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Decision Support for Bus Operations under Uncertainty: a Fuzzy Expert System Approach

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This paper presents a novel approach to handle uncertainty in the daily operation of urban bus transit systems using a straightforward framework. A fuzzy expert system is developed to properly formulate the current bus operation problem in Taiwan. To facilitate the construction of the fuzzy knowledge base and to allow feasible action alternatives of multiple action types to be generated for multiple abnormal conditions on multiple bus routes, the system is composed of three decision support modules: demand, supply, and decision. The demand module determines the extra trip requirement to cope with multiple abnormal conditions. The supply module works out the capacity available for providing extra trips. The decision module generates action alternatives to meet the extra trip requirement, and ranks them based on the degree of feasibility modelled by the willingness/ability of drivers available for providing extra trips. As a result, the best feasible action alternatives can always be generated if supply exceeds demand. In cases where supply is less than demand, the best possible action alternatives based on the extra available capacity are suggested. Examples are given to illustrate how the system can support decision making under different situations. © 1998 Elsevier Science Ltd. All rights reserved

Key words—bus operations, decision support, expert systems, fuzzy sets, uncertainty

1. INTRODUCTION

THE PUBLIC BUS system is the most common means of urban mass transit world wide [1]. The business of bus operating companies is to provide service to meet travel demand. The supply of bus service is realised by assigning drivers and vehicles to designated routes for pre-planned trips based on a bus schedule. The bus schedule is determined and maintained by a planning process which involves four phases [2]: (1) strategic planning for setting the service level by determining bus line networks and offered trips; (2) operational planning for determining vehicle and driver schedules to carry out bus operations; (3) implementation for controlling the execution of bus schedules and dealing with daily variations in operation; and (4) critical appraisal for analysing planned and actual operations to provide feedback for planning future schedules.

The operational planning phase has attracted most of the research attention in the past three decades. Computer techniques, mathematical programming and simulation models are commonly used to determine bus schedules by optimisation or heuristic algorithms [3–6], with little reported work on generating bus schedules in a real-time [7, 8] or decision support [9] fashion. Despite a tremendous effort being spent and significant progress being made in generating an optimum bus schedule, bus companies realise that their daily bus schedules are seldom followed on their bus routes. Taking appropriate action for managing day-to-day variations in bus operations remains a challenging job for practitioners. The dynamic operating environment of a bus system makes this a routine operation, to which considerable operational efforts have to be devoted in order to maintain a given service level.

This paper addresses the daily decision problem of handling uncertainty in carrying out and controlling the bus schedule for the urban bus system in Taiwan. A new approach incorporating a fuzzy expert system is proposed to generate feasible alternatives of multiple action types for coping with multiple abnormal conditions on multiple bus routes, based on the current operational practice in Taiwan. In subsequent sections, we first discuss the uncertainty associated with bus operations and then present the approach with illustrations and examples.

2. UNCERTAINTY IN BUS OPERATIONS

Uncertainty associated with bus operations is twofold. First, the operating environment causes unexpected fluctuations in both travel demand and service supply, resulting in frequent variations in operation. Second, data available to describe the abnormal conditions are often imprecise, and knowledge required to handle the problem is usually imperfect. As a result, the decision making process is often not a precise one.

The service level of a bus route is determined by the number of trips offered for a given travel demand. The planned trips may be affected by unexpected or abnormal conditions, such as driver absence, vehicle breakdown, and temporary road congestion, resulting in a lower service level. An unexpected increase in demand may also lower the service level. An effective bus operations system should detect these abnormal conditions and take appropriate action promptly. To maintain a given service level, in addition to planned trips, extra trips may be needed to cope with these abnormal conditions. The flexibility and adaptability of a bus system facilitate prompt action be taken to adjust the bus schedule by providing extra trips to a dynamic operating environment.

To determine the number of extra trips required, information on abnormal conditions is needed. In most situations, abnormal conditions are expressed by vague statements (such as the demand is very high), or by imprecise data (such as the operating speed is about 15 km/h). This is because human judgement on abnormal conditions is often intrinsically imprecise.

In the current bus operations practice in Taiwan, driver assignment is associated with vehicle deployment which is carried out in accordance with the bus schedule under the control of human dispatchers. In this daily operation, experienced dispatchers use their intuition and knowledge to manage abnormal conditions by a rule of thumb. However, the current emergency management of abnormal conditions handling is sometimes not totally certain or consistent, due to the imprecise nature of human knowledge. In particular, there are limitations on the amount of information that humans can effectively handle [10]. This makes it difficult for the human dispatchers to explore the whole problem space for all possible solutions in a complex situation where multiple abnormal conditions occur simultaneously, or multiple bus routes are involved. This uncertainty is increased when experienced dispatchers are not available. It is evident that a more structured approach for handling the uncertainty of bus operations is required in order to support managerial judgements and to improve the effectiveness of decision making.

3. UNCERTAINTY MANAGEMENT WITH FUZZY EXPERT SYSTEMS

Conventional mathematical programming and simulation models have never handled uncertain information satisfactorily. The development of expert system techniques has provided an effective way of reasoning with uncertain information [11]. An expert system in a problem domain emulates the decision making process of a human expert. It is best applicable to problems that have limited domains and well-defined expertise. In particular, the knowledge it incorporates is acquired through experience and observation. The decision making process for taking action to manage bus operations under unpredictable or uncertain conditions is ideally suited for emulation by an expert system approach.

Most classic expert systems have modelled uncertainty informally [11]. The methods most commonly used are based on Bayesian probabilities [12] and some form of confidence or certainty factors [13]. While these methods prove their usefulness in handling uncertainty of probabilistic or stochastic nature, they are essentially carried out in an *ad hoc* manner with unpredictable interpretations, and the uncertainty values are assigned outside the model rather than as part of the knowledge representation itself [14].

