

Decision Trees Using TreePlan

Add-In for Microsoft Excel

Contents

Chapter 1 Quick Start.....	2
TreePlan Installation.....	2
Occasional Use.....	2
Selective Use.....	2
Steady Use.....	2
Building a Decision Tree in TreePlan.....	3
The Anatomy of a TreePlan Decision Tree.....	5
Chapter 2 Step-by-Step TreePlan Tutorial	6
DriveTek Problem.....	6
Nodes and Branches	7
Terminal Values.....	7
Building the Tree Diagram.....	8
Interpreting the Results.....	13
Formatting the Tree Diagram.....	14
Displaying Model Inputs	16
Printing the Tree Diagram	17
Alternative Model.....	19
Chapter 3 Decision Tree Solution.....	20
Strategy.....	20
Payoff Distribution	20
DriveTek Strategies	20
Strategy Choice	25
Certainty Equivalent	25
Rollback Method.....	27
Optimal Strategy	28
Chapter 4 Sensitivity Analysis.....	30
One-Variable Sensitivity Analysis using an Excel data table	30
Two-Variable Sensitivity Analysis using an Excel data table	32

Chapter 1 Quick Start

TreePlan is an Excel add-in that allows you to build decision trees in Excel. It was developed by Professor Michael R. Middleton at the University of San Francisco and modified for use at Fuqua (Duke) by Professor James E. Smith.

TreePlan Installation

All of TreePlan's functionality is in a single file, TreePlan.xla. Depending on your preference, there are three ways to install TreePlan. (These instructions also apply to the other Decision ToolPak add-ins: SensIt.xla and RiskSim.xla.)

Occasional Use

If you plan to use TreePlan on an irregular basis, simply use Excel's File | Open command to load TreePlan.xla each time you want to use it. You may keep the TreePlan.xla file on a floppy disk, your computer's hard drive, or a network server.

Selective Use

You can use Excel's Add-In Manager to install TreePlan. First, copy TreePlan.xla to a location on your computer's hard drive. Second, if you save TreePlan.xla in the Excel or Office Library subdirectory, go to the third step. Otherwise, run Excel, choose Tools | Add-Ins; in the Add-Ins dialog box, click the Browse button, use the Browse dialog box to specify the location of TreePlan.xla, and click OK. Third, in the Add-Ins dialog box, note that TreePlan is now listed with a check mark, indicating that its menu command will appear in Excel, and click OK.

If you plan to not use TreePlan and you want to free up main memory, uncheck the box for TreePlan in the Add-In Manager. When you do want to use TreePlan, choose Tools | Add-Ins and check TreePlan's box.

To remove TreePlan from the Add-In Manager, use Windows Explorer or another file manager to delete TreePlan.xla from the Library subdirectory or from the location you specified when you used the Add-In Manager's Browse command. The next time you start Excel and choose Tools | Add-Ins, a dialog box will state "Cannot find add-in ... treeplan.xla. Delete from list?" Click Yes.

Steady Use

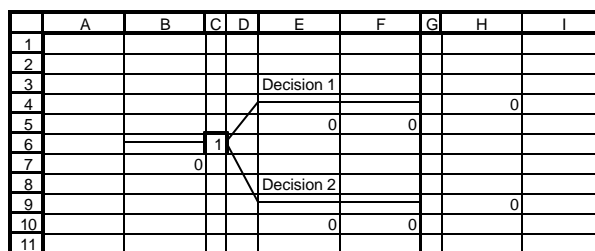
If you want TreePlan's options immediately available each time you run Excel, use Windows Explorer or another file manager to save TreePlan.xla in the Excel XLStart directory. Alternatively, in Excel you can use Tools | Options | General to specify an alternate startup file location and use a file manager to save TreePlan.xla there. When you start Excel, it tries to open all files in the XLStart directory and in the alternate startup file location.

For additional information visit "TreePlan FAQ" at www.treeplan.com.

After opening TreePlan.xla in Excel, the command "Decision Tree" appears at the bottom of the Tools menu (or, if you have a customized main menu, at the bottom of the sixth main menu item).

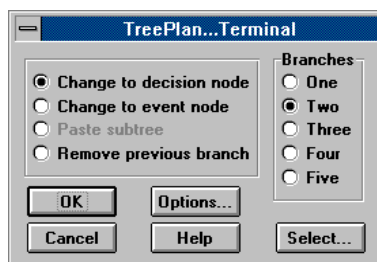
Building a Decision Tree in TreePlan

You can start TreePlan either by choosing **Tools | Decision Tree** from the menu bar or by pressing **Ctrl+t** (hold down the Ctrl key and press t). If the worksheet doesn't have a decision tree, TreePlan prompts you with a dialog box with three options; choose **New Tree** to begin a new tree. TreePlan draws a default initial decision tree with its upper left corner at the selected cell. For example, the figure below shows the initial tree when \$B\$2 is selected. (Note that TreePlan writes over existing values in the spreadsheet: begin your tree to the *right* of the area where your data is stored, and *do not subsequently add or delete rows or columns in the tree-diagram area.*) In Excel 5 and 95 a terminal node is represented by a triangle instead of a vertical bar.

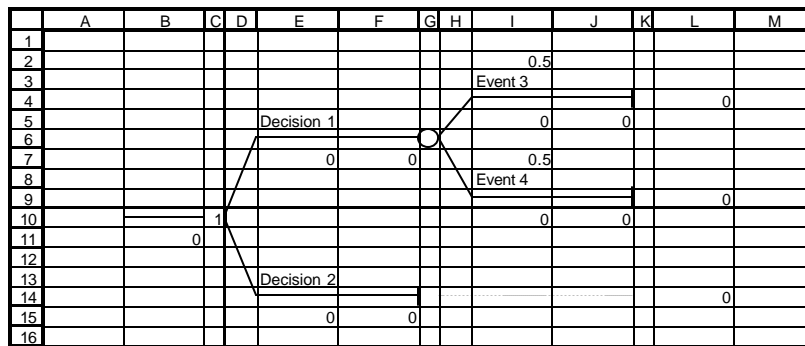


Build up a tree by adding or modifying branches or nodes in the default tree. To change the branch labels or probabilities, click on the cell containing the label or probability and type the new label or probability. To modify the structure of the tree (e.g., add or delete branches or nodes in the tree), select the node or the cell containing the node in the tree to modify, and choose **Tools | Decision Tree** or press **Ctrl+t**. TreePlan will then present a dialog box showing the available commands.

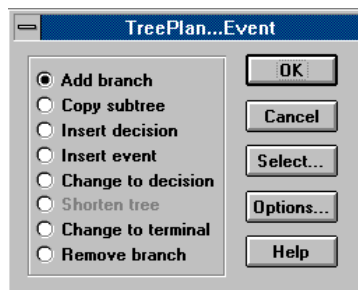
For example, to add an event node to the top branch of the tree shown above, select the square cell (cell G4) next to the vertical line at the end of a terminal branch and press **Ctrl+t**. TreePlan then presents this dialog box.



To add an event node to the branch, we change the selected terminal node to an event node by selecting **Change to event node** in the dialog box, selecting the number of branches (here two), and pressing **OK**. TreePlan then redraws the tree with a chance node in place of the terminal node.



The dialog boxes presented by TreePlan vary depending on what you have selected when you choose **Tools | Decision Tree** or press **Ctrl+t**. The dialog box shown below is presented when you press **Ctrl+t** with an event node selected; a similar dialog box is presented when you select a decision node. If you want to add a branch to the selected node, choose **Add branch** and press **OK**. If you want to insert a decision or event node before the selected node, choose **Insert decision** or **Insert event** and press **OK**. To get a description of the available commands, click on the **Help** button.

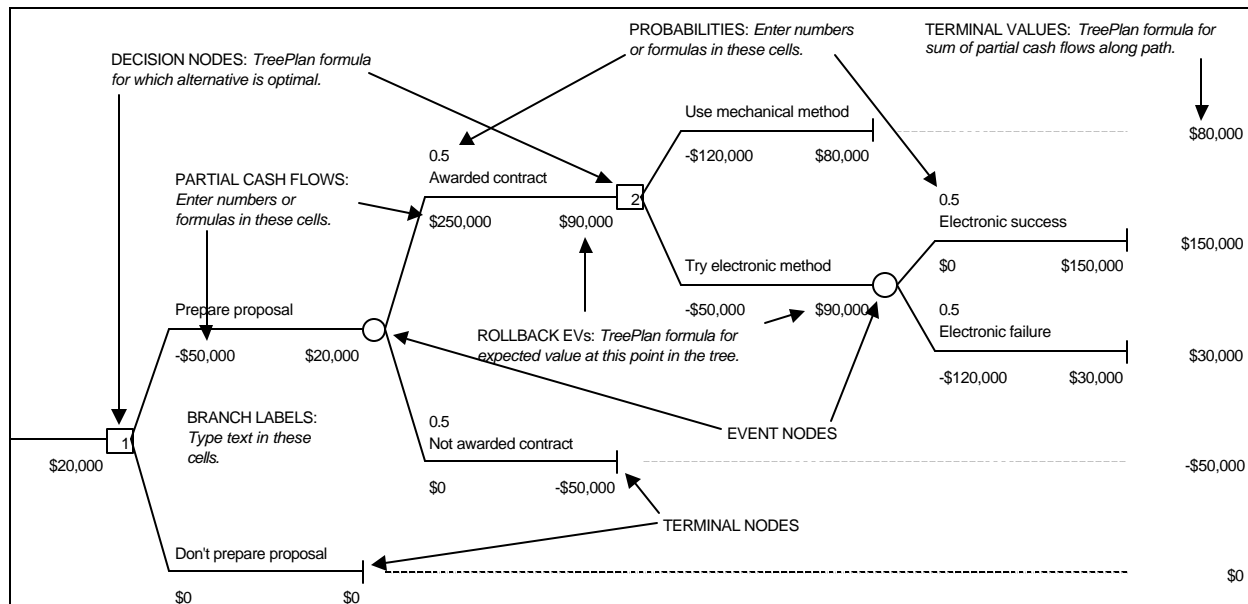


The **Copy subtree** command is particularly useful when building large trees. If two or more parts of the tree are similar, you can copy and paste "subtrees" rather than building up each part separately. To copy a subtree, select the node at the root of the subtree and choose **Copy subtree**. This tells TreePlan to copy the selected node and everything to the right of it in the tree. To paste this subtree, select a terminal node and choose **Paste subtree**. TreePlan then duplicates the specified subtree at the selected terminal node.

