APM1111: Statistical Theory
Formative Assessment 2

Lindsay Faith C. Bazar

3.49 Prove that $\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$.
First, let's expand the LHS,

Expanding $(X_j - 1)^2$ using the square of a binomial, we have:

$$(X_j - 1)^2 = X_j^2 - 2X_j + 1$$

Applying the summation:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2X_j + 1$$

Using the linearity of summation, we can separate the terms on the RHS:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + \sum_{j=1}^{N} 1$$

Simplifying each term,

The first term, $\sum_{j=1}^{N} X_j^2$, remains as it is because it's just the sum of the squared values.

The second term, $-2\sum_{j=1}^{N} X_j$ also remains as it is because it's simply summing the X_j's and multiplying it by -2.

The third term, $\sum_{j=1}^{N} 1$, is simply N, because summing 1 for N terms is the same as multiplying 1 by N, which gives N.

Thus, we have:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N. \qquad \square.$$

3.51 Two variables, U and V, assume the values $U_1 = 3, U_2 = -2, U_3 = 5,$ and $V_1 = -4, V_2 = -1, V_3 = 6$, respectively. Calculate $(a) \sum UV$, $(b) \sum(U+3)(V-4)$, $(c) \sum V^2$, $(d)(\sum U)(\sum V)^2$, $(e) \sum UV^2$, $(f) \sum(U^2 - 2V^2 + 2)$, and $(g) \sum(U/V)$.

$(a) \sum UV$

Calculating each pair:

$$\begin{aligned} U_1V_1 &= 3 \cdot (-4) &&= -12 \\ U_2V_2 &= (-2) \cdot (-1) &&= 2 \\ U_3V_3 &= 5 \cdot 6 &&= 30 \end{aligned}$$

Thus,

$$\sum UV = -12 + 2 + 30$$
$$\boxed{\sum UV = 20}$$

$(b) \sum(U+3)(V-4)$

Computing $(U+3)(V-4)$ for each pair:

$$\begin{aligned}
U_1 = 3, V_1 = -4: &\quad = (3+3)(-4-4) = 6\cdot(-8) && = -48\\
U_2 = -2, V_2 = -1: &\quad = (-2+3)(-1-4) = 1\cdot(-5) && = -5\\
U_3 = 5, V_3 = 6: &\quad = (5+3)(6-4) = 8\cdot 2 && = 16
\end{aligned}$$

Thus,

$$\sum(U+3)(V-4) = -48 - 5 + 16$$
$$\boxed{\sum(U+3)(V-4) = -37}$$

$(c) \sum V^2$

Calculating V^2 for each V:

$$\begin{aligned}
V_1^2 &= (-4)^2 && = 16\\
V_2^2 &= (-1)^2 && = 1\\
V_3^2 &= 6^2 && = 36
\end{aligned}$$

Thus,

$$\sum V^2 = 16 + 1 + 36$$
$$\boxed{\sum V^2 = 53}$$

(d) $(\sum U)(\sum V)^2$

Computing for $\sum U$ and $\sum V$:

$$\begin{aligned}
\sum U &= 3 + (-2) + 5 && = 6\\
\sum V &= -4 + (-1) + 6 && = 1
\end{aligned}$$

Then,

$$(\sum V)^2 = 1^2 \qquad = 1$$
$$\boxed{(\sum U)(\sum V)^2 = 6\cdot 1 = 6}$$

$(e) \sum UV^2$

Computing UV^2 for each pair:

$$\begin{aligned} U_1 = 3, V_1 = -4: \quad &= 3 \cdot (-4)^2 = 3 \cdot 16 && = 48 \\ U_2 = -2, V_2 = -1: \quad &= -2 \cdot (-1)^2 = -2 \cdot 1 && = -2 \\ U_3 = 5, V_3 = 6: \quad &= 5 \cdot 6^2 = 5 \cdot 36 && = 180 \end{aligned}$$

Thus,

$$\sum UV^2 = 48 - 2 + 180$$
$$\boxed{\sum UV^2 = 226}$$

$(f) \sum (U^2 - 2V^2 + 2)$

Computing $(U^2 - 2V^2 + 2)$ for each pair:

$$\begin{aligned} U_1 = 3, V_1 = -4: \quad &= 3^2 - 2(-4)^2 + 2 = 9 - 32 + 2 && = -21 \\ U_2 = -2, V_2 = -1: \quad &= (-2)^2 - 2(-1)^2 + 2 = 4 - 2 + 2 && = 4 \\ U_3 = 5, V_3 = 6: \quad &= 5^2 - 2 \cdot 6^2 + 2 = 25 - 72 + 2 && = -45 \end{aligned}$$

Thus,

$$\sum (U^2 - 2V^2 + 2) = -21 + 4 - 45$$
$$\boxed{\sum (U^2 - 2V^2 + 2) = -62}$$

$(g) \sum \frac{U}{V}$

Computing $\frac{U}{V}$ for each pair:

$$\begin{aligned} U_1 = 3, V_1 = -4: \quad &= \frac{3}{-4} && = -\frac{3}{4} \\ U_2 = -2, V_2 = -1: \quad &= \frac{-2}{-1} && = 2 \\ U_3 = 5, V_3 = 6: &= \frac{5}{6} \end{aligned}$$

Thus,

$$\sum \frac{U}{V} = -\frac{3}{4} + 2 + \frac{5}{6}$$
$$\sum \frac{U}{V} = -\frac{3}{4} + \frac{12}{6} + \frac{5}{6}$$
$$\sum \frac{U}{V} = -\frac{3}{4} + \frac{17}{6}$$

To combine the fractions, we find a common denominator (12):

$$\sum \frac{U}{V} = -\frac{9}{12} + \frac{34}{12}$$
$$\boxed{\sum \frac{U}{V} = \frac{25}{12}}$$

3.90 Find the geometric mean of the sets $(a)\ 3, 5, 8, 3, 7, 2\ and\ (b)\ 28.5, 73.6, 47.2, 31.5, 64.8.$

The geometric mean of a set of numbers is calculated by taking the n-th root of the product of all the numbers in the set, where n is the number of values in the set

(a) Set: $3, 5, 8, 3, 7, 2$

Multiplying all the numbers:

$$3 \cdot 5 \cdot 8 \cdot 3 \cdot 7 \cdot 2 = 5040$$

Since there are 6 values in the set, we set n = 6. Therefore, we take the 6th root of 5040:

$$\boxed{\text{Geometric mean} = \sqrt[6]{5040} \approx 4.14}$$

(b) Set: $28.5, 73.6, 47.2, 31.5, 64.8$

Multiplying all the numbers:

$$28.5 \cdot 73.6 \cdot 47.2 \cdot 31.5 \cdot 64.8 \approx 202,092,516.86$$

Since there are 5 values in the set, we set n = 5. Therefore, we take the 5th root of $202,092,516.86$:

$$\boxed{\text{Geometric mean} = \sqrt[5]{202,092,516.86} \approx 45.83}$$