DSA/ISE 5113 Advanced Analytics and Metaheuristics
Homework #2

Instructor: Charles Nicholson

Due: See course website for due date

Requirement details

1. Homeworks should be submitted in a clean, clear, concise electronic format. You must show your logic, work, and/or code where appropriate. Note: any code (e.g., AMPL model and/or data files) is a part of your solution (turn in as separate files)
2. For mathematical programming problems, ensure you clearly define the following elements: (i) **necessary assumptions**, (ii) **decision variables**, (iii) **objective and objective function**, and (iv) **constraints**. Points may be deducted if proper definitions/documentation is missing.
3. For problems denoted as *Case Studies*, there may be a lot of information, some of which you need, some which you do not; and, you may have to make assumptions to complete the problem. Additionally, for case studies you may have to learn new concepts (e.g., marketing or finance); also, I want you to think about the solutions in terms of more than simply the "math", i.e., what are the implications and/or what do the solutions really tell us. Have fun!

Question 1: Golden Canning Co (30 points)

On Monday, February 8, 2021, Mr. Ernesto Ochandio, Vice-President of Operations, asked the Controller, the Sales Manager, and the Production VP to meet with him to discuss the amount of tomato products to pack that season. The tomato crop, which had been purchased at planting, was beginning to arrive at the cannery, and packing operations would have to be started in the next two weeks. Golden Canning Co. cans and distributes a variety of fruit and vegetable products under private brands in the US.

Mr. Dale Thomas, the Controller, and Ms. Michelle Bollman, the Sales Manager, were the first to arrive in Mr. Ochandio's office. Brett Jaggers, the Production VP, came in a few minutes later and said that he had picked up Produce Inspection's latest estimate of the quality of the incoming tomatoes. According to their report, about 20% of the crop was Grade "A" quality and the remaining portion of the 3,000,000-pound crop was Grade "B."

Ochandio asked Bollman about the demand for tomato products for the coming year. Bollman replied that they could sell all of the whole canned tomatoes they could produce. The expected demand for tomato juice and tomato paste, on the other hand, was limited. The Sales Manager then passed around the latest demand forecast, which is shown in Table 1. She reminded the group that the selling prices had been set in light of the long-term marketing strategy of the company, and potential sales had been forecast at these prices.

Dale Thomas, after looking at Bollman's estimates of demand, said that it looked like the company "should do quite well (on the tomato crop) this year." With the new accounting system that had been set up, he had been able to compute the contribution for each product, and according to his analysis the incremental profit on the whole tomatoes was greater than for any other tomato product. In May, after Golden Canning had signed contracts agreeing to purchase the grower's production at an average delivered price of 6 cents per pound, Thomas had computed the tomato products' contributions (see Table 2).

Brett Jaggers brought to Thomas' attention that, although there was ample production capacity, it was impossible to produce all whole tomatoes as too small a portion of the tomato crop was "A" quality. Golden Canning used a numerical scale to record the quality of both raw produce and prepared products. This scale ran from zero to ten, the higher number representing better quality. Rating tomatoes according to this scale, "A" tomatoes averaged nine points per pound and "B" tomatoes averaged five points per pound. Jaggers noted that the minimum average input quality for canned whole tomatoes was eight and for juice it was six points per pound. Paste could be made entirely from "B" grade tomatoes. This meant that whole tomato production was limited to 800,000 pounds.

Mr. Ochandio stated that this was not a real limitation. He has been recently solicited to purchase 80,000 pounds of Grade "A" tomatoes at 8.5 cents per pound and at that time had turned down the offer. He felt, however, that the tomatoes were still available.

Ms. Bollman, who had been doing some calculations, said that although she agreed that the company "should do quite well this year," it would not be by canning whole tomatoes. It seemed to her that the tomato cost should be allocated on the basis of quality and quantity rather than by quantity only as Thomas had done. Therefore, she had recomputed the marginal profit on this basis (see Table 3), and from her results, Golden Canning should use 2,000,000 pounds of the "B" tomatoes for paste, and the remaining 400,000 pounds of "B" tomatoes and all of the "A" tomatoes for juice – no tomatoes should be sold has whole canned. If the demand expectations were realized, a contribution of $48,000 would be made on this year's tomato crop.

