MONASH University

Information Technology

FIT5186 Intelligent Systems

Lecture 12

Decision Trees and Decision Making Using Sample Information

Learning Objectives

- Understand
  - the construction and evaluation of decision trees as a decision tool
  - the use of sample information in decision making
  - the principles of Bayes' Theorem for decision making

Lecture 11 Review

- A decision problem is characterised by decision alternatives, states of nature, and resulting payoffs.
- The decision alternatives are the different possible actions the decision maker can take.
- The states of nature refer to future events, not under the control of the decision maker, which may occur.
- A payoff is the consequence resulting from a specific combination of a decision alternative and a state of nature.
- A Payoff Table or Payoff Matrix shows payoffs for all combinations of decision alternatives and states of nature.

Lecture 11 Review

Decision Making under Uncertainty

- Decision rules without probability information
  - Maximax
  - Maximin
  - Minimax Regret
- Decision rules with probability information
  - Expected Monetary Value (EMV)
  - Expected Regret or Opportunity Loss (EOL)
- Expected Value of Perfect Information (EVPI)

EVPI = Expected Value with Perfect Information – Maximum EMV

Outline of Lecture 12

- Decision Trees
- Decision Making Using Sample Information
  - Expected Value of Sample Information
- Bayes' Theorem
- Using TreePlan*

* *Slides included for information but not discussed in lecture.*

Decision Trees

- A **decision tree** is a graphical representation of a decision problem.

  Decision trees use the following convention:

- Square nodes □ = **Decision nodes**
  - Correspond to the decision alternatives.
- Round nodes ○ = **Event nodes** or chance nodes
  - Correspond to the states of nature.
- **Branch**
  - The branches leaving each decision (square) node represent the different decision alternatives.
  - The branches leaving each event (round) node represent the possible states of nature or the possible outcomes of an uncertain event.
  - The various branches in a decision tree end at a **terminal node** (also called **leaf**) which corresponds to a payoff obtained from the set of branches leading to the leaf.

Evaluating a Decision Tree

- Decision trees enable us to represent sequential decision problems in a way that allows for systematic evaluation.
- <u>The evaluation process</u> (referred to as “<u>rolling back</u>” or “folding back” a decision tree, or backward induction) requires us to <u>work backward from the leaves</u> (payoffs) through the decision tree <u>to the decision node</u>.
- By evaluating the payoff of the decision tree at each decision node using any of the decision rules (discussed in Lecture 11), sub-optimal courses of action are identified and discarded.
  - <u>The EMV decision rule</u> is most often used with decision trees.
- By convention, when the decision tree is ‘pruned’, discarded courses of action are indicated by //.

Example 1: Magnolia Inns (from Lecture 11)

Magnolia Inns
Real Estate Acquisition Analysis

	Parcel of Land Near Location	
	A	B
Current purchase price	$18	$12
Present value of future cash flows if hotel and airport are built at this location	$31	$23
Present value of future sales price of parcel if the airport is not built at this location	$6	$4

(Note: All values are in millions of dollars.)

Decision Alternatives:

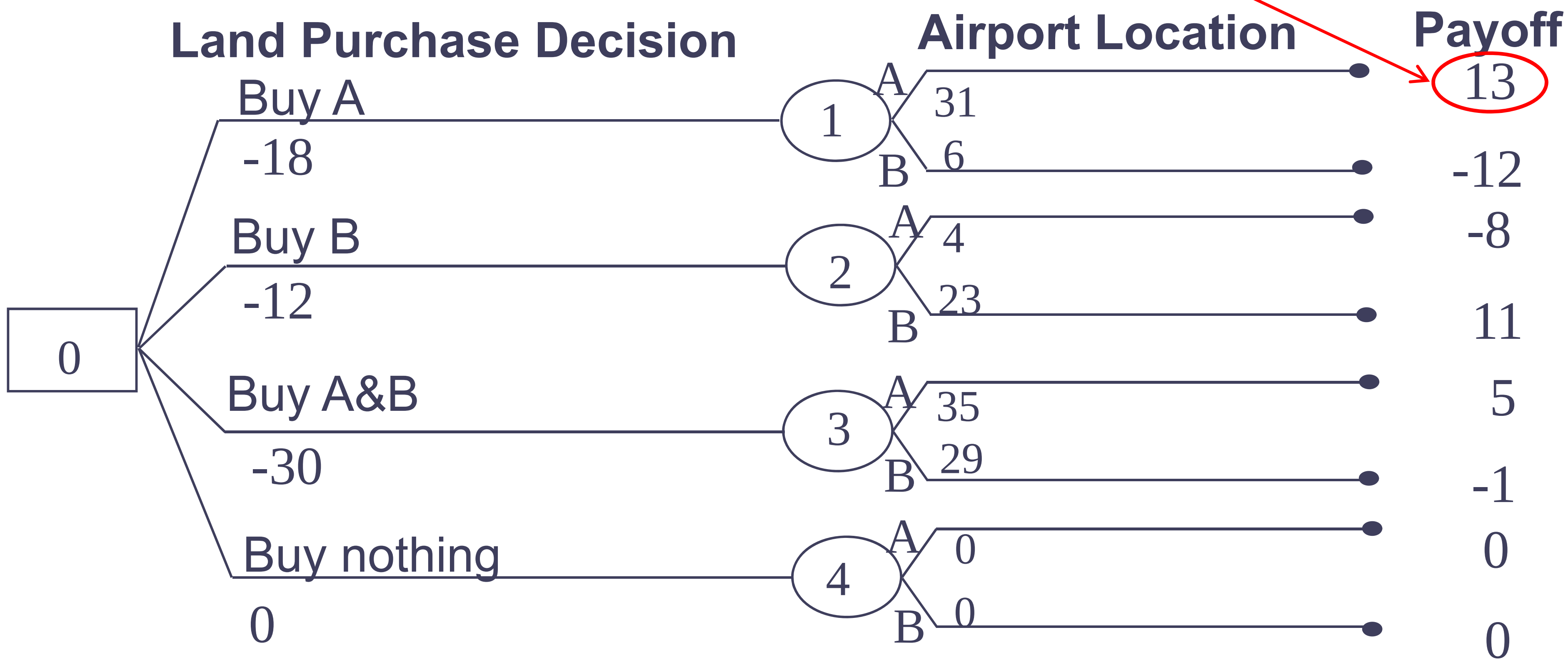
1) Buy parcel of land near location A
2) Buy parcel of land near location B
3) Buy both parcels
4) Buy nothing

Possible States of Nature

1) New airport is built at location A
2) New airport is built at location B

Payoff Matrix and the Decision Tree

	F	G	H
1		**Payoff Matrix**	
2			
3	**Land Purchased**	**Airport is Built at Location**	
4	**at Location(s)**	**A**	**B**
5	**A**	\$13 = \$31 - \$18	-\$12 = \$6 - \$18
6	**B**	-\$8 = \$4 - \$12	\$11 = \$23 - \$12
7	**A&B**	\$5 = \$31 + \$4 - \$18 - \$12	-\$1 = \$23 + \$6 - \$18 - \$12
8	**None**	\$0	\$0



Expected Monetary Value (EMV)

Probability information for the future states of nature is available.

	A	B	C	D	E
1		Payoff Matrix &			
2		EMV Decision Rule			
3	Land Purchased	Airport is Built at Location			
4	at Location(s)	A	B	EMV	
5	A	$13	($12)	($2.0)	
6	B	($8)	$11	$3.4	
7	A&B	$5	($1)	$1.4	
8	None	$0	$0	$0.0	
9					
10	Probability	0.4	0.6		
11					
12	D5	=SUMPRODUCT($B$10:$C$10,B5:C5)			

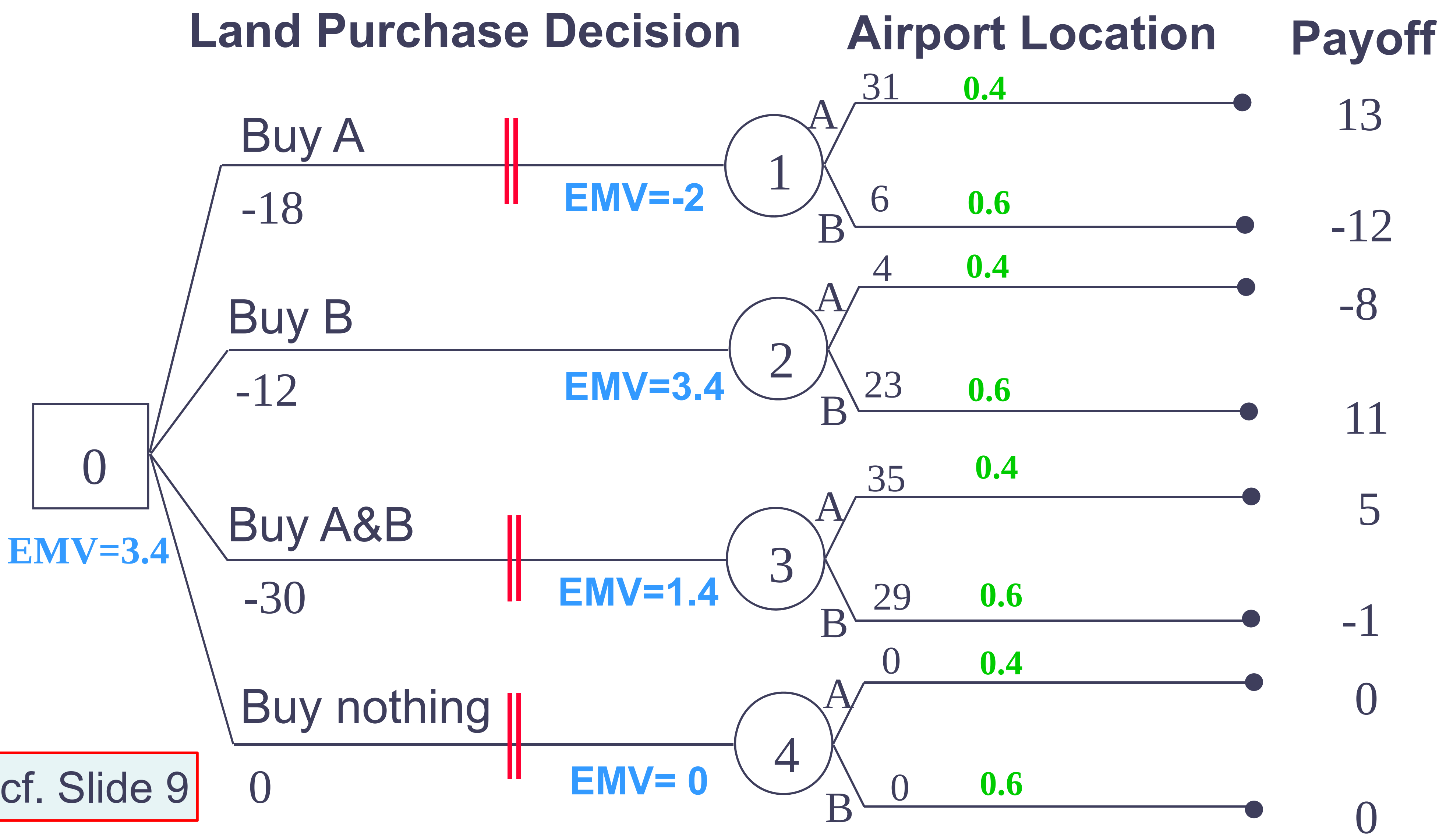
$$EMV_i = \sum_j r_{ij} p_j$$

($13×0.4) + (–$12×0.6) = –$2.0
(–$8×0.4) + ($11×0.6) = $3.4
($5×0.4) + (–$1×0.6) = $1.4

The Decision Tree with Probability Information

The probabilities on the branches at any event node must always sum to 1.

Evaluating the Decision Tree – Rolling Back



cf. Slide 9

- See file Lecture 12 Example 1.xls for the decision tree.

