Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 194 (2009) 464–473
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Decision Support

Modeling subjective evaluation for fuzzy group multicriteria decision making

Chung-Hsing Yeh [a,b,*], Yu-Hern Chang [b]

[a] *Clayton School of Information Technology, Faculty of Information Technology, Monash University, Clayton, Victoria 3800, Australia*
[b] *Department of Transportation and Communications Management, National Cheng Kung University, Tainan 70101, Taiwan*

Received 5 April 2007; accepted 27 December 2007
Available online 9 January 2008

Abstract

This paper presents a new fuzzy multicriteria decision making (MCDM) approach for evaluating decision alternatives involving subjective judgements made by a group of decision makers. A pairwise comparison process is used to help individual decision makers make comparative judgements, and a linguistic rating method is used for making absolute judgements. A hierarchical weighting method is developed to assess the weights of a large number of evaluation criteria by pairwise comparisons. To reflect the inherent imprecision of subjective judgements, individual assessments are aggregated as a group assessment using triangular fuzzy numbers. To obtain a cardinal preference value for each decision alternative, a new fuzzy MCDM algorithm is developed by extending the concept of the degree of optimality to incorporate criteria weights in the distance measurement. An empirical study of aircraft selection is presented to illustrate the effectiveness of the approach.

Keywords: Fuzzy multicriteria decision making; Group decisions; Comparative judgement; Absolute judgement; TOPSIS

1. Introduction

Decision making in the public and private sectors often involves the evaluation and ranking of available courses of action or decision alternatives based on multiple criteria. Multicriteria decision making (MCDM) has proven to be an effective methodology for solving a large variety of multicriteria evaluation and ranking problems (Dyer et al., 1992; Hwang and Yoon, 1981). The most widely used theory in solving MCDM problems probably is the multiattribute utility theory or multiattribute value theory (MAVT) (Dyer and Sarin, 1979; Keeney and Raiffa, 1993). With simplicity in both concept and computation, MAVT-based MCDM methods are intuitively appealing to the decision makers in practical applications. These methods are particularly suited to decision problems where a cardinal preference or ranking of the decision alternatives is required. In addition, these methods are the most appropriate quantitative tools for group decision support systems (Bose et al., 1997; Matsatsinis and Samaras, 2001).

Despite their diversity, MAVT-based MCDM problems often share the following common characteristics: (a) a finite number of comparable alternatives, (b) multiple criteria for evaluating the alternatives, (c) non-commensurable units for measuring the performance rating of the alternatives on each criterion, and (d) criteria weights for representing the relative importance of each criterion. The performance ratings of the alternatives on all evaluation criteria are to be aggregated with the criteria weights using an MCDM method in order to obtain an overall preference value for each alternative. The resultant overall preference values provide a cardinal ranking of the alternatives.

* Corresponding author. Address: Clayton School of Information Technology, Faculty of Information Technology, Monash University, Clayton, Victoria 3800, Australia. Tel.: +61 3 99055808; fax: +61 3 99055159.
E-mail address: ChungHsing.Yeh@infotech.monash.edu.au (C.-H. Yeh).

0377-2217/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2007.12.029

In evaluating decision alternatives in new problem settings, the assessment data for the criteria weights and for the performance ratings of the alternatives on qualitative criteria are often not available and have to be assessed subjectively by the decision makers, the stakeholders or the experts. This subjective assessment process may involve two types of judgement: comparative judgement and absolute judgement (Blumenthal, 1977; Saaty, 2006). Making comparative judgements involves the identification of some relation between two stimuli both present to the decision makers. In MCDM, comparative judgements may be used to compare the relative importance of evaluation criteria and to compare the performance of each alternative with that of other alternatives on each evaluation criterion with relative measurement. In making absolute judgements, the decision makers rate a single stimulus by assessing the relation between it and some information about former comparison stimuli or about previously experienced measurement scale. In MCDM, absolute judgements are commonly used to rate the alternatives individually with respect to each evaluation criterion with an absolute measurement scale.

The subjective evaluation process is intrinsically imprecise, due to the characteristics of new problem settings, particularly in relation to newly generated alternatives or vaguely defined qualitative evaluation criteria. To reflect the inherent subjectiveness and imprecision involved in the evaluation process, the concept of fuzzy sets is often used (Zadeh, 1965). Modelling using fuzzy numbers has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise (Chang and Yeh, 2004; Zimmermann, 1996).

In this paper, we present a novel approach for modeling subjective assessments made by a group of decision makers as fuzzy numbers. To reduce the cognitive demand from the decision makers, individual judgements are aggregated as a group judgement using a triangular fuzzy number instead of using fuzzy numbers for individual judgements as commonly used in traditional approaches. To effectively solve the resultant fuzzy MAVT-based MCDM problem, we develop a new algorithm by extending the concept of the degree of optimality (Hwang and Yoon, 1981; Zeleny, 1982) to incorporate criteria weights in the distance measurement. To deal with a large number of lower-level criteria for the decision problem, we develop a new hierarchical weighting method to obtain the relative weights for all lower-level criteria. The method effectively reduces the total number of paired comparisons required, thus facilitating the application of the new fuzzy MCDM algorithm to large-sized problems.

In subsequent sections, we first present the fuzzy modeling processes for aggregating decision makers' subjective assessments using comparative and absolute judgements, respectively. We then discuss the general MAVT-based fuzzy group MCDM problem and develop an effective algorithm to solve it. Finally, we present an empirical study of aircraft selection to illustrate the new fuzzy group MCDM approach.

2. Modeling group subjective evaluation with comparative judgements

In group MCDM, decision makers often make comparative judgements about the criteria weights for representing the relative importance of each criterion (Saaty, 1980). Each criterion needs to be compared with other criteria in terms of their relative importance for achieving the overall objective of the MCDM problem. To facilitate comparative judgements, a pairwise comparison process is commonly used. The concept of pairwise comparisons has been known since the work of Thurstone (1927) and has been popularly implemented in the analytic hierarchy process (AHP) of Saaty (1980). AHP has been applied to a wide variety of practical decision problems (Vaidya and Kumar, 2006).

As suggested by AHP, a 1–9 ratio scale is used to compare two criteria for indicating the strength of their relative importance; e.g. 1 = equally important, 3 = moderately more important, 5 = strongly more important, 7 = very strongly more important, 9 = extremely more important. The ratio scale associates qualitative judgements with absolute numbers, thus enabling different criteria to be weighted with a homogeneous measurement scale (Vreeker et al., 2002). The 1–9 ratio scale has proven to an effective measurement scale for reflecting the qualitative information of a decision problem and for enabling the unknown weights to be approximated. Applying this paired comparison judgement to all n criteria will result in a positive $n \times n$ reciprocal matrix with all its elements $x_{ij} = 1/x_{ji}$ ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, n$). The relative importance of all the criteria can be obtained by applying the eigenvector method of AHP (Saaty, 1980). The eigenvector method possesses a number of desirable merits and can be easily implemented (Harker and Vargas, 1987). However, in order for the eigenvector method to generate correct solutions, the requirement of consistent judgement matrices has to be met (Barzilai et al., 1987). Inconsistencies in pairwise comparisons can be handled practically or methodologically (Gonzalez-Pachon and Romero, 2004).