(a) How does Bollman compute the tomato costs in Table 3? How does she reach the conclusion that the company should use 2,000,000 pounds "B" tomatoes for paste, the remaining 400,000 pounds of "B" tomatoes, and all of the "A" in juice? Is anything wrong with her reasoning?

(b) Ignoring for the moment the chance to buy additional A-grade tomatoes. Formulate the production question as a LP problem, solve with AMPL and answer the following questions:
   i. How much whole, juice, and paste should be made?
   ii. What is the contribution to profit?
   iii. Are there any tomatoes left over? If so, of what grade?
   iv. What is the average quality point count of whole, juice, and paste?
   v. What would be the worth of one additional pound of A-grade tomatoes?
   vi. Should Golden Canning Co buy the extra 80,000 pounds of A-grade tomatoes at the offered price?

(c) For the following questions, assume that the 80,000 pounds of A-grade is not in the formulation.
   i. What product mix would result if Thomas's contribution figures were used? Is Thomas's contribution more or less than the contribution to your (earlier) Part (c) solution? Why the difference?
   ii. Alternatively, what product mix would result if Bollman's profit figures were used? What would the profit be? Is this more or less than you found in Part (c)?
   iii. Suppose an unlimited supply of A-grade tomatoes were available at $0.085 per lb. How much should Golden Canning Co buy? What would be the product mix?

Table 1: Demand Forecasts

Product	Selling Price per Case	Demand Forecast (Cases)
24 - $2\frac{1}{2}$ whole tomatoes	\$4.00	800,000
24 - $2\frac{1}{2}$ choice peach halves	\$5.40	10,000
24 - $2\frac{1}{2}$ peach necture	\$4.60	5,000
24 - $2\frac{1}{2}$ tomato juice	\$4.50	50,000
24 - $2\frac{1}{2}$ cooking apples	\$4.90	15,000
24 - $2\frac{1}{2}$ tomato paste	\$3.80	80,000

Table 2: Product Item Profitability

Product	24 - $2\frac{1}{2}$ Canned Whole Tomatoes	24 - $2\frac{1}{2}$ Peach Halves	24 - $2\frac{1}{2}$ Peach Nectar	24 - $2\frac{1}{2}$ Tomato Juice	24 - $2\frac{1}{2}$ Cooking Apples	24 - $2\frac{1}{2}$ Tomato Paste
Selling Price (\$)	4.00	5.40	4.60	4.50	4.90	3.80
Variable Costs (\$)						
Direct Labor	1.18	1.40	1.27	1.32	1.70	0.54
Variable OHD.	0.24	0.32	0.23	0.36	0.22	0.26
Variable Selling	0.40	0.30	0.40	0.85	0.28	0.38
Packaging Material	0.70	0.56	0.60	0.65	0.70	0.77
Fruit*	1.08	1.80	1.70	1.20	0.90	1.50
Total Variable Costs	3.60	4.38	4.20	4.38	3.80	3.45
Contribution to profit	0.40	1.02	0.40	0.12	1.10	0.35
Less Allocated OHD	0.28	0.70	0.52	0.21	0.75	0.23
Net Profit	0.12	0.32	-0.12	-0.09	0.35	0.12

*Product usage is given as follows:

Product	Pounds per case
whole tomatoes	18
peach halves	18
peach nectar	17
tomato juice	20
cooking apples	27
tomato paste	25

Table 3: Bollman's marginal analysis of tomato products

1. Let z = cost per pound of A tomatoes in cents
2. Let y = cost per pound of B tomatoes in cents
3. (600,000 lb. $\times z$) + (2,400,000 lb. $\times y$) = (3,000,000 lb. $\times$ 6)
4. $\frac{z}{9} = \frac{y}{5}$
5. Therefore, $z = 9.32$ cents per pound; $y = 5.18$ cents per pound.

