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Control

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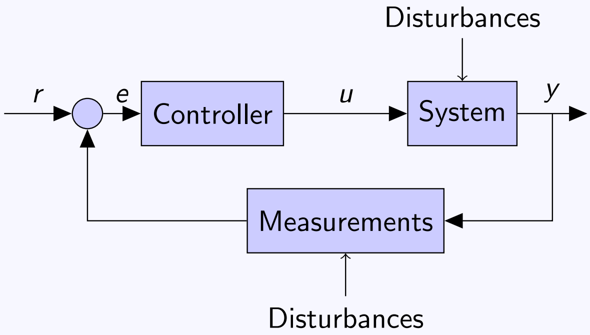
Feedback & Control

Basic Architecture



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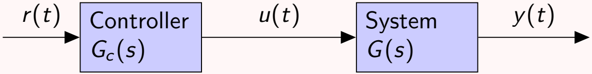
Feedback & Control

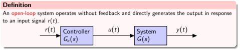
Open-loop systems

Definition

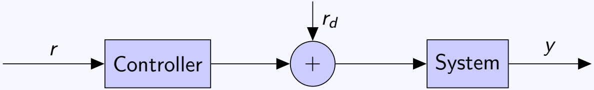
An open-loop system operates without feedback and directly generates the output in response to an input signal $r(t)$.



Tracking error



Adding disturbances



Definition

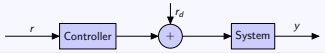
The tracking error is defined as follows:

$$
\begin{aligned}
E(s) &= R(s) - Y(s) \\
&= R(s) - \left[R(s)G_c(s) + R_d(s)\right]G(s),
\end{aligned}
$$

with $G_c(s)$ being the controller transfer function and $G(s)$ being the system transfer function.

Steady state error

Adding disturbances



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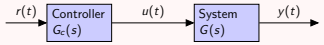
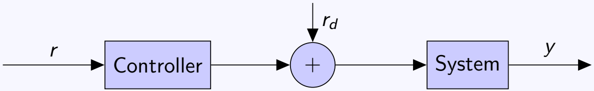


Open-loop control system



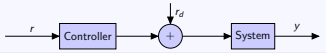
Definition

The steady state error is defined as follows:

$$\begin{aligned} e_0(\infty) &= \lim_{t\to\infty} e(t) \\ &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0} [sR(s) - sR(s)G_c(s)G(s) - sR_d(s)G(s)] \end{aligned}$$

Steady state error

Open-loop control system



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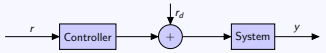
Adding disturbances



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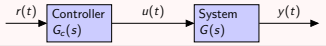
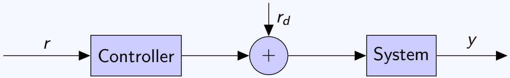


Open-loop control system



Properties

Using the unit step function as a comparable input, one gets:

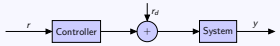
$$\begin{aligned} e_0(\infty) &= \lim_{s\to 0}\left[s\frac{1}{s} - s\left(\frac{1}{s}G_c(s) + R_d(s)\right)G(s)\right] \\ &= 1 - \lim_{s\to 0}\left[G_c(s)G(s) + sR_d(s)G(s)\right] \end{aligned}$$

For $R_d(s) = 0$ (no disturbance):

$$e_0(\infty) = 1 - \lim_{s\to 0}\left[G_c(s)G(s)\right]$$

Drawbacks

Open-loop control system



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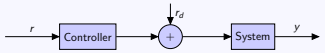
Open-loop control system



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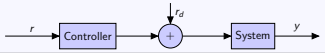
Adding disturbances



Definition

The tracking error is defined as follows:

$$\begin{aligned} E(s) &= R(s) - Y(s) \\ &= R(s) - \left[R(s)G_c(s) + R_d(s)\right]G(s), \end{aligned}$$

with $G_c(s)$ being the controller transfer function and $G(s)$ being the system transfer function.

The open-loop system

... can not react to disturbances

... may be calibrated so that $G(0)G_c(0) = 1$

... but during the operation $G(s)$ might change over time

Feedback & Control

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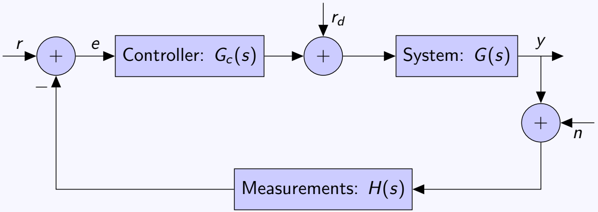
Closed-loop systems

Closed-loop systems

Basic structure



- r_d: Disturbance
- n: Measurement noise

Note: Also called feedback control

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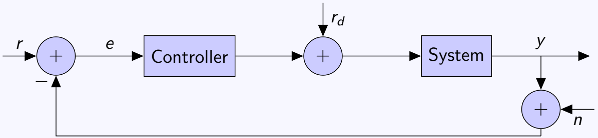
Closed-loop systems

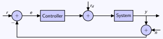
Unity feedback



For $H(s) = 1$ one gets

$$\begin{aligned} Y(s) &= \frac{G_c(s)G(s)}{1+G_c(s)G(s)}R(s) + \frac{G(s)}{1+G_c(s)G(s)}R_d(s) \\ &\quad - \frac{G_c(s)G(s)}{1+G_c(s)G(s)}N(s) \\ &= C(s)R(s) + \frac{G(s)}{1+G_c(s)G(s)}R_d(s) - C(s)N(s) \end{aligned}$$

Unity feedback



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Loop gain

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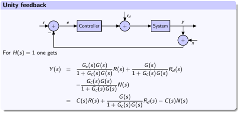

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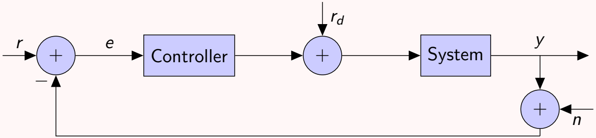
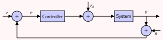


The loop gain is defined by

$$L(s) = G_c(s)G(s).$$

Definition



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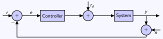
Loop gain

Sensitivity

Unity feedback

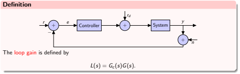


For $H(s) = 1$ one gets

$$\begin{aligned} Y(s) &= \frac{G_c(s)G(s)}{1+G_c(s)G(s)}R(s) + \frac{G(s)}{1+G_c(s)G(s)}R_d(s) \\ &\quad - \frac{G_c(s)G(s)}{1+G_c(s)G(s)}N(s) \\ &= C(s)R(s) + \frac{G(s)}{1+G_c(s)G(s)}R_d(s) - C(s)N(s) \end{aligned}$$

Sensitivity

Sensitivity



Definition

System sensitivity is the ratio of the change e.g. in the system transfer function to the change of a process transfer function (or parameter) for a small incremental change

Dorf, Bishop

Loop gain

Sensitivity

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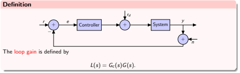
Dorf, Bishop

Sensitivity



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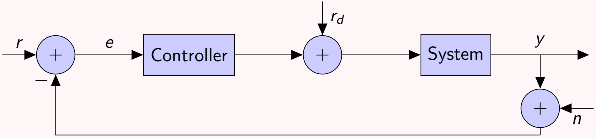


With

$$F(s) = 1 + L(s)$$

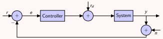
one can define the sensitivity as follows[a]

$$S(s) = \frac{G_C}{F^2(s)}.$$

[a] This describes how relative errors in the plant affect the closed loop transfer function. One can define other sensitivity functions as well. E.g. S_G^E describes the sensitivity of the tracking error to small changes in the transfer function of the plant $G(s)$: $S_G^E = \frac{-G(s)G_c(s)}{1+G_c(s)G(s)}$.

Without distortion

Definition



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Dorf, Bishop

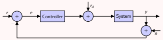
Without distortion

Sensitivity

Without disturbance

Without distortion

Definition



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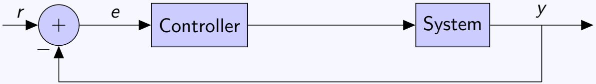
Dorf, Bishop

Tracking error



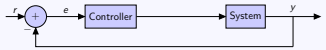
$$Y(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)}R(s)$$

Tracking error:

$$\begin{aligned} E(s) &= R(s)\left[1 - \frac{G_c(s)G(s)}{1+G_c(s)G(s)}\right] = R(s)(1-C(s)) \\ &= R(s)\left[\frac{1}{1+G_c(s)G(s)}\right] = R(s)S(s). \end{aligned}$$

Unity feedback without disturbance

Tracking error



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Without distortion

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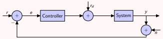

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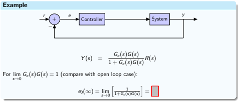
Example

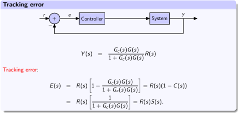
$$Y(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)}R(s)$$

For $\lim_{s\to 0} G_c(s)G(s) = 1$ (compare with open loop case):

$$e_0(\infty) = \lim_{s\to 0}\left[\frac{1}{1+G_c(s)G(s)}\right] = \boxed{}$$

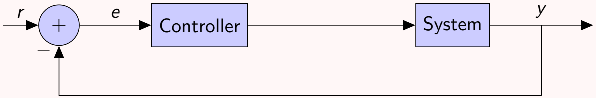
DC loop gain





Without distortion

Definition



The DC loop gain $L(0)$ is defined as follows:

$$L(0) = G_c(0)G(0)$$

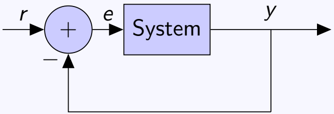
The steady state error will be *small* if the DC loop gain is *reasonably* large:

$$e_0(\infty) = \frac{1}{1+L(0)}$$

Closed-loop systems

Closed-loop system

Stability criterion



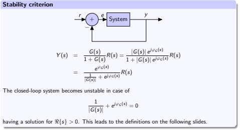
$$
\begin{aligned}
Y(s) &= \frac{G(s)}{1+G(s)}R(s) = \frac{|G(s)|\, e^{j\varphi_G(s)}}{1+|G(s)|\, e^{j\varphi_G(s)}}R(s) \\
&= \frac{e^{\varphi_G(s)}}{\frac{1}{|G(s)|}+e^{j\varphi_G(s)}}R(s)
\end{aligned}
$$

The closed-loop system becomes unstable in case of

$$
\frac{1}{|G(s)|}+e^{j\varphi_G(s)}=0
$$

having a solution for $\Re\{s\}>0$. This leads to the definitions on the following slides.

Closed-loop system


Stability criterion

$$Y(s) = \frac{G(s)}{1+G(s)}R(s) = \frac{|G(s)|\, e^{j\varphi_G(s)}}{1+|G(s)|\, e^{j\varphi_G(s)}}R(s) = \frac{e^{j\varphi_G(s)}}{\frac{1}{|G(s)|}+e^{j\varphi_G(s)}}R(s)$$

The closed-loop system becomes unstable in case of

$$\frac{1}{|G(s)|}+e^{j\varphi_G(s)} = 0$$

having a solution for $\Re\{s\} > 0$. This leads to the definitions on the following slides.

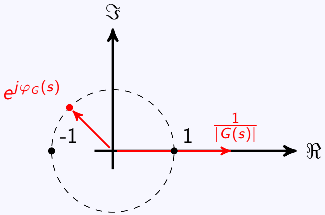
Stability criterion

The closed-loop system becomes unstable in case of

$$1 + |G(s)|\, e^{j\varphi_G(s)} = 0$$

having a solution for $\Re\{s\} > 0$.
For linear systems: The closed loop is stable if the gain of the loop transfer function is less than one for all frequencies

Complex plane



Closed-loop systems

Phase margin

Margins

In practice it is not enough that the system is stable: There must also be some margins of stability. See also elective *SiCo 2*.

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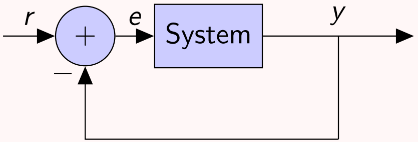


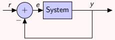
The phase margin is defined as 180° plus the phase of the open-loop transfer function at unity gain.