Example 2: (from Lecture 11)

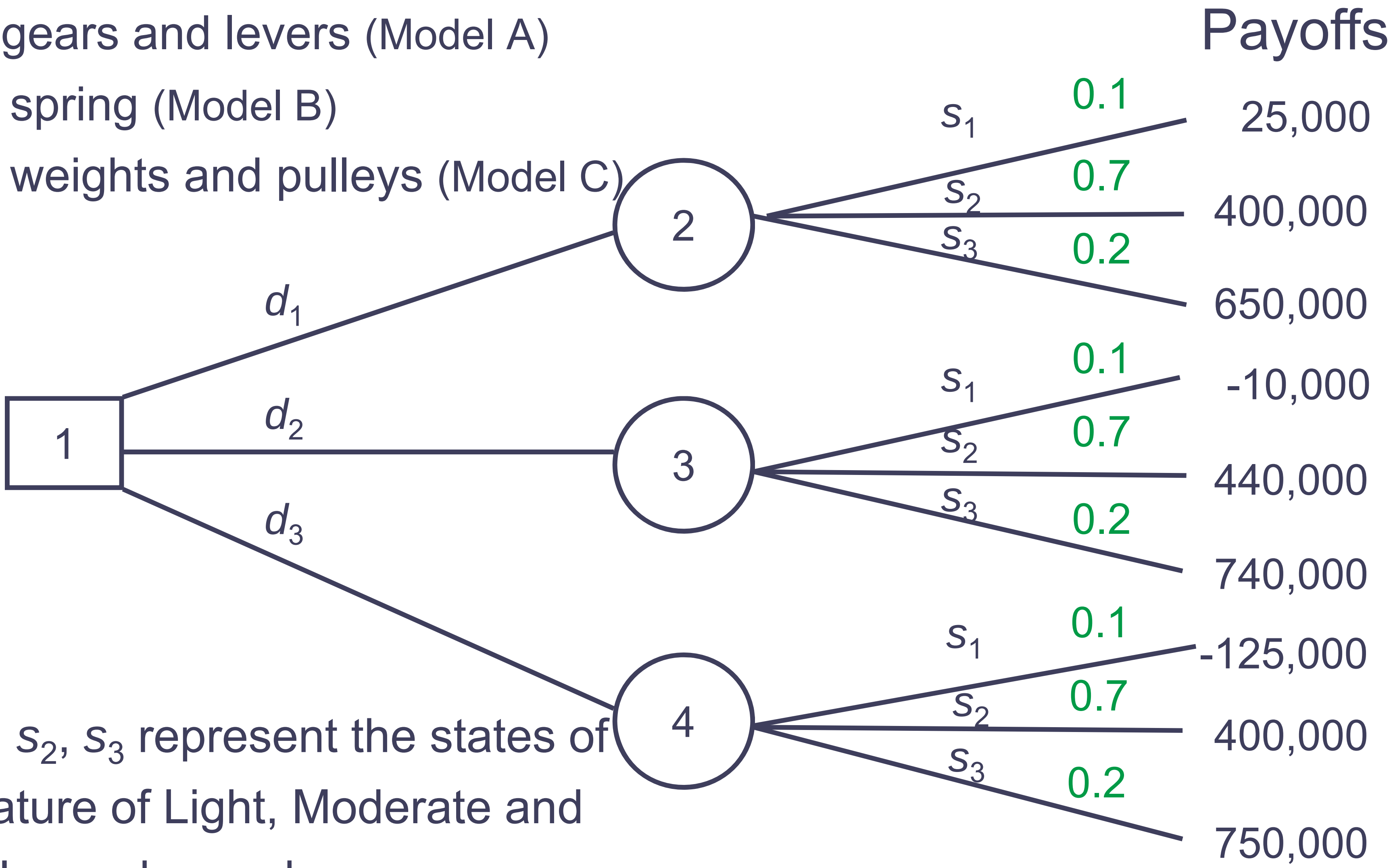
- A company is choosing a motorised mechanism for a new toy.
- Assuming that the probability of Light demand is 0.1, Moderate demand is 0.7, and Heavy demand is 0.2, the expected value under each choice is calculated below.

		Action: Choice of Mechanism		
Event	Probability	Gears and Levers	Spring	Weights and Pulleys
Light	0.10	25,000	-10,000	-125,000
Moderate	0.70	400,000	440,000	400,000
Heavy	0.20	650,000	740,000	750,000
	Expected Payoff	412,500	455,000	417,500

- The action of Spring mechanism has the largest payoff and should be chosen under the EMV decision rule.

Example 2: The Decision Tree

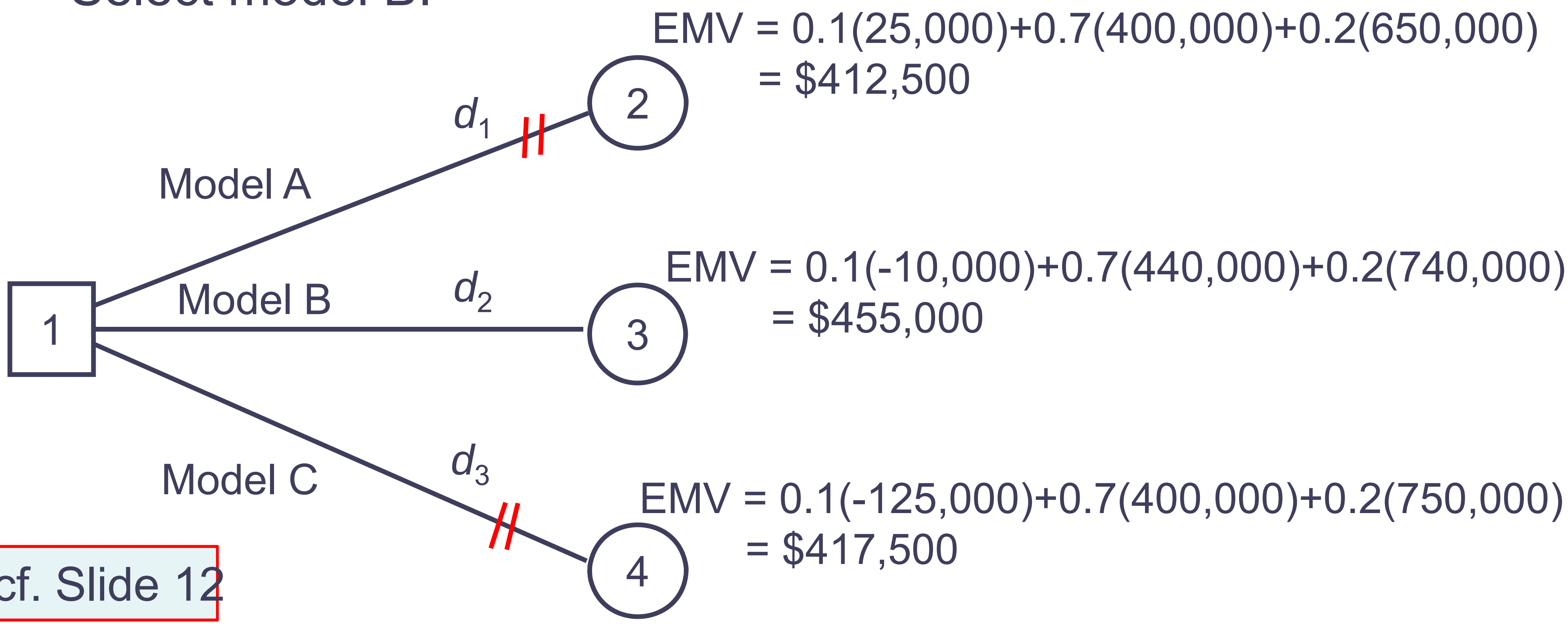
- d_1, d_2, d_3 represent the decision alternatives of models.
  d_1= gears and levers (Model A)
  d_2 = spring (Model B)
  d_3 = weights and pulleys (Model C)



- s_1, s_2, s_3 represent the states of nature of Light, Moderate and Heavy demand.
- $P(s_1)$= 0.1, $P(s_2)$= 0.7, $P(s_3)$= 0.

Example 2: Evaluating the Decision Tree

- Calculate the Expect Monetary Value (EMV) for each decision.
- At the decision node, select the alternative with the highest EMV.
- Select model B.



cf. Slide 12

// is used to indicate pruning.

Example 3: Ponderosa Record Company

- This example is from Lapin & Whisler (2002), Chapter 5.
- The Ponderosa Record Company has signed a contract with a new band. A recording has been made and the company must decide whether or not to market the recording nationally.
- The CD may be marketed nationally without testing the market, in which case 50,000 CDs will be made.
- Alternatively, the company may produce a limited run of 5,000 CDs and test the market locally. If the product is a success (meaning all CDs are sold), the CD will then be marketed nationally with additional 45,000 CDs produced.
- Probabilities and payoffs are given on the following slide.

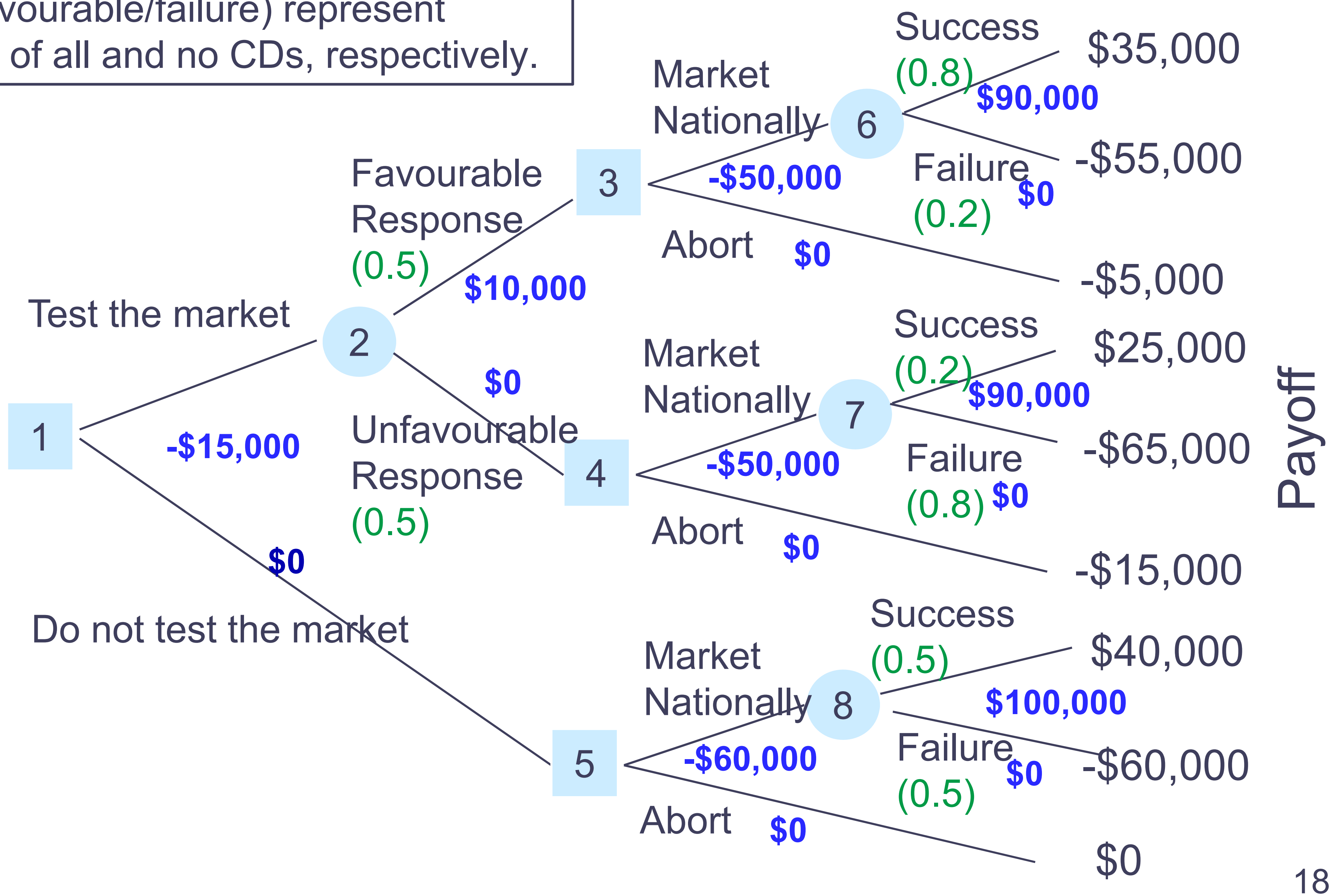
Example 3: The Decision Alternatives

- Without testing the market, the band's CD has a 50% chance of success. If the CD is successful locally (50% chance), this is a good indicator of success nationally estimated to be 80%.
- If the CD fails locally (50% chance), the chance of success nationally is only 20%.
- The cost of producing CDs are a $5,000 fixed cost + $1 variable cost per CD.
- The company receives $2 for each CD sold.
- The company pays a fixed fee of $5,000 to the band.
- **The decision alternatives:**

  The company has to decide whether to press 50,000 units without testing the market, or whether to press 5,000 first to test the market and then decide to enter the national market or decide not to enter it, depending on sales in the test market. → **decision nodes**
- The next slide shows the decision tree with partial cash flows, net payoffs and probabilities on the branches.
  - The net payoffs at the terminal node (leaf) are obtained by adding the partial cash flows on the set of branches leading to the terminal node.