Product	Canned Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	\$4.00	\$4.50	\$3.80
Variable Costs (excluding tomato costs)	2.52	3.18	1.95
	\$1.48	\$1.32	\$1.85
Tomato cost	1.49	1.24	1.30
Marginal profit	-\$0.01	\$0.08	\$0.55

Question 2: Titan Enterprises Case Study (26 points)

Mr. Jonathon Lee, President of Titan Enterprises, called a special meeting of the company's Investment Review Committee in January 2020. Members of the committee are Mr. Will Zhang, the Treasurer; Ms. Brandi Phillips, the Controller; and Mr. Amin Shirazi, Operations director. The committee spent its time reviewing different methods for evaluating investment projects. During the meeting, Mr. Zhang said he believed it would be appropriate to use some method that takes into account the value of funds over the entire project life, and Ms. Phillips suggested the use of discounting[1] to accomplish this.

Unfortunately, the committee could not agree on a hurdle rate[2] that reflected the company's current financial position. Indeed, they had found it hard to accept any rate that would stay constant over the life of projects that might be considered. They had considered using a 10% hurdle rate for after-tax cash flows, typical of practice in their industry, but had not felt comfortable about that figure.

Mr. Shirazi suggested that by use of mathematical programming they could determine a portfolio of projects and decide on the amount to be invested in each. In addition, he knew that a such a solution would help determine what hurdle rate or rates would be appropriate for the company.

Overall, there had been much argument and little progress because they had been discussing the methods in abstract terms, and finally Mr. Lee suggested that Mr. Shirazi prepare a list of projects that were typical of the projects they had evaluated that year. This list was to be circulated and used to focus the discussion at the next meeting. Subsequently, Mr. Shirazi had circulated the email shown in Figure 1. The email details 5 projects with respect to their returns, net present value[3] (at time of cash outflow),

[1]Discounting is the process of determining the present value of a payment or a stream of payments that is to be received in the future. Given the time value of money, a dollar is worth more today than it would be worth tomorrow.

[2]A hurdle rate is the minimum required rate of return that investors are expecting to receive on an investment. The rate is determined by assessing the cost of capital, risks involved, current opportunities in business expansion, rates of return for similar investments, and other factors that could directly affect an investment. If a project or investment strategy has a return rate greater than the hurdle rate, then it is considered a worthwhile; otherwise, it is rejected.

[3]Net Present Value (NPV) is the value of all future cash flows (positive and negative) over the entire life of an investment discounted to the present. NPV analysis is a form of intrinsic valuation and is used extensively across finance and accounting for determining the value of a business, investment security, capital project, new venture, cost reduction program, and anything

and internal rate of return[4].

(a) Formulate and solve Titan's investment decision problem as a linear program (use AMPL).

(b) Provide an interpretation of the shadow prices in the specific context of the Titan investment problem, e.g., how might the shadow prices be used to determine hurdle rates consistent with Titan's available investments? Note: I know that most of you are not Finance specialists, nonetheless, I am asking you to provide thoughtful commentary on how the shadow prices can support decision-making in this context.

(c) How might you determine how sensitive the investment decision is to changes in the projects' final payouts? In particular consider Projects D and E. Would the portfolio change if E pays only \$1.34 per dollar invested? if D pays only \$1.70 per dollar invested? Can you determine this without re-running your model?

(d) How might you use results to determine whether new projects should be included in the portfolio? In particular, consider the two projects, F and G depicted in Table 4, which return \$1.25 per dollar invested. Would you recommend that the portfolio be changed if F were available? if G were available? if both were available?

