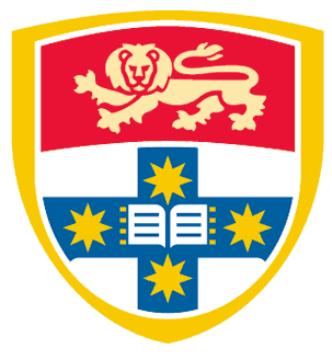


香港中文大學  
The Chinese University of Hong Kong



香港浸會大學  
HONG KONG BAPTIST UNIVERSITY



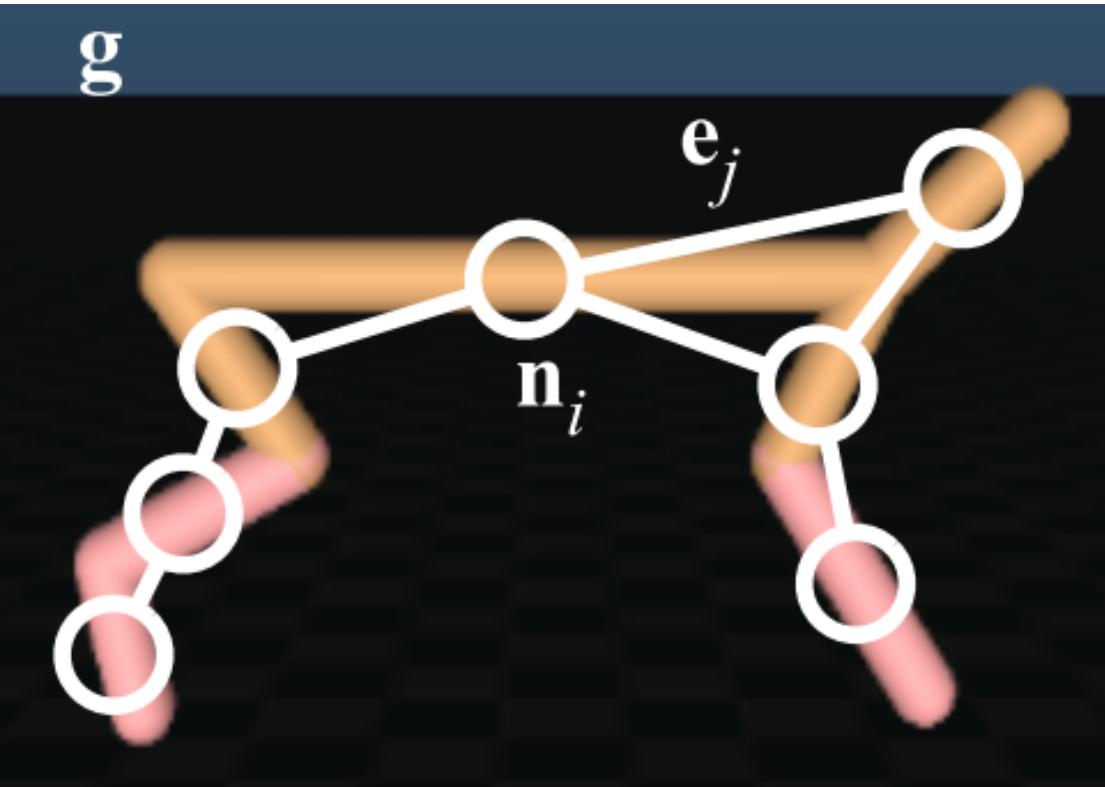
THE UNIVERSITY OF  
SYDNEY

# Understanding and Improving Graph Injection Attack by Promoting Unnoticeability

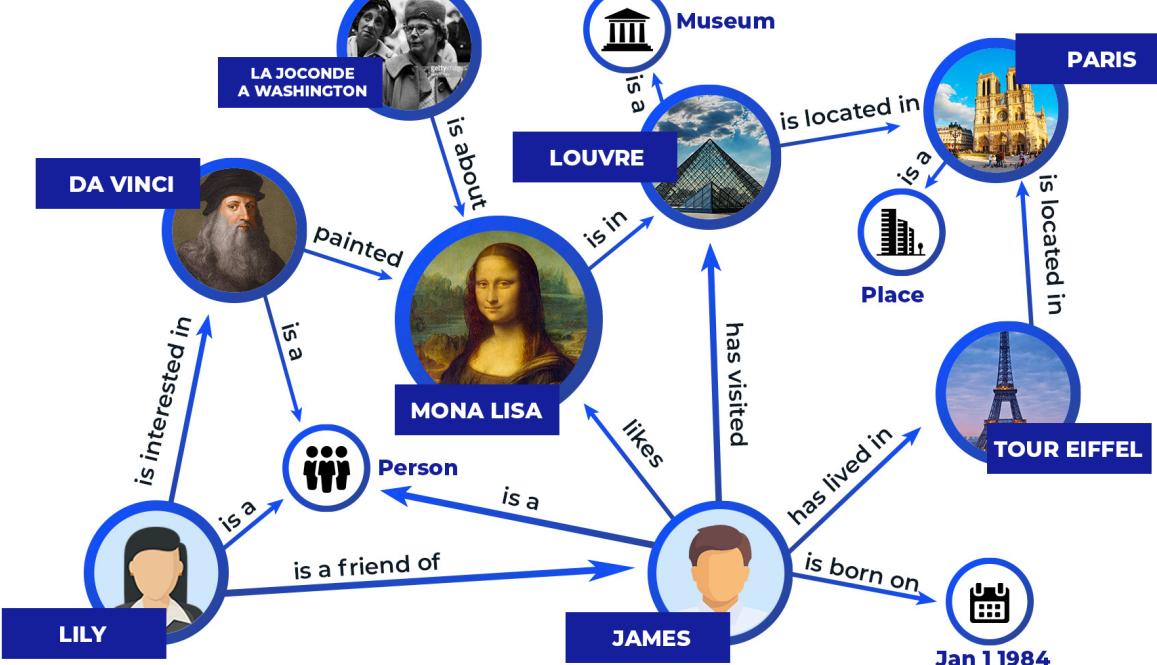
Yongqiang Chen  
CUHK

*with Han Yang, Yonggang Zhang, Kaili Ma, Tongliang Liu, Bo Han, and James Cheng*

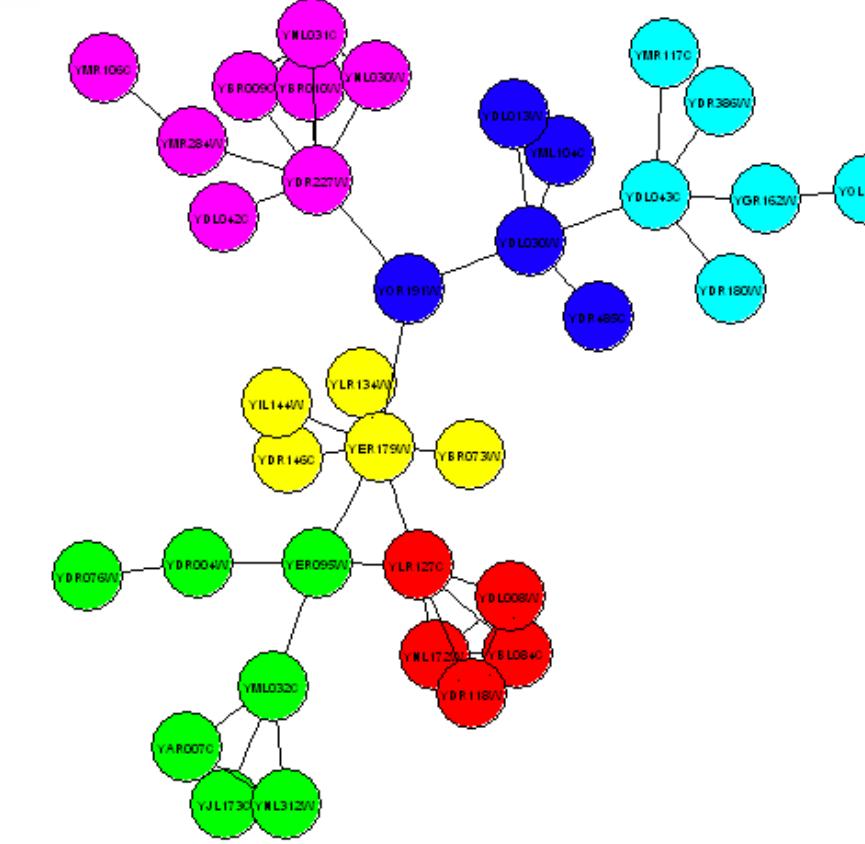
# Graph Neural Networks (GNNs) Are Widely Applied



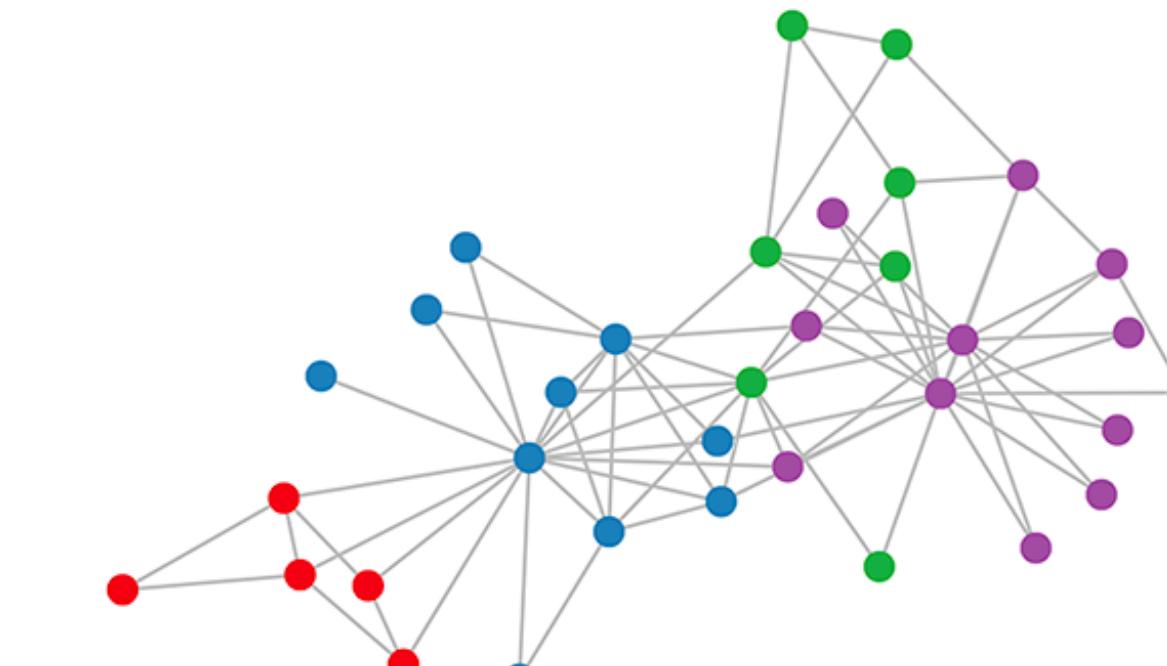
Model & Inference over  
the physical world



Knowledge Graph  
Completion & Analysis

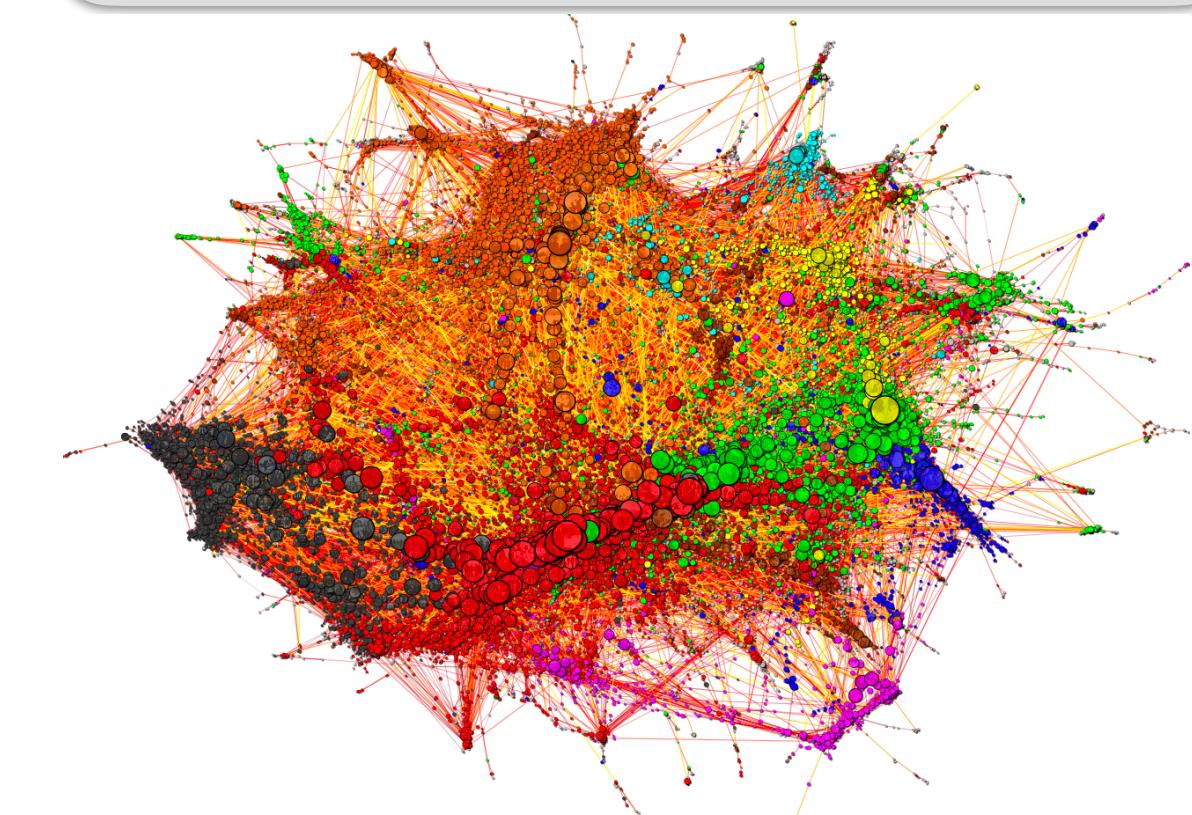


Protein Interaction  
Predictions



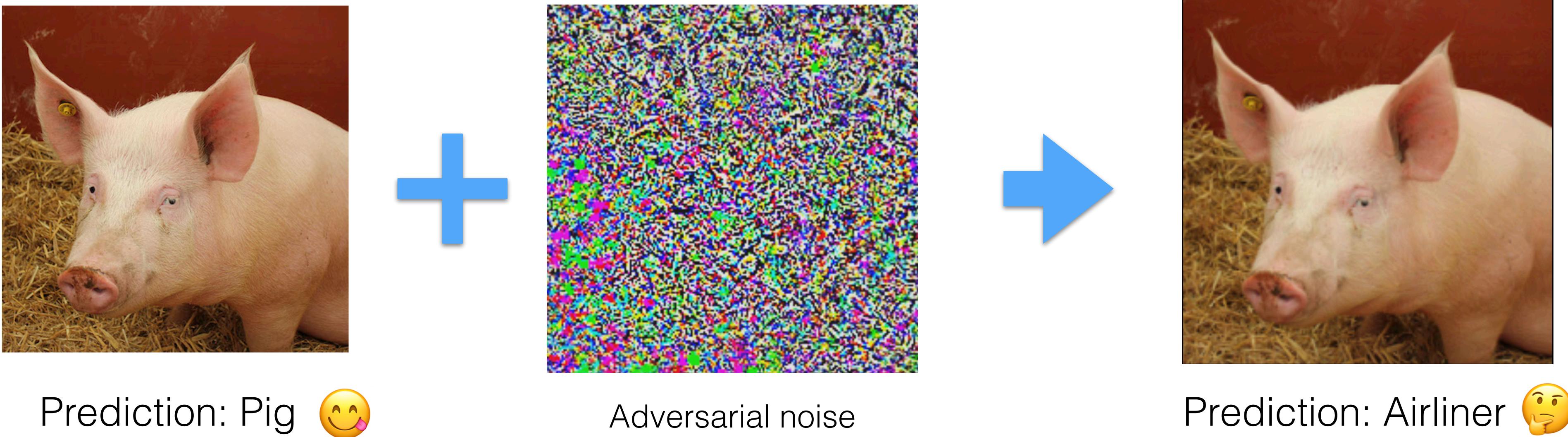
Social Network Analysis

Besides, GNNs  
can also process  
structures like  
image and text...

