MONASH University

Information Technology

FIT5186 Intelligent Systems

Lecture 11

Business Intelligence Modelling - Decision Analysis under Uncertainty

Learning Objectives

- Understand
  - the principles of business intelligence modelling for decision making
  - the characteristics of decision analysis under uncertainty
  - how to construct payoff tables and regret tables
  - the use of decision rules for decision making under uncertainty
  - the expected value method for decision making with probabilities

Business Intelligence Modelling for Decision Making

- Business intelligence aims to support better business decision making.
- Business intelligence often involves the development of mathematical models and analysis methodologies that exploit the available data to generate information and knowledge useful for complex decision making processes.

The Modelling Approach to Decision Making

- A model is
  - A structure which has been built purposely to exhibit features and characteristics of some other objects
- We often use models to make decisions.
- Types of models:
  - Mental (arranging furniture)
  - Visual (blueprints, road maps)
  - Physical/Scale (aerodynamics, buildings)
  - **Mathematical** – These are what we'll be studying.

> Do not worry about your difficulties in mathematics.
> I can assure you mine are still greater.
>
> Albert Einstein (1879 - 1955)

Models for Decision Making

May include:

- Mathematical equations
  - Mathematical Models
- Structured set of rules or steps to follow
  - Algorithms and Discrete Event processes
- Simulation of real events
  - Queuing Models, Waiting Lines
- Statistical techniques
  - Data Analysis, Equation Fitting

Modelling: Why Bother?

- The process may reveal hidden inner relationships within the system.
- Formal analysis (assumptions/relationships)
- Experimentation - can be used repeatedly
- To large extent, independent of the data
- Variety of models for the same problem

Benefits of Modelling

- Economy
  - It is often less costly to analyse decision problems using models.
- Timeliness
  - Models often deliver needed information more quickly than their real-world counterparts.
- Feasibility
  - Models can be used to do things that would be impossible in reality.
- Insight and understanding
  - Models improves decision making.

Advantages of Using Models

- Models:
  - Are less expensive and disruptive than experimenting with real world system;
  - Enable us to ask ‘what if’ questions and test the sensitivity of particular values;
  - Force a consistent, systematic approach to a problem;
  - Require managers to be specific about constraints and goals;
  - Can reduce the time taken to make decisions;
  - Can increase our understanding of a particular problem and lead to the acquisition of managerial insights.

Disadvantages of Using Models

- Models:
  - May be expensive to develop and test;
  - May be misused and misunderstood because of their complexity;
  - May down play the value of non-quantifiable information;
  - May oversimplify the variables of the real world.

The Modelling Process to Problem Solving

1. Begin with the real world problem.
2. Simplify and abstract the important elements.
3. Develop a mathematical (spreadsheet) model.
4. Apply the model, using parameters observed or estimated from the real world.
5. Analyse the results. Do they make sense? If not repeat Steps 2 – 4 until they do.
6. Apply the conclusions of the final model (Step 5) to the original problem, always being mindful of the problems with modelling.

How to Use a Model

- Don’t accept an answer without further analysis and questioning.
- A mathematical model is only one of many tools for decision making.
- If the answer is unacceptable, the reasons for unacceptability should be spelled out and incorporated in a modified model.
- An acceptable answer is only an option.
- By successive questioning of the answers and altering the model, it is possible to clarify the options available and obtain a greater understanding of what is possible in a real situation.

Psychology of Decision Making

- Models can be used for structurable aspects of decision problems.
- Other aspects cannot be structured easily, requiring intuition and judgment.
- *Caution*: Human judgment and intuition are not always rational!
- Errors in human judgment often arise because of *anchoring* and *framing* effects associated with decision problems.

Anchoring Effects

- Arise when trivial factors influence initial thinking about a problem.
- Decision-makers usually under-adjust from their initial “anchor”.
- Example:
  – What is 1x2x3x4x5x6x7x8? Median estimate = 512
  – What is 8x7x6x5x4x3x2x1? Median estimate = 2,250

The product is the same in both cases: 40,320

Framing Effects

- Refers to how decision-makers view a problem from a win-loss perspective.
- The way a problem is framed often influences choices in irrational ways…

Framing Effects 1

- Suppose you’ve been given $1000 and must choose between:

  A1. Receive $500 more immediately $\boxed{\text{=> Most popular}}$

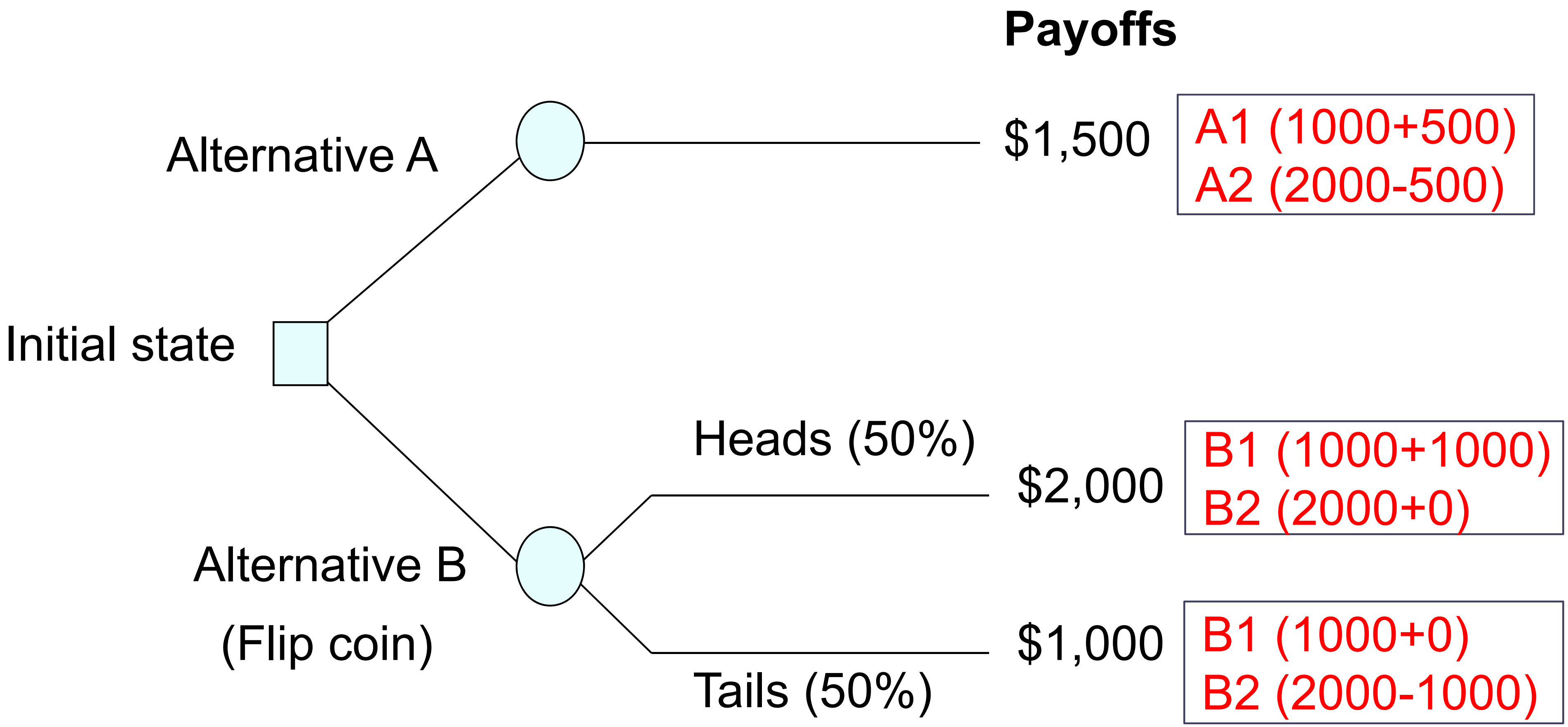
  B1. Flip a coin and receive $1000 more if heads occurs or $0 more if tails occurs.

- Now suppose you’ve been given $2000 and must choose between:

  A2. Give back $500 immediately

  B2. Flip a coin and give back $0 if heads occurs or give back $1000 if tails occurs. $\boxed{\text{=> Most popular}}$

A Decision Tree for Both Examples



Framing Effects 2

Which option would you select?

This food is 95% fat free

vs

This food consists of 5% fat

You generally obtain different answers from the same person.

Good Decisions *vs* Good Outcomes

- Good decisions do not always lead to good outcomes ... for example …

Andre-Francois Raffray thought he had a great deal in 1965 when he agreed to pay a 90-year-old woman named Jeanne Calment $500 a month until she died to acquire her grand apartment in Arles, south of France. But in December 1995, Raffray died at age 77, having paid more than $180,000 for an apartment he never got to live in.

Calment does not need to worry about losing her $500 monthly income. Although the amount Raffray already paid is twice the apartment's current market value, his widow is obligated to keep sending the monthly check to Calment. If Calment also outlives her, then the Raffray children will have to pay. **"In life, one sometimes makes bad deals"** said Calment of the outcome of Raffray's decision. (Source: The Savannah Morning News, 12/29/95) http://www.cnn.com/WORLD/9708/04/obit.oldest/

Good Decisions *vs* Good Outcomes

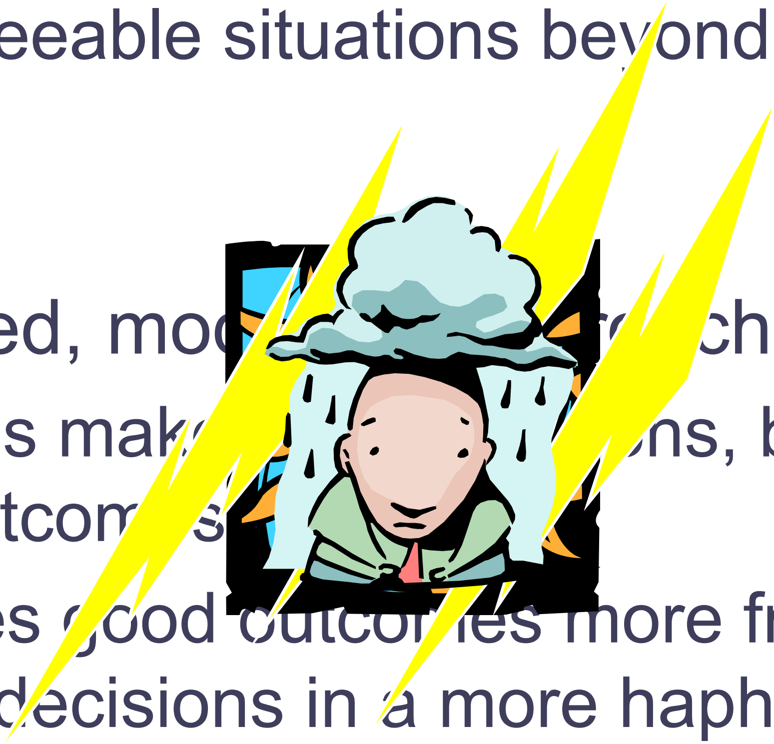
- Suppose … sharks have been spotted along the beach where you are vacationing with a friend. You and your friend have been informed of the shark sightings and are aware of the damage a shark attack can inflict on human flesh. You both decide (individually) to go swimming anyway.

- You are promptly attacked by a shark while your friend has a nice time body surfing in the waves.

- *Did you make a good or bad decision?*
- *Did your friend make a good or bad decision?*

Good Decisions *vs* Good Outcomes

- Good decisions do not always lead to good outcomes...
  - Luck often plays a role in determining whether a good or bad outcome occurs.
  - Unforeseeable situations beyond our control may occur.



- A structured, mod ... ch to decision making
  - Helps us mak ... ns, but cannot guarantee good outcomes.
  - Produces good outcomes more frequently than making decisions in a more haphazard manner.