> *The phase margin is the amount of phase shift of $L(j\omega)$ at unity magnitude that will result in a marginally stable system with intersection of the $-1 + j0$ point on the Nyquist plot.*

Dorf, Bishop

Phase crossover frequency

Definition



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>
> *Dorf, Bishop*

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In practice it is not enough that the system is stable: There must also be some margins of stability. See also elective *SiCo 2*.

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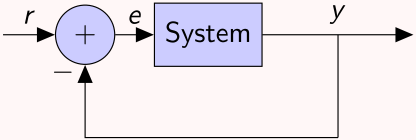
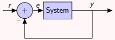


The angular frequency ω_π for which $\arg\left(G(s = j\omega_\pi)\right) = \pi$ holds true is called the phase crossover frequency.

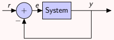
Gain margin

Definition



The angular frequency ω_π for which $\arg(G(s=j\omega_\pi))=\pi$ holds true is called the phase crossover frequency.

Definition



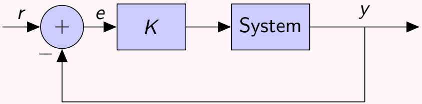
The phase margin is defined as 180° plus the phase of the open-loop transfer function at unity gain.

> *The phase margin is the amount of phase shift of $L(j\omega)$ at unity magnitude that will result in a marginally stable system with intersection of the $-1+j0$ point on the Nyquist plot.*
>
> *Dorf, Bishop*

Margins

In practice it is not enough that the system is stable: There must also be some margins of stability. See also elective *SiCo 2*.

Definition



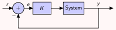
The gain margin is defined as the reciprocal of the open-loop transfer function at $s = j\omega_\pi$:

$$\mathrm{GM} = \frac{1}{|G(s = j\omega_\pi)|}.$$

The gain margin tells you how much you can increase a gain factor K until the closed loop system becomes unstable.

Gain margin

Definition



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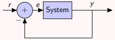
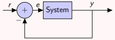


The angular frequency ω_π for which $\arg\left(G(s = j\omega_\pi)\right) = \pi$ holds true is called the phase crossover frequency.

Definition

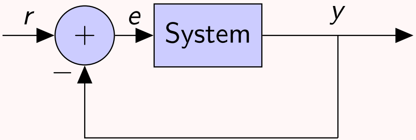


The phase margin is defined as 180° plus the phase of the open-loop transfer function at unity gain.

The phase margin is the amount of phase shift of $L(j\omega)$ at unity magnitude that will result in a marginally stable system with intersection of the $-1 + j0$ point on the Nyquist plot.

Dorf, Bishop

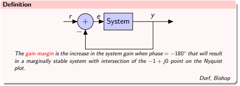
Definition

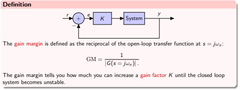


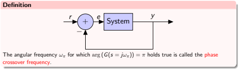
The gain margin is the increase in the system gain when phase $= -180^\circ$ that will result in a marginally stable system with intersection of the $-1 + j0$ point on the Nyquist plot.

Dorf, Bishop

Gain and phase margin



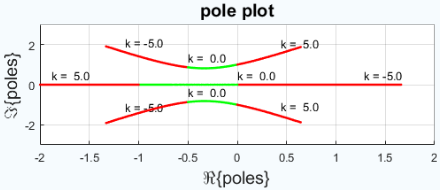




Example

Given is a system with a transfer function

$$G(s) = \frac{1}{s^3 + s^2 + s + k}$$

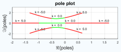


Open loop: Stable for $0 < k < 1$.

Example

Given is a system with a transfer function

$$G(s) = \frac{1}{s^3 + s^2 + s + k}$$



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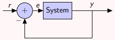
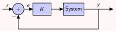


The gain margin is the increase in the system gain when phase $= -180°$ that will result in a marginally stable system with intersection of the $-1 + j0$ point on the Nyquist plot.

Dorf, Bishop

Definition



The gain margin is defined as the reciprocal of the open-loop transfer function at $s = j\omega_\pi$:

$$\mathrm{GM} = \frac{1}{|G(s = j\omega_\pi)|}.$$

The gain margin tells you how much you can increase a gain factor K until the closed loop system becomes unstable.

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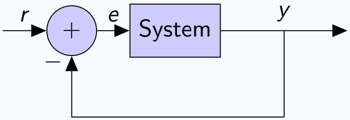
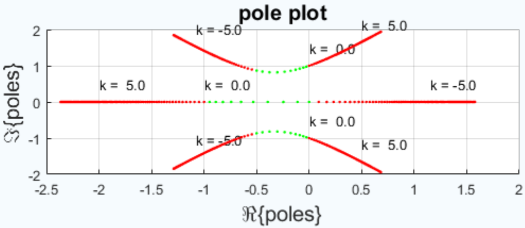


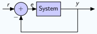
$$G(s) = \frac{1}{s^3 + s^2 + s + k}$$

Stable for $-1 < k < 0$.



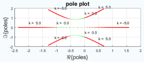
Gain and phase margin

Example



$G(s) = \frac{1}{s^3+s^2+s+k}$

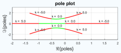
Stable for $-1 < k < 0$.



Example

Given is a system with a transfer function

$G(s) = \frac{1}{s^3+s^2+s+k}$



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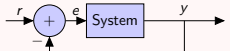

The gain margin is the increase in the system gain when phase $= -180°$ that will result in a marginally stable system with intersection of the $-1 + j0$ point on the Nyquist plot.

Dorf, Bishop

Example

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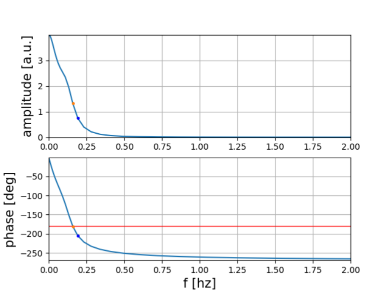
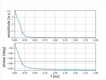


Gain and phase margin

Example

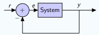
$G(s) = \frac{1}{s^3+s^2+s+0.25}$: Open loop stable

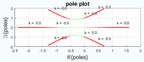


Example

$G(s) = \frac{1}{s^3+s^2+s+k}$

Stable for $-1 < k < 0$.





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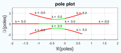

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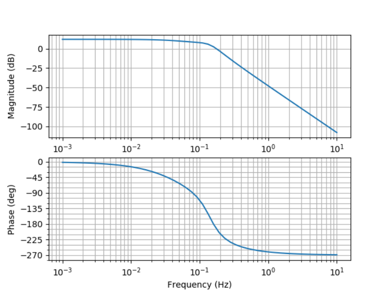
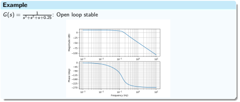
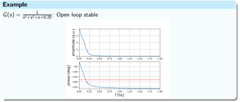
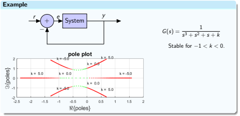


Gain and phase margin







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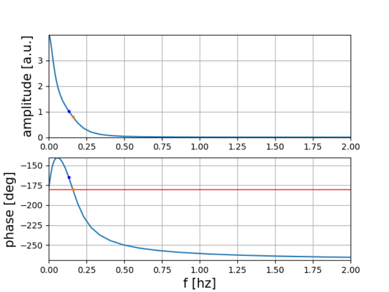
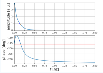
$G(s) = \frac{1}{s^3+s^2+s-0.25}$: Open loop unstable

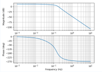


Gain and phase margin

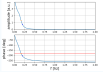
Example
$G(s) = \frac{1}{s^3+s^2+s-0.25}$: Open loop unstable



Example
$G(s) = \frac{1}{s^3+s^2+s+0.25}$: Open loop stable



Example
$G(s) = \frac{1}{s^3+s^2+s+0.25}$: Open loop stable



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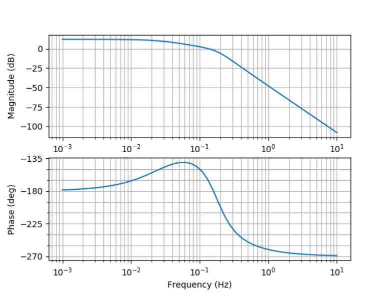
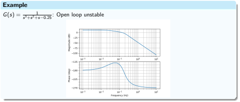
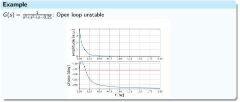
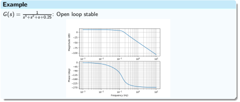
$G(s) = \frac{1}{s^3+s^2+s-0.25}$: Open loop unstable



Gain and phase margin







Properties

- ▷ Negative gain **and** phase margin: Closed loop system (with negative feedback) is unstable

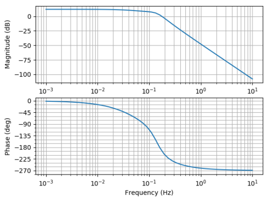


Figure 15: Unstable closed loop

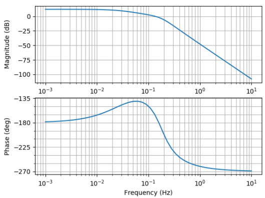
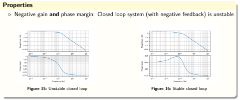
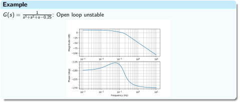
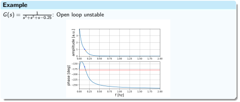


Figure 16: Stable closed loop







Exercise (#5.1)

Plot the bode diagrams and check open and closed loop (unity feedback, $G_C(s) = 1$) stability for

▷
$$G(s) = \frac{1}{s^3 + s^2 + s - 0.25}$$

▷
$$G(s) = \frac{1}{s^3 + s^2 + s + 0.25}$$

by making use of NUMPY or MATLAB. Plot the unit step responses of open and closed loop systems.

Exercise

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Properties

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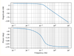


Figure 15: Unstable closed loop

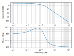
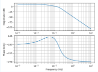


Figure 16: Stable closed loop

Example

$G(s) = \frac{1}{s^3+s^2+s-0.25}$: Open loop unstable



Exercise (#5.2)

Skech the bode diagrams and check open and closed loop (unity feedback, $G_C(s) = 1$) stability for

- $$G(s) = \frac{1}{s^2 + 2s + 3},$$
- $$G(s) = \frac{1}{s + 0.25},$$
- $$G(s) = \frac{1}{s - 0.25}.$$
- $$G(s) = \frac{1}{s^3 + 2.25s^2 + 3.5s + 0.75}$$

Sketch the unit step responses of open and closed loop systems.

Exercise

Exercise (#5.2)

Skech the bode diagrams and check open and closed loop (unity feedback, $G_C(s) = 1$) stability for

- $$G(s) = \frac{1}{s^2 + 2s + 3},$$
- $$G(s) = \frac{1}{s + 0.25},$$
- $$G(s) = \frac{1}{s - 0.25}.$$
- $$G(s) = \frac{1}{s^3 + 2.25s^2 + 3.5s + 0.75}$$

Sketch the unit step responses of open and closed loop systems.

Closed-loop systems

Nyquist diagrams

Using Nyquist diagrams to check stability: Stable plant

Checking stability for the unity feedback closed loop system with

$$C(s) = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)}$$

using the Nyquist diagram in case of $G(s)$ being stable:

1. Plot the Nyquist diagram for the open loop system $G(s)G_c(s)$
2. Travel the Nyquist contour from $\omega = 0$ to $\omega = \infty$.
3. The closed loop system is (still) stable, if the point $(-1, 0)$ is always to the left.

Using Nyquist diagrams to check stability: Stable plant
Checking stability for the unity feedback closed loop system with
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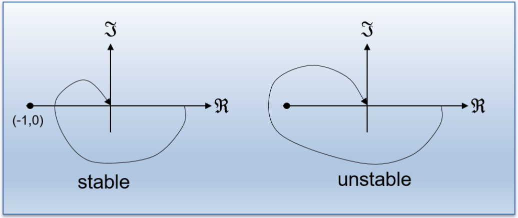

Nyquist diagrams

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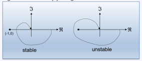

Using Nyquist diagrams to check stability: Stable plant

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Using Nyquist diagrams to check stability: Unstable plant

Checking stability for the unity feedback closed loop system with

$$C(s) = \frac{G(s)G_c(s)}{1+G(s)G_c(s)}$$

using the Nyquist diagram (general case):

1. Plot the Nyquist diagram for the **open loop** system $G(s)G_c(s)$
2. Count the number of poles r_k with a real part larger than zero
3. Count the number of poles i_k with a real part of zero.