With an effective ratio scale and the eigenvector method, the primary drawback of the pairwise comparison process is the time required to make the paired comparisons (Dyer and Forman, 1992). To compare n criteria, $n(n-1)/2$ judgements are required. It would be tedious and impractical if a large number of criteria are to be considered. For example, in the empirical study to be presented in Section 5, each decision maker needs to make 55 (=11(11 − 1)/2) comparisons to assess the relative weight of the 11 evaluation criteria if the pairwise comparison process is directly applied. In addition to the problem with the time required, there is an upper limit on the amount of information that humans can effectively handle (Miller, 1956). From the perspective of judgement consistency, the number of elements in a group for pairwise comparisons should be not more than seven (Saaty and Ozdemir, 2003).

We develop a hierarchical weighting method to facilitate the pairwise comparison process for assessing the relative weighs of many (e.g. more than seven) criteria, on which the performance ratings of the decision alternatives are to be measured or assessed. As part of the problem formulation, these criteria are grouped into super-criteria (criteria categories) that meet the overall objective of the decision problem. The relationship between super-criteria and criteria forms a two-level hierarchical structure, similar to that for criteria and sub-criteria in AHP. As required of AHP, each super-criterion will have not more than seven criteria. In line with the overall objective of the decision problem, the concept for grouping criteria into super-criteria on a two-level hierarchical structure is in essence the same as that for grouping sub-criteria into criteria (or dividing criteria into sub-criteria) in AHP. However, the idea and the method used in this paper for obtaining and using the weights of criteria and super-criteria to solve the fuzzy group MCDM problem is different from that of AHP and of existing MCDM methods. This is due to the fact that the new approach developed in this paper extends the concept of the degree of optimality by incorporating criteria weights in the distance measurement.

In existing MCDM methods including AHP, the criteria weights mainly serve as a channel through which different performance ratings of the alternatives on the criteria can be aggregated (Chen and Hwang, 1992; Deng et al., 2000). With a two-level criteria hierarchy, the weights of the lower-level criteria are used only to aggregate the performance ratings of an alternative as the weighted performance ratings for the alternative with respect to the corresponding upper-level super-criteria (Saaty, 1980; Yeh et al., 2000). As such, the weights of the lower-level measurable criteria under different super-criteria are not related. This is because these weights are assessed within their corresponding super-criterion only, not with respect to the overall objective of the decision problem.

Based on the concept of the degree of optimality, the overall preference value of an alternative is determined by its distance to the positive ideal solution and to the negative ideal solution. This distance is thus interrelated with the weights of the criteria (or the lower-level measurable criteria on a two-level hierarchy) (Zeleny, 1982) and should be incorporated in the distance measurement (Shipley et al., 1991). This is because all alternatives are compared with the positive and negative ideal solutions rather than directly among themselves. As such, in the algorithm developed in this paper, we use the relative weights of the lower-level measurable criteria across all super-criteria to weight the distance between the alternatives and the positive (or negative) ideal solution. When the number of the lower-level measurable criteria is more than seven, the hierarchical weighting method presented below can be applied to obtain the relative weights for the lower-level measurable criteria with respect to the overall objective of the decision problem.

With the two-level hierarchy of criteria and super-criteria, the pairwise comparison process is conducted (a) between super-criteria to obtain the relative weights of all super-criteria and (b) between criteria within each super-criterion to obtain the relative weight of the criteria with respect to their corresponding super-criterion. As such, the number of comparisons required is greatly reduced. For example, the 11 criteria used in the empirical study are grouped into three super-criteria, which consist of 4, 2, and 5 criteria, respectively. When this hierarchical weighting method is applied to the empirical study, only 20 ($=3+6+1+10$) paired comparisons are required.

With the hierarchical weighting method, a criterion is associated with a local weight and a global weight. The local weight of a criterion is referred to the weight relative to other criteria under the same super-criterion, which is to be assessed by the decision makers using the pairwise comparison process for the corresponding super-criterion. The global weight of a criterion is referred to the weight relative to all other criteria across all super-criteria for the overall objective of the decision problem. The global weight of a criterion is obtained by converting from its local weight, together with the local weights of other criteria under the same super-criterion. For the criteria under a super-criterion, their local weights are converted to global weights by making the weight of their corresponding super-criterion (relative to other super-criteria) be the geometric mean of these global weights. In other words, the weight of a super-criterion (relative to other super-criteria) can be derived by synthesizing the global weights of their associated criteria (relative to all other criteria across all super-criteria) using the geometric mean method.

The use of the geometric mean instead of the arithmetic mean for synthesizing ratio judgements has been well justified (Aczel and Saaty, 1983; Barzilai et al., 1987; Gass and Rapcsak, 1998). The geometric mean is best suited to synthesize individual judgements obtained from pairwise comparisons as it preserves the reciprocal property of the judgement matrix. Without violating the Pareto principle in aggregating individual judgements, the geometric mean is more consistent with the meaning of judgements in the pairwise comparison process (Forman and Peniwati, 1998). For a sufficiently large group size, the geometric mean guarantees the consistency of the aggregate judgement matrix, regardless of the consistency measures of the individual judgement matrices (Aull-Hyde et al., 2006). For pairwise comparisons using a ratio scale with geometric progression as in the multiplicative AHP, the geometric mean is the only suitable method for aggregating individual judgements (Lootsma, 1999). For the problem of eliciting weights from inconsistent judgement matrices, the geometric mean is the only solution satisfying consistency axioms (Barzilai et al., 1987).

If the weight of a super-criterion S_g $(g = 1, 2, \ldots, h)$, relative to other $(h-1)$ super-criteria, is w_{s_g} and the local weights of its q_g associated criteria (within the super-criteria S_g) are $w^g_{S_{gl}}$ $(l = 1, 2, \ldots, q_g)$, then the global weights of these q_g criteria among all the criteria for the decision problem are calculated as

$$w_{S_{gl}} = \frac{w_{S_g} \times w_{S_{gl}}^g}{\left(\prod_{l=1}^{q_g} w_{S_{gl}}^g\right)^{1/q_g}}, \quad l = 1, 2, \ldots, q_g. \tag{1}$$

Eq. (1) can be applied to obtain the global weights of the lower-level criteria which are more than seven and are to be measured quantitatively or assessed qualitatively. In the empirical study, each decision maker assesses the global weights of the 11 criteria (grouped into three super-criteria) by making 20 paired comparisons and then applying Eq. (1). Although the case of a two-level hierarchy is exemplified in this paper, the hierarchical weighting method can be easily applied to decision problems involving multi-level criteria.

