=\infty} H_a\left(\omega - \frac{2\pi k}{T}\right).$$

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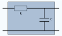

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Properties

- **Note:** Aliasing may occur
- Impulse response shows IIR behaviour
- **But:** Number N of poles defines necessary elements for standard form implemenation

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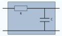

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Properties

- Assuming that all N poles have an order/multiplicity of one and applying partial fraction expansion leads to:

$$H_a(s) = \sum_{k=1}^{N} \frac{A_k}{s - s_k} \Leftrightarrow h_a(t) = \sum_{k=1}^{N} A_k e^{s_k t} u(t).$$

- Sampled impulse response will become

$$h_n = Th_a(nT) = \sum_{k=1}^{N} TA_k e^{s_k nT} u(nT)$$

Impulse invariance method

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Properties

- Sampled impulse response

$$h_n = T \sum_{k=1}^{N} A_k p_k^n u_n$$

- Z transform

$$H_d(z) = \sum_{k=1}^{N} \frac{TA_k}{1 - p_k z^{-1}}$$

- Bring it to a common denominator to derive the standard form

$$\boxed{H_d(z) = \frac{a_0 z^0 + a_1 z^{-1} + \ldots a_{M-1} z^{-(M-1)}}{1 + b_1 z^{-1} + \ldots b_N z^{-N}}}$$

Impulse invariance method

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- **But:** Number N of poles defines necessary elements for standard form implemenation

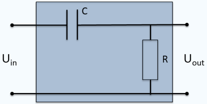
Example

Re-consider the low-pass ($T = 1$ for simplicity):

$$H(s) = \frac{1}{RC}\frac{1}{s+\frac{1}{RC}}$$

$$h(t) = \frac{1}{RC}e^{-\frac{1}{RC}t}u(t) \rightarrow h_n = \frac{1}{RC}e^{-\frac{1}{RC}n}u_n \rightarrow H(z) = \frac{1}{RC}\frac{1}{1-e^{-\frac{1}{RC}}z^{-1}}$$

And thus $a_0 = \frac{1}{RC}$ and $b_1 = e^{-\frac{1}{RC}}$.



Impulse invariance method

Example

Re-consider the low-pass ($T = 1$ for simplicity):

$$H(s) = \frac{1}{RC}\frac{1}{s+\frac{1}{RC}}$$
$$h(t) = \frac{1}{RC}e^{-\frac{1}{RC}t}u(t) \to h_n = \frac{1}{RC}e^{-\frac{1}{RC}n}u_n \to H(z) = \frac{1}{RC}\frac{1}{1-e^{-\frac{1}{RC}}z^{-1}}$$

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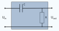

Properties

- Sampled impulse response

$$h_n = T\sum_{k=1}^{N} A_k p_k^n u_n$$

- Z transform

$$H_d(z) = \sum_{k=1}^{N}\frac{TA_k}{1-p_k z^{-1}}$$

- Bring it to a common denominator to derive the standard form

$$\boxed{H_d(z) = \frac{a_0 z^0 + a_1 z^{-1} + \ldots a_{M-1}z^{-(M-1)}}{1 + b_1 z^{-1} + \ldots b_N z^{-N}}}$$

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- Assuming that all N poles have an order/multiplicity of one and applying partial fraction expansion leads to:

$$H_a(s) = \sum_{k=1}^{N}\frac{A_k}{s-s_k} \Leftrightarrow h_a(t) = \sum_{k=1}^{N} A_k e^{s_k t}u(t).$$

- Sampled impulse response will become

$$h_n = Th_a(nT) = \sum_{k=1}^{N} TA_k e^{s_k nT}u(nT)$$

Some identities

In case of poles of higher order, on can make use of the following identities:

$H(s)$	$h(t)$	h_n	$H(z)$
1	$\delta(t)$	$T\delta[k]$	T
$\frac{1}{s}$	u(t)	1	$\frac{Tz}{z-1}$
$\frac{1}{s+\alpha}$	$e^{-\alpha t}u(t)$	$Te^{-\alpha kT}u_n$	$\frac{Tz}{z-e^{-\alpha T}}$
$\frac{1}{s^2}$	$tu(t)$	$nT^2u(k)$	$\frac{T^2z}{(z-1)^2}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}u(t)$	$nT^2e^{-\alpha kT}u_n$	$\frac{T^2e^{-\alpha T}z}{[z-e^{-\alpha T}]^2}$
$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$e^{-\alpha t}\cos(\beta t)u(t)$	$Te^{-\alpha nT}\cos(\beta kT)u_n$	$\frac{Tz[z-e^{-\alpha T}\cos(\beta T)]}{z^2-2e^{-\alpha T}\cos(\beta T)z+e^{-2\alpha T}}$
$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$e^{-\alpha t}\sin(\beta t)u(t)$	$Te^{-\alpha nT}\sin(\beta kT)u_n$	$\frac{Tze^{-\alpha T}\sin(\beta T)}{z^2-2e^{-\alpha T}\cos(\beta T)z+e^{-2\alpha T}}$