Nyquist diagrams

Using Nyquist diagrams to check stability: Unstable plant

Checking stability for the unity feedback closed loop system with

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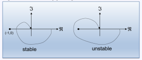

Using Nyquist diagrams to check stability: Stable plant

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$$C(s) = \frac{G(s)G_c(s)}{1+G(s)G_c(s)}$$

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4. Travel the Nyquist contour from $\omega = 0$ to $\omega = \infty$.
5. Sum up the angles of $\triangle\varphi$ of $G(s)G_c(s)$ encircling the point $(-1,0)$. If the curve goes counter-clockwise: Count angles positive. Else: Count them negative. In other words: Examine the number of encirclements of the point $(-1,0)$
6. The closed loop system is stable if $\triangle\varphi = i_k\frac{\pi}{2} + r_k\pi$.

Gain and phase margin

Using Nyquist diagrams to check stability: Unstable plant

Checking stability for the unity feedback closed loop system with

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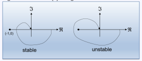
1. Plot the Nyquist diagram for the **open loop** system $G(s)G_c(s)$
2. Count the number of poles r_k with a real part larger than zero
3. Count the number of poles i_k with a real part of zero.

Using Nyquist diagrams to check stability: Stable plant

Checking stability for the unity feedback closed loop system

$$C(s) = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)}$$

using the Nyquist diagram in case of $G(s)$ being stable:



Properties

- Negative gain and phase margin: Closed loop system (with negative feedback) is unstable

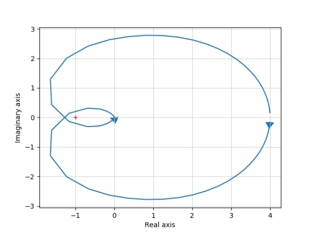


Figure 17: Transfer function $G(s)$ (unstable closed loop)

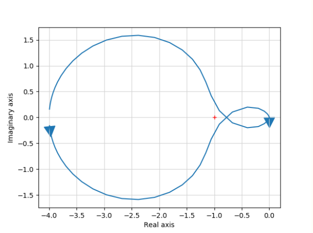


Figure 18: Transfer function $G(s)$ (stable closed loop)

Nyquist diagrams

Properties

- Negative gain and phase margin: Closed loop system (with negative feedback) is unstable

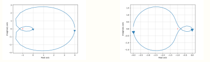


Figure 17: Transfer function $G(s)$ (unstable closed loop)

Figure 18: Transfer function $G(s)$ (stable closed loop)

Using Nyquist diagrams to check stability: Unstable plant

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Using Nyquist diagrams to check stability: Unstable plant

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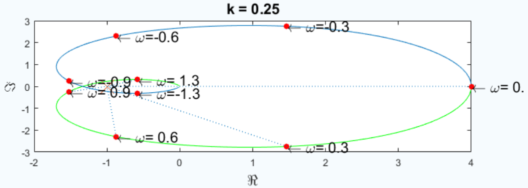
Example: Stable open loop system

Given is a system with a transfer function

$$G(s) = \frac{1}{s^3 + s^2 + s + 0.5}$$



Poles: $(-0.34 + j0.82)$, $(-0.34 - j0.82)$, $(-0.32) \rightarrow r_k = 0 = i_k = 0 \rightarrow$ closed loop is unstable.

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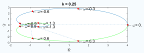

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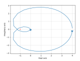

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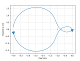

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Using Nyquist diagrams to check stability: Unstable plant

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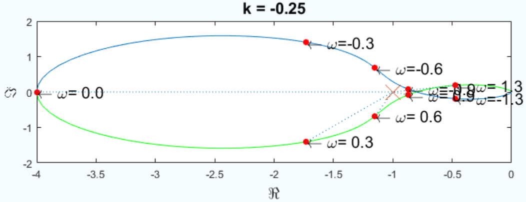

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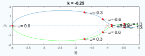

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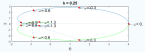

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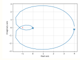

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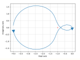

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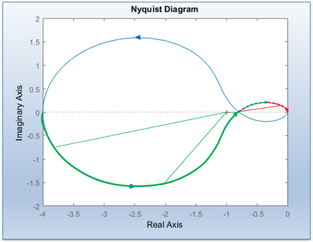

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Closed-loop systems

Poles

Consider the transfer function

$$G(s) = \frac{1}{s^2 + ps + q}$$

With poles at

$$s = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}.$$

For $p^2 - 4q < 0$ one gets poles at

$$s = -\frac{p}{2} \pm j\sqrt{q - \left(\frac{p}{2}\right)^2}.$$

Second-order systems

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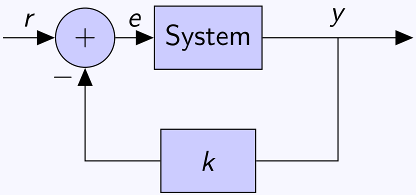
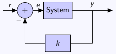
With feedback (gain k)



$$C(s) = \frac{1}{ms^2 + ps + q + k}$$

Second-order systems

With feedback (gain k)



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Routh array:

s^2	m	q+k
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A second-order system is stable, if all of the coefficients are either positive or negative (no sign change). See Routh-Hurwitz criterion.

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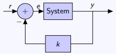

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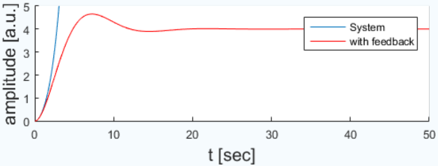
Example

$$G(s) = \frac{1}{s^2 + 0.5s - 0.75}$$

This system is stabilized by using feedback:

$$C(s) = \frac{1}{s^2 + 0.5s - 0.75 + 1.}$$



Second-order systems

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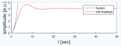

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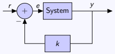

$$C(s) = \frac{1}{ms^2 + ps + q + k}$$

With feedback (gain k)

Consider

$$G(s) = \frac{\omega_0^2}{s^2 + 2\sigma\omega_0 s + \omega_0^2}$$

with feedback

$$C(s) = \frac{\omega_0^2}{s^2 + 2\sigma\omega_0 s + (k+1)\omega_0^2}$$

The Routh array gives the condition that $2\sigma\omega_0 > 0$ and $w_0^2(1+k) > 0$ needs to be fulfilled for stability. $\Rightarrow$ Stability depends only on σ.

Second-order systems

With feedback (gain k)

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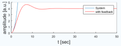

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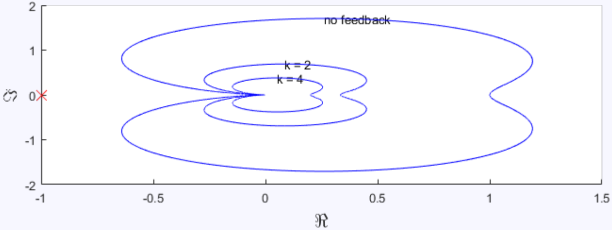
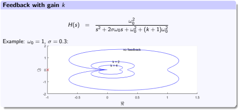
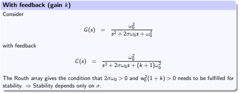
Feedback with gain k

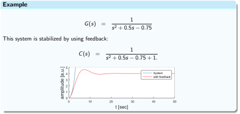
$$H(s) = \frac{\omega_0^2}{s^2 + 2\sigma\omega_0 s + \omega_0^2 + (k+1)\omega_0^2}$$

Example: $\omega_0 = 1$, $\sigma = 0.3$:







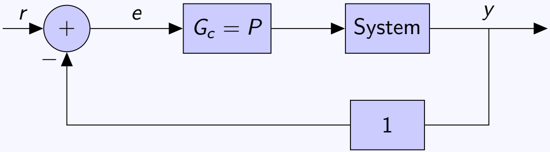


Increasing gain factor P

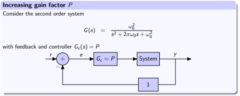
Consider the second order system

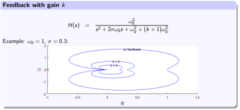
$$G(s) = \frac{\omega_0^2}{s^2 + 2\sigma\omega_0 s + \omega_0^2}$$

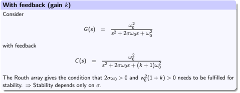
with feedback and controller $G_c(s) = P$



second order systems







Properties: second order systems

The phase of second order systems never crosses -180 degree and thus, they never provide positive feedback. Consequently, the gain margin is infinite.

Second-order systems

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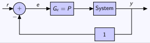

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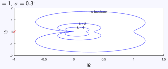

Increasing gain factor P

$$C(s) = \frac{P\omega_0^2}{s^2 + 2\sigma\omega_0 s + \omega_0^2(1+P)}$$

The Routh array gives the condition that

$$2\sigma\omega_0 > 0$$

and

$$\omega_0^2 + A > 0$$

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$\Rightarrow$ Stability depends only on σ and not on P.

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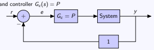

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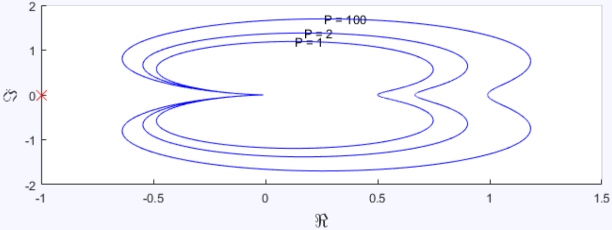

Second-order systems

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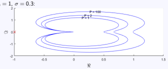

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The phase of second order systems never crosses -180 degree and thus, they never provide positive feedback. Consequently, the gain margin is infinite.

Phase crossover frequency

$$\begin{aligned} -\pi &= \arg\left\{(j\omega_\pi)^2 + jp\omega_\pi + q\right\} \\ &= \arg\left\{-\omega_\pi^2 + jp\omega_\pi + q\right\} \end{aligned}$$

A second-order system ($p \neq 0$) does not have a phase crossover frequency and thus the gain margin is infinite.

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Exemplary step response

Step response: Some properties

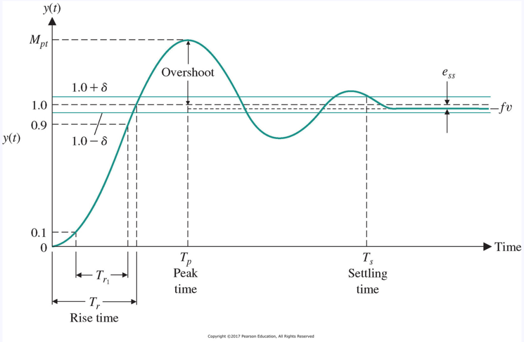
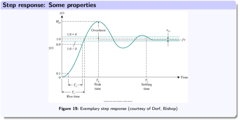


Figure 19: Exemplary step response (courtesy of Dorf, Bishop)

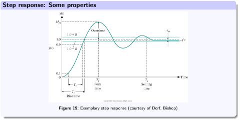


Exemplary step response

Steady state response

Steady state response

Steady state response

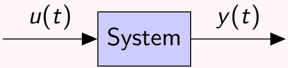


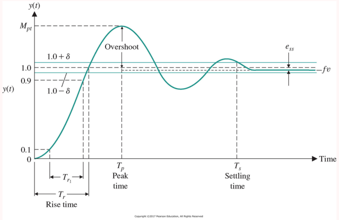
Exemplary step response

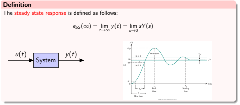
Definition

The steady state response is defined as follows:

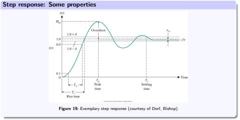
$$e_{SS}(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$







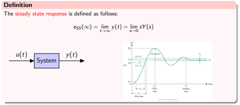
Steady state response



Overshoot

Overshoot

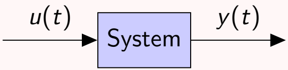
Overshoot

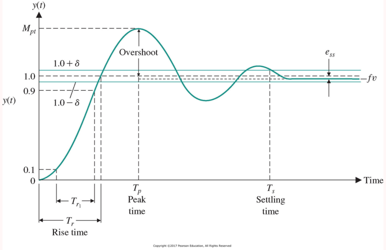


Steady state response

Definition

The overshoot is the maximum difference between the transient and steady state response towards a unit step input. It is often used as a measure for the relative stability.