In modeling the inherent subjectiveness and imprecision involved in the pairwise comparison process, existing studies use fuzzy numbers (Buckley, 1985) or intervals (Salo and Hamalainen, 1992, 1995) to represent the ratio value. These studies have also developed efficient algorithms to solve the resultant fuzzy or interval judgement matrix. In the case of fuzzy judgements, no universally accepted method is available to solve the fuzzy positive reciprocal matrix (Buckley, 1985; Mikhailov, 2003).

In addition to the issue of selecting a universally accepted method, decision makers may not have enough knowledge of using fuzzy ratios directly in their pairwise comparison judgements. To model decision makers' subjective evaluation without getting them involved directly in using fuzzy numbers, we apply the concept of fuzzy numbers only to aggregate their individual judgements. As such, decision makers use a conventional (crisp) pairwise comparison process to assess the criteria weights individually, which can be solved easily using a commonly accepted method. The criteria weights obtained from an individual decision maker are normalized to sum to 1, so that the weights given by different decision makers for a criterion can be aggregated on a common scale into a group weight for the criterion, relative to other criteria. To reflect the subjectiveness and imprecision of the evaluation process, the group weight for a criterion is represented by a triangular fuzzy number.

A triangular fuzzy number is a convex fuzzy set (Zadeh, 1965), often expressed as a triple (a_1, a_2, a_3) with its membership function defined as

$$\mu_A(x) = \begin{cases} (x - a_1)/(a_2 - a_1), & a_1 \leqslant x \leqslant a_2, \\ (a_3 - x)/(a_3 - a_2), & a_2 \leqslant x \leqslant a_3, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where a_2 is the most possible (modal) value of the fuzzy number A, and a_1 and a_3 are the lower and upper bounds, respectively used to reflect the fuzziness of the subjective evaluation.

The triangular fuzzy number (a_{1j}, a_{2j}, a_{3j}) for representing the group weight of a criterion C_j $(j = 1, 2, \ldots, n)$ assessed by all s decision makers DM_d $(d = 1, 2, \ldots, s)$ is given as

$$\begin{aligned} &a_{1j} = \min\{w_{1j}, w_{2j}, \ldots, w_{dj}\}, \quad d = 1, 2, \ldots, s, \\ &a_{2j} = \left(\prod_{d=1}^{s} w_{dj}\right)^{1/s}, \\ &a_{3j} = \max\{w_{1j}, w_{2j}, \ldots, w_{dj}\}, \quad d = 1, 2, \ldots, s, \\ &\sum_{j=1}^{n} w_{dj} = 1, \quad d = 1, 2, \ldots, s, \end{aligned} \tag{3}$$

where $w_{1j}, w_{2j}, \ldots, w_{dj}$ are the normalized global weights of the criterion C_j $(j = 1, 2, \ldots, n)$ given by the decision makers DM_d $(d = 1, 2, \ldots, s)$, respectively. The most possible value, the lower bound, and the upper bound of the fuzzy group weight of the criterion are given by the geometric mean, the smallest value, and the largest value of the individual weights, respectively. As a measure of central tendency, the geometric mean is well suited to represent the most possible value of a triangular fuzzy number. In addition to its merits for synthesizing ratio judgements as used in Eq. (1), the geometric mean is a meaningful way of dealing with situations where a consensus cannot be obtained and the group is not willing to compromise on a judgement (Dyer and Forman, 1992).

In practical applications, the triangular form of the membership function is used most often for representing fuzzy numbers that characterize linguistic information (Klir and Yuan, 1995; Yeh and Deng, 2004). The popular use of triangular fuzzy numbers is mainly attributed to their simplicity in both concept and computation. Theoretically, the merits of using triangular fuzzy numbers in fuzzy modeling have been well justified (Pedrycz, 1994). With the simplest form of the membership function, triangular fuzzy numbers constitute an immediate solution to the optimization problems in fuzzy modeling.

3. Modeling group subjective evaluation with absolute judgements

In group MCDM, decision makers frequently express their absolute judgement about the performance of the alternatives by choosing a value between a predetermined lower limit for the worst alternative and a predetermined upper limit for the best alternative (Lootsma, 1999). This absolute judgement procedure is known as direct rating, which is often expressed in grades on a numerical scale. With absolute judgement, decision makers can rate the performance of an alternative, independent of other alternatives, with respect to a qualitative criterion. The performance rating reflects the degree to which the alternative satisfies the criterion. In this paper, we use a set of linguistic terms such as {very low, low, medium, high, very high} to express decision makers' absolute judgement about the performance rating of the alternatives with respect to a qualitative criterion. When associated with a numerical scale, these linguistic terms have a strong qualitative connotation and can be used in MCDM (Lootsma, 1999).

Linguistic terms have been found intuitively easy to use in expressing the subjectiveness and imprecision of the decision maker's evaluation (Zimmermann, 1996; Yeh et al., 1999). In existing studies and applications using fuzzy set theory, each linguistic term is conventionally characterized by a fuzzy number for representing its approximate value range (Chen and Hwang, 1992; Olcer and Odabasi, 2005; Yeh et al., 2000). However, decision makers may not have sufficient knowledge of using fuzzy numbers for representing their absolute judgement. Often they are more familiar with a point (crisp) estimate measurement system such as a Likert type scale. As such, instead of being characterized by fuzzy numbers, the linguistic terms are associated with a corresponding set of absolute numbers such as $\{1,2,3,4,5\}$, as commonly used in Likert type surveys. Decision makers rate each alternative with respect to a qualitative criterion by using one of the linguistic terms.

Instead of using a fuzzy number for representing a linguistic term in individual decision makers' evaluation, the subjectiveness and imprecision of the direct rating process is reflected by aggregating individual judgements as a group judgement using a triangular fuzzy number. As such, decision makers do not deal with fuzzy numbers directly, as fuzzy numbers are not used in their subjective judgements for assessing the performance rating of an alternative. Fuzzy numbers are generated only for aggregating individual performance ratings of an alternative with respect to a qualitative criterion into a group performance rating for the alternative.