In contrast to stochastic uncertainty, a large fraction of uncertainties in real-life situations are in the class of fuzziness, which is concerned with the vague description of the semantic meaning of the events, phenomena, or statements in a natural language [15]. With a strict mathematical framework, fuzzy set theory [16] has proven to be a very powerful modelling tool for coping with fuzziness. Amongst its wide range of applications, fuzzy set theory has shown advantages in modelling traffic and transportation systems [17, 18], and solving dispatching and assignment problems [19, 20], characterised by subjectivity, ambiguity, uncertainty and imprecision. In the applications of expert systems, fuzzy set theory has demonstrated its applicability in modelling the fuzziness of uncertain information manipulated by an expert system [21, 22], constituting a fuzzy expert system. It provides a natural framework for the management of uncertainty in expert systems for representing imprecise knowledge and inferring from knowledge bases which are imprecise, incomplete, or not totally reliable [23].

The uncertainty in bus operations is of a fuzzy nature. Expert dispatchers often do not express uncertainty and imprecision in their knowledge and observations in probability values, but in vague linguistic terms. This suggests the use of a fuzzy expert system approach for effective modelling of the uncertain bus operations problem, in which fuzzy sets and fuzzy logic are incorporated into its knowledge representation and reasoning process to formulate the imprecise way the expert dispatchers communicate their knowledge and make their decisions. The application of fuzzy sets and fuzzy logic can broaden the usefulness of expert systems, allowing operation in situations where precise values are not known or are not necessary for drawing conclusions [24]. This approach can thus reduce the dispatchers' burden for precise data gathering and manipulation, and enable them to focus on the monitoring of abnormal conditions.

4. THE FUZZY EXPERT SYSTEM APPROACH

There are a number of methods available for representing expert knowledge in an expert system [25–27]. Based on the current practice of bus operations in Taiwan, the production rule method can best represent how the expert dispatcher takes action to handle abnormal operating conditions. Knowledge of expert dispatchers is represented as a set of conditional fuzzy rules. Each rule takes the form of:

$$\text{IF } <\textit{condition}> \text{ THEN } <\textit{action}> \quad (1)$$

or

$$\text{IF } X \textit{ is } A \text{ THEN } Y \textit{ is } B \quad (2)$$

where X and Y are linguistic variables representing fuzzy variables in the antecedent and consequent statements respectively, and A and B are linguistic values or terms taken by X and Y respectively. For instance, in the fuzzy rule of "IF travel demand is high THEN extra trip requirement is medium", the linguistic variable 'travel demand' in the antecedent takes the linguistic term 'high', and the linguistic variable 'extra trip requirement' in the consequent takes the linguistic term 'medium'.

Linguistic terms A and B in (2) are modelled by fuzzy sets whose membership functions are denoted by $\mu_A(x)$ and $\mu_B(y)$ as

$$\mu_A(x):X \longrightarrow [0,1], x \in X \text{ and}$$
$$\mu_B(y):Y \longrightarrow [0,1], \quad y \in Y. \qquad (3)$$

which represent the grade of membership of the element x in A, and y in B respectively. A membership function is defined on a linguistic variable by assigning a value between 0 and 1 to each element in the domain to indicate the degree of membership, which can be interpreted as a possibility distribution over the real line. For example, for the linguistic variable 'travel demand', μ_{high} (0.65) = 0.5 means that the degree of membership of the travel demand 0.65 (a value within the range of values for representing the travel demand) belonging to the term 'high' is 0.5. In other words, the possibility of the travel demand 0.65 being high is 0.5.

This set of fuzzy rules constitutes the knowledge base of the system which expresses all of the system's expertise on what action to take for given conditions. A good discussion on the use of fuzzy production rules is given by Negoita [28]. The use of linguistic terms modelled by fuzzy sets has been found intuitively easy to express the imprecision or uncertainty of human knowledge and judgement in the knowledge base [29], and will provide accurate solutions [11].

What is stated in Equation (2) is the simple form of fuzzy rules used in the system. To handle multiple dependent conditions which occur simultaneously, rules with multiple antecedents combined with the intersection operator 'AND' are used. For example, IF X *is* A AND Z *is* C THEN Y *is* B. In actual bus operations, a combination of different action types may be required to provide sufficient extra trips to cope with abnormal conditions. It would create complications if all action types were embedded in the rules as the consequents. To facilitate the knowledge acquisition process, and to simplify the decision rules needed, all the rules used have only one single consequent. The common consequent statement used in the rules, determines the extra trip requirement expressed by a linguistic value.

To generate action alternatives of multiple action types for meeting the extra trip requirement and to examine their feasibility, the system is composed of three decision support modules which incorporate both the demand and supply factors of bus operations, as shown in Fig. 1. In addition to the generation of multiple feasible alternatives, this feature also facilitates the handling of multiple bus routes in which interlining assignments are a common practice. Interlining allows multiple routes to support each other by assigning a driver to work on two or more routes during a day's work.

Abnormal conditions in each route are individually inputted into the demand module and the extra trip requirement for each route is independently determined. In the supply module, the total number of extra trips that can be provided by drivers from all routes are aggregated for each route based on action types, in which drivers may take extra runs or a duty change, and may be assigned to any route based on their willingness/ability. The decision module then generates all possible action alternatives for each route based on the extra trip requirement obtained in the demand module. These action alternatives are associated with the degree of feasibility given by the possible extra capacity obtained in the supply module, on which a ranking order is used to support decision making.

In what follows, we describe how the basic elements of the three modules are developed and implemented, and then present a numerical example to illustrate how they work.

5. THE DEMAND MODULE

The demand module involves the development of a fuzzy knowledge base for determining the number of extra trips required for specific abnormal conditions affecting the service level. Among the abnormal conditions to be dealt with by the system, the dispatcher can clearly determine the number of trips to be affected by driver absence and vehicle breakdown. The schedule disturbance problem of this kind is of a crisp nature, and has been well addressed and dealt with by a number of research papers [30–32]. There is no need to include these two crisp conditions in the fuzzy knowledge base, as they are independent of other fuzzy conditions. They are thus calculated separately and added to the output of the fuzzy knowledge base.

