

# The Selection of Multiattribute Decision Making Methods for Scholarship Student Selection

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Selecting scholarship students from a number of competing candidates is a complex decision making process, in which multiple selection criteria have to be considered simultaneously. Multiattribute decision making (MADM) has proven to be an effective approach for ranking or selecting one or more alternatives from a finite number of alternatives with respect to multiple, usually conflicting criteria. This paper formulates the scholarship student selection process as an MADM problem, and presents suitable compensatory methods for solving the problem. A new empirical validity procedure is developed to deal with the inconsistent ranking problem caused by different MADM methods. The procedure aims at selecting a ranking outcome which has a minimum expected value loss, when true attribute weights are not known. An empirical study of a scholarship student selection problem in an Australian university is conducted to illustrate how the selection procedure works.

## 1. Introduction

Granting scholarships to students is an important decision to be made regularly in a higher education environment. To ensure that scholarships are allocated to the best-qualified candidates in a fair and informed manner, a comprehensive assessment of the candidates' performance in terms of a set of selection criteria is required. The assessment of the overall performance of the candidates with respect to multiple selection criteria is a complex decision-making process.

Multiattribute decision making (MADM) has been widely used in ranking or selecting one or more alternatives from a finite number of alternatives with respect to multiple, usually conflicting criteria or attributes (Zeleny, 1982; Yoon and Hwang, 1995; Belton and Stewart, 2002). Among its broad range of applications, MADM has shown advantages in evaluating the performance of the resources and operations of higher education sectors in various decision contexts, with respect to conflicting performance measures or selection criteria (e.g. Saaty and Ramarujam, 1983; Blanchard et al., 1989; Davey et al., 1994; Mustafa and Goh, 1996). In these applications, MADM provides a

systematic means for assisting the decision makers in making more informed decisions about the comparative performance of the resources and operations.

Numerous MADM methods have been proposed for a large variety of selection and evaluation problems (e.g. Hwang and Yoon, 1981; Zeleny, 1982; Colson and de Bruyn, 1989; Dyer et al., 1992; Stewart, 1992; Olson, 1996; Kasanen et al., 2000; Yeh et al., 2000). Due to the multiplicity and complexity of multicriteria decisions, there is no best method for the general MADM problem, and the validity of the decision outcome remains an open issue. As suggested by existing studies (Voogd, 1983; Zanakakis et al., 1998; Hobbs and Meier, 2000) and evidenced in the empirical study of this paper, different MADM methods often produce inconsistent rankings for the same problem. The ranking inconsistency of MADM methods increases as the number of alternatives to be ranked or selected increases, or when the alternatives have similar performances (Olson et al., 1995). It is thus desirable to apply more than one method to an MADM problem, followed by a validity check for selecting the most valid ranking outcome (Hobbs et al., 1992).

In subsequent sections, we first describe a scholarship student selection problem and formulate it as an MADM problem, followed by a discussion of suitable MADM methods. We then develop a new validity procedure for choosing the most valid ranking outcome for situations

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where the rankings produced by different MADM methods differ. Finally we conduct an empirical study to demonstrate the effectiveness of the validity procedure.

## 2. The Scholarship Student Selection Problem

An Australian university has recently offered a number of industry-sponsored scholarships to the first year students in one of its business-oriented undergraduate degrees on a yearly basis. The scholarships are for a duration of three years, subject to the scholarship holders' satisfactory progress in their studies. During their studies, scholarship students are required to work with industry sponsors for a total period of one year under an industry-based learning program. Despite the fact that there is no "correct" decision and the performance of scholarship students is generally beyond the university's control, the university needs to justify that scholarships are granted to the best-qualified candidates in a fair and informed manner.