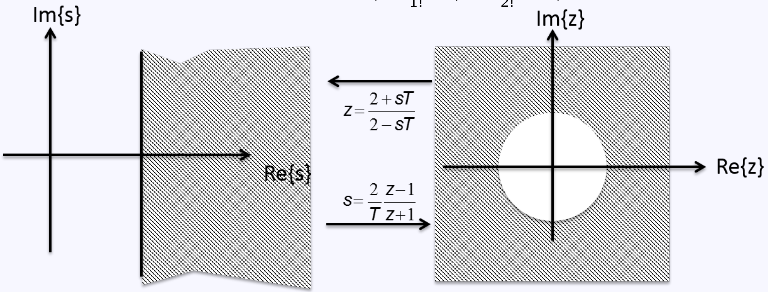
Digital filters

Bilinear transform

Using the Taylor series

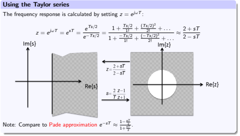
The frequency response is calculated by setting $z = e^{j\omega T}$:

$$z = e^{j\omega T} = e^{sT} = \frac{e^{Ts/2}}{e^{-Ts/2}} = \frac{1 + \frac{Ts/2}{1!} + \frac{(Ts/2)^2}{2!} + \ldots}{1 + \frac{-Ts/2}{1!} + \frac{(-Ts/2)^2}{2!} + \ldots} \approx \frac{2+sT}{2-sT}$$



Note: Compare to Pade approximation $e^{-sT} \approx \frac{1-\frac{sT}{2}}{1+\frac{sT}{2}}$.

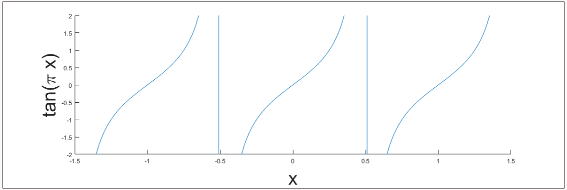
Bilinear transform



Definition

Frequency warping describes the effect of the bilinear transform onto the frequency response[a]:

$$\omega_a = \frac{2}{T}\tan\left(\omega_d \frac{T}{2}\right) \Leftrightarrow \omega_d = \frac{2}{T}\tan\left(\omega_a \frac{T}{2}\right)$$



[a] $s_a = j\omega_a = \frac{2}{T}\frac{1-e^{-j\omega_d T}}{1+e^{-j\omega_d T}} = \frac{2}{T}\frac{e^{j\frac{1}{2}\omega_d T}-e^{-j\frac{1}{2}\omega_d T}}{e^{j\frac{1}{2}\omega_d T}e^{-j\frac{1}{2}\omega_d T}} = j\frac{2}{T}\tan\left(\omega_d \frac{T}{2}\right)$

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Digital filters

Window method

Definition

The window method as applied as follows:

- ▷ Define a filter in the spectral domain
- ▷ Use inverse Fourier transform to get the impulse response
- ▷ Take a subset of the impulse repsonse
- ▷ Shift the impulse response in time to derive a causal system
- ▷ FIR filter coefficients are given by the impulse response
- ▷ Use a window function to remove ripples

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Example

Continuous impulse response $h(t)$ for an ideal low-pass with cut-off frequency w_c:

$$h(t) = \frac{\sin(\omega_c t)}{\pi t}$$

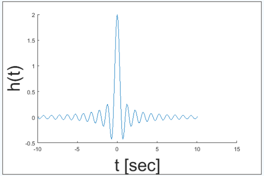

Figure 37: $h(t)$

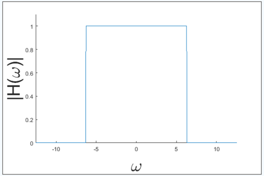

Figure 38: $H(\omega)$

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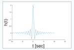

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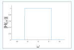

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- ▷ Non-causal
- ▷ Infinite number of coefficients

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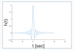

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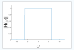

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Solution

- ▷ Shift all coefficients by a reasonable number k (e.g. $k = 10$)
- ▷ Take only the first $2k$ values

$$h'(n) = \sum_{i=0}^{2k} a_i \delta_i$$

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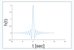

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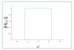