The triangular fuzzy number (a_{1i}, a_{2i}, a_{3i}) for representing the group performance rating of an alternative A_i $(i = 1, 2, \ldots, m)$ on a qualitative criterion assessed by all s decision makers is given as

$$\begin{aligned} a_{1i} &= \min\{x_{1i}, x_{2i}, \ldots, x_{di}\}, \quad d = 1, 2, \ldots, s, \\ a_{2i} &= \left(\prod_{d=1}^{s} x_{di}\right)^{1/s}, \\ a_{3i} &= \max\{x_{1i}, x_{2i}, \ldots, x_{di}\}, \quad d = 1, 2, \ldots, s, \end{aligned} \tag{4}$$

where $x_{1i}, x_{2i}, \ldots, x_{di}$ are the performance ratings of the alternative A_i $(i = 1, 2, \ldots, m)$ given by the decision makers DM_d $(d = 1, 2, \ldots, s)$, respectively. The most possible value, the lower bound, and the upper bound of the fuzzy group performance rating of the alternative on the criterion are given by the geometric mean, the smallest value, and the largest value of the individual performance ratings, respectively. The rationale for using the triangular fuzzy number and the geometric mean is in line with that for obtaining the group criteria weights as given in Eq. (3).

4. Fuzzy group multicriteria decision making

With the methods for modeling and aggregating subjective evaluation of multiple decision makers as fuzzy numbers, we formulate the evaluation problem with a two-level hierarchy of both quantitative and qualitative criteria as an MAVT-based fuzzy group MCDM problem. To solve the problem effectively, we develop a new]algorithm based on the concept of the degree of optimality.

4.1. The fuzzy group MCDM problem

The problem involves a finite set of m decision alternatives A_i $(i = 1, 2, \ldots, m)$, which are to be evaluated by a group of s decision makers DM_d $(d = 1, 2, \ldots, s)$ with respect to a set of n criteria C_j $(j = 1, 2, \ldots, n)$. These evaluation criteria are measurable quantitatively or assessable qualitatively, and are independent of each other. These criteria are grouped into h super-criteria S_g $(g = 1, 2, \ldots, h)$, each of which has q_g criteria C_{gl} $(l = 1, 2, \ldots, q_g)$, where $C_{gl} \in \{C_1, C_2, \ldots, C_n\}$. Assessments are to be made to determine (a) the weight vector $W_S = (w_{S1}, w_{S2}, \ldots, w_{Sg}, \ldots, w_{Sh})$, (b) the weight vectors $W_{S_g} = (w^g_{S_{g1}}, w^g_{S_{g2}}, \ldots, w^g_{S_{gl}}, \ldots, w^g_{S_{gq_g}})$ $(g = 1, 2, \ldots, h;\ l = 1, 2, \ldots, q_g)$, and (c) the decision matrix $X = \{x_{ij},\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n\}$.

The weight vector W_S represents the weights (relative importance) of h super-criteria S_g $(g = 1, 2, \ldots, h)$ with respect to the overall objective of the decision problem. The weight vectors W_{S_g} $(g = 1, 2, \ldots, h)$ represent the local weights of q_g criteria C_{gl} $(l = 1, 2, \ldots, q_g)$ with respect to their corresponding super-criteria S_g $(g = 1, 2, \ldots, h)$. These weight vectors are subjectively assessed by s decision makers DM_d $(d = 1, 2, \ldots, s)$ using pairwise comparisons. For each decision maker DM_d $(d = 1, 2, \ldots, s)$ involved, a weight vector $W_d = (w_{d1}, w_{d2}, \ldots, w_{dj}, \ldots, w_{dn})$ for the global weights of all n criteria C_j $(j = 1, 2, \ldots, n)$ can be generated by applying Eq. (1). A group weight vector $W = (w_1, w_2, \ldots, w_j, \ldots, w_n)$ can thus be obtained by applying Eq. (3) to aggregate all weight vectors W_d $(d = 1, 2, \ldots, s)$ assessed by all decision makers. As such, the group weight vector W represents the fuzzy group weights of all n criteria C_j $(j = 1, 2, \ldots, n)$ for the decision problem, characterized by triangular fuzzy number (a_{1j}, a_{2j}, a_{3j}) $(j = 1, 2, \ldots, n)$.

The decision matrix X represents the performance ratings (x_{ij}) of alternative A_i $(i = 1, 2, \ldots, m)$ with respect to criteria C_j $(j = 1, 2, \ldots, n)$. These performance ratings are given based on either quantitative or qualitative assessments. While the quantitative assessments are straightforward based on the data available, the qualitative assessments require decision makers' absolute judgements to rate the performance of each alternative with respect to each qualitative criterion. To reflect the subjectiveness and imprecision involved in the absolute judgements, the performance ratings given by all decision makers for a qualitative criterion are aggregated and represented by a triangular fuzzy number using Eq. (4).

Given the fuzzy weight vector W and the fuzzy decision matrix X, the objective of the problem is to rank all the alternatives by giving each of them an overall preference value with respect to all criteria.

4.2. The algorithm

A two-phase approach has been typically used to solve the fuzzy MAVT-based MCDM problem defined above (Chen and Hwang, 1992; Deng and Yeh, 2006; Olcer and Odabasi, 2005; Zimmermann, 1996). The first phase aggregates the fuzzy assessments with respect to all criteria for each alternative by a value function. The second phase ranks the alternatives by comparing their aggregated overall preference values, characterized by fuzzy numbers. To aggregate the fuzzy assessments, we develop a new algorithm based on the concept of the degree of optimality rooted in an alternative where multiple criteria characterize the notion of the best (Hwang and Yoon, 1981; Zeleny, 1982). The concept suggests that the most preferred alternative should not only have the shortest distance from the positive ideal solution (or the best possible alternative), but also have the longest distance from the negative ideal solution (or the worst possible alternative).

This concept has been implemented by a widely used MCDM method called the technique for order preference by similarity to ideal solution (TOPSIS) (Hwang and Yoon, 1981; Deng et al., 2000) and has been applied under different decision contexts (e.g. Chang and Yeh, 2002; Chen and Hwang, 1992; Liang, 1999; Kuo et al., 2007; Olcer and Odabasi, 2005; Zavadskas and Antucheviciene, 2006; Zeleny, 1998). The advantages of using this concept have been highlighted by (a) its intuitively appealing logic, (b) its simplicity and comprehensibility, (c) its computational efficiency, (d) its ability to measure the relative performance of the alternatives with respect to individual or all evaluation criteria in a simple mathematical form, and (e) its applicability in solving various practical MAVT-based MCDM problems (Deng et al., 2000; Yeh et al., 2000; Zanakis et al., 1998). Despite its merits in comparison with other MCDM methods, the TOPSIS method does not consider the relative importance (weight) of the distances from the positive and the negative ideal solutions (Opricovic and Tzeng, 2004, 2007). This issue is addressed in the new algorithm presented below.

In the new algorithm, the concept of the degree of optimality is used (a) to transform the fuzzy decision matrix into a relative performance value matrix, and (b) to obtain an overall fuzzy preference value for each alternative. In line with this concept, the criteria weights are incorporated in the distance measurement in order to weight the distance between the alternative and the positive (or negative) ideal solution. As discussed in Section 2, the hierarchical weighting method developed in this paper can be used to deal with this weighted distance issue for a large number of lower-level measurable criteria.