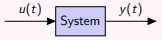


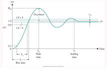


Settling time

Definition

The overshoot is the maximum difference between the transient and steady state response towards a unit step input. It is often used as a measure for the relative stability.





Overshoot

Definition

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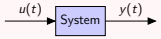
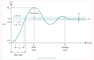
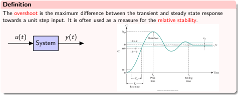




Settling time

Settling time

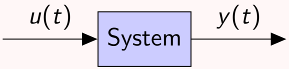
Settling time

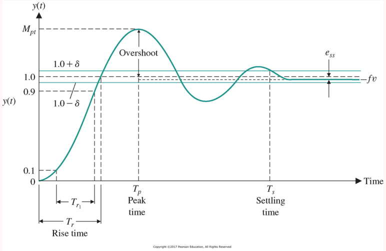


Overshoot

Definition

The settling time is usually defined as the time required for the response to a unit step input to reach and remain within a specific distance to the steady state value (e.g. 2% or 5%).





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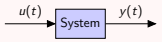
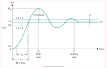
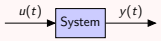


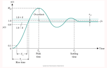


Settling time

Rise time

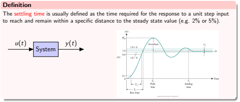
Definition
The overshoot is the maximum difference between the transient and steady state response towards a unit step input. It is often used as a measure for the relative stability.





Rise time

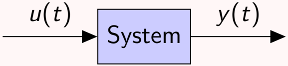
Rise time

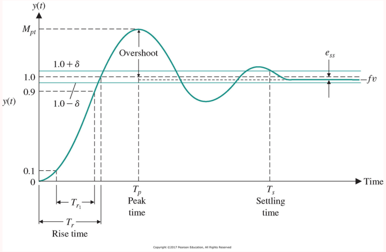


Settling time

Definition

The rise timeis frequently defined as the time required for the response to a unit step input to rise e.g. from 10% to 90% or 0% to 100% of the steady state value.





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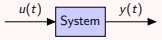
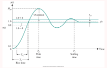


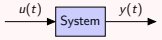


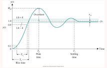
Rise time

Time to peak

Definition

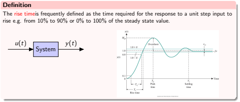
The settling time is usually defined as the time required for the response to a unit step input to reach and remain within a specific distance to the steady state value (e.g. 2% or 5%).





Time to peak

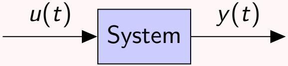
Time to peak

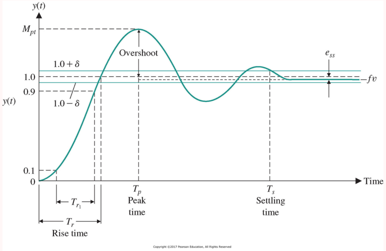


Rise time

Definition

The time to peak or peak time is the time required to reach the maximum of the response to a unit step input.





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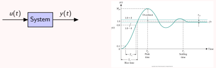
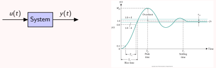


Time to peak

Delay-time

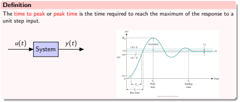
Definition

The rise timeis frequently defined as the time required for the response to a unit step input to rise e.g. from 10% to 90% or 0% to 100% of the steady state value.



Delay-time

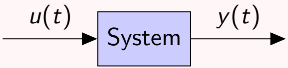
Delay-time

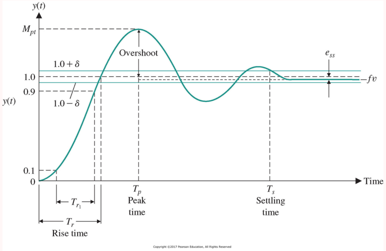


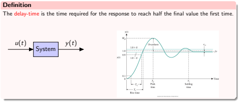
Time to peak

Definition

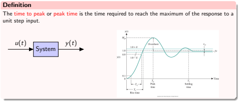
The delay-time is the time required for the response to reach half the final value the first time.







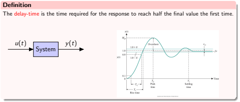
Delay-time



Resonant peak

Resonant peak

Resonant peak



Delay-time

Definition

The resonant peak G_r is the maximum value of $|G(j\omega)|$. The resonant frequency is the frequency at which the peak resonance G_r occurs.

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Resonant peak

Tracking error

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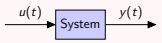
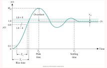
The delay-time is the time required for the response to reach half the final value the first time.





Tracking error

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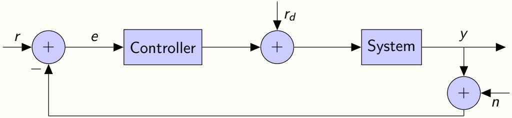
Resonant peak

Properties

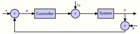


The tracking error:

$$
\begin{aligned}
E(s) &= R(S) - Y(s) \\
&= \frac{1}{1+L(s)}R(s) - \frac{G(s)}{1+L(s)}R_d(s) + \frac{L(s)}{1+L(s)}N(s) \\
&= \frac{1}{F(s)}R(s) - \frac{G(s)}{F(s)}R_d(s) + \frac{F(s)-1}{F(s)}N(s).
\end{aligned}
$$

Tracking error

Properties



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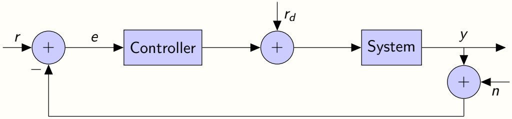


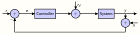
$$E(s) \quad = \quad S(S)R(s) - S(s)G(s)R_d(s) + C(s)N(s)$$

Minimizing the tracking error is equivalent to minimizing $S(s)$ and $C(s)$ at the same time. Obviously, due to

$$C(s) = 1 - \frac{1}{F(s)} = 1 - S(s)$$

compromises are necessary.

Properties



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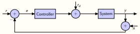


The tracking error:

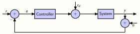
$$\begin{aligned} E(s) &= R(S) - Y(s) \\ &= \frac{1}{1+L(s)}R(s) - \frac{G(s)}{1+L(s)}R_d(s) + \frac{L(s)}{1+L(s)}N(s) \\ &= \frac{1}{F(s)}R(s) - \frac{G(s)}{F(s)}R_d(s) + \frac{F(s)-1}{F(s)}N(s). \end{aligned}$$

Tracking error

Steady state error

Steady state error

Properties



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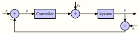

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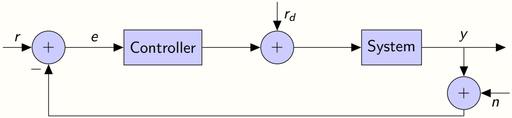
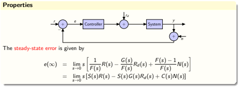
Properties



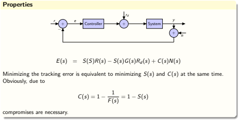
The steady-state error is given by

$$\begin{aligned} e(\infty) &= \lim_{s\to 0} s\left[\frac{1}{F(s)}R(s) - \frac{G(s)}{F(s)}R_d(s) + \frac{F(s)-1}{F(s)}N(s)\right] \\ &= \lim_{s\to 0} s\left[S(s)R(s) - S(s)G(s)R_d(s) + C(s)N(s)\right] \end{aligned}$$

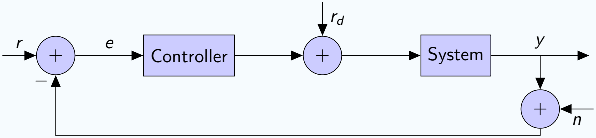
Steady-state error



Steady state error



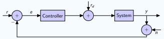
Example: Unit step input



The steady-state error is given by

$$
\begin{aligned}
e(\infty) &= \lim_{s\to 0} s\left[\frac{1}{F(s)}\frac{1}{s} - \frac{G(s)}{F(s)}R_d(s) + \frac{F(s)-1}{F(s)}N(s)\right] \\
&= \lim_{s\to 0}\left[\frac{1}{1+G_c(0)G(0)} - \frac{G(s)}{F(s)}R_d(s)s + \frac{F(s)-1}{F(s)}N(s)s\right]
\end{aligned}
$$

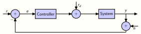
Example: Unit step input



The steady-state error is given by

$$\begin{aligned} e(\infty) &= \lim_{s\to 0} s\left[\frac{1}{F(s)}\frac{1}{s} - \frac{G(s)}{F(s)}R_d(s) + \frac{F(s)-1}{F(s)}N(s)\right] \\ &= \lim_{s\to 0}\left[\frac{1}{1+G_c(0)G(0)} - \frac{G(s)}{F(s)}R_d(s)s + \frac{F(s)-1}{F(s)}N(s)s\right] \end{aligned}$$

Properties



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$$\begin{aligned} e(\infty) &= \lim_{s\to 0} s\left[\frac{1}{F(s)}R(s) - \frac{G(s)}{F(s)}R_d(s) + \frac{F(s)-1}{F(s)}N(s)\right] \\ &= \lim_{s\to 0} s\left[S(s)R(s) - S(s)G(s)R_d(s) + C(s)N(s)\right] \end{aligned}$$

Steady state error

Exercise (#5.3)

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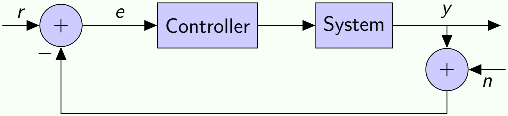

$$\begin{aligned} E(s) &= S(S)R(s) + C(s)N(s) \\ G(s) &= \frac{10}{s+10} \\ G_c(s) &= K \end{aligned}$$

Plot the tracking error for the unit step response for several values of K assuming n to be white noise with standard deviation of 0, 0.1 and 1, respectively. Make use of MATLAB or NUMPY. Hint: In MATLAB make use of `lsim`.

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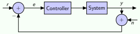

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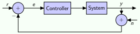

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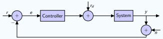
Example: Unit step input



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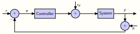
Properties



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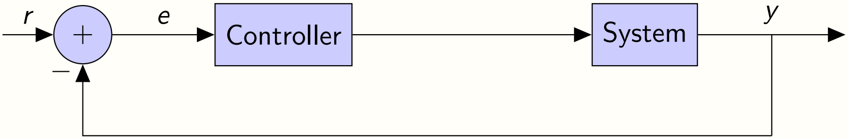

Steady state error for unit step input and absence of disturbances:

$$\begin{aligned} e_0(\infty) &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0}\left[\frac{1}{1+G_c(s)G(s)}\right] \end{aligned}$$

Closed-loop systems

Lag compensator

Lag compensator

Definition

The lag compensator or phase-lag compensator has a transfer function of the form

$$\begin{aligned} G_c(s) &= K\frac{s+z}{s+p}, \text{ with } \alpha = z/p > 1 \text{ or } z > p \\ &= \frac{Kz}{p}\frac{\frac{s}{z}+1}{\frac{s}{p}+1} \\ &= V\frac{T_2 s+1}{T_1+1}. \end{aligned}$$

Note that the choice of T_1 and T_2 will lead to lead or lag behaviour. See also Dorf, Bishop, 13th edition, page 548.