Example 3: The Decision Tree with Payoffs

The **events** (favourable/success) and (unfavourable/failure) represent sales of all and no CDs, respectively.



Example 3: Calculating the Payoffs

- **Decision node 1**

  Test the market -\$5,000 (payment to band)

  -\$5,000 (fixed cost of production)

  -\$5,000 (variable costs of 5,000 CDs at \$1 each)

  Total: **-\$15,000**

  Don't test the market **\$0**

- **Event node 2** (local market)

  Favourable **\$10,000** (sales of all 5,000 CDs at \$2 each) 50% chance (success)

  Unfavourable **\$0** (no sales) 50% chance (failure)

- **Decision node 5**

  Market nationally (without test the market) -\$5,000 (payment to band)

  -\$5,000 (fixed cost of production)

  -\$50,000 (variable costs of 50,000 CDs at \$1 each)

  Total: **-\$60,000**

  Abort **\$0**

Example 3: Calculating the Payoffs (continued)

- **Decision nodes 3 and 4**

  Market nationally -$5,000 (fixed cost of production)
  -$45,000 (variable costs of 45,000 CDs at $1 each)
  Total: **-$50,000**

  Abort **$0**

- **Event nodes 6 and 7** (national market)

			local market: success	local market: failure
Success	**$90,000**	(sales of all 45,000 CDs at $2 each)	80%	20%
Failure	**$0**	(no sales)	20%	80%

- **Event node 8** (national market)

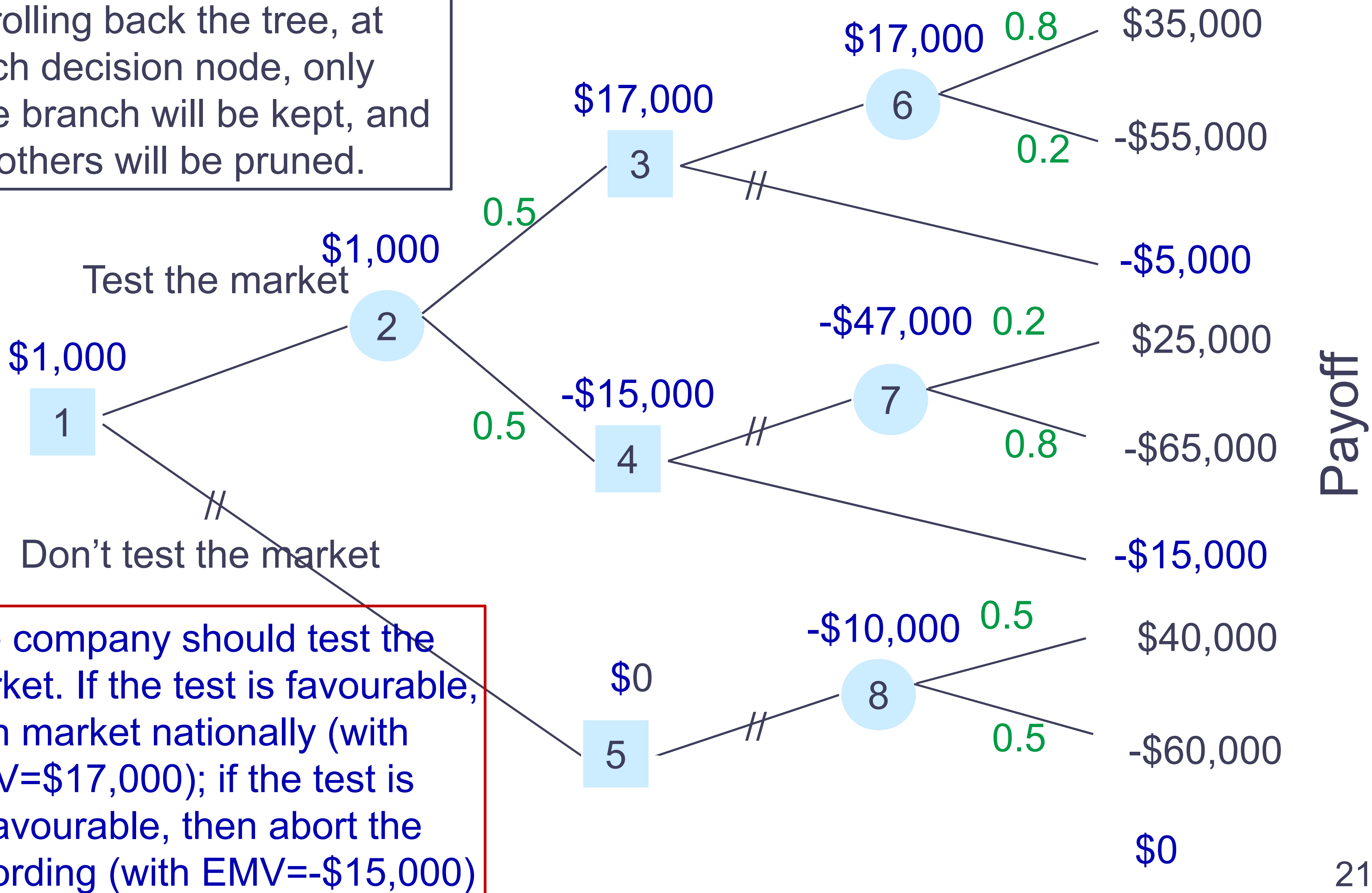
  Success **$100,000** (sales of all 50,000 CDs at $2 each) 50% chance
  Failure **$0** (no sales) 50% chance

- The net payoffs at the terminal node (leaf) are computed by adding the partial cash flows on the set of branches leading to the terminal node.

  e.g. $35,000 = – $15,000 + $10,000 – $50,000 + $90,000

Example 3: Evaluating the Decision Tree

In rolling back the tree, at each decision node, only one branch will be kept, and all others will be pruned.



The company should test the market. If the test is favourable, then market nationally (with EMV=$17,000); if the test is unfavourable, then abort the recording (with EMV=-$15,000)

Using Sample Information in Decision Making

- We have so far assumed that the probabilities of future events occurring are completely accurate.
- We now look at decision making when there is some uncertainty in survey or **sample information**.
- We can sometimes obtain additional sample information about the possible outcomes of decisions (e.g. by surveys or tests) before the decisions are made.
- This sample information allows us to refine probability estimates for the possible states of nature that are associated with various survey results.

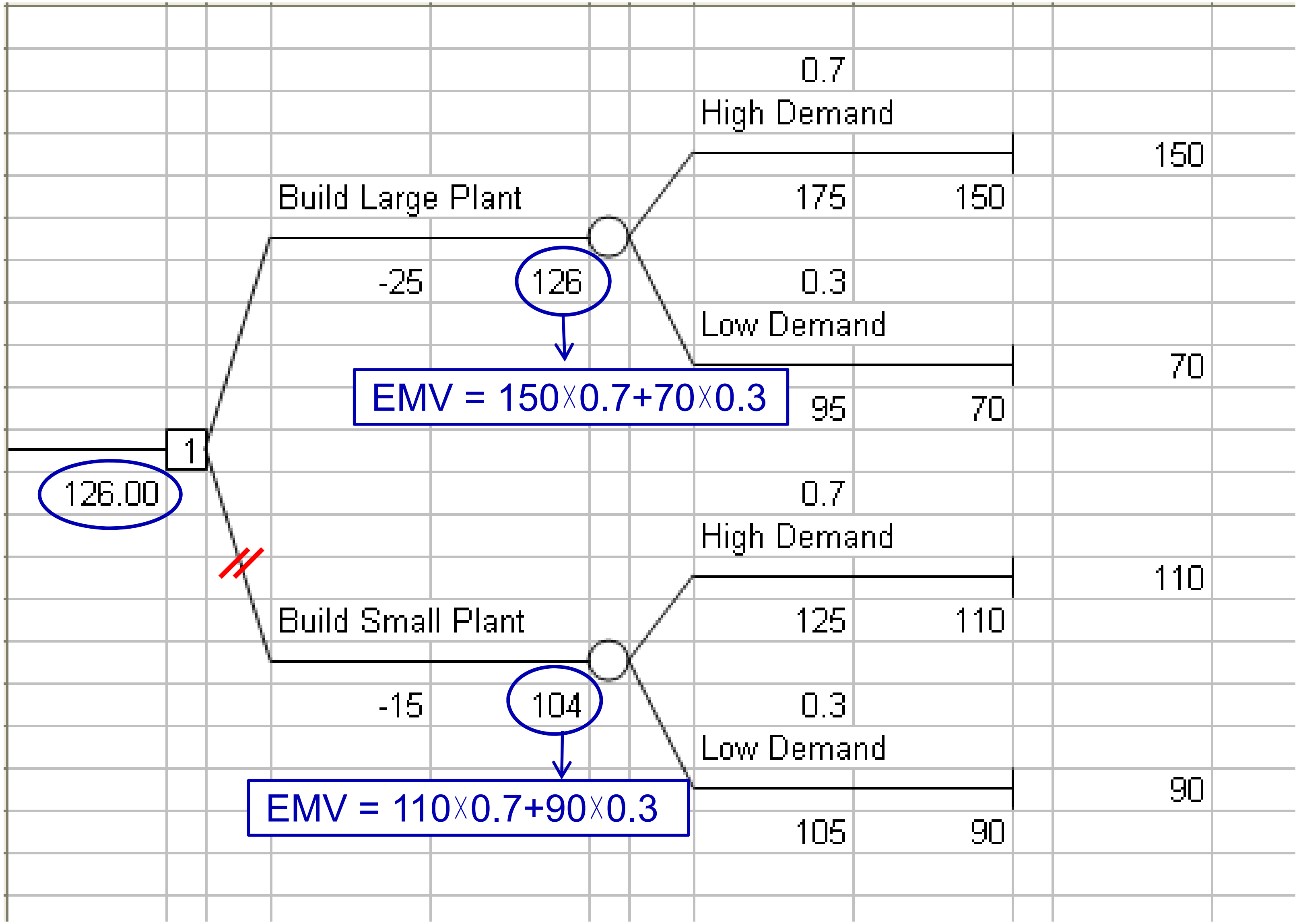
Example 4: Colonial Motors

- Colonial Motors (CM) needs to determine whether to build a large or small plant for a new car it is developing.
- The cost of constructing a large plant is $25 million and the cost of constructing a small plant is $15 million.
- CM believes a 70% chance exists that demand for the new car will be high and a 30% chance that it will be low.
- The payoffs (in millions of dollars) are summarised below.

	Demand	
Factory Size	High	Low
Large	$175	$95
Small	$125	$105

- See file Lecture 12 Example 4.xls for the decision tree.

Example 4: Decision Tree with No Sampling



Example 4: Including Sample Information

- Before making a decision, suppose CM conducts a consumer attitude survey (with zero cost).
- The survey can indicate favourable or unfavourable attitudes toward the new car. Assume:

  P(favorable response) = 0.67 (i.e. 2/3)
  P(unfavorable response) = 0.33 (i.e. 1/3)

> Probability estimates without survey:
> P(high demand) = 0.7
> P(low demand) = 0.3

- If the survey response is favourable, this should increase CM's belief that demand will be high. Assume:

Given "favourable response", the probability of "high demand" is 0.9.