Introduction to Decision Analysis

- Models help managers gain insight and understanding, but they cannot make decisions.
- A structured approach to decision making
  - Can help us make good decisions, but cannot guarantee good outcomes.
- It is important to appreciate that
  - Good decisions sometimes result in bad outcomes.
- Decision making often remains a difficult task due to:
  - Conflicting values or objectives
  - Uncertainty regarding the future
- Consider the following example...

Deciding between Job Offers

- Company A
  - In a new industry that could boom or bust.
  - Low starting salary, but could increase rapidly.
  - Located near friends, family and favorite sports team.
- Company B
  - Established firm with financial strength and commitment to employees.
  - Higher starting salary but slower advancement opportunity.
  - Distant location, offering few cultural or sporting activities.
- Which job would you take?

Characteristics of Decision Problems

- **Alternatives**: Different courses of action intended to solve a problem.
- **Criteria**: Factors that are important to the decision maker.
- **States of Nature** (State-of-the-World): Possible future events not under the decision maker's control.

Alternatives	Criteria	States of Nature
Work for Co. A	Salary	Co. A grows
Work for Co. B	Career potential	Co. A goes bust
Reject both offers and keep looking	Location	etc.
	etc.	

Assumptions of Decision Problems

The decision maker knows what states-of-nature are possible:

- The number of possible states is finite;
- States-of-nature are mutually exclusive;
- The number of actions available is finite;
- One, and only one, action must be chosen;
- For each combination of an action and a state-of-nature, there is a unique payoff.

Classification of Decision Problems

Due to degree of uncertainty

- Decision making under certainty
  The true state-of-nature is known to the decision maker before he/she makes a decision.
- Decision making under risk
  - Although the true state-of-nature is not known with certainty, uncertainty can be quantified by means of probability distribution.
- Decision making under strict uncertainty
  - The decision maker knows nothing about the true state-of-nature, except for which states are generally possible.

N.B. The uncertainty dealt with in this lecture is of stochastic nature, commonly based on Bayesian probabilities. In contrast to stochastic uncertainty, a large fraction of uncertainties in real-life situations are in the class of fuzziness which is concerned with the vague description of the semantic meaning of the events, phenomena, or statements in a natural language, characterised by subjectivity and imprecision.

Fuzzy logic - modelling the subjectivity and imprecision of uncertain information.

Decision Rules

Two categories:

- Non-probabilistic decision rules
  - Maximax
  - Maximin
  - Minimax Regret
- Probabilistic decision rules
  - Expected Monetary Value (EMV)
  - Expected Regret or Opportunity Loss (EOL)

Problem Formulation

- A decision problem is characterised by decision alternatives, states of nature, and resulting payoffs.
- The decision alternatives are the different possible actions the decision maker can take.
- The states of nature refer to future events, not under the control of the decision maker, which may occur.
  - States of nature should be defined so that they are mutually exclusive and collectively exhaustive.

Payoff Tables

- The consequence resulting from a specific combination of a decision alternative and a state of nature is a payoff.
- A table showing payoffs for all combinations of decision alternatives and states of nature is a Payoff Table or Payoff Matrix.
- Payoffs can be expressed in terms of profit, cost, time, distance or any other appropriate measure that is used as the decision criteria.

Example 1: Magnolia Inns

Background information:

- Hartsfield International airport in Atlanta Georgia – one of the busiest in the world.
- Analysts predict that traffic will continue to increase well into the future.
- Commercial development in surrounding area prevents the construction of extra runways.
- To solve the problems, plans are being developed to <u>build another airport</u> outside the city limits.
- <u>Two possible locations</u> have been identified and the decision as to where to build will not be made for another year (<u>location A and location B</u>).

Example 1: Magnolia Inns (continued)

Meanwhile….

- Magnolia Inns hotel chain intends building a new hotel near the airport once the site is determined.
- Land values near the two possible sites for the airport are increasing due to speculation.
- Magnolia Inns have information regarding:
  - The price of a suitable parcel of land for building a hotel near each possible airport site
  - The estimated present value of future cash flows that a hotel would generate at each site if the airport is ultimately located at that site
  - The present value of the amount that Magnolia Inns believes it can sell the site for if the airport is NOT to built there

Magnolia Inns: Data

See file FIT5186 Lecture 11.xls (Data)

Magnolia Inns
Real Estate Acquisition Analysis

	Parcel of Land Near Location	
	A	B
Current purchase price	$18	$12
Present value of future cash flows if hotel and airport are built at this location	$31	$23
Present value of future sales price of parcel if the airport is not built at this location	$6	$4

(Note: All values are in millions of dollars.)

The Decision Alternatives and The Possible States of Nature

Decision Alternatives:

(1) Buy the parcel of land near location A

(2) Buy the parcel of land near location B

(3) Buy both parcels

(4) Buy nothing

Possible states of Nature:

(1) New airport is built at location A

(2) New airport is built at location B

Constructing a Payoff Matrix (1)

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	?	?
B	?	?
A&B	?	?
None	?	?

States of nature

Decision alternatives

Payoff for each combination of decision alternative and state of nature

Constructing a Payoff Matrix (2)

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	?	
B		
A&B		
None		

Present value of future cash flows if hotel and airport are built at location A — $31,000,000

minus

Current purchase price of hotel site at location A — - $18,000,000

$13,000,000

Constructing a Payoff Matrix (3)

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$13	?
B		
A&B		
None		

Present value of future sales price of parcel if airport is not built at location A	$6,000,000
minus	
Current purchase price of hotel site at location A	- $18,000,000
	-$12,000,000

Constructing a Payoff Matrix (4)

	Payoff Matrix		
Land Purchased	Airport is Built at Location		
at Location(s)	A	B	
A	$13	($12)	
B	?		
A&B			
None			

Present value of future sales price of parcel if airport is not built at location B — $4,000,000

minus

Current purchase price of hotel site at location B — - $12,000,000

-$8,000,000

Constructing a Payoff Matrix (5)

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	\$13	(\$12)
B	(\$8)	?
A&B		
None		

Present value of future cash flows if hotel and airport are built at location B — \$23,000,000

minus

Current purchase price of hotel site at location B — - \$12,000,000

\$11,000,000

Constructing a Payoff Matrix (6)