Since TreePlan decision trees are built directly in Excel, you can use Excel's commands to format your tree. For example, you can use bold or italic fonts for branch labels: select the cells you want to format and change them using Excel's formatting commands. To help you, TreePlan provides a **Select** dialog box that appears when you choose **Tools Decision Tree** or press **Ctrl+t** without a node selected. You can also bring up this dialog box by pressing the **Select** button on the **Node** dialog box. From here, you can select all items of a particular type in the tree. For example, if you choose **Probabilities** and press **OK**, TreePlan selects all cells containing probabilities in the tree. You can then format all of the probabilities simultaneously using Excel's formatting commands. (Because of limitations in Excel, the **Select** dialog box will not be available when working with very large trees.)

The Anatomy of a TreePlan Decision Tree

An example of a TreePlan decision tree is shown below. In the example, a firm must decide (1) whether to prepare a proposal for a possible contract and (2) which method to use to satisfy the contract. The tree consists of *decision nodes*, *event nodes* and *terminal nodes* connected by *branches*. Each branch is surrounded by cells containing formulas, cell references, or labels pertaining to that branch. You may edit the *labels*, *probabilities*, and *partial cash flows* associated with each branch. The partial cash flows are the amount the firm "gets paid" to go down that branch. Here, the firm pays \$50,000 if it decides to prepare the proposal, receives \$250,000 up front if awarded the contract, spends \$50,000 to try the electronic method, and spends \$120,000 on the mechanical method if the electronic method fails.



The trees are "solved" using formulas embedded in the spreadsheet. The *terminal values* sum all the partial cash flows along the path leading to that terminal node. The tree is then "rolled back" by computing expected values at event nodes and by maximizing at decision nodes; the *rollback EVs* appear next to each node and show the expected value at that point in the tree. The numbers in the decision nodes indicate which alternative is optimal for that decision. In the example, the "1" in the first decision node indicates that it is optimal to prepare the proposal, and the "2" in the second decision node indicates the firm should try the electronic method because that alternative leads to a higher expected value, \$90,000, than the mechanical method, \$80,000.

TreePlan has a few options that control the way calculations are done in the tree. To select these options, press the **Options** button in any of TreePlan's dialog boxes. The first choice is whether to **Use Expected Values** or **Use Exponential Utility Function** for computing certainty equivalents. The default is to rollback the tree using expected values. If you choose to use exponential utilities, TreePlan will compute utilities of endpoint cash flows at the terminal nodes and compute expected utilities instead of expected values at event nodes. Expected utilities are calculated in the cell below the certainty equivalents. You may also choose to **Maximize (profits)** or **Minimize (costs)** at decision nodes; the default is to maximize profits. If you choose to minimize costs instead, the cash flows are interpreted as costs, and decisions are made by choosing the minimum expected value or certainty equivalent rather than the maximum. See the Help file for details on these options.

Chapter 2 Step-by-Step TreePlan Tutorial

A decision tree can be used as a model for a sequential decision problems under uncertainty. A decision tree describes graphically the decisions to be made, the events that may occur, and the outcomes associated with combinations of decisions and events. Probabilities are assigned to the events, and values are determined for each outcome. A major goal of the analysis is to determine the best decisions.

Decision tree models include such concepts as nodes, branches, terminal values, strategy, payoff distribution, certainty equivalent, and the rollback method. The following problem illustrates the basic concepts.

DriveTek Problem

DriveTek Research Institute discovers that a computer company wants a new tape drive for a proposed new computer system. Since the computer company does not have research people available to develop the new drive, it will subcontract the development to an independent research firm. The computer company has offered a fee of \$250,000 for the best proposal for developing the new tape drive. The contract will go to the firm with the best technical plan and the highest reputation for technical competence.

DriveTek Research Institute wants to enter the competition. Management estimates a cost of \$50,000 to prepare a proposal with a fifty-fifty chance of winning the contract.

However, DriveTek's engineers are not sure about how they will develop the tape drive if they are awarded the contract. Three alternative approaches can be tried. The first approach is a mechanical method with a cost of \$120,000, and the engineers are certain they can develop a successful model with this approach. A second approach involves electronic components. The engineers estimate that the electronic approach will cost only \$50,000 to develop a model of the tape drive, but with only a 50 percent chance of satisfactory results. A third approach uses magnetic components; this costs \$80,000, with a 70 percent chance of success.

DriveTek Research can work on only one approach at a time and has time to try only two approaches. If it tries either the magnetic or electronic method and the attempt fails, the second choice must be the mechanical method to guarantee a successful model.

The management of DriveTek Research needs help in incorporating this information into a decision to proceed or not.

[Source: The tape drive example is adapted from Spurr and Bonini, *Statistical Analysis for Business Decisions*, Irwin.]

Nodes and Branches

Decision trees have three kinds of nodes and two kinds of branches. A decision node is a point where a choice must be made; it is shown as a square. The branches extending from a decision node are decision branches, each branch representing one of the possible alternatives or courses of action available at that point. The set of alternatives must be mutually exclusive (if one is chosen, the others cannot be chosen) and collectively exhaustive (all possible alternatives must be included in the set).

There are two major decisions in the DriveTek problem. First, the company must decide whether or not to prepare a proposal. Second, if it prepares a proposal and is awarded the contract, it must decide which of the three approaches to try to satisfy the contract.

An event node is a point where uncertainty is resolved (a point where the decision maker learns about the occurrence of an event). An event node, sometimes called a "chance node," is shown as a circle. The event set consists of the event branches extending from an event node, each branch representing one of the possible events that may occur at that point. The set of events must be mutually exclusive (if one occurs, the others cannot occur) and collectively exhaustive (all possible events must be included in the set). Each event is assigned a subjective probability; the sum of probabilities for the events in a set must equal one.

The three sources of uncertainty in the DriveTek problem are: whether it is awarded the contract or not, whether the electronic approach succeeds or fails, and whether the magnetic approach succeeds or fails.

In general, decision nodes and branches represent the controllable factors in a decision problem; event nodes and branches represent uncontrollable factors.

Decision nodes and event nodes are arranged in order of subjective chronology. For example, the position of an event node corresponds to the time when the decision maker learns the outcome of the event (not necessarily when the event occurs).

The third kind of node is a terminal node, representing the final result of a combination of decisions and events. Terminal nodes are the endpoints of a decision tree, shown as the end of a branch on hand-drawn diagrams and as a triangle on computer-generated diagrams.

The following table shows the three kinds of nodes and two kinds of branches used to represent a decision tree.

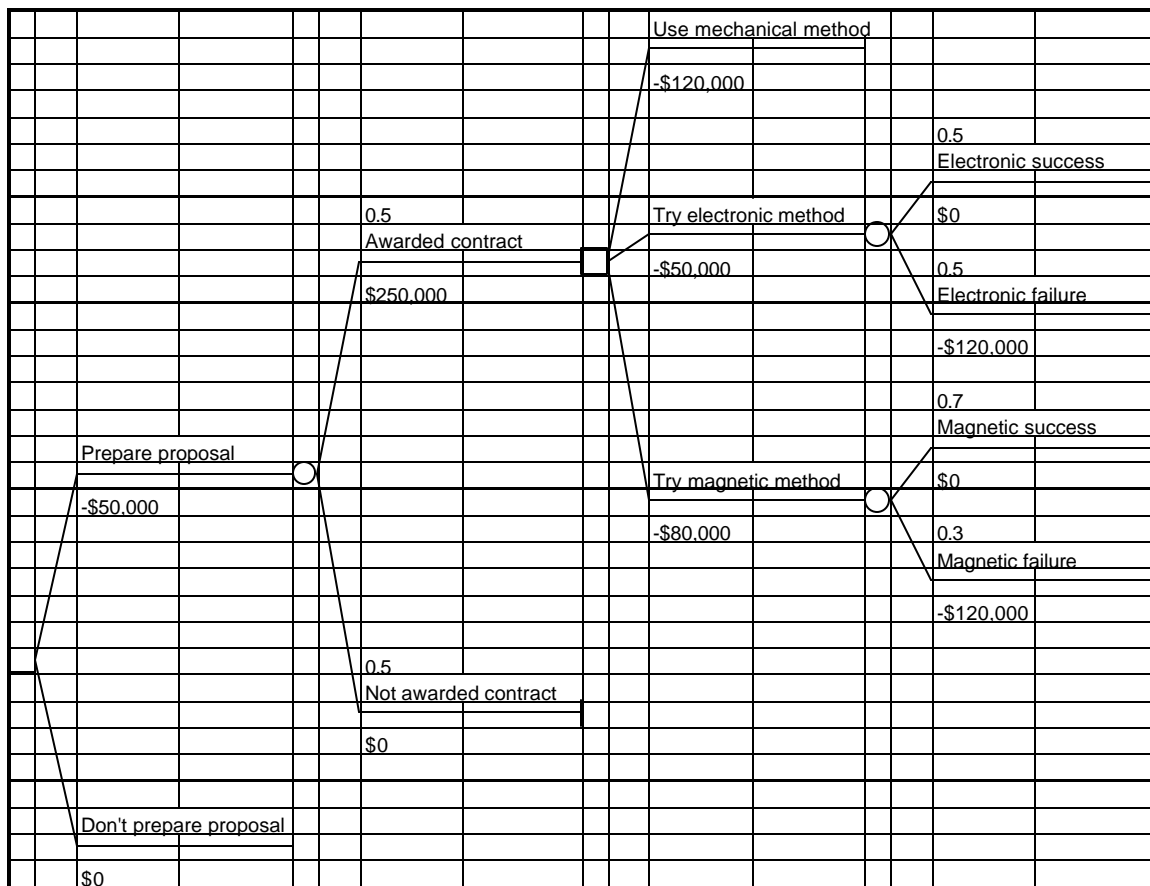
<u>Type of Node</u>	<u>Written Symbol</u>	<u>Computer Symbol</u>	<u>Node Successor</u>
Decision	square	square	decision branches
Event	circle	circle	event branches
Terminal	endpoint	triangle or bar	terminal value

Terminal Values

Each terminal node has an associated terminal value, sometimes called a payoff value, outcome value, or endpoint value. Each terminal value measures the result of a *scenario*: the sequence of decisions and events on a unique path leading from the initial decision node to a specific terminal node.

To determine the terminal value, one approach assigns a cash flow value to each decision branch and event branch and then sum the cash flow values on the branches leading to a terminal node to determine the terminal value. In the DriveTek problem, there are distinct cash flows associated with many of the decision and event branches. Some problems require a more elaborate value model to determine the terminal values.

The following diagram shows the arrangement of branch names, probabilities, and cash flow values on an unsolved tree.



To build the decision tree, you use TreePlan's dialog boxes to develop the structure. You enter a branch name, branch cash flow, and branch probability (for an event) in the cells above and below the left side of each branch. As you build the tree diagram, TreePlan enters formulas in other cells.

