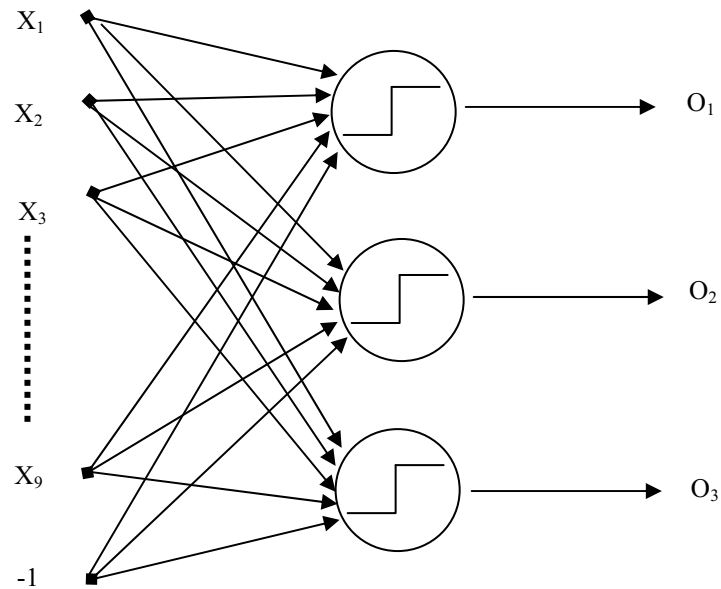


# FIT5186 Intelligent Systems

## Tutorial 2 Solution

### Design Classifier



### Train Classifier

Inputs ( $x_1, x_2, \dots, x_9$ )	1	1	0
	1	0	1
	1	0	0
	1	1	0
	0	0	1
	0	0	0
	1	1	0
	1	1	1
	1	1	0
	-1	-1	-1
Outputs (desired)	1	0	0
	0	1	0
	0	0	1

Initialise weights to zero,  $w^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Present input 1

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \quad \text{with } \mathbf{d} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Calculate output vector

$$\mathbf{o} = f \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Adapt weights since  $\mathbf{d} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  and  $\mathbf{o} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$  (errors in neurons 2 and 3)

$$\mathbf{w}_2^1 = \mathbf{w}_2^0 + c(d_2 - o_2)\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (1)(0-1) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\mathbf{w}_3^1 = \mathbf{w}_3^0 + c(d_3 - o_3)\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (1)(0-1) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\mathbf{w}_1^1 = \mathbf{w}_1^0 \text{ (unchanged)}$$

So after adaptation:  $w^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \end{pmatrix}$

Present input 2  $x = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$  with  $d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Calculate output vector  $o = f \begin{pmatrix} 0 \\ -6 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Adapt weights since  $d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$  and  $o = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  (errors in neurons 1 and 2)

$w_1^2 = w_1^1 + c(d_1 - o_1)x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (1)(0-1) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$

$$w_2^2 = w_2^1 + c(d_2 - o_2)x = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + (1)(1-0) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$w_3^2 = w_3^1 \text{ (unchanged)}$$

$$\text{So after adaptation: } w^2 = \begin{pmatrix} -1 & 0 & 0 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \end{pmatrix}$$

$$\text{Present input 3} \quad x = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \quad \text{with } d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{Calculate output vector} \quad o = f \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Adapt weights since } d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ and } o = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ (error in neuron 3)}$$

$$w_3^3 = w_3^2 + c(d_3 - o_3)x = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + (1)(1-0) \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -1 \\ -1 \\ 1 \\ 0 \\ -1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$w_1^3 = w_1^2 \text{ and } w_2^3 = w_2^2 \text{ (unchanged)}$$

$$\text{So after adaptation: } w^3 = \begin{pmatrix} -1 & 0 & 0 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}$$