Payoff Matrix		
Land Purchased at Location(s)	**Airport is Built at Location A**	**Airport is Built at Location B**
A	$13	($12)
B	($8)	$11
A&B	?	
None		

	PV of future CFs if hotel & airport are built at location A	$31,000,000
+	PV of future sales price of parcel if airport is not built at location B	+ $ 4,000,000
–	Current purchase price of hotel site at location A	- $18,000,000
–	Current purchase price of hotel site at location B	- $12,000,000
		$ 5,000,000

Constructing a Payoff Matrix (7)

	Payoff Matrix		
Land Purchased	**Airport is Built at Location**		
at Location(s)	**A**	**B**	
A	$13	($12)	
B	($8)	$11	
A&B	$5	?	
None			

	PV of future CFs if hotel & airport are built at location B	$23,000,000
+	PV of future sales price of parcel if airport is not built at location A	+ $ 6,000,000
−	Current purchase price of hotel site at location A	- $18,000,000
−	Current purchase price of hotel site at location B	- $12,000,000
		-$1,000,000

Constructing a Payoff Matrix (8)

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	?	?

Zero in both cases

How to Make a Decision under Uncertainty?

- If the future state of nature (airport location) were known, it would be easy to make a decision.
- If the future state of nature is uncertain and the **probability information** regarding the likelihood of the states of nature **is unavailable,** three decision rules can be applied:

(a) The **Maximax** decision rule (the optimistic approach);

(b) The **Maximin** decision rule (the conservative approach);

(c) The **Minimax Regret** decision rule (the opportunity loss approach).

- No decision rule is always best and each has its own weaknesses.
- It depends on the company’s attitudes and criteria.

The Maximax Decision Rule: The Optimistic Approach

- This is a strategy for a risk-taking decision maker.
- The optimistic approach would be used by a decision maker who believed that the future event had a high probability of success.
- The decision alternative with the **largest possible payoff** is chosen.
- If the payoff table was in terms of costs, the decision alternative with the lowest cost would be chosen.

The Maximax Decision Rule

- Identify the maximum payoff for each decision alternative.
- Choose the alternative with the largest maximum payoff.
  - **Maxi**mum of the **maxi**mum payoffs.
- Purchase land at Location A.

	A	B	C
1		**Payoff Matrix**	
2			
3	**Land Purchased**	**Airport is Built at Location**	
4	**at Location(s)**	**A**	**B**
5	**A**	\$13	(\$12)
6	**B**	(\$8)	\$11
7	**A&B**	\$5	(\$1)
8	**None**	\$0	\$0

Max {13, 11, 5, 0} = 13

The Maximax Decision Rule

- Weakness
  - Consider the following payoff matrix:

	State of Nature			
Decision	1	2	MAX	
A	30	-10000	30	← maximum
B	29	29	29	

The Maximin Decision Rule: The Conservative Approach

- This is a strategy for a cautious decision maker.
- The conservative approach would be used by a decision maker who thought the future event had a low chance of success.
- For each decision alternative, the minimum payoff is listed and then the decision alternative corresponding to the **maxi**mum of these **min**imum payoffs is selected.
  - Hence, the minimum possible payoff is maximised.
- If the payoff was in terms of costs, the maximum costs would be determined for each decision alternative and then the decision alternative corresponding to the minimum of these maximum costs is selected.
  - Hence, the maximum possible cost is minimised.

The Maximin Decision Rule

- Identify the minimum payoff for each alternative.
- Choose the alternative with the largest minimum payoff.
- Do not purchase land at either location.

	A	B	C
1	Payoff Matrix		
2			
3	**Land Purchased**	**Airport is Built at Location**	
4	**at Location(s)**	**A**	**B**
5	**A**	\$13	(\$12)
6	**B**	(\$8)	\$11
7	**A&B**	\$5	(\$1)
8	**None**	\$0	\$0

Max {-12, -8, -1, 0} = 0

The Maximin Decision Rule

- Weakness
  - Consider the following payoff matrix:

	State of Nature			
Decision	1	2	MIN	
A	1000	28	28	
B	29	29	29	← maximum

The Minimax Regret Decision Rule: The Opportunity Loss Approach

- This is a strategy to avoid “I told you…” discussions.
- Regret = opportunity loss
- The minimax regret approach requires the construction of a **regret table** (matrix) or an opportunity loss table.
- This is done by calculating *for each state of nature* the difference between each payoff and the largest payoff for that state of nature.
- Then, using this regret table, the maximum regret for each possible decision alternative is identified.
- The decision alternative chosen is the one corresponding to the **mini**mum of the **max**imum regrets.

The Minimax Regret Rule

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	$0	$0

Regret Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	?	?
B		
A&B		
None		

Assume purchased land at location A

- If airport is built at location A
  - No regret
- If airport is built at location B
  - Made a loss of $12 million
  - If bought in location B, would have earned $11 million (the largest payoff)
  - Opportunity loss of $23 million

The Minimax Regret Rule

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	$0	$0

Regret Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$0	$23
B	?	?
A&B		
None		

Assume purchased land at location B

- If airport is built at location A
  - Made a loss of $8 million
  - If bought in location A, would have earned $13 million (the largest payoff)
  - Opportunity loss of $21 million
- If airport is built at location B
  - No regret

The Minimax Regret Rule

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	$0	$0

Regret Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$0	$23
B	$21	$0
A&B	?	?
None		

Assume purchased land at locations A & B

- If airport is built at loc A
  - Made a gain of $5 million.
  - If bought only in location A, would have earned $13 million (the largest payoff)
  - Opportunity loss of $8 million
- If airport is built at loc B
  - Made a loss of $1 million
  - If bought in location B, would have earned $11 million (the largest payoff)
  - Opportunity loss of $12 million

The Minimax Regret Rule

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	Airport is Built at Location B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	$0	$0

Regret Matrix

Land Purchased at Location(s)	Airport is Built at Location A	Airport is Built at Location B
A	$0	$23
B	$21	$0
A&B	$8	$12
None	?	?