Building the Tree Diagram

1. Start with a new worksheet. (If no workbook is open, choose File | New. If a workbook is open, choose Insert | Worksheet.)
2. Select cell A1. From the Tools menu, choose Decision Tree. In the TreePlan New dialog box, click the New Tree button. A decision node with two branches appears.

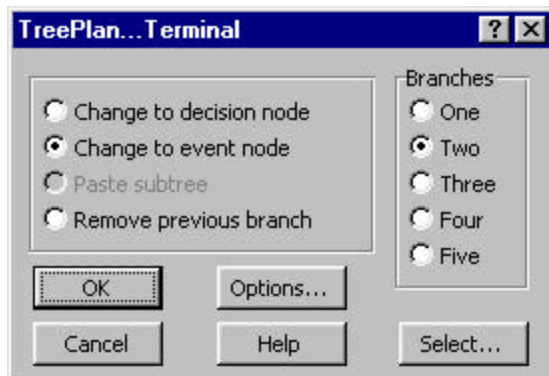


	A	B	C	D	E	F	G
1							
2				Decision 1			
3							0
4				0	0		
5			1				
6		0					
7				Decision 2			
8							0
9				0	0		

3. Do not type the quotation marks in the following instructions. Select cell D2, and enter **Prepare proposal**. Select cell D4, and enter **-50000**. Select cell D7, and enter **Don't prepare proposal**.

	A	B	C	D	E	F	G
1							
2				Prepare proposal			
3							-50000
4				-50000	-50000		
5			2				
6		0					
7				Don't prepare proposal			
8							0
9				0	0		

4. Select cell F3. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Event Node, select Two Branches, and click OK. The tree is redrawn.



	A	B	C	D	E	F	G	H	I	J	K
1								0.5			
2								Event 3			
3											-50000
4				Prepare proposal				0	-50000		
5											
6				-50000	-50000			0.5			
7								Event 4			
8											-50000
9			2					0	-50000		
10			0								
11											
12				Don't prepare proposal							
13											0
14					0	0					

5. Select cell H2, and enter **Awarded contract**. Select cell H4, and enter **250000**. Select cell H7, and enter **Not awarded contract**.

	A	B	C	D	E	F	G	H	I	J	K
1								0.5			
2								Awarded contract			
3											200000
4				Prepare proposal				250000	200000		
5											
6				-50000	75000			0.5			
7								Not awarded contract			
8											-50000
9			1					0	-50000		
10			75000								
11											
12				Don't prepare proposal							
13											0
14					0	0					

6. Select cell J3. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Decision Node, select Three Branches, and click OK. The tree is redrawn.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2												Decision 5			
3															200000
4											0	200000			
5															
6								0.5							
7								Awarded contract				Decision 6			
8															200000
9								250000	200000		1	0	200000		
10															
11															
12				Prepare proposal								Decision 7			
13															200000
14				-50000	75000						0	200000			
15															
16								0.5							
17								Not awarded contract							
18			1												-50000
19			75000					0	-50000						
20															
21															
22				Don't prepare proposal											
23															0
24					0	0									

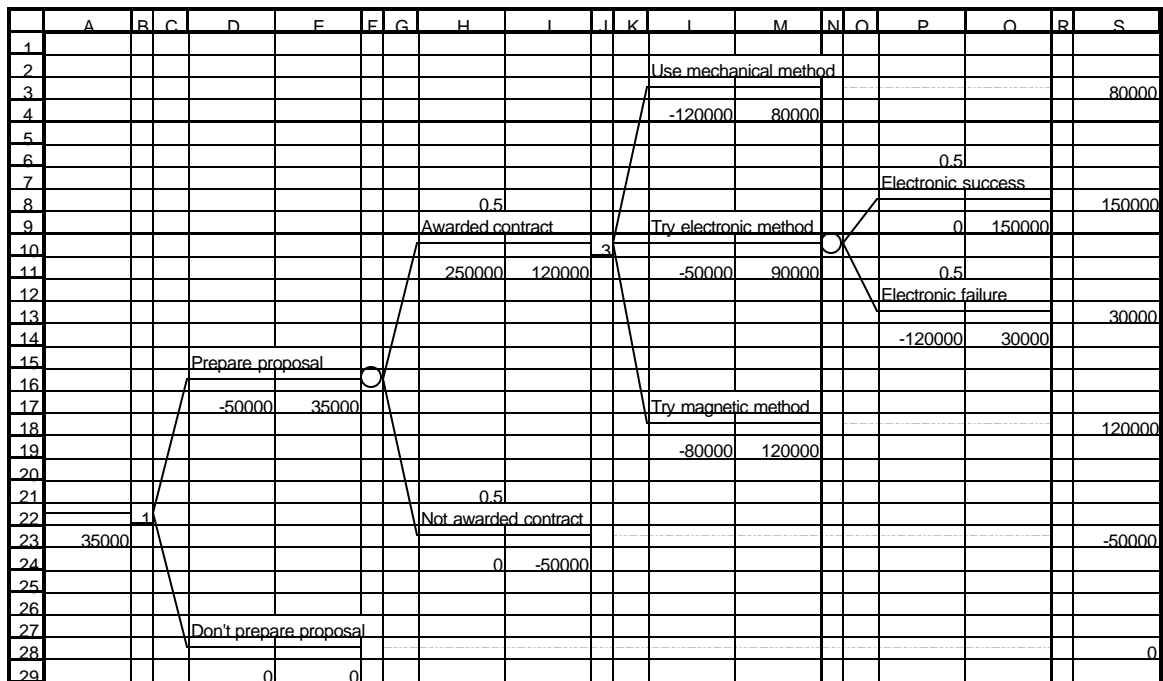
7. Select cell L2, and enter **Use mechanical method**. Select cell L4, and enter **-120000**. Select cell L7, and enter **Try electronic method**. Select cell L9, and enter **-50000**. Select cell L12, and enter **Try magnetic method**. Select cell L14, and enter **-80000**.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2												Use mechanical method			
3															80000
4												-120000	80000		
5															
6								0.5							
7								Awarded contract				Try electronic method			
8										2					150000
9								250000	150000			-50000	150000		
10															
11															
12				Prepare proposal								Try magnetic method			
13															120000
14				-50000	50000							-80000	120000		
15															
16								0.5							
17								Not awarded contract							
18															-50000
19									0	-50000					
20															
21															
22				Don't prepare proposal											
23															0
24					0	0									

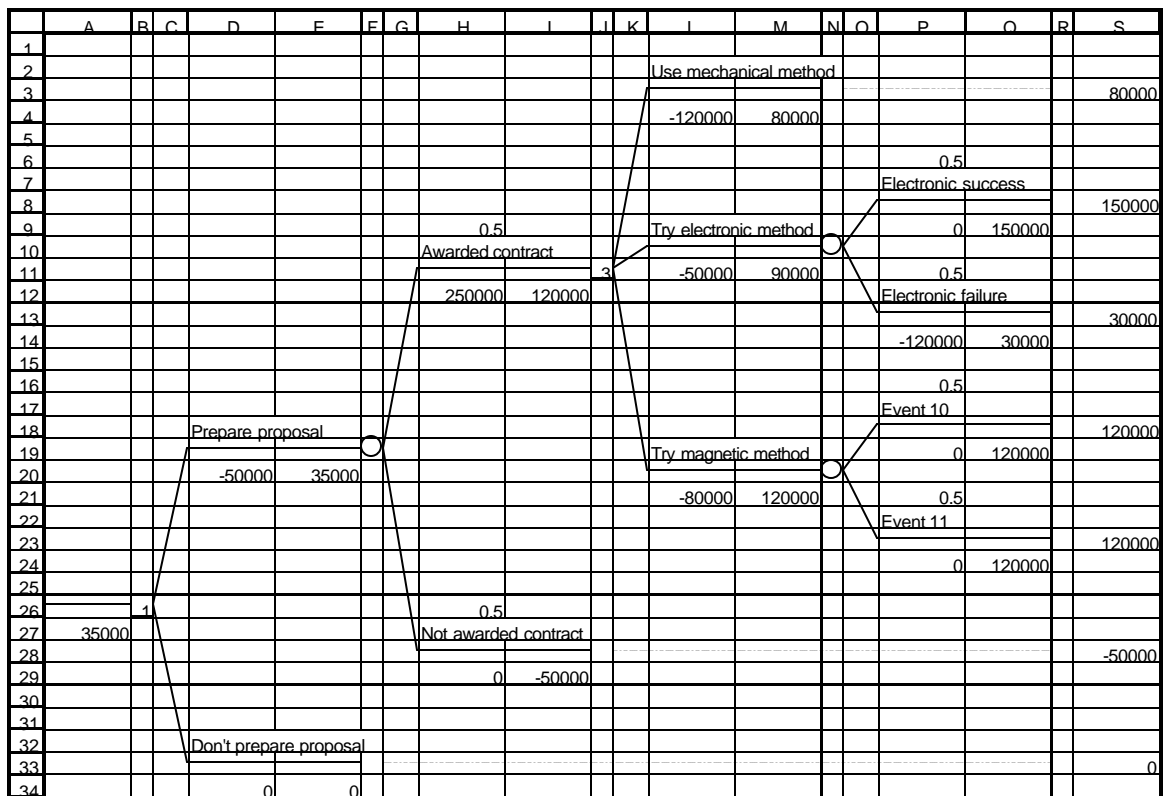
8. Select cell N8. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Event Node, select Two Branches, and click OK. The tree is redrawn.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2												Use mechanical method							
3																			80000
4												-120000	80000						
5																			
6																			
7														0.5					
8								0.5						Event 8					150000
9								Awarded contract				Try electronic method			0	150000			
10										2									
11								250000	150000			-50000	150000						
12																			
13																			150000
14																0	150000		
15				Prepare proposal															
16																			
17				-50000	50000							Try magnetic method							
18																			120000
19												-80000	120000						
20																			
21								0.5											
22								Not awarded contract											
23																			-50000
24									0	-50000									
25																			
26																			
27				Don't prepare proposal															
28																			0
29					0	0													

