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DSA 5113
Advanced Analytics & Metaheuristics
Group 32 - HW 1

Problem 1 – Smullyan's Island Revisited

Gregor, Tywin, and Catelyn are either always lying or telling the truth. Gregor was asked, "how many of you are truth-tellers?"
Gregor : *something unintelligible*
Tywin: "He said only one of us is truth-teller."
Catelyn: "No he (Gregor) didn't!"

Propositions:
p: Gregor is a truth-teller.
q: Tywin is a truth-teller.
r: Catelyn is a truth-teller.

p	q	r	Gregor told a true statement	Tywin: "He said only one of us is truth-teller."	Catelyn: "No he didn't!"
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	**T**	**F**	**T**
T	F	F	T	F	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	**F**	**F**	**T**
F	F	F	F	F	T

Since we are not sure what Gregor was saying, Tywin's column is the same as the assumption, *q*. Because of that and due to their statements, we can say that Catelyn is lying if Tywin is telling the truth and she is telling the truth whenever Tywin is lying. For Gregor, when Tywin is telling the truth but there is more than one truth-teller, Gregor's statement is false. Whenever Tywin is lying, since his statement could be anything, Gregor's statement is based on the assumption, *p*, whether or not he's telling the truth.

Therefore, from the table above, we found two possible cases. We can conclude:

- **Tywin is a liar,**
- **Catelyn is a truth-teller, and**
- **Gregor is unknown (he could either be a truth-teller or a liar).**

Problem 2 – Baby Yoda

Baby Yoda only say gurmp and pvlork when talking to Mando since he does not speak Mandalorian's language. Mand knows that one means *yes* and the other means *no*. He tried to figure out which one is which. It should be noted that Baby Yoda could either be a liar or a truth-teller.

Their conversation goes as follows:
Mando : "Does gurmp means yes?"
Child : "Pvlork"

Propositions:
p: Gurmp means Yes.
q: Baby Yoda is a truth-teller.

p	**q**	**Mando: "Does Gurmp means yes?"**	Child: "Pvlork"	**Child is telling the truth**
T	T	T	Yes	F
T	F	**T**	No	**F**
F	T	F	No	F
F	F	**F**	Yes	**F**

At first, looking at the statement "Does gurmp means yes?" and check it against the proposition, *p*. The child's answer is used to determine if the assumption of the child as a truth-teller is true or not. So, if the proposition, *q*, is telling truth then Pvlork will be Yes or No, according to combination of *p* and *q* propositions. Since Gurmp and Pvlork cannot be both 'Yes' or both 'No' and also since the two other cases contradict the truth, all of the cases shows that the child is a liar. **Therefore, baby Yoda is an evil dark side liar. With two possible cases to be true, it is still unknown what gurmp means.**

Problem 3 – Portfolio Selection

Chapter 1 - Section 1.3 from the Applied Mathematical Programming textbook

For part a, without borrowing any money, the objective (maximum total bond earnings) is $0.2984 million. The Bonds for each bond type are listed below:

The amount of investment for Bond A: $2.1818 million

The amount of investment for Bond B: $0.00

The amount of investment for Bond C: $7.3636 million

The amount of investment for Bond D: $0.00

The amount of investment for Bond E: $0.4545 million

```
ampl: model Group32-HW1-3a.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 0.2983636364
3 dual simplex iterations (1 in phase I)

The amount of investment for Bond A: 2.1818 million
The amount of investment for Bond B: 0.0000 million
The amount of investment for Bond C: 7.3636 million
The amount of investment for Bond D: 0.0000 million
The amount of investment for Bond E: 0.4545 million
```

For part b, with up to 1 million dollar loan, the objective (maximum total bond earnings) is $0.3007 million. The Bonds for each bond type are listed below:

The amount of investment for Bond A: $2.4000 million

The amount of investment for Bond B: $0.00

The amount of investment for Bond C: $8.1000 million

The amount of investment for Bond D: $0.00

The amount of investment for Bond E: $0.5000 million

The amount of investment borrowed: $1.0000 million

```
ampl: model Group32-HW1-3b.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 0.3007
The amount of investment for Bond A: 2.4000 million
The amount of investment for Bond B: 0.0000 million
The amount of investment for Bond C: 8.1000 million
The amount of investment for Bond D: 0.0000 million
The amount of investment for Bond E: 0.5000 million

The amount of investment borrowed: 1.0000 million
```

Code files:
Group32-HW1-3a.mod, Group32-HW1-3a.dat,
Group32-HW1-3b.mod, and Group32-HW1-3b.dat.

Problem 4 – Advertising

Exercise 1-1 from AMPL textbook (Chapter1)

a) **How should you spend your budget to maximize your audience? Formulate the problem in AMPL and solve it. Check the solution by hand using at least one of the approaches described in Section 1.1.**

The solutions below depend on the assumption that an additional page or minute of advertisement increases the potential customer reach linearly. In real life much of this reach would probably be to the same people, so it is not actually linear and diminishing returns would have to be taken into account. Therefore, can maximize the audience by finding the maximum of the combination of total customer reached for every minutes on TV and the total customer reached for every pages on the magazines. Using AMPL, we found that with the 1 million dollar budget, we can reach 98 million potential customers by advertising 10 minutes on TV and 80 pages in the magazines.

```
-----Solution Question 4a: maximize audience at $1.00 million budget-----

Potential customers reached: 98 million.

This is achieved by advertising in:
 10 TV minutes
 80 magazine pages

Cost by advertising in:
 TV      : 0.20 million
 Magazine: 0.80 million
```

Check solution by hand:

Potential TV customers reached per dollar spent: 1.8 million / 20,000 = 90

Potential Magazine customers reached per dollar spent: 1 million / 10,000 = 100

From that calculation, magazine page advertising is the most "profitable" in terms of customer reach per dollar, therefore it makes sense to maximize the advertising in magazine pages. Since the instructions say we must buy at least 10 minutes of TV advertising, once we done that, putting the rest into magazine pages is the logical solution and agrees with our answer above.

b) You have only 100 person-weeks available. Add this constraint to the model and determine how you should now spend your budget.

By adding 100-person weeks available as a constraint, the number of minutes on TV and the number of pages will be restricted. Because for TV minute only takes 1 person-week where magazine page needed 3 person-week, now, $800,000 is being used for 10 minutes of TV advertisement and $200,000 is being used to create 20 pages of advertisement for the magazines.

```
-----Solution Question 4b: limiting person-weeks available to 100-----

Potential customers reached: 92 million.

This is achieved by advertising in:
 40 TV minutes
 20 magazine pages

Cost by advertising in:
 TV      : 0.80 million
 Magazine: 0.20 million
```

c) How does this medium (radio) affect your solutions?

Adding radio into the mix of possible medium to use for advertising in this case help reach even more potential customers. The potential customers go up to 118 million. Although it reaches fewer potential customers compare to TV and magazine since it has lower cost and less person-week to work on it, it ended up being a better medium to do the advertising. It even ended up eliminating magazine advertising.

```
-----Solution Question 4c: adding radio-----

Potential customers reached: 118 million.

This is achieved by advertising in:
 10 TV minutes
 0 magazine pages
 400 radio minutes

Cost by advertising in:
 TV      : 0.20 million
 Magazine: 0.00 million
 Radio   : 0.80 million
```

d) **How does the solution change if you have to sign up for at least two magazine pages? A maximum of 120 minutes of radio?**

By having a limit of 120 minutes of radio advertisement, magazine come back into play. The number of potential customers come down to 100 million instead of 118 million when there is no limit of radio advertisement. From part c, it shows that 400 minutes of radio was being used. Therefore, by having the 120 minutes of radio and at least two magazine pages, the advertising goes to 29 minutes of TV, 17 pages of magazines, and 120 minutes of radio.

```
-----Solution Question 4d: adding magazine and radio constraints-----

Potential customers reached: 100 million.

This is achieved by advertising in:
 29 TV minutes
 17 magazine pages
 120 radio minutes

Cost by advertising in:
 TV      : 0.58 million
 Magazine: 0.18 million
 Radio   : 0.24 million
```

Code files:
Group32-HW1-4a.mod, Group32-HW1-4b.mod, Group32-HW1-4c.mod,
Group32-HW1-4d.mod, and Group32-HW1-4.dat.

