FIT5186 Intelligent Systems

Lecture 10 Fuzzy Logic

Fuzzy Decision Making Using Fuzzy Rules with Discrete Problem Space

- A fuzzy rule:

IF ***Price* is *Low*** AND ***Quality* is *Good***

THEN ***Buy More***

- Define:

$$\mu_{Low}(x) = \frac{1}{1} + \frac{0.4}{2} + \frac{0}{3} + \frac{0}{4} \qquad x\text{: } Price$$

$$\mu_{Good}(y) = \frac{1}{Good} + \frac{0.5}{Fair} + \frac{0}{Bad} \qquad y\text{: } Quality$$

$$\mu_{More}(z) = \frac{0}{10} + \frac{0}{20} + \frac{0.8}{30} + \frac{1}{40} \qquad z\text{: } Buy$$

- The fuzzy rule is represented by a fuzzy Relation **R**, given as:

$$\mu_R(x, y, z) =$$

	(10)	(20)	(30)	(40)
$(1, G)$	0	0	0.8	1
$(1, F)$	0	0	0.5	0.5
$(1, B)$	0	0	0	0
$(2, G)$	0	0	0.4	0.4
$(2, F)$	0	0	0.4	0.4
$(2, B)$	0	0	0	0
$(3, G)$	0	0	0	0
$(3, F)$	0	0	0	0
$(3, B)$	0	0	0	0
$(4, G)$	0	0	0	0
$(4, F)$	0	0	0	0
$(4, B)$	0	0	0	0

The fuzzy Relation **R** implied by the fuzzy rule:

$$\mu_R(x, y, z) = \begin{array}{c} \\ (1,G) \\ (1,F) \\ (1,B) \\ (2,G) \\ (2,F) \\ (2,B) \\ (3,G) \\ (3,F) \\ (3,B) \\ (4,G) \\ (4,F) \\ (4,B) \end{array} \begin{array}{c} \begin{array}{cccc} (10) & (20) & (30) & (40) \end{array} \\ \begin{bmatrix} 0 & 0 & 0.8 & 1 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.4 \\ 0 & 0 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Case 1.

Given Condition: Price is *2* and Quality is *Fair*.

$$A' = \overset{(1,G)(1,F)(1,B)(2,G)(2,F)(2,B)(3,G)(3,F)(3,B)(4,G)(4,F)(4,B)}{[0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]}$$

Action: $B' = A' \circ R = \overset{(10)\ (20)\ (30)\ (40)}{[0,\ 0, 0.4, 0.4]} \Rightarrow$ Buy 30 or 40?

Case 2.

Given Condition:

Price is *2 or a little more* and Quality is *Not Bad*.

$$A' = \overset{(1,G)\ (1,F)\ (1,B)\ (2,G)\ (2,F)\ (2,B)\ (3,G)\ (3,F)(3,B)(4,G)(4,F)(4,B)}{[1, 0.9, 0, 0.9, 0.7, 0, 0.2, 0, 0, 0, 0, 0]}$$

Action: $B' = A' \circ R = \overset{(10)\ (20)\ (30)\ (40)}{[0,\ 0, 0.8, 1]} \Rightarrow$ Buy 40.

- **Key Issues:**
  - (a) Definition of linguistic terms
  - (b) Defuzzification of the consequent fuzzy set

Fuzzy Decision Making Using Fuzzy Rules with Continuous Problem Space

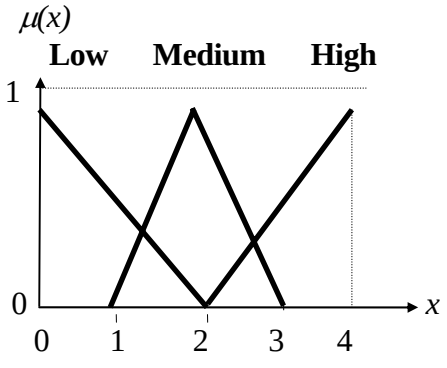
- **Three fuzzy rules:**

Rule 1:
IF ***Price*** **is** ***Low*** AND ***Quality*** **is** ***Good***
THEN ***Buy More***