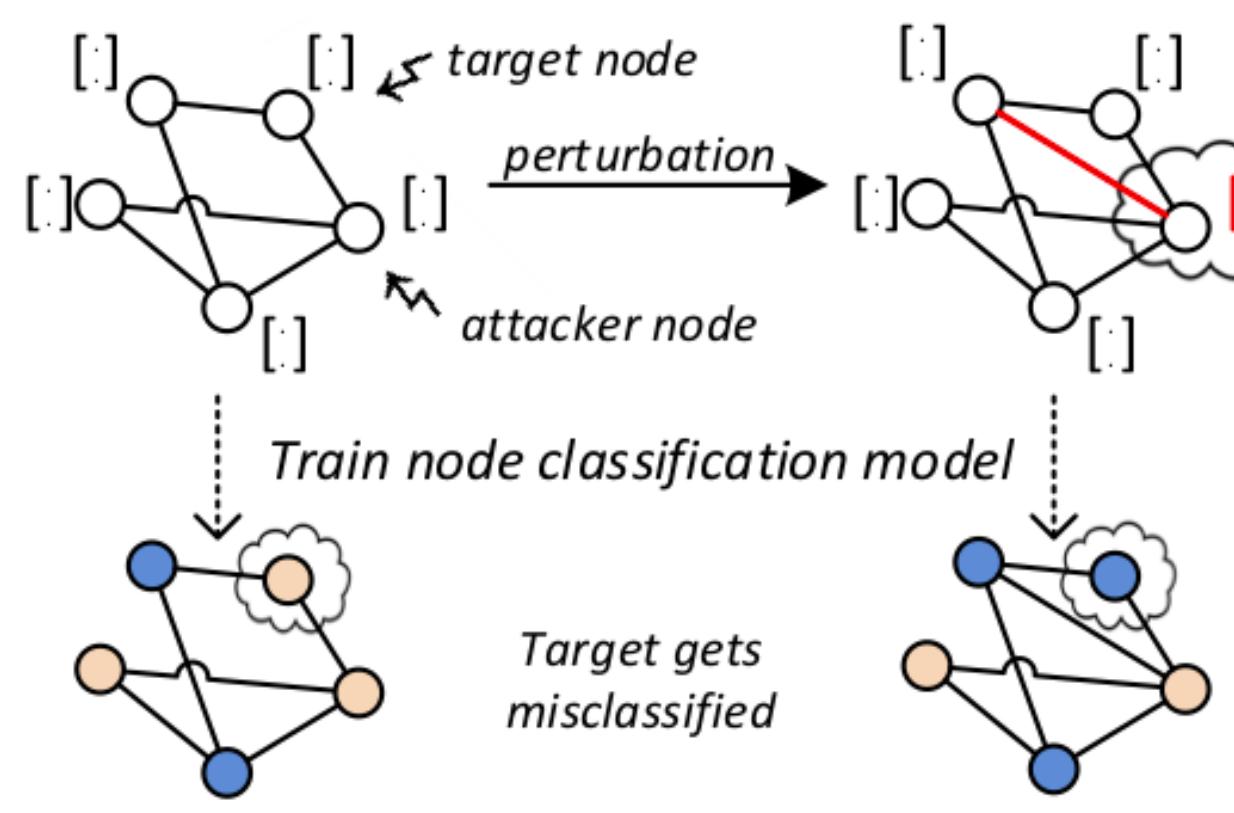


Recommender Systems

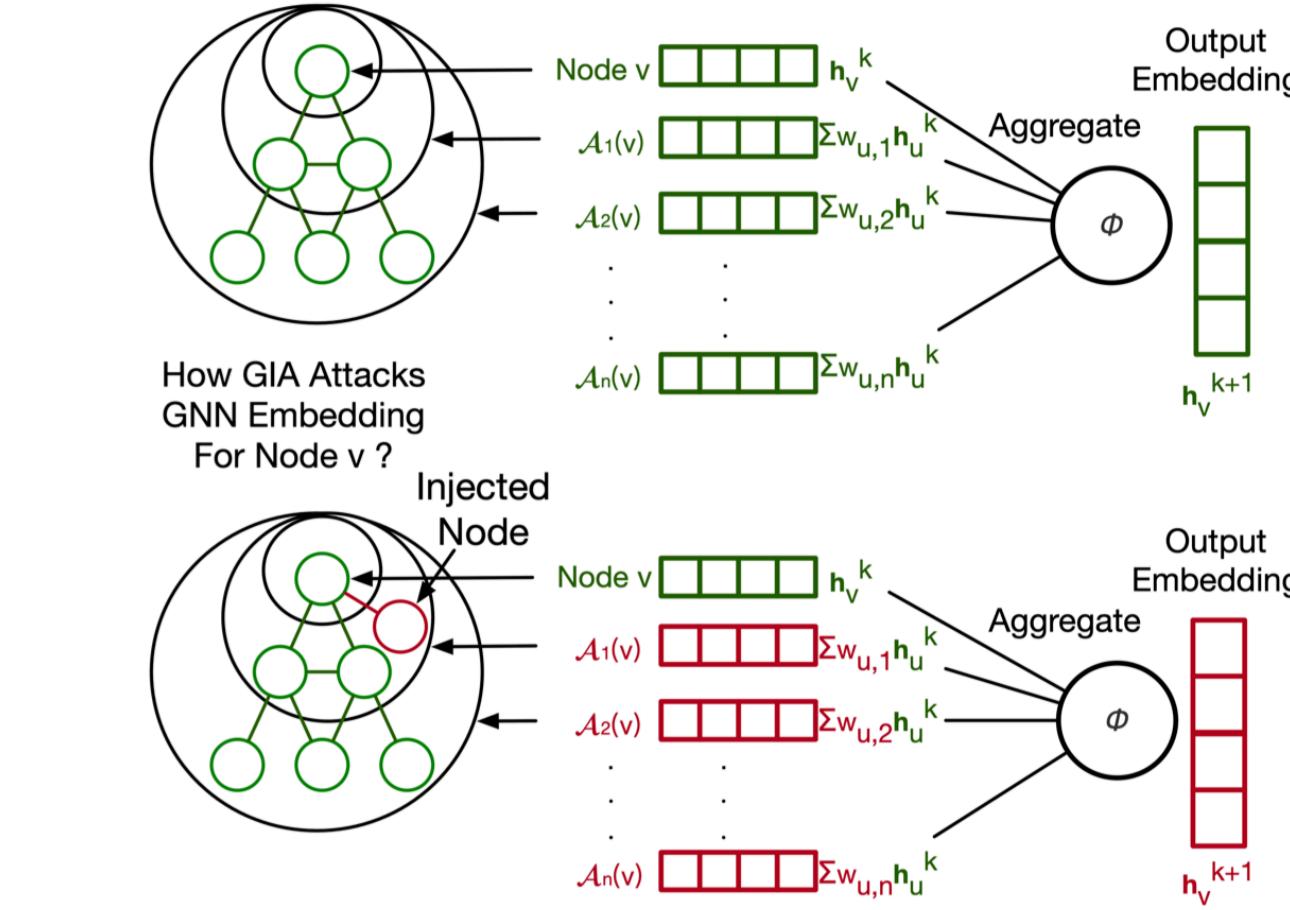
# GNNs Are Inherently Vulnerable



(Szegedy et al., 2014; Goodfellow et al., 2015; Kolter and Madry et al. 2019)

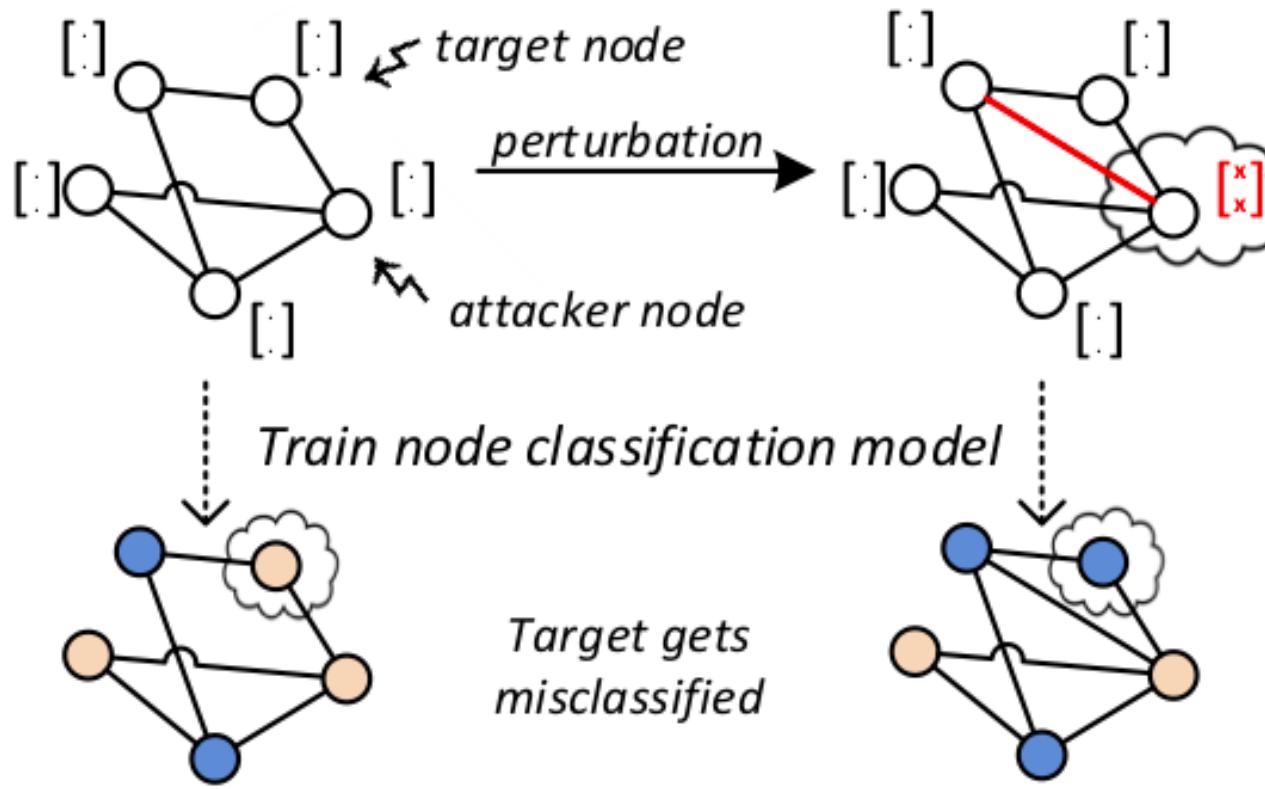


(Zügner et al., 2018)

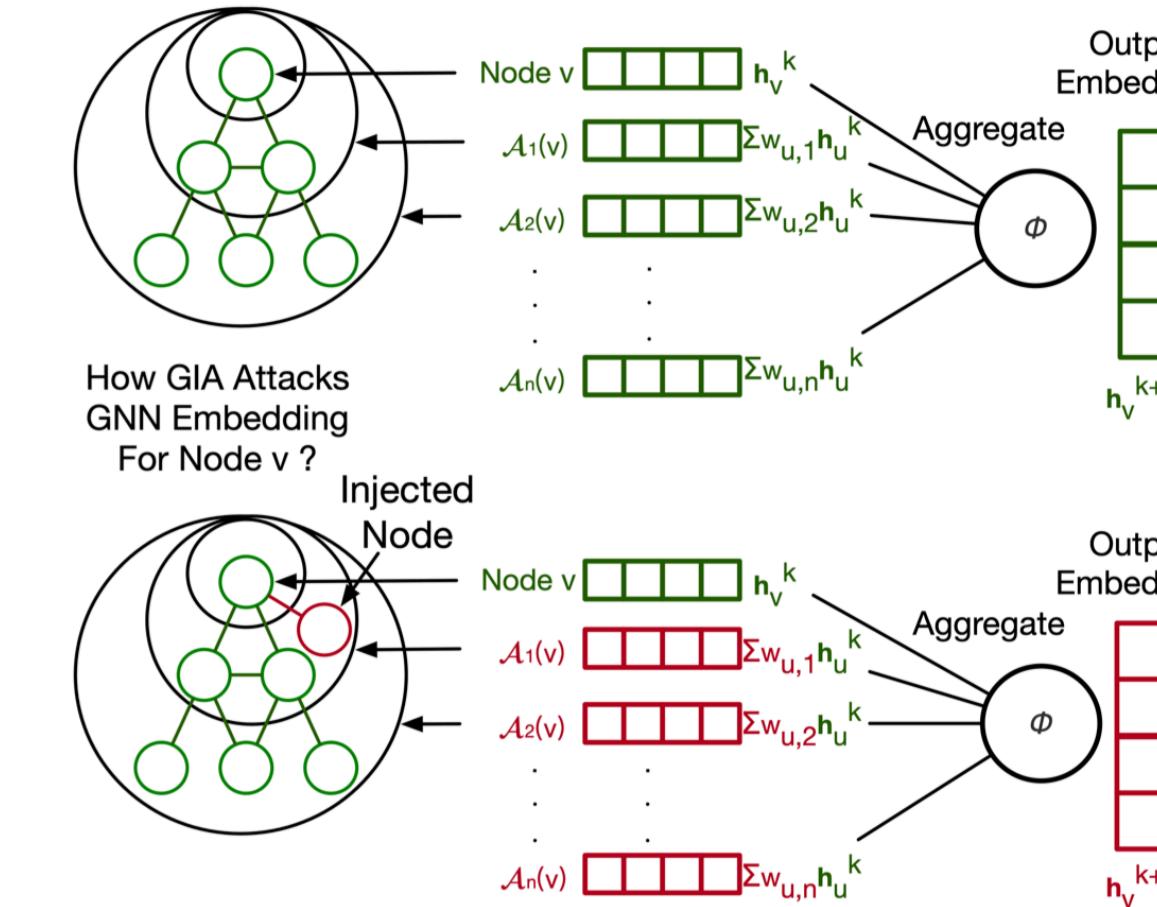


(Zou et al., 2020)