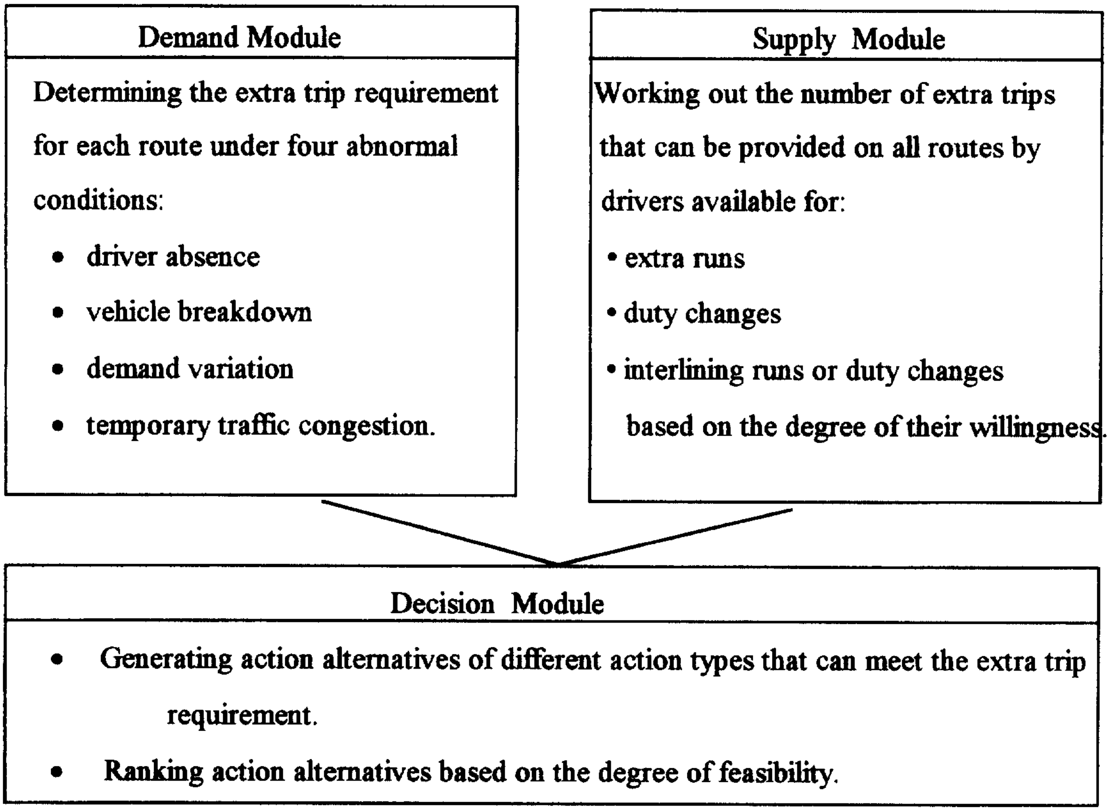

Fig. 1. Decision support modules of the system

Fuzzy conditions to be considered in the system are the states of travel demand, demand variation, and temporary road congestion. These conditions are dependent, and are therefore incorporated into a single knowledge base. Travel demand is represented by the average load factor (the ratio of passengers actually carried versus the total passenger capacity of a vehicle). Demand variation is measured by the load factor variation, given as $(LF_c - LF_p)/LF_p$, where LF_c and LF_p are the load factors of the currently and previously completed trips respectively. Road congestion is indicated by the average effective operating speed (km/h) of a vehicle. All these three fuzzy conditions are judged by the knowledge and general observations of the dispatcher by making reference to the operating conditions reported by drivers returning to the depot. They are the input variables to the knowledge base.

A fuzzy condition is described by means of a linguistic variable whose states are denoted by a given set of linguistic terms. In the demand module, very low (VL), low (L), medium (M), high (H), and very high (VH) are the term set used for the load factor (travel demand) and operating speed (road congestion). Negative big (NB), negative small (NS), almost zero (Z), positive small (PS), and positive big (PB) are used to describe the five states of the load factor variation (demand variation). For the output linguistic variable, the terms used to describe the extra trip requirement are very low (VL), low (L), medium (M), high (H), and very high (VH).

The linguistic terms used for each linguistic variable described above, are characterised by a group of overlapping fuzzy sets whose membership functions cover the complete range of values for the variable. To properly transform the expert knowledge into the fuzzy rules using these linguistic terms, their corresponding membership functions are defined through a manual knowledge acquisition process by questionnaires and interviews.

