

DSA/ISE 5113 Advanced Analytics and Metaheuristics

Homework #3

Instructor: Charles Nicholson

Due: See course website for due date

Requirement details

1. Homeworks should be submitted in a clean, clear, concise electronic format. You must show your logic, work, and/or code where appropriate. Note: any code (e.g., AMPL model and/or data files) is a part of your solution (turn in as separate files)
2. For mathematical programming problems, ensure you clearly define the following elements: (i) **necessary assumptions**, (ii) **decision variables**, (iii) **objective and objective function**, and (iv) **constraints**. Points may be deducted if proper definitions/documentation is missing.
3. For problems denoted as *Case Studies*, there may be a lot of information, some of which you need, some which you do not; and, you may have to make assumptions to complete the problem. Additionally, for case studies you may have to learn new concepts (e.g., marketing or finance); also, I want you to think about the solutions in terms of more than simply the “math”, i.e., what are the implications and/or what do the solutions really tell us. Have fun!

Question 1: CASE STUDY: BOOMER SOONER AIR SERVICES (28 points)

BACKGROUND

Li Chase set about the task of preparing a fuel plan for her upcoming five-leg flight to Nantucket, MA; New York City, NY; Nashville, TN; Tulsa, OK; and back. Like the other corporate pilots she works with, Ms. Chase enjoys flying a lot more than doing paperwork. But unlike some of her colleagues she rather enjoys the challenge of constructing a fuel plan.

Boomer Sooner Air Services (BSAS) operates multiple aircraft to serve the transportation needs of the corporate headquarters of Boomer, Inc. Located in Cedar Rapids, IA, the headquarters houses the executive and administrative staff of Boomer’s divisions along with a wide array of company-wide functions. Company executives routinely visit BSAS to fly to company factories, marketing facilities, and customer locations throughout the world.

BSAS currently operates two Cessna Citation X aircraft (CE750), among others. The CE750 often flies from Cedar Rapids to as far as South America, Europe, and western Russia. Its fuel burn rate of 310 gallons per hour coupled with its 14,000 pound capacity tank meant that it requires a fuel stop to reach these more distant destinations. It carries up to eight passengers and two pilots.

FLIGHT PLAN

In two days, the CEO and CFO of Boomer, Inc. has a trip scheduled for Cedar Rapids to Nantucket, the New York City area, Nashville, Tulsa, and then back to Cedar Rapids. The purpose of the trip is to



Figure 1: Cessna Citation X aircraft (CE750)

pick up some key analysts and mutual fund managers in Massachusetts and New York and show them the new factories in Nashville and Tulsa and the new distribution center in Cedar Rapids. They would be picking up two passengers in Nantucket and four in New York.

As usual, BSAS will use the airport in Morriston, New Jersey, as their destination for the New York City area. Each U.S. airport carries a four-letter identifier beginning with the letter K. The upcoming four-leg flight would go from KCID to KACK to KMMU to KBNA to KTUL and back to KCID. Pilots at BSAS were responsible for creating and filing their own flight plans with the U.S. Federal Aviation Administration (FAA).

One important element of the flight plan is the takeoff and landing weight of the aircraft. To calculate these, one starts with the basic operating weight (BOW) of the aircraft and added the weights of the passengers and fuel. The BOW includes the structure of the aircraft, a stocked galley, emergency equipment, and the crew. The only weight components that vary from flight to flight are passengers and fuel. The only component that varies from takeoff to landing on a given flight is fuel.

FUEL PLAN AND TANKERING

This means that one of Chase's first tasks is to determine a fueling plan for the upcoming flights. Coming up with a fuel plan is not a joyful task for pilots because there is no straightforward ways to calculate how much fuel to take on or "upload" at the beginning of each leg. One question is whether or not to "tanker."

Tankering refers to a practice in which extra fuel is uploaded initially to avoid having to purchase higher-priced fuel at destination airports. BSAS operated its own fuel farm at Cedar Rapids's KCID, which kept its fuel costs low. Fuel at KCID at the time cost \$4.00 a gallon. In contrast, fuel purchased at KACK cost \$8.32 a gallon. As a simple example of tankering, Chase could decide to upload enough fuel at KCID to carry him through both of the first two legs, thereby avoiding buying fuel at KACK. In essence, BSAS would carry or tanker from KCID the fuel needed to fly from KACK to KMMU.

