FIT5186 Intelligent Systems

Week 8 Tutorial Solution

x_1 = (10000 01000 00100 00010 00001)

x_2 = (10001 01010 00100 01010 10001)

x_3 = (10001 01010 10101 01010 10001)

x_4 = (10001 11011 11111 11011 10001)

Initialise weights as $w_{ij} = \frac{1}{26}$ $v_{ij} = 1$ for all i = 1, 2, ….., 25 and j = 1

Present input 1 x_1 = (10000 01000 00100 00010 00001)

$$y_1 = \sum_{i=1}^{25} w_{i1} x_i = \frac{1}{26} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26} = 0.1923$$

Neuron 1 is the winner (only choice)

Vigilance Test: Is $\frac{1}{\| x_1 \|} \| v_1 x_1 \| > \rho$? $\frac{1}{5}$ x 5 = 1 > 0.7 So the test is passed.

Update Weights:

$$w_{ij} = \frac{v_{ij} x_i}{0.5 + \sum v_{ij} x_i} \text{ so } w_{1\,1} = w_{7\,1} = w_{13\,1} = w_{19\,1} = w_{25\,1}$$

$$= \frac{1}{0.5 + 5} = \frac{2}{11} = 0.1818$$

and all other weights $w_{i1} = \frac{1}{26}$ as initialised

$v_{ij} \leftarrow x_i v_{ij}$ so $v_{1\,1} = v_{7\,1} = v_{13\,1} = v_{19\,1} = v_{25\,1} = 1$

and all others are equal to zero

$$w = [\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26} \quad \frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26} \quad \frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26} \quad \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26} \quad \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}]^T$$

(long term memory)

and v = [10000 01000 00100 00010 00001]^T (short term memory)

Present input 2 x_2 = (10001 01010 00100 01010 10001)

$$y_1 = \sum_{i=1}^{25} w_{i1} x_i = \frac{2}{11} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} = 1.063$$

Neuron 1 is the winner (again, it's the only choice)

Vigilance Test: Is $\frac{1}{\| x_2 \|} \| v_1 x_2 \| > \rho$? $\frac{1}{9}$ x 5 = 0.56 < 0.7 So the test is failed.

There are no more neurons to test, so we create a new neuron, $w_{i2} = \frac{1}{26}$, $v_{i2} = 1 \; \forall \; i$

Update Weights:

$$w_{ij} = \frac{v_{ij} x_i}{0.5 + \sum v_{ij} x_i} \text{ so } w_{1\,2} = w_{5\,2} = w_{7\,2} = w_{9\,2} = w_{13\,2} = w_{17\,2} = w_{19\,2}$$

$$= w_{21\,2} = w_{25\,2} = \frac{1}{0.5+9} = \frac{2}{19} = 0.1053$$

and all other weights $w_{i2} = \frac{1}{26}$ as initialised

$v_{ij} \leftarrow x_i v_{ij}$ so $v_{1\,2} = v_{5\,2} = v_{7\,2} = v_{9\,2} = v_{13\,2} = v_{17\,2} = v_{19\,2} = v_{13\,2}$
$= v_{21\,2} = v_{25\,2} = 1$
and all others are equal to zero

$$w = \begin{bmatrix} \frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11} \\ \frac{2}{19}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{19} & \frac{1}{26}\frac{2}{19}\frac{1}{26}\frac{2}{19}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{2}{19}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{2}{19}\frac{1}{26}\frac{2}{19}\frac{1}{26} & \frac{2}{19}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{19} \end{bmatrix}^T$$

(long term memory)

and $v = \begin{bmatrix} 10000\ 01000\ 00100\ 00010\ 00001 \\ 10001\ 01010\ 00100\ 01010\ 10001 \end{bmatrix}^T$ (short term memory)

Present input 3 x_3 = (10001 01010 10101 01010 10001)

$$y_1 = \sum_{i=1}^{25} w_{i1} x_i = \frac{2}{11} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{2}{11} = 1.1398$$

$$y_2 = \sum_{i=1}^{25} w_{i2} x_i = \frac{2}{19} + \frac{2}{19} + \frac{2}{19} + \frac{2}{19} + \frac{1}{26} + \frac{2}{19} + \frac{1}{26} + \frac{2}{19} + \frac{2}{19} + \frac{2}{19} + \frac{2}{19} = 1.0246$$

Neuron 1 is the winner

Vigilance Test: $\frac{1}{\| x_3 \|} \| v_1 x_3 \| > \rho$? $\frac{1}{11}$ x 5 = 0.45 < 0.7 So the test is failed.