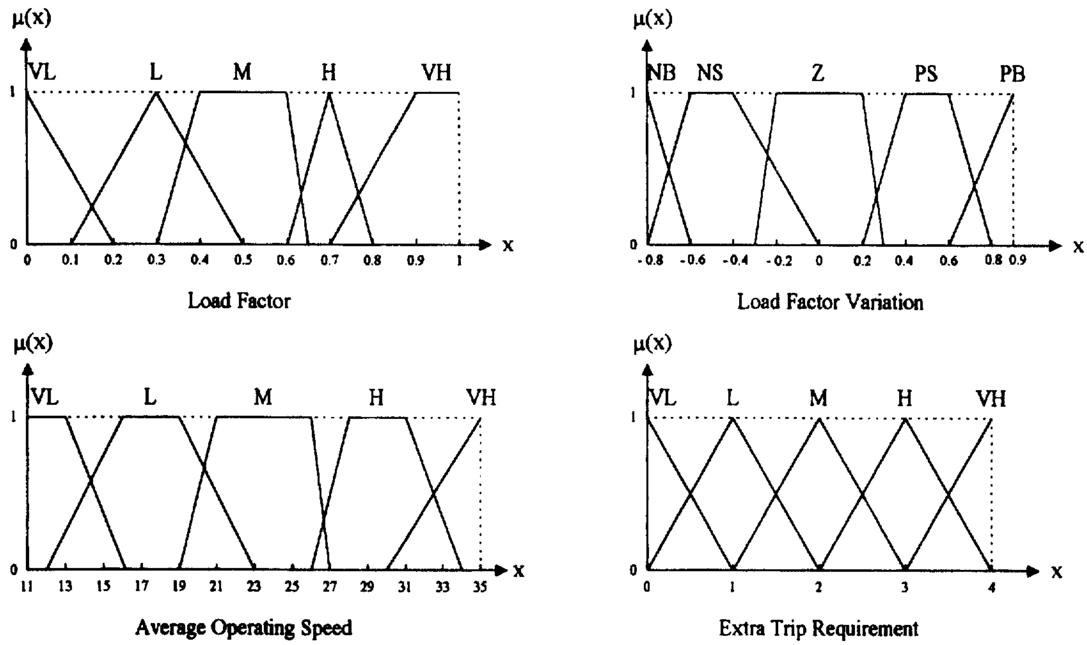

Fig. 2. Membership functions used in the fuzzy knowledge base

5.1. Knowledge representation

Experimental methods for the acquisition of membership values and for the construction of membership functions from membership values obtained are well reviewed by Turksen [33], who regards the direct rating as a preferred acquisition method. For computational efficiency and ease of data acquisition, trapezoidal or triangular fuzzy sets are used to describe the linguistic terms used in the fuzzy knowledge base. Their values, or intervals of values, are obtained by asking expert dispatchers through questionnaires.

A questionnaire survey was sent out to 20 expert dispatchers in Taiwan and 18 effective responses were received. For each linguistic term A to be used in a linguistic variable X for an input fuzzy condition, with a range of values given, the question asked was in the following form:

"What is the interval of possible values of A in X, and what is the most possible value or the interval of the most possible values?"

The intervals obtained from the experts were used to construct their individual membership functions of a triangular or trapezoidal shape for the linguistic term. The median of 18 membership functions was selected for use in the fuzzy knowledge base. The membership functions for the extra trip requirement were obtained by interviewing experts to reflect the current operational practice in Taiwan. Figure 2 shows the results.

There are basically four methods used in the literature for deriving a fuzzy rule base [34]. The most natural and straightforward method is to obtain the rules directly from the human expert, since the fuzzy set theory can effectively model and reason with vague linguistic information that the human expert uses to make decisions.

With the linguistic terms and their membership functions defined in Fig. 2, the fuzzy rules used in the demand module were constructed from the knowledge of expert dispatchers through interviews, which were conducted at a number of bus companies in Taiwan. All information was recorded and reviewed for accuracy and consistency based on our understanding of the process. A set of 25 fuzzy rules was thus constructed for determining the number of extra trips required on a route under unexpected conditions of demand variation and/or temporary traffic congestion. Table 1 shows these fuzzy rules. For example, Rule 1 is "If the load factor variation is negative big (NB), then the extra trip requirement is very low (VL)". Rule 25

Table 1. Fuzzy rules in the knowledge base

Rule	Load factor	IF (Antecedent) Load factor variation	Operating speed	THEN (Consequent) Extra trip requirement
1		NB		VL
2		NS		VL
3			H	VL
4			VH	VL
5			M	VL
6	M	PS		L
7	M	PB		L
8	M		L	L
9	M		VL	L
10	M	Z	L	VL
11	M	PB	VL	M
12	H		L	M
13	H		VL	M
14	H	Z		L
15	H	PS		M
16	H	PB		M
17	H	Z	L	L
18	H	PB	VL	H
19	VH	Z		M
20	VH	PS		H
21	VH	PB		H
22	VH		L	H
23	VH		VL	H
24	VH	Z	L	M
25	VH	PB	VL	VH

states that "If the load factor is very high (VH), the operating speed is very low (VL), and the load factor variation is positive big (PB), then the extra trip requirement is very high (VH)".

The fuzzy rules in Table 1 reflect how the expert dispatchers manipulate their knowledge under different situations, in which not all three input conditions are used in each rule. The first five rules are designed to represent situations where no action is needed, and which sets the lowest bound of the output to 0.3. These fuzzy rules are readily understood and can be easily modified if necessary. They are generic in nature and can be adapted to a specific operating environment by incorporating operation-dependent or operator-dependent knowledge.

To achieve the overall expert conclusion from these fuzzy rules under given conditions, an inference process is required. With the fuzzy rules or implications used for reflecting the system's behaviour, this process can be easily implemented.

5.2. *Inference process*

The inference process used is based on the composition of a given fact (the input) with a given set of fuzzy rules, known as the compositional rule of inference [35] or the generalised modus ponens [36]. There are quite a few compositional methods that have been developed, each of which represents a special inference engine and has its own significance and applications [37]. The max-min compositional method is used, as it can closely reflect the approximate and interpolative reasoning used by humans based on linguistic IF-THEN rules.

In the inference process, all fuzzy rules are effectively executed in parallel. The given input determines the degree of truth (or match) for the corresponding antecedents in each fuzzy rule. This limits the degree of truth for the consequent in the rule to no more than the same degree for any of its antecedents. The conclusion (the output) of each rule is thus obtained by truncating the fuzzy set for the consequent in the rule by the minimum degree of truth for its antecedents. For example, Fig. 3 shows the degree of truth for the three antecedents of Rule 25 are 0.5, 0.33 and 0.67 respectively, given that: (1) the load factor is 0.8; (2) the load factor variation is 0.7; and (3) the average operating speed is 14 km/h. The hatched area in the consequent in Fig. 3 is the resultant fuzzy conclusion.