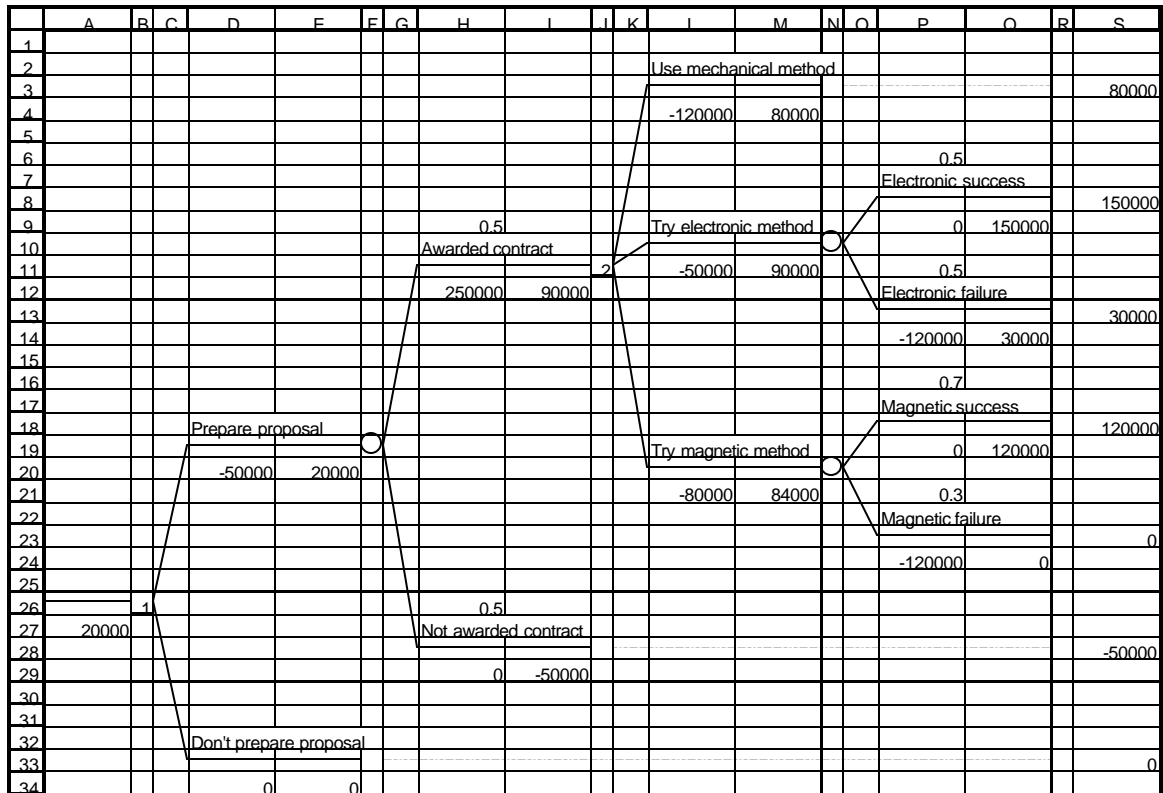
9. Select cell P7, and enter **Electronic success**. Select cell P12, and enter **Electronic failure**. Select cell P14, and enter **-120000**.



10. Select cell N18. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Event Node, select Two Branches, and click OK. The tree is redrawn.



11. Select cell P16, and enter **.7**. Select cell P17, and enter **Magnetic success**. Select cell P21, and enter **.3**. Select cell P22, and enter **Magnetic failure**. Select cell P24, and enter **–120000**.



12. Double-click the sheet tab (or right-click the sheet tab and choose Rename from the shortcut menu), and enter **Original**. Save the workbook.

Interpreting the Results

The \$30,000 terminal value on the far right of the diagram in cell S13 is associated with the following scenario:

Branch Type	Branch Name	Cash Flow
Decision	Prepare proposal	–\$50,000
Event	Awarded contract	\$250,000
Decision	Try electronic method	–\$50,000
Event	Electronic failure (Use mechanical method)	–\$120,000
	Terminal value	\$30,000

TreePlan put the formula =SUM(P14,L11,H12,D20) into cell S13 for determining the terminal value.

Other formulas, called rollback formulas, are in cells below and to the left of each node. These formulas are used to determine the optimal choice at each decision node.

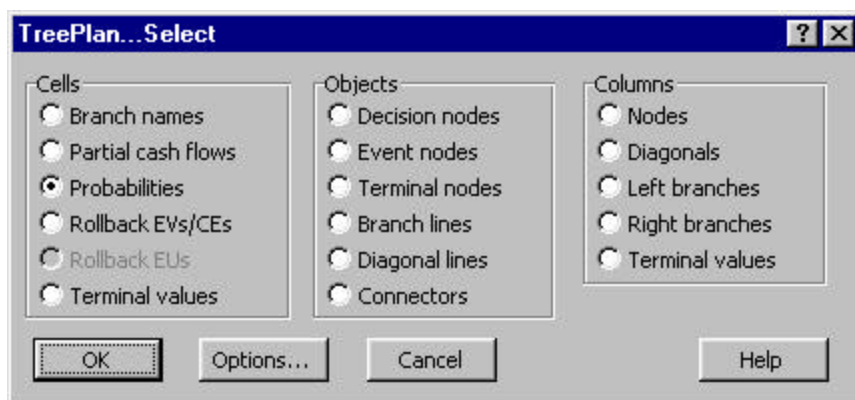
In cell B26, a formula displays 1, indicating that the first branch is the optimal choice. Thus, the initial choice is to prepare the proposal. In cell J11, a formula displays 2, indicating that the second branch (numbered 1, 2, and 3, from top to bottom) is the optimal choice. If awarded the contract, DriveTek should try the electronic method. A subsequent chapter provides more details about interpretation.

Formatting the Tree Diagram

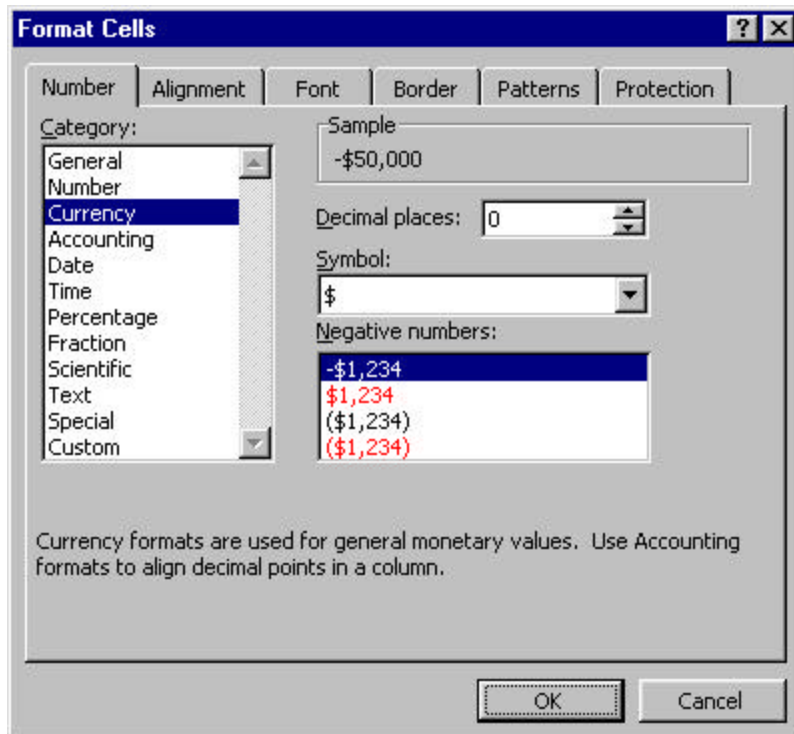
The following steps show how to use TreePlan and Excel features to format the tree diagram. You may choose to use other formats for your own tree diagrams.

13. From the Edit menu, choose Move or Copy Sheet (or right-click the sheet tab and choose Move Or Copy from the shortcut menu). In the lower left corner of the Move Or Copy dialog box, check the Create A Copy box, and click OK.

14. On sheet Original (2), select cell H9. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Probabilities is selected, and click OK. With all probability cells selected, click the Align Left button.



15. Select cell H12. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Partial Cash Flows is selected, and click OK. With all partial cash flow cells selected, click the Align Left button. With those cells still selected, choose Format | Cells. In the Format Cells dialog box, click the Number tab. In the Category list box, choose Currency; type 0 (zero) for Decimal Places; select \$ in the Symbol list box; select -\$1,234 for Negative Numbers. Click OK.



16. Select cell I12. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Rollback EVs/CEs is selected, and click OK. With all rollback cells selected, choose Format | Cells. Repeat the Currency formatting of step 16 above.

17. Select cell S3. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Terminal Values is selected, and click OK. With all terminal value cells selected, choose Format | Cells. Repeat the Currency formatting of step 16 above.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2												Use mechanical method							
3																			\$80,000
4												-\$120,000	\$80,000						
5																			
6																			
7																0.5			
8																Electronic success			
9																	\$150,000		
10																			
11																			
12																			
13																			
14																			
15																			
16																			
17																			
18																			
19																			
20																			
21																			
22																			
23																			
24																			
25																			
26																			
27																			
28																			
29																			
30																			
31																			
32																			
33																			
34																			

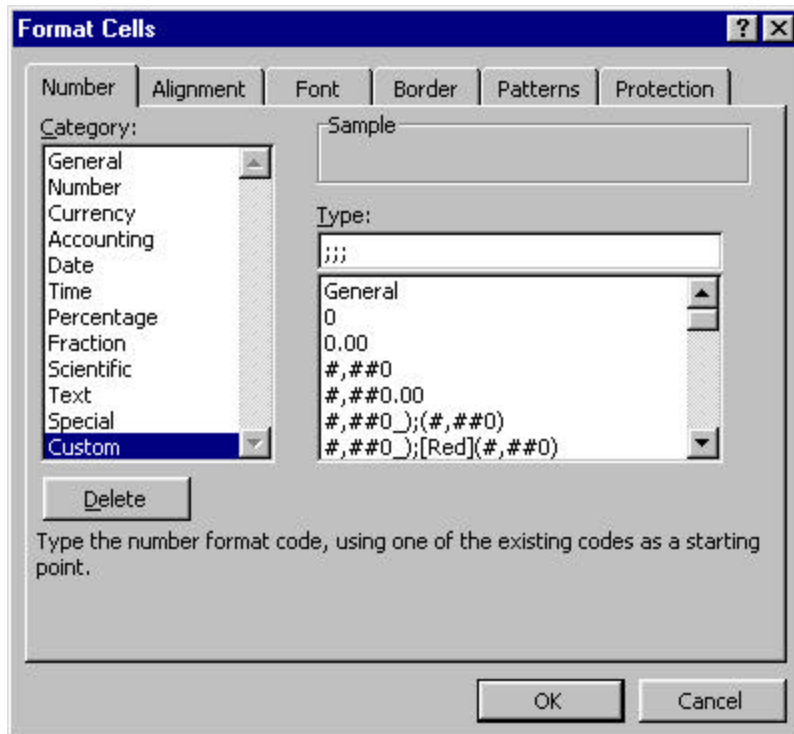
18. Double-click the Original (2) sheet tab (or right-click the sheet tab and choose Rename from the shortcut menu), and enter **Formatted**. Save the workbook.