The candidates for scholarships are selected based on their performance on non-academic, qualitative attributes (selection criteria), assessed via an interview process. The reason for excluding the academic attributes is that all the candidates have overcome a considerable academic hurdle to become eligible for study in the program. Therefore they are expected to have the capability to complete all the specified academic requirements of the scholarship. Based on comprehensive discussions with industry sponsors, a set of eight attributes relevant to the industry-based learning program is determined. These attributes are briefly discussed below:

- (1) Community services. Voluntary work within the community by candidates is viewed favorably. Examples include activities involved in social welfare, coaching, peer support, etc.
- (2) Sports/Hobbies. Non-work related activities that the candidates are involved in are deemed as beneficial to the candidates' "well-roundedness". Candidates with a wider range of interests are favored.
- (3) Work experience. This is concerned with the degree of the candidates' participation in any paid activities. Experiences in more relevant areas and/or with higher responsibility are preferred.
- (4) Energy. Future demands placed on the candidates will require energy that indicates a positive attitude and a willingness to participate in demanding tasks.
- (5) Communication skills. The candidates' ability to communicate is important, as they need to interact with other individuals in their industry-based learning placements. Their manner of speaking, writing ability and appearance are all communication enablers or disablers.
- (6) Attitude to business. Most candidates, after finishing their studies, will work in the business world. Their

attitude to, and ambitions in the corporate world are crucial in indicating what kinds of employees they will make.

- (7) Maturity. This is related to the candidates' willingness and ability to take on responsibility for their current situations. The candidates' performance in academic studies and industry-based learning placements is highly dependent on the degree of responsibility they undertake.
- (8) Leadership. Potential leadership qualities are preferred, as they reflect the candidates' overall performance for their academic studies and industry-based learning placements.

The decision makers (interviewers) and the stakeholders (industry sponsors) have decided to use equal weights for the attributes, due to the fact that no other weights can be agreed upon in a fair and convincing manner. This setting is in accordance with the principle of insufficient reason (Starr and Greenwood, 1977), which suggests the use of equal weights if the decision maker has no reason to prefer one attribute to another. This view is highly advocated in the psychological decision-making field (Dawes, 1979). In addition, no single attribute weighting method can guarantee a more accurate result, and the same decision makers may elicit different weights using different methods (Weber and Borcherting, 1993; Doyle et al., 1997; Yeh et al., 1999). In practical applications, this implies that there is no easy way for determining attribute weights and there are no criteria for determining what the true weight is (Weber and Borcherting, 1993). As such, assigning equal weights can be considered for decision situations where (a) it is difficult to obtain the importance weight judgments from the stakeholders (such as the scholarship student selection problem examined) or (b) widely differing weights are obtained from conflicting stakeholders (Edwards and Newman, 1982).

The performance of the candidates on the eight attributes will be assessed on a 6-point Likert-type scale, ranging from 5 (extremely high) to 0 (extremely low). With this problem setting and the use of equal attribute weights, a traditional scoring method, referred to as the total sum (TS) method, can be used. The TS method simply sums the performance ratings of each candidate on all attributes to a total score. The TS method implies trade-offs between attributes – a high value on one attribute can compensate for low values on other attributes (Davey et al., 1994).

Research in MADM has suggested the use of simple, understandable, and usable approaches for solving practical MADM problems (Dyer et al., 1992). In the context of MADM, the TS method is in line with the compensatory methods based on multiattribute value theory (MAVT) (Keeney and Raiffa, 1993), which is the most widely used theory in solving MADM problems (Weber and Borcherting, 1993). In practical applications, MAVT-based methods, using an additive value function, are intuitively

appealing to the decision makers (Zanakis et al., 1998; Cabrera and Raju, 2001). This is mainly due to their simplicity in both concept and computation. In addition, MAVT-based MADM is the most appropriate quantitative tool for group decision support systems (Bose et al., 1997).

To facilitate the presentation of suitable MAVT-based MADM methods for the scholarship student selection problem, we first introduce MADM in the context of MAVT and then describe the selection problem as an MADM problem.

### 3. The MADM Problem and Methods

MADM is a methodology that aids the decision maker in making preference decisions (e.g. assessment, ranking, selection) over a finite set of available alternatives (courses of action) characterized by multiple, potentially conflicting attributes (Yoon and Hwang, 1995; Mollaghasemi and Pet-Edwards, 1997). It provides a formal framework for modeling multiattribute decision problems, in particular when the nature of the problem demands a systematical analysis, such as the complexity of the decision, the regularity of the decision, the significant consequences, and the need for accountability (Belton and Stewart, 2002). As an aid to decision making, its intent is to improve the quality of decisions by making choices more explicit, rational, and efficient (Hobbs and Meier, 2000).