Problem 5 – Steel Mill

Exercise 1-2 from AMPL textbook (Chapter1)

a) **How would you change the constraints so that total hours used by all products must equal the total hours available for each stage?**

We would change the original constraint to be:

subject to Time {s in STAGE}:

sum {p in PROD} (1/rate[p,s]) * Make[p] = avail[s]

INSTEAD OF the original constraint:

subject to Time {s in STAGE}:

sum {p in PROD} (1/rate[p,s]) * Make[p] **<=** avail[s];

By using an equal sign instead of less than or equal to sign, the total of hours used for all of products on each stage MUST EQUAL to the hours available during the corresponding stage.

```
ampl: model Group32-HW1-5.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 190071.4286
2 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

The maximum total profit from the original code:
 190071.43
ampl:
ampl: model Group32-HW1-5.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 190071.4286
2 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

The maximum total profit from the modified constraint code:
 190071.43
```

As shown, there is no difference in the maximum of the total profit. This is because the model was already using all of the available hours of each stage to make as many profitable tons of products as possible to begin with (original constraint) even though the constraint of using less than available hours was a possibility.

b) How would you add to the model to restrict the total weight of all products to be less than a new parameter, max_weight? Solve the linear program for a weight limit of 6500 tons, and explain how this extra restriction changes the results.

We would create a new constraint with the ***max_weight*** as the upper limit of the total weight of all products. Since this new restriction, the optimal solution change to $183,791.67, which is lower than before there is any total weight restriction. The previous results are most likely to be heavier than 6500 tons.

```
CPLEX 12.9.0.0: optimal solution; objective 183791.6667
3 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

The maximum total profit for 6500 tons:
 $183791.67
```

c) **The incentive system for mill managers may tend to encourage them to produce as many tons as possible. How would you change the objective function to maximize total tons? For the data of our example, does this make a difference to the optimal solution?**

In order to maximize total tons, we changed the objective function by removing the profit factor from the original equation and just maximizing the ***Make*** var. The maximum total tons come out to 7,000 tons. From the solution we get in **part a**, this would not be the optimal solution, since profit comes out to $180,500, which is almost $10,000 less than when we maximized profit.

```
Maximum total products being produced: 7000.00 tons

Total weight of each product:
 5750 tons of bands
  500 tons of coils
  750 tons of plate
 Profit would be $180500.00
```

d) **Suppose that instead of the lower bounds represented by commit[p] in our model, we want to require that each product represent a certain share of the total tons produced. How would you change the AMPL model to use this constraint in place of the lower bounds commit[p]? If the minimum shares are 0.4 for bands and plate, and 0.1 for coils, what is the solution? Verify that if you change the minimum shares to 0.5 for bands and plate, and 0.1 for coils, the linear program gives an optimal solution that produces nothing, at zero profit. Explain why this makes sense.**