Displaying Model Inputs

When you build a decision tree model, you may want to discuss the model and its assumptions with co-workers or a client. For such communication it may be preferable to hide the results of formulas that show rollback values and decision node choices. The following steps show how to display only the model inputs.

19. From the Edit menu, choose Move or Copy Sheet (or right-click the sheet tab and choose Move Or Copy from the shortcut menu). In the lower left corner of the Move Or Copy dialog box, check the Create A Copy box, and click OK.

20. On sheet Formatted (2), select cell B1. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Columns with Nodes is selected, and click OK. With all node columns selected, choose Format | Cells | Number. In the Category list box, select Custom. Select the entry in the Type edit box, and type ;;; (three semicolons). Click OK.



Explanation: A custom number format has four sections of format codes. The sections are separated by semicolons, and they define the formats for positive numbers, negative numbers, zero values, and text, in that order. When you specify three semicolons without format codes, Excel does not display positive numbers, negative numbers, zero values, or text. The formula remains in the cell, but its result is not displayed. Later, if you want to display the result, you can change the format without having to enter the formula again. Editing an existing format does not delete it. All formats are saved with the workbook unless you explicitly delete a format.

21. Select cell A27. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Rollback EVs/CEs is selected, and click OK. With all rollback values selected, choose Format | Cells | Number. In the Category list box, select Custom. Scroll to the bottom of the Type list box, and select the three-semicolon entry. Click OK.

22. Double-click the Formatted (2) sheet tab (or right-click the sheet tab and choose Rename from the shortcut menu), and enter **Model Inputs**. Save the workbook.

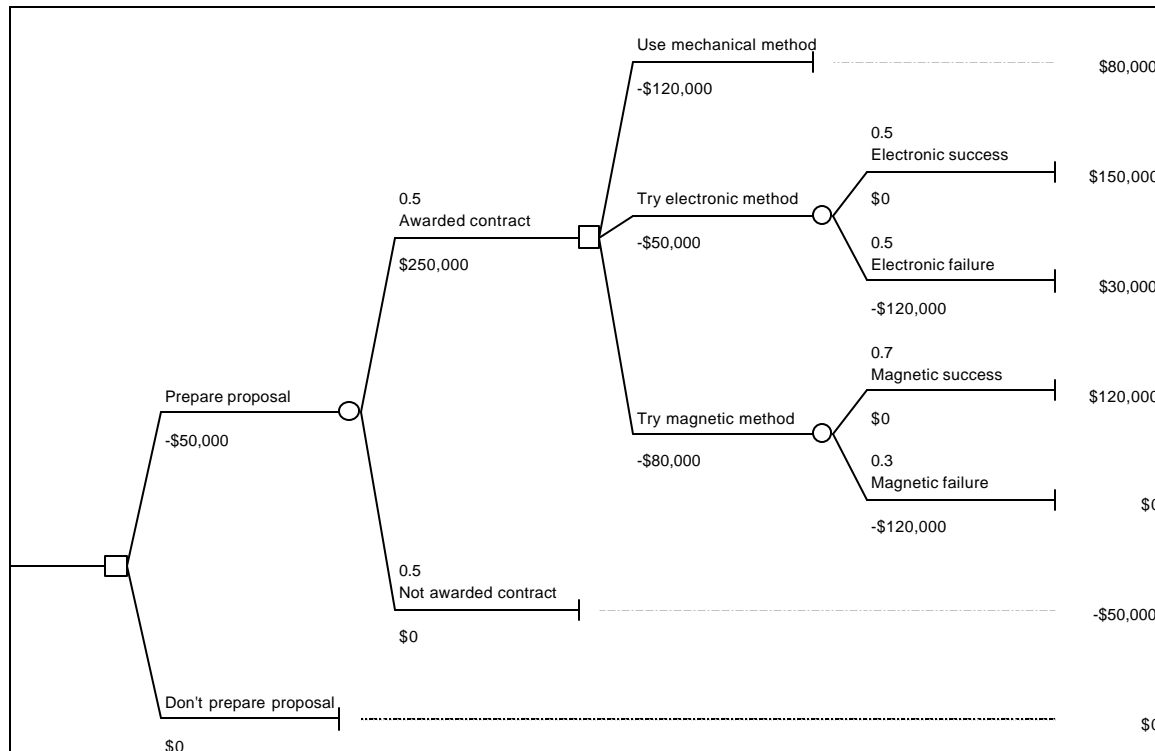
Printing the Tree Diagram

23. In the Name Box list box, select TreeDiagram (or select cells A1:S34).

24. To print the tree diagram from Excel, with the tree diagram range selected choose File | Print Area | Set Print Area. Choose File | Page Setup. In the Page Setup dialog box, click the Page tab; for Orientation click the option button for Landscape, and for Scaling click the option button for Fit To 1 Page Wide By 1 Page Tall. Click the Header/Footer tab; in the Header list box select None, and in the Footer

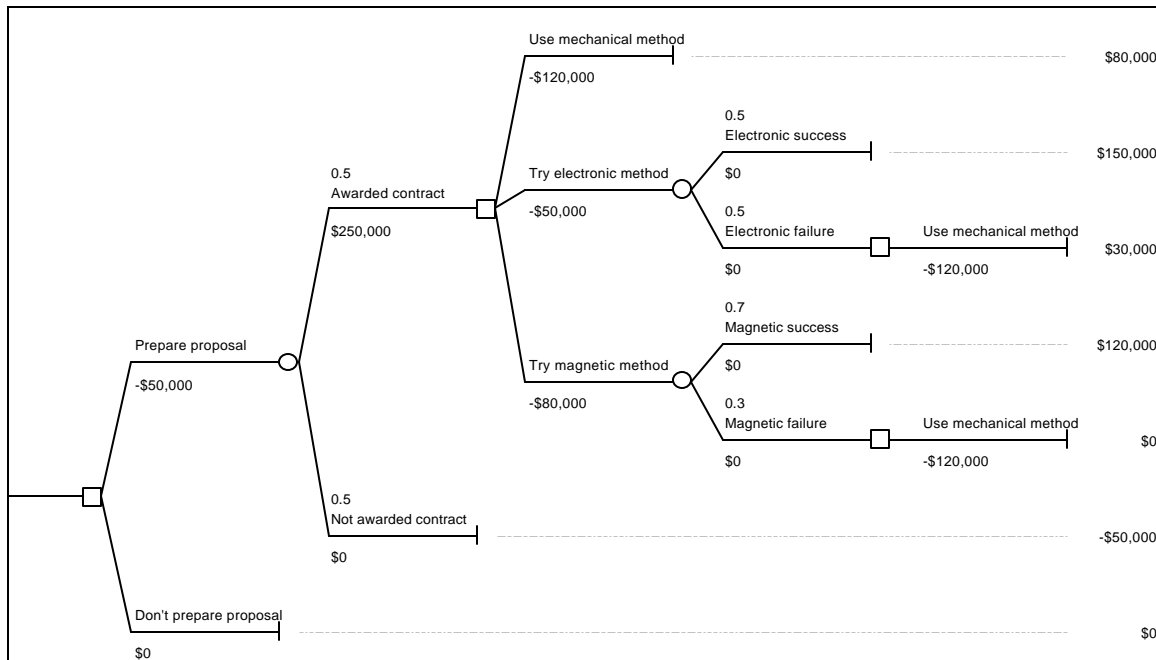
list box select None (or select other appropriate headers and footers). Click the Sheet tab; clear the check box for Gridlines, and clear the check box for Row And Column Headings. Click OK. Choose File | Print and click OK.

25. To print the tree diagram from Word, clear the check boxes for Gridlines and for Row And Column Headings on Excel's Page Setup dialog box Sheet tab. Select the tree diagram range. Hold down the Shift key and from the Edit menu choose Copy Picture. In the Copy Picture dialog box, click the option button As Shown When Printed, and click OK. In Word select the location where you want to paste the tree diagram and choose Edit | Paste.



Alternative Model

If you want to emphasize that the time constraint forces DriveTek to use the mechanical approach if they try either of the uncertain approaches and experience a failure, you can change the terminal nodes in cells R13 and R23 to decision nodes, each with a single branch.



Chapter 3 Decision Tree Solution

Strategy

A strategy specifies an initial choice and any subsequent choices to be made by the decision maker. The subsequent choices usually depend upon events. The specification of a strategy must be comprehensive; if the decision maker gives the strategy to a colleague, the colleague must know exactly which choice to make at each decision node.

Most decision problems have many possible strategies, and a goal of the analysis is to determine the optimal strategy, taking into account the decision maker's risk attitude. There are four strategies in the DriveTek problem. One of the strategies is: Prepare the proposal; if not awarded the contract, stop; if awarded the contract, try the magnetic method; if the magnetic method is successful, stop; if the magnetic method fails, use the mechanical method. The four strategies will be discussed in detail below.

Payoff Distribution

Each strategy has an associated payoff distribution, sometimes called a risk profile. The payoff distribution of a particular strategy is a probability distribution showing the probability of obtaining each terminal value associated with a particular strategy.