Assume purchased none

- If airport is built at loc A
  - If bought in location A, would have earned $13 million (the largest payoff)
  - Opportunity loss of $13 million
- If airport is built at loc B
  - If bought in location B, would have earned $11 million (the largest payoff)
  - Opportunity loss of $11 million

The Minimax Regret Rule – Excel Formula

See file FIT5186 Lecture 11.xls (Minimax Regret)

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	$0	$0

Regret Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$0	$23
B	$21	$0
A&B	$8	$12
None	$13	$11

Cell B14: = MAX(B$5:B$8)-B5

The largest payoff for the state of nature

Decision with the Minimax Regret Rule

- Identify maximum possible regret for each alternative.
- Choose alternative with the minimum of the maximum regrets.
- Purchase land at both Locations A and B.

	A	B	C	D	E
1	Regret Matrix &				
2	Minimax Regret Decision Rule				
3	**Land Purchased**	**Airport is Built at Location**			
4	**at Location(s)**	**A**	**B**	**Max**	
5	**A**	$0	$23	$23	
6	**B**	$21	$0	$21	
7	**A&B**	$8	$12	$12	
8	**None**	$13	$11	$13	
9					
10	**E7**	=IF(D7=MIN($D$5:$D$8),"<--minimum","")			
11					

Anomalies with Minimax Regret Rule

- Consider the following payoff matrix

	State of Nature	
Decision	1	2
A	9	2
B	4	6

- The regret matrix is:

	State of Nature			
Decision	1	2	MAX	
A	0	4	4	<-- minimum
B	5	0	5	

- Note that we prefer A to B.
- Now let's add an alternative...

Adding an Alternative

- Consider the following payoff matrix:

Decision	State of Nature 1	State of Nature 2
A	9	2
B	4	6
C	3	**9**

- The regret matrix is:

Decision	State of Nature 1	State of Nature 2	MAX	
A	0	7	7	
B	5	3	5	<--minimum
C	6	0	6	

- Now we prefer B to A???

Decision Making with Probabilities

- Sometimes, the states of nature may be assigned probabilities that represent their likelihood of occurrence.
- For decision problems that occur more than once
  - Estimate probabilities from historical data.
- For decision problems that occur once off
  - Subjective probabilities are often assigned based on interviews with one or more domain experts.
- We will focus on methods that can be used once appropriate probability estimates have been obtained.

The Expected Value Method

- This is a strategy based on averages.
- If probability information regarding the states of nature is available, we may use the expected value approach.
- The expected value for each decision alternative is calculated by summing the products of the payoff under each state of nature and the probability of the respective state of nature occurring.
- The decision alternative with the **best expected value** is chosen.
- The **expected value** may be positive or negative.
- The **expected value** refers to the long run average value, not necessarily the values for individual cases.
  - Often expressed in monetary terms, as **expected monetary value (EMV).**

Expected Monetary Value (EMV)

- The **expected monetary value (EMV)** of a decision alternative is the sum of weighted payoffs for the decision alternative.
- EMV_i is the **average payoff** we'd received if we faced the same decision problem numerous times and always selected alternative *i.*

$$EMV_i = \sum_j r_{ij} p_j$$

where

r_{ij} = payoff for alternative *i* under the *j*th state of nature

p_j = probability of the *j*th state of nature

- The EMV decision rule selects the alternative with the largest expected monetary value.

Expected Monetary Value (EMV)

See file FIT5186 Lecture 11.xls (Expected Value (EMV))

	A	B	C	D	E
1		**Payoff Matrix &**			
2		**EMV Decision Rule**			
3	**Land Purchased**	**Airport is Built at Location**			
4	**at Location(s)**	**A**	**B**	**EMV**	
5	**A**	\$13	(\$12)	(\$2.0)	
6	**B**	(\$8)	\$11	\$3.4	
7	**A&B**	\$5	(\$1)	\$1.4	
8	**None**	\$0	\$0	\$0.0	
9					
10	**Probability**	0.4	0.6		
11					
12	**D5**	=SUMPRODUCT(\$B\$10:\$C\$10,B5:C5)			
13					

$$EMV_i = \sum_j r_{ij} p_j$$

(\$13 × 0.4) + (–\$12 × 0.6) = –\$2.0
(–\$8 × 0.4) + (\$11 × 0.6) = \$3.4
(\$5 × 0.4) + (–\$1 × 0.6) = \$1.4

EMV Caution

- The EMV decision rule should be used with caution in one-time decision problems.
- Weakness
  - Consider the following payoff matrix:

	State of Nature			
Decision	1	2	EMV	
A	15,000	–5,000	5,000	<--maximum
B	5,000	4,000	4,500	
Probability	0.5	0.5		

Expected Regret or Opportunity Loss (EOL)

- EOL_i is the average **regret** or **opportunity loss** if we'd face the same decision problem numerous times and always selected alternative *i.*

$$EOL_i = \sum_j g_{ij} p_j$$

where

g_{ij} = regret for alternative *i* under the *j*th state of nature

p_j = probability of the *j*th state of nature

- The EOL decision rule selects the alternative with the smallest expected regret or opportunity loss.
- The decision alternative with the largest EMV will also have the smallest EOL.
- The EMV and EOL decision rules give the same decision alternative.

Expected Regret or Opportunity Loss (EOL)

See file FIT5186 Lecture 11.xls (Expected Regret (EOL))

	A	B	C	D	E
1		Regret Matrix &			
2		EOL Decision Rule			
3	**Land Purchased**	**Airport is Built at Location**			
4	**at Location(s)**	**A**	**B**	**EOL**	
5	**A**	$0	$23	$13.8	
6	**B**	$21	$0	$8.4	← **minimum**
7	**A&B**	$8	$12	$10.4	
8	**None**	$13	$11	$11.8	
9					
10	**Probability**	0.4	0.6		
11					
12	**D5**	=SUMPRODUCT($B$10:$C$10,B5:C5)			

$$EOL_i = \sum_j g_{ij} p_j$$

($0 × 0.4) + ($23 × 0.6) = $13.8
($21 × 0.4) + ($0 × 0.6) = $8.4
($8 × 0.4) + ($12 × 0.6) = $10.4
($13 × 0.4) + ($11 × 0.6) = $11.8

