ISE 5113 Advanced Analytics and Metaheuristics
Exam #1

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- This exam is *not* a team effort: Do not discuss the exam problems or solutions with anyone until after the exam has been submitted by all students. Any evidence of academic integrity violations will be investigated. Using sites like Chegg.com, etc. to find answers is considered cheating. Don't cheat!
- Otherwise, you may use notes, books, and/or general computer resources you have available.
- If something in a problem is not clear, you are allowed to state your assumptions. If your assumptions are reasonable, they may be accepted.
- Your submission will be a single PDF file. If you choose to scan or take a picture of your handwritten work to include in the PDF, make sure it is a high quality image. Make sure all of your work is legible! If AMPL code is required, copy and paste the code into your submission.
- NOTE: *Nonlinear formulations are NOT acceptable for this exam.*

Question 1: TEAM WORK! (14 points)

You and your teammate have just been assigned a new homework in ISE/DSA 5113. While the incredibly handsome and intelligent instructor of the course has begged you to not divide the homework into separate problems per team member, you have decided to do so anyway. There are 8 problems on the new assignment. Using binary programming you want to decide on which problems you will work, i.e., $x_i = 1$, if you choose to work on problem i, and 0, otherwise.

Model the following related constraints mathematically (no AMPL code required!). Each must be a correct linear constraint using the techniques discussed in the course. Before formulating, please **clearly define any variables, sets, parameters that you are using**. You must use correct mathematical notation in responding.

(a) (2 points) You cannot work on all the problems.

(b) (2 points) You must choose at least two of the problems to work on.

(c) (2 points) If you choose problem 2, then you must choose problem 7 or 8.

(d) (2 points) Problem 2 cannot be chosen if problem 4 is chosen.

(e) (2 points) Problem 5 can be chosen only if problem 3 is also chosen.

(f) (2 points) You must choose either *both* problems 1 and 8 or *neither*.

(g) (2 points) You must choose at least one of the problems 1, 2, and 5 *or* at least two problems from 3, 4, 6, and 8.

Question 2: Post COVID party planning (24 points)

As more people are getting vaccinated from COVID-19, there is hope that, yes soon, we will no longer have to social distance and life can get back to normal. With this new found hope for tomorrow, you decide to begin planning a party.

You know how to make two delicious party beverages (beverage A and beverage B) using a unique and secret blend of three different types of ingredients (1, 2, and 3). Unfortunately, you don't remember the exact instructions on how to make the secret blend... But who cares, you remember enough: you know that at least 45% of the mix must be from ingredient #1; at least 10% should be from ingredient #2, and absolutely no more than 30% can be from ingredient #3. You also remember, that after you make the blend, you will use exactly 40% of it to make beverage A, exactly 25% of it for beverage B, and the remaining 35% is a toxic waste by-product that must be discarded (carefully).

The disposal fee is $8 per liter of the toxic waste by-product.

Ingredient #1 costs $12 per liter; ingredient #2 costs $25 per liter; and ingredient #3 cost $9 per liter. You have a total of $300 dollars and want to know much consumable beverage can you make.

(a) (10 points) Formulate this problem mathematically using proper notation.

(b) (10 points) Solve using AMPL – what is the total amount of each beverage? the amount of each ingredient?

(c) (4 points) Interpret the shadow price of the $300 budget constraint.

Question 3: MYSTERIOUS CONSTRAINTS (10 points)

Sitting in the dark recesses of the OU library studying for an upcoming exam in 5113, you come across an old, dusty book that catches your attention. Flipping through you notice all kinds cryptic language, ancient symbols, mathematical notations, and early 1960's computer code. The book is filled with outlandish claims and wild prophecies. Skipping past sections "Birds aren't real" and "$5 \times \mathrm{G} + \mathrm{C(OV)} = 19_{\mathrm{jnj}}$" you come to chapter that has several pages torn out. You can only read a little bit of what it says something about "...mixed integer...solution...ultimate question...life, the universe...everything." It is then followed by what appears to be a mixed integer program. A lot of the formulation is missing, but below is the excerpt that you are able read:

Decision variables

$$\begin{aligned} x_i &\quad \forall i \in I \\ y_i \text{ binary} &\quad \forall i \in I \\ Z & \end{aligned}$$

MIP formulation

$$\begin{aligned} \text{max life-happiness-success:} &\quad \ldots \\ \text{s.t.} & \\ & \ldots \\ & 0 \leq x_i \leq \left(\frac{1+\sqrt{5}}{2}\right) \quad \forall i \in I \\ & Z \leq x_i \ \forall i \in I \\ & Z \geq x_i - \left(\frac{1+\sqrt{5}}{2}\right)(1-y_i) \ \forall i \in I \\ & \sum_{i \in I} y_i = 1 \\ & y_i \in \{0,1\} \quad \forall i \in I \\ & \ldots \end{aligned}$$

What the heck is "Z"? You are sure if you answer this you will succeed in life.

(a) (10 points) How do the MIP constraints logically define Z in terms of x_i and/or y_i and/or $\frac{1+\sqrt{5}}{2}$? Be precise as possible in your response.

Question 4: Work crews (20 points)

You have been hired as the operations manager for city of Norman. There are 4 construction projects that need to be completed across the city and you have a total of 6 work crews available. All projects must be completed, but not all work crews have to be assigned. Each crew can do at most one project. No project can use more than one crew. Table 1 details the dollar costs associated with possible crew-project assignments (in terms of thousands). If a "." is shown instead of cost, then that work crew is not able to work on the associated project. For instance, work crew 1 cannot be assigned to project 1, but if assigned to project 2 there is an associated cost of $8,000. Use a network flow approach to find a minimal cost feasible assignment.

Table 1: Crew-project assignment costs

		Projects			
		1	2	3	4
Crews	1	.	8	6	12
	2	15	12	7	.
	3	10	.	5	.
	4	12	.	12	16
	5	18	17	.	.
	6	.	13	.	14

(a) (10 points) Formulate the problem as a minimum cost network flow problem. Make sure your diagram is legible and all nodes and arcs are appropriately labeled.

(b) (10 points) Solve the problem using AMPL. Provide your data file, the minimal cost, and optimal solution.

Question 5: Mix Tape (32 points)

Back in the old days, people recorded music on a piece of technology known as a "cassette tape" (see Figure 1). Music could be stored on both sides of the device; generally referred to as Side A and Side B. The music storage amount is quite small with respect to current standards and is related to the length of tape held in the cassette device.

Figure 1: Ancient music storage device

Nostalgia Music, Inc. is in the process of releasing a cassette tape with 11 songs on it. The length of tape required for each song is given in the table below. The tape will have 2 sides, A and B. Some of the songs will be put on A and others on B. A song cannot be split across both sides A and B.

As an example, if the company puts all songs on Side A and leaves Side B blank, they would need 569 cm of tape. But if they put the first 10 songs on Side A, and only song 11 on Side B, then they would only need 532 cm of tape.

(a) (16 points) Provide a correct mathematical formulation to assign songs to each side of the tape so that the total length of the tape is minimized. Correct notation and clear definitions are required.

(b) (16 points) Solve the problem using AMPL and provide the optimal objective, solution, and your commented code in your submission.

Table 2: Tape length for songs

Song	Length (cm.)
1	44
2	67
3	37
4	54
5	79
6	56
7	35
8	37
9	53
10	70
11	37