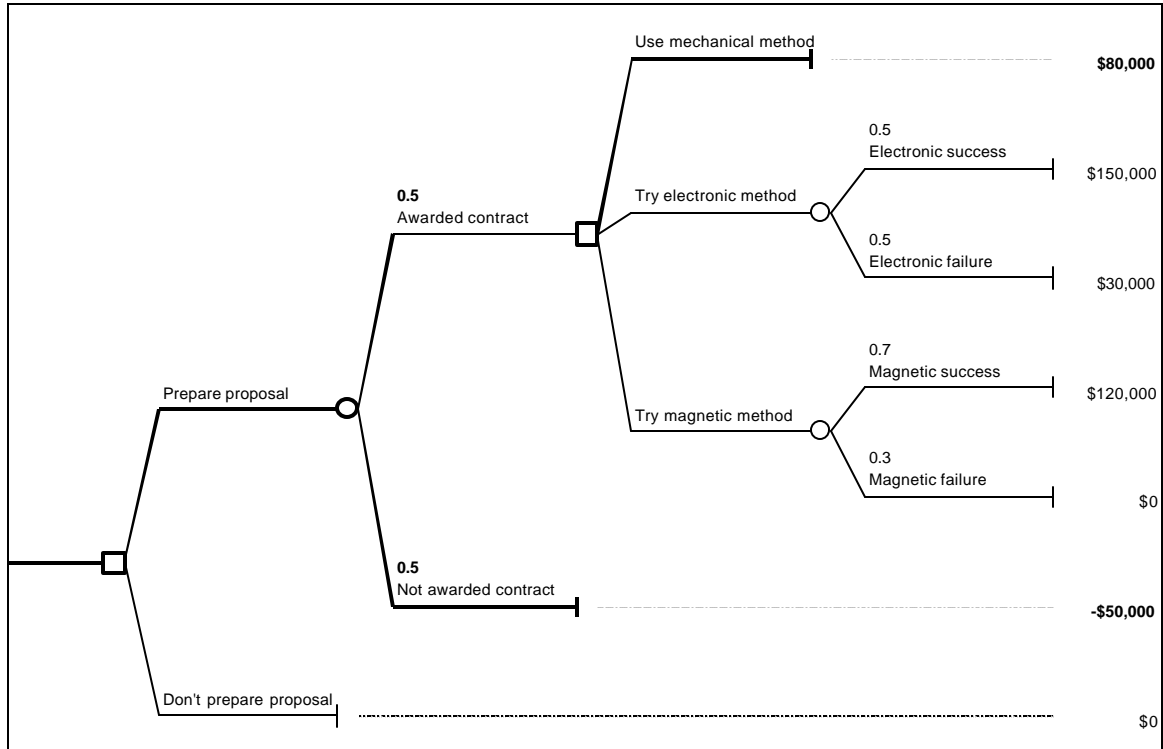
In decision tree models, the payoff distribution can be shown as a list of possible payoff values, x , and the discrete probability of obtaining each value, $P(X=x)$, where X represents the uncertain terminal value associated with a strategy. Since a strategy specifies a choice at each decision node, the uncertainty about terminal values depends only on the occurrence of events. The probability of obtaining a specific terminal value equals the product of the probabilities on the event branches on the path leading to the terminal node.

DriveTek Strategies

In this section each strategy of the DriveTek problem is described by a shorthand statement and a more detailed statement. The possible branches following a specific strategy are shown in decision tree form, and the payoff distribution is shown in a table with an explanation of the probability calculations.

Strategy 1 (Mechanical): Prepare; if awarded, use mechanical.

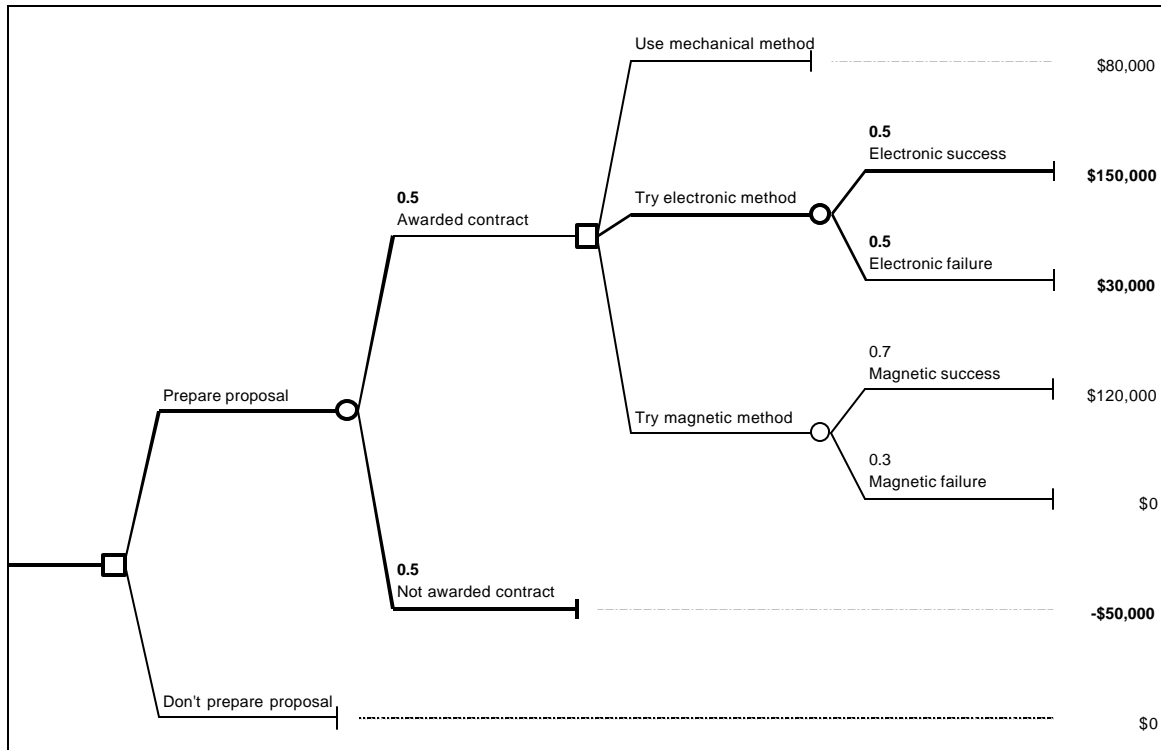
Details: Prepare the proposal; if not awarded the contract, stop (payoff = -\$50,000); if awarded the contract, use the mechanical method (payoff = \$80,000).



Value, x	Probability $P(X=x)$
\$80,000	0.50
-\$50,000	0.50
	1.00

Strategy 2 (Electronic): Prepare; if awarded, try electronic.

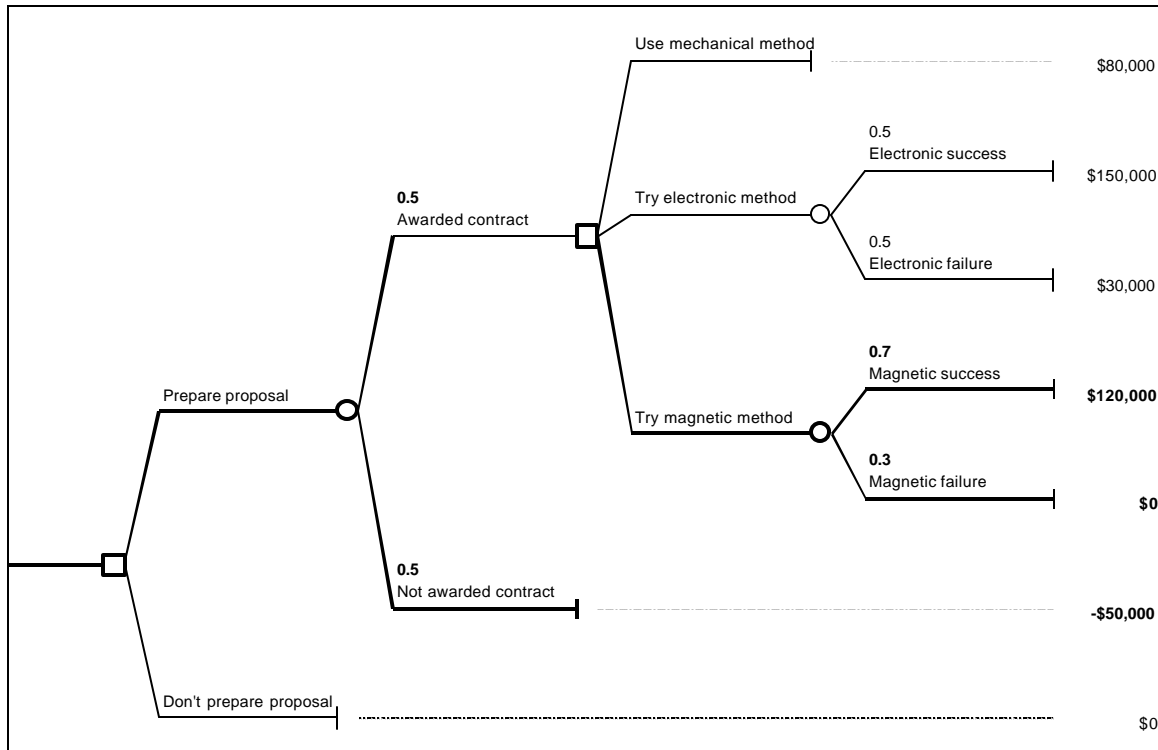
Details: Prepare the proposal; if not awarded the contract, stop (payoff = -\$50,000); if awarded the contract, try the electronic method; if the electronic method is successful, stop (payoff = \$150,000); if the electronic method fails, use the mechanical method (payoff = \$30,000).



Value, x	Probability P(X=x)	
\$150,000	0.25	= 0.5 * 0.5
\$30,000	0.25	= 0.5 * 0.5
-\$50,000	0.50	
	1.00	

Strategy 3 (Magnetic): Prepare; if awarded, try magnetic.

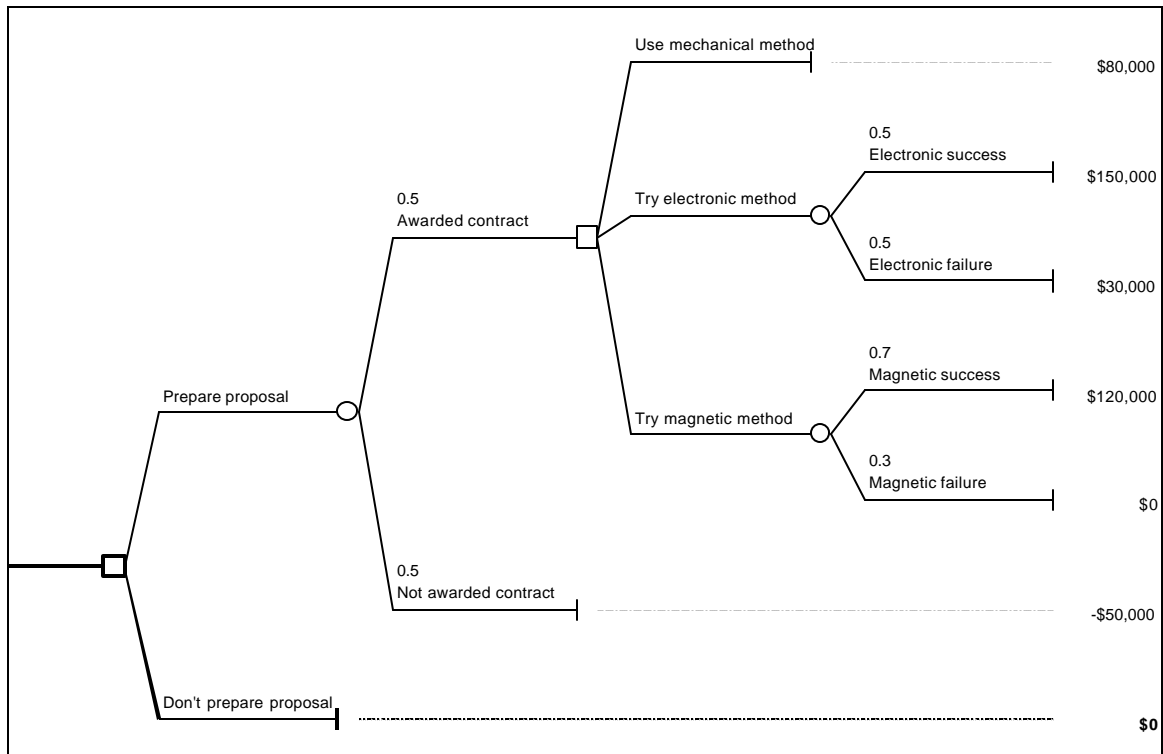
Details: Prepare the proposal; if not awarded the contract, stop (payoff = -\$50,000); if awarded the contract, try the magnetic method; if the magnetic method is successful, stop (payoff = \$120,000); if the magnetic method fails, use the mechanical method (payoff = \$0).



