The Implications of the Holy Ghost in Turing's Machine

Is mathematics objective? Brian Rotman's *Ad Infinitum, The Ghost in Turing's Machine: Taking God Out of Mathematics and Putting the Body Back In. An Essay in Corporeal Semiotics* views mathematics literally as a language, with all the human baggage that comes with the word. This in contrast to mathematicians who strictly colloquially refer to it as a language, as to them it's far outside the bounds of human communication and culture. Viewing an academic subject from an unorthodox angle is always interesting in the postmodern sense, but mathematics is especially interesting as its foundational for the rest of science. A change in the eyepiece of mathematics recalibrates the telescope of science, and hence what we perceive in the universe. Whilst Rotman never looks back into the eyepiece after recalibration, the revised telescope is published for anyone to hold a wink at, and the cosmos is delightfully unclear. His essay has consequences on both mathematics' relationship with philosophy and scientism, which he never outright mentions but alludes to and sets his audience up to tackle.

In the eyes of the philosopher, mathematics and philosophy have been two sides of the same coin. Backed by Greek titans of ideology like Aristotle and Pythagoras, philosophy is by and large the only major which can break into industrial mathematical fields. Western logical philosophy has its distinct schools backed by patterns of thinking which all fall under the same 'meta-proof,' or 'meta-meta-proof' for that matter, which gives way to a formal education of philosophy. Mathematics is not one of these schools. Mathematics sits aside from these philosophical schools. In the venn-diagram between the two; their two-dimensional overlap is slim in the proof system they exist under. It's not a school, but is recognized and respected by philosophers as the ultimate system of reasoning, pure logic supposedly unswayed by cultural influence, a formal wrangling of the messy meta-code which brings about inspiration and garners

intuition in mathematicians (Rotman 70). Mathematics deals with logical flights in no way deficient to its humanity counterpart, but is never explicitly named as superior. There is a sometimes addressed but rarely confronted prejudice towards mathematics as an inhuman logical thought experiment for the sake of thought experiments, but on the other side of the coin it is silently feared as the truest form of a philosophical undertaking. It's both objectified and admired. It's philosophy by root but not quite in the branches and certainly not the leaves. Its inspiration, purpose, and applications differ so starkly that being a professional philosopher can lock you outside the glass prison of mathematics.

Ask the mathematician and their plights are of a higher flight than their coworker in the analytics department of Northwestern Mutual, or if the philosopher is really lucky, the quant think tank at Vanguard. The mathematician stands outside a concrete prison of 'thinkers' constructed of cultural imposition, the human condition, and their disagreements over it. And as self-aware as they are, they are still inebriated by language, wavering on the tightrope of reason. Because of this the mathematician finds himself closer to the Platonist God that the philosophers discovered. His discoveries are effable in language only because application necessitates it. His discoveries are objective truths absent of human influence and hence closer to the nature of the universe and God. He not only looks at prison, but understands the logic behind the laws of the atoms that form the bars. By extension, the applied sciences sit outside with the mathematician as well. The demi-urge physicist spends his whole life trying to usurp his mathematician father and disprove the God he so often finds staring him in the face. The chemist and biologist do not concern themselves with the theory of the mathematician, and biologist not with that of the physicist either, though the physicist and the mathematician take full responsibility for the two's plights, and the psychologist keeps accidentally locking himself in the prison with the

philosopher. Aside from the prison allegory, the mathematician finds his venn-diagram with the philosopher having even less an overlap. They both pursue logic and have nothing more in common.

Brian Rotman writes *Ad-Infinitum* as an attempt to bring the mathematician down from their divine perch by interrogating one of the most central concepts in mathematics, infinity. In doing so, Rotman inadvertently attempts to (but may be aware of) critique scientism. Critical to understanding this critique is understanding Rotman's two innovative ideas about mathematics and their consequences, the agent and the trans-iterates.

The first two thirds of *Ad-Infinitum* are spent setting up mathematics as a subject with external influence. He details the code of math against the meta-code of math, and the signifiers themselves being subjective in nature. The meta-code being all the work-up and thought surrounding formal mathematics such as illustrations, real world motivations, and applications; the code being the formal system of proof robust against human influence and perception (Rotman 69). The subjectivity of the signifiers stems from the meta-code. Whilst the code is designed to be free of worldly influence and strictly concerning mathematics, ideas in mathematics, how we perceive the world and choose to put it in on paper via functions, matrices, and the like is a result of ideas and assumptions about how humans perceive the world; mathematical signifiers and the code spring out of the meta-code, they are not born simultaneously.