Neuron 2 is the next winner (only other choice)

Vigilance Test: $\frac{1}{\| x_3 \|} \| v_2 x_3 \| > \rho$? $\frac{1}{11}$ x 9 = 0.82 > 0.7 So the test is passed.

Update Weights:

$$w_{ij} = \frac{v_{ij} x_i}{0.5 + \sum v_{ij} x_i} \text{ so } w_{1\,2} = w_{5\,2} = w_{7\,2} = w_{9\,2} = w_{13\,2} = w_{17\,2} = w_{19\,2}$$

$$= w_{21\,2} = w_{25\,2} = \frac{1}{0.5+9} = \frac{2}{19} = 0.1053$$

and all other weights $w_{i3} = \frac{1}{26}$ as initialised

$v_{ij} \leftarrow x_i v_{ij}$ so $v_{1\,2} = v_{5\,2} = v_{7\,2} = v_{9\,2} = v_{13\,2} = v_{17\,2} = v_{19\,2} = v_{21\,2}$
$= v_{25\,2} = 1$ and all others are equal to zero

$$w = \begin{bmatrix} \frac{2}{11} \frac{1}{26} \frac{1}{26} \frac{1}{26} \frac{1}{26} & \frac{1}{26} \frac{2}{11} \frac{1}{26} \frac{1}{26} \frac{1}{26} & \frac{1}{26} \frac{1}{26} \frac{2}{11} \frac{1}{26} \frac{1}{26} & \frac{1}{26} \frac{1}{26} \frac{1}{26} \frac{2}{11} \frac{1}{26} & \frac{1}{26} \frac{1}{26} \frac{1}{26} \frac{1}{26} \frac{2}{11} \\ \frac{2}{19} \frac{1}{26} \frac{1}{26} \frac{1}{26} \frac{2}{19} & \frac{1}{26} \frac{2}{19} \frac{1}{26} \frac{2}{19} \frac{1}{26} & \frac{1}{26} \frac{1}{26} \frac{2}{19} \frac{1}{26} \frac{1}{26} & \frac{1}{26} \frac{2}{19} \frac{1}{26} \frac{2}{19} \frac{1}{26} & \frac{2}{19} \frac{1}{26} \frac{1}{26} \frac{1}{26} \frac{2}{19} \end{bmatrix}^T$$

(long term memory)

and $v = \begin{bmatrix} 10000\ 01000\ 00100\ 00010\ 00001 \\ 10001\ 01010\ 00100\ 01010\ 10001 \end{bmatrix}^T$ (short term memory)

Present input 4 x_4 = (10001 11011 11111 11011 10001)

$$y_1 = \sum_{i=1}^{25} w_{i1} x_i = \frac{2}{11} + \frac{1}{26} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26} + \frac{2}{11} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26}$$
$$+ \frac{2}{11} + \frac{1}{26} + \frac{1}{26} + \frac{2}{11} = 1.37$$
$$y_2 = \sum_{i=1}^{25} w_{i2} x_i = \frac{2}{19} + \frac{2}{19} + \frac{1}{26} + \frac{2}{19} + \frac{2}{19} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26} + \frac{2}{19} + \frac{1}{26} + \frac{1}{26} + \frac{1}{26} + \frac{2}{19}$$
$$+ \frac{2}{19} + \frac{1}{26} + \frac{2}{19} + \frac{2}{19} = 1.26$$

Neuron 1 is the winner

Vigilance Test: $\frac{1}{\| x_4 \|} \| v_1 x_4 \| > \rho$? $\frac{1}{17}$ x 5 = 0.29 < 0.7 So the test is failed.