Value, x	Probability $P(X=x)$	
\$120,000	0.35	$= 0.5 * 0.7$
\$0	0.15	$= 0.5 * 0.3$
-\$50,000	0.50	
	1.00	

Strategy 4 (Don't): Don't.

Details: Don't prepare the proposal (payoff = \$0).



Value, x	Probability P(X=x)
\$0	1.00
	1.00

Strategy Choice

Since each strategy can be characterized completely by its payoff distribution, selecting the best strategy becomes a problem of choosing the best payoff distribution.

One approach is to make a choice by direct comparison of the payoff distributions.

Strategy 1 (Mechanical)		Strategy 2 (Electronic)	
Value, x	Probability P(X=x)	Value, x	Probability P(X=x)
\$80,000	0.50	\$150,000	0.25
-\$50,000	0.50	\$30,000	0.25
	1.00	-\$50,000	0.50
			1.00

Strategy 3 (Magnetic)		Strategy 4 (Don't)	
Value, x	Probability P(X=x)	Value, x	Probability P(X=x)
\$120,000	0.35	\$0	1.00
\$0	0.15		1.00
-\$50,000	0.50		
	1.00		

Another approach for making choices involves certainty equivalents.

Certainty Equivalent

A certainty equivalent is a certain payoff value which is equivalent, for the decision maker, to a particular payoff distribution. If the decision maker can determine his or her certainty equivalent for the payoff distribution of each strategy, then the optimal strategy is the one with the highest certainty equivalent.

The certainty equivalent is the minimum selling price for a payoff distribution; it depends on the decision maker's personal attitude toward risk. A decision maker may be risk preferring, risk neutral, or risk avoiding.

If the terminal values are not regarded as extreme (relative to the decision maker's total assets), if the decision maker will encounter other decision problems with similar payoffs, and if the decision maker has the attitude that he or she will "win some and lose some," then the decision maker's attitude toward risk may be described as risk neutral.

If the decision maker is risk neutral, the expected value is the appropriate certainty equivalent for choosing among the strategies. Thus, for a risk neutral decision maker, the optimal strategy is the one with the highest expected value.

The expected value of a payoff distribution is calculated by multiplying each terminal value by its probability and summing the products. The expected value calculations for each of the four strategies of the DriveTek problem are shown below.

Strategy 1 (Mechanical)		
Value, x	Probability $P(X=x)$	$x * P(X=x)$
\$80,000	0.50	\$40,000
-\$50,000	0.50	-\$25,000
		\$15,000

Strategy 2 (Electronic)		
Value, x	Probability $P(X=x)$	$x * P(X=x)$
\$150,000	0.25	\$37,500
\$30,000	0.25	7,500
-\$50,000	0.50	-\$25,000
		\$20,000

Strategy 3 (Magnetic)		
Value, x	Probability $P(X=x)$	$x * P(X=x)$
\$120,000	0.35	\$42,000
\$0	0.15	\$0
-\$50,000	0.50	-\$25,000
		\$17,000

Strategy 4 (Don't)		
Value, x	Probability $P(X=x)$	$x * P(X=x)$
\$0	1.00	\$0
		\$0

The four strategies of the DriveTek problem have expected values of \$15,000, \$20,000, \$17,000, and \$0. Strategy 2 (Electronic) is the optimal strategy with expected value \$20,000.

A risk neutral decision maker's choice is based on the expected value. However, note that if strategy 2 (Electronic) is chosen, the decision maker does not receive \$20,000. The actual payoff will be \$150,000, \$30,000, or -\$50,000, with probabilities shown in the payoff distribution.

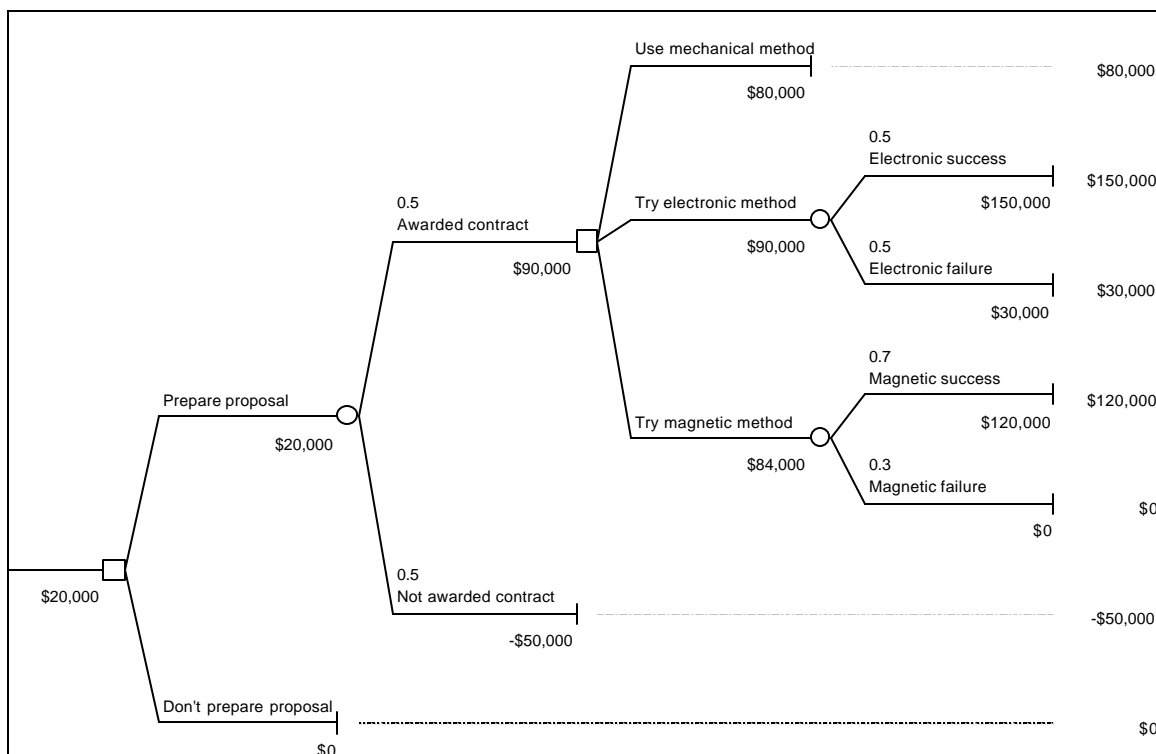
Rollback Method

If we have a method for determining certainty equivalents (expected values for a risk neutral decision maker), we don't need to examine every possible strategy explicitly. Instead, the method known as rollback determines the single best strategy.

The rollback algorithm, sometimes called backward induction or "average out and fold back," starts at the terminal nodes of the tree and works backward to the initial decision node, determining the certainty equivalent rollback values for each node. Rollback values are determined as follows:

- At a terminal node, the rollback value equals the terminal value.
- At an event node, the rollback value for a risk neutral decision maker is determined using expected value; the branch probability is multiplied times the successor rollback value, and the products are summed.
- At a decision node, the rollback value is set equal to the highest rollback value on the immediate successor nodes.

In TreePlan tree diagrams the rollback values are located to the left and below each decision, event, and terminal node. Terminal values and rollback values for the DriveTek problem are shown below.



Optimal Strategy

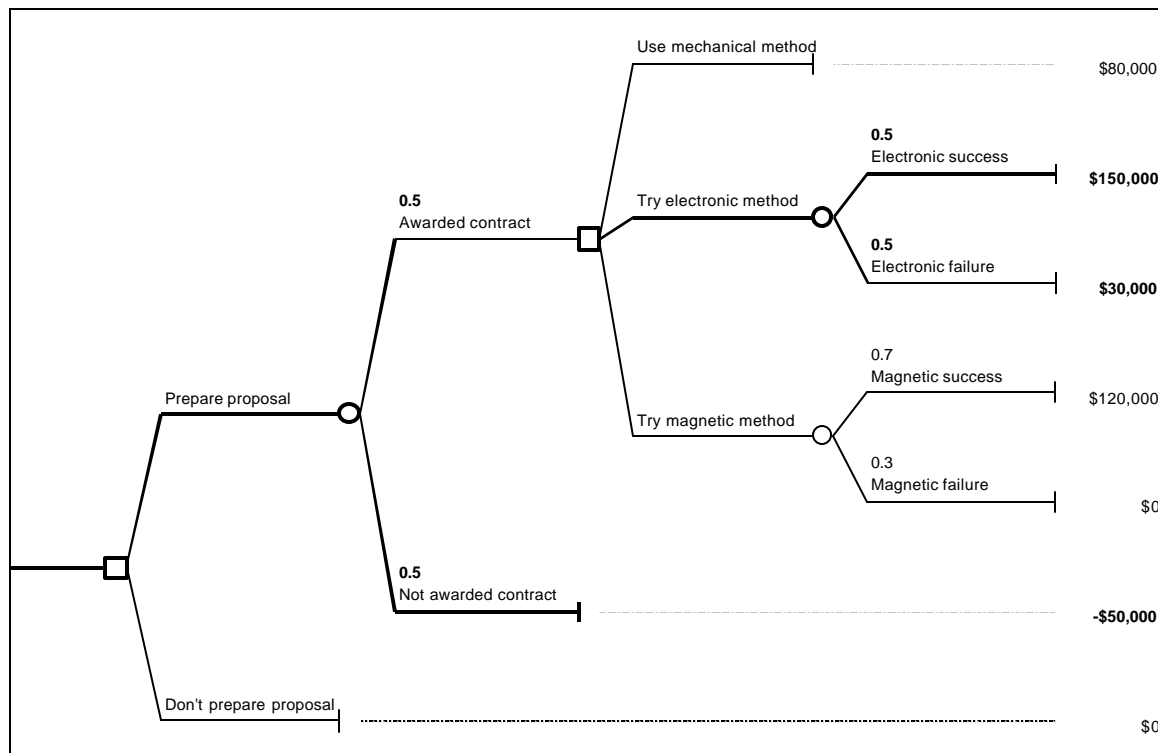
After the rollback method has determined certainty equivalents for each node, the optimal strategy can be identified by working forward through the tree. At the initial decision node, the \$20,000 rollback value equals the rollback value of the "Prepare proposal" branch, indicating the alternative that should be chosen. DriveTek will either be awarded the contract or not; there is a subsequent decision only if DriveTek obtains the contract. (In a more complicated decision tree, the optimal strategy must include decision choices for all decision nodes that might be encountered.) At the decision node following "Awarded contract," the \$90,000 rollback value equals the rollback value of the "Try electronic method" branch, indicating the alternative that should be chosen. Subsequently, if the electronic method fails, DriveTek must use the mechanical method to satisfy the contract.