Given the fuzzy decision matrix X and the fuzzy weight vector W, the algorithm works as follows:

Step 1: Normalize the fuzzy decision matrix X to allow a comparable scale for all criteria, on which performance ratings (x_{ij}) may be assessed by different measurement units, by

$$y_{ij} = x_{ij} \Bigg/ \sqrt{\sum_{i=1}^{m} x_{ij}^2}. \tag{5}$$

With the use of triangular fuzzy numbers, the arithmetic operations on fuzzy numbers are based on interval arithmetic (Kaufmann and Gupta, 1991).

Step 2: Obtain the relative degree of optimality for the performance of each alternative A_i ($i = 1, 2, \ldots, m$) with respect to each criterion C_j ($j = 1, 2, \ldots, n$) by first calculating the Hamming distances between the normalized fuzzy performance rating (y_{ij}) and a fuzzy maximum ($M^j_{\max}$) and a fuzzy minimum ($M^j_{\min}$) (Chen, 1985; Zadeh, 1998), respectively by

$$h^+_{ij} = H(y_{ij}, M^j_{\max}) \quad \text{and} \quad h^-_{ij} = H(y_{ij}, M^j_{\min}). \tag{6}$$

The fuzzy maximum ($M^j_{\max}$) and the fuzzy minimum ($M^j_{\min}$) represent the positive ideal (best possible) performance rating and the negative ideal (worst possible) performance rating of all alternatives with respect to each criterion C_j ($j = 1, 2, \ldots, n$), respectively, whose membership functions are defined as

$$\mu_{M^j_{\max}}(y_{ij}) = \frac{y_{ij} - y^j_{\min}}{y^j_{\max} - y^j_{\min}}, \quad \mu_{M^j_{\min}}(y_{ij}) = \frac{y^j_{\max} - y_{ij}}{y^j_{\max} - y^j_{\min}}, \tag{7}$$

where

$$i = 1, 2, \ldots, m;$$

$$j = 1, 2, \ldots, n; \quad y^j_{\max} \sup \bigcup_{i=1}^{m} (y_{ij}); \quad y^j_{\min} = \inf \bigcup_{i=1}^{m} (y_{ij}). \tag{8}$$

With the use of triangular fuzzy numbers, the Hamming distance between two fuzzy numbers $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ is calculated as (Klir and Yuan, 1995):

$$H(A, B) = |a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3|. \tag{9}$$

For each criterion C_j, h^+_{ij} and h^-_{ij} in Eq. (6) represent the closeness of alternatives A_i's performance rating to the positive ideal performance rating and the negative ideal performance rating, respectively. The degree of optimality (or preferability) of alternative A_i ($i = 1, 2, \ldots, m$) relative to other alternatives with respect to each criterion C_j ($j = 1, 2, \ldots, n$) is thus determined by

$$z_{ij} = \frac{h^-_{ij}}{h^+_{ij} + h^-_{ij}}. \tag{10}$$

The matrix given in Eq. (10) indicates the relative performance value of each alternative A_i, with respect to each criterion C_j.

Step 3: Determine the positive ideal solution A^+ and the negative ideal solution A^- for representing the best possible and the worst possible results of the alternatives respectively, by

$$A^+ = (z_1^+, z_2^+, \ldots, z_n^+), \quad A^- = (z_1^-, z_2^-, \ldots, z_n^-), \tag{11}$$

where

$$z_j^+ = \max\{z_{1j}, z_{2j}, \ldots, z_{mj}\}, \quad z_j^- = \min\{z_{1j}, z_{2j}, \ldots, z_{mj}\}, \quad j = 1, 2, \ldots, n. \tag{12}$$

The positive (or negative) ideal solution consists of the best (or worst) performance values attainable from all the alternatives if each criterion takes a monotonically increasing or decreasing value (Hwang and Yoon, 1981).

Step 4: Calculate the weighted Euclidean distances, between A_i and A^+, and between A_i and A^-, respectively by

$$d_i^+ = \sqrt{\sum_{j=1}^{n} w_j (d_{ij}^+)^2} \quad \text{and} \quad d_i^- = \sqrt{\sum_{j=1}^{n} w_j (d_{ij}^-)^2}, \tag{13}$$

where

$$d_{ij}^+ = z_j^+ - z_{ij}, \quad d_{ij}^- = z_{ij} - z_j^-, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n. \tag{14}$$

Step 5: Obtain an overall fuzzy preference value for each alternative A_i $(i = 1, 2, \ldots, m)$, relative to other alternatives, by

$$P_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{15}$$

The larger the preference value, the more preferred the alternative.

To compare the overall fuzzy preference value of the alternatives, we use the concept of α-cut in fuzzy set theory (Klir and Yuan, 1995). The α-cut of a fuzzy set is the ordinary (crisp) set that contains all the values with a membership degree (possibility) of at least α (where $0 \leqslant \alpha \leqslant 1$). By using an α-cut on a triangular fuzzy number, a value interval $[x_l^\alpha, x_r^\alpha]$ is derived. For a given α, x_l^α and x_r^α are the average of the lower bounds and upper bounds of the crisp intervals, respectively, resulted from all the α-cuts using the α values equal to or greater than the specified value of α. The value of α can be used to represent the decision makers' degree of confidence about their assessments of criteria weights. A larger α value indicates that the decision makers are more confident, as the interval is smaller and has a higher possibility and lower uncertainty (Yeh and Kuo, 2003).

To reflect the decision makers' relative preference between x_l^α and x_r^α, an attitude index λ in the range of 0 and 1 can be incorporated. As a result, a crisp value can be obtained as

$$x_\alpha^\lambda = \lambda x_r^\alpha + (1 - \lambda) x_l^\alpha, \quad 0 \leqslant \lambda \leqslant 1. \tag{16}$$

The value of λ can be used to reflect the decision makers' attitude towards risk (Yeh and Deng, 2004). In actual decision settings, $\lambda = 1$, $\lambda = 0.5$, or $\lambda = 0$ can be used to indicate that the decision makers have an optimistic, moderate, or pessimistic view, respectively about their assessments. If the decision makers are optimistic about the results, they would prefer higher values; otherwise, they would favor lower values. In a group decision environment, the setting of $\alpha = 0$ and $\lambda = 0.5$ is often used to reflect that no particular preference is given for the fuzzy assessment results. $\alpha = 0$ implies that the mean value of a fuzzy number (i.e. the average of value intervals of all α-cuts on the fuzzy number) is used. $\lambda = 0.5$ indicates that all the values derived from fuzzy assessments are weighted equally.