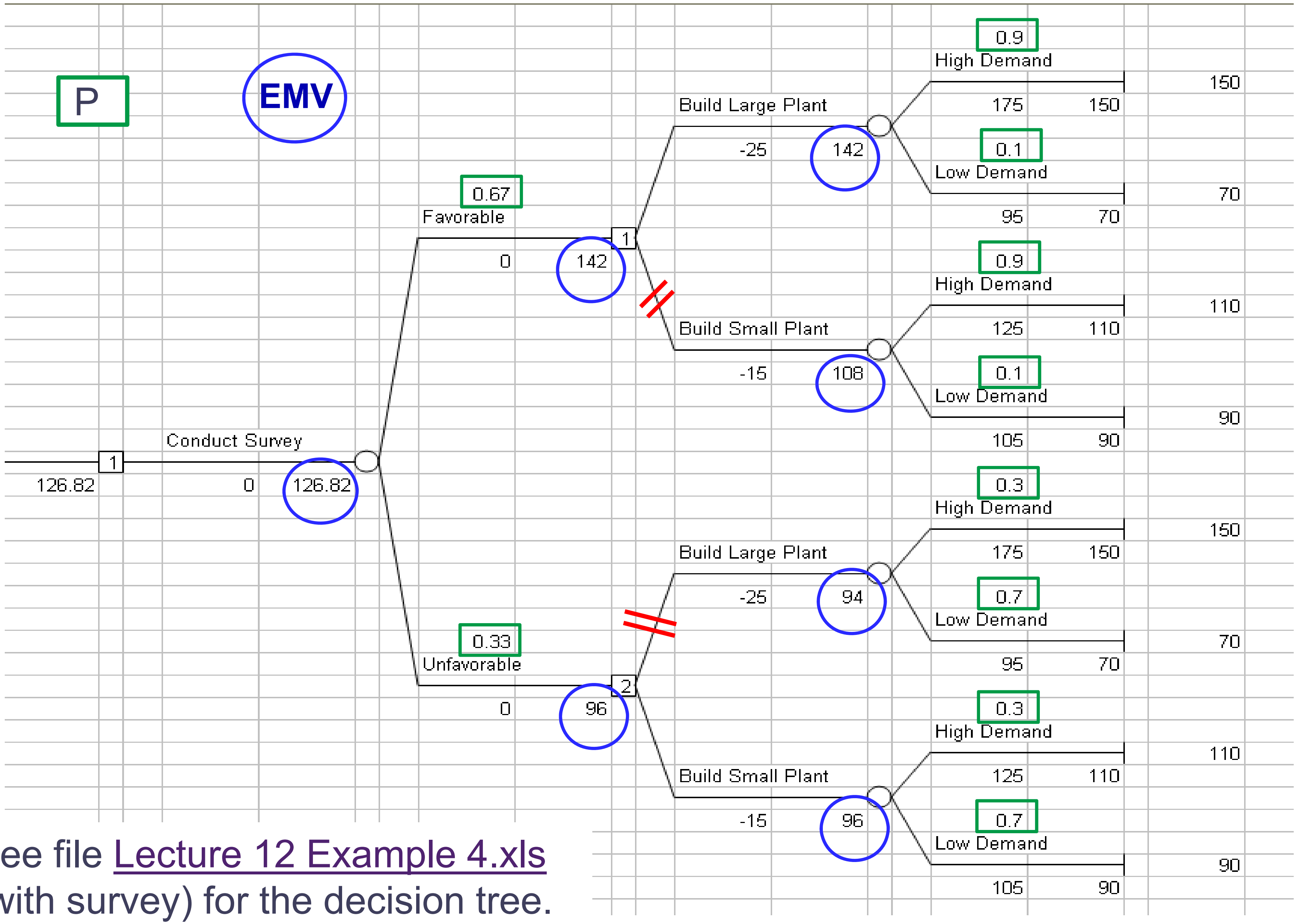
P(high demand | favourable response) = 0.9 → **Conditional probability**
P(low demand | favourable response) = 0.1 → **Conditional probability**

- If the survey response is unfavourable, this should increase CM's belief that demand will be low. Assume:

Given "unfavourable response", the probability of "low demand" is 0.7.

P(low demand | unfavourable response)= 0.7 → **Conditional probability**
P(high demand | unfavourable response)= 0.3 → **Conditional probability**

Example 4: Decision Tree with Sample Info.



See file Lecture 12 Example 4.xls (with survey) for the decision tree.

The Expected Value of Sample Information

- How much should CM be willing to pay to conduct the consumer attitude survey?

$$\text{Expected Value of Sample Information} = \text{Expected Value with Sample Information} - \text{Expected Value without Sample Information}$$

- In the CM example,

  Expected Value of Sample Information (EVSI)

  = \$126.82 - \$126.00

  = \$0.82 million

Computing Probabilities Using Historical Data

- Let H = high demand L = low demand

  F = favourable response U = unfavourable response
- To complete the Example 4 decision tree with sample information (Slide 25), we need the values for the following six probabilities:

  P(F), P(U), P(H | F), P(L| F), P(H | U), P(L | U).
- So far we assumed that the conditional probabilities were assigned subjectively by CM.
- However, this company may have been in business for many years and completed many market surveys and have data available from which they can compute these probabilities.
- The following slides show how CM can use the joint probability table obtained from historical data to compute the six probabilities required.

Computing Conditional Probabilities

- Conditional probabilities (like those in the CM example) are often computed from joint probability tables.

	High Demand (H)	Low Demand (L)	Total
Favorable Response (F)	0.600	0.067	0.667
Unfavorable Response (U)	0.100	0.233	0.333
Total	0.700	0.300	1.000

- The joint probabilities indicate that

  Of all the new car models CM developed, 60% received a favourable survey response and subsequently had high demand.

$$P(F \cap H) = 0.600,\ P(F \cap L) = 0.067$$

$$P(U \cap H) = 0.100,\ P(U \cap L) = 0.233$$

- The marginal probabilities indicate:

$$P(H) = 0.700, \quad P(L) = 0.300$$

$$P(F) = 0.667, \quad P(U) = 0.333$$

Joint probabilities obtained from historical data

Because they are located in the *margins* of the table.

Computing Conditional Probabilities (cont'd)

Joint probabilities

	High Demand (H)	Low Demand (L)	Total
Favorable Response (F)	0.600	0.067	0.667
Unfavorable Response (U)	0.100	0.233	0.333
Total	0.700	0.300	1.000

- In general, $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Conditional probability formula

- So we have

$$\boxed{P(H \mid F) = \frac{P(H \cap F)}{P(F)} = \frac{0.60}{0.667} = 0.90}$$

$$\boxed{P(H \mid U) = \frac{P(H \cap U)}{P(U)} = \frac{0.10}{0.333} = 0.30}$$

$$\boxed{P(L \mid F) = \frac{P(L \cap F)}{P(F)} = \frac{0.067}{0.667} = 0.10}$$

$$\boxed{P(L \mid U) = \frac{P(L \cap U)}{P(U)} = \frac{0.233}{0.333} = 0.70}$$

Conditional Probabilities

For a Given Survey Response

	High Demand (H)	Low Demand (L)
Favourable Response (F)	0.900	0.100
Unfavourable Response (U)	0.300	0.700

Bayes' Theorem

- Bayes' Theorem provides another definition of conditional probability that is sometimes helpful. Here, A and B represent any 2 events and
  $\bar{A}$ means "not A" or the complement of A.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})}$$

$\overline{A}$ means "not A"

- For example, in Example 4, we want to find P(H|F) – i.e. the probability of a high demand given a favourable response.
- Using Bayes' theorem:

$$P(H \mid F) = \frac{P(F \mid H)P(H)}{P(F \mid H)P(H) + P(F \mid L)P(L)}$$

Note that $\overline{H} = L$

Bayes' Theorem and Posterior Probabilities

- Survey or test results (sample information) can be used to revise the probability estimates for future events (the states of nature).
- Prior to obtaining this information, the probability estimates for the events are called **prior probabilities**.
- With knowledge of **conditional probabilities** for the survey results, these prior probabilities can be revised by using **Bayes' Theorem**.
- The outcomes of this analysis are called **posterior probabilities** or branch probabilities for decision trees.

P(future event | survey result)

The given event is the survey result on which the revised probability for the future event is based. e.g. Slide 30: P(H|F) - the probability of a high demand given a favourable response.

Application of Bayes' Theorem

- Often we begin the probability analysis with initial or prior probabilities.
- Then, from a survey, sample, special report, market research, or a product test we obtain some additional information.
- Given this additional information, we calculate revised or posterior probabilities.
- Bayes' theorem provides the means for revising the prior probabilities.

Prior Probabilities → New Information → Application of Bayes' Theorem → Posterior Probabilities

Computing Posterior Probabilities

Posterior Probabilities Calculation

- Step 1: For each state of nature, multiply the prior probability by its conditional probability for the survey result – this gives the joint probabilities for the state of nature and the survey result.
- Step 2: Sum these joint probabilities over all states of nature – this gives the marginal probability for the survey result .
- Step 3: For each state, divide its joint probability by the marginal probability for the survey result – this gives the posterior probability.

Example 5: Oil Wildcatting

- This example is from Lapin & Whisler (2002), Chapter 3.
- Lucky Luke is an Oil Wildcatter (a person who searches for oil). Based on 20 years of experience he can estimate the probability of oil beneath Crockpot Dome. Let:

  A_1 = oil below Crockpot Dome

  A_2 = no oil below Crockpot Dome

- Prior Probabilities:

  Using his subjective judgment: $P(A_1) = 0.2$, $P(A_2) = 0.8$

  – Based on this subjective information, there is a 20% probability of finding oil.

Example 5: Conducting a Survey

New Information:

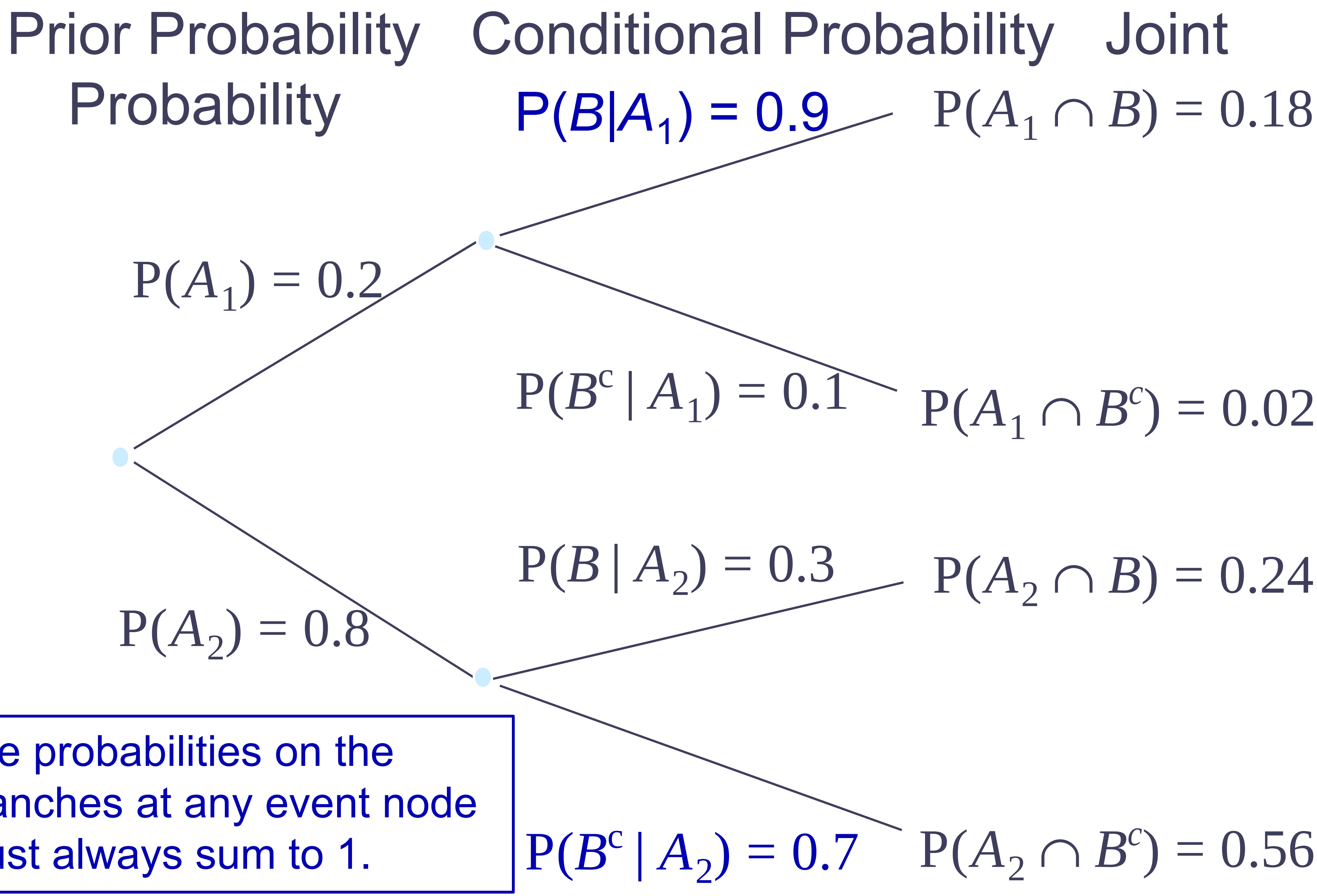
- Lucky Luke orders a seismic survey. The petroleum engineering consultant is 90% reliable in confirming oil when there actually is oil, but only 70% reliable in predicting that there is no oil when there actually is no oil.
- Let B and B^c be the events that the survey confirms oil and does not confirm oil respectively, then we can represent these conditional probabilities for the survey results (B and B^c) as:

$$P(B|A_1) = 0.9 \qquad P(B^c|A_2) = 0.7$$

(A_1=oil; A_2=no oil)

- We now consider how a positive/negative survey result affects the probability of finding oil (i.e. based on posterior probability).