Table 4: New investment options

	Project	
Year	**F**	**G**
2021	(1.00)	(1.00)
2022	0.80	1.10
2023	0.45	0.00
2024	0.00	0.15

(e) Assuming that Project F is available, use the computer output to consider the sensitivity of the portfolio decision to the changes in projects D and E considered in question (c). Would the portfolio change if Project E pays only \$1.34 per dollar invested? How? Does that make sense? Would the portfolio change if D pays only \$1.70 per dollar invested (and E retains original payout)? How? Does that make sense?

that involves cash flow.

$$NPV = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t}$$

where R_t is the net cash inflow-outflow during a single period t, i is the discount rate, t is the time period, and n is the total number of time periods.

[4] The internal rate of return (IRR) is a metric used in capital budgeting to estimate the profitability of potential investments. The internal rate of return is a discount rate that makes the net present value of all cash flows from a particular project equal to zero. IRR calculations rely on the same formula as NPV does.

From: Sharizi, Amin
Sent: Friday, January 24, 2020 12:37 PM
To: Lee@Titan.com, Zhang@Titan.com, Phillips@Titan.com
Subject: Investment Review: Project List

Investment Review Committee:

Mr. Lee asked me to prepare a list of typical investment projects for us to consider in our next meeting. The table below depicts five projects that might compete for our investment dollar. The table details the cash flow resulting from investing a single dollar.

	Project				
Year	**A**	**B**	**C**	**D**	**E**
2021	(1.00)	0.00	(1.00)	(1.00)	0.00
2022	0.30	(1.00)	1.10	0.00	0.00
2023	1.00	0.30	0.00	0.00	(1.00)
2024	0.00	1.00	0.00	1.75	1.40
Limit	$500,000	$500,000	N/A	N/A	$750,000

- Project A is a two-year investment available at the beginning of 2021, which pays 30 cents per dollar invested at the end of the first year and returns an additional dollar per dollar invested at the end of the second. At most $500,000 can be invested in Project A.
- Project B is identical to A except that it is available a year later.
- Project C is a one-year investment available only at the beginning of 2021, which pays $1.10 per dollar invested at the end of that year.
- Project D is a three-year investment available at the beginning of 2021, which pays $1.75 per dollar invested at the beginning of 2024.
- Project E will become available at the beginning of 2023 and will, after a year, pay $1.40 per dollar invested. Project E is limited to a maximum of $750,000.

Of course, the cash we receive from any of these projects may be reinvested in others that are available at the time. In addition, we could obtain 6% via short-term bank accounts for any money not invested in a given year.

For purpose of discussion, let's assume we want to invest $1,000,000 into some mix of these projects at the beginning of 2021 but no more thereafter, although we will reinvest throw-offs. All cash received at the beginning of 2024 will be withdrawn.

The table below gives the results of the discounting these projects as we had previously considered.

Project	Value @ 10%*	IRR
A or **B**	$0.099	16.1%
C	$0.000	10.0%
D	$0.315	20.5%
E	$0.273	40.0%

*A hypothetical hurdle rate similar to ones we have seen used in the past. NPV's are calculated to the beginning of the year in which the outflow occurs.

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Figure 1: Shirazi's email

Question 3: Outdoor grilling (12 points)

"Immerse yourself in the pleasures of unique flavor creations and the gratification of outdoor cooking" says the salesman of a particular brand of high-end open-air culinary system. The proposition is simply too good, your passion has been ignited, and as visions of lazy Summer afternoons grilling delicious meats and veggies in your backyard with your friends, colleagues, and Advanced Analytics professor, you simply cannot resist purchasing this amazing piece of equipment. And thus, the following homework problem is motivated...

As equipment ages, it typically degrades, resulting in rising maintenance costs and decreasing salvage values. These conditions provide ample motivation to periodically replace such equipment.

Consider the specific case described above concerning a new gas griller you purchased to host parties at your home during the next five years. The cost of the grill is $7,600. The cost of maintaining such a high-end item per year is significant if you wish to keep it in the best condition as possible (which, of course, you do). Assume the maintenance costs during a year depend on its age at the beginning of the year, as given in Table 5.