5. Empirical study

To illustrate the fuzzy group MCDM approach, we present an aircraft selection problem faced by a Taiwan's domestic airline for its major route. To maintain its dominant position in Taiwan's domestic airline market, the airline needs to upgrade its services for its competitive routes. To match the increasing demand for its major route, the airline has to make rational decisions about which type of aircraft to purchase for its fleet. In addition to the traditional selection criteria such as the purchase cost and the operation efficiency, other factors such as future demand and passenger preference also need to be considered (Taneja, 1982). This makes the process of selecting an aircraft type a complex MCDM problem.

Five types of aircraft, including B757-200 (A_1), A-321 (A_2), B767-200 (A_3), MD-82 (A_4), and A310-300 (A_5), are

Table 1
Evaluation criteria for the aircraft selection problem

Super-criteria		Criteria (measure)	
S_1	Technological advance	C_1	Maintenance requirements (subjective assessment)
		C_2	Pilot adaptability (subjective assessment)
		C_3	Aircraft reliability (subjective assessment)
		C_4	Maximum range (kilometers)
S_2	Social responsibility	C_5	Passenger preference (subjective assessment)
		C_6	Noise level (Effective perceived noise level in decibels – EPNdB)
S_3	Economical efficiency	C_7	Operational productivity (seat-kilometer per hour)
		C_8	Airline fleet economy of scale (subjective assessment)
		C_9	Direct operating cost (US cents per available seat miles)
		C_{10}	Purchasing price (US millions)
		C_{11}	Consistency with corporate strategy (subjective assessment)

to be evaluated with respect to 11 criteria, which are grouped into three categories (super-criteria). Eight decision makers, representing the interests of all the stakeholders, are involved in the evaluation process. Table 1 shows the 11 evaluation criteria and their corresponding super-criteria, together with the criteria measures. The criteria measures involve both quantitative and qualitative assessments, for which numerical data and triangular fuzzy numbers are used, respectively. Since the qualitative assessments are to be subjectively made by the eight decision makers and the 11 criteria are independent of each other, this aircraft selection problem can be solved by the fuzzy group MCDM approach developed. We briefly discuss the evaluation criteria within each super-criterion below.

The technological advance of an aircraft (S_1) is a reflection of technological features that a specific aircraft type has. These features are directly related to the design of the aircraft, the quality of the post-sale service, and the specific situation of the airline. This super-criterion is underlain by 4 criteria. The aircraft maintenance capability (C_1) is concerned with the availability and the level of standardization of spare parts and post-sale services. The pilot adaptability (C_2) to a specific type of aircraft is related to the skills of available pilots and the specific features of the aircraft. The aircraft reliability (C_3) is a reflection of the historical performance demonstrated by a specific type of aircraft. The maximum range (C_4) of an aircraft is determined by the maximum kilometers that the aircraft can travel at the maximum payload.

The social responsibility super-criterion (S_2) is a reflection of the airline's desire to establish a positive image in public and of the requirements imposed by various environment protection laws and regulations. This super-criterion is represented by the passengers' preference (C_5) towards a specific aircraft type and the level of noise (C_6) that an aircraft type produces.

The economical efficiency super-criterion (S_3) is concerned with the overall benefits that the aircraft type can bring to the airline in relation to its overall costs. It is formed by five criteria. The aircraft's operational productivity (C_7) is determined by the number of seats available, the load rate, the travel frequency, and the aircraft travel speed. The airline fleet economy of scale (C_8) is related to the number of aircraft available in the airline, the number of aircraft in various types, and the characteristics of each aircraft type. The aircraft direct operating cost (C_9) is a measure that reflects the normal cost in running the aircraft. The aircraft purchasing price (C_{10}) is the price to be paid for a new aircraft. The consistency of the purchasing decision with the corporate strategy (C_{11}) is a reflection of the impact of the purchase decision on the airline with respect to the future market demand and the overall direction of the airline.

The eight decision makers use a set of five linguistic terms {very low, low, medium, high, very high} to assess the performance ratings of the five aircraft types with respect to the 6 qualitative criteria in Table 1. These terms are associated with the corresponding numbers 1, 2, 3, 4 and 5, respectively, as in a 5-point Likert scale. To reflect the imprecision involved in the subjective assessment

Table 2
Performance ratings of five aircraft types

Criteria	A_1	A_2	A_3	A_4	A_5
C_1	(2, 3.064, 4)	(4, 4.229, 5)	(3, 3.224, 4)	(1, 1.929, 3)	(3, 3.464, 4)
C_2	(2, 2.852, 3)	(2, 2.000, 2)	(2, 2.852, 3)	(4, 4.113, 5)	(2, 2.000, 2)
C_3	(4, 4.599, 5)	(2, 3.413, 4)	(4, 4.599, 5)	(3, 4.314, 5)	(3, 4.314, 5)
C_4	5,522	4,350	5,856	4,032	7,968
C_5	(4, 4.000, 4)	(2, 2.852, 3)	(4, 4.000, 4)	(3, 3.591, 4)	(3, 3.342, 4)
C_6	97.7	97.0	101.6	94.6	98.6
C_7	116,279	109,063	129,465	87,662	130,664
C_8	(3, 3.000, 3)	(2, 2.000, 2)	(3, 3.000, 3)	(4, 4.000, 4)	(2, 2.000, 2)
C_9	3.08	3.28	3.62	3.79	4.10
C_{10}	56	54	69	33	80
C_{11}	(4, 4.349, 5)	(2, 2.852, 3)	(3, 3.859, 4)	(1, 1.834, 2)	(3, 3.859, 4)

Table 3
Normalized super-criteria weights by individual decision makers

Super-criteria	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7	DM_8
S_1	0.400	0.389	0.448	0.434	0.363	0.379	0.325	0.329
S_2	0.269	0.262	0.281	0.248	0.274	0.275	0.349	0.295
S_3	0.331	0.349	0.271	0.318	0.363	0.346	0.326	0.376

Table 4
Normalized local criteria weights by individual decision makers

Super-criteria	Criteria	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7	DM_8
S_1	C_1	0.286	0.266	0.296	0.283	0.218	0.223	0.269	0.255
	C_2	0.251	0.266	0.238	0.247	0.257	0.268	0.246	0.255
	C_3	0.279	0.266	0.267	0.256	0.286	0.293	0.272	0.281
	C_4	0.184	0.202	0.199	0.214	0.239	0.218	0.213	0.209
S_2	C_5	0.539	0.500	0.473	0.473	0.531	0.500	0.568	0.635
	C_6	0.461	0.500	0.527	0.527	0.469	0.500	0.432	0.365
S_3	C_7	0.232	0.225	0.251	0.230	0.225	0.226	0.248	0.232
	C_8	0.232	0.223	0.221	0.227	0.193	0.200	0.215	0.214
	C_9	0.195	0.203	0.191	0.194	0.189	0.189	0.180	0.212
	C_{10}	0.179	0.148	0.144	0.146	0.170	0.169	0.153	0.145
	C_{11}	0.163	0.201	0.193	0.203	0.223	0.217	0.204	0.198