Example 5: Probability Tree with a Survey



The probabilities on the branches at any event node must always sum to 1.

Bayes' Theorem

- To find the posterior probability that event A_i will occur given that event (e.g. survey result) B has occurred, we apply Bayes' theorem.

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

$$= \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \ldots + P(A_n)P(B|A_n)}$$

- Bayes' theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and their union is the entire sample space.

Example 5: Posterior Probabilities with a Survey Result

Posterior Probabilities

- Given that the survey result of the consultant confirms the existence of oil (i.e. event (survey result) ***B*** occurs), we revise the prior probabilities as follows:

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1)+P(A_2)P(B|A_2)} = \frac{(0.2)(0.9)}{(0.2)(0.9)+(0.8)(0.3)}$$

$$= 0.429 \approx 0.43$$

(A_1=oil; A_2=no oil)

- Consequently, when the survey for the presence of oil is positive, the probability of oil increases from 0.2 to 0.43.

- Note that we expect that a negative survey result (i.e. B^c occurs – does not confirm oil) would decrease the probability of the presence of oil.

Example 5: Calculating Posterior Probabilities

- The simplest way of calculating posterior probabilities P(A_1 | B) is to use the following tabular format.

B: survey result (confirms oil)

	States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
oil	A1	0.2	0.9	0.18	0.43
no oil	A2	0.8	0.3	0.24	0.57
				0.42	

	States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
oil	A1	$P(A_1)$	$\mathrm{P}(B \mid A_1)$	$\mathrm{P}(A_1 \cap B)$	$P(A_1 \mid B)$
no oil	A2	$P(A_2)$	$\mathrm{P}(B \mid A_2)$	$\mathrm{P}(A_2 \cap B)$	$P(A_2 \mid B)$
				$\mathrm{P}(B)$	

$$\mathrm{P}(A_1 \mid B) = \frac{\mathrm{P}(A_1 \cap B)}{\boxed{\mathrm{P}(B)}} = \frac{\mathrm{P}(A_1)\mathrm{P}(B \mid A_1)}{\mathrm{P}(A_1)\mathrm{P}(B \mid A_1) + \mathrm{P}(A_2)\mathrm{P}(B \mid A_2)} = \frac{0.18}{0.42} = 0.43$$

Example 5: Expected Value of Sample Information

- So what is the value of the petrol consultant's information?
- Let's assume that if Lucky Luke is successful in finding oil, then he will earn $100,000. If he drills and does not find oil then he loses $30,000.
- Assuming that he will not proceed if the survey result is negative, we see the expected returns after a positive test result. In this case, the expected value of sample information (EVSI) is $29,900.

=$25,900–(–$4,000)

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
A1	0.2	0.9	0.18	0.43
A2	0.8	0.3	0.24	0.57
			0.42	

See Slide 26

If oil found	$ 100,000			$ 100,000	EVSI
If no oil found	-$ 30,000			-$ 30,000	
Payoff	-$ 4,000			$ 25,900	$ 29,900

=($100,000 × 0.2) + (–$30,000 × 0.8)   =($100,000 × 0.43) + (–$30,000 × 0.57)

About TreePlan *

- *TreePlan* is an Excel add-in designed to draw and calculate decision trees.
- *TreePlan* is a *shareware* product developed by Dr. Mike Middleton at the Univ. of San Francisco and distributed with Ragsdale textbook at no charge to you.
- If you like this software package and plan to use for more than 30 days, you are expected to pay a nominal registration fee. Details on registration are available near the end of the *TreePlan* help file.
- See: Treeplan.xla, Treeplan.pdf, Treeplan tutorial.xls.
- * ***= Slides included for information but not discussed in lecture.***

Treeplan steps

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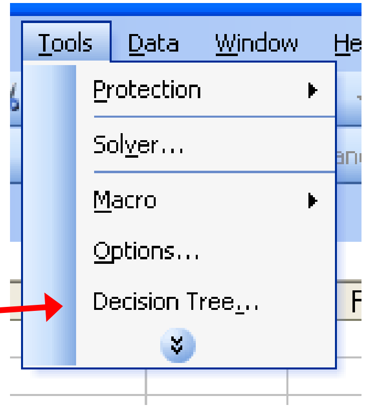
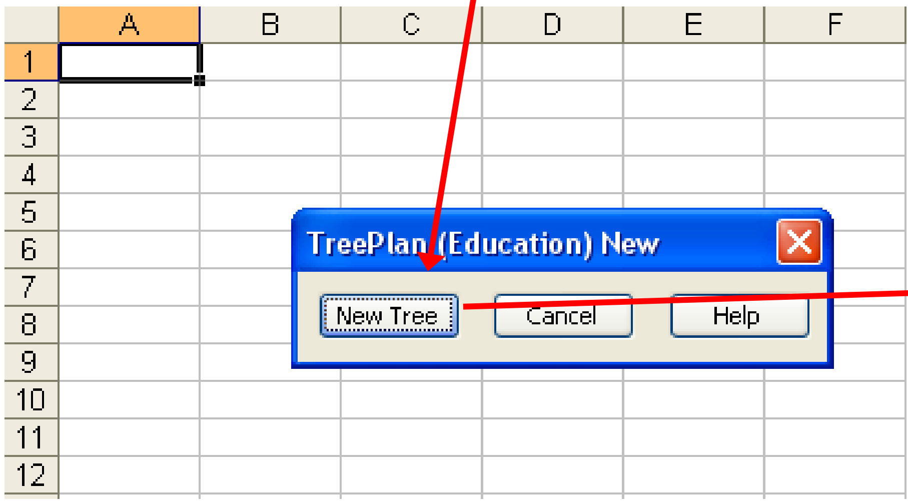
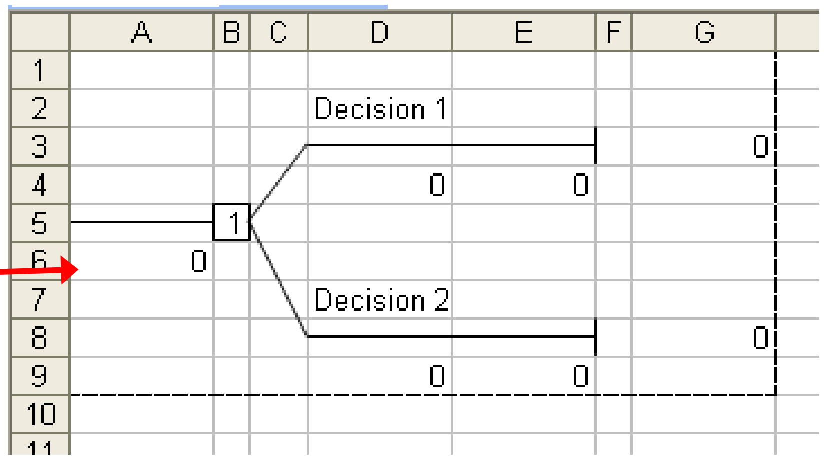
- To attach the Treeplan add in, open Excel, then open treeplan.xla



- To create a new tree:
  - Open a new file
  - Select Decision Tree from the Tools menu





Using Treeplan

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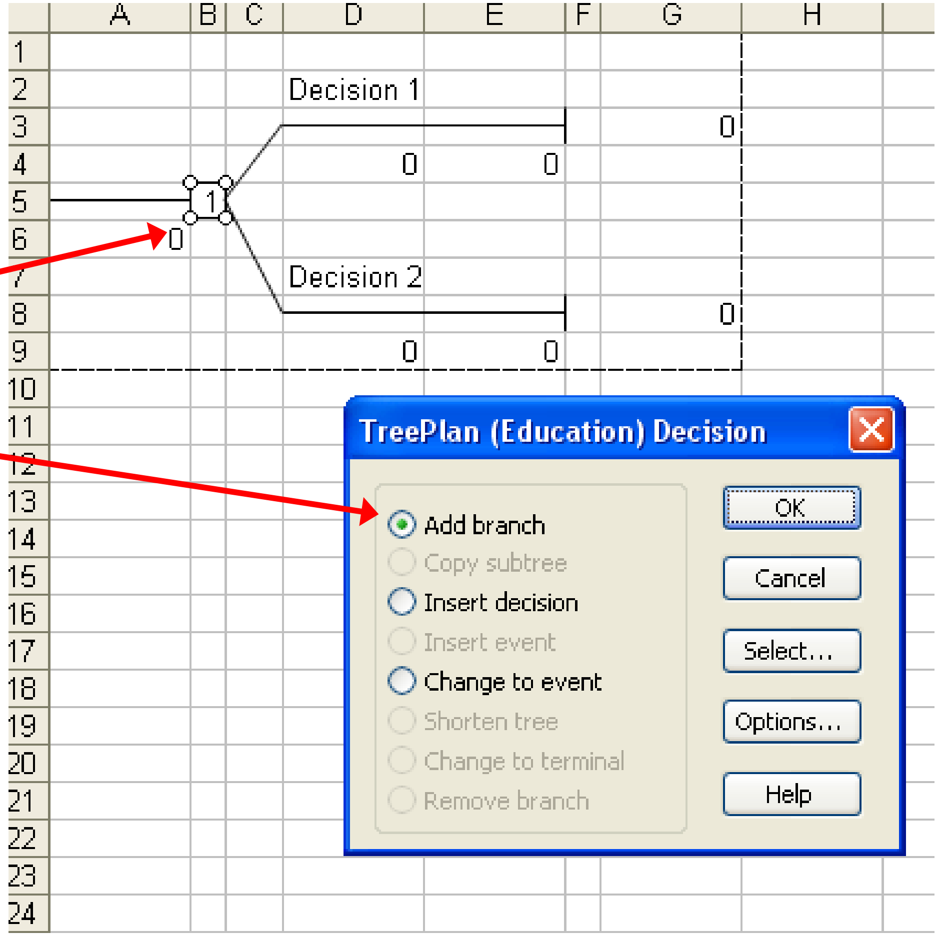
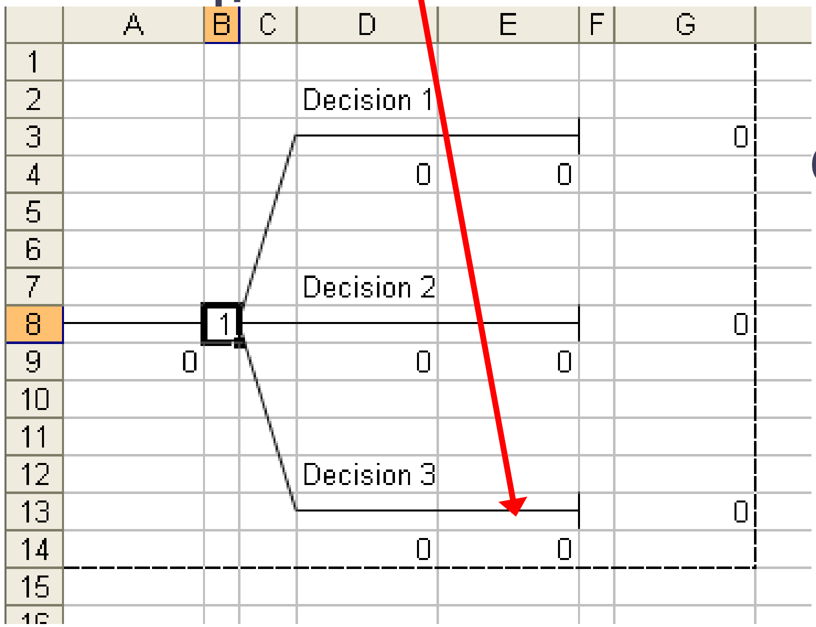
- Adding branches
  - Click the appropriate node
  - Press [Ctrl][t]
  - Select Add Branch