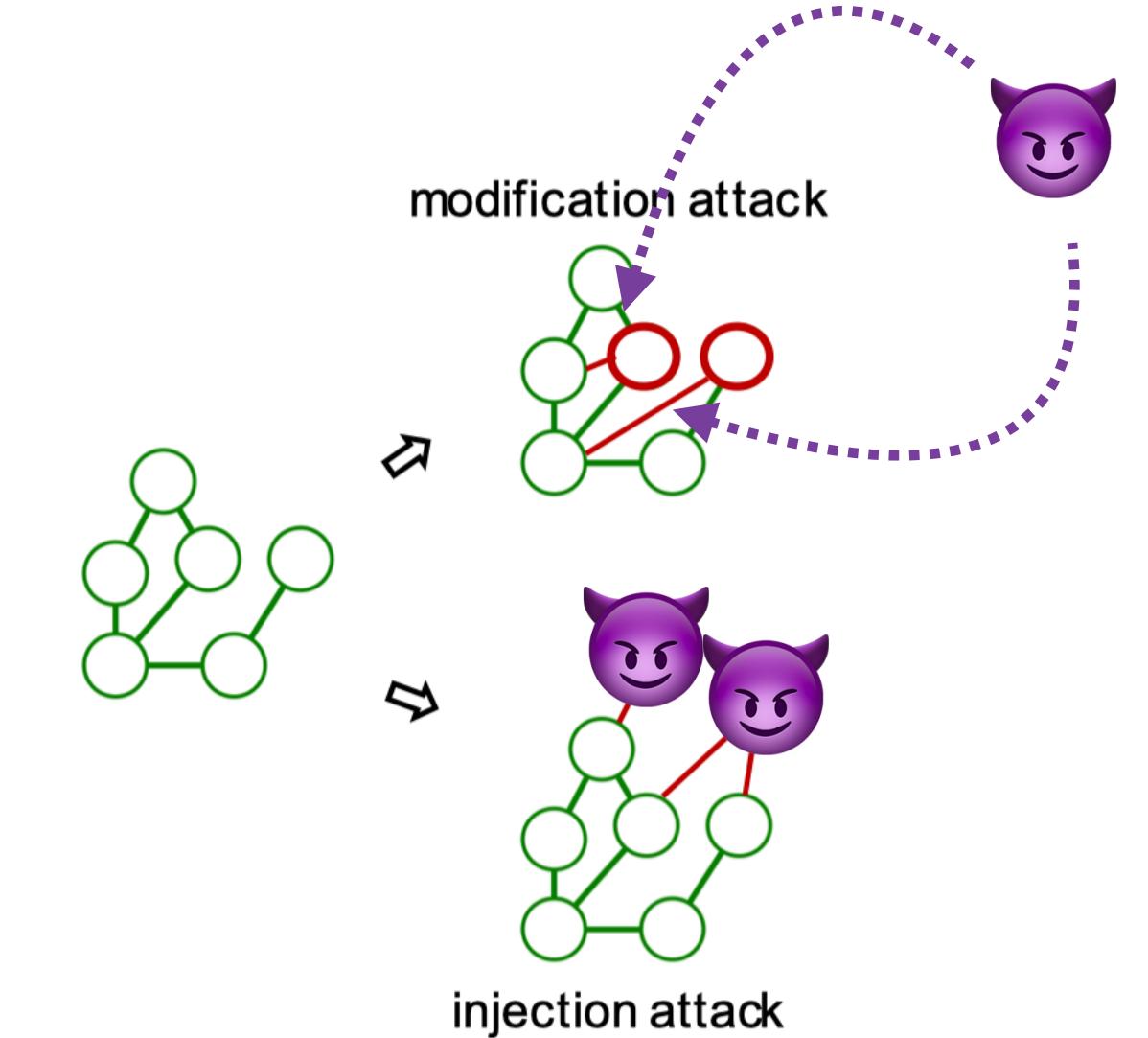
# Adversarial Attacks on GNNs



(Zügner et al., 2018)



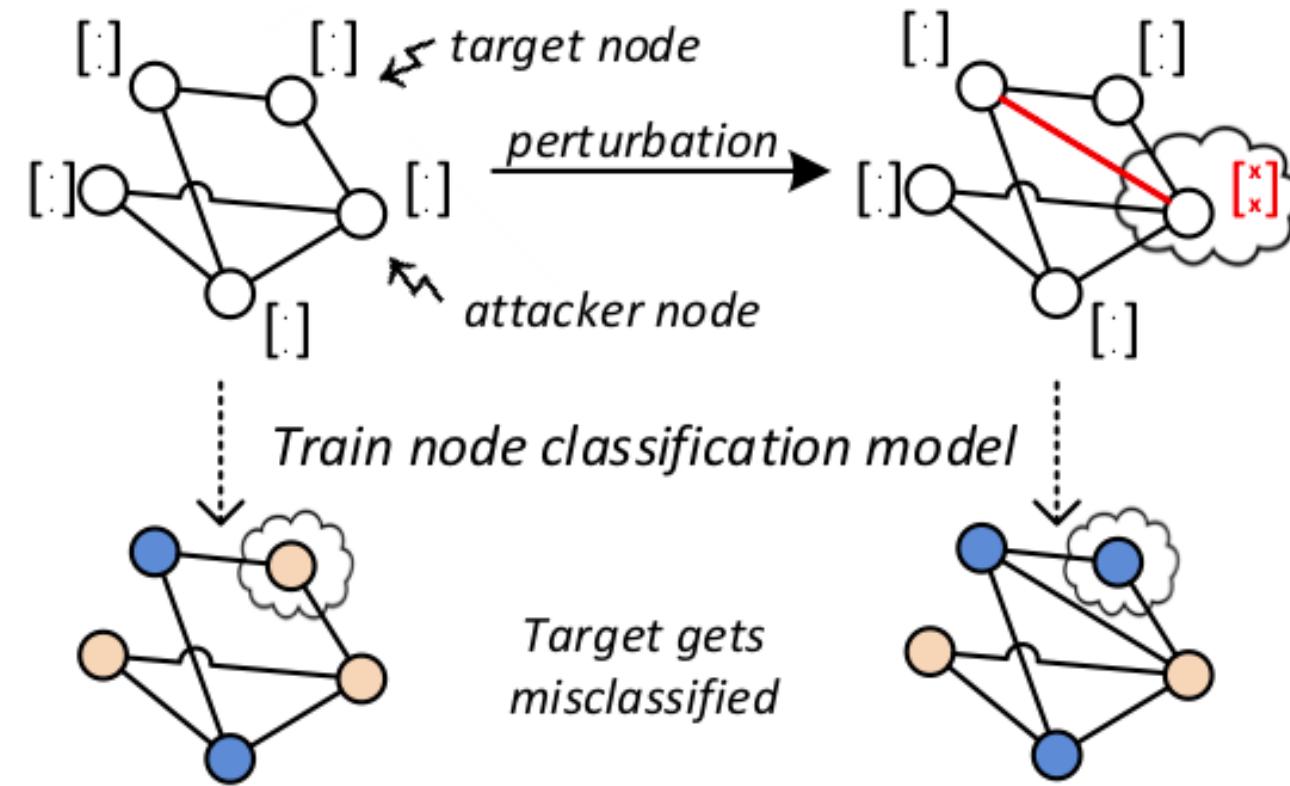
(Zou et al., 2020)



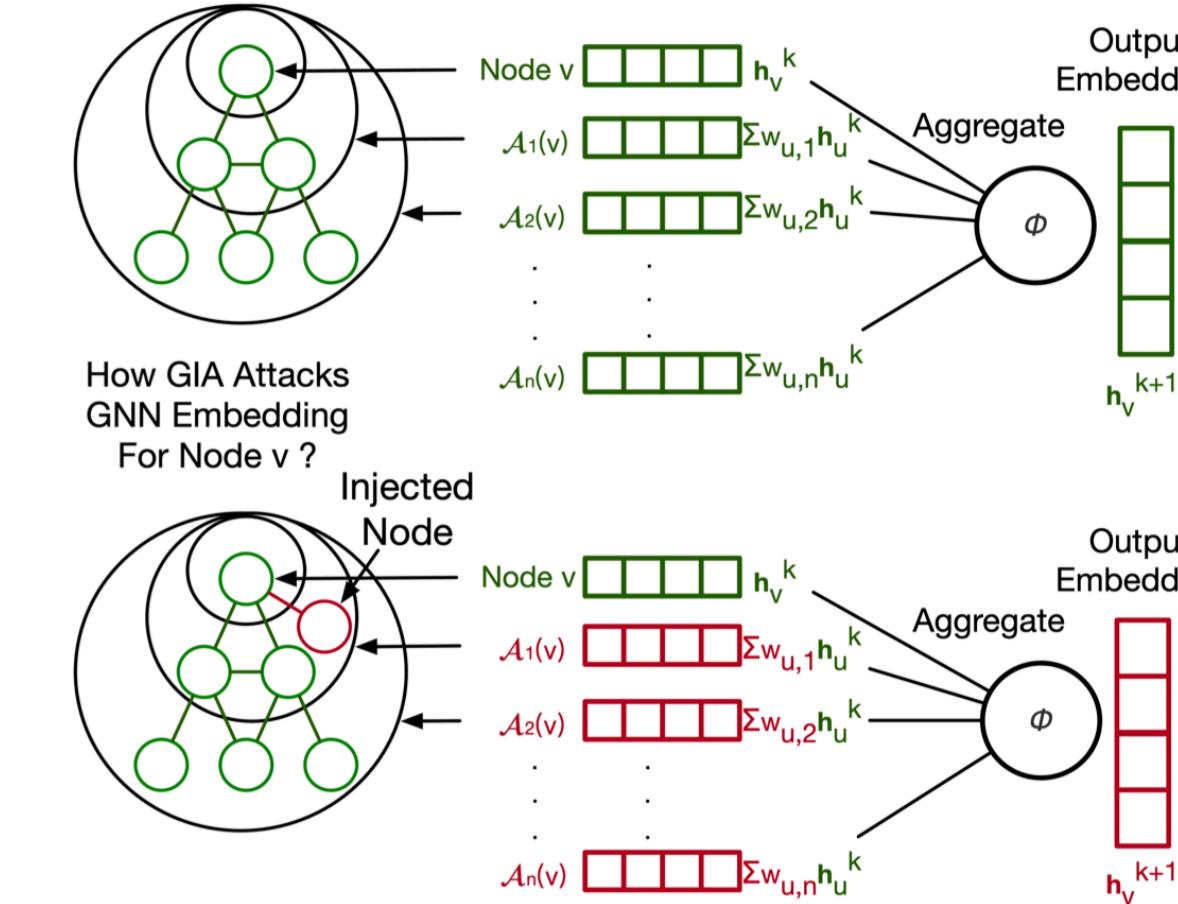
## Adversarial Objective:

$$\min \underbrace{\mathcal{L}_{\text{atk}}(f_{\theta^*}(G'))}_{\text{Usually } -\mathcal{L}_{\text{sup}}}, \text{ s.t. } \|G' - G\| \leq \underbrace{\Delta}_{\text{perturbation budgets}}$$

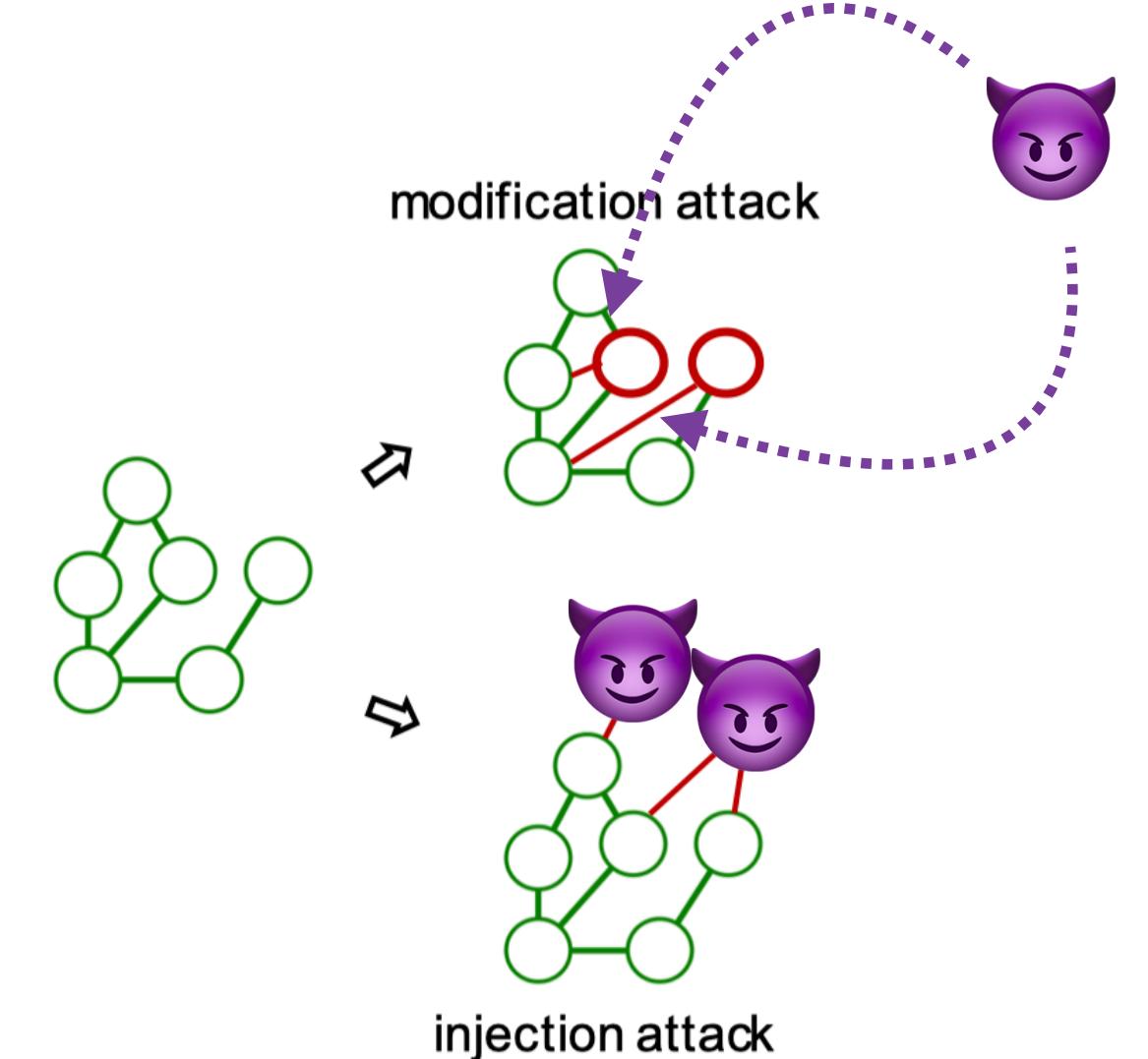
# Adversarial Attacks on GNNs



(Zügner et al., 2018)



(Zou et al., 2020)



GMA vs. GIA

## Adversarial Objective:

$$\min \mathcal{L}_{\text{atk}}(f_{\theta^*}(G')), \text{ s.t. } \|G' - G\| \leq \underbrace{\Delta}_{\text{perturbation budgets}}$$

## Graph Modification Attack (GMA):

$$\Delta_A + \Delta_X \leq \Delta \in \mathbb{Z}, \|A' - A\|_0 \leq \Delta_A \in \mathbb{Z}, \|X' - X\|_\infty \leq \epsilon \in \mathbb{R}$$