Taking the union (or the maximum operation) of all the truncated fuzzy sets for all

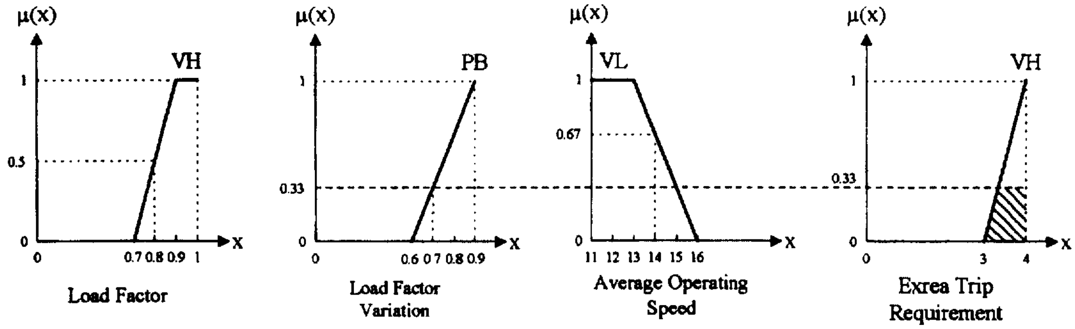

Fig. 3. Output of Rule 25 for a given input

effective fuzzy rules constitutes the overall inference conclusion, which is also a fuzzy set. This aggregated fuzzy set thus reflects the degree of contribution from each antecedent in the given set of fuzzy rules. To translate the aggregated fuzzy set obtained by the inference process into an executable action, a defuzzification method is required. The defuzzification of a fuzzy set will result in a single scalar value. Although there exists no optimal method, we can choose an appropriate one based on an axiomatic and/or empirical justification and the computational effort required [15]. The mean of maxima (MOM) method is used because, (1) it is generally preferable for decision analysis problems where the prescribed action is to be selected from a set of discrete alternatives [38]; (2) it will not produce unplausible results due to the use of unimodal fuzzy sets [15]; and (3) it is computationally fast. The concept of the MOM method is to take the most plausible action that corresponds to the maximum membership, and the result is the mean of the corresponding interval of most possible actions. This defuzzification result is to be rounded up, since an integer solution is necessary for the extra trip requirement.

5.3. Verification and testing

To examine whether the expert's knowledge has been transformed properly, a simulation process has been conducted. It is designed to verify the effectiveness of the fuzzy knowledge base and the inference process in determining the extra trip requirement under various conditions. In an attempt to cover samples of all possible extreme or special conditions for verifying the system [39], the states of the three input variables are changed from the lowest to the highest, and from the highest to the lowest respectively, resulting in eight scenarios examined. The states of the three input variables are equally divided into 100 scales. As a result, 100 test conditions of different input values are examined in each scenario. All the eight scenarios tested, with 800 different conditions altogether, have a similar result in terms of the effectiveness of the conclusion. Therefore, we present only 100 conditions tested in one scenario here as an example.

Figure 4 shows the number of extra trips required for the scenario in which the load factor is increased from 0 to 1, the load factor variation percentage is increased from -0.8 to 0.9, and the average operating speed is decreased from 40 km/h to 11 km/h. The simulation result shows that the extra trip requirement increases from 0.3 to 3.1 as the overall operating condition is changed from best to worst. The extra trip requirement for quite a few conditions at the initial stages is 0.3, which is the lowest bound of the system output. This is interpreted by the system as the option of 'do-nothing', that is, no action is required. In this way, the system is not too sensitive to small changes in operating conditions, which exactly reflects how the expert dispatchers make decisions in actual settings. One interesting point that can be observed from Fig. 4 is that the integer solution tends to remain steady for more conditions than the non-integer solutions. This is because the integer value has the highest possibility in its corresponding linguistic value for the linguistic variable of extra trip requirement. We regard this phenomenon as favorable, because an integer solution is eventually required.

In addition to the simulation analysis, the knowledge base and the inference process have

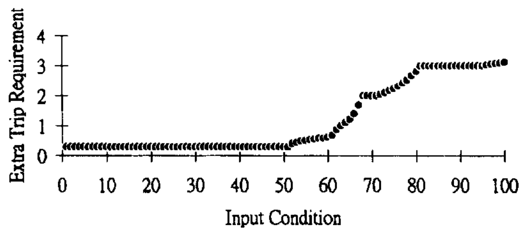

Fig. 4. Extra trip requirements for simulated conditions

also been tested on a number of actual conditions on bus routes in Taipei, Taiwan. The results have suggested that the output of the system is consistent with the decisions made by the expert dispatchers.

6. THE SUPPLY MODULE

To ensure feasible action is always taken, the supply module works out the capacity available for providing extra trips. With no spare drivers available, the supply of extra trips for a route can be made by extra runs or duty changes performed by drivers currently scheduled on the route or temporarily transferred from another route.

Depending on when the action is to be taken during a day, there are certain ways for a driver with morning or evening straight duty, or split duty, to provide extra trips. In the bus system examined in Taipei, Taiwan, a driver taking straight duty will provide four scheduled trips continuously for the whole morning or evening, which covers the morning or evening peak. A driver taking split duty will provide two scheduled trips each for both the morning and evening peaks. During a day's normal work, a driver takes only either one morning or one evening straight duty, or one split duty.

A driver can provide extra trips by performing extra runs at unscheduled times or taking a duty change. A duty change means that (1) a morning straight duty driver takes split duty for the evening peak; (2) an evening straight duty driver takes split duty for the morning peak; or (3) a split duty driver takes either morning or evening straight duty. Thus, a driver taking a proper duty change will provide six trips altogether during a day's work, resulting in two extra trips provided. Due to the work time restrictions within a day, a driver cannot take both extra runs and a duty change, and can make only one duty change. The time-dependent ways of extra capacity supply will affect the willingness of the drivers available. For simplicity, the willingness of providing extra trips by drivers also includes their performing ability in terms of timing and in particular, interlining working, which reflects the possible extra capacity supply.