Despite their diversity, MADM problems share the following common characteristics: (a) a finite number of comparable alternatives, (b) multiple attributes for comparison among alternatives, (c) measurable units for measuring performance rating of alternatives on each attribute, (d) attribute weights for representing the relative importance of each attribute. The performance ratings of alternatives are to be aggregated with the attribute weights by MADM methods to give an overall assessment of each alternative indicative of the decision maker's preferences. The resulting overall preference values provide a cardinal ranking of the alternatives.

#### 3.1 The selection problem formulated as an MADM problem

The MADM problem involves a set of  $m$  alternatives (candidates)  $A_i$  ( $i = 1, 2, \dots, m$ ). These alternatives are to be assessed by a cardinal scale with respect to a set of  $n$  attributes (selection criteria)  $C_j$  ( $j = 1, 2, \dots, n$ ), via an interview process. The result of the assessment forms a decision matrix, where rows and columns indicate  $m$  alternatives and  $n$  attributes respectively, given as

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \quad (1)$$

where  $x_{ij}$  are the performance ratings of alternative  $A_i$  ( $i = 1, 2, \dots, m$ ) with respect to attribute  $C_j$  ( $j = 1, 2, \dots, n$ ). A weighting vector representing the attribute weights is to be given by a cardinal scale as

$$W = (w_1, w_2, \dots, w_n) \quad (2)$$

Cardinal weights are usually normalized to sum to 1, in order to allow the weight value to be interpreted as the percentage of the total importance weight (Belton and Stewart, 2002). The cardinal values given in the decision matrix and the weighting vector represent the absolute preferences of the decision maker. The objective of the problem is to rank all the alternatives in terms of their overall preference value, based on the cardinal values given in (1) and (2).

With the selection problem formulated above, the TS method and three MAVT-based MADM methods described below can be used. These methods assume that the attribute weights and the performance ratings of alternatives are given on an interval scale. Another widely used MADM method is the analytic hierarchy process (AHP) (Saaty, 1994), with which the values for  $w_j$  and  $x_{ij}$  are given based on a ratio scale of preferences. In practical applications where effective guidance is not available, the use of simple MAVT-based MADM methods may be more justifiable than AHP (Stewart, 1992).

#### 3.2 The Simple Additive Weighting (SAW) Method

The SAW method, also known as the weighted sum method, is probably the best known and most widely used MADM method (Hwang and Yoon, 1981). The basic logic of the SAW method is to obtain a weighted sum of the performance ratings of each alternative over all attributes. The SAW method normally requires normalizing the decision matrix ( $X$ ) to allow a comparable scale for all ratings in  $X$  by

$$r_{ij} = \begin{cases} \frac{x_{ij}}{\max_i x_{ij}}, & \text{if } j \text{ is a benefit attribute} \\ \frac{\min_i x_{ij}}{x_{ij}}, & \text{if } j \text{ is a cost attribute} \end{cases}; \quad (3)$$

$$i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n.$$

where  $r_{ij}$  ( $0 \leq r_{ij} \leq 1$ ) is defined as the normalized performance rating of alternative  $A_i$  on attribute  $C_j$ . This normalization process transforms all the ratings in a linear (proportional) way, so that the relative order of magnitude of the ratings remains equal. The overall preference value of each alternative ( $V_i$ ) is obtained by

$$V_i = \sum_{j=1}^n w_j r_{ij}; \quad i = 1, 2, \dots, m. \quad (4)$$

The greater the value ( $V_i$ ), the more preferred the alternative ( $A_i$ ). Research results have shown that the

linear form of trade-offs between attributes used by the SAW method produces extremely close approximations to complicated nonlinear forms, while remaining far easier to use and understand (Hwang and Yoon, 1981).

The TS method discussed in the previous section is the same as the SAW method, except for the normalization process. To facilitate the comparison between the TS method and the other three MADM methods, the overall preference value of each alternative ( $V_i$ ) by the TS method is given as

$$V_i = \sum_{j=1}^n w_j \frac{x_{ij}}{M}; \quad i = 1, 2, \dots, m. \quad (5)$$

where  $M$  is a constant which equals the maximum score on the measure scale (e.g.  $M = 5$  in the problem setting).