Sometimes Expensive

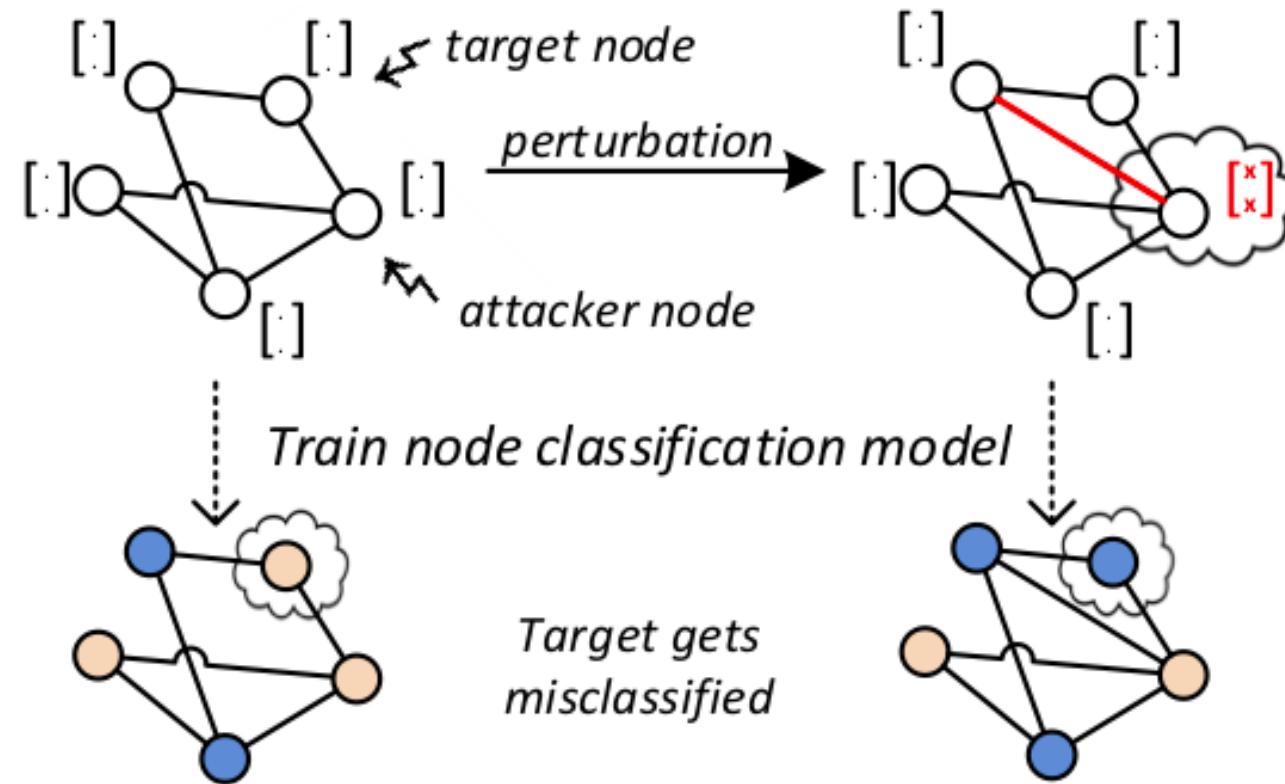


Modifying edges

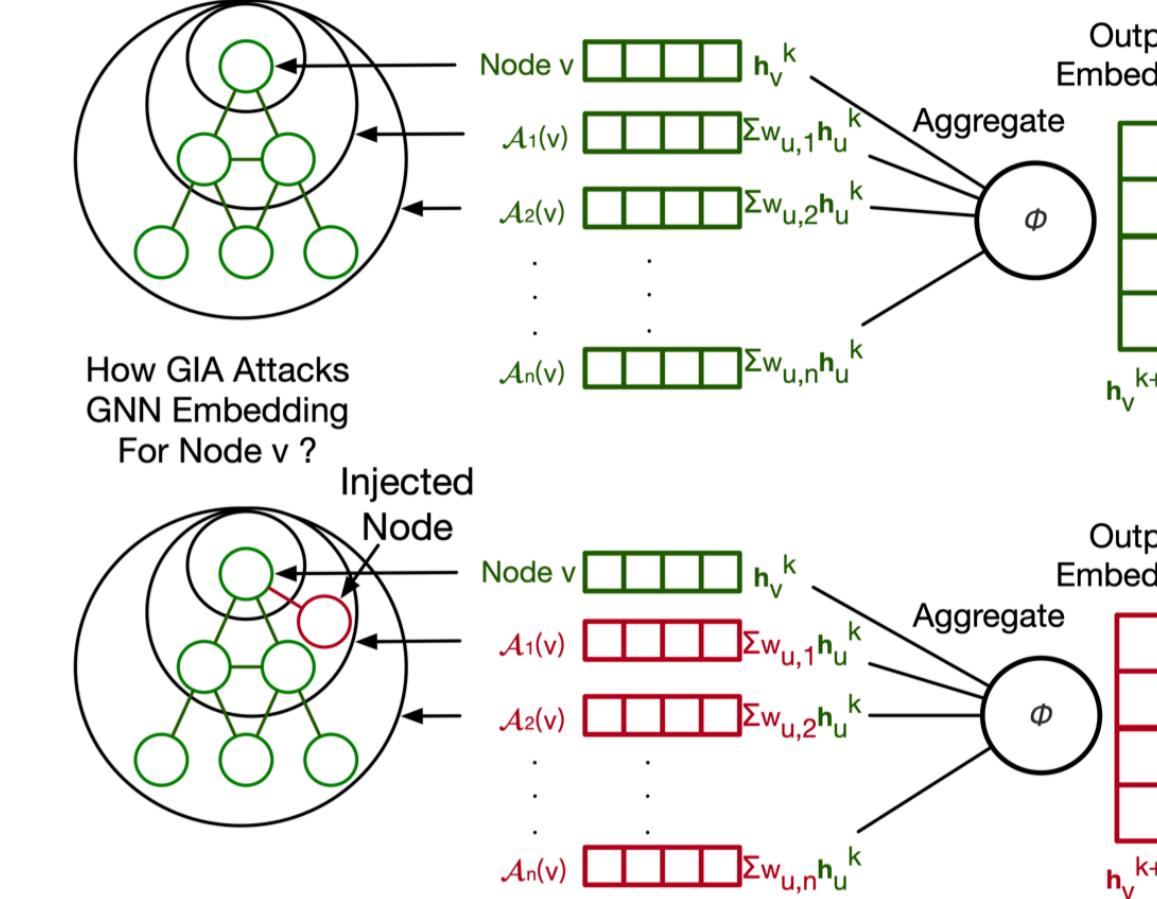


Perturbing node features

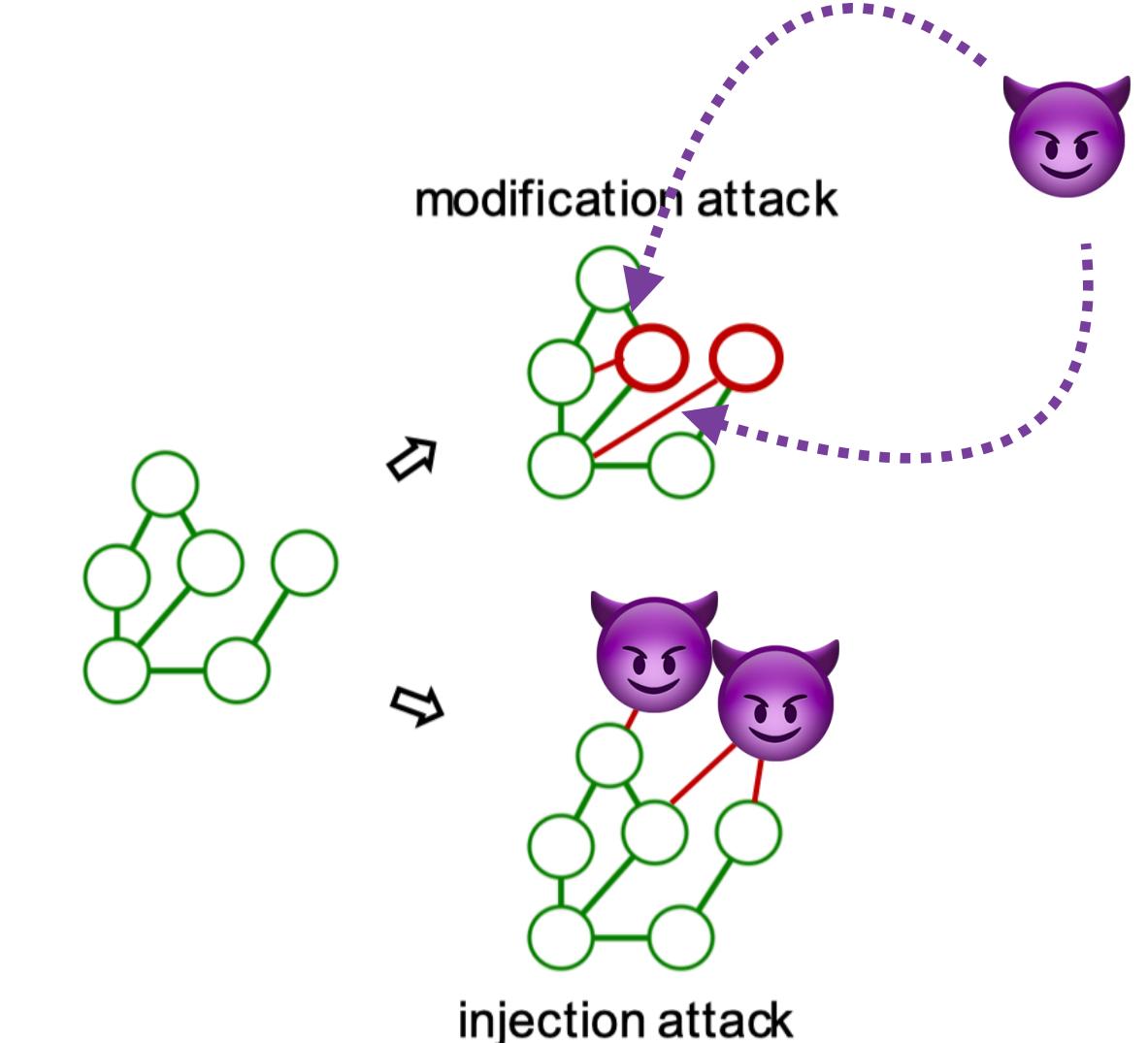
# Adversarial Attacks on GNNs



(Zügner et al., 2018)



(Zou et al., 2020)



GMA vs. GIA

## Adversarial Objective:

$$\min \mathcal{L}_{\text{atk}}(f_{\theta^*}(G')), \text{ s.t. } \|G' - G\| \leq \underbrace{\Delta}_{\text{perturbation budgets}}$$

## Graph Injection Attack (GIA):

$$X' = \begin{bmatrix} X \\ X_{\text{atk}} \end{bmatrix}, A' = \begin{bmatrix} A & A_{\text{atk}} \\ A_{\text{atk}}^T & O_{\text{atk}} \end{bmatrix}, \quad |V_{\text{atk}}| \leq \Delta \in \mathbb{Z}, 1 \leq d_u \leq b \in \mathbb{Z}, X_u \in \mathcal{D}_X \subseteq \mathbb{R}^d, \forall u \in V_{\text{atk}}$$



Practical

Injecting nodes Carefully injected connections



Carefully crafted node features

# Let's find out more about GIA!

- *through a friendly comparison* -



GIA



GMA



# The Power of Graph Injection Attack

## Definition 1 (Threats)

Consider an adversary  $\mathcal{A}$ , given a perturbation budget  $\Delta$ , the threat of  $\mathcal{A}$  to a GNN  $f_\theta$  is defined as  $\min_{\|G'-G\| \leq \Delta} \mathcal{L}_{\text{atk}}(f_{\theta^*}(G'))$ , i.e., the **optimal objective value**.

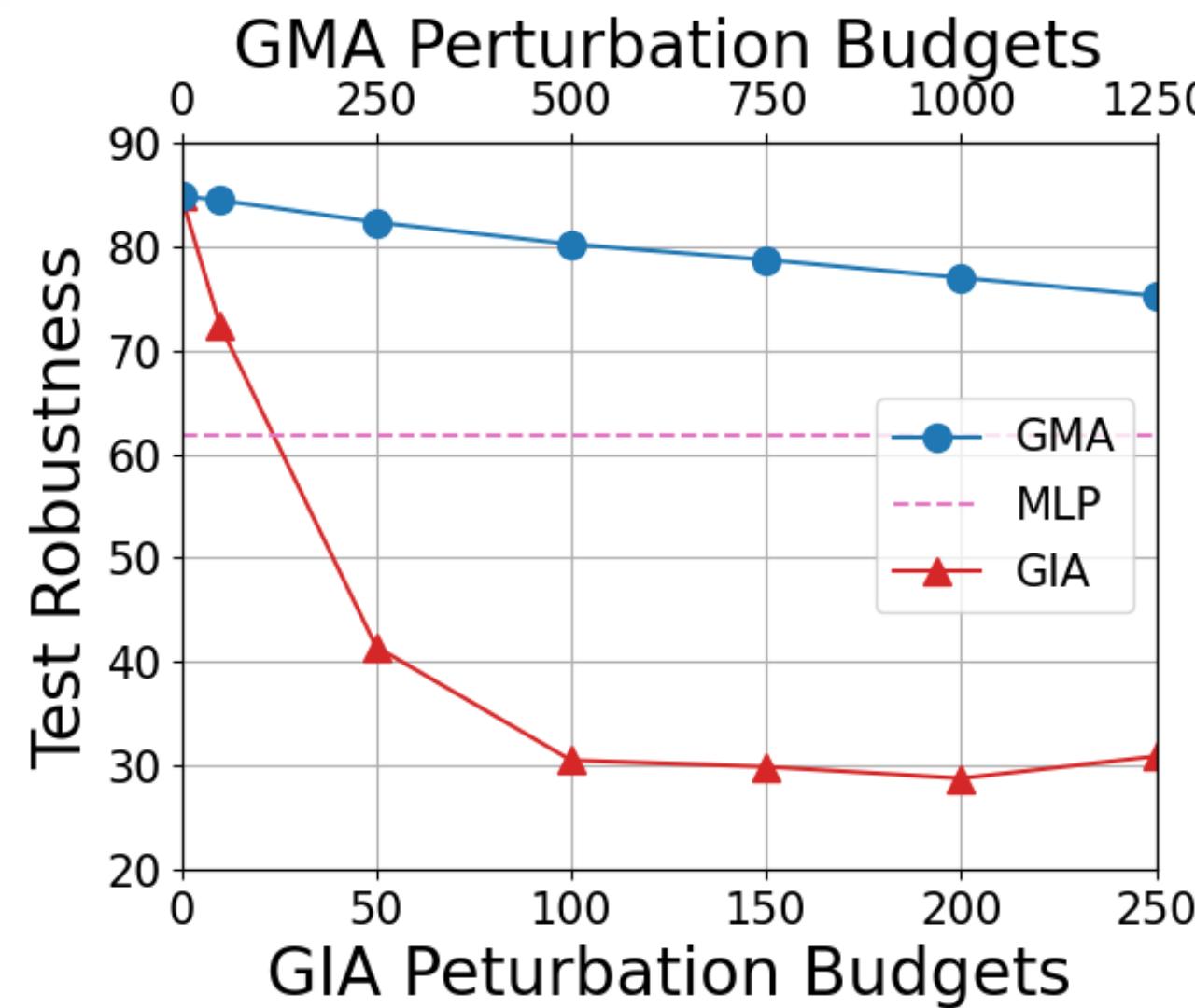
## Theorem 1 (GIA is more harmful than GMA)

Given moderate perturbation budgets  $\Delta_{\text{GIA}}$  for GIA and  $\Delta_{\text{GMA}}$  for GMA, that is, let  $\Delta_{\text{GIA}} \leq \Delta_{\text{GMA}} \ll |V| \leq |E|$ , for a fixed linearized GNN  $f_\theta$  trained on  $G$ , assume that  $G$  has no isolated nodes, and both GIA and GMA follow the **optimal strategy**, then,  
 $\forall \Delta_{\text{GMA}} \geq 0, \exists \Delta_{\text{GIA}} \leq \Delta_{\text{GMA}},$

$$\mathcal{L}_{\text{atk}}(f_\theta(G'_{\text{GIA}})) - \mathcal{L}_{\text{atk}}(f_\theta(G'_{\text{GMA}})) \leq 0,$$

where  $G'_{\text{GIA}}$  and  $G'_{\text{GMA}}$  are perturbed graphs generated by GIA and GMA, respectively.