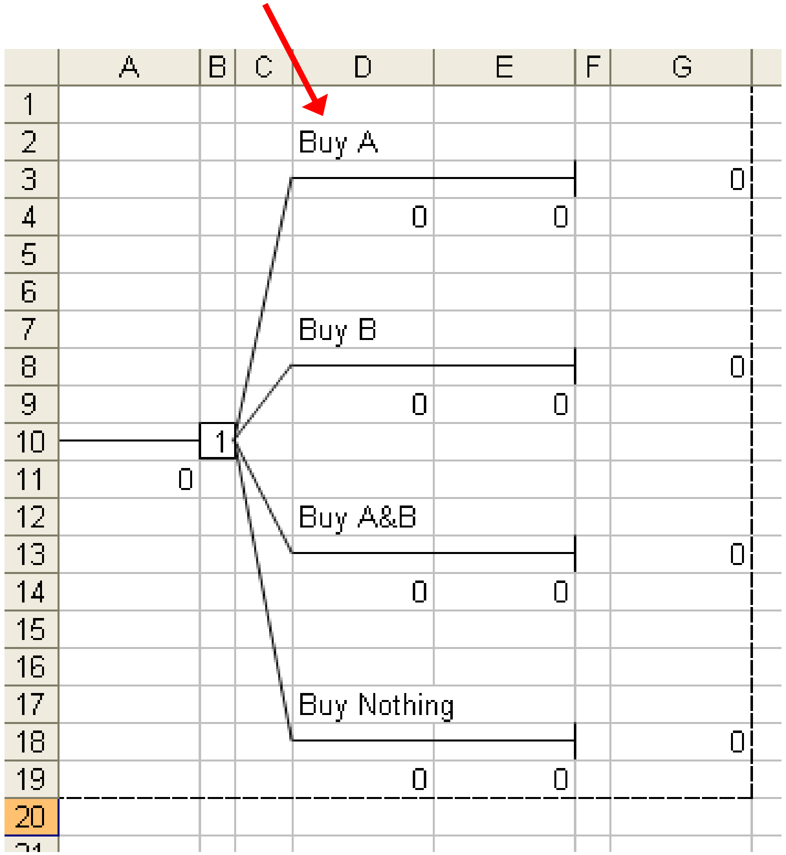


More branches can be added in the same way

Using Treeplan

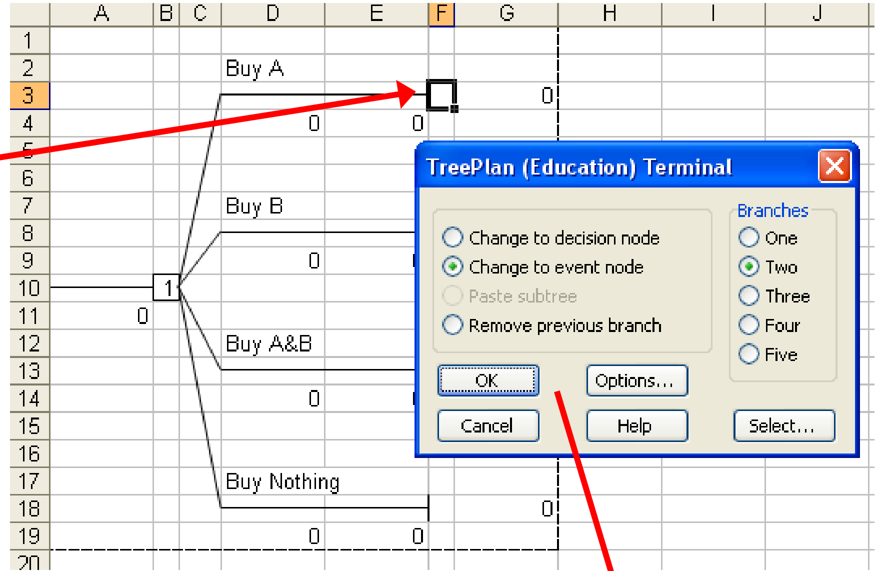
*

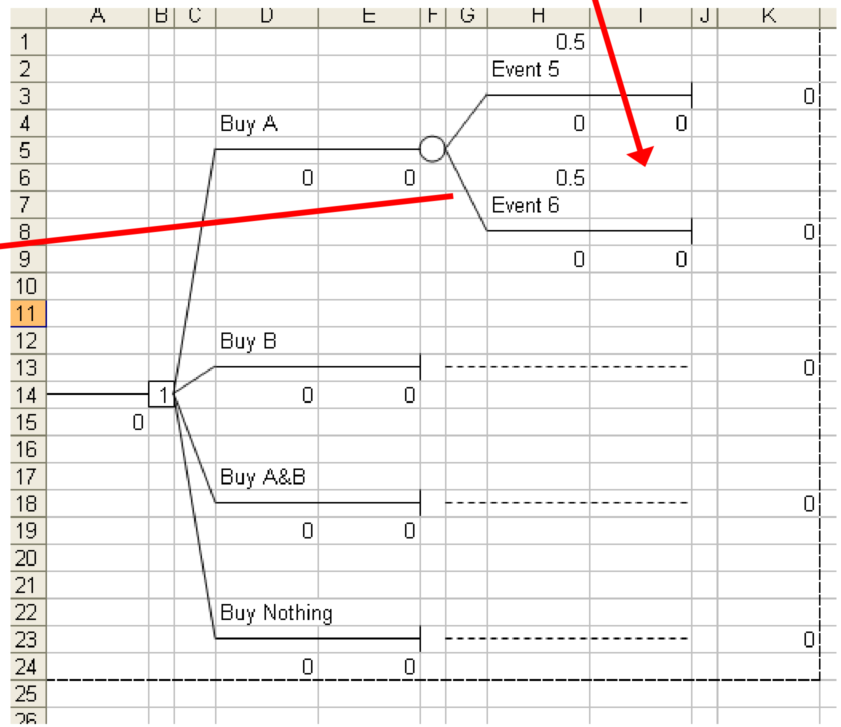
- Default branch labels can then be modified

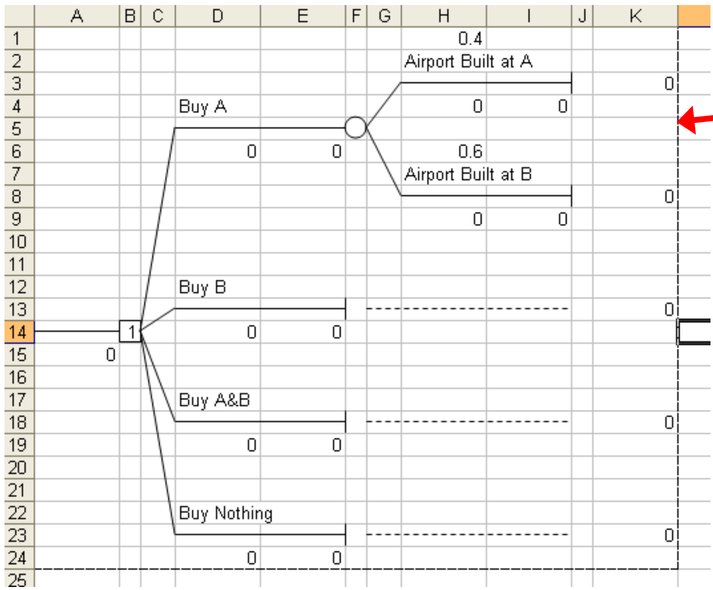


Using Treeplan

- Adding event nodes:
- Select the terminal node for the branch.
- Press [Ctrl] [t] to invoke Treeplan.
- Select Change to Event Node, and select Two Branches.
- Change the labels and probabilities as desired.

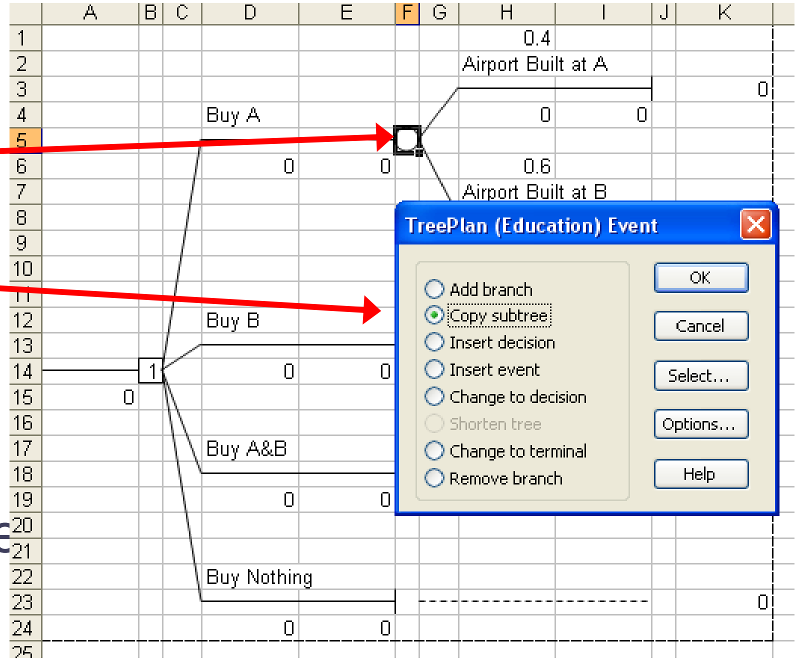


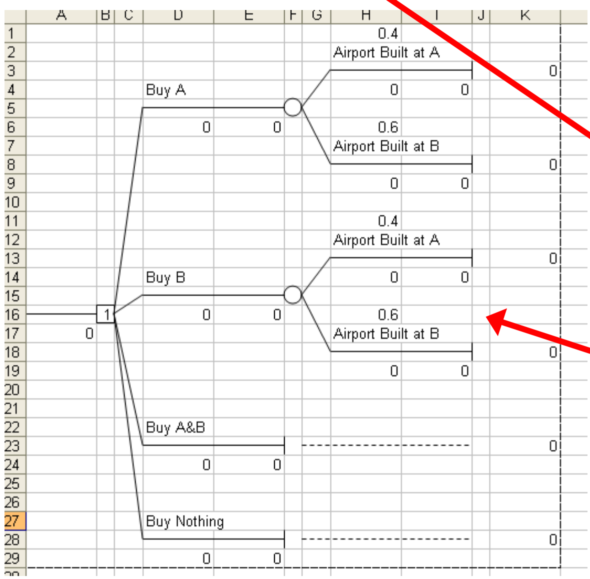


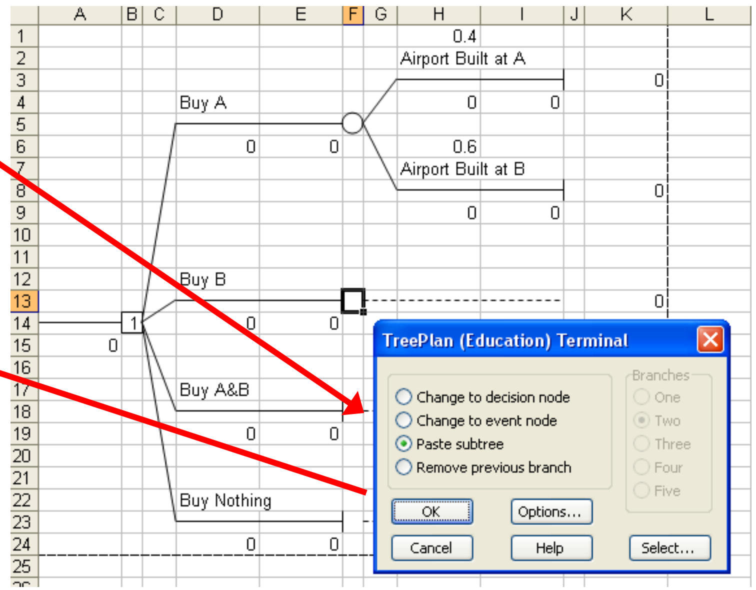


Using Treeplan

- Copying a subtree.
- Select the node you want to copy
- Press [Ctrl] [t] and select Copy subtree, press OK
- Select the target cell location for the copied subtree.
- Press [Ctrl] [t], then select Paste subtree
- Repeat as necessary



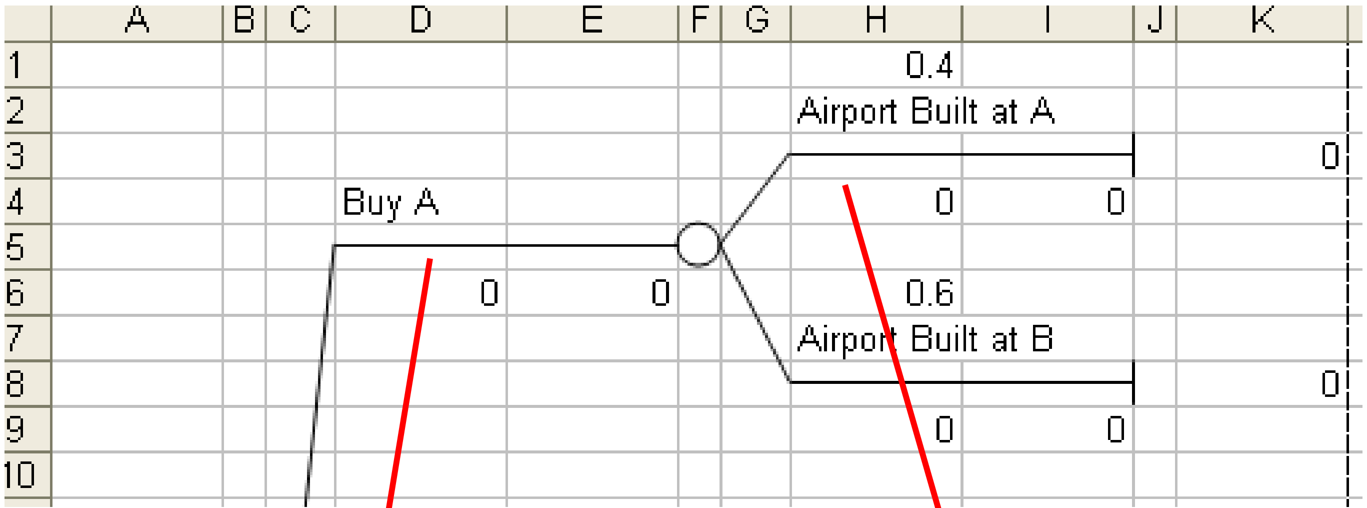




Using Treeplan

*

- Adding the cash flows
- Treeplan reserves the first cell below each branch for the partial cashflow associated with that branch. e.g.