The extra capacity to be provided by an available driver k ($k = 1,2,\ldots,m_i$) currently assigned on a route i ($i = 1,2,\ldots,n$) is expressed by the membership function of the driver's willingness as $\mu_{S(i,j,k,l)}(x)$, indicating that it is to be performed on route l ($l \in \{1,2,\ldots n\}$) by performing action j ($j = 1$ for extra runs, or $j = 2$ for a duty change). The membership function is obtained by asking each driver about their willingness to take either extra runs or a duty change, and the preferred route to be assigned. In the case of extra runs, the driver is asked to specify two additional crisp values, *a1* and *a2*. This specification indicates that the driver would like to take *a1* or less extra runs, and take no more than *a2* extra runs. As a result, the membership function of taking either extra runs or a duty change is shown in Fig. 5 (a) or (b), defined as

$$\mu_{S(i,1,k,l)}(x) = \begin{cases} 1; & x \le a1 \\ \dfrac{a2 - x}{a2 - a1}; & a1 < x \le a2 \\ 0; & x > a2 \end{cases} \qquad (4)$$

$$\mu_{S(i,2,k,l)}(x) = \begin{cases} 1; & x \le 1 \\ 0; & x > 1 \end{cases} \qquad (5)$$

As a trapezoidal fuzzy number, $\mu_{S(i,j,k,l)}(x)$ of all drivers k ($k = 1,2,\ldots,m_i$) on all routes i ($i = 1,2,\ldots,n$) can be aggregated as

$$\mu_{TA(i,j)}(x) = \sum_{k=1}^{m_i} \mu_{s(i,j,k,i)}(x) \qquad (6)$$

and

$$\mu_{TI(i,j)}(x) = \sum_{l=1}^{n} \sum_{k=1}^{m_i} \mu_{s(i,j,k,l)}(x) j\; l \neq i. \qquad (7)$$

$\mu_{TA(i,j)}$ and $\mu_{TI(i,j)}$ are the total extra capacity available on each route i ($i = 1,2,\ldots,n$) which are performed by each action type j ($j = 1,2$) and provided by currently assigned drivers

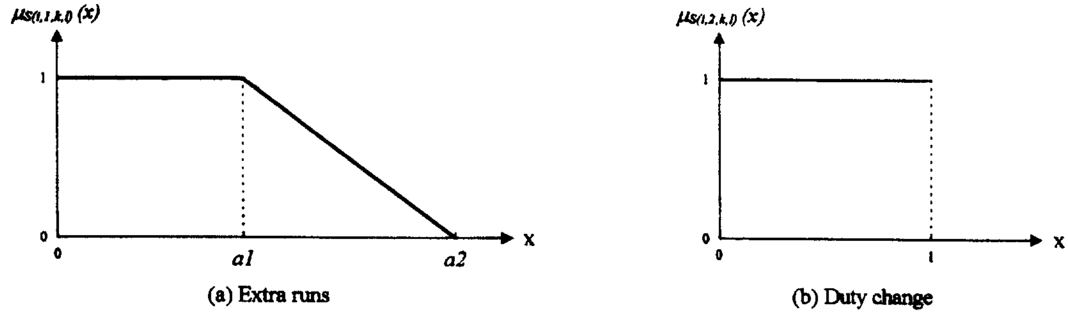

Fig. 5. Membership functions of extra capacity supply

(TA) and interlining drivers (TI) respectively. The summation of fuzzy sets in Equation (6) and Equation (7) is based on fuzzy interval arithmetic [40].

7. THE DECISION MODULE

The decision module generates p action alternatives for each route i ($i = 1,2,\ldots,n$) based on its corresponding extra trip requirement ($R(i)$) obtained in the demand module. Each action alternative h ($h = 1,2,\ldots,p$) consists of a combination of four action types: (1) extra runs by assigned drivers ($ra(i,1,h)$), (2) duty changes by assigned drivers ($ra(i,2,h)$), (3) extra runs by interlining drivers ($ri(i,1,h)$), and (4) duty changes by interlining drivers ($ri(i,2,h)$), which satisfies

$$R(i) = ra(i,1,h) + ra(i,2,h) \times 2 + ri(i,1,h) + ri(i,2,h) \times 2 \qquad (8)$$

where a duty change is equivalent to the supply of two extra trips. This is exercised by the bus system modelled, as explained in the previous section.

The degree of feasibility ($F(i,h)$) of each alternative h ($h = 1,2,\ldots,p$) for each route i ($i = 1,2,\ldots,n$) is determined by the possible extra capacity obtained in the supply module (given by Equation (6) and Equation (7)), defined as

$$F(i,h) = j = 1{,}2\mu_{TA(i,j}(ra(i,j,h) \wedge \mu_{TI(i,j}(ra(i,j,h) \qquad (9)$$

where $\wedge$ denotes the minimum operation.

All p alternatives for route i are ranked based on $F(i,h)$, where $h = 1,2,\ldots,p$. The ranking result facilitates the dispatcher's selection of an appropriate action from the best feasible alternatives. This can always be expected if supply ($\mu_{TA(i,j)}$ and $\mu_{TI(i,j)}$) exceeds demand ($R(i)$). However, there may be no feasible alternative which completely meets the extra trip requirement under situations where supply is less than demand, meaning that extra trips to be offered by available drivers cannot meet the extra trip requirement. In this case, the decision module will generate the best possible feasible alternatives that can partially meet the requirement.

8. EXAMPLE

The fuzzy expert system presented above can generate alternatives of multiple action types, to deal with multiple abnormal conditions on multiple routes. To illustrate how the best solutions can be obtained under different circumstances, we use a two-route bus system as an example.

Assume that abnormal conditions are detected on the two bus routes separately at the same time, as follows:

(a) Route 1. A driver is taking one hour off in an emergency which will affect one assigned trip. There is currently a vehicle breakdown on the road which is expected to affect two assigned trips. The current travel demand is very high (the load factor is about 0.9). The demand increase is high (the load factor variation is about 0.3). The road congestion is high (the average operating speed is about 18 km/h).