# The Power of Graph Injection Attack



GMA vs. GIA

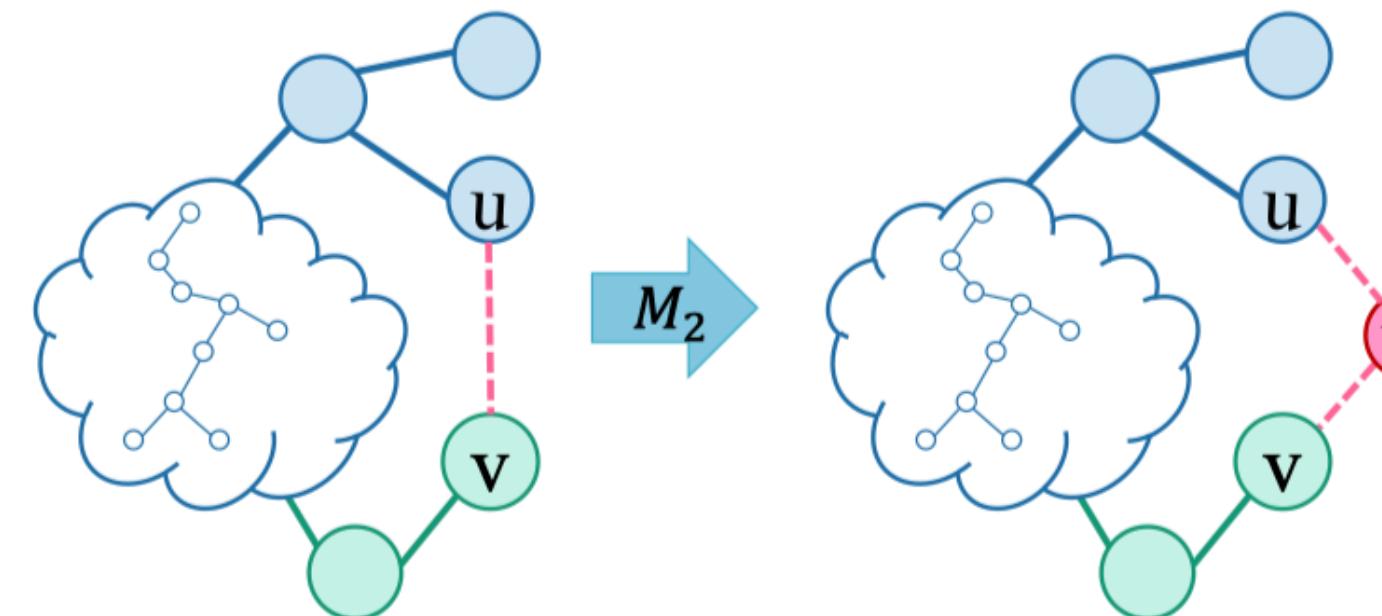
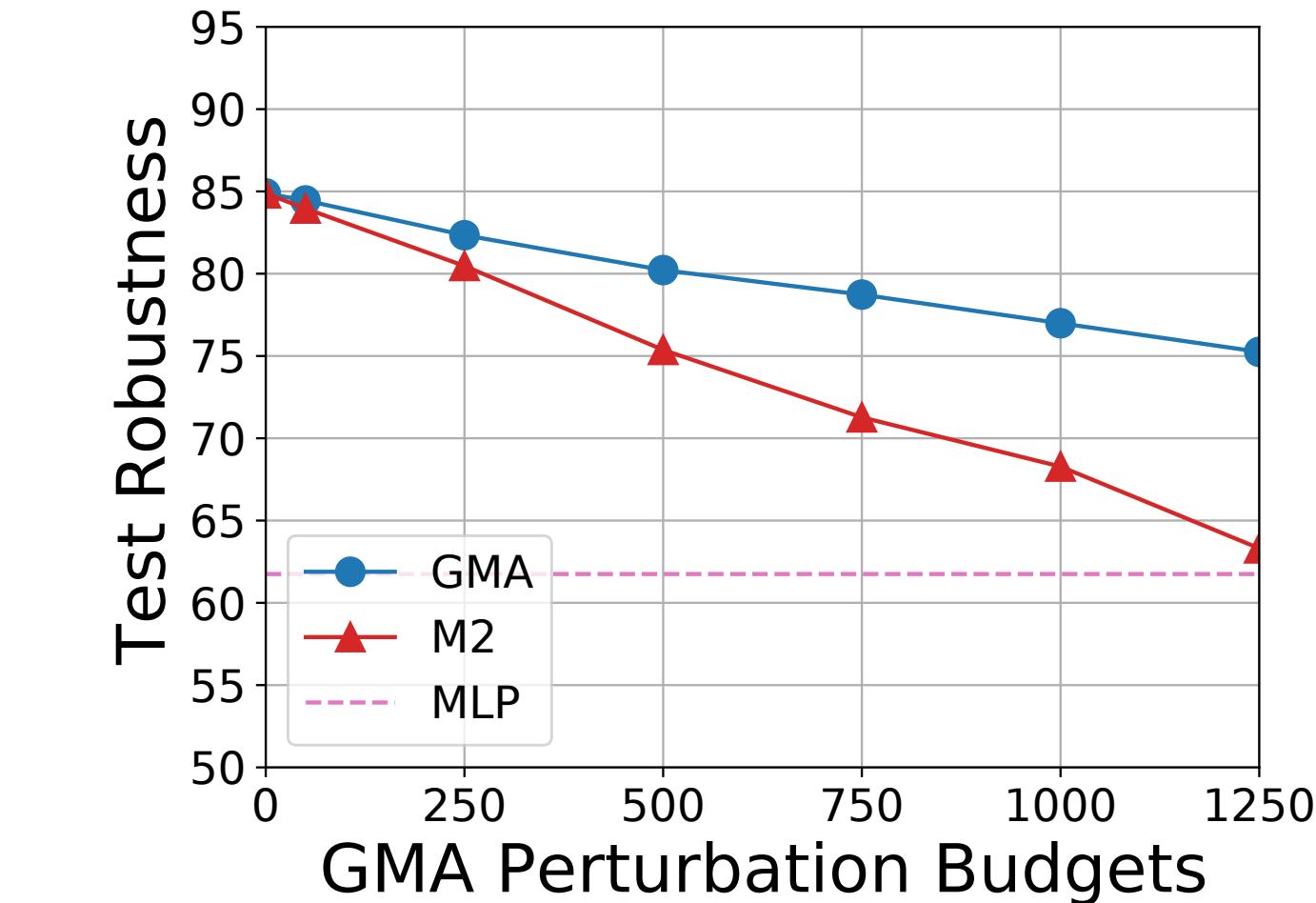


Illustration of  $\mathcal{M}_2$  mapping



GMA vs. GIA with  $\mathcal{M}_2$

## Definition 2 (Plural Mapping $\mathcal{M}_2$ )

A plural mapping  $\mathcal{M}_2$  maps a perturbed graph  $G'_{\text{GMA}}$  generated by GMA with only edge addition perturbations\*, to a GIA perturbed graph  $G'_{\text{GIA}} = \mathcal{M}_2(G'_{\text{GMA}})$ , such that:

$$f_\theta(G'_{\text{GIA}})_u = f_\theta(G'_{\text{GMA}})_u, \forall u \in V.$$

\*We can also find such mappings for other perturbation actions of GMA.

# Is The Power of GIA A Free Lunch? 🤔

It turns out to be NO. 😅

# The Pitfalls in Graph Injection Attack

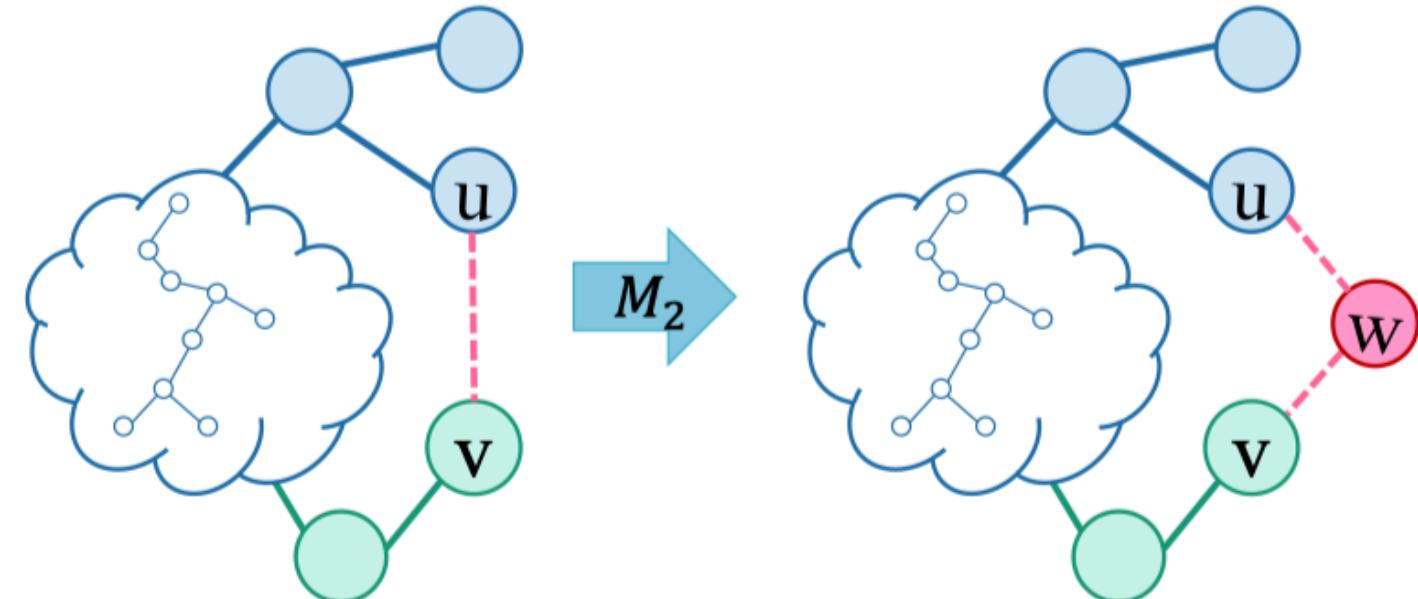


Illustration of  $\mathcal{M}_2$  mapping

Given the example of  $\mathcal{M}_2$ , assume GIA uses PGD to optimize  $X_w$  iteratively, we find:

$$\text{sim}(X_u, X_w)^{(t+1)} \leq \text{sim}(X_u, X_w)^{(t)},$$

where  $t$  is the number of optimization steps and  $\text{sim}(\cdot)$  is the cosine similarity.

## Definition 3 (Node-Centric Homophily)

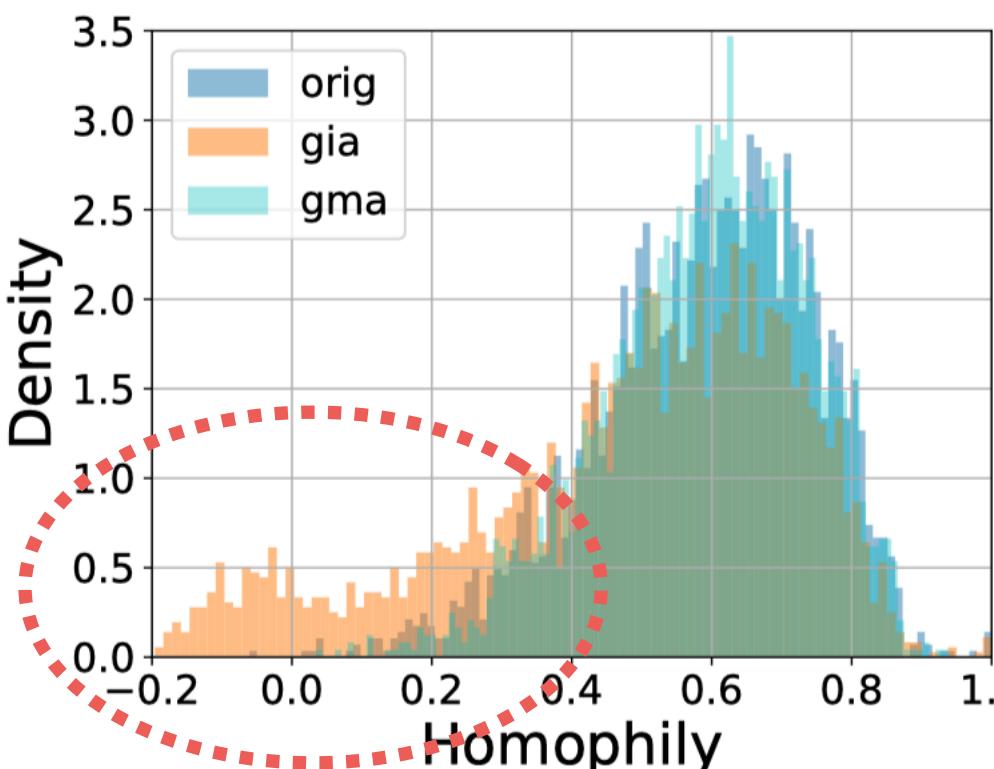
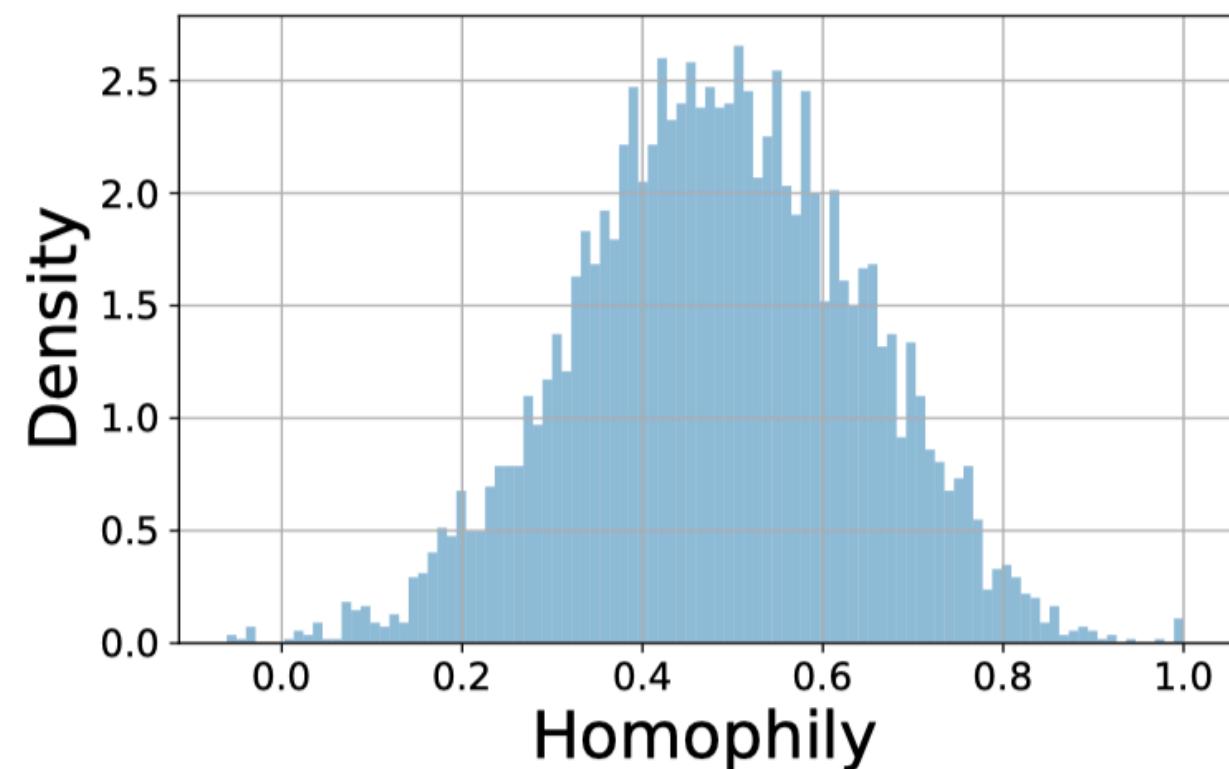
The homophily of a node  $u$  can be defined with the similarity between the features of node  $u$  and the aggregated features of its neighbors\*:

$$h_u = \text{sim}(r_u, X_u), r_u = \sum_{j \in \mathcal{N}(u)} \frac{1}{\sqrt{d_j d_u}} X_j,$$

where  $d_u$  is the degree of node  $u$  and  $\text{sim}(\cdot)$  is a similarity metric, e.g., cosine similarity.

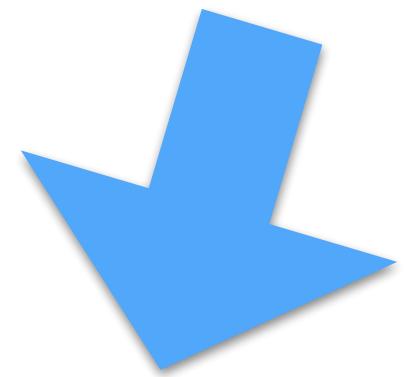
\*We can also define edge-centric homophily, while we will focus on node-centric homophily.