### 3.3 The Weighted Product (WP) Method

The WP method uses multiplication for connecting attribute ratings, each of which is raised to the power of the corresponding attribute weight (Bridgman, 1922; Starr, 1972). This multiplication process has the same effect as the normalization process for handling different measurement units. The overall preference score of each alternative ( $S_i$ ) is given by

$$S_i = \prod_{j=1}^n x_{ij}^{w_j}; \quad i = 1, 2, \dots, m. \quad (6)$$

where  $\sum_{j=1}^n w_j = 1$ .  $w_j$  is a positive power for benefit attributes and a negative power for cost attributes. In this study, for easy comparison with other methods, the relative overall preference value of each alternative ( $V_i$ ) is given by

$$V_i = \frac{\prod_{j=1}^n x_{ij}^{w_j}}{\prod_{j=1}^n (x_j^*)^{w_j}}, \quad i = 1, 2, \dots, m. \quad (7)$$

where  $x_j^* = \max x_{ij}$  and  $0 \leq V_i \leq 1$ . The greater the value ( $V_i$ ), the more preferred the alternative ( $A_i$ ).

### 3.4 The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS is based on the concept that the most preferred alternative should not only have the shortest distance from the positive ideal solution, but also have the longest distance from the negative ideal solution (Hwang and Yoon, 1981; Zeleny, 1982). This concept has been widely used in various MADM models for solving practical decision problems (e.g. Parkan and Wu, 1999; Deng et al., 2000; Yeh et al., 2000). This is due to (a) its simplicity and comprehensibility in concept, (b) its computational efficiency, and (c) its ability to measure the relative performance of the decision alternatives in a simple mathematical form.

TOPSIS normally requires normalizing the performance ratings of alternative  $A_i$  on attribute  $C_j$  by

$$r_{ij} = x_{ij} / \sqrt{\sum_{i=1}^m x_{ij}^2}; \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \quad (8)$$

The positive ideal solution  $A^+$  and the negative ideal solution  $A^-$  can be determined based on the weighted normalized ratings ( $y_{ij}$ ) by

$$y_{ij} = w_j r_{ij}; \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \quad (9)$$

$$A^+ = (y_1^+, y_2^+, \dots, y_n^+); \quad A^- = (y_1^-, y_2^-, \dots, y_n^-) \quad (10)$$

where

$$y_j^+ = \begin{cases} \max_i y_{ij}, & \text{if } j \text{ is a benefit attribute} \\ \min_i y_{ij}, & \text{if } j \text{ is a cost attribute} \end{cases};$$

$$y_j^- = \begin{cases} \min_i y_{ij}, & \text{if } j \text{ is a benefit attribute} \\ \max_i y_{ij}, & \text{if } j \text{ is a cost attribute} \end{cases};$$

$$j = 1, 2, \dots, n. \quad (11)$$

The distance between alternatives  $A_i$  and the positive ideal solution and the negative ideal solution can be calculated respectively by

$$D_i^+ = \sqrt{\sum_{j=1}^n (y_i^+ - y_{ij})^2}; \quad D_i^- = \sqrt{\sum_{j=1}^n (y_{ij} - y_i^-)^2};$$

$$i = 1, 2, \dots, m. \quad (12)$$

The overall preference value of each alternative ( $V_i$ ) is given by

$$V_i = \frac{D_i^-}{D_i^+ + D_i^-}; \quad i = 1, 2, \dots, m. \quad (13)$$

The greater the value ( $V_i$ ), the more preferred the alternative ( $A_i$ ).

### 3.5 Differences between MADM Methods

The main differences between the four methods described above lie in (a) the normalization process for comparing all performance ratings on a common scale, and (b) the aggregation of the normalized decision matrix and weighting vector for obtaining an overall preference value for each alternative. Due to these structural differences, the ranking outcome produced by the four methods may not always be consistent for a given decision matrix and weighting vector. In fact, the empirical study presented in this paper shows that the rankings are so different that the relative effectiveness of the methods used needs to be examined.