One factor working against tankering is the airport ramp fee. Ramp fees are fixed fees charged to each landing jet by the destination airport's general-aviation terminal; the fees cover the costs of operating the terminal. For example, the ramp fee at KACK is \$800, but the fee is waived with the purchase of 600 or more gallons of fuel.

To begin the process of constructing a fuel plan, Ms. Chase assembled the information in **Table 1**. The fuel burn numbers were fairly easy to calculate based on flight miles and aircraft. (The burn numbers included the fuel used during taxiing at the departing airport.) Although the calculation is more complicated than just multiplying miles by average gallons per mile (because extra fuel is used at takeoff), most pilots could do the calculation in their heads. Fuel prices, ramp fees, and minimum gallons needed to waive the ramp fees could all be found on the Internet.

In addition to the cost of fuel and ramp fees, Chase needs to consider the limitation of the CE750 **Table 2**). The fuel tank capacity is a firm physical limit, and the departure ramp and lading weight limit are

structural limits developed by the manufacturer and approved by the FAA during aircraft certification. To calculate departing ramp or arrival weight, Chase added BOW to the weight of the fuel and the weight of the passengers (passenger weight calculations are based on a company-mandated figure of 200 pounds per person, including luggage).

Table 1: Flight Details

Leg	Depart	Arrive	Miles	Duration (hh:mm)	Fuel burn incl. taxi (pounds)	Fuel price (\$/gallon)	Ramp fee	Minimum gallons to waive fee
1	KCID	KACK	1,090	2:56	5,100	\$4.00		
2	KACK	KMMU	196	0:59	2,200	\$8.32	\$800	600
3	KMMU	KBNA	814	2:35	4,700	\$8.99	\$750	500
4	KBNA	KTUL	550	1:57	3,800	\$6.48	\$600	650
5	KTUL	KCID	485	1:48	3,600	\$9.27	\$800	500

Table 2: Aircraft Limitations (in pounds)

Aircraft	CE750
Maximum Ramp Weight	36,400
Maximum Landing Weight	31,800
BOW	22,800
Fuel Tank Capacity	14,000

There are two final considerations. The company requires that aircraft always land with at least 2,500 pounds of fuel. Any valid fuel plan Chase could create would have to be one in which the weight of fuel at arrival met or exceeded 2,500 pounds. This “safety stock” is there to ensure jets had enough fuel to make it to an alternate airport should there be bad weather at the destination airport. The second consideration is that the company dictated immediately bringing the fuel level up to 7,000 pounds upon arrival back at KCID. The rationale for this is that the aircraft would always be ready to go at a moment’s notice. This meant the Chase’s fuel plan should begin with the CE750 containing 7,000 pounds of fuel.

As Chase prepared to put pencil to paper to create a fuel plan for the upcoming KCID to KACK to STEB to KTUL to KCID trip, she paused to ponder why aircraft gauges measured fuel in pounds and yet fuel is sold in gallons. Like every other pilot at BSAS, she knew the importance of the number 6.7 – the weight in pounds of a gallon of jet fuel.

- (a) Formulate the problem and solve for a minimum cost fuel plan for the upcoming trip.
 - i. (16 points) What is the optimal fueling plan and minimal cost?
 - ii. (4 points) Compare your results with a no “tankering” solution.
- (b) Suppose the BSAS department manager wished to modify the model to require that “if you buy any gas, you must buy at least 200 gallons”.
 - i. (6 points) Formulate the problem with this modification.
 - ii. (2 points) How does this change the solution and cost for Ms. Chase’s current optimal plan?

Question 2: WE GOT GAS! (20 points)

A local oil and gas refinery company, We Got Gas! (WGG) manufactures 5 types of gasoline. WGG has several thousands liters of each which must be stored in 8 different storage tanks. Each storage tank can contain at most one type of gasoline. Associated with each gasoline type and storage tank combination is a pumping cost. Each tank has a finite capacity so some gasoline types may have to be split over several storage tanks. Table 3 provides the relevant data for each type of gasoline. Formulate and solve this problem to find a minimum cost storage plan.