# The Pitfalls in Graph Injection Attack



Homophily changes before and after attacks

GIA provably leads more damage to the homophily of the original graph than GMA



## Definition 3 (Homophily Defenders)

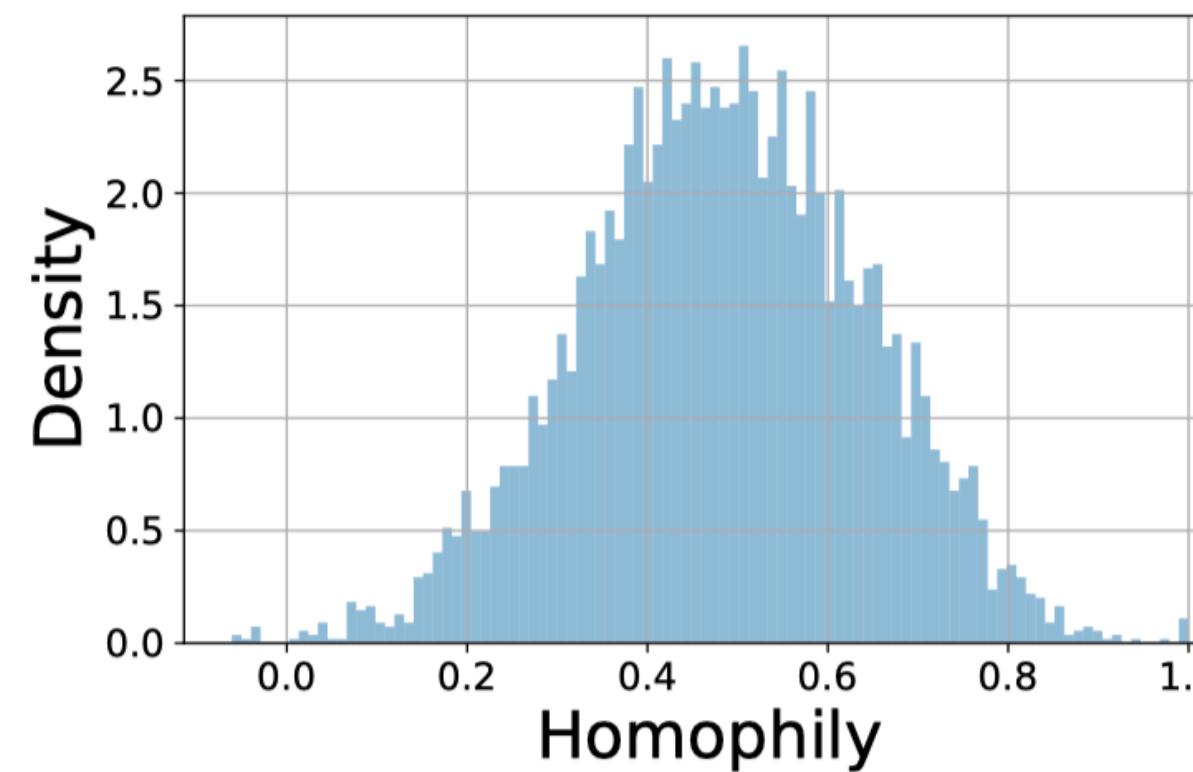
The homophily defenders can be implemented via edge pruning\*:

$$H_u^{(k)} = \text{READOUT}(W_k \cdot \text{AGG}(\mathbb{I}_{\text{con}}(u, v)\{H_v^{(k-1)}\} \mid v \in \mathcal{N}(u) \cup \{u\})),$$

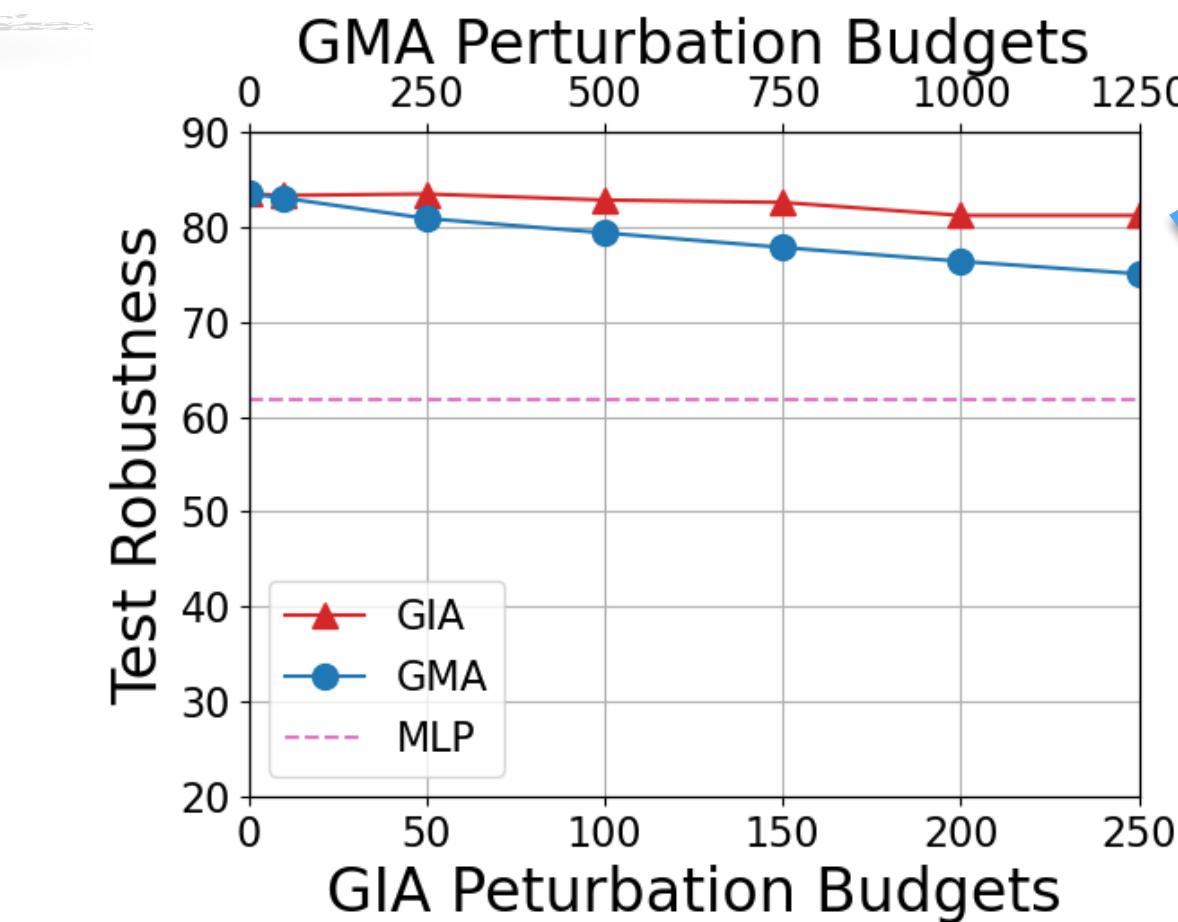
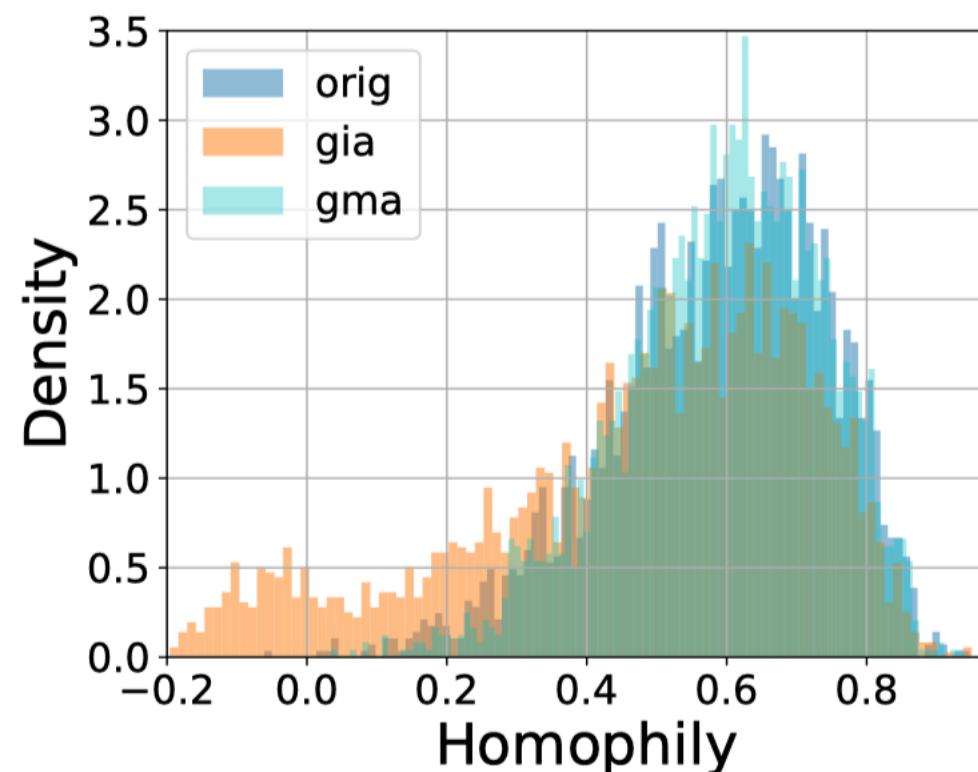
where  $\mathbb{I}_{\text{con}}(u, v)$  elaborates the pruning condition for edge  $(u, v)$ .

\*Essentially, homophily defenders can have other implementations than edge pruning.

# The Pitfalls in Graph Injection Attack



Homophily changes before and after attacks



GMA vs. GIA when with defense

GIA almost loses its power!

## Theorem 2 (GIA loses power when against homophily defenders)

Given conditions in Theorem 1, consider a GIA attack, which **(i)** is mapped by  $\mathcal{M}_2$  from a GMA attack that only performs edge addition perturbations, and **(ii)** uses a linearized GNN trained with at least one node from each class in  $G$  as the surrogate model, and **(iii)** optimizes the malicious node features with PGD. Assume that  $G$  has no isolated node, and has node features as  $x_u = \frac{C}{C-1}e_{Y_u} - \frac{1}{C-1}\mathbf{1} \in \mathbb{R}^d$  where  $Y_u$  is the label of node  $u$  and  $e_{Y_u} \in \mathbb{R}^d$  is a one-hot vector with the  $Y_u$ -th entry being 1 and others being 0. Let the minimum similarity for any pair of nodes connected in  $G$  be  $s_G = \min_{(u,v) \in E} \text{sim}(X_u, X_v)$  implemented with cosine similarity. For a homophily defender  $g_\theta$  that prunes edges  $(u, v)$  if  $\text{sim}(X_u, X_v) \leq s_G$ , we have:

$$\mathcal{L}_{\text{atk}}(g_\theta(\mathcal{M}_2(G'_{\text{GMA}}))) - \mathcal{L}_{\text{atk}}(g_\theta(G'_{\text{GMA}})) \geq 0.$$

# Unnoticeability in Graph Adversarial Attack

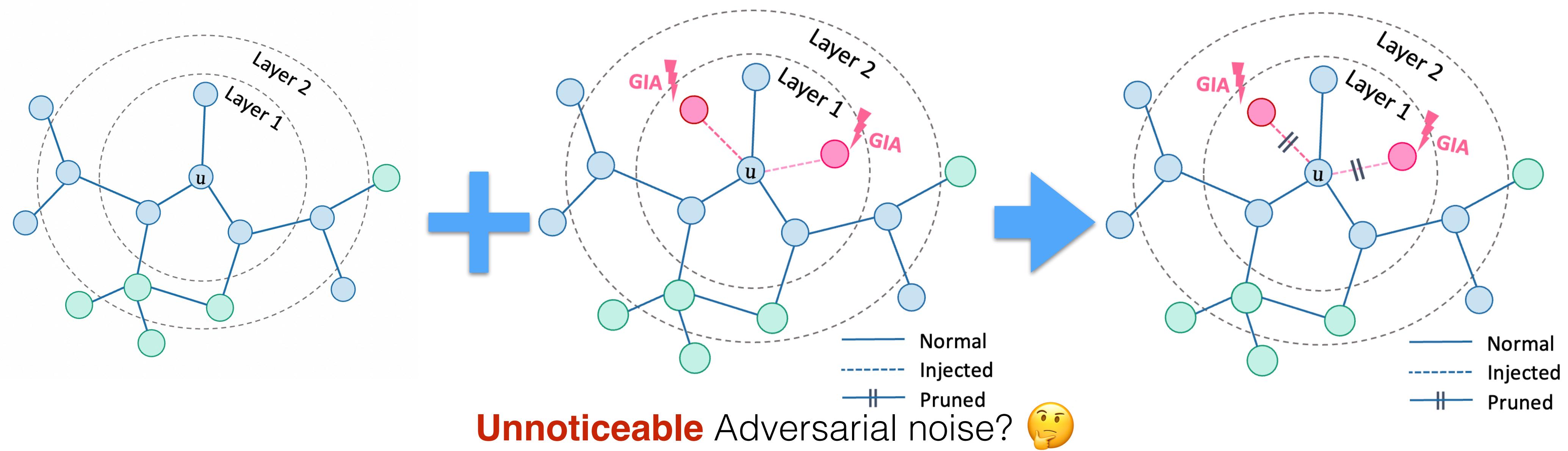


Prediction: Pig

(Szegedy et al., 2014; Goodfellow et al., 2015; Kolter and Madry et al. 2019)

**Unnoticeable** Adversarial noise 😊

Prediction: Airliner



# Homophily Unnoticeable Graph Injection Attack

## Definition 4 (Homophily Unnoticeability)

Let the node-centric homophily distribution for a graph  $G$  be  $\mathcal{H}_G$ . Given the upper bound for the allowed homophily distribution shift  $\Delta_{\mathcal{H}} \geq 0$ , an attack  $\mathcal{A}$  is **homophily unnoticed** if:

$$m(\mathcal{H}_G, \mathcal{H}_{G'}) \leq \Delta_{\mathcal{H}},$$

where  $G'$  is the perturbed graph generated  $\mathcal{A}$  and  $m(\cdot)$  is a distribution distance measure.

Homophily Unnoticeability measures how likely the new connections between the malicious nodes and target nodes will appear *naturally*.

Homophily Defender provides efficient check for homophily unnoticeability serving as *external examiners*.

# Homophily Unnoticeable Graph Injection Attack

## Definition 5 (Harmonious Adversarial Objective (HAO))

Observing the homophily (Definition. 4) is differentiable with respect to  $X$ , we can integrate it into the original adversarial objective as\*:

$$\min_{\|G'-G\|\leq\Delta} \mathcal{L}_{\text{atk}}^h(f_{\theta^*}(G')) = \mathcal{L}_{\text{atk}}(f_{\theta^*}(G')) - \lambda C(G, G'),$$

where  $C(G, G')$  is a regularization term based on homophily and  $\lambda \geq 0$  is the corresponding weight.

\*We only use HAO to solve for  $G'$  while still using the original objective to evaluate the threats.