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Lag compensator

Properties

- Provides attenuation at high frequencies
- Slows system
- Gain crossover frequency increases
- Steady state accuracy and disturbance rejection increases
- Idealized element with $T_1 \to 0$: PD-element (see later)
- German: $\mathrm{PDT_1}$-Glied
- Approximates PI control as $b \to 0$
- Decreases phase around $\sqrt{zp}$

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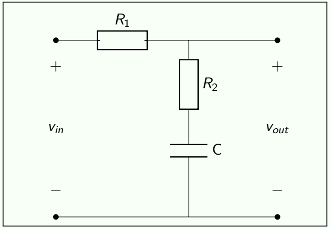

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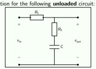

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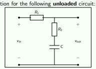

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Exercise (#5.5)

Given are

$$\begin{aligned} G(s) &= \frac{1}{(s+1-jk)(s+1+jk)} \\ \text{and} \\ G_c(s) &= \frac{2(s+4)}{4(s+2)} \end{aligned}$$

Plot the root locus for the closed-loop systems (unity feedback, compensated/uncompensated, parameter k). Make use of NUMPY or MATLAB.

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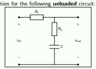

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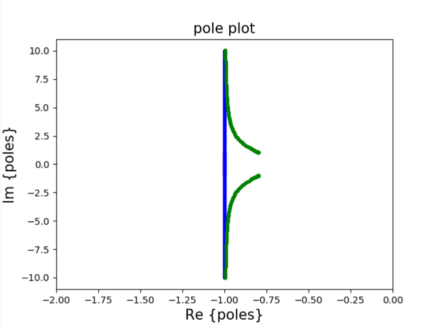

The lag compensator moves the poles to the right and increases DC gain.

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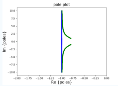

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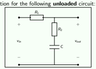

Lead compensator

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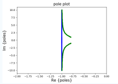

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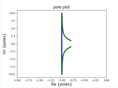

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- Adds positive phase (gain margin increases)
- System stability improves
- System responds faster
- Steady state accuracy does not increase
- Maximum phase lead occurs at $\omega_m = \sqrt{zp}$
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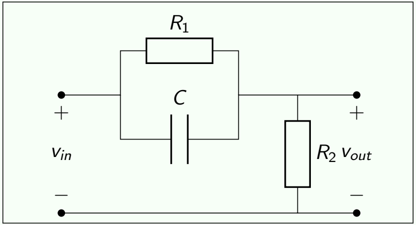

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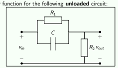

Lead compensator

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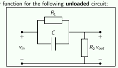

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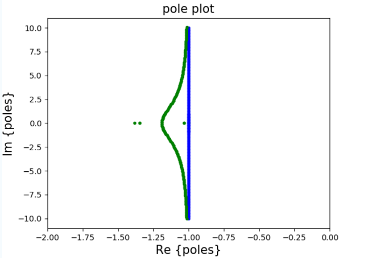

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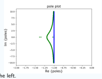

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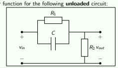

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Exercise (#5.7)

Given is a system and a compensator as follows:

$$G(s) = \frac{20}{s(s+2)}$$
$$G_c(s) = 3\frac{s+4.8}{s+14.4}$$

- ▷ Plot the Bode plots of compensated and uncompensated system.
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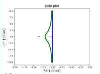

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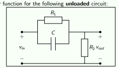

Lead lag compensator

Lead-lag compensator

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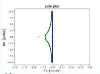

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Definition

Cascading a lead and a lag compensator yields the lead-lag compensator with transfer function of the form

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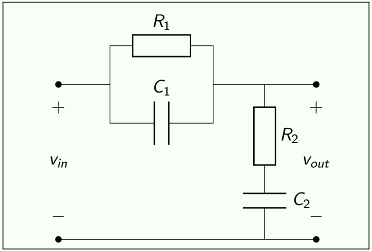

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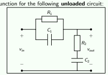

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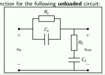

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Lead lag compensator

Exercise (#5.9)

An industrial grinding process is given by[a] the transfer function

$$G_p(s) = \frac{10}{s(s+5)}.$$

The plant shall be controlled using a digital controller based upon an analog prototype of the form

$$G_c(s) = K\frac{s+a}{s+b}.$$

Ensure a phase marging of at least 45^o and a settling time of less than 1 second (2% criterion).

[a] This exercise is partially based upon Dorf, Bishop. The corresponding source is: Predictive Control of a Robotic Grinding System, Journal of Engineering for Industry, ASME, November 1992, pp. 412-420. See also MATLAB

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PID controller

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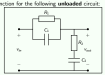


Definition

Cascading a lead and a lag compensator yields the lead-lag compensator with transfer function of the form

$$G_c(s) = \frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}, \text{ with } p_1 > z_1, z_2 > p_2.$$

PID controller

PID controller

PID controller

Exercise (#5.9)

An industrial grinding process is given by[a] the transfer function

$$G_p(s) = \frac{10}{s(s+5)}.$$

The plant shall be controlled using a digital controller based upon an analog prototype of the form

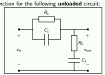
$$G_c(s) = K\frac{s+a}{s+b}.$$

Ensure a phase marging of at least 45° and a settling time of less than 1 second (2% criterion).

[a] This exercise is partially based upon Dorf, Bishop. The corresponding source is: Predictive Control of a Robotic Grinding System, Journal of Engineering for Industry, ASME, November 1992, pp. 412-420. See also MATLAB

Exercise (#5.8)

Determine the transfer function for the following **unloaded** circuit:

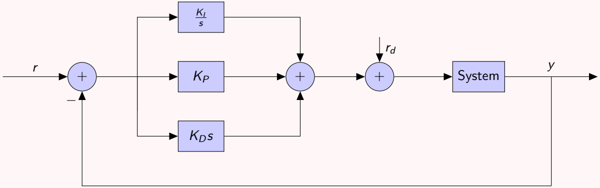


Definition

A controller with a transfer function of the form

$$G_c(s) = K_P + sK_D + \frac{K_I}{s}$$

is called PID controller.



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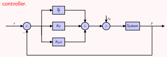

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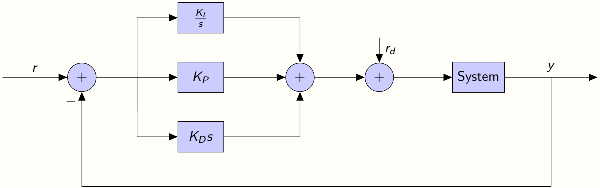
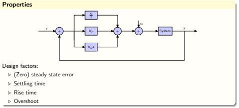
Properties



Design factors:

- (Zero) steady state error
- Settling time
- Rise time
- Overshoot

PID controller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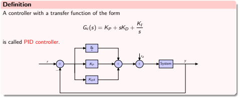
Definition
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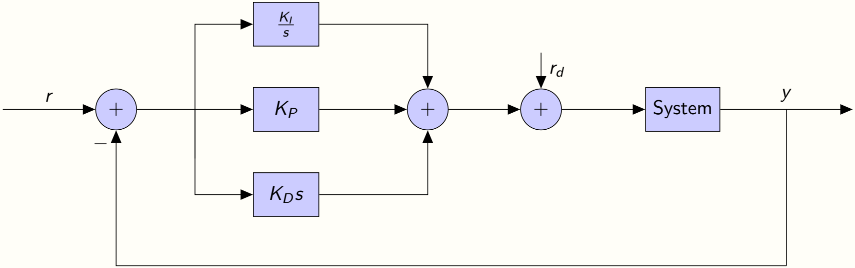
PID controller

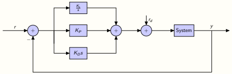
Properties



- P term is used to adjust the speed.
- I term provides zero error.
- D term introduces damping.

PID controller

Properties



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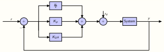

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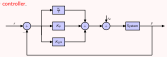
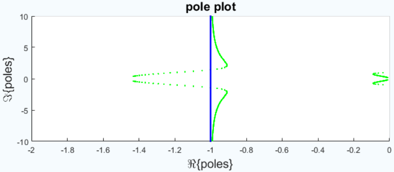


Example

$$G(s) = \frac{1}{(s+1-jk)(s+1+jk)}$$

$$G_c(s) = 2 + \frac{3}{s} + 0.1s$$



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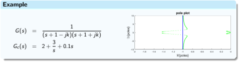

Properties

The PID controller combines properties of the lead and the lag compensator.

- ▷ A lead compensator approximates PD control
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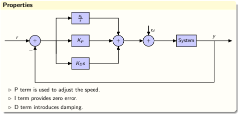

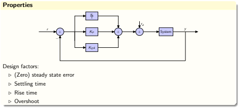

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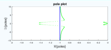

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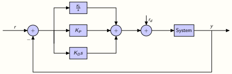

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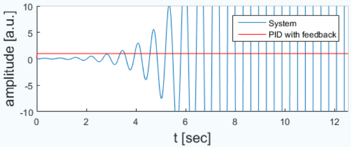
Example

Consider

$$G(s) = \frac{5}{s^2 - 2s + 101}$$



At least in theory, this system can be stabilized using a PID controller. But: Usually actuators are limited.

Ziegler–Nichols method

Example

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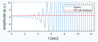

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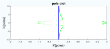

Heuristic tuning

Heuristic method of tuning a PID controller. Works for unkown plants as well! Workflow:

1. Set $K_I = K_D = 0$.
2. Increase K_P up to K_0 until the output becomes unstable
3. Measure the period T_0 of oscillation (frequency f_0)
4. Use the following table to determine K_I and K_D.

Control type	K_P	K_I/K_P	K_D/K_P
P	$0.5K_0$	–	–
PI	$0.45K_0$	$1.2f_0$	–
PD	K_0	–	0
PID tight	$0.6K_0$	$2f_0$	$0.125T_0$
Some overshoot	$0.33K_0$	$2f_0$	$0.33T_0$
No Overshoot	$0.2K_0$	$2f_0$	$0.5T_0$

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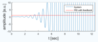

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Exercise (#5.10)

Given is a system (plant) with following transfer function:

$$H(s) = \frac{10}{s^3 + 6s^2 + 11s + 16}.$$

Plot the step response of a unity feedback system with PID controller and the following parameters:

Control type	K_P	K_I	K_D
P	5	–	–
P	5.5	–	–
P	3	–	–
PI	2.7	1.8	–
PID	2	2.22	1.2

Note: Make use of NUMPY and the functions `tf`, `step`, `series` and `feedback`.

Exercise

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P	5.5	–	–
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PI	2.7	1.8	–
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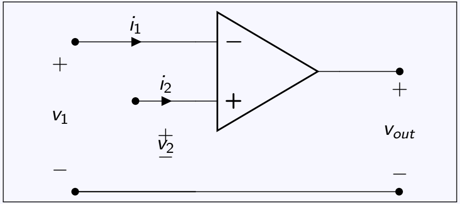
Note: Make use of NUMPY and the functions `tf`, `step`, `series` and `feedback`.

Closed-loop systems

OP-amps

OP-amps

Basic operation

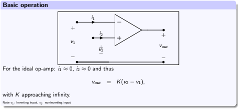


For the ideal op-amp: $i_1 \approx 0$, $i_2 \approx 0$ and thus

$$v_{out} = K(v_2 - v_1),$$

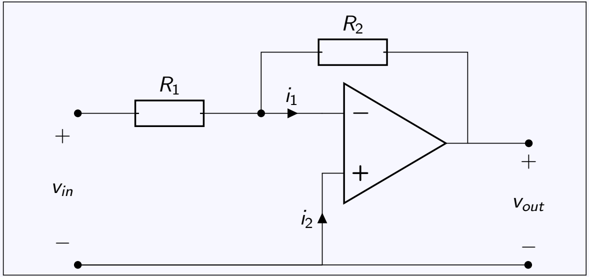
with K approaching infinity.