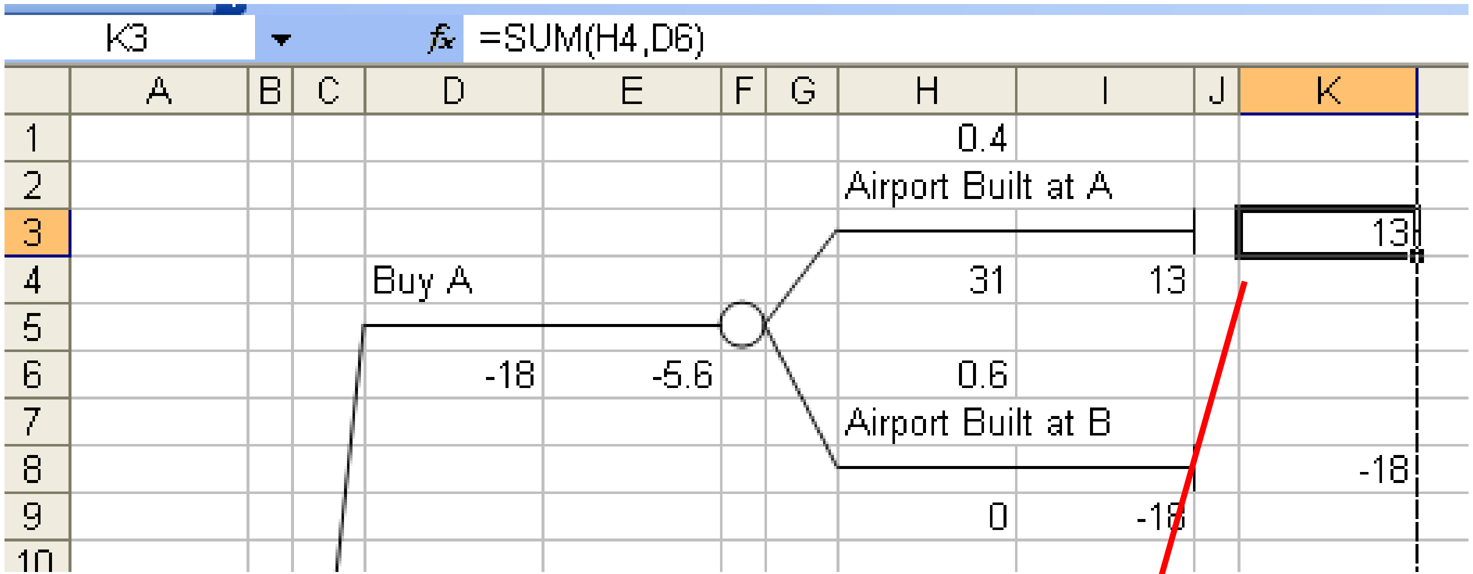
Partial cash flow if Magnolia Inns buys the land near location A – $18 million

Partial cash flow if Magnolia Inns buys the land near location A and the airport is built at that location $31 million

Using Treeplan

*

- Determining Payoffs and EMVs
- Treeplan creates a formula next to each terminal node that sums the payoffs along the branches that lead to that node. e.g.

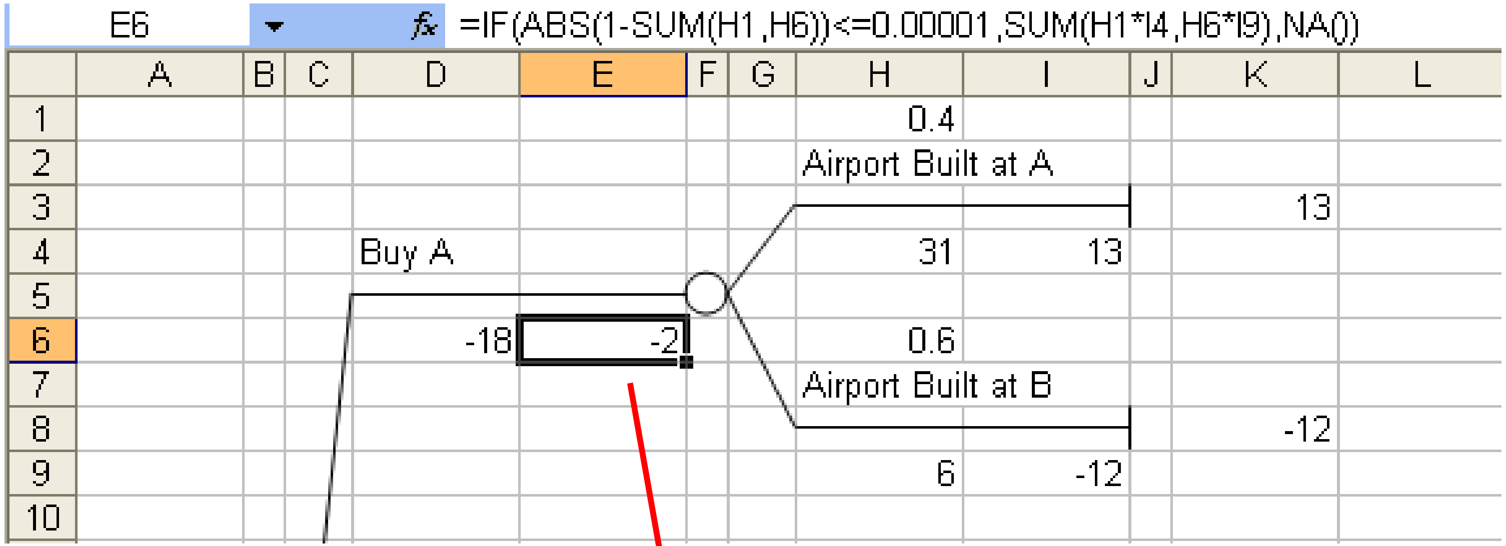


Total payoffs leading to this terminal node (=SUM(H4,D6))

Using Treeplan

*

- Determining Payoffs and EMVs
- Below and to the left of each node, Treeplan creates formulas that compute the EMV at each event node



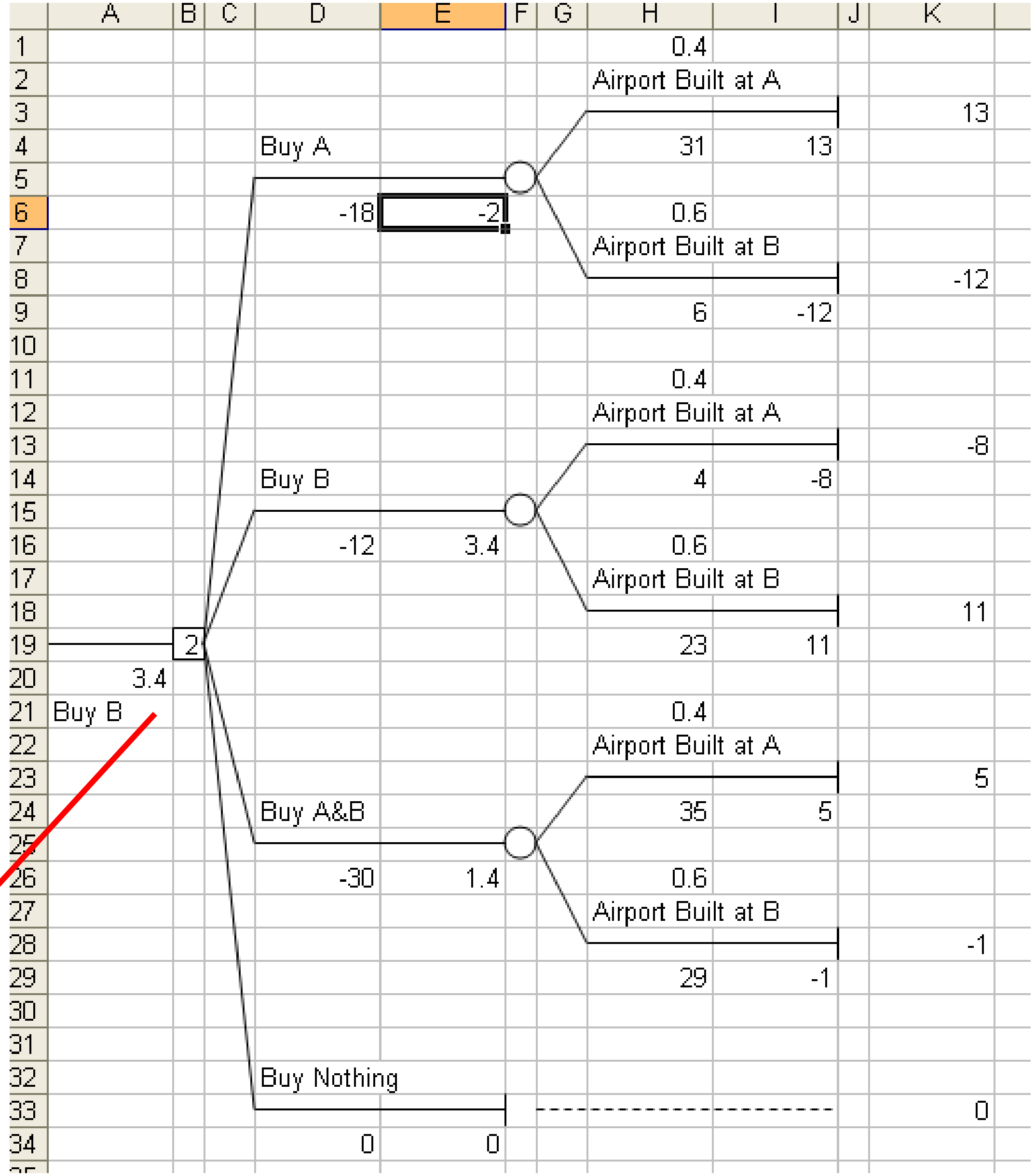
Calculated EMV at event node = SUM(H1*I4,H6*I9)

Using Treeplan

*

- Determining Payoffs and EMVs
- At a decision node Treeplan calculated the branch with the highest EMV. Both the value and the branch number with the highest EMV are included
- The Choose function can be used to display the label of the branch with the highest

EMV = CHOOSE(B19,D4,D14,D24)