Neuron 2 is the next winner

Vigilance Test: $\frac{1}{\| x_4 \|} \| v_2 x_4 \| > \rho$? $\frac{1}{17}$ x 9 = 0.53 < 0.7 So the test is failed.

There are no more neurons to test, so we create a new neuron, $w_{i3} = \frac{1}{26}$, $v_{i3} = 1 \ \forall \ i$

Update Weights:

$$w_{ij} = \frac{v_{ij} x_i}{0.5 + \sum v_{ij} x_i} \text{ so } w_{1\,3} = w_{5\,3} = w_{6\,3} = w_{7\,3} = w_{9\,3} = w_{10\,3} = w_{11\,3}$$
$$= w_{12\,3} = w_{13\,3} = w_{14\,3} = w_{15\,3} = w_{16\,3} = w_{17\,3} = w_{19\,3} = w_{20\,3}$$
$$= w_{21\,3} = w_{25\,3} = \frac{1}{0.5 + 17} = \frac{2}{35} = 0.0571$$

and all other weights $w_{i3} = \frac{1}{26}$ as initialised

$$v_{ij} \leftarrow x_i v_{ij} \text{ so } v_{1\,3} = v_{5\,3} = v_{6\,3} = v_{7\,3} = v_{9\,3} = v_{10\,3} = v_{11\,3} = v_{12\,3} = v_{13\,3}$$
$$= v_{14\,3} = v_{15\,3} = v_{16\,3} = v_{17\,3} = v_{19\,3} = v_{20\,3} = v_{21\,3} = v_{25\,3} = 1$$

and all others are equal to zero

FINAL WEIGHTS:

$$w = \begin{bmatrix} \frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11} \\ \frac{2}{19}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{19} & \frac{1}{26}\frac{2}{19}\frac{1}{26}\frac{2}{19}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{2}{19}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{2}{19}\frac{1}{26}\frac{2}{19}\frac{1}{26} & \frac{2}{19}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{19} \\ \frac{2}{35}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{35} & \frac{2}{35}\frac{2}{35}\frac{1}{26}\frac{2}{35}\frac{2}{35} & \frac{2}{35}\frac{2}{35}\frac{2}{35}\frac{2}{35}\frac{2}{35} & \frac{2}{35}\frac{2}{35}\frac{1}{26}\frac{2}{35}\frac{2}{35} & \frac{2}{35}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{35} \end{bmatrix}^T$$

(long term memory)

$$\text{and } v = \begin{bmatrix} 10000\ 01000\ 00100\ 00010\ 00001 \\ 10001\ 01010\ 00100\ 01010\ 10001 \\ 10001\ 11011\ 11111\ 11011\ 10001 \end{bmatrix}^T \text{ (short term memory)}$$

Vigilance Test Reduced to ρ = 0.3

Present input 1, $y_1 = 0.1923$
Neuron 1 is the winner.

Vigilance Test: Is $\frac{1}{\| x_1 \|} \| v_1 x_1 \| > \rho$? $\frac{1}{5}$ x 5 = 1 > 0.3 So the test is passed.

Update weights (same as part 1)

Present input 2, $y_1 = 1.063$
Neuron 1 is the winner.

Vigilance Test: Is $\frac{1}{\| x_2 \|} \| v_1 x_2 \| > \rho$? $\frac{1}{9}$ x 5 = 0.56 > 0.3 So the test is passed.

Update Weights:

$$w_{ij} = \frac{v_{ij} x_i}{0.5 + \sum v_{ij} x_i} \text{ so } w_{1\,1} = w_{7\,1} = w_{13\,1} = w_{19\,1} = w_{25\,1}$$

$$= \frac{1}{0.5+5} = \frac{2}{11} = 0.1818$$

and all other weights $w_{i2} = \frac{1}{26}$ as initialised

$v_{ij} \leftarrow x_i v_{ij}$ so $v_{1\,1} = v_{7\,1} = v_{13\,1}\ v_{19\,1} = v_{25\,1} = 1$
and all others are equal to zero

$$w = [\ \frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26}\quad \frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26}\quad \frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26}\quad \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26}\quad \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}\ ]^T$$

(long term memory)

and v = [10000 01000 00100 00010 00001]^T (short term memory)

Present input 3, $y_1 = 1.14$
Neuron 1 is the winner.