He details long standing thought experiments designed to strain the concept of infinity, especially as it relates to the natural numbers (1, 2, 3 and so forth), and by extension, all integers (back forth, -2, -1, 0, 1, 2, and so forth). To Rotman, infinity derived from the natural intuition of *Ad Infinitum* (counting forever) is pervasive in all of mathematics, not just analysis and other

fields where infinity is necessary to take a limit. Any proposition which asks to consider an arbitrary value from an infinite set, the integers or otherwise, to validate or negate said proposition relies on the *Ad Infinitum* principle; holding for an arbitrary value implies it holds for infinite values, and hence all must be tested.

But who is doing such testing? Rotman responds to this question with a dessert fork for an answer; three entities evaluate a mathematics problem, the person, the subject, and the agent. The person is the individual who exists in the context of the world, and hence outside of the context of pure mathematics. They develop inspiration and intuition for mathematics, and come about their understanding in abstract and comparatively 'illogical' ways. They exist in the meta-code. The subject on the other hand exists in the formal-code of mathematics, made up of universal signifiers and accepted patterns of logic. They think strictly in the bounds of mathematics, they have no motivation or inspiration other than understanding the thought experiment and to translate the signifiers of mathematics to palatable reasoning and feed for the agent. Lastly is the agent, the computer of the process. The role of the agent is to verify every computational statement infinitely that holds in a proof. If the supposition projects 'for all $\alpha \in S$,' the agent's job is to verify the supposition for every α (Rotman 73).

The split between the person and the subject is normative; it's the foundation of formal proofs. The split of the agent is unusual; it's not that it doesn't take the logic of the proof for granted; but if we are to propose a theorem which is true for infinitely many values, someone must do the verifying. Put simply, the agent is the computer that has eternally existed in mathematics, verifying the logic of an 'infinitely-true' statement with said infinitely many values.

The perch of which mathematics may be knocked off now arises. Rotman brings up many other manners of questioning infinity, but most importantly, if the role of the agent is to verify infinitely, when does it stop? It may verify incredibly quickly, but there will always be values it doesn't reach in its verification. My goal is not to investigate when the death of the agent occurs, but rather the bound for which it does not reach. Rotman denotes this bound as $, and the numbers which come after it as 'trans-iterates' (as opposed to the iterates 1, 2, 3, …$). The theorems for which have been established for mathematics don't apply for transiterates, as the agent never reaches them in its proof. The bound of the iterates also implies a breakdown of the idea of fields for the integers, as they are no longer closed under addition or multiplication.

A more concrete analog of the transiterates can be found in the comparison of the signifiers of the real numbers and the natural numbers, and is also where I suspect some of Rotman's inspiration for this thinking may come from. Consider the sequence of real numbers 1.1, 1.11, 1.111, 1.1111…. It's obvious by definition of the real numbers that such a sequence is infinite, but we also know 1.2 to be greater than every value in this increasing sequence, yet it will never be reached. Now, consider representing every natural number as tally marks, 1 as 1, 2 as 11, 3, as 111, and so forth. Return to the sequence of real numbers and cover up the '1.' on each number with your left hand (or right if you prefer the sequence starting farther out of your view), and you now have the iterates. Not only does this idea suppose the natural numbers and real numbers being different only by signifiers, but the $ value Rotman proposes can be thought of here as 2. The iterates never reach it, but it exists and is greater than the iterates, and because the iterates never reach it, the agent can't verify statements about it, and values greater than '2' constitute the 'trans-iterates'.

A questioning of the existence of infinity as an element of the natural numbers, or real numbers for that matter in mathematical theory obviously uproots most of mathematics, and a breakdown of field theory certainly does. Does this then invalidate all of mathematics and its consequences up to this point? No, not quite. Rather it purports mathematics into a framework more similar to the study of philosophy. We can consider *Ad Infinitum* mathematics and *Transiterate* mathematics as two different schools of thought. Questions about the validity of infinity have existed for thousands of years. Like Rotman suggests, how many grains of sand do you remove from a heap until it is no longer a heap? After what point of running a half-mile, half of that, and half that, do you eventually complete the mile? It just so happens, whether for cognitively limiting reasons or otherwise, an infinitely iterative school of thought caught fire rather than a finite one. But such a human assumption or presupposition is a product of human inference, not a fact of the universe. Whilst constantly expanding, even the universe at any given moment is finite, and eventually will reach a conclusion of expansion at heat death, should the math used to support these theories hold as well. Infinity is not empirical, but theoretical, and the math that follows from it should be treated as such.

For this reason we can break up mathematics into two separate schools (though the latter may have no students), those who operate under the assumption of infinity, and those who do not. Both have logical bases and vastly different consequences on mathematical theory. Neither one is necessarily ‘correct’, and the former at current seems far more applicable.