#### 4. Validity of MADM Methods

In higher education selection research, particularly for admission decisions, the validity issue has been conventionally addressed along the lines of the predictive validity of the selection instruments (criteria). A typical study of the predictive validity is to examine the correlation between the selected students' results on the chosen selection criteria and an indicator of subsequent academic performance (Wolming, 1999). The predictive validity study may help make admission or selection procedures more efficient and effective (Powers and Lehman, 1983; Dobson et al., 1999; Lievens and Coetsier, 2002). However, the selection criteria used in higher education admission processes varies widely among programs and no consistent conclusions can be reached on the predictive values of these criteria (Wilson, 1999). This may partly be due to the fact that the predictive validity of the selection instruments is not in itself sufficient for an assessment of the validity of a selection, although it can be a critical factor (Wolming, 1999). In some cases, the predictive values of the selection criteria may be affected by background variables such as gender and race (Dunlap et al., 1998).

In this paper, prediction is not the stated purpose for the scholarship student selection problem, partly because we do not have sufficient empirical data to conduct a predictive validity study. Thus, the selection of scholarship students is made on the grounds of the candidates' merits (performance ratings) assessed by an interview process, based on a given set of criteria in accordance with the requirements of the industry-based learning program for which the scholarships are designed. Although the construct validity of an interview can be examined with sufficient empirical data (Moscato, 2000), we still need to select among eligible MADM methods for the overall ranking of the candidates. As such, in this paper we focus the validity issue on the selection of suitable MADM methods, given the selection criteria and candidates' performance ratings.

MADM research on the validity issue has focused on the problem of selecting among MADM methods under various decision contexts along two lines of development: (a) experimental comparisons of MADM methods for examining their appropriateness of use and/or theoretical validity (Zanakis et al., 1998), and (b) method selection procedures for specific characteristics of the decision problem and distinct features of available methods in the form of decision support systems (Ozernoy, 1992; Siskos and Spyridakos, 1999) or as general selection principles (Guitouni and Martel, 1998). Studies of experimental comparisons can reveal when and by how much the solutions of different MADM methods may differ under various problem settings. However, these experimental studies cannot result in a set of guidelines that enable a decision maker to select a proper MADM method for a specific problem (Ozernoy, 1992). When the type of the

decision problem or required preference information can be explicitly specified, the method selection procedures equipped with decision support capabilities would facilitate the selection of a proper MADM method that is operational from the decision maker's point of view. However, the method selection procedures may not always make a clear unequivocal choice (Guitouni and Martel, 1998) among suitable methods, in particular among methods of the same category such as MAVT-based compensatory methods to be used for the scholarship student selection problem. Due to the implicit and explicit assumptions of these method selection procedures, the applicability of the methods selected remains uncertain (Nijkamp and Blaas, 1994), as evidenced by the fact that these selection procedures do not normally examine the validity of the decision outcome.

Despite the significant development in MADM research, the validity of the ranking outcome remains an open issue. This is mainly due to the fact that there are no objective measures of a decision maker's values (e.g. scales, ratings, weights) to which the results of an MADM method can be compared (Hobbs and Meier, 2000). In addition, there is no such thing as the "right answer" as the concept of an optimum does not exist in a multicriteria framework (Belton and Stewart, 2002). This implies that the "true" cardinal ranking of alternatives is not known or cannot be obtained in a universally accepted way. To address this problem for the MADM methods used in this paper, we develop a new empirical validity procedure for selecting the most valid ranking outcome which has the minimum expected distance or deviation from the unknown true one. The true ranking outcome is assumed to be associated with the true weighting vector, which is not known but its possible bounds can be set. Each of these allowable weighting vectors represents a feasible solution (ranking outcome) to the problem. As a result, for a given weighting vector (e.g. equal weights) used for the MADM selection problem, we should choose the ranking outcome of an MADM method, which has a minimum average deviation from all the feasible solutions of the method, as compared to that of other methods. This implies that the ranking outcome selected will have a minimum expected value loss if the true weighting vector is not the one used. This validity procedure for obtaining the average expected deviation of the ranking outcome produced by an MADM method will be illustrated with the empirical study.