Note v_1: Inverting input, v_2: noninverting input



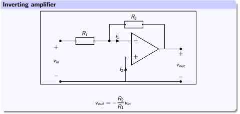
OP-amps

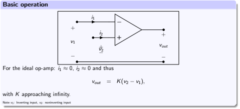
Inverting amplifier



$$v_{out} = -\frac{R_2}{R_1} v_{in}$$

Integrating circuit





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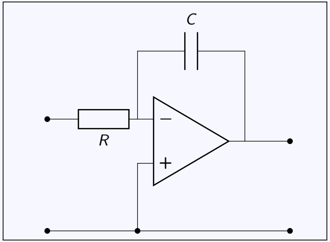
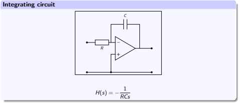
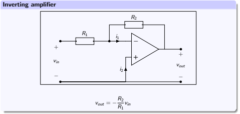
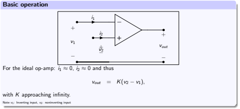


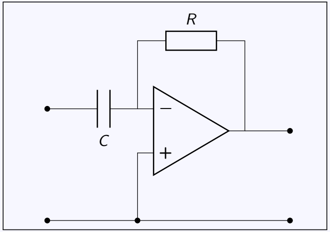
$$H(s) = -\frac{1}{RCs}$$







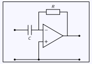
Differentiating circuit



$$H(s) = -RCs$$

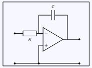
Accelerometer

Differentiating circuit



$H(s) = -RCs$

Integrating circuit



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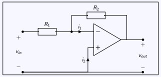
Inverting amplifier



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Accelerometer

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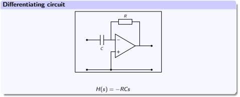
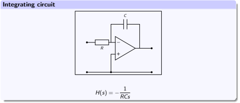
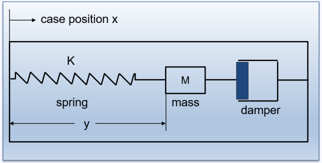




Accelerometer



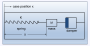
$$-B\frac{\mathrm{d}y}{\mathrm{d}t} - Ky = M\frac{\mathrm{d}^2}{\mathrm{d}t^2}(y - x)$$

With $a(t)$ being the input acceleration, one gets

$$\frac{Y}{A} = \frac{1}{s^2 + B/Ms + K/M}$$

DC motor

Accelerometer



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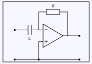

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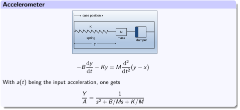


DC motor

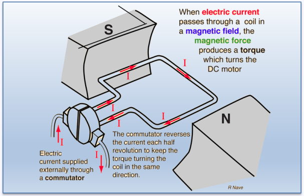


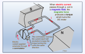
Figure 20: DC motor. Source: R. Nave, Georgia State University

Approximation for armature controlled DC motor (e.g. Dorf, Bishop, 13th edition, page 105):

$$H(s) = \frac{\phi(s)}{V(s)} = \frac{K}{s(\tau s + 1)}$$

Lever

DC motor



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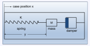

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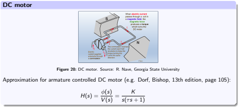

DC motor

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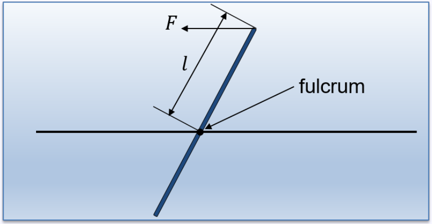
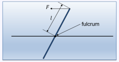


The angular momentum (torque, Drehmoment):

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F},$$

with $\boldsymbol{r}$ being the vector pointing to the fulcrum.

Lever



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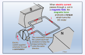

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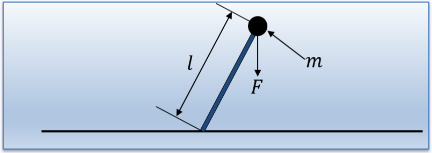
$$H(s) = \frac{\phi(s)}{V(s)} = \frac{K}{s(\tau s + 1)}$$

Lever



Resistance against rotation (moment of interia, Trägheitsmoment) for a simple pendulum:

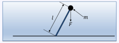
$$I = ml^2.$$

Relationship between torque and rotation (shape of body **not** changing):

$$\tau = I\dot{\omega} = ml^2\dot{\omega}$$

Exercise

Lever



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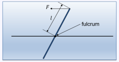

The angular momentum (torque, Drehmoment):

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F},$$

with $\boldsymbol{r}$ being the vector pointing to the fulcrum.

Exercise (#5.11)

- ▷ Design a PID controler for the inverted pendulum using NUMPY.
- ▷ Use an initial value of $\phi(t) = 0$ and plot the response to $\phi_c(t) = 0$

Sources:

System modeling

PID controller desing with MATLAB

Lever

Exercise

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System modeling

PID controller desing with MATLAB

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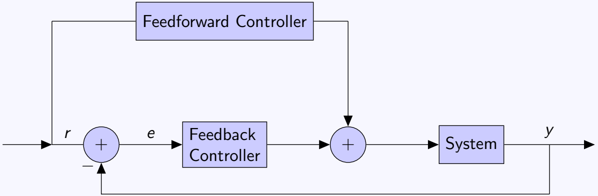
Feedback & Control

Feed-forward Control

Not covered here



FF controller can be designed by using inverse model of system which leads to rapid response.

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Feedback & Control

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State Variable Feedback Systems

Closed-loop feedback system

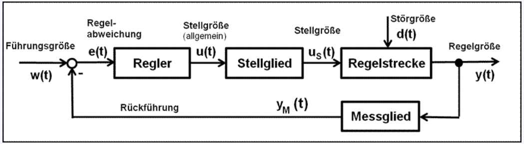
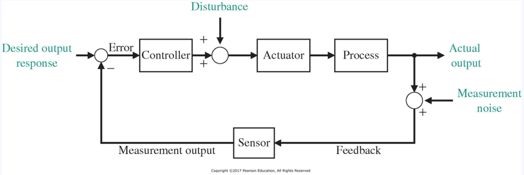


Figure 21: Source: Wikipedia (HeinrichKÜ)



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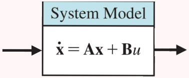
State Variable Feedback Systems

Definition

We consider a system fulfilling the equation

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u.$$

This system



is said to be completely controllable if for each initial state $\boldsymbol{x}(t_0)$ there exists a control signal $u(t)$ that can transfer the system state to any other desired location $\boldsymbol{x}(t_0 + T)$ in a finite time T.

Controllability

Definition

We consider a system fulfilling the equation

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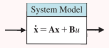
This system



is said to be completely controllable if for each initial state $\boldsymbol{x}(t_0)$ there exists a control signal $u(t)$ that can transfer the system state to any other desired location $\boldsymbol{x}(t_0 + T)$ in a finite time T.

Definition

The $(n \times n)$ controllability matrix is defined as follows:

$$\boldsymbol{P}_c = \left[\boldsymbol{b} \;\; \boldsymbol{A}\boldsymbol{b} \;\; \boldsymbol{A}^2\boldsymbol{b} \ldots \boldsymbol{A}^{n-1}\boldsymbol{b}\right].$$

The system is completely controllable if $\boldsymbol{P}_c$ is of full rank (determinant unequal to zero). Note that controllabilty is a concept based upon unconstrained inputs.

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State Variable Feedback Systems

Full-state feedback

Task

In this section we deal with the control of controllable systems where all internal states area available.

Task
In this section we deal with the control of controllable systems where all internal states area available.

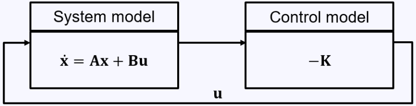
Model

Full-state feedback

Model

Task
In this section we deal with the control of controllable systems where all internal states area available.

System state



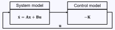
Goal: Determination of the $(1 \times n)$ matrix $\boldsymbol{K}$

- ▷ Assume that all states are available (can be measured)
- ▷ $u = -\boldsymbol{K}\boldsymbol{x}$
- ▷

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\,\boldsymbol{x}$$

Characteristic equation

System state



Goal: Determination of the $(1 \times n)$ matrix $\boldsymbol{K}$

- Assume that all states are available (can be measured)
- $u = -\boldsymbol{K}x$
-

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u = \boldsymbol{A}x + \boldsymbol{BK}x = (\boldsymbol{A} - \boldsymbol{BK})\,x$$

Model

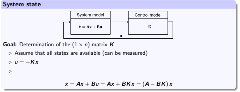
Task

In this section we deal with the control of controllable systems where all internal states area available.

Characteristic equation

Full-state feedback

Characteristic equation

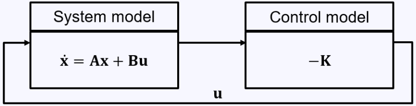


Model

Characteristic equation

The characteristic equation of the full-state feedback system shown below is given by

$$\det\left(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{BK})\right) = 0.$$



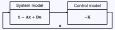
If all roots of the characteristic equation lie in the left half-plane, then the closed-loop is stable.

Pole placement

Characteristic equation

The characteristic equation of the full-state feedback system shown below is given by

$$\det\left(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\right) = 0.$$



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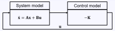
Characteristic equation

Pole placement

System state



Goal: Determination of the $(1 \times n)$ matrix $\boldsymbol{K}$

- Assume that all states are available (can be measured)
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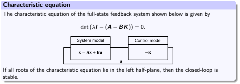
-

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\,\boldsymbol{x}$$

Pole placement

Pole placement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$$\det(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})) = 0.$$

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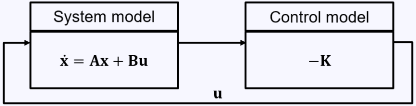
Characteristic equation

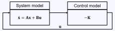
System state



If the system is completely controllable, then $\boldsymbol{K}$ can be determined to place all poles in the left half-plane, so that the transient performance meets the desired response.

Ackermann's formula

System state



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Pole placement

Characteristic equation

The characteristic equation of the full-state feedback system shown below is given by

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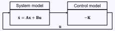

If all roots of the characteristic equation lie in the left half-plane, then the closed-loop is stable.

Definition

For a single input system with $u = -\boldsymbol{K}\boldsymbol{x}$ and a desired characteristic equation

$$q(\lambda) = \lambda^n + \alpha_{n-1}\lambda^{n-1} \cdots + \alpha_0$$

one can calculate the gain matrix with Ackermann's formula as follows:

$$\boldsymbol{K} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} \boldsymbol{P}_c^{-1} q(\boldsymbol{A}),$$

with

$$q(\boldsymbol{A}) = \boldsymbol{A}^n + \alpha_{n-1}\boldsymbol{A}^{n-1} + \cdots \alpha_1 \boldsymbol{A}^1 + \alpha_0 \boldsymbol{I}.$$

and $\boldsymbol{P}_c$ being the controllability matrix.

Ackermann's formula

Definition

For a single input system with $u = -\boldsymbol{K}\boldsymbol{x}$ and a desired characteristic equation

$$q(\lambda) = \lambda^n + \alpha_{n-1}\lambda^{n-1} \cdots + \alpha_0$$

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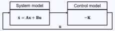
System state



If the system is completely controllable, then $\boldsymbol{K}$ can be determined to place all poles in the left half-plane, so that the transient performance meets the desired response.

Pole placement

Definition

Also used: For a single input system with $u = -\boldsymbol{K}\boldsymbol{x}$ and a desired characteristic equation

$$q(\lambda) = \lambda^n + \alpha_1\lambda^{n-1} \cdots + \alpha_n$$

one can calculate the gain matrix with Ackermann's formula as follows:

$$\boldsymbol{K} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} \boldsymbol{P}_c^{-1} q(\boldsymbol{A}),$$

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$$q(\boldsymbol{A}) = \boldsymbol{A}^n + \alpha_1\boldsymbol{A}^{n-1} + \cdots \alpha_{n-1} \boldsymbol{A}^1 + \alpha_n \boldsymbol{I}.$$

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State Variable Feedback Systems

Full-state feedback

Task

In this section we deal with the control of controllable systems with measurement of only some internal states.

Task
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Observability

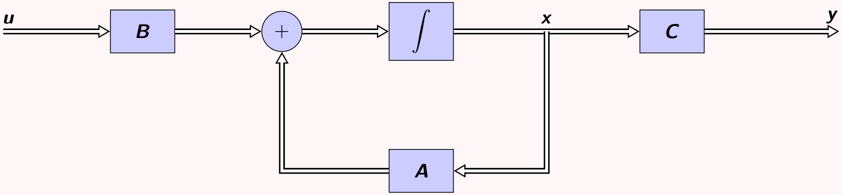
Measurement

Observability

Task
In this section we deal with the control of controllable systems with measurement of only some internal states.