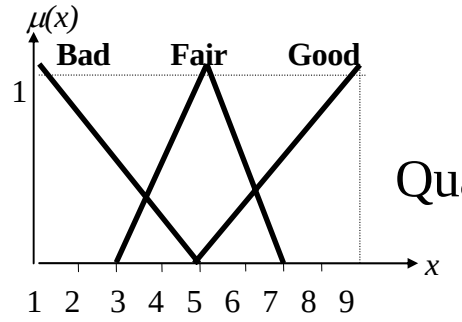
Rule 2:
IF ***Price*** **is** ***Medium*** AND ***Quality*** **is** ***Fair***
THEN ***Buy Moderate***

Rule 3:
IF ***Price*** **is** ***High*** AND ***Quality*** **is** ***Bad***
THEN ***Buy Less***

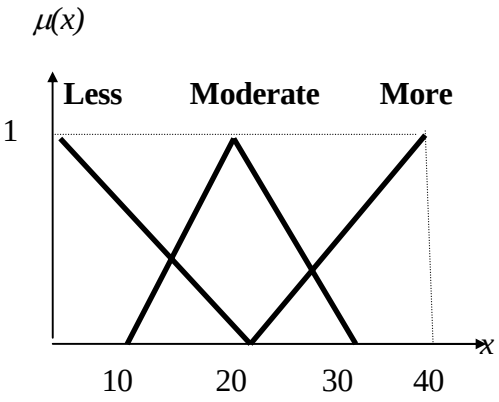
- **Term sets for linguistic variables used:**



Price



Quality

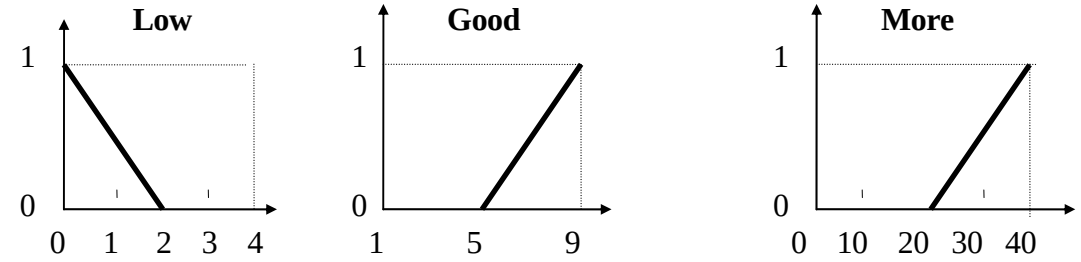


Buy

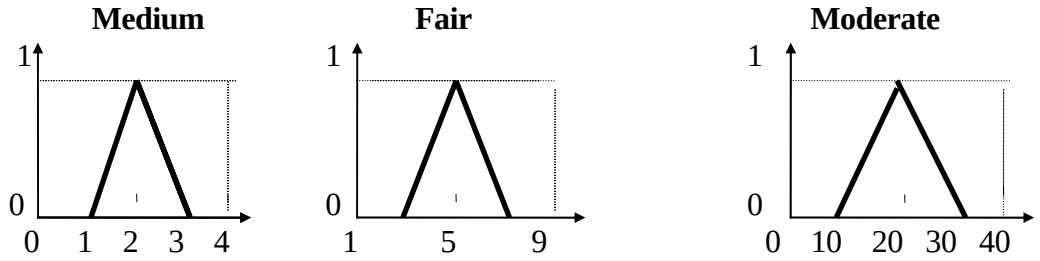
Fuzzy Interpolative Reasoning with Crisp Input

IF **Price** AND **Quality** THEN **Buy**

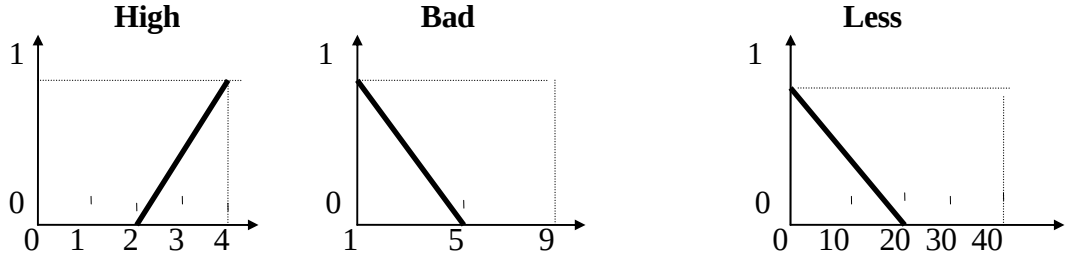
Rule 1:



Rule 2:



Rule 3:



- Given crisp input condition:

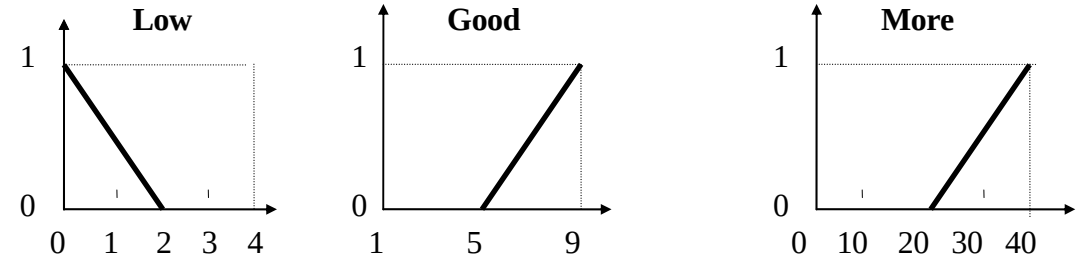
  Price is **2.8**
  Quality is **4**

- Conclusion:
  Buy ?

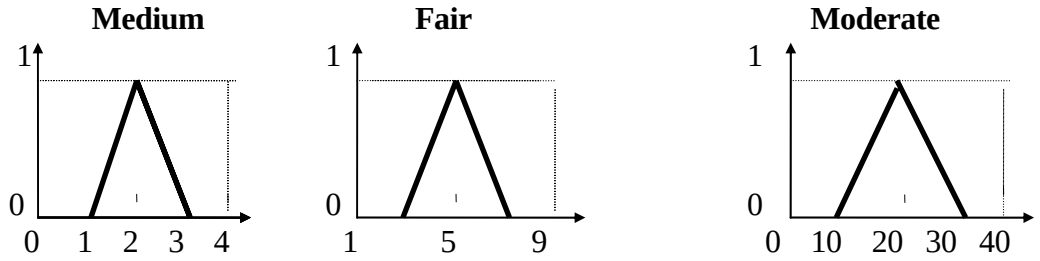
Fuzzy Interpolative Reasoning with Fuzzy Input

IF **Price** AND **Quality** THEN **Buy**

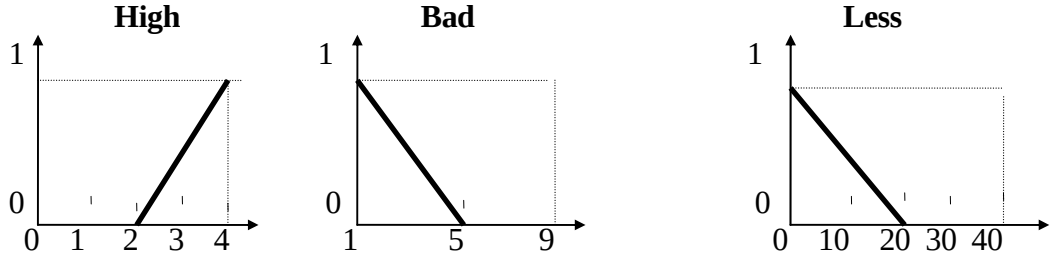
Rule 1:



Rule 2:



Rule 3:



- Given fuzzy input condition:

  Price is ***Rather Low***
  Quality is ***Not Bad***

- Conclusion:
  Buy ?

Linguistic Hedges

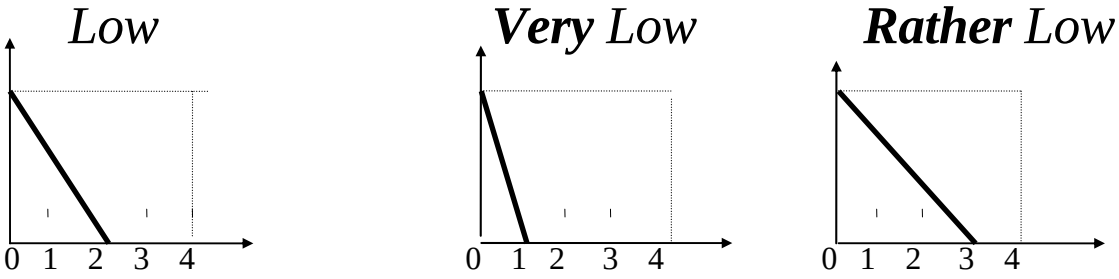
Linguistic hedges are used to modify linguistic terms, i.e. modify the membership function of a fuzzy set.
For example,