#### 5. The Empirical Study

To illustrate how the validity procedure can be used to select the most valid ranking outcome when using the four MADM methods to solve the scholarship student selection problem, we examine a recently completed selection round. A total of 57 candidates attended the interview. The result of this interview process constituted the decision matrix  $X$ ,

as expressed in (1) with  $m = 57$  and  $n = 8$ . With the use of equal weights, the weighting vector  $W$  used was (0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125) which sum up to 1 (i.e.  $\sum_{j=1}^n w_j = 1$ ). The rankings of the candidates obtained by the four methods were not consistent, leading to some decision difficulties. As an illustration, Table 1 shows the top 10 rankings with the four methods. For easy comparison, candidates  $A_i$  ( $i = 1, 2, \dots, 57$ ) are denoted in order of their overall preference value  $V_i$  ( $i = 1, 2, \dots, 57$ ) by the TS method. If there were only 10 candidates to be selected,  $A_6$  would not be selected using TOPSIS, and  $A_{11}$  would not be selected using TS, SAW, or WP. For most decision situations where the number of candidates to be selected varies, there will be some candidates being included using some methods, and being excluded with other methods. To examine whether the rankings of the four methods differ significantly, Friedman's nonparametric analysis of variance by ranks was conducted. The result indicated a significant variation ( $\chi^2 = 193.31$ ,  $df = 3$ ,  $p = 0.000$ ).

To select among the inconsistent rankings in Table 1, we apply the validity procedure to each method individually for obtaining the average expected deviation of its ranking outcome. Although the scholarship student selection problem uses equal weights for all attributes (selection criteria) with the reasons given in Section 2, a perception commonly shared by the stakeholders is that no single attribute is more than twice important as any other attributes. This perception makes the possible upper bound of attribute weights be 2, when the lower bound is set to 1. To construct the space of all feasible solutions, we use an increment of 0.05 for the weight changes between 1 and 2. This increment level is chosen based on two reasons. First, any smaller value would not make any practical sense in terms of the relativeness of attribute weights. That is, with a lower bound being 1, the increment level 0.05 represents the smallest value that makes meaningful implications in terms of the relative difference between any two attribute weights (e.g. between 1.00 and 1.05). Second, the

computation required (i.e. the number of weighting vectors considered) is manageable.

Based on the problem settings described above, the validity procedure for obtaining the average expected deviation of the ranking outcome produced by an MADM method can be carried out as follows:

- Step 1: Obtain the overall preference values  $V_i^*$  ( $i = 1, 2, \dots, 57$ ) for 57 candidates, using equal weights for 8 attributes (i.e.  $w_j = 1/8$ ,  $j = 1, 2, \dots, 8$ ).
- Step 2: Assign a weight value of 1 ( $e_j = 1$ ,  $j = 1, 2, \dots, 8$ ) to all attributes  $C_j$  ( $j = 1, 2, \dots, 8$ ).
- Step 3: Increase the weight value for one attribute  $C_j$  at a time ( $j = 1, 2, \dots, 8$ ) by an increment of 0.05 until the value reaches the upper bound of 2. This results in  $8^{21}$  weighting vectors  $(e_1^t, e_2^t, \dots, e_8^t)$ ;  $t = 1, 2, \dots, 8^{21}$ .
- Step 4: Normalize each weighting vector obtained at Step 3 by  $w_j^t = e_j^t / \sum_{j=1}^8 e_j^t$  to satisfy  $\sum_{j=1}^8 w_j^t = 1$ .
- Step 5: Obtain the overall preference values  $V_i^t$  ( $i = 1, 2, \dots, 57$ ) for 57 candidates, using each normalized weighting vector  $((w_1^t, w_2^t, \dots, w_8^t)$  obtained at step 4.
- Step 6: Repeat step 4 and step 5 for each weighting vector obtained at Step 3 until all  $8^{21}$  corresponding preference values  $V_i^t$  ( $i = 1, 2, \dots, 57$ ;  $t = 1, 2, \dots, 8^{21}$ ) are obtained.
- Step 7: Calculate the average deviation by

$$d = \sum_{t=1}^{8^{21}} \sqrt{\sum_{i=1}^8 (V_i^t - V_i^*)^2} / 8^{21} \quad (14)$$

Based on the principle of insufficient reason, (14) assumes a uniform probability distribution over all possible weighting vectors defined by the specified bounds. The ranking outcome  $V_i^*$ , which has the smallest average deviation in comparison with that of other methods, is the one to be selected.