(b) Route 2. There is no driver absence or vehicle breakdown. The current travel demand is high (the load factor is about 0.7). The demand increase is high (the load factor variation is about 0.3). The road congestion is medium (the average operating speed is about 22 km/h).

Three fuzzy conditions of Routes 1 and 2 are inputted separately into the 25 fuzzy rules of the knowledge base in the demand module. The output results are 2.8 and 1.5 trips respectively. After rounding up the output results, and adding up the extra trips required for driver absence and vehicle breakdown, the extra trip requirements for Routes 1 and 2 are 6 and 2 trips respectively.

We first consider the situation where supply exceeds demand. After inputting the willingness of all drivers on both routes who would like to take extra trips by performing either extra runs or a duty change, the total extra capacity available on Routes 1 and 2 can be calculated by Equation (6) and Equation (7). The membership functions of those action types that can provide extra capacity on each route are shown in Fig. 6.

Based on the extra trips required for Route 1 ($R(1) = 6$) and Route 2 ($R(2) = 2$), the decision module generates 28 and 5 possible alternatives respectively based on Equation (8). Each of these possible alternatives is given a degree of feasibility ($F(i,h)$) by Equation (9). Table 2 shows the ranking of all feasible alternatives within each route whose degree of feasibility is greater than zero.

With the understanding of the current operation, the dispatcher can easily select an action alternative for each route from a limited set of alternatives whose possibility of being feasible is 1.0, as shown in Table 2. For example, if interlining is generally not preferred unless necessary, Alternative 2 in Table 2 can well be the choice as it involves no interlining assignment. If the current operation prefers extra runs to duty changes, Alternative 1 can be considered.

We now consider the situation where supply is less than demand. For example, Table 3

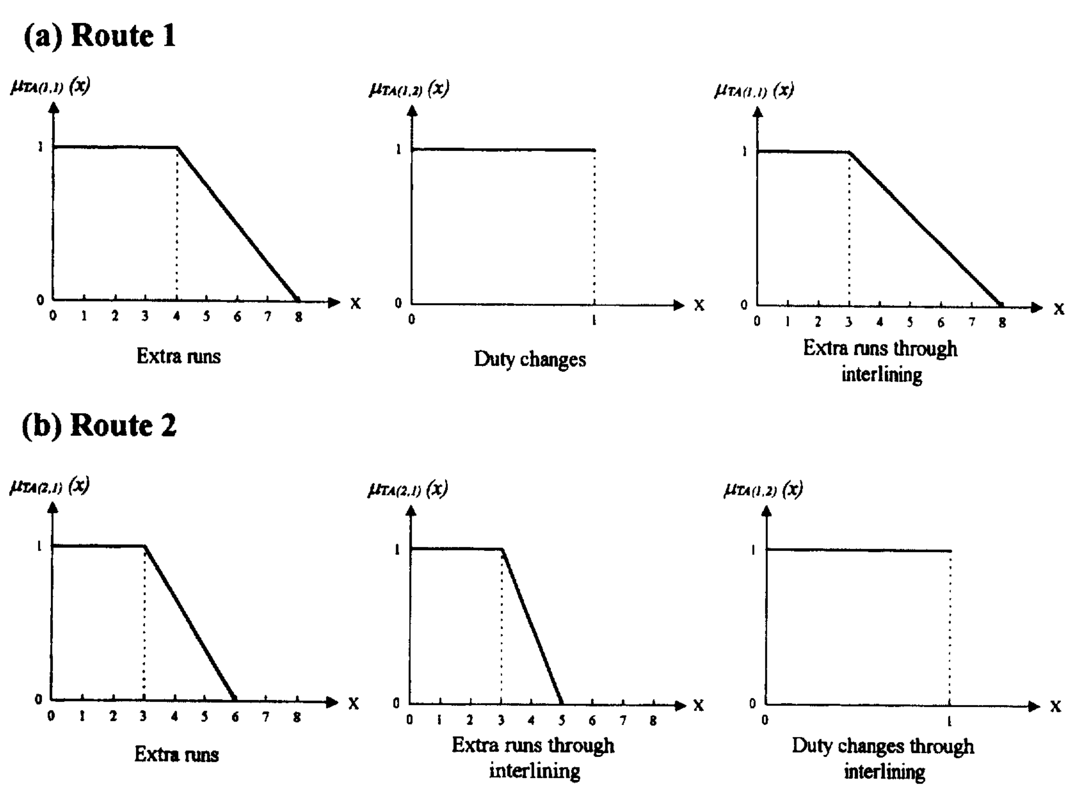

Fig. 6. Membership functions of total extra capacity available which is in over-supply

Table 2. Ranking of feasible alternatives with supply exceeding demand

Route i	Alternative h	Extra run $ra(I,1,h)$	Duty change $ra(i,2,h)$	Interlining extra run $ri(i,1,h)$	Interlining duty change $ri(i,2,h)$	Degree of feasibility $F(i,h)$
1	1	4	0	2	0	1.0
1	2	4	1	0	0	1.0
1	3	3	0	3	0	1.0
1	4	3	1	1	0	1.0
1	5	2	1	2	0	1.0
1	6	1	1	3	0	1.0
1	7	2	0	4	0	0.8
1	8	0	1	4	0	0.8
1	9	5	0	1	0	0.75
1	10	1	0	5	0	0.6
1	11	6	0	0	0	0.5
1	12	0	0	6	0	0.4
2	1	2	0	0	0	1.0
2	2	0	0	0	1	1.0
2	3	1	0	1	0	1.0
2	4	0	0	2	0	1.0

shows the only feasible alternative, among all possible alternatives for each route, if the membership functions of all action types that can provide extra capacity on each route are given as in Fig. 7.

The result of Table 3 suggests that no possible alternatives can completely meet the extra trip requirement. In such situations, other measures, such as negotiating with drivers involved, can be sought to increase the degree of feasibility of the feasible alternative to 1.0. If this cannot be achieved, the dispatcher has to accept the best possible feasible alternatives which can only partially meet the requirement, as shown in Table 4. These best possible feasible alternatives indicate the maximum extra capacity that can be provided, which is one trip short of the extra trip requirement on each route in the example.