Taking out the *commit* variable from the decision variable and adding a new constraint to calculate the new lower bound based on the minimum shares rates associated with the products and number of products being produced. On this problem there are two scenarios used for the minimum shares of the products. For minimum shares of 0.4 for bands and plates and 0.1 for coils, the optimal solution is $189,700. For minimum shares of 0.5 for bands and plates and 0.1 for coils, the optimal solution is $0. That is because the number of products made was zero due to the new constraint.

```
ampl: model Group32-HW1-5d.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 189700
5 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

The minimum shares of bands: 0.4000
The minimum shares of coils: 0.1000
The minimum shares of plate: 0.4000
The amount of bands made   : 3500.00 ton(s)
The amount of coils made   : 700.00 ton(s)
The amount of plate made   : 2800.00 ton(s)

ampl: model Group32-HW1-5d.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 0
4 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

The minimum shares of bands: 0.5000
The minimum shares of coils: 0.1000
The minimum shares of plate: 0.5000
The amount of bands made   : 0.00 ton(s)
The amount of coils made   : 0.00 ton(s)
The amount of plate made   : 0.00 ton(s)
```

e) Suppose there is an additional finishing stage for plates only, with a capacity of 20 hours and a rate of 150 tons per hour. Explain how you could modify the data, without changing the model, to incorporate this new stage.

This can done by adding *finishing* to the stage set and updating other variables related to that stage in the data file. It should be noted that the band and coil will need to be set to infinity since *finishing* stage is for plates only. The infinity is important since it will help take the time constraint for the other products to zero.

```
#Group32 - Rumainum & Navarrete
#HW 1
#AMPL model file for Problem 1-5e
#Steel Mill

#indicate it's a data file----------------------------------------
data;

#Code from:
#Figure1-6b from AMPL book pg 17

#parameters and sets----------------------------------------------
set PROD := bands coils plate;
set STAGE := reheat roll finishing; #add finishing to stage

#add rates for finishing
param rate:  reheat roll finishing :=
bands           200  200 Infinity
coils           200  140 Infinity
plate           200  160      150;

#no change here
param: profit commit market :=
bands      25    1000    6000
coils      30     500    4000
plate      29     750    3500;

#add available time for finishing
param avail := reheat 35 roll 40 finishing 20;
```

Code files:

Group32-HW1-5a.mod, Group32-HW1-5b.mod, Group32-HW1-5c.mod, Group32-HW1-5d.mod, Group32-HW1-5.dat, and Group32-HW1-5e.dat.

Problem 6 – Portfolio Selection Revisited

Exercise 3 on page 25 from Chapter 1 of Applied Mathematical Programming textbook

By modifying the bond to only and A and D and putting $3 million limit on municipal bond investment, here are the results:

a) What is the optimal solution?

The objective (maximum total bond earnings) is $0.2830 million. The Bonds for each bond type are listed below:

The amount of investment for Bond A : $3.0000 million
The amount of investment for Bond D : $7.0000 million
The amount of investment borrowed : $0.00

```
ampl: model Group32-HW1-6.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 0.283
0 dual simplex iterations (0 in phase I)
The amount of investment for Bond A: 3.0000 million(s)
The amount of investment for Bond D: 7.0000 million(s)

The amount of investment borrowed: 0.0000 million
```

b) What is the shadow price on the municipal limit?

By looking at the total investment in municipal bond constraint, the shadow price on the municipal limit can be found, which is $0.021 million.

```
The shadow price on the municipal limit and its range of feasibility:
TotalInvestmentInMunicipal = 0.021
TotalInvestmentInMunicipal.up = 0
TotalInvestmentInMunicipal.down = 0
```

c) How much can the municipal limit be relaxed before it becomes a nonbinding constraint?

The municipal bond, which is Bond A, can be relaxed to $3.33 million before it becomes a nonbinding constraint since it will break the other constraints if it is greater than that.

d) Below what interest rate is it favorable to borrow funds to increase the overall size of the portfolio?

The interest rate that is favorable to borrow funds to increase the overall size of the portfolio needs to be lower than the earning rate. Preferably less than the lowest earning rates. In this problem, that would be the 2.2% after-tax yield of the government type bond.

e) Why is this rate less than the earnings rate on the portfolio as a whole?

This rate is less than the earnings rate on the portfolio as a whole because it guarantees to be covered by those earnings, which means it definitely will be profitable even after paying interest rate.

Code files:
Group32-HW1-6.mod and Group32-HW1-6.dat.