Cell B26 has the formula $=IF(A27=E20,1,IF(A27=E34,2))$ which displays 1, indicating that the first branch is the optimal choice. Thus, the initial choice is to prepare the proposal. Cell J11 has the formula $=IF(I12=M4,1,IF(I12=M11,2,IF(I12=M21,3)))$ which displays 2, indicating that the second branch (numbered 1, 2, and 3, from top to bottom) is the optimal choice. If awarded the contract, DriveTek should try the electronic method.

The pairs of rollback values at the relevant decision nodes (\$20,000 and \$90,000) and the preferred decision branches are shown below in bold.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2												Use mechanical method							
3																			\$80,000
4													\$80,000						
5																			
6																0.5			
7																Electronic success			
8																			\$150,000
9							0.5					Try electronic method					\$150,000		
10							Awarded contract												
11											2		\$90,000						
12									\$90,000							0.5			
13																Electronic failure			
14																			\$30,000
15																			\$30,000
16																			
17																0.7			
18																Magnetic success			
19																			\$120,000
20							Prepare proposal					Try magnetic method							\$120,000
21																			
22																			\$84,000
23																0.3			
24																Magnetic failure			
25																			\$0
26																			\$0
27																			
28																			
29																			
30																			
31																			
32																			
33																			\$0
34																			

Taking into account event branches with subsequent terminal nodes, all branches and terminal values associated with the optimal risk neutral strategy are shown below.

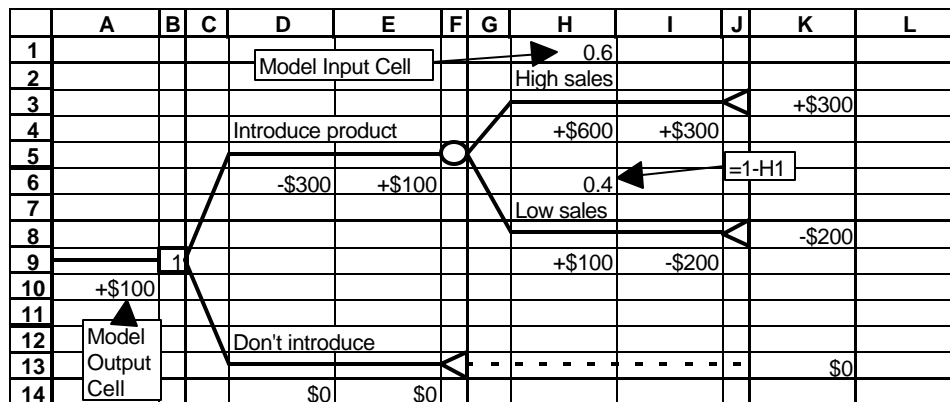


The rollback method has identified strategy 2 (Electronic) as optimal. The rollback value on the initial branch of the optimal strategy is \$20,000, which must be the same as the expected value for the payoff distribution of strategy 2. Some of the intermediate calculations for the rollback method differ from the calculations for the payoff distributions, but both approaches identify the same optimal strategy with the same initial expected value. For decision trees with a large number of strategies, the rollback method is more efficient.

Chapter 4 Sensitivity Analysis

One-Variable Sensitivity Analysis using an Excel data table

1. Construct a decision tree model or financial planning model.
2. Identify the model input cell (H1) and model output cell (A10).
3. Modify the model so that probabilities will always sum to one. (That is, enter the formula =1-H1 in cell H6.)



4. Enter a list of input values in a column (N3:N13).
5. Enter a formula for determining output values at the top of an empty column on the right of the input values (=A10 in cell O2).
6. Select the data table range (N2:O13).
7. From the Data menu choose the Table command.

	M	N	O	P
1				
2			+\$100	=A10
3		0.00		
4		0.10		
5		0.20		
6		0.30		
7		0.40		
8		0.50		
9		0.60		
10		0.70		
11		0.80		
12		0.90		
13		1.00		
14				

8. In the Data Table dialog box, select the Column Input Cell edit box. Type the model input cell (H1), or point to the model input cell (in which case the edit box displays \$H\$1). Click OK.

The image shows the 'Table' dialog box. It has a title bar 'Table'. Below it, there are two input fields: 'Row Input Cell:' and 'Column Input Cell:'. The 'Row Input Cell' field is empty. The 'Column Input Cell' field contains the text '\$H\$1'. To the right of these fields are three buttons: 'OK', 'Cancel', and 'Help'.

9. The Data Table command substitutes each input value into the model input cell, recalculates the worksheet, and displays the corresponding model output value in the table.
10. Optional: Change the formula in cell O2 to =CHOOSE(B9,"Introduce","Don't").

	M	N	O	P
1	P(High Sales) Exp. Value			
2				
3		0.00	0	
4		0.10	0	
5		0.20	0	
6		0.30	0	
7		0.40	0	
8		0.50	50	
9		0.60	100	
10		0.70	150	
11		0.80	200	
12		0.90	250	
13		1.00	300	
14				

Two-Variable Sensitivity Analysis using an Excel data table

Strategy Region Table

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2												Use mechanical method							
3																			
4												-\$120,000	+\$80,000						+\$80,000
5																			
6																0.50			
7																Electronic success			
8																			+\$150,000
9								0.50				Try electronic method				\$0	+\$150,000		
10								Awarded contract											
11																0.50			
12								+\$250,000	+\$90,000			-\$50,000	+\$90,000			Electronic failure			
13																			+\$30,000
14																-\$120,000	+\$30,000		
15																			
16																0.70			
17																Magnetic success			
18								Prepare proposal				Try magnetic method							+\$120,000
19																\$0	+\$120,000		
20								-\$50,000	+\$20,000										
21												-\$80,000	+\$84,000			0.30			
22																Magnetic failure			
23																			\$0
24																-\$120,000	\$0		
25																			
26																0.50			
27																Not awarded contract			
28																			-\$50,000
29																\$0	-\$50,000		
30																			
31																			
32								Don't prepare proposal											
33																			\$0
34								\$0	\$0										

Optional: Activate the Base Case worksheet. From the Edit menu, choose Move Or Copy Sheet. In the Move Or Copy dialog box, check the box for Create A Copy, and click OK. Double-click the new worksheet tab and enter **Strategy Region Table**.

Setup for Data Table

Select cell P11, and enter the formula =**1-P6**. Select cell P21, and enter the formula =**1-P16**.

In cell U3 enter **P(Elec OK)**. In cell V3 enter **1**, and in cell V4 enter **0.9**. Select cells V3:V4. In the lower right corner of cell V4, click the fill handle and drag down to cell V13. With cells V3:V13 still selected, click the Increase Decimal button once so that all values are displayed with one decimal place.

Select columns V:AG. (Select column V. Click and drag the horizontal scroll bar until column AG is visible. Hold down the Shift key and click column AG.) From the Format menu choose Column | Width. In the Column Width edit box type **5** and click OK.

In cell W1 enter **P(Mag OK)**. In cell W2 enter **0** (zero), and in cell X2 enter **0.1**. Select cells W2:X2. In the lower right corner of cell X2, click the fill handle and drag right to cell AG2. With cells W2: AG2 still selected, click the Increase Decimal button once so that all values are displayed with one decimal place.

Select cell V2 and enter the formula =**CHOOSE(J11,"Mech","Elec","Mag")**. With the base case assumptions the formula shows Elec.

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG
1			P(Mag OK)										
2		Elec	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
3	P(Elec OK)	1.0											
4		0.9											
5		0.8											
6		0.7											
7		0.6											
8		0.5											
9		0.4											
10		0.3											
11		0.2											
12		0.1											
13		0.0											

Obtaining Results Using Data Table Command

Select the entire data table, cells V2:AG13.

From the Data menu, choose Table. In the Table dialog box, type **P16** in the Row Input Cell edit box, type **P6** in the Column Input Cell edit box, and click OK.

With cells V2:AG13 still selected, click the Align Right button.

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG
1			P(Mag OK)										
2		Elec	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
3	P(Elec OK)	1.0	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
4		0.9	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
5		0.8	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
6		0.7	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag
7		0.6	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag
8		0.5	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag	Mag
9		0.4	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
10		0.3	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
11		0.2	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
12		0.1	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
13		0.0	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag

Embellishments

Select cells U1:AG13, and click the Copy button. Select cell AI1, right-click, and from the shortcut menu choose Paste Special. In the Paste Special dialog box, click the Values option button, and click OK. Right-click again, choose Paste Special, click the Formats option button, and click OK.

Select columns AJ:AU. Choose Format | Cells | Width, type **5**, and click OK.

Select cell AJ2, right-click, and from the shortcut menu choose Clear Contents. Select cells AK2:AU2, move the cursor near the border of the selection until it becomes an arrow, click and drag the selection down to cells AK14:AU14. Similarly, select cell AK1 and move its contents down to cell AP15. Also, move the contents of cell AI3 to cell AI8. Select cell AN1, and enter **Strategy Region Table**.

	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU
1						Strategy Region Table							
2													
3		1.0	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
4		0.9	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
5		0.8	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
6		0.7	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag
7		0.6	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag
8	P(Elec OK)	0.5	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag	Mag
9		0.4	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
10		0.3	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
11		0.2	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
12		0.1	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
13		0.0	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
14			0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
15								P(Mag OK)					

Apply borders to appropriate ranges and cells to show the strategy regions. Apply shading to cell AR8 to show the base case strategy.

	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU
1						Strategy Region Table							
2													
3		1.0	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
4		0.9	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
5		0.8	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
6		0.7	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag
7		0.6	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag
8	P(Elec OK)	0.5	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag	Mag
9		0.4	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
10		0.3	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
11		0.2	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
12		0.1	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
13		0.0	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
14			0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
15								P(Mag OK)					

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