Definition

Frequently, not all states can be measured. This is taken into account by introducing the measurement matrix $\boldsymbol{C}$ (also called output matrix):



$$\boxed{\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}}$$

Observalibility

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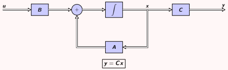


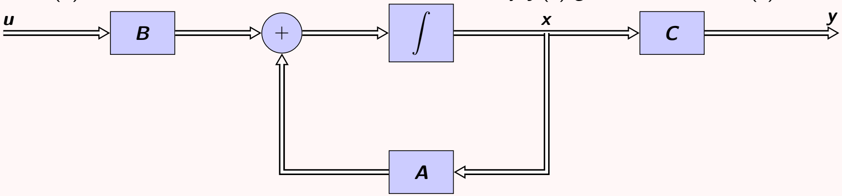
Observability

Task

In this section we deal with the control of controllable systems with measurement of only some internal states.

Definition

A system is completely observable if and only if there exists a finite time T such that the initial state $\boldsymbol{x}(0)$ can be determined from the observation history $\boldsymbol{y}(t)$ given the control $\boldsymbol{u}(t)$.[a]



[a] Dorf, Bishop

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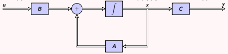

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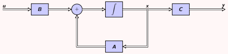

$\boxed{\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}}$

Definition

The observability matrix $\boldsymbol{P}_O$ is given by

$$\boldsymbol{P}_O = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C}\boldsymbol{A} \\ \vdots \\ \boldsymbol{C}\boldsymbol{A}^{n-1} \end{bmatrix}.$$

The system is completely observable in case of $\boldsymbol{P}_O$ having full rank $(\det(\boldsymbol{P}_O) \neq 0)$.

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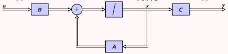

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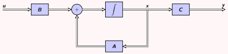

$$\boxed{\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}}$$

Full-state observer

Full-state observer

Full-state observer

Definition

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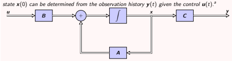

[a] Dorf, Bishop

System state determination

If the system is completely observable, then one can determine (estimate) the states that are not directly measurable. Idea: Use an estimated state $\hat{\boldsymbol{x}}$ with

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u},$$

where $\boldsymbol{u}$ is the (known) control input. Due the fact that the initial state $\hat{\boldsymbol{x}}(t=0)$ is unknown (needs to be *guessed*), a correction is needed:

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{L}\tilde{\boldsymbol{y}},$$

where $\tilde{\boldsymbol{y}}$ is a **time dependent** correction factor.

Full-state observer

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The system is completely observable in case of $\boldsymbol{P}_O$ having full rank ($\det(\boldsymbol{P}_O) \neq 0$).

Correction factor

Making use the difference between observed state and estimated observed state:

$$\begin{aligned} \tilde{\boldsymbol{y}} &= \boldsymbol{C}\boldsymbol{x} - \boldsymbol{C}\hat{\boldsymbol{x}} \\ &= \boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}} \end{aligned}$$

and thus

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{L}\left(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}}\right),$$

Full-state observer

Correction factor

Making use the difference between observed state and estimated observed state:

$$\begin{aligned}\tilde{y} &= Cx - C\hat{x} \\ &= y - C\hat{x}\end{aligned}$$

and thus

$$\dot{\hat{x}} = A\hat{x} + Bu + L\left(y - C\hat{x}\right).$$

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Full-state observer

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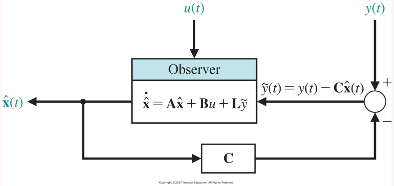
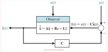


This a called an observer with L being the observer gain matrix.

Full-state observer

Definition

$$\boxed{\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})}.$$



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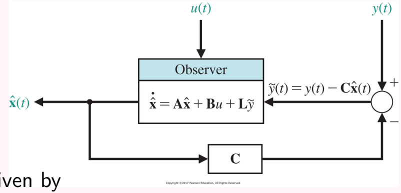
Definition



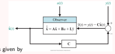
The estimation error is given by

$$\begin{aligned}\boldsymbol{e(t)} &= \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{e}}(t) &= \boldsymbol{Ax} + \boldsymbol{Bu} - \boldsymbol{A}\hat{\boldsymbol{x}} - \boldsymbol{Bu} - \boldsymbol{L}\left(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}}\right)\\ &= \left(\boldsymbol{A} - \boldsymbol{LC}\right)\boldsymbol{e}(t)\end{aligned}$$

Note that the error does not depend on $u(t)$.

Full-state observer

Definition



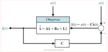
The estimation error is given by

$$\begin{aligned} \boldsymbol{e}(t) &= \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{e}}(t) &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u - \boldsymbol{A}\hat{\boldsymbol{x}} - \boldsymbol{B}u - \boldsymbol{L}(y - \boldsymbol{C}\hat{\boldsymbol{x}}) \\ &= (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\,\boldsymbol{e}(t) \end{aligned}$$

Note that the error does not depend on $u(t)$.

Definition

$$\boxed{\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}(y - \boldsymbol{C}\hat{\boldsymbol{x}})}.$$



This a called an observer with $\boldsymbol{L}$ being the observer gain matrix.

Correction factor

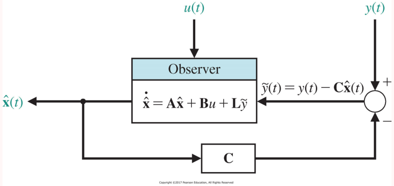
Making use the difference between observed state and estimated observed state:

$$\begin{aligned} \tilde{y} &= \boldsymbol{C}\boldsymbol{x} - \boldsymbol{C}\hat{\boldsymbol{x}} \\ &= y - \boldsymbol{C}\hat{\boldsymbol{x}} \end{aligned}$$

and thus

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}(y - \boldsymbol{C}\hat{\boldsymbol{x}}),$$

Definition



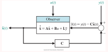
The estimation error $\boldsymbol{e}(t)$ will approach zero for $t \to \infty$ in case of

$$\det\left(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\right) = 0$$

having all roots in the left half-space.

Observer design

Definition

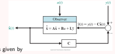


The estimation error $\boldsymbol{e}(t)$ will approach zero for $t \to \infty$ in case of

$$\det\left(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{LC})\right) = 0$$

having all roots in the left half-space.

Definition



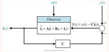
The estimation error is given by

$$\begin{aligned} \boldsymbol{e}(t) &= \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{e}}(t) &= \boldsymbol{Ax} + \boldsymbol{Bu} - \boldsymbol{A}\hat{\boldsymbol{x}} - \boldsymbol{Bu} - \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}}) \\ &= (\boldsymbol{A} - \boldsymbol{LC})\,\boldsymbol{e}(t) \end{aligned}$$

Note that the error does not depend on $u(t)$.

Definition

$$\boxed{\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{Bu} + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}})}.$$

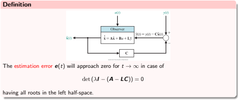


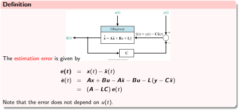
This a called an observer with $\boldsymbol{L}$ being the observer gain matrix.

Observer design

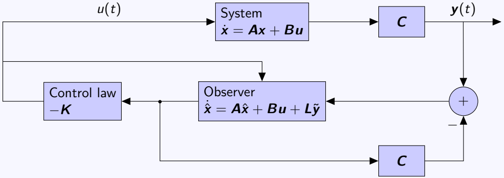
Full-state observer

Observer design





Full-state observer

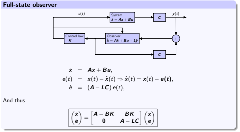


$$
\begin{aligned}
\dot{x} &= Ax + Bu, \\
e(t) &= x(t) - \hat{x}(t) \Rightarrow \hat{x}(t) = x(t) - e(t), \\
\dot{e} &= (A - LC)\,e(t),
\end{aligned}
$$

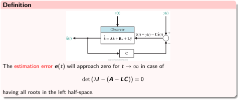
And thus

$$
\boxed{\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix}}
$$

Design process



Observer design



Steps

1. Determine $\boldsymbol{K}$
2. Determine $\boldsymbol{L}$
3. Connect the observer

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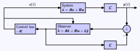
Design process

Steps
1. Determine $\boldsymbol{K}$
2. Determine $\boldsymbol{L}$
3. Connect the observer

Full-state observer



$$
\begin{aligned}
\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}, \\
e(t) &= \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t) \Rightarrow \hat{\boldsymbol{x}}(t) = \boldsymbol{x}(t) - \boldsymbol{e}(t), \\
\dot{\boldsymbol{e}} &= (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\,\boldsymbol{e}(t),
\end{aligned}
$$

And thus

$$
\boxed{\begin{pmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{e}} \end{pmatrix} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K} & \boldsymbol{B}\boldsymbol{K} \\ \boldsymbol{0} & \boldsymbol{A} - \boldsymbol{L}\boldsymbol{C} \end{bmatrix} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{e} \end{pmatrix}}
$$

Observer design

Steps

1. Determine $\boldsymbol{K}$ by making use of Ackermann's formula:

$$
\boldsymbol{K} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} \boldsymbol{P}_c^{-1} q(\boldsymbol{A})
$$

and the desired characteristic equation

$$
\det\left(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\right) = \lambda^n + \alpha_{n-1}\lambda^{n-1} \cdots + \alpha_0 \quad = \quad 0,
$$

with

$$
q(\boldsymbol{A}) = \boldsymbol{A}^n + \alpha_{n-1}\boldsymbol{A}^{n-1} + \cdots \alpha_1 \boldsymbol{A}^1 + \alpha_0 \boldsymbol{I}.
$$

and $\boldsymbol{P}_c$ being the controllability matrix.

2. Determine $\boldsymbol{L}$
3. Connect the observer

Design process

Steps

1. Determine $\boldsymbol{K}$ by making use of Ackermann's formula:

$$\boldsymbol{K} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} \boldsymbol{P}_c^{-1} q(\boldsymbol{A})$$

and the desired characteristic equation

$$\det\left(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{BK})\right) = \lambda^n + \alpha_{n-1}\lambda^{n-1} \cdots + \alpha_0 = 0,$$

with

$$q(\boldsymbol{A}) = \boldsymbol{A}^n + \alpha_{n-1}\boldsymbol{A}^{n-1} + \cdots \alpha_1 \boldsymbol{A}^1 + \alpha_0 \boldsymbol{I}.$$

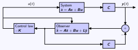
and $\boldsymbol{P}_c$ being the controllability matrix.

2. Determine $\boldsymbol{L}$
3. Connect the observer

Steps

1. Determine $\boldsymbol{K}$
2. Determine $\boldsymbol{L}$
3. Connect the observer

Full-state observer



$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{Ax} + \boldsymbol{Bu}, \\ e(t) &= \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t) \Rightarrow \hat{\boldsymbol{x}}(t) = \boldsymbol{x}(t) - \boldsymbol{e}(t), \\ \dot{\boldsymbol{e}} &= (\boldsymbol{A} - \boldsymbol{LC})\,\boldsymbol{e}(t), \end{aligned}$$

And thus

$$\boxed{\begin{pmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{e}} \end{pmatrix} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{BK} & \boldsymbol{BK} \\ \boldsymbol{0} & \boldsymbol{A} - \boldsymbol{LC} \end{bmatrix} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{e} \end{pmatrix}}$$

Steps

1. Determine $\boldsymbol{K}$
2. Determine $\boldsymbol{L}$ by making use of Ackermann's formula:

$$\boldsymbol{L} \;=\; p(\boldsymbol{A})\boldsymbol{P}_0^{-1}\begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}^T$$

and the desired characteristic equation

$$\det\left(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{LC})\right) = \lambda^n + \beta_{n-1}\lambda^{n-1} \cdots + \beta_0 = 0$$

describing the observer dynamics and

$$q(\boldsymbol{A}) = \boldsymbol{A}^n + \beta_{n-1}\boldsymbol{A}^{n-1} + \cdots \beta_1 \boldsymbol{A}^1 + \beta_0 \boldsymbol{I},$$

where $\boldsymbol{P}_0$ is the observability matrix.