This is where mathematics's additional similarity with philosophy arises. It has schools, just philosophy, and hence loses its objectivity. Much like philosophy uses base assumptions and then sequences of logic to discuss the human condition, culture, mortality, etc., mathematics assumes infinity (as well as other axioms) and uses sequences of logic to discuss the state of the

observable, and even unobservable universe. And much like how philosophy has some easily applicable and widely adopted schools of thought, math does too, although dirac in nature.

Furthermore, mathematics is not necessarily just separated into these two schools of thought. There may be many more schools to be discovered from the deconstruction of assumptions made, and many more of which are cognitively inaccessible. Rotman even parallels this in discussing mathematical operations. We start with addition, then multiplication, exponentiation, hyper-exponentiation, hyper-hyper-exponentiation, and so forth. There exists a cognitive limit to this process however, but the operation still exists. Similarly, there are schools of math outside of our cognitive limit, but they can still exist (Rotman 107).

In practice, this consequence does nothing for the practice and application of mathematics. Who could know where to begin with the *transiterate* system? The *Ad Infinitum* school of mathematics that has been bought into has worked its way up the chain to physics and then split to chemistry and engineering successfully in a way that develops medicine and creates the internet. Our applications of physics often work in bounds of estimation, never nearing Rotman's $ bound of the *transiterate*. The agent has checked and verified the statements for the numbers we deem necessary to have built the world around us, and we have double checked the agent in creating it. What I am suggesting is a reality check for mathematics. To propose that mathematicians and their understanding of the world is not objective. It is a practical subjective that allows humans to explain the universe in a logical manner. This doesn't mean it is the 'truth' though, just one way of looking at numbers; mathematicians do not have the be-all end-all way of observing the world, they aren't discovering ultimate truths, and getting no closer to god, but rather a fuller picture of the human interpretation of the universe. In separating mathematical schools of thought and breaking down mathematics into the person, subject, and the agent,

Rotman has taken God out of the mathematician, put him back in heaven, and handed the mathematician a computer.

What Rotman does not touch on heavily is the implication of mathematical schools of thought on the rest of science. He touts mathematics and science as "inextricable from each other," but science operating in what is "real" and math often operating in what is "imagined" (Rotman 86). For this reason, a breakdown of current mathematical theory as objective has a null effect on practical science in his opinion. He even discusses physics as operating in known numerical bounds, and as such not having to worry about the possibility of *transiterates*. Whilst in the application of science as an endeavor of human curiosity and sometimes progress, a reframing of mathematics as subjective has no impetus on the practice of physics, and hence chemistry, and hence biology, and really even mathematics practically. Once again, where would one even cognitively start in a *transiterate* system? But like the effect on math, it should reframe how scientists subconsciously understand and digest their science; mathematics is at best subjective to the human experience, and at worst a totality which is forever cognitively out of reach of humanity.

To harden the tone a little, I am outright suggesting that scientists view their work as subjective, as when non-observational or observational with statistically quantification, it's a consequence of a subjective, human influenced mathematical system. The downstream effects of mathematical laws as subjective should purport the laws of physics at the edges of numerical approximation of which we are yet to reach as subjective, and hence the chemistry and biology that follow subjective as well. The statistical system which measures outcomes in these fields is deeply rooted in limits and infinity, and should certainly purport subjectively into the results of

the field, an extra ounce of subjectivity beyond the randomness the field of statistics already tries to measure.

Taking the current mathematical system to be subjective, what consequences does this have on scientism? If mathematics is subjective, then the downstream consequences are that physics, chemistry, biology, and other natural sciences, and any epistemic undertaking which uses mathematical forms of measurement for that matter are then also subjective. This includes but is not limited to political science, sociology, and psychology. Hence, the breakdown is twofold; scientific knowledge rooted in mathematical proofs, like brownian motion, should not be taken for granted as impartial, nor should the statistical theories which quantify observational science. Certainly then the principles of strong scientism are violated, if the foundations of the scientific process, the quantification of common sense is subjective, then the modern practice of the scientific method is also subjective. (Hietanan 526). Current science may not be an objective truth, and hence not the *only* way to obtain knowledge. How can a subjective foundation lead to an objective result?

From this, scientism starts to look more like religion and less like philosophy. When *Ad Infinitum* mathematics is seen as objective, scientists are putting their belief in it, but with subjectivity, they have faith. The clergymen of top mathematicians possess an esoteric knowledge of the system which whilst public in that anyone can come to understand it, it is shrouded in the weeds of language and a cognitive barrier such that when it reaches the lay scientist, the orthodoxy is lost and the orthopraxy on faith remains. Axioms exist as fundamental truths, like life after death, and the proofs that follow constitute the holy book of scientism. There is even a seminary through the university for those who wish to become part of the clergy.