Figure 38: $H(\omega)$

Digital filter with 20 coefficients

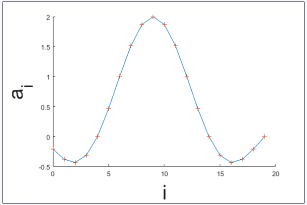

Figure 39: Coefficients

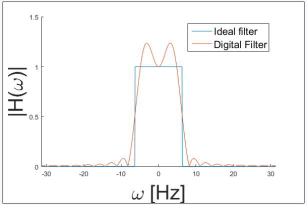

Figure 40: Spectrum

- The truncation of the impulse response is equivalent to multiplying it with a rectangle
- Overshoot and ripples (Gibb's phenomenon)

Window method

Digital filter with 20 coefficients

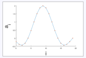

Figure 39: Coefficients

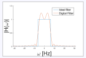

Figure 40: Spectrum

- ▷ The truncation of the impulse response is equivalent to multiplying it with a rectangle
- ▷ Overshoot and ripples (Gibb's phenomenon)

Solution

- ▷ Shift all coefficients by a reasonable number k (e.g. $k = 10$)
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Properties

- ▷ Non-causal
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Example

Digital filter with $2k$ coefficients

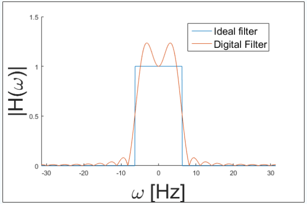


Figure 41: 20 taps

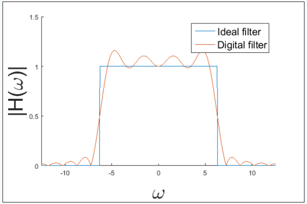


Figure 42: 40 taps

- ▷ Amplitude of ripples does not decrease with the number of coefficients

Example
Digital filter with $2k$ coefficients

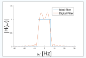
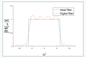



Figure 41: 20 taps

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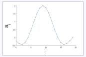
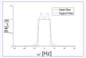


Figure 39: Coefficients

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Example

Weighting with a window function:

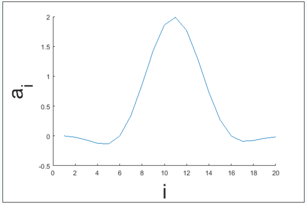

Figure 43: 20 coefficients multiplied with a Hamming window

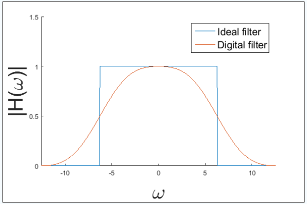

Figure 44: Spectrum

- ▷ The use of a window function can reduce ripples
- ▷ Usually, this is for the sake of getting a less steep cut-off slope

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Digital filters

First order FIR filter

Example

$$
\begin{aligned}
y_n &= x_n + \alpha x_{n-1} \\
Y(z) &= X(z) - \alpha Z^{-1} X(z) \\
H(z) &= 1 - \alpha z^{-1}
\end{aligned}
$$

The choice of α determines if the filter shows a low pass or high pass behavior.

First order FIR filter

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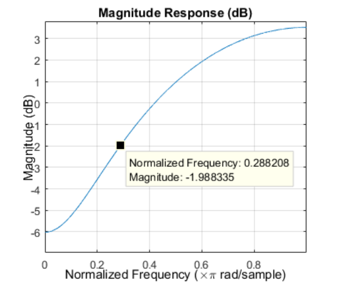
The choice of α determines if the filter shows a low pass or high pass behavior.

Example

```
clear;close all;clc;
s = sin((1:512)*2*pi*10/512);
myzeros = 0.5;
poles = 0;
gain = 1;
[NUM,DEN] = zp2tf(myzeros,poles,
    gain);
s1 = filter(NUM,DEN,s);
plot(abs(fft(s)),'red');
hold on
plot(abs(fft(s1)),'green');
fvtool(NUM,DEN);
```



DC blocker

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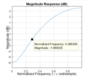

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y_n &= x_n + \alpha x_{n-1} \\
Y(z) &= X(z) - \alpha Z^{-1} X(z) \\
H(z) &= 1 - \alpha z^{-1}
\end{aligned}
$$

The choice of α determines if the filter shows a low pass or high pass behavior.

Example

DC-Blocker:

$$
\begin{aligned}
y_n &= x_n - x_{n-1} + \alpha y_{n-1} \\
Y(z) &= X(z) - Z^{-1} X(z) + \alpha Z^{-1} Y(z) \\
H(z) &= \frac{1 - z^{-1}}{1 - \alpha z^{-1}}
\end{aligned}
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A typical choice for α is $0.9 < \alpha < 1$.