Table 5
Normalized global criteria weights by individual decision makers and their aggregated fuzzy group weights

Criteria	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7	DM_8	Fuzzy group weight
C_1	0.121	0.108	0.143	0.128	0.083	0.089	0.096	0.088	(0.083, 0.105, 0.143)
C_2	0.106	0.108	0.115	0.112	0.098	0.107	0.088	0.088	(0.088, 0.102, 0.115)
C_3	0.118	0.108	0.129	0.116	0.109	0.117	0.097	0.097	(0.097, 0.111, 0.129)
C_4	0.078	0.082	0.096	0.097	0.091	0.087	0.076	0.072	(0.072, 0.084, 0.097)
C_5	0.076	0.068	0.071	0.061	0.076	0.072	0.109	0.101	(0.061, 0.078, 0.109)
C_6	0.065	0.068	0.079	0.068	0.067	0.072	0.083	0.058	(0.058, 0.070, 0.083)
C_7	0.101	0.103	0.092	0.096	0.107	0.103	0.112	0.115	(0.092, 0.103, 0.115)
C_8	0.101	0.102	0.081	0.095	0.092	0.091	0.097	0.106	(0.081, 0.095, 0.106)
C_9	0.085	0.093	0.070	0.081	0.090	0.086	0.081	0.105	(0.07, 0.086, 0.105)
C_{10}	0.078	0.068	0.053	0.061	0.081	0.077	0.069	0.072	(0.053, 0.069, 0.081)
C_{11}	0.071	0.092	0.071	0.085	0.106	0.099	0.092	0.098	(0.071, 0.088, 0.106)

Table 6
Preference value and ranking of five aircraft types

Aircraft type	Fuzzy preference value	Crisp preference value	Ranking
A_1 (B757-200)	(0.459, 0.628, 0.881)	0.649	1
A_2 (A-321)	(0.325, 0.469, 0.690)	0.488	5
A_3 (B767-200)	(0.389, 0.581, 0.811)	0.591	3
A_4 (MD-82)	(0.392, 0.598, 0.823)	0.603	2
A_5 (A310-300)	(0.336, 0,525, 0.763)	0.537	4

process, the group performance ratings are represented by triangular fuzzy numbers, which are obtained using Eq. (4). Table 2 shows the result of the fuzzy group performance ratings of the five aircraft types, together with their performance ratings on quantitative criteria, which are collected from objective sources. Table 2 constitutes the decision matrix for the aircraft selection problem.

The eight decision makers use a pairwise comparison process with a 1–9 ratio scale to assess the weights between the three super-criteria, and between criteria within each super-criterion, respectively. Tables 3 and 4 show the result for each decision maker, which is normalized to sum to 1 so that the weight value can be interpreted as the percentage of the total importance weight (Belton and Stewart, 2002). To obtain the global weights of the 11 criteria assessed by each decision maker for the problem, Eq. (1) is applied using the data in Tables 3 and 4. The global weights given individually by the eight decision makers are normalized (to sum to 1) and aggregated using Eq. (3) to obtain the fuzzy group weights for the 11 criteria. Table 5 shows the result.

With the fuzzy group decision matrix in Table 2 and the fuzzy group weight vector in the last column of Table 5, an overall fuzzy preference value for each aircraft type can be obtained by applying the algorithm given in Eqs. (5)–(15). A crisp preference value for each aircraft type can be generated by applying Eq. (16) with $\alpha = 0$ and $\lambda = 0.5$. Table 6 shows the result and the corresponding ranking order. The relative ranking remains unchanged if different α and λ values are used. This result would provide the decision makers with reasonable evidence in supporting their decisions.

6. Conclusion

Evaluating decision alternatives in a new and complex problem setting often involves subjective evaluation by a group of decision makers with respect to a set of qualitative criteria. To address this decision problem, we have developed a new fuzzy group MCDM approach with an effective algorithm that extends the concept of the degree of optimality. The approach allows individual decision makers to make comparative and absolute judgements in a conventional manner. Individual judgements are aggregated as a group judgement using triangular fuzzy numbers to reflect the inherent imprecision involved. In particular, we have developed a hierarchical weighting method to help assess the weights of a large number of criteria using pairwise comparisons. The method facilitates the application of the new fuzzy MCDM approach to large-sized problems. To illustrate how the approach works, we have presented an aircraft selection problem. In line with conventionally used evaluation processes, the approach provides a simple and effective mechanism for modeling MCDM problems involving subjective evaluation in a group decision environment. The algorithm developed is applicable to the general fuzzy MCDM problem where a cardinal ranking is required.

Acknowledgements

This research was supported by the National Science Council of Taiwan (Grant NSC95-2811-H-006-001) and the National Cheng Kung University (Grant OUA 95-21 T-6-151). We are grateful to anonymous referees for their valuable comments and advice.

References

Aczel, J., Saaty, T.L., 1983. Procedures for synthesizing ratio judgements. Journal of Mathematical Psychology 27, 93–102.

Aull-Hyde, R., Erdogan, S., Duke, J.M., 2006. An experiment on the consistency of aggregated comparison matrices in AHP. European Journal of Operational Research 171 (1), 290–295.

Barzilai, J., Cook, W.D., Golany, B., 1987. Consistent weights for judgements matrices of the relative importance of alternatives. Operations Research Letters 6 (3), 131–134.