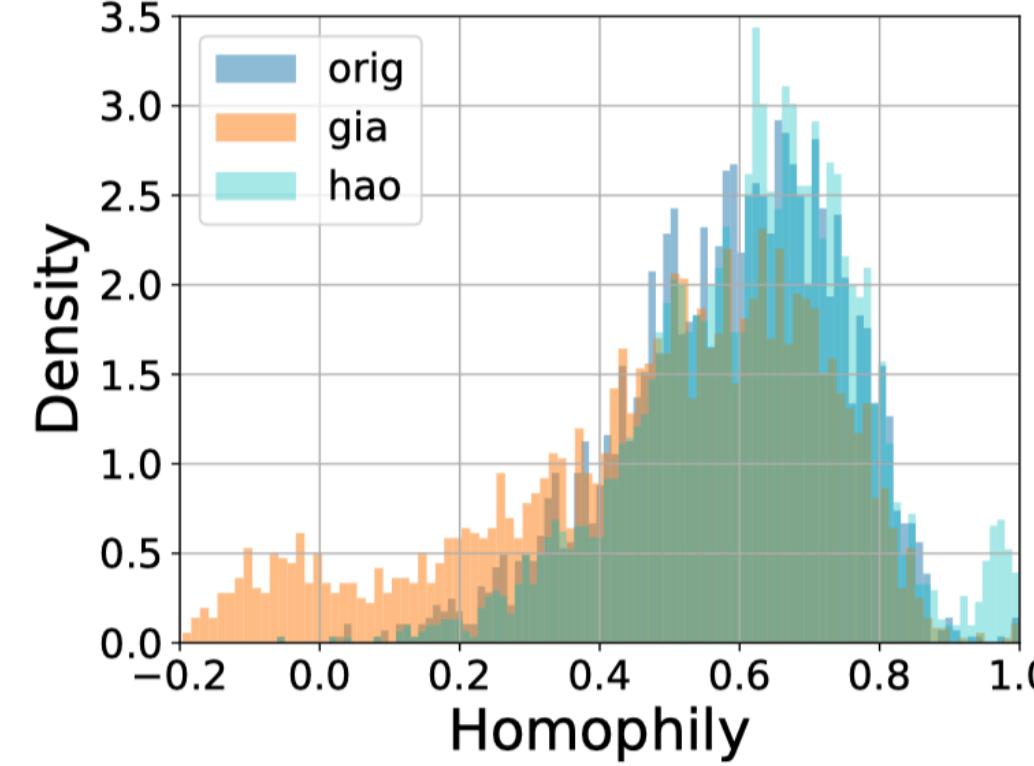
## Theorem 3 (HAO re-empowers GIA)

Given conditions in Theorem 2, we have  $m(\mathcal{H}_G, \mathcal{H}_{G'_{\text{HAO}}}) \leq m(\mathcal{H}_G, \mathcal{H}_{G'_{\text{GIA}}})$ , hence:

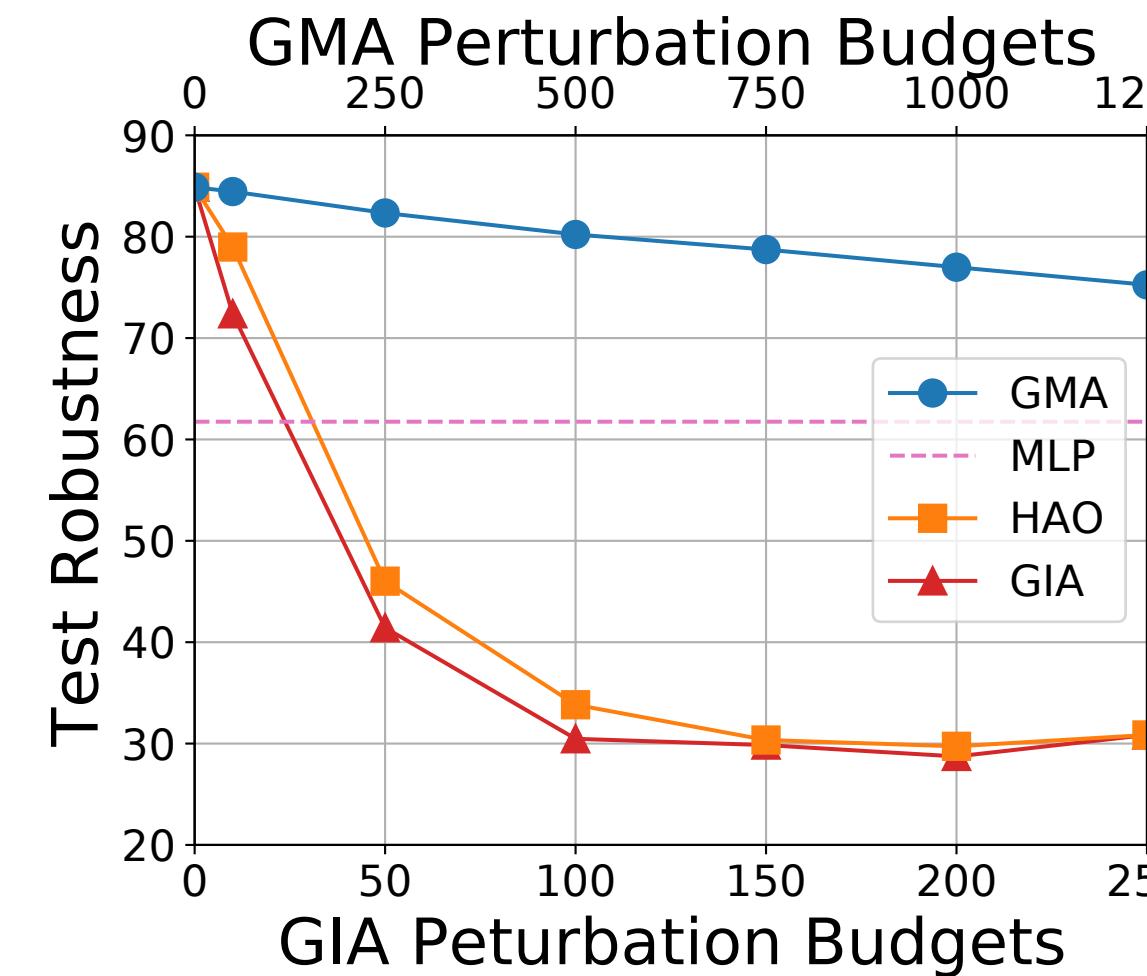
$$\mathcal{L}_{\text{atk}}(g_\theta(G'_{\text{HAO}})) - \mathcal{L}_{\text{atk}}(g_\theta(G'_{\text{GIA}})) \leq 0,$$

where  $G'_{\text{HAO}}$  and  $G'_{\text{GIA}}$  are perturbed graphs generated by GIA with and without HAO, respectively..

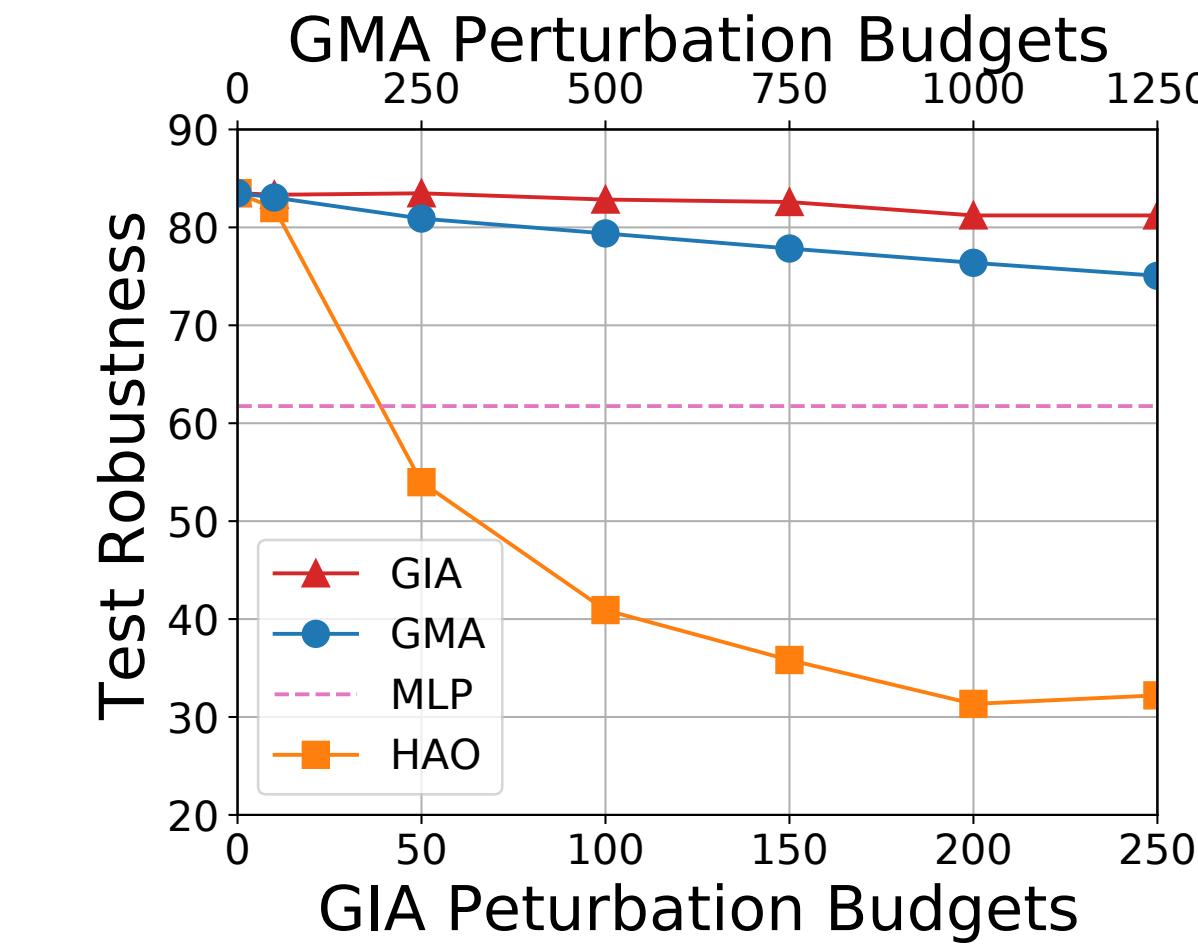
# Homophily Unnoticeable Graph Injection Attack



Homophily changes



GMA vs. GIA without defense



GMA vs. GIA when with defense

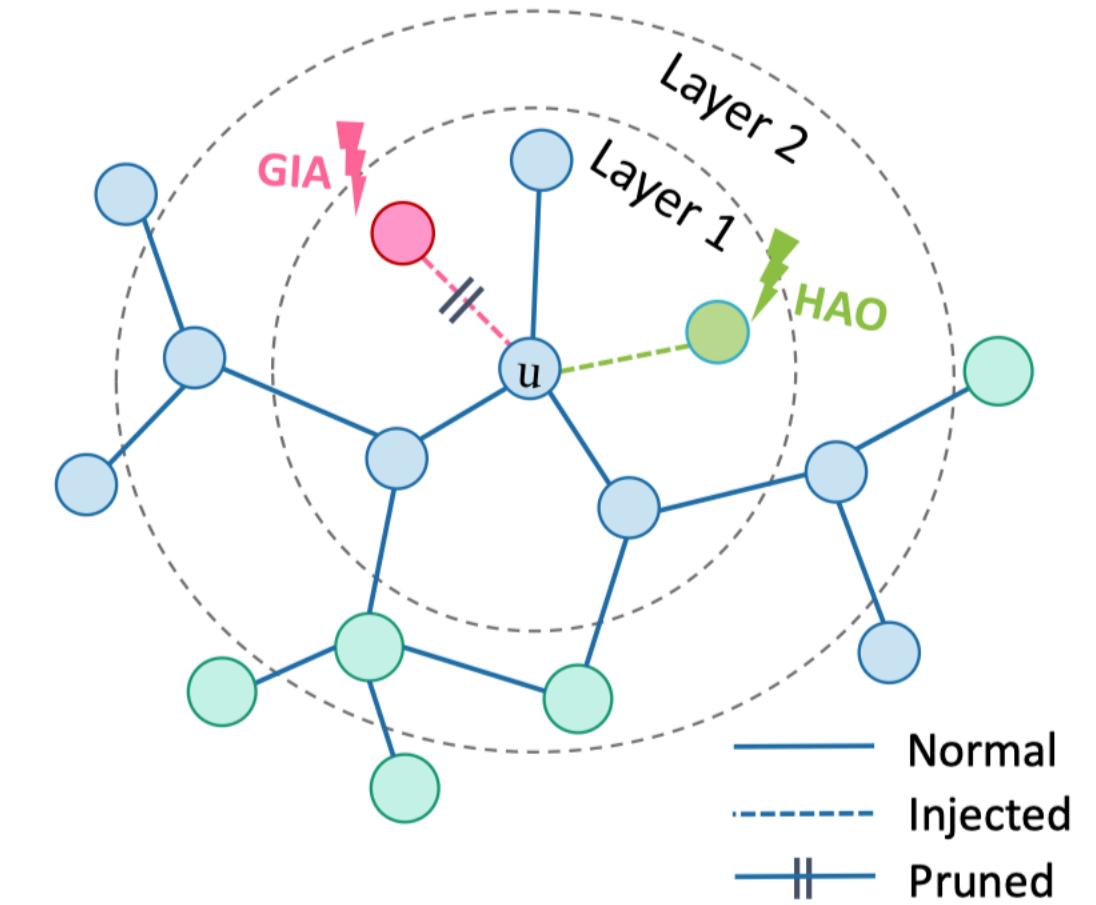


Illustration of GIA at node  $u$

## Theorem 3 (HAO re-empowers GIA)

Given conditions in Theorem 2, we have  $m(\mathcal{H}_G, \mathcal{H}_{G'_{\text{HAO}}}) \leq m(\mathcal{H}_G, \mathcal{H}_{G'_{\text{GIA}}})$ , hence:

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where  $G'_{\text{HAO}}$  and  $G'_{\text{GIA}}$  are perturbed graphs generated by GIA with and without HAO, respectively..

# HAO Re-empowers GIA

HAO significantly improves the performance of ***all attacks on all datasets up to 30%***. Adaptive injection strategies can further advance the state of the art.

**Homo:** Homophily Defenders

**Vanilla:** Vanilla GNNs,  
e.g., GCN, GAT, GraphSage.

**Robust:** Robust GNN  
models, or GNN models with  
robust tricks such as layer  
normalisation, or adversarial  
training.

**Combo:** Robust GNN  
models with robust tricks  
such as layer normalisation,  
or adversarial training.

Table 1: Performance of non-targeted attacks against different models

	HAO	Cora ( $\downarrow$ )			Citeseer( $\downarrow$ )			Computers( $\downarrow$ )			Arxiv( $\downarrow$ )		
		Homo	Robust	Combo	Homo	Robust	Combo	Homo	Robust	Combo	Homo	Robust	Combo
Clean		85.74	86.00	87.29	74.85	75.46	75.87	93.17	93.17	93.32	70.77	71.27	71.40
PGD		83.08	83.08	85.74	74.70	74.70	75.19	84.91	84.91	91.41	68.18	68.18	71.11
PGD	✓	52.60	62.60	77.99	69.05	69.05	73.04	79.33	79.33	87.83	55.38	62.89	68.68
MetaGIA <sup>†</sup>		83.61	83.61	85.86	74.70	74.70	75.15	84.91	84.91	91.41	68.47	68.47	71.09
MetaGIA <sup>†</sup>	✓	49.25	69.83	76.80	68.04	68.04	71.25	78.96	78.96	90.25	57.05	63.30	69.97
AGIA <sup>†</sup>		83.44	83.44	85.78	74.72	74.72	75.29	85.21	85.21	91.40	68.07	68.07	71.01
AGIA <sup>†</sup>	✓	47.24	61.59	75.25	70.24	70.24	71.80	75.14	75.14	86.02	59.32	65.62	69.92
TDGIA		83.44	83.44	85.72	74.76	74.76	75.26	88.32	88.32	91.40	64.49	64.49	70.97
TDGIA	✓	56.95	73.38	79.45	60.91	60.91	72.51	74.77	74.77	90.42	49.36	60.72	63.57
ATDGIA		83.07	83.07	85.39	74.72	74.72	75.12	86.03	86.03	91.41	66.95	66.95	71.02
ATDGIA	✓	42.18	70.30	76.87	61.08	61.08	71.22	80.86	80.86	84.60	45.59	63.30	64.31
MLP		61.75			65.55			84.14			52.49		

<sup>†</sup>The lower number indicates better attack performance. <sup>†</sup>Runs with SeqGIA framework on Computers and Arxiv.

We evaluate with **38** defense models and report the *maximum* mean test robustness from multiple runs.