**Table 1. Comparison of top 10 rankings between four MADM methods**

Ranking	TS		SAW		WP		TOPSIS	
	$A_i$	$V_i$	$A_i$	$V_i$	$A_i$	$V_i$	$A_i$	$V_i$
1	$A_1$	1.000	$A_1$	1.000	$A_1$	1.000	$A_1$	1.000
2	$A_2$	0.975	$A_2$	0.975	$A_2$	0.973	$A_2$	0.915
3	$A_3$	0.950	$A_3$	0.950	$A_3$	0.946	$A_3$	0.892
4	$A_4$	0.925	$A_4$	0.925	$A_4$	0.920	$A_4$	0.868
5	$A_5$	0.900	$A_5$	0.900	$A_5$	0.895	$A_7$	0.855
6	$A_6$	0.900	$A_6$	0.900	$A_7$	0.895	$A_5$	0.847
7	$A_7$	0.900	$A_7$	0.900	$A_9$	0.895	$A_9$	0.846
8	$A_8$	0.900	$A_8$	0.900	$A_6$	0.887	$A_{10}$	0.831
9	$A_9$	0.900	$A_9$	0.900	$A_8$	0.887	$A_{11}$	0.815
10	$A_{10}$	0.875	$A_{10}$	0.875	$A_{10}$	0.870	$A_8$	0.811

**Table 2. Average expected value loss of four MADM methods under various settings**

Upper bound	Increment	TS	SAW	WP	TOPSIS
1.5	0.05	0.099	0.089	0.131	0.165
1.5	0.10	0.099	0.088	0.130	0.159
2.0	0.05	0.107	0.098	0.137	0.178
2.0	0.10	0.106	0.098	0.137	0.175
2.5	0.10	0.131	0.104	0.151	0.193
2.5	0.20	0.136	0.108	0.159	0.183
3.0	0.10	0.156	0.133	0.185	0.212
3.0	0.20	0.152	0.130	0.177	0.203

By applying the TS, SAW, WP, and TOPSIS methods individually to the validity procedure presented above, the average expected deviations or value losses of the ranking outcome by these methods are 0.106, 0.098, 0.137 and 0.175 respectively. This result indicates that the ranking outcome of SAW has a minimum expected value loss. To examine the effect of various sets of bounds (lower bound being 1.0 and upper bound being between 1.5 and 3.0) and increments (between 0.05 and 0.2) on the selection of four MADM methods, further experimental study has been conducted. The bound values are chosen to sufficiently examine whether the level of attribute weight variation will affect the degree of ranking deviations. The various increment values are used to examine whether the level of ranking deviations is consistent within a given weight bound range, independent of the number of all possible values considered. Table 2 shows some representative results, which are consistent in terms of the average expected value losses by the four methods. This suggests that SAW should be used for the scholarship student selection problem.

Clearly, the validity procedure can be easily adjusted and applied to any given weighting vector and any set of bounds on the weights. In other practical applications, the weighting vector can be obtained by a reliable weighting method commonly accepted by the stakeholders (not necessarily using equal weights) for determining the ranking outcome. The weight bounds can be set based on the overall perception of the stakeholders about the degree of relative importance of the attributes as a whole. In decision situations where attribute weights assessed by individual stakeholders are available, the weight bounds can be based on the range of stakeholder weights.

## 6. Conclusion

Selecting scholarship students involves a candidate assessment process based on multiple selection criteria. MADM has shown advantages in ranking the performance of a set of decision alternatives with respect to multiple attributes in various decision contexts. In this paper, we have

formulated a scholarship student selection problem as an MADM problem and explored how MAVT-based MADM methods can be used to assess scholarship candidates. Given the selection criteria and candidates' performance ratings, an empirical validity procedure has been developed to deal with the ranking inconsistency problem resulting from the use of different MADM methods. Among the four methods (TS, SAW, WP and TOPSIS) suitable for the selection problem formulated, SAW is the most appropriate method as its ranking outcome will have a minimum expected value loss when using equal weights for the selection criteria. With its simplicity in both concept and computation, the validity procedure can be applied to the general multiattribute selection problem solvable by MAVT-based MADM methods, particularly in situations where attribute weighting is a great concern and no reliable subjective weights can be obtained. It is particularly suited to large-scale selection problems where the ranking outcome produced by different methods differs significantly.

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