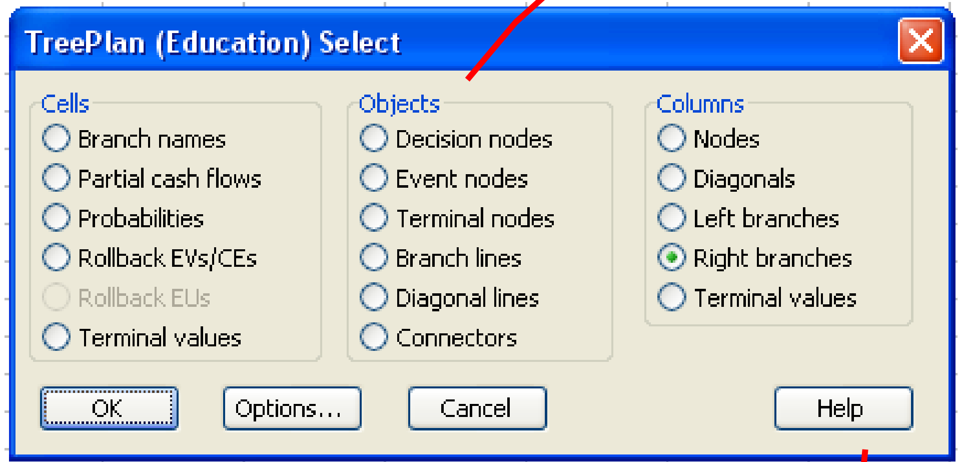
Highest EMV

Using Treeplan

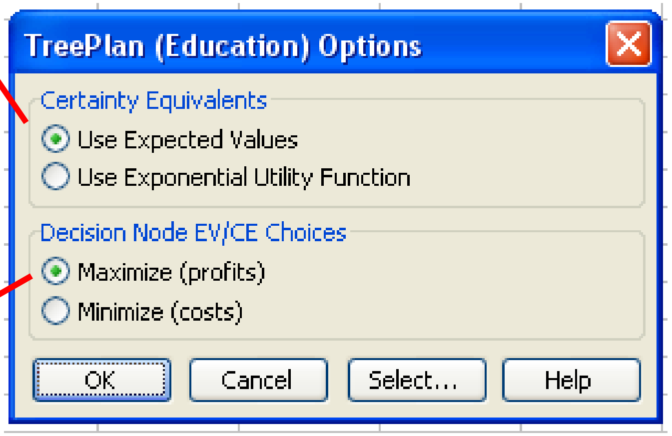
*

- Other features:

e.g. Select all objects of a certain type and apply formatting



EMVs most commonly used, but can use exponential utility function



Choose whether to maximise or minimise

Use Help button

Choose function *

- CHOOSE(index_num,value1,value2,...)
- Uses index_num to return a value from the list of value arguments. Use CHOOSE to select one of up to 29 values based on the index number.
- For example, if value1 through value7 are the days of the week, CHOOSE returns one of the days when a number between 1 and 7 is used as index_num.
- i.e. = CHOOSE(4,"mon","tues","wed","thurs","fri") returns “thurs”

Multi-Stage Decision Example: COM-TECH

*

- Steve Hinton, owner of COM-TECH, is considering whether to apply for a $85,000 OSHA research grant for using wireless communications technology to enhance safety in the coal industry.
- Steve would spend approximately $5,000 preparing the grant proposal and estimates a 50-50 chance of receiving the grant.
- If awarded the grant, Steve would need to decide whether to use microwave, cellular, or infrared communication technology
- Steve would need to acquire some new equipment depending on which technology is used …

  Technology Equipment Cost

  – Microwave $4,000, Cellular $5,000, Infrared $4,000.

COM-TECH (continued) *

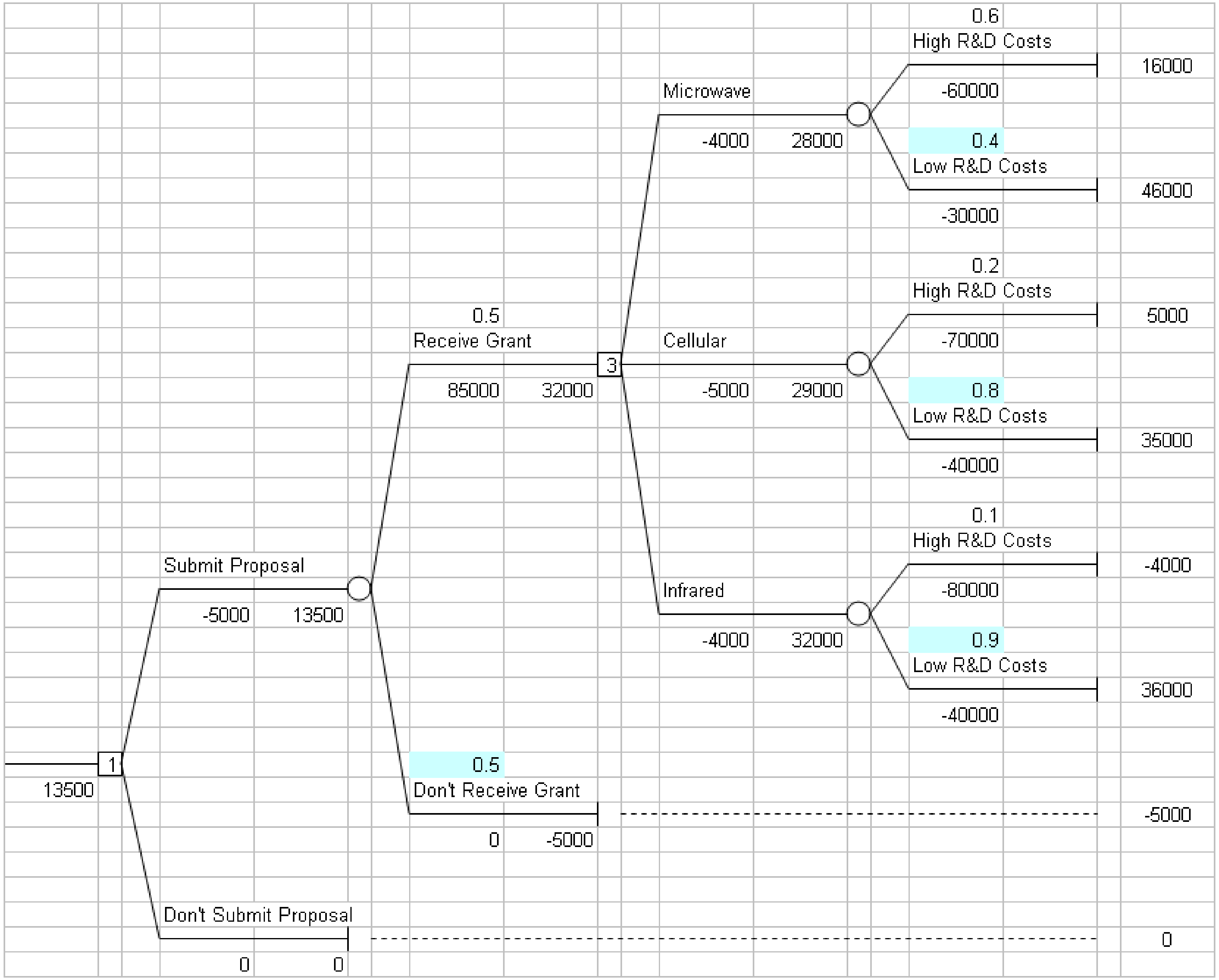
- Steve knows he will also spend money in R&D, but he doesn't know exactly what the R&D costs will be. Steve estimates the following best case and worst case R&D costs and probabilities, based on his expertise in each area.

	Best Case		Worst Case	
	Cost	Prob.	Cost	Prob.
Microwave	$30,000	0.4	$60,000	0.6
Cellular	$40,000	0.8	$70,000	0.2
Infrared	$40,000	0.9	$80,000	0.1

- Steve needs to synthesize all the factors in this problem to decide whether or not to submit a grant proposal to OSHA.
- See file Lecture 12 Risk Profile.xls (GrantProposal worksheet).

COM-TECH Decision Tree

*



Risk Profiles

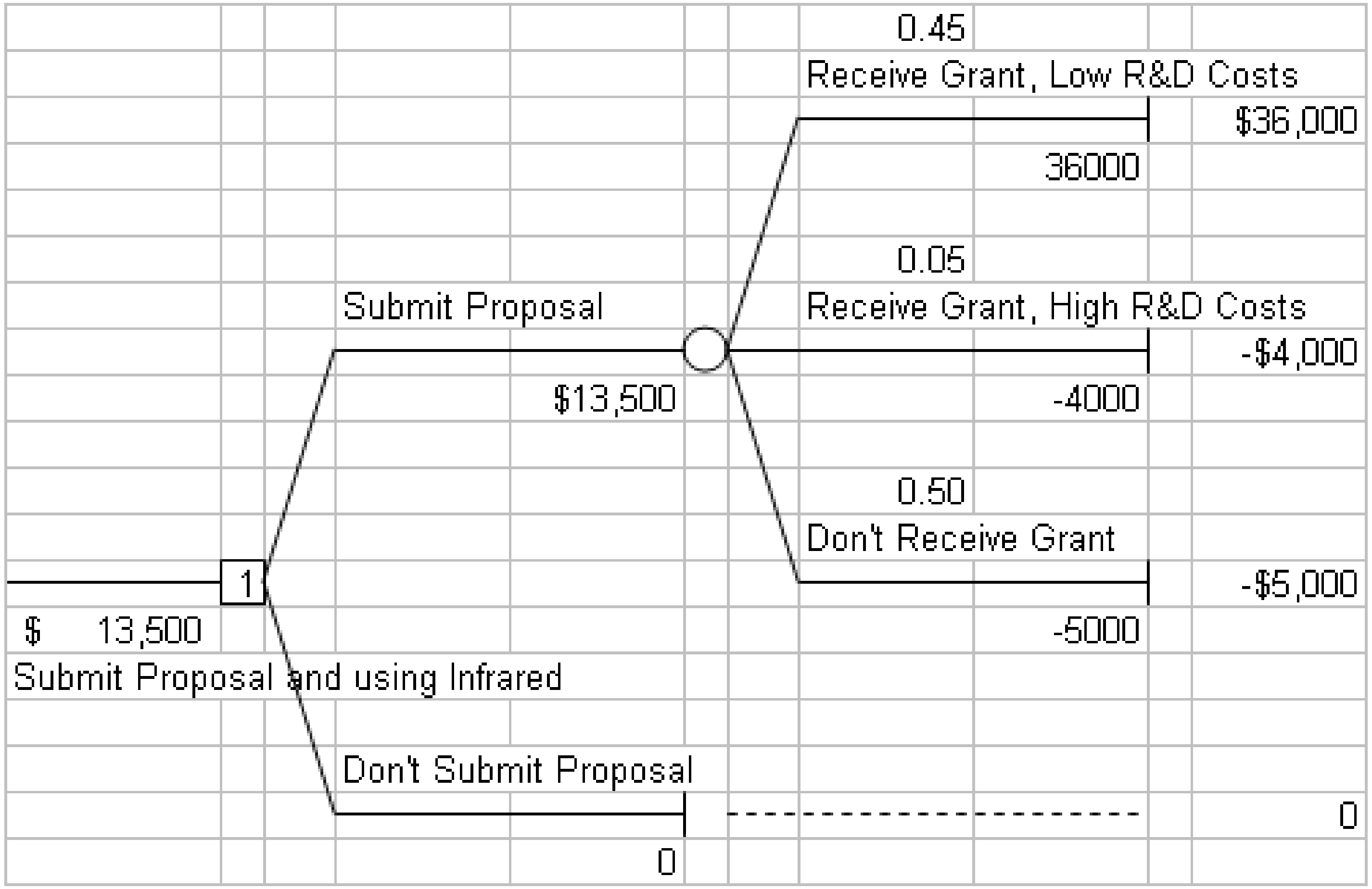
*

- A risk profile summarizes the make-up of an EMV.

Event	Probability	Payoff
Receive grant, Low R&D costs	0.5*0.9=0.45	$36,000
Receive grant, High R&D costs	0.5*0.1=0.05	-$4,000
Don’t receive grant	0.5	-$5,000
EMV		$13,500

Risk Profiles (as a Decision Tree) *

- See Lecture 12 Risk Profile.xls



Analysing Risk in a Decision Tree *

- How sensitive is decision in COM-TECH problem to changes in one probability estimate?
- Use Solver to determine smallest probability of receiving grant for which Steve should still be willing to submit proposal.
- Answer 13.51%

- See Lecture 12 Risk Profile.xls.

Analysing Risk in a Decision Tree *

- How sensitive is decision in COM-TECH problem to changes in two probability estimates?
- How optimal strategy changes as: Probability of receiving grant varies from 0 to 1, Probability of encountering high infrared R&D costs varies from 0 to 0.5. Create a strategy table.

		Prob. of High Infrared R&D Costs					
	Infrared	0.0	0.1	0.2	0.3	0.4	0.5
Prob. of Grant	0.0	Don't	Don't	Don't	Don't	Don't	Don't
	0.1	Don't	Don't	Don't	Don't	Don't	Don't
	0.2	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.3	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.4	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.5	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.6	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.7	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.8	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.9	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	1.0	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular

Conclusion

- Reference:
  - Lapin, L. and Whisler, W. (2002). *Quantitative Decision Making with Spreadsheet Applications*, 7th Ed., Wadsworth/Thomson Learning, Belmont.
- Go through the lecture examples to understand
  - The construction and evaluation of decision trees;
  - Using sample information in decision making;
  - Conditional probabilities;
  - Bayes' Theorem.
- This week's tutorial
  - Decision Trees – Risk and Uncertainty