Sensitivity Analysis and EMV

- Changing the probability of an airport at Location A using a data table.
- See file FIT5186 Lecture 11.xls (EMV Decision Table)

	A	B	C	D	E
1		Payoff Matrix &			
2		EMV Decision Rule			
3	Land Purchased	Airport is Built at Location			
4	at Location(s)	A	B	EMV	
5	A	$13	($12)	($2.0)	
6	B	($8)	$11	$3.4	<--maximum
7	A&B	$5	($1)	$1.4	
8	None	$0	$0	$0.0	
9					
10	Probability	0.4	0.6		
11					
12		EMV A	EMV B	EMV A&B	EMV None
13	Prob. of Location A	($2.0)	$3.4	$1.4	$0.0
14	0.0	-12	11	-1	0
15	0.1	-9.5	9.1	-0.4	0
16	0.2	-7	7.2	0.2	0
17	0.3	-4.5	5.3	0.8	0
18	0.4	-2	3.4	1.4	0
19	0.5	0.5	1.5	2	0
20	0.6	3	-0.4	2.6	0
21	0.7	5.5	-2.3	3.2	0
22	0.8	8	-4.2	3.8	0
23	0.9	10.5	-6.1	4.4	0
24	1.0	13	-8	5	0
25					

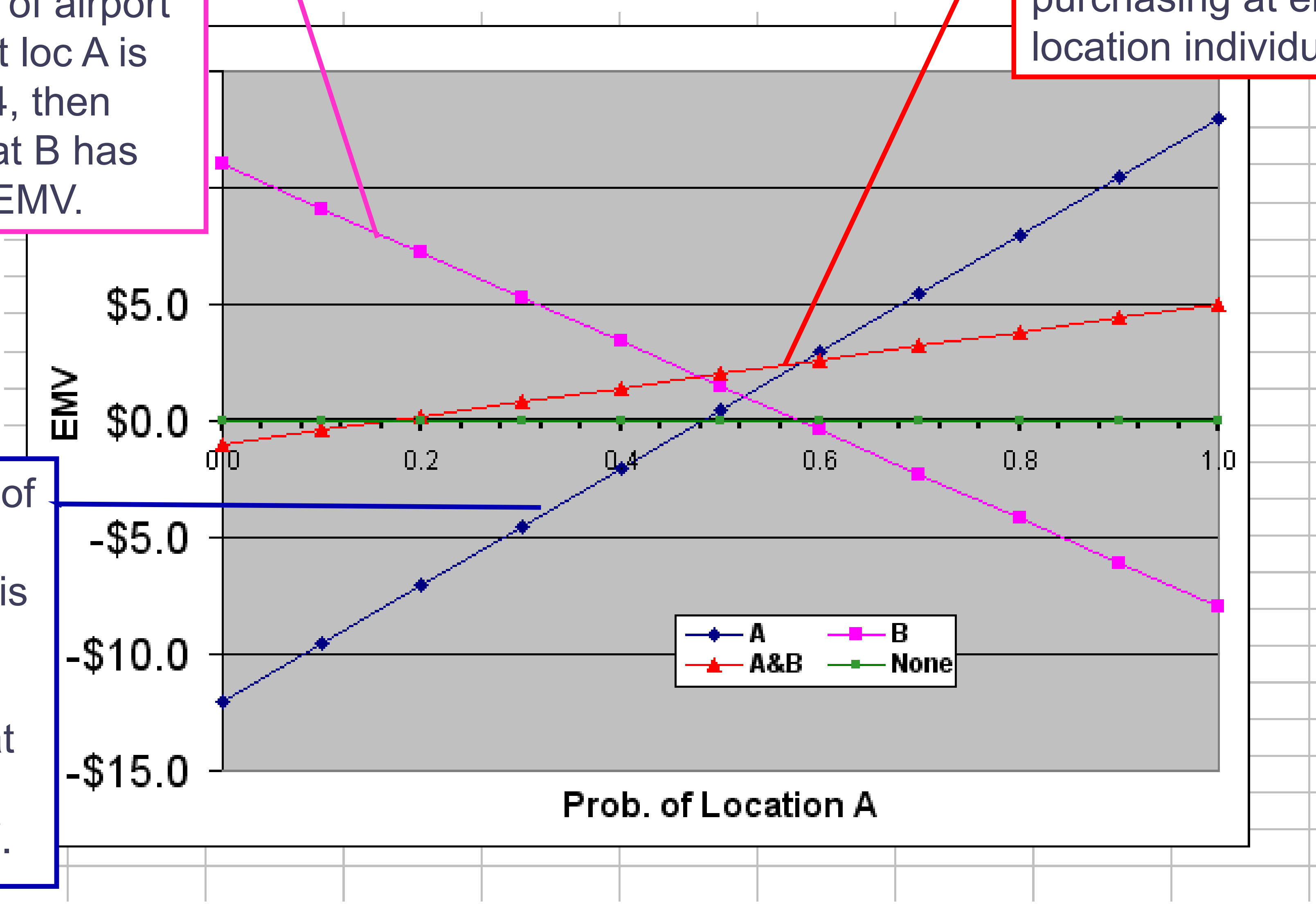
Sensitivity Analysis and EMV

- See file FIT5186 Lecture 11.xls (EMV Decision Table)

If probability of airport being built at loc A is less than 0.4, then purchasing at B has the highest EMV.

If uncertainty is high, purchase at both A&B. Purchasing at A&B is less risky than purchasing at either location individually.

If probability of airport being built at loc A is greater than 0.6, then purchasing at A has the highest EMV.



Expected Value with Perfect Information

- Often the information is available which can improve the probability estimates for the states of nature.
- For the example problem, suppose we could hire a consultant who could predict the future with 100% accuracy. This means that the consultant knows the outcome, but cannot influence the probabilities of the outcome.
- 0.4 of the time the outcome will be that the location is A, so the best choice is A with a payoff of $13.
- 0.6 of the time the outcome will be that the location is B, so the best choice is B with a payoff of $11.