Table 5: Open-air culinary system costs and resale values

	Project	
Age (years)	Maintenance costs	eBay value
0	800	.
1	1250	5000
2	2000	4100
3	2900	1500
4	4800	950
5	.	0

To avoid the high maintenance costs associated with an older grill, you may sell the grill on eBay and purchase a new grill. The price you receive on eBay depends on the age of the grill at the time of sale (see Table 5). Assume that at any time, it costs $7,600 to purchase a new grill.

The goal is to minimize the net cost (purchasing costs + maintenance costs - money received from eBay) for the next five years. Note: there is no option to simply sell the grill and not replace it: remember, your passion has been ignited and you must grill! Also, to simplify things, you will only make decisions at the beginning of each year..

For instance, when you first purchase the grill at the beginning of year 1, it costs you $7600. You will keep the grill for at least one year, so you would incur $800 of maintenance costs. Now, at the beginning of year 2, you can either decide to keep it for another year (and incur the additional maintenance costs) or sell it on eBay for $5,000 and buy a new grill for $7,600. Then you would need to make the same decision the next year, etc.

Formulate this problem as a shortest-path problem and solve.

Question 4: RACECAR TIRES (16 points)

An automobile association is organizing a series of car races that will last four days. The organizers know that $r_j \geq 0$ special race tires in good condition will be required on each of the four successive days, $j = 1, 2, 3, 4$. They can meet these needs either by buying new tires at P dollars apiece or by reshaping[5] used tires[6] after the day's race or some combination of both. Two kinds of reshaping service are available: *normal* service, which takes one full day at N dollars per tire, and *quick* service, which is an overnight service and costs Q dollars per tire. How should the association, which starts out with no special tires, meet the daily requirements at minimal cost?

(a) Formulate a network flow model to address the problem.

(b) Using the following values, solve an instance of problem: $P = \$600, N = \$95, Q = \$250$; and r_j for $j = 1, 2, 3, 4$ is 320, 240, 400, and 520, respectively.

Question 5: DUNDER MIFFLIN PAPER COMPANY (16 points)

You have been hired as a director in the *Paper Products* division of Dunder Mifflin to support operations for a certain B2B[7] product line.

Specifically, you have two major business customers, Goggle and Faceblock, who have firm contracts for your product each month of 1000 and 600 units, respectively.

SBI has three different production plants that can produce the product, one in Scranton, PA; one in Utica, NY, and one in Stamford, CT. However, each of the plants is limited on how much they can produce during regular time. Additionally, each plant has a different cost (not including labor) to produce one unit of the product. The transportation costs to ship the product from each plant to each customer is also different. All of these details are provided in Table 6.

Table 6: Plant informations and relevant costs

Plant	Max capacity (units)	Costs per unit ($)		
		Manufacture	Transportation	
			Goggle	Faceblock
Scranton	505	90	8	15
Utica	465	105	14	18
Stamford	570	115	24	20

If overtime is used, then each plant has an additional capacity of 100 units per month. The cost for overtime (not including labor) is 1.5 times the regular time cost.

Given your previous success at determining workforce levels, SBI has also decided to ask you to deal with the labor options available to this operations group. There are two sources of labor: certified specialists (100 available in the area available at \$2000 per month) and generalists (200 available in the area at \$1700 per month). You do not have to hire all of the workers. The specialists are able to produce 12 units of product each month, whereas the generalists only produce 10 units per month.

SBI must provide transportation for all workers to the plants. The cost per month of transporting a worker to Scranton is \$300, to Utica is \$250, and to Stamford is \$275.

Create a network flow formulation of this problem and solve to minimize costs while meeting demand. Hint: you may need to consider the *generalized* network flow model.

[5]Reshaping is a technique by which the grooves on the tire are deepened, using a special profile-shaped tool.

[6]You can safely assume that after a day of racing any used tire is no longer in "good" condition.

[7]Business-to-business