To ensure its implementation on existing bus systems, the system developed has been tested on a number of bus routes, with off-line data provided by the computer information system of a bus company in Taipei, Taiwan. After inputting the data required, the feasible action alternatives for a given abnormal operating situation were generated within seconds of the computer time. The action actually taken by the human dispatchers was included in the feasible alternatives generated, which were not all considered when the decision was made. The human dispatchers confirmed that the system output of all feasible alternatives would have given them more options and flexibility in negotiating with available drivers before reaching a decision, because no alternative could be identified as the best in most situations due to the uncertainty associated with demand and supply. In particular, the specification of the willingness of available drivers for providing extra trips would associate each alternative with a degree of feasibility. This would help the human dispatchers focus on the most promising alternatives, with a better understanding of actual extra capacity available. They also confirmed that there would be no difficulty in practice in collecting data required by the system.

As a decision aid to the human dispatchers, the system does provide all possible solutions with necessary information for decision making. This has led us to believe that the system can be incorporated into existing computer-based bus information systems, with the devel-

Table 3. Feasible alternative with supply less than demand

Route I	Alternative h	Extra run $ra(i,1,h)$	Duty change $ra(i,2,h)$	Interlining extra run $ri(i,1,h)$	Interlining duty change $ri(i,2,h)$	Degree of feasibility $F(i,h)$
1	1	2	1	2	0	0.5

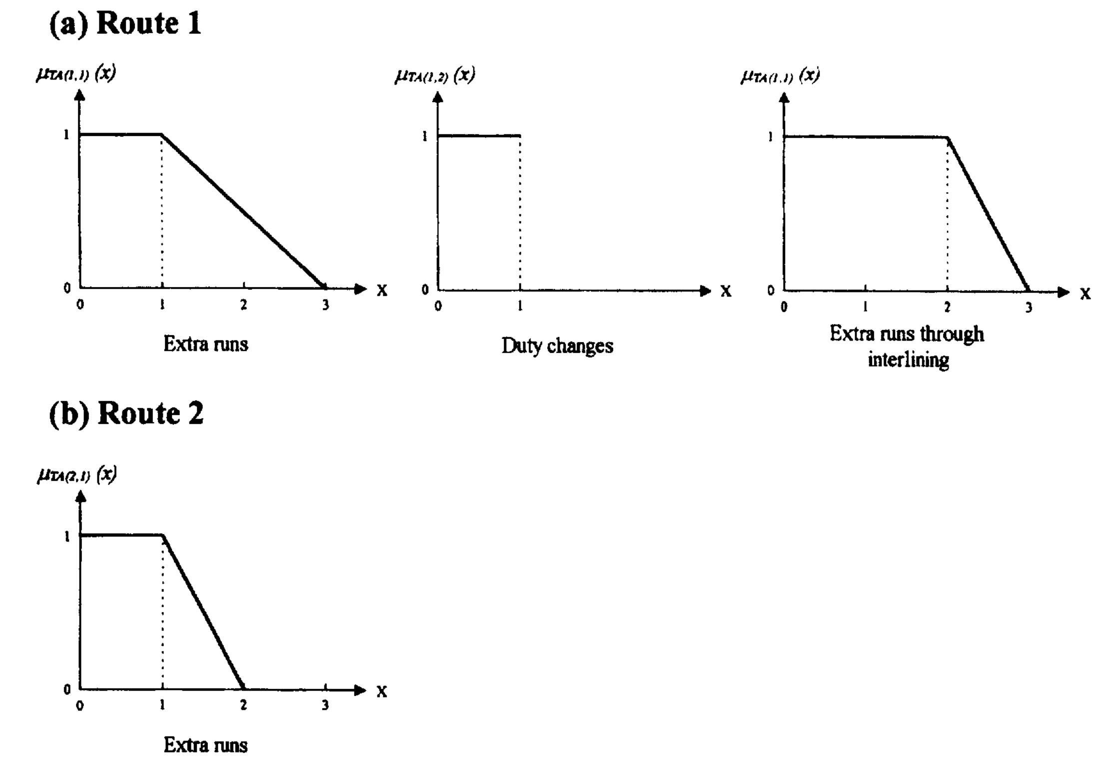

Fig. 7. Membership functions of total extra capacity available which is in under-supply

opment of user interfaces for entering data describing the abnormal conditions and specifying the willingness of available drivers. With a small amount of data required, it is expected that the system can provide the human dispatchers with feasible action alternatives within minutes, including time for data entry, once an abnormal operating situation is identified.

9. CONCLUSION

Bus operation practitioners face the driver and vehicle dispatching problem with day-to-day fluctuations in supply and demand. Unexpected or abnormal conditions can occur simultaneously, and human judgement on abnormal conditions is often intrinsically imprecise. Existing approaches such as mathematical programming, simulation models or classical expert systems cannot handle these practical bus operation problems satisfactorily.

In this paper we have presented a novel approach to provide the best possible solutions for dealing with uncertainty in daily bus operations. Equipped with a rule-based fuzzy expert system, the approach is designed to handle multiple abnormal conditions on multiple bus routes. By incorporating both the demand and supply factors into the system, feasible alternatives of multiple action types are generated and ranked for each route to help the human dispatcher make effective decisions. Although the system has been developed based on the operational practice in Taiwan, the underlying approach provides a conceptual framework to handle uncertainty

Table 4. Best possible alternatives with supply less than demand

Route I	Alternative h	Extra run $ra(i,1,h)$	Duty change $ra(i,2,h)$	Interlining extra run $ri(i,1,h)$	Interlining duty change $ri(i,2,h)$	Degree of feasibility $F(i,h)$
1	29	1	1	2	0	1.0
2	6	1	0	0	0	1.0

in the daily operation of any urban bus system.

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