Vigilance Test: Is $\frac{1}{\| x_2 \|} \| v_1 x_2 \| > \rho$? $\frac{1}{11}$ x 5 = 0.45 > 0.3 So the test is passed.

Update Weights:

$$w_{ij} = \frac{v_{ij} x_i}{0.5 + \sum v_{ij} x_i} \text{ so } w_{1\,1} = w_{7\,1} = w_{13\,1} = w_{19\,1} = w_{25\,1}$$

$$= \frac{1}{0.5 + 5} = \frac{2}{11} = 0.1818 \text{ and all other weights } w_{i2} = \frac{1}{26} \text{ as initialised}$$

$$v_{ij} \leftarrow x_i v_{ij} \text{ so } v_{1\,1} = v_{7\,1} = v_{13\,1} = v_{19\,1} = v_{25\,1} = 1$$

and all others are equal to zero

$$w = [\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26} \quad \frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26} \quad \frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26} \quad \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26} \quad \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}]^T$$

(long term memory)

and v = [10000 01000 00100 00010 00001]^T (short term memory)

Present input 4, $y_1 = 1.37$
Neuron 1 is the winner.

Vigilance Test: Is $\frac{1}{\| x_2 \|} \| v_1 x_2 \| > \rho$? $\frac{1}{17}$ x 5 = 0.29 < 0.3 So the test is failed.

There are no more neurons to test, so we create a new neuron, $w_{i2} = \frac{1}{26}$, $v_{i2} = 1 \ \forall\, i$

Update Weights:

$$w_{ij} = \frac{v_{ij} x_i}{0.5 + \sum v_{ij} x_i} \text{ so } w_{1\,2} = w_{5\,2} = w_{6\,2} = w_{7\,2} = w_{9\,2} = w_{10\,2} = w_{11\,2}$$

$$= w_{12\,2} = w_{13\,2} = w_{14\,2} = w_{15\,2} = w_{16\,2} = w_{17\,2} = w_{19\,2} = w_{20\,2}$$

$$= w_{21\,2} = w_{25\,2} = \frac{1}{0.5 + 17} = \frac{2}{35} = 0.0571$$

and all other weights $w_{i2} = \frac{1}{26}$ as initialised

$$v_{ij} \leftarrow x_i v_{ij} \text{ so } v_{1\,2} = v_{5\,2} = v_{6\,2} = v_{7\,2} = v_{9\,2} = v_{10\,2} = v_{11\,2} = v_{12\,2} = v_{13\,2}$$

$$= v_{14\,2} = v_{15\,2} = v_{16\,2} = v_{17\,2} = v_{19\,2} = v_{20\,2} = v_{21} = v_{25\,2} = 1$$

and all others are equal to zero

$$w = \begin{bmatrix} \frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11}\frac{1}{26} & \frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{11} \\ \frac{2}{35}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{35} & \frac{2}{35}\frac{2}{35}\frac{1}{26}\frac{2}{35}\frac{2}{35} & \frac{2}{35}\frac{2}{35}\frac{2}{19}\frac{2}{35}\frac{2}{35} & \frac{2}{35}\frac{2}{35}\frac{1}{26}\frac{2}{35}\frac{2}{35} & \frac{2}{35}\frac{1}{26}\frac{1}{26}\frac{1}{26}\frac{2}{35} \end{bmatrix}^T$$

(long term memory)

$$\text{and } v = \begin{bmatrix} 10000\ 01000\ 00100\ 00010\ 00001 \\ 10001\ 11011\ 11111\ 11011\ 10001 \end{bmatrix}^T \text{ (short term memory)}$$