END OF 1<sup>ST</sup> EPOCH

$$\text{Present input 1} \quad x = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \text{ with } d = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Calculate output vector} \quad o = f \begin{pmatrix} -6 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Adapt weights since } d = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } o = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ (error in neuron 1)}$$

$$w_1^4 = w_1^3 + c(d_1 - o_1)x = \begin{pmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + (1)(1-0) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$w_2^4 = w_2^3 \text{ and } w_3^4 = w_3^3 \text{ (unchanged)}$$

$$\text{So after adaptation: } w^4 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}$$

$$\text{Present input 2} \quad x = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \text{ with } d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\text{Calculate output vector} \quad o = f \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\text{Adapt weights since } d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } o = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \text{ (error in neuron 1)}$$

$$w_1^5 = w_1^4 + c(d_1 - o_1)x = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (1)(0-1) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$w_2^5 = w_2^4 \text{ and } w_3^5 = w_3^4 \text{ (unchanged)}$$

$$\text{So after adaptation: } w^5 = \begin{pmatrix} -1 & 1 & 1 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}$$

$$\text{Present input 3} \quad x = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \text{ with } d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{Calculate output vector} \quad o = f \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{No need to adapt weights since } d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ and } o = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ (no error)}$$

$$w_1^6 = w_1^5, w_2^6 = w_2^5 \text{ and } w_3^6 = w_3^5 \text{ (unchanged)}$$

$$\text{So after adaptation: } w^6 = \begin{pmatrix} -1 & 1 & 1 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}$$

END OF 2<sup>ND</sup> EPOCH

Present input 1

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \text{ with } \mathbf{d} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Calculate output vector

$$\mathbf{o} = f \begin{pmatrix} -4 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Adapt weights since  $\mathbf{d} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  and  $\mathbf{o} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  (error in neuron 1)

$$\mathbf{w}_1^7 = \mathbf{w}_1^6 + c(d_1 - o_1)\mathbf{x} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + (1)(1-0) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\mathbf{w}_2^7 = \mathbf{w}_2^6$  and  $\mathbf{w}_3^7 = \mathbf{w}_3^6$  (unchanged)

So after adaptation:  $\mathbf{w}^7 = \begin{pmatrix} 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}$



Present input 2

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \quad \text{with } \mathbf{d} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Calculate output vector

$$\mathbf{o} = f \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Adapt weights since  $\mathbf{d} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$  and  $\mathbf{o} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$  (error in neuron 1)

$$\mathbf{w}_1^8 = \mathbf{w}_1^7 + c(d_1 - o_1)\mathbf{x} = \begin{pmatrix} 0 \\ 2 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (1)(0-1) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 2 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$\mathbf{w}_2^8 = \mathbf{w}_2^7$  and  $\mathbf{w}_3^8 = \mathbf{w}_3^7$  (unchanged)

So after adaptation:  $\mathbf{w}^8 = \begin{pmatrix} -1 & 2 & 2 & -1 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}$

Present input 3

$$\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \quad \text{with } \mathbf{d} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Calculate output vector

$$\mathbf{o} = f \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Adapt weights since  $\mathbf{d} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$  and  $\mathbf{o} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$  (error in neuron 1)

$$\mathbf{w}_1^9 = \mathbf{w}_1^8 + c(d_1 - o_1)\mathbf{x} = \begin{pmatrix} -1 \\ 2 \\ 2 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + (1)(0-1) \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \\ -1 \\ -1 \\ 0 \\ -1 \\ -2 \\ -1 \\ 2 \end{pmatrix}$$

$\mathbf{w}_1^2 = \mathbf{w}_1^1$ ,  $\mathbf{w}_2^2 = \mathbf{w}_2^1$  and  $\mathbf{w}_3^2 = \mathbf{w}_3^1$  (unchanged)

So after adaptation:  $\mathbf{w}^9 = \begin{pmatrix} -1 & 1 & 2 & -1 & -1 & 0 & -1 & -2 & -1 & 2 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}$

END OF 3<sup>RD</sup> EPOCH

# Classify Corrupted Characters

$$f\left(\begin{pmatrix} 0 & 2 & 3 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}\right) = f\left(\begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow C$$

$$f\left(\begin{pmatrix} 0 & 2 & 3 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}\right) = f\left(\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow C$$

$$f\left(\begin{pmatrix} 0 & 2 & 3 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}\right) = f\left(\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \text{Not classified}$$