- Intensifying a linguistic term (a fuzzy set):
  very, *extremely*
- Diluting a linguistic term (a fuzzy set)
  rather, *quite*, *fairly*, *somewhat*, *more or less*

Two widely used hedges "*very*" and "*rather*" are often defined as

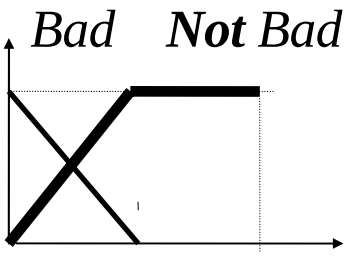
$$Very(\mu(x)) = \mu^2(x)$$
$$Rather(\mu(x)) = \mu^{1/2}(x)$$

Example:



- Negation of a linguistic term is represented by the complement of its corresponding fuzzy set, given as

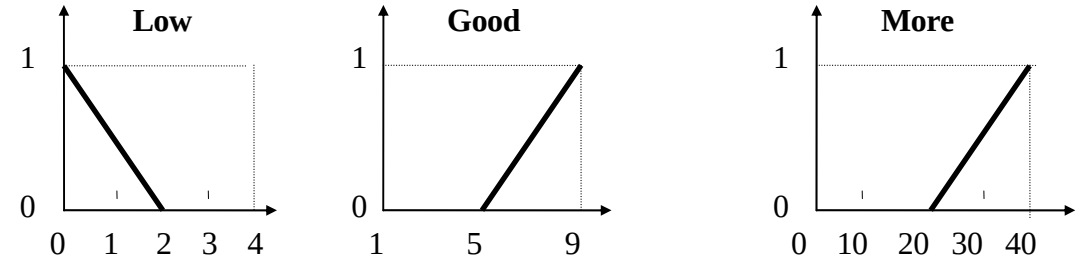
$$\textbf{Not}\ (\mu(x)) = 1 - \mu(x)$$



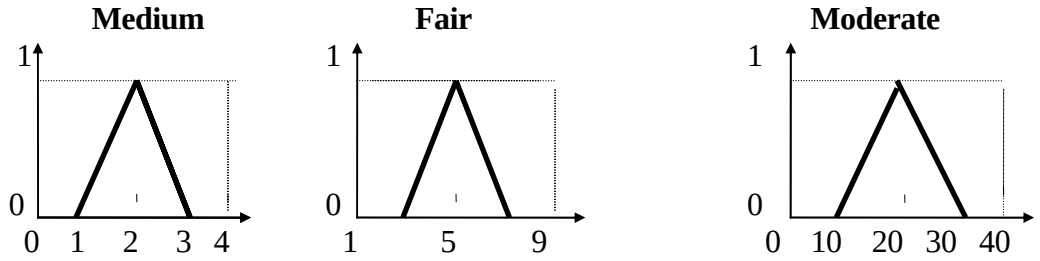
Fuzzy Interpolative Reasoning with Crisp Input and Fuzzy Input

IF **Price** AND **Quality** THEN **Buy**

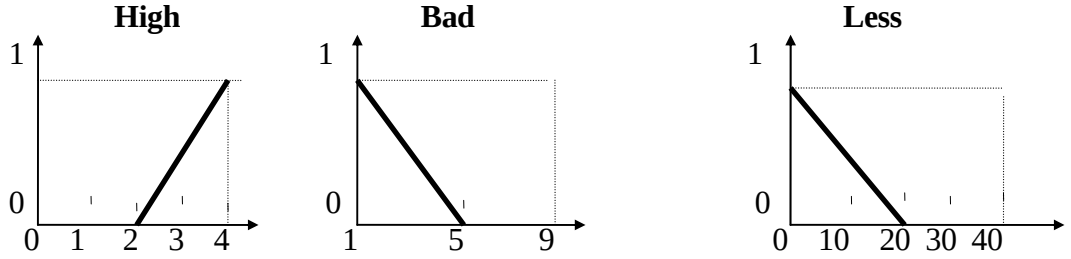
Rule 1:



Rule 2:



Rule 3:



- Given hybrid input condition:

  Price is ***1.5***
  Quality is ***a fuzzy number (2, 4, 6)***

- Conclusion:
  Buy ?