	A	B	C	D	E
1		Payoff Matrix &			
2		EMV Decision Rule			
3	**Land Purchased**	**Airport is Built at Location**			
4	**at Location(s)**	**A**	**B**	**EMV**	
5	**A**	$13	($12)	($2.0)	
6	**B**	($8)	$11	$3.4	
7	**A&B**	$5	($1)	$1.4	
8	**None**	$0	$0	$0.0	
9					
10	**Probability**	0.4	0.6		

Expected Value with Perfect Information

	A	B	C	D	E
1	Payoff Matrix &				
2	EMV Decision Rule				
3	Land Purchased	Airport is Built at Location			
4	at Location(s)	A	B	EMV	
5	A	$13	($12)	($2.0)	
6	B	($8)	$11	$3.4	
7	A&B	$5	($1)	$1.4	
8	None	$0	$0	$0.0	
9					
10	Probability	0.4	0.6		

(Expected Monetary Value)

- **With** perfect information about where the airport is going to be built, the expected payoff would be:

  EV with PI = $(0.4 \times \$13) + (0.6 \times \$11) = \$11.8$

  – The expected value with perfect information provides an upper bound on the expected value of any sample or survey information.

- How much would the company be willing to pay the consultant for such information?

The Expected Value of Perfect Information (EVPI)

- The expected value of perfect information (EVPI) is the increase in the expected profit that would result if we knew with certainty which state of nature would occur.
- **Without** perfect information, maximum EMV = $3.4
- **EVPI = EV with PI - maximum EMV**

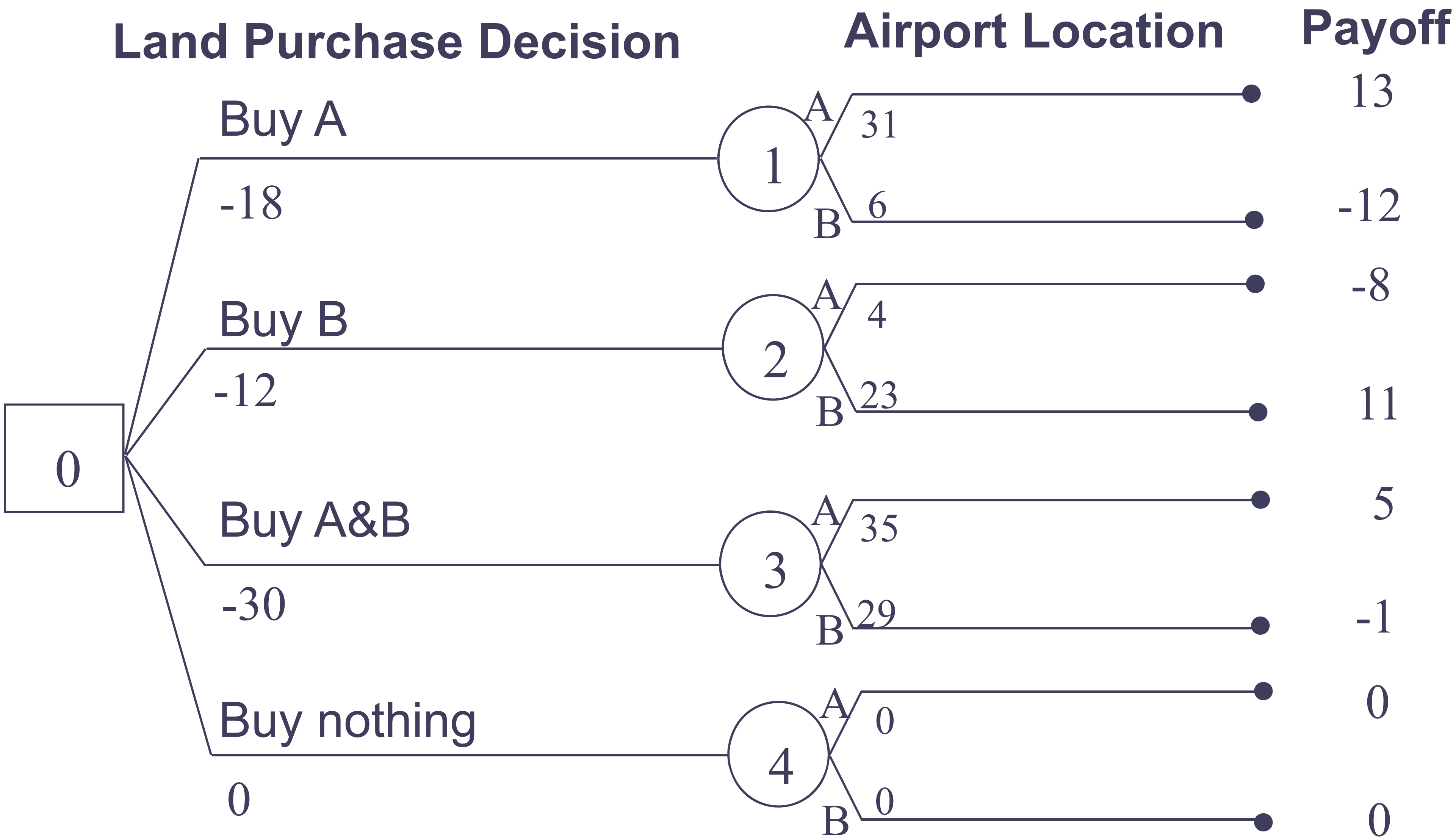
  = $11.8 - $3.4 = $8.4 => Maximum price to pay for PI

- It will always the case that,

  EVPI = minimum EOL

	A	B	C	D	E
1		Regret Matrix &			
2		EOL Decision Rule			
3	Land Purchased	Airport is Built at Location			
4	at Location(s)	A	B	EOL	
5	A	$0	$23	$13.8	
6	B	$21	$0	$8.4	<--minimum
7	A&B	$8	$12	$10.4	
8	None	$13	$11	$11.8	
9					
10	Probability	0.4	0.6		
11					
12	D5	=SUMPRODUCT($B$10:$C$10,B5:C5)			

A Decision Tree for Magnolia Inns



Decision trees (a graphical representation of decision problems) will be discussed in Lecture 12.