Belton, V., Stewart, T.J., 2002. Multiple Criteria Decision Analysis: An Integrated Approach. Kluwer, Boston.
Blumenthal, A., 1977. The Process of Cognition. Prentice-Hall, Englewood Cliffs, NJ.
Bose, U., Davey, A.M., Olson, D.L., 1997. Multi-attribute utility methods in group decision making: Past applications and potential for inclusion in GDSS. Omega 25 (6), 691–706.
Buckley, J.J., 1985. Fuzzy hierarchical analysis. Fuzzy Sets and Systems 17 (3), 233–247.
Chang, Y.-H., Yeh, C.-H., 2002. A survey analysis of service quality for domestic airlines. European Journal of Operational Research 139 (1), 166–177.
Chang, Y.-H., Yeh, C.-H., 2004. A new airline safety index. Transportation Research Part B: Methodological 38 (4), 369–383.
Chen, S.H., 1985. Ranking fuzzy numbers with maximising set and minimising set. Fuzzy Sets and Systems 17, 113–129.
Chen, S.J., Hwang, C.L., 1992. Fuzzy Multiple Attribute Decision Making: Methods and Applications. Springer-Verlag, New York.
Deng, H., Yeh, C.-H., 2006. Simulation-based evaluation of defuzzification-based approaches to fuzzy multicriteria analysis. IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans 36 (5), 968–977.
Deng, H., Yeh, C.-H., Willis, R.J., 2000. Inter-company comparison using modified TOPSIS with objective weights. Computers & Operations Research 27 (10), 963–973.
Dyer, J.S., Fishburn, P.C., Steuer, R.E., Wallenius, J., Zionts, S., 1992. Multiple criteria decision making, multiattribute utility theory: The next ten years. Management Science 38, 645–653.
Dyer, R.F., Forman, E.H., 1992. Group decision support with the analytic hierarchy process. Decision Support Systems 8 (2), 99–124.
Dyer, J.S., Sarin, R.K., 1979. Measurable multi-attribute value functions. Operations Research 27, 810–822.
Forman, E., Peniwati, K., 1998. Aggregating individual judgments and priorities with the analytic hierarchy process. European Journal of Operational Research 108 (1), 165–169.
Gass, S.I., Rapcsak, T., 1998. A note on synthesizing group decisions. Decision Support Systems 22 (1), 59–63.
Gonzalez-Pachon, J., Romero, C., 2004. A method for dealing with inconsistencies in pairwise comparisons. European Journal of Operational Research 158, 351–361.
Harker, P.T., Vargas, L.G., 1987. Theory of ratio scale estimation: Saaty's analytic hierarchy process. Management Science 33, 1383–1403.
Hwang, C.L., Yoon, K., 1981. Multiple Attribute Decision Making: Methods and Applications. Springer-Verlag, New York.
Kaufmann, A., Gupta, M.M., 1991. Introduction to Fuzzy Arithmetic, Theory and Applications. International Thomson Computer Press, Boston.
Keeney, R., Raiffa, H., 1993. Decisions with Multiple Objectives, Preferences and Value Tradeoffs. Cambridge University Press, New York.
Klir, G.R., Yuan, B., 1995. Fuzzy Sets and Fuzzy Logic Theory and Applications. Prentice-Hall, Upper Saddle River, NJ.
Kuo, M.-S., Tzeng, G.-H., Huang, W.-C., 2007. Group decision-making based on concepts of ideal and anti-ideal points in a fuzzy environment. Mathematical and Computer Modelling 45, 324–339.
Liang, G.S., 1999. Fuzzy MCDM based on ideal and anti-ideal concepts. European Journal of Operational Research 112 (3), 682–691.
Lootsma, F.A., 1999. Multi-Criteria Decision Analysis via Ratio and Difference Judgement. Kluwer Academic Publishers, Dordrecht, The Netherlands.
Matsatsinis, N.F., Samaras, A.P., 2001. MCDA and preference disaggregation in group decision support systems. European Journal of Operational Research 130 (2), 414–429.
Mikhailov, L., 2003. Deriving priorities from fuzzy pairwise comparison judgements. Fuzzy Sets and Systems 134, 365–385.
Miller, G.A., 1956. The magic number seven, plus or minus two: Some limits on our capacity for processing information. Psychological Review 63 (2), 81–97.
Olcer, A.I., Odabasi, A.Y., 2005. A new fuzzy multiple attribute group decision making methodology and its application to propulsion/manoeuvring system selection problem. European Journal of Operational Research 166 (1), 93–114.
Opricovic, S., Tzeng, G.-H., 2004. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. European Journal of Operational Research 156 (2), 445–455.
Opricovic, S., Tzeng, G.-H., 2007. Extended VIKOR method in comparison with outranking methods. European Journal of Operational Research 178 (2), 514–529.
Pedrycz, W., 1994. Why triangular membership functions?. Fuzzy Sets and Systems 64 21–30.
Saaty, T.L., 1980. The Analytic Hierarchy Process. McGraw-Hill, New York.
Saaty, T.L., 2006. Rank from comparisons and from ratings in the analytic hierarchy/network processes. European Journal of Operational Research 168 (2), 557–570.
Saaty, T.L., Ozdemir, M.S., 2003. Why the magic number seven plus or minus two. Mathematical and Computer Modelling 38, 233–244.
Salo, A.A., Hamalainen, R.P., 1992. Preference assessment by imprecise ratio statements. Operations Research 40 (6), 1053–1061.
Salo, A.A., Hamalainen, R.P., 1995. Preference programming through approximate ratio comparisons. European Journal of Operational Research 82 (3), 458–475.
Shipley, M.F., de Korvin, A., Obid, R., 1991. A decision making model for multi-attribute problems incorporating uncertainty and bias measures. Computers & Operations Research 18, 335–342.
Taneja, N.K., 1982. Airline in Transition. Lexington Books, Berlin.
Thurstone, L.L., 1927. The method of paired comparisons for social values. Journal of Abnormal Psychology 21, 384–400.
Vaidya, O.S., Kumar, S., 2006. Analytic hierarchy process: An overview of applications. European Journal of Operational Research 169 (1), 1–29.
Vreeker, R., Nijkamp, P., Welle, C.T., 2002. A multicriteria decision support methodology for evaluating airport expansion plans. Transportation Research Part D 7 (1), 27–47.
Yeh, C.-H., Deng, H., 2004. A practical approach to fuzzy utilities comparison in fuzzy multi-criteria analysis. International Journal of Approximate Reasoning 35 (2), 179–194.
Yeh, C.-H., Deng, H., Chang, Y.-H., 2000. Fuzzy multicriteria analysis for performance evaluation of bus companies. European Journal of Operational Research 126 (3), 459–473.
Yeh, C.-H., Kuo, Y.-L., 2003. Evaluating passenger services of Asia-Pacific international airports. Transportation Research Part E 39 (1), 35–48.
Yeh, C.-H., Willis, R.J., Deng, H., Pan, H., 1999. Task oriented weighting in multi-criteria analysis. European Journal of Operational Research 119 (1), 130–146.
Zadeh, L.A., 1965. Fuzzy sets. Information and Control 8, 38–353.
Zadeh, L.A., 1998. Maximizing sets and fuzzy markoff algorithms. IEEE Transactions on Systems, Man, and Cybernetics 28 (1), 9–15.
Zanakis, S.H., Solomon, A., Wishar, N., Dublish, S., 1998. Multi-attribute decision making: A simulation comparison of select methods. European Journal of Operational Research 107 (3), 507–529.
Zavadskas, E.K., Antucheviciene, J., 2006. Development of an indicator model and ranking of sustainable revitalization alternatives of derelict property: A Lithuanian case study. Sustainable Development 14, 287–299.
Zeleny, M., 1982. Multiple Criteria Decision Making. McGraw-Hill, New York.
Zeleny, M., 1998. Multiple criteria decision making: Eight concepts of optimality. Human Systems Management 17 (2), 97–107.
Zimmermann, H.-J., 1996. Fuzzy Set Theory and Its Applications, third ed. Kluwer Academic Publishers, Boston.