3. Connect the observer

Desing process

Steps

1. Determine $\boldsymbol{K}$
2. Determine $\boldsymbol{L}$ by making use of Ackermann's formula:

$$\boldsymbol{L} = p(\boldsymbol{A})\boldsymbol{P}_o^{-1}\begin{bmatrix}0 & 0 & \cdots & 1\end{bmatrix}^T$$

and the desired characteristic equation

$$\det(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})) = \lambda^n + \beta_{n-1}\lambda^{n-1} \cdots + \beta_0 = 0$$

describing the observer dynamics and

$$q(\boldsymbol{A}) = \boldsymbol{A}^n + \beta_{n-1}\boldsymbol{A}^{n-1} + \cdots \beta_1 \boldsymbol{A}^1 + \beta_0 \boldsymbol{I}.$$

where $\boldsymbol{P}_o$ is the observability matrix.

3. Connect the observer

Steps

1. Determine $\boldsymbol{K}$ by making use of Ackermann's formula:

$$\boldsymbol{K} = \begin{bmatrix}0 & 0 & \cdots & 1\end{bmatrix}\boldsymbol{P}_c^{-1}q(\boldsymbol{A})$$

and the desired characteristic equation

$$\det(\lambda \boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})) = \lambda^n + \alpha_{n-1}\lambda^{n-1} \cdots + \alpha_0 \;=\; 0,$$

with

$$q(\boldsymbol{A}) = \boldsymbol{A}^n + \alpha_{n-1}\boldsymbol{A}^{n-1} + \cdots \alpha_1 \boldsymbol{A}^1 + \alpha_0 \boldsymbol{I}.$$

and $\boldsymbol{P}_c$ being the controllability matrix.

2. Determine $\boldsymbol{L}$
3. Connect the observer

Steps

1. Determine $\boldsymbol{K}$
2. Determine $\boldsymbol{L}$
3. Connect the observer

Steps

1. Determine $\boldsymbol{K}$
2. Determine $\boldsymbol{L}$
3. Connect the observer by making use of

$$\boldsymbol{u}(t) = -\boldsymbol{K}\hat{\boldsymbol{x}}(t).$$

$$\begin{pmatrix}\dot{\boldsymbol{x}} \\ \dot{\boldsymbol{e}}\end{pmatrix} = \begin{bmatrix}\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K} & \boldsymbol{B}\boldsymbol{K} \\ \boldsymbol{0} & \boldsymbol{A} - \boldsymbol{L}\boldsymbol{C}\end{bmatrix}\begin{pmatrix}\boldsymbol{x} \\ \boldsymbol{e}\end{pmatrix}$$

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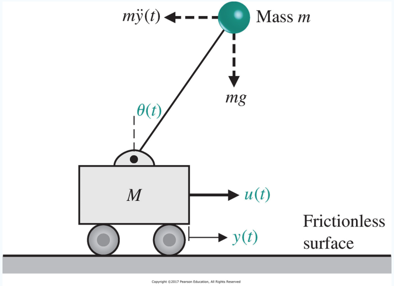
State Variable Feedback Systems

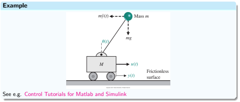
Inverted pendulum

Example

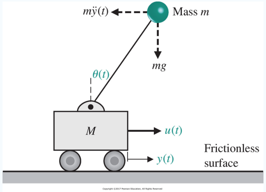


See e.g. Control Tutorials for Matlab and Simulink

Inverted pendulum



Example



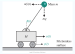
Sum of forces in horizontal direction:

$$M\ddot{y}(t) - \cos(\pi + \theta)ml\ddot{\theta}(t) - ml\dot{\theta}^2 \sin(\pi + \theta) - u(t) = 0$$

Sum of torques at pivot point:

$$-\cos(\pi + \theta)ml\ddot{y}(t) + ml^2\ddot{\theta}(t) + \sin(\pi + \theta)mlg\theta = 0$$

Example



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Sum of torques at pivot point:

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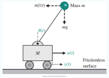
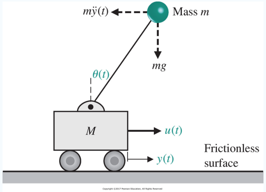
Example



See e.g. Control Tutorials for Matlab and Simulink

Example



Idea: Linearize for $\theta \approx 0$. Sum of forces in horizontal direction:

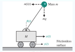
$$M\ddot{y}(t) + ml\ddot{\theta}(t) - u(t) = 0$$

Sum of torques at pivot point:

$$ml\ddot{y}(t) + ml^2\ddot{\theta}(t) - mlg\theta = 0$$

Inverted pendulum

Example



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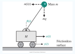
Example



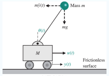
Sum of forces in horizontal direction:

$$M\ddot{y}(t) - \cos(\pi+\theta)ml\ddot{\theta}(t) - ml\dot{\theta}^2\sin(\pi+\theta) - u(t) = 0$$

Sum of torques at pivot point:

$$-\cos(\pi+\theta)ml\ddot{y}(t) + ml^2\ddot{\theta}(t) + \sin(\pi+\theta)mlg\theta = 0$$

Example



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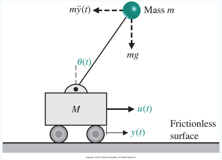


Using state variables

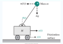
$$\boldsymbol{x}\,\begin{bmatrix} y & \dot{y} & \theta & \dot{\theta} \end{bmatrix}^T$$

one gets

$$\begin{aligned} M\dot{x}_2(t) + ml\dot{x}_4(t) - u(t) &= 0 \\ \dot{x}_2(t) + l\dot{x}_4(t) - gx_3(t) &= 0 \end{aligned}$$

Inverted pendulum

Example



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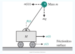
Example



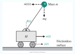
Idea: Linearize for $\theta \approx 0$. Sum of forces in horizontal direction:

$$M\ddot{y}(t) + ml\ddot{\theta}(t) - u(t) = 0$$

Sum of torques at pivot point:

$$ml\ddot{y}(t) + ml^2\ddot{\theta}(t) - mlg\theta = 0$$

Example



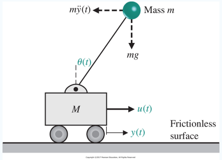
Sum of forces in horizontal direction:

$$M\ddot{y}(t) - \cos(\pi+\theta)ml\ddot{\theta}(t) - ml\dot{\theta}^2\sin(\pi+\theta) - u(t) = 0$$

Sum of torques at pivot point:

$$-\cos(\pi+\theta)ml\ddot{y}(t) + ml^2\ddot{\theta}(t) + \sin(\pi+\theta)mlg\theta = 0$$

Example

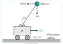


Assuming $M \gg m$ one gets the state variables and systems matrices as follows:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad \boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g/l & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ -1/(Ml) \end{bmatrix}$$

Inverted pendulum

Example



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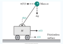
Example



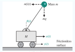
Using state variables

$$\boldsymbol{x}\,[y \ \ \dot{y} \ \ \theta \ \ \dot{\theta}]^T$$

one gets

$$\begin{aligned} M\dot{x}_2(t) + ml\dot{x}_4(t) - u(t) &= 0 \\ \dot{x}_2(t) + l\dot{x}_4(t) - gx_3(t) &= 0 \end{aligned}$$

Example



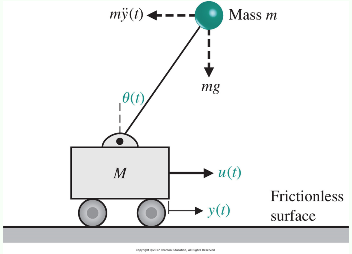
Idea: Linearize for $\theta \approx 0$. Sum of forces in horizontal direction:

$$M\ddot{y}(t) + ml\ddot{\theta}(t) - u(t) = 0$$

Sum of torques at pivot point:

$$ml\ddot{y}(t) + ml^2\ddot{\theta}(t) - mlg\theta = 0$$

Exercise (#5.12)



Given is the system shown above.

- Develop a state variable model for the system shown above and $l = 0.098\,\mathrm{m}$, $g = 9.8\,\mathrm{m\,s^{-2}}$, $m = 825\,\mathrm{g}$ and $M = 8085\,\mathrm{g}$.

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Feedback & Control

Exercise (#5.13)

Given is the system shown in figure 22 with

$$H(s) = \frac{1}{s^3 + s^2 + 10s + 2}$$

and the corresponding step response for different PID controllers shown in figure 23. The coefficients of the PID controllers are given in table 1. Fill the table.

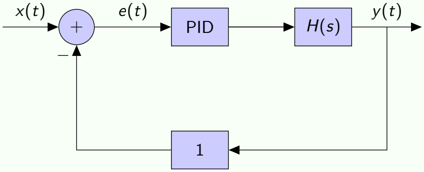


Figure 22: A system

Exercise (#5.13)

Given is the system shown in figure 22 with

$$H(s) = \frac{1}{s^3 + s^2 + 10s + 2}$$

and the corresponding step response for different PID controllers shown in figure 23. The coefficients of the PID controllers are given in table 1. Fill the table.

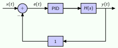


Figure 22: A system

Continued exercise (#5.13)

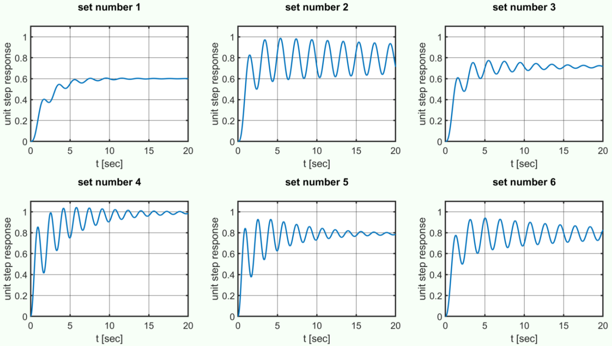
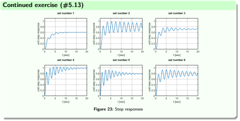
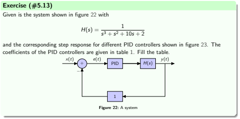


Figure 23: Step responses





Continued exercise (#5.13)

P	I	D	Set number	Justification
3	0	0		
5	0	0		
7.5	0	0		
7.5	0	1		
7.5	0	5		
7.5	1	5		

Table 1: PID coefficients

Exercise

Continued exercise (#5.13)

P	I	D	Set number	Justification
3	0	0		
5	0	0		
7.5	0	0		
7.5	0	1		
7.5	0	5		
7.5	1	5		

Table 1: PID coefficients

Continued exercise (#5.13)

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Table 1: PID coefficients

Continued exercise (#5.13)

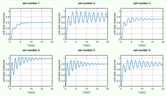

Figure 23: Step responses

Exercise (#5.13)

Given is the system shown in figure 22 with

$$H(s) = \frac{1}{s^3 + s^2 + 10s + 2}$$

and the corresponding step response for different PID controllers shown in figure 23. The coefficients of the PID controllers are given in table 1. Fill the table.

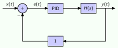

Figure 22: A system

Exercise (#5.14)

Given is a plant with the following system parameters:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 100 & 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0.1237 \\ 0 \\ -1.2621 \end{bmatrix}$$

Calculate $\boldsymbol{K}$ for poles at $s = -8 \pm 6i$ and $-0.4 \pm 0.3i$. Simulate the system for an initial state of $\boldsymbol{x} = \begin{bmatrix} 0 & 0 & 0.1 & 0 \end{bmatrix}^T$ using MATLAB and SIMULINK.
Hint: Useful functions are `acker`, `lsim`, `zp2tf` and `ss`.

Exercise

Exercise (#5.14)

Given is a plant with the following system parameters:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 100 & 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0.1237 \\ 0 \\ -1.2621 \end{bmatrix}$$

Calculate $\boldsymbol{K}$ for poles at $s = -8 \pm 6i$ and $-0.4 \pm 0.3i$. Simulate the system for an initial state of $\boldsymbol{x} = \begin{bmatrix} 0 & 0 & 0.1 & 0 \end{bmatrix}^T$ using MATLAB and SIMULINK.
Hint: Useful functions are `acker`, `lsim`, `zp2tf` and `ss`.

Table of Contents

Feedback & Control

Translations

Englisch	Deutsch
Overshoot	Überschwingweite
Settling time	Ausregelzeit
Rise time	Anstiegszeit
Peak time	$t_{max} - Zeit$
Steady state error	stationäre Regeldifferenz

Source: Europa-Lehrmittel