When mathematics no longer tells an objective truth about the universe, the scientist who relies on it becomes faithful to it, they no longer have certainty but rather approximations of such.

Whilst unexplored, a school of *transiterate* mathematics could create another sect of such a religion. Similar axioms would hold, but the proofs would be different, and the clergy between the two wouldn't agree; the existence of infinity would undoubtedly be an irreparable difference, but still one based on faith. If mathematics isn't objective, it isn't God, it plays God, and those that follow it are playing along. Such a framework makes strong scientism's strongest advocates' arguments against religion hypocritical. The metaphysical cannot be proved corporeally, or at least in a scientifically sufficient manner, but neither can infinity. The dogma that religion enforces on its members scientism forces on its practitioners: peer review, repeatability, statistical significance; all are arbitrary but deemed necessary and regulated. The blind obedience religion instills (though liberation theology would have its say) is the same blind obedience that scientists place in mathematics, if not in proven mathematics than the ability to discover foundations through mathematics. The structure of modern scientific practice and the belief in strong scientism becomes only nominally different from the religion of science. Scientism and religion do not have to butt heads; they have more in common than they care to admit, and in the present market economy of religions, it's no harm to subscribe to both.

It's important to distinguish that an entire breakdown of science does not follow from a breakdown of mathematics. Hypothesis, experiment, conclusion, and repeat, the scientific method, still holds as a structure without numerical measurement. Binary outcomes can be collected, and conclusions drawn at arbitrary levels of significance to the scientist without ever divulging into statistical theory.

It's also important to distinguish that such a discussion and subsequent reframing of science as epistemologically subjective does not ruin science as we practice it today. In reference to the aforementioned 'playing God', I make such a statement incredibly lightly; scientists are playing the absolute best game we have with the cards humanity was given. Weak scientism still holds weight irrespective of one's view on mathematics. Aside from the argument on mathematics, many titans of science history were devoutly religious, and the development of the two at odds with each other is a recent consequence of religious fundamentalism (Richard 53). Science was a religious exploration rather than a religious usurpation. A view of modern science as an extension of spirituality would have been a more natural evolution in view of the standoff considering the marketplace of religions today compared to its actual evolution as an enemy of religion. In fact, calling into question mathematics' status as an objective truth is both contradictory towards strong scientism, and exactly at the heart of the science itself. Science's purpose is to ask and answer questions, and is most interesting and world changing when it does so of the status quo. Why not question its very mathematical foundation?

To loop back towards the mathematician, they have little control over how science interprets their work downstream, but they too should recognize the assumptions in their work and the subjectivity that follows. While it may not lead to a development of a separate school of mathematics, seeing themselves as numerical philosophers rather than divine translators may bring about a new intuition and breakthroughs in mathematics that otherwise would not have come about; seeing existing mathematical axioms, proofs, and of course meta-code as subjective, critiqueable especially, has no negative other than the expenditure of time.

Brian Rotman discusses the semiotics, culture of the thought experiment, and inductive reasoning on the natural numbers which fly under the radar in mathematics. Breaking

mathematics into an interaction between three individuals, the person, subject, and agent, he forces any mathematically trained reader to rethink any proof they have 'proved,' and consider the assumptions they blindly lean on, most importantly the assumption of an infinity. He never truly breaks out the topic of mathematics, touching on physics and considering computer science, but not the natural or soft sciences. Nor is he able to set an axiomatic foundation for his *transiterate* thinking but his questions and propositions have arms which rope in all scientific disciplines. His breakdown of mathematics into its core assumptions, and the questioning of their validity implicates a fragmenting of mathematics into schools based on assumptions, similar to philosophy. This purports mathematics as a subjective quantification of the human experience, rather than discovered objective truths about the universe, and hence calls into question the objectivity of the ensuing sciences which rely on mathematical theory. Breaking mathematical assumptions transforms math from divine to philosophical, and divulges scientism from philosophical into religious; curious as scientism finds itself often at odds with religion. However, the conversation around mathematical assumptions is not a trickle down which takes science to the flame. What it deserves is a reframing of how we look at mathematics and consequently science. Mathematics and science may not be objective truths, but rather subjective readings of the universe palatable to human cognition, but for our purposes, whether it be in medicine, technology, or architecture, this has proved to be more than suitable.

What still needs investigating is the *transiterate* school of mathematics. Its inspiration and purpose differ so starkly that while an individual possessing the skills to study *Ad Infinitum* mathematics would more than likely bear fruit studying the other with help and training. But in a world of such specialized academics, where no one takes a risk in funding such a thought

experiment with murky applications, we are stuck, possibly cognitively limited and certainly otherwise, in one school of mathematics.

Works Cited

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