DSA/ISE 5113
Advanced Analytics and Metaheuristics
Final Exam

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- This exam is *not* a team effort: Do not discuss the exam problems or solutions with anyone until after the exam has been submitted by all students. Any evidence of academic integrity violations will be investigated. Don't cheat!
- You may use notes, books, and/or computer resources you have available. This excludes sites like chegg.com, etc.
- If something in a problem is not clear, you are allowed to state your assumptions. If your assumptions are reasonable, they may be accepted.
- Make sure I can read and understand your work. It should be legible.
- Your submission will be a single PDF file. If you choose to scan or take a picture of your handwritten work to include in the PDF, make sure it is a sufficiently high quality image. If code is required, copy and paste the code into your submission.

Question 1: OK Highway Patrol (32 points)

The Oklahoma state highway patrol desires to better allocate its 20 officers along segments of highway I-35 to improve safety by ticketing more people who are exceeding the speed limits. There are six segments along the highway that have been identified as important areas and evaluated in terms of the expected number of speeders that could be caught each day *per* officer assigned to that segment. Due to various considerations, there is an upper limit on the number of officers that can be assigned to any given highway segment. See Table 1 for specific data.

Figure 1: Oklahoma Highway Patrol

(a) (6 points) Provide a written mathematical formulation of this problem as an integer linear program to *maximize* the expected number of speeders caught across the segments. Make sure to define any sets, indices, and decision variables; use correct mathematical notation.

Table 1: Highway patrol problem data

	Highway segment, j					
	1	2	3	4	5	6
Upper bound, u_j, on officers assigned to segment j	5	9	5	6	8	7
Expected speeders per officer, s_j for segment j	11	6	14	4	2	19

(b) (6 points) Write AMPL code using a *model* file and a separate *data* file (please provide the code, copied and pasted, in your PDF submission). Run your code and clearly detail the optimal solution and identify how many speeders are expected to be caught.

(c) (10 points) Consider the following problem variation: The highway patrol captain likes to drive out and observe the patrol officers throughout the week. She visits one highway segment per day – assuming that the highway segment has at least one officer assigned to it. The time it takes her to visit the segment and return to the office is provided in Table 2. The captain only has a total of 5 hours to dedicate to these visits each week. Reformulate the problem to maximize the expected number of speeders caught while accounting for the new constraint from the captain.

Table 2: Patrol captain drive time

	Highway segment, j					
	1	2	3	4	5	6
round trip time (minutes) to visit segment j	20	200	30	90	50	110

i. (6 points) Specify the changes to the original written mathematical model (e.g., new variables? updated constraints? revised objective?) – remember to use correct mathematical notation.

ii. (4 points) Modify your AMPL code and determine the new optimal solution and total number of speeders caught?

(d) (10 points) Consider the following problem variation, independent of part (c): The various counties along the highway want to ensure they are not overlooked in officer assignments for the various segments. Reformulate the original problem and use all officers in such a way to *maximize the minimum* expected number of speeders caught across the segments.

i. (6 points) Specify the changes to the original written mathematical model (e.g., new variables? updated constraints? revised objective?) – remember to use correct mathematical notation.

ii. (4 points) Modify your AMPL code and determine the new optimal solution and total number of speeders caught?

Question 2: Branch-and-bound (16 points)

Consider the following problem:

$$\begin{aligned} &\min\ x_4 \\ &\text{s.t. } 2x_1 + 2x_2 + 2x_3 + x_4 = 5 \\ &x_i \in \{0,1\}, i = 1, \dots, 4 \end{aligned}$$

(a) (8 points) Solve this problem by drawing out the complete branch-and-bound tree. Make sure to label the upper and lower bounds, the branches, the LP-relaxation optimal solution, and which nodes are fathomed and why.

(b) (4 points) Define the best *valid inequaltity* you can for this problem.

(c) (4 points) Draw and label the branch-and-bound tree again, but this time, incorporate your valid inequality. *Note: This is should reduce the size of your B&B tree!*

Question 3: Neighborhood-based search questions (32 points)

Consider the following 2D problem:

$$\begin{aligned} &\min\ (x_1 - 1)^2 + (3x_2 - 1)^2 - x_2 x_1 \\ &\text{s.t. } -4 \le x_i \le 4,\ i = 1, 2 \\ &x_i \text{ integer},\ i = 1, 2 \end{aligned}$$

(a) (4 points) What is the size of the solution space. Explain.

(b) (4 points) Define and explain a valid neighborhood definition that could be used in a hill-climber algorithm for this problem. What is the size of the neighborhood you have defined?

(c) (8 points) Apply the `iterated hill-climber` algorithm from Chapter 2 (page 43-44) of the *How to Solve It* textbook to this problem assuming an initial location of $(-2, 3)$ with an evaluation of 79. Complete 2 iterations of the algorithm by hand and demonstrate the steps.

(d) (8 points) Assume that throughout a search algorithm you come accross two particular solutions, $A = (-1, -3)$ and $B = (1, -1)$. Demonstrate the evaluations and moves from path-relinking starting at solution A and ending at solution B.

(e) (8 points) Assume you are using Simulated Annealing to solve this problem. The current temperature is 15.0 and the current solution is $(3, 2)$.

   i. What is the probability of accepting the candidate solution $(3, 1)$?

   ii. What is the probability of accepting the candidate solution $(3, 3)$?

Question 4: POPULATION-BASED ALGORITHMS (12 points)

Assume the two 3-dimensional continuous valued encoded solutions given in Table 3 are valid for some continuous optimization problem. For GA, conduct crossover between solutions A and B using the approach in the continuous-valued crossover from lecture and provided in the related homework Python base code. (The method is also described in the resource "The Continuous Genetic Algorithm", pages 58-60.) Assume the crossover point is between the first and second element of the chromosome. Assume the random value β equals 0.25. Clearly show each step.

Table 3: Solutions for continuous value crossover

Solution	Encoding		
A	12.9	27.1	33.4
B	13.5	76.2	19.1

Question 5: MULTIPLE OBJECTIVE OPTIMIZATION (8 points)

Management wants to produce products quickly, cheaply, and with the highest quality. Unfortunately, minimizing *time*, minimizing *cost*, and maximizing *quality* cannot be achieved in any solution. Nonetheless, some solutions are better than others. See the corresponding objective values for candidate solutions in Table 4. Identify all non-dominated solutions.

Table 4: Candidate solutions and their objective values

Solution ID	Time (minutes)	Cost (dollars)	Quality (100 point scale)
1	54.4	2951.5	12
2	102.4	2361.9	56
3	52.2	1762.2	63
4	122.9	3049.6	58
5	66.6	1934.3	79
6	47.2	1984.1	33
7	49.1	1800.7	57