# HAO Re-empowers GIA

HAO significantly improves the performance of ***all attacks on all datasets up to 15%***. Adaptive injection strategies can further advance the state of the art.

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normalisation, or adversarial  
training.

**Combo:** Robust GNN  
models with robust tricks  
such as layer normalisation,  
or adversarial training.

Table 2: Performance of targeted attacks against different models

	HAO	Computers(↓)			Arxiv(↓)			Aminer(↓)			Reddit(↓)		
		Homo	Robust	Combo									
Clean		92.68	92.68	92.83	69.41	71.59	72.09	62.78	66.71	66.97	94.05	97.15	97.13
PGD		88.13	88.13	91.56	69.19	69.19	71.31	53.16	53.16	56.31	92.44	92.44	93.03
PGD	✓	71.78	71.78	85.81	<b>36.06</b>	<b>37.22</b>	69.38	34.62	34.62	39.47	<u>56.44</u>	<u>86.12</u>	<b>84.94</b>
MetaGIA <sup>†</sup>		87.67	87.67	91.56	69.28	69.28	71.22	48.97	48.97	52.35	92.40	92.40	93.97
MetaGIA <sup>†</sup>	✓	<u>70.21</u>	<u>71.61</u>	85.83	38.44	<u>38.44</u>	48.06	41.12	41.12	45.16	<b>46.75</b>	90.06	90.78
AGIA <sup>†</sup>		87.57	87.57	91.58	66.19	66.19	70.06	50.50	50.50	53.69	91.62	91.62	93.66
AGIA <sup>†</sup>	✓	<b>69.96</b>	<b>71.58</b>	85.72	38.84	38.84	68.97	35.94	35.94	42.66	80.69	88.84	90.44
TDGIA		87.21	87.21	91.56	63.66	63.66	71.06	51.34	51.34	54.82	92.19	92.19	93.62
TDGIA	✓	71.39	71.62	<b>77.15</b>	42.56	42.56	<u>42.53</u>	<u>25.78</u>	<u>25.78</u>	<u>29.94</u>	78.16	<b>85.06</b>	<u>88.66</u>
ATDGIA		87.85	87.85	91.56	66.12	66.12	71.16	50.87	50.87	53.68	91.25	91.25	93.03
ATDGIA	✓	72.00	72.53	<u>78.35</u>	<u>38.28</u>	40.81	<b>39.47</b>	<b>22.50</b>	<b>22.50</b>	<b>28.91</b>	64.09	89.06	88.91
MLP		84.11			52.49			32.80			70.69		

↓The lower number indicates better attack performance. <sup>†</sup>Runs with SeqGIA framework.

We evaluate with **38** defense models and report the *maximum* mean test robustness from multiple runs.

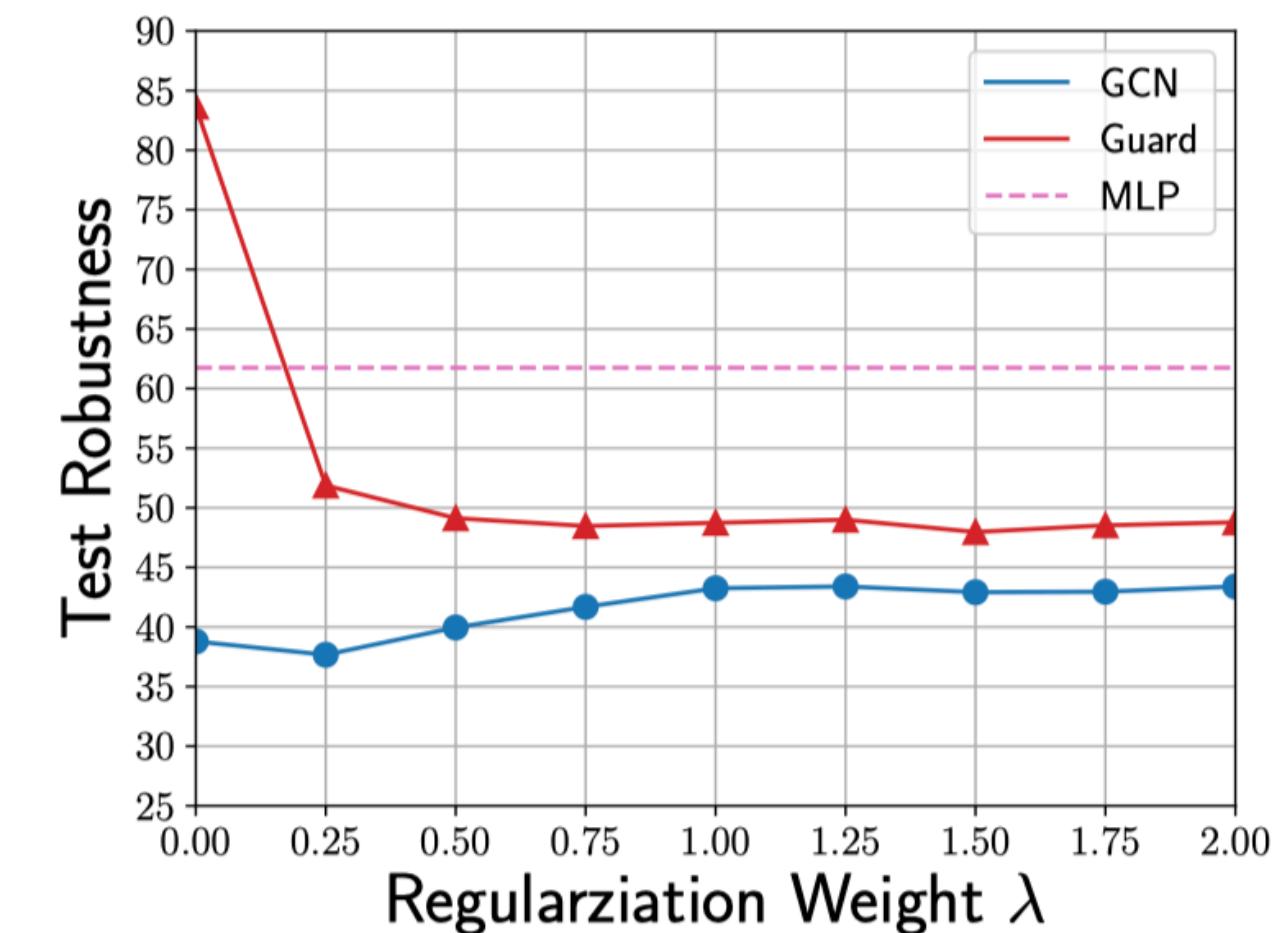
# HAO Re-empowers GIA

HAO consistently improves the performances of ***all attacks on all datasets up to 5%***. Adaptive injection strategies can further advance the state of the art.

Table 9: Full Averaged performance across all defense models

Model	Cora <sup>†</sup>	Citeseer <sup>†</sup>	Computers <sup>†</sup>	Arxiv <sup>†</sup>	Arxiv <sup>‡</sup>	Computers <sup>‡</sup>	Aminer <sup>‡</sup>	Reddit <sup>‡</sup>
Clean	84.74	74.10	92.25	70.44	70.44	91.68	62.39	95.51
PGD	61.09	54.08	61.75	54.23	36.70	62.41	26.13	62.72
+HAO	<u>56.63</u>	48.12	<u>59.16</u>	<u>45.05</u>	28.48	59.09	<u>22.15</u>	56.99
MetaGIA	60.56	53.72	61.75	53.69	28.78	62.08	32.78	60.14
+HAO	58.51	47.44	60.29	48.48	<u>24.61</u>	<u>58.63</u>	29.91	<b>54.14</b>
AGIA	60.10	54.55	60.66	48.86	32.68	61.98	31.06	59.96
+HAO	<b>53.79</b>	48.30	<b>58.71</b>	48.86	29.52	<b>58.37</b>	26.51	56.36
TDGIA	66.86	52.45	66.79	49.73	31.68	62.47	32.37	57.97
+HAO	65.22	<u>46.61</u>	65.46	49.54	<b>22.04</b>	59.67	22.32	<u>54.32</u>
ATDGIA	61.14	49.46	65.07	46.53	32.08	64.66	24.72	61.25
+HAO	58.13	<b>43.41</b>	63.31	<b>44.40</b>	29.24	59.27	<b>17.62</b>	56.90

The lower is better. <sup>†</sup>Non-targeted attack. <sup>‡</sup>Targeted attack.



Varying  $\lambda$  in HAO

We evaluate with **38** defense models and report the mean test robustness of all models from multiple runs.

# Summary

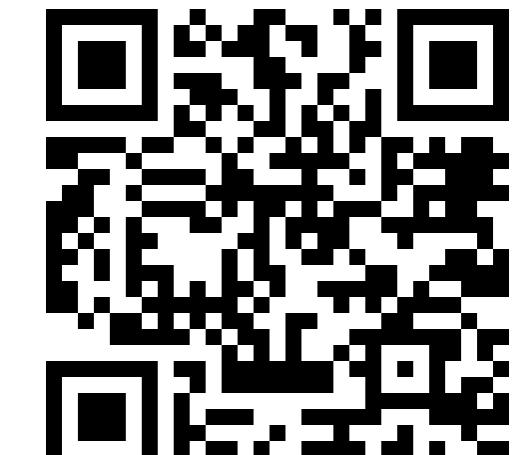
We provide a formal comparison between GIA and GMA in a unified setting and find that GIA can be provably more harmful than GMA due to its high flexibility (Theorem 1).

However, the flexibility of GIA will cause severe damage to the homophily which makes GIA easily defendable by homophily defenders (Theorem 2).

To mitigate the issue, we introduce the concept of homophily unnoticeability and a novel objective HAO to conduct homophily unnoticeable attacks (Theorem 3).



Paper

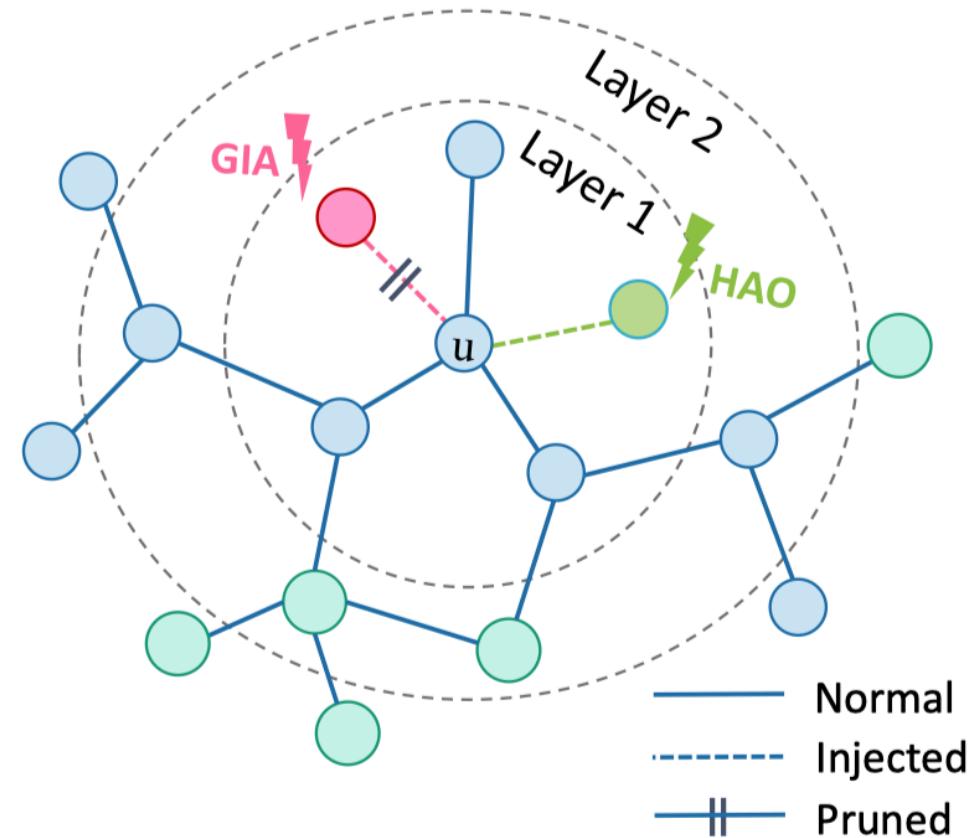


Code

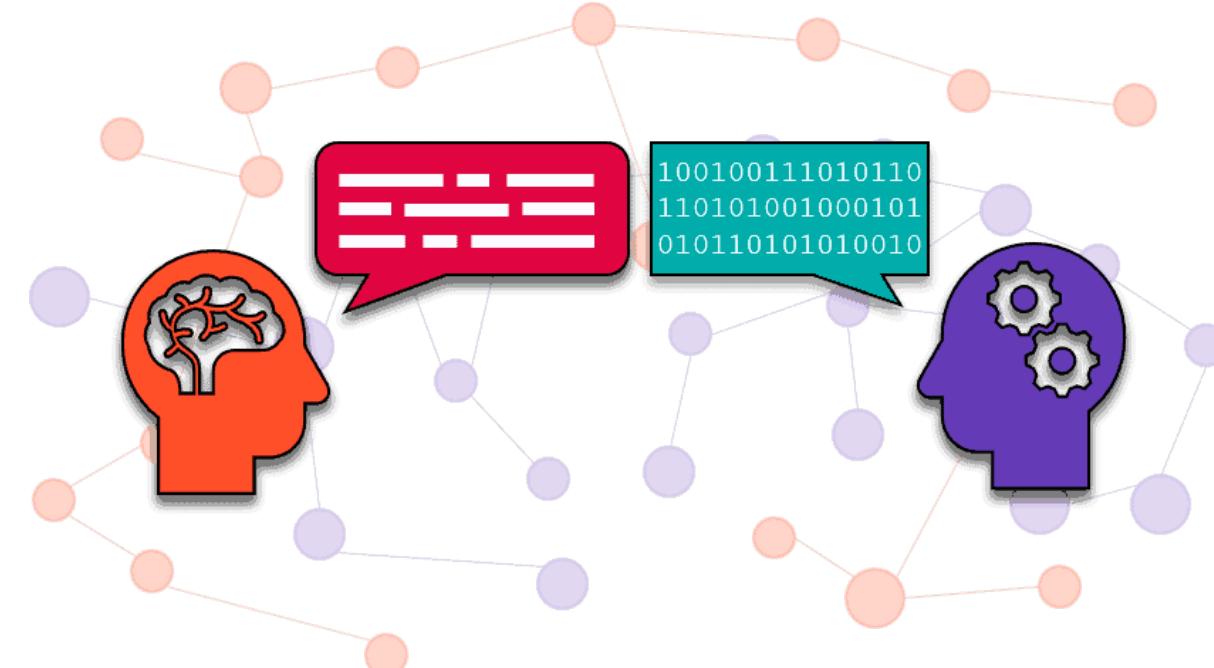
## Thank you!

Contact: [yqchen@cse.cuhk.edu.hk](mailto:yqchen@cse.cuhk.edu.hk)

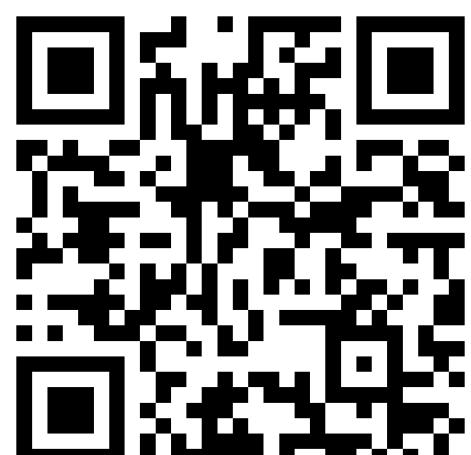
# Future Navigations



If you focus on graph domain: more advanced injection strategies, more downstream tasks, more attack scenarios, more robust GNNs...



If you focus on other domains: more unnoticeability constraints & the corresponding external examiners...



Paper



Code

# Thank you!

Contact: [yqchen@cse.cuhk.edu.hk](mailto:yqchen@cse.cuhk.edu.hk)