Parametric equalizer

Example

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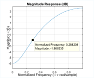
H(z) &= \frac{1 - z^{-1}}{1 - \alpha z^{-1}}
\end{aligned}
$$

A typical choice for α is $0.9 < \alpha < 1$.

Example

```
clear;close all;clc;
s = sin((1:512)*2*pi*10/512);
myzeros = 0.5;
poles = 0;
gain = 1;
[NUM,DEN] = zp2tf(myzeros,poles,
    gain);
s1 = filter(NUM,DEN,s);
plot(abs(fft(s)),'red');
hold on
plot(abs(fft(s1)),'green');
fvtool(NUM,DEN);
```



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The choice of α determines if the filter shows a low pass or high pass behavior.

Example

Equalizer with gain g and bandwidth B:

$$
\begin{aligned}
H(s) &= 1 + (g-1)\frac{Bs}{s^2 + Bs + 1} \\
&= \frac{s^2 + gBs + 1}{s^2 + Bs + 1}
\end{aligned}
$$

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Digital filters

Linear-phase filters

Properties

- Linear-phase filters have a symmetric impulse response:

$$h(n) = h(N-1-n),$$

  with $n = 0 \cdots N-1$.
- Linear-phase filters must be FIR-filters.

FIR vs. IIR

Properties

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Properties

FIR	IIR
▷ No feedback (only zeros) $\rightarrow$ always stable ▷ Can have linear phase	▷ Feedback (poles and zeros) $\rightarrow$ may be unstable ▷ Phase is difficult to control
▷ Higher order (assuming same performance) ▷ No analog prototype	▷ Smaller order (rule of thumb: 1/10) ▷ Can be based upon analog prototypes

Summary

Properties

FIR	IIR
▷ No feedback (only zeros) → always stable ▷ Can have linear phase	▷ Feedback (poles and zeros) → may be unstable ▷ Phase is difficult to control
▷ Higher order (assuming same performance) ▷ No analog prototype	▷ Smaller order (rule of thumb: 1/10) ▷ Can be based upon analog prototypes

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Summary

- ▷ Use an analog prototype
  - ▷ Impulse invariance
  - ▷ Bilinear transform
- ▷ Window method

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Exercise (#7.1)

Calculate the impulse response of ideal time discrete lowpass, highpass and bandpass filters.

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Exercise (#7.3)

Does the filter with the impulse response

$$h_n = 2\delta_n + \delta_{n-1} + C\delta_{n-2}$$

show a linear phase response for $C = 1$ and $C = 2$, respectively? **Proof** your answer mathematically.

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Exercise (#7.4)

Proof that a symmetrical impulse response (odd filter length)

$$h_n = h_{N-1-n}$$

and a antisymmetrical impulse response (odd filter length)

$$h_n = -h_{N-1-n}$$

of a causal filter leads to a linear phase behaviour

$$H(\omega) = G(\omega)e^{j(-\alpha\omega+\beta)},$$

with $G(\omega) \in \mathbb{R}$.

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Design a digital filter using a sample interval of $T = 0.1$ sec and the following analog prototypes:

- $$H(s) = \frac{s+2}{(s+4)(s^2+4s+3)}$$
- $$H(s) = \frac{s^2+9s+20}{(s+2)(s^2+4s+3)}$$

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Use two different approaches to design the filter.

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$$H(\omega) = \mathrm{tria}\left(\frac{\omega}{2\pi}\right).$$

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Given is the circuit shown in figure 45.

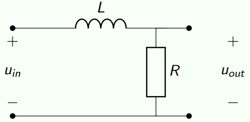

Figure 45: Circuit

- ▷ Derive the transfer function and discuss the frequency behavior of this circuit.
- ▷ Use this prototype to design a digital filter making use of the bilinear transform (assuming a time step of $T = 2$).
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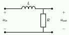

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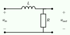

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Given is the transfer function

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of a filter.

- ▷ Determine the poles and zeros.
- ▷ Discuss stability.
- ▷ Sketch a standard implementation of the filter and determine the corresponding coefficients $a_0, a_1, \ldots$ and $b_1, b_2, \ldots$.
- ▷ Sketch the magnitude response for $-\frac{2\pi}{T} < \omega < \frac{2\pi}{T}$.

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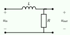

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Using the bilinear-transformation method with frequency prewarping, find the digital filter transfer function $H_d(z)$ corresponding to the following analog filter transfer function

$$H_a(s) = \frac{0.5}{s+0.5}.$$

Assume that the sampling frequency is $f_s = 1\,\text{Hz}$.

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