Example 2

- A company is choosing a motorised mechanism for a new toy, a dancing ballerina (Tippi-Toes), that is currently under development. Demand is uncertain, but the company is confident that demand will be: Light (25,000 units), Moderate (100,000 units) or Heavy (150,000) units.
- The payoff table for each mechanism (in $) is given below.
- What mechanism should the company choose?

	Action: Choice of Mechanism			
Event	Gears and Levers	Spring	Weights and Pulleys	Pneumatic
Light	25,000	-10,000	-125,000	-300,000
Moderate	400,000	440,000	400,000	300,000
Heavy	650,000	740,000	750,000	700,000

Dominated Alternatives

- Comparing the payoffs for Weights and Pulleys with the Pneumatic mechanism in the payoff table below, it is evident that no matter what the outcome, a greater payoff is always obtained by using Weights and Pulleys.
- In this case, Weights and Pulleys dominate the Pneumatic mechanism.
- Thus choosing the Pneumatic movement is an inadmissible act. We won't consider this option further in the following analysis.

	Action: Choice of Mechanism			
Event	Gears and Levers	Spring	Weights and Pulleys	Pneumatic
Light	25,000	-10,000	-125,000	-300,000
Moderate	400,000	440,000	400,000	300,000
Heavy	650,000	740,000	750,000	700,000

- Concept of dominated alternatives (actions):
  - If action *a(i)* is dominated, then in no state of the nature is *a(i)* better than other action *a(k)*, and in at least one state of the nature *a(i)* is inferior to *a(k)*.
  - Thus, if action *a(i)* is dominated, there is no reason to choose *a(i)*, since under any scenario *a(k)* would be at least as good a choice, or better choice.

Example 2: Maximax Rule
(Optimistic Approach)

- An optimistic decision maker would choose the decision alternative that has the largest single value in the payoff table. In this case, we choose to use Weights and Pulleys.

	Action: Choice of Mechanism		
Event	Gears and Levers	Spring	Weights and Pulleys
Light	25,000	-10,000	-125,000
Moderate	400,000	440,000	400,000
Heavy	650,000	740,000	750,000
Maximum Payoff	**650,000**	**740,000**	**750,000**

Example 2: Maximin Rule
(Conservative Approach)

- A conservative decision maker would identify the minimum payoff for each decision alternative and choose the alternative with the maximum of these minimum payoffs.

	Action: Choice of Mechanism		
Event	Gears and Levers	Spring	Weights and Pulleys
Light	25,000	-10,000	-125,000
Moderate	400,000	440,000	400,000
Heavy	650,000	740,000	750,000
Minimum Payoff	**25,000**	**-10,000**	**-125,000**

Example 2: Minimax Regret Rule
(Opportunity Loss Approach)

- For the Minimax Regret approach, first compute an opportunity loss table by subtracting each payoff for each event from the largest payoff for each event, then calculate the maximum regret for each decision alternative.
- In the example, choosing the Spring mechanism minimises the maximum regret that results from the wrong decision.

		Regret (Opportunity Loss)		
Event	**Event Maximum**	Gears and Levers	Spring	Weights and Pulleys
Light	25,000	0	35,000	150,000
Moderate	440,000	40,000	0	40,000
Heavy	750,000	100,000	10,000	0
	Maximum Regret	**100,000**	**35,000**	**150,000**

Example 2: EMV Decision Rule

- Assuming that the probability of Light demand is 0.1, Moderate demand is 0.7, and Heavy demand is 0.2, the expected value under each choice is calculated below.

		Action: Choice of Mechanism		
Event	Probability	Gears and Levers	Spring	Weights and Pulleys
Light	0.10	25,000	-10,000	-125,000
Moderate	0.70	400,000	440,000	400,000
Heavy	0.20	650,000	740,000	750,000
	Expected Payoff	412,500	455,000	417,500

- The action of Spring mechanism has the largest payoff and should be chosen under the EMV decision rule.

Example 2: EVPI Calculation

- Step 1: Determine the maximum payoff for each state of nature (event) (i.e. with perfect information) → 25,000, 440,000, 750.000
- Step 2: Compute the expected value with perfect information

  (EV with PI) →(25,000 × 0.1)+(440,000 × 0.7)+(750,000 × 0.2)=460,500
- Step 3: Compute the expected value (EMV) for each alternative (action)

  → 412,500; 455,000; 417,000
- Step 4: EVPI = EV with PI – maximum EMV

  = \$460,500 – \$455,000 = \$5,500

		Action: Choice of Mechanism			
Event	**Probability**	Gears and Levers	Spring	Weights and Pulleys	**Maximum Payoff**
Light	0.10	25,000	-10,000	-125,000	25,000
Moderate	0.70	400,000	440,000	400,000	440,000
Heavy	0.20	650,000	740,000	750,000	750,000
	EMV	412,500	455,000	417,500	460,500

Lecture Exercise

Question:

- News vendor Phyllis Pauley sells newspapers, and each day she must determine how many newspapers to order
- Phyllis pays the company 20c for each newspaper and sells the paper for 25c each. Newspapers that are unsold by the end of the day are worthless.
- Phyllis knows that each day she can sell between 6 and 10 papers, with each possibility being equally likely.

- Set up a payoff table for the problem.
- Find the optimal order quantity for Phyllis, using:

  (a) Maximax rule, (b) Maximin rule, and (c) Minimax Regret rule.

Lecture Exercise

- Payoff Table

		Papers Demanded			
Papers Ordered	6	7	8	9	10
6	30	30	30	30	30
7	10	35	35	35	35
8	-10	15	40	40	40
9					
10					

- Finish off the exercise in the tutorial.

Conclusion

- Go through the lecture examples to understand
  - The construction of payoff tables and regret tables;
  - The use of Maximax, Maximin, Minimax Regret, EMV and EOL decision rules;
  - The EVPI calculation.
- This week’s tutorial
  - Decision Making under Uncertainty
- Week 12’s lecture (Lecture 12) and tutorial:
  - Decision Trees, Expected Values of Sample Information, Bayes’ Theorem