Table 3: Storage data

Gas Type	Tank-Pumping Cost per 1,000 liters (\$)								Gas (liters)
	1	2	3	4	5	6	7	8	
A	1	2	2	1	4	4	5	3	75,000
B	2	3	3	3	1	4	5	2	50,000
C	3	4	1	2	1	4	5	1	25,000
D	1	1	2	2	3	4	5	2	80,000
E	1	1	1	1	1	1	5	5	20,000
Tank Capacity (liters)	25,000	25,000	30,000	60,000	80,000	85,000	100,000	50,000	

- Formulate and solve this problem to find a minimum cost storage plan.
- Extra credit [10 pts] Some of the executive management team at WGG has decided they do not like partially filling tanks. Even though that they realize this certainly must happen from time to time, they still just don't like doing it, even if it is more cost effective. Other executives are perfectly fine with partially filled tanks, as long as the storage plan is as inexpensive as possible. Everyone agrees that if all costs were equal, then, for example, it'd be better to store 20,000 liters in a 25,000 liter capacity tank instead of using their 100,000 liter capacity tank. How could you modify your formulation to deal with the management teams wishes? Implement your revised model in AMPL to address this issue and discuss the ramifications.

Question 3: ED ON-CALL (18 points)

An Emergency Department needs to keep doctors on call over the weekend so that a qualified individual is available to perform every medical procedure that might be required. For each of several doctors available for on-call duty, the additional weekend salary they need to be paid, and which procedures they can perform, is known (see table below). The goal is to choose doctors so that each procedure is covered at a minimum total cost.

- (10 points) Formulate the general Emergency Department problem as a mathematical programming problem for n procedures and m doctors. Clearly define and label all decision variables and constraints. If you choose to use parameters, sets, and indices, make sure you define them. Note: This is not AMPL code, but a written mathematical formulation of the problem.
- (8 points) Code the problem in AMPL and solve the instance provided in Table 4.

Table 4: ED Doctors

	Doc 1	Doc2	Doc3	Doc4	Doc 5	Doc 6
	\$3100	\$4800	\$2500	\$3750	\$2950	\$1400
Procedure 1	X	X	.	.	X	.
Procedure 2	.	X	.	X	X	.
Procedure 3	.	X	X	X	.	.
Procedure 4	X	.	X	X	.	X
Procedure 5	X	.	.	X	X	.
Procedure 6	.	X	.	.	X	X
Procedure 7	.	X	X	.	.	X
Procedure 8	X	.	X	X	.	.

Question 4: GALAXY INDUSTRIES REVISITED (26 points)

Reformulate the Galaxy Industries problem from lecture – this time with all of the IP magic you heart desires (i.e., *piecewise linear cost functions*, *disjunctive constraints*, and *integer solutions*).

Galaxy manufactures two toy guns:

Space Rays:

- \$8 revenue per unit
- Total per unit costs: \$3.75 for 0 to 500 units; \$3.25 for the next 500 units; \$2.80 for the next 1,000 units; \$1.90 for any more

Zappers:

- \$5 revenue per unit
- Total per unit costs: \$1.95 for 0 to 1500 units; \$2.15 for the next 500 units; \$2.95 for any more units

Management is seeking a production schedule to maximize the company's profit per week.

- Resources are limited:
 - 7,000 pounds of special plastic per week
 - 250 hours of production time per week
- Space Rays require 2 pounds of plastic and 3 minutes of labor per unit.
- Zappers require 1 pound of plastic and 4 minutes of labor per unit.

Additionally, based on recommendations from the marketing team, if Galaxy Industries makes more than 3,000 total units, at least 45% of all units made should be Zappers. (For example, if a total of 1,000 units are made, then all of them can be Space Rays; but, if 10,000 units are produced, at least 4,500 should be Zappers.)

Question 5: VALID INEQUALITIES (8 points)

Identify two valid inequalities for the following mathematical program. Explain how each is a valid inequality.

$$\begin{aligned}
 &\text{minimize } 14x_1 + 2x_2 + 11x_3 + 9x_4 + x_5 \\
 &\text{s.t. } 3x_1 - 4x_2 + 2x_3 - 3x_4 + x_5 \leq -2 \\
 &\quad x_i \in \{0, 1\} \text{ for } i = 1, \dots, 5
 \end{aligned}$$