Fuzzy Decision Making Using Fuzzy Rules with Continuous Problem Space

- **A complete set of fuzzy rules:**

Rule 1:
IF ***Price*** **is** ***Low*** AND ***Quality*** **is** ***Good*** THEN ***Buy More***

Rule 2:
IF ***Price*** **is** ***Medium*** AND ***Quality*** **is** ***Fair*** THEN ***Buy Moderate***

Rule 3:
IF ***Price*** **is** ***High*** AND ***Quality*** **is** ***Bad*** THEN ***Buy Less***

Rule 4:
IF ***Price*** **is** ***Low*** AND ***Quality*** **is** ***Bad*** THEN ***Buy Moderate***

Rule 5:
IF ***Price*** **is** ***High*** AND ***Quality*** **is** ***Good*** THEN ***Buy Moderate***

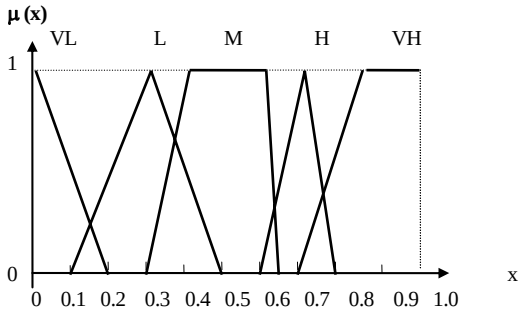
Application to Bus Operations under Uncertainty

Fuzzy Rules in the Knowledge Base

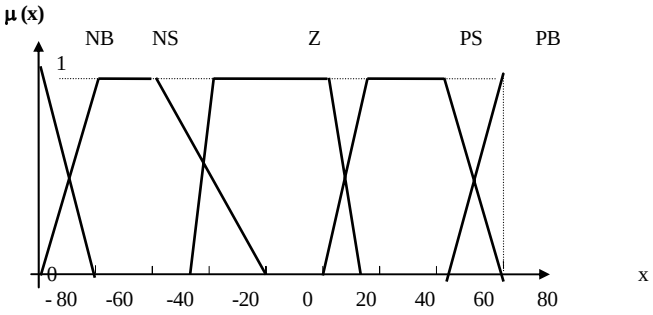
Rule	IF (Antecedent)			THEN (Consequent)
	Load Factor	Load Factor variation	Operating Speed	Extra Trip Requirement
1		NB		VL
2		NS		VL
3			H	VL
4			VH	VL
5			M	VL
6	M	PS		L
7	M	PB		L
8	M		L	L
9	M		VL	L
10	M	Z	L	VL
11	M	PB	VL	M
12	H		L	M
13	H		VL	M
14	H	Z		L
15	H	PS		M
16	H	PB		M
17	H	Z	L	L
18	H	PB	VL	H
19	VH	Z		M
20	VH	PS		H
21	VH	PB		H
22	VH		L	H
23	VH		VL	H
24	VH	Z	L	M
25	VH	PB	VL	VH

Application to Bus Operations under Uncertainty

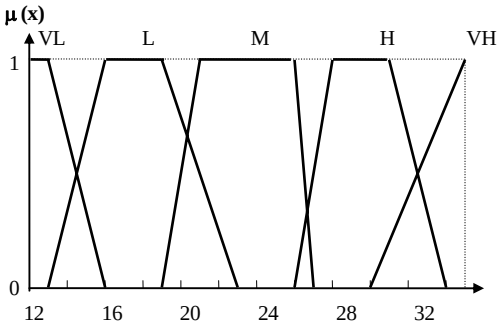
Membership functions used in the fuzzy knowledge base



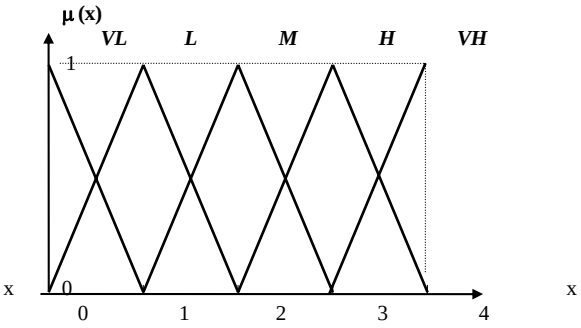
Load Factor



Load Factor Variation



Average Operating Speed



Extra Trip Requirement

- Extra trip requirements for simulated cases.