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DSA 5113
Advanced Analytics & Metaheuristics
Group 11 - HW 2

Problem 1 – Golden Canning Co

a) Bollman compute the tomato costs in Table 3 by calculating the cost per pound of grade A and grade B tomatoes because she thinks that tomato cost should be based by not only quality but quantity as well. She reached the conclusion that the company should use 2,000,000 pounds “B” tomatoes for paste, the remaining 400,000 pounds of “B” tomatoes, and all of the “A” in juice because according to her marginal profit calculation, canned tomatoes does not bring profit. Her reasoning would be wrong in this case since for canned tomatoes grade “B” tomatoes is the standard so there is no need to use grade “A” tomatoes, especially if grade “A” tomatoes are more expensive.

b)

Problem 2 – Titan Enterprises Case Study

Assumptions:

- The return of investment is received in the beginning of the year and have to be invested to the available project or the bank within that year.
- Bank is divided into three different projects since each one depends on the amount of throw-off for that year.

The objective is to maximize the return investment at the end of year 2024

$$\sum_{\mathrm{p}}^{\mathrm{P}} r_{2024,p}\, x_p$$

Where P = {Bank_{2021}, Bank_{2022}, Bank_{2023}, A, B, C, D, E}

Decision Variables:

X_p : the amount of investments of project P

Constraints:

(Maximum investment of Project A) $X_A \leq 500{,}000$

(Maximum investment of Project B) $X_B \leq 500{,}000$

(Maximum investment of Project E) $X_E \leq 750{,}000$,

(initial investment on 2021)

$$x_{\mathrm{Bank}_{2021}} + x_{\mathrm{A}} + x_{\mathrm{C}} + x_{\mathrm{D}} = 1{,}000{,}000$$

(total investment on 2022)

$$\sum_{\mathrm{p}}^{\mathrm{P}} r_{2022,p}\, x_p = x_{\mathrm{B}} + x_{\mathrm{Bank}_{2022}}$$

(total investment on 2023)

$$\sum_{p}^{P} r_{2023,p}\, x_p \;= x_{\mathrm{E}} + x_{\mathrm{Bank}_{2023}}$$

(non-negative rate of returns) $r_{y,p} \geq 0$, where y is year, p is project

(non-negative investment) $X_p \geq 0$, where p is project

Below are the data file and mod file for part a, b, and c:

```
#sets----------------------------------------------------------------
#Bank investments are seperated since each one are used to balance any money
#   not invested in a given year
#Bank_2021 indicate the investment to the Bank on year 2021
#Bank_2022 indicate the investment to the Bank on year 2022
#Bank_2023 indicate the investment to the Bank on year 2023
#Other project A, B, C, D, and E
set PROJECT = BANK_2021 BANK_2022 BANK_2023 A B C D E;

#parameters----------------------------------------------------
#maximum investment for each project
param maxInvestment :=
    BANK_2021 Infinity
    BANK_2022 Infinity
    BANK_2023 Infinity
        A       500000
        B       500000
        C     Infinity
        D     Infinity
        E       750000;

#the return rate on year 2022
param secondYearRate :=
    BANK_2021 0.06
    BANK_2022 0.00
    BANK_2023 0.00
        A     0.30
        B     0.00
        C     1.10
        D     0.00
        E     0.00;

#the return rate on year 2023
param thirdYearRate :=
    BANK_2021 0.00
    BANK_2022 0.06
    BANK_2023 0.00
        A     1.00
        B     0.30
        C     0.00
        D     0.00
        E     0.00;

#the return rate on year 2024
param fourthYearRate :=
    BANK_2021 0.00
    BANK_2022 0.00
    BANK_2023 0.06
        A     0.00
        B     1.00
        C     0.00
        D     1.75
        E     1.40;

#the initial investment in 2021, which is a million dollar
param initalInvestment = 1000000;
```

```
set PROJECT; # set of the project investments (including Banks)

param maxInvestment{PROJECT}  >= 0; #maximum amount of investment for each project
param secondYearRate{PROJECT} >= 0; #the rate of return for each project in 2022
param thirdYearRate{PROJECT}  >= 0; #the rate of return for each project in 2023
param fourthYearRate{PROJECT} >= 0; #the rate of return for each project in 2024
param initalInvestment        >= 0; #the initial investment in 2021

#-----------------------Decision Variables-----------------------
var x{PROJECT} >=0; #total amount invested in each project

#-----------------------Objective Function-----------------------
#maximizing the total return in 2024
maximize totaReturnIn2024: sum {p in PROJECT} (fourthYearRate[p] * x[p]);

#-----------------------Constraints-----------------------
#The total of investment for available projects has to be equal to 1 million in 2021
subject to FirstYearInvestment: x['BANK_2021'] + x['A'] + x['C'] + x['D'] = initalInvestment;

#the throw-off from 2021 needs to equal the amount of project B + the bank
subject to SecondYearInvestmentReturn: sum {p in PROJECT} (secondYearRate[p] * x[p]) = x['B'] + x['BANK_2022'] ;

#the throw-off from 2022 needs to equal the amount of project E + the bank
subject to ThirdYearInvestmentReturn:  sum {p in PROJECT} (thirdYearRate[p] * x[p]) = x['E'] + x['BANK_2023'];

#Maximum amount that can be invested in a project
subject to projectMaxInvestment {p in PROJECT}: x[p] <= maxInvestment[p];

#-----------------------Data-----------------------
data Group11-HW2-p2.dat;

#-----------------------Commands-----------------------
#solve the LP problem
solve;
```

a) The amount of investment for each project with the maximum return of investment:

```
ampl: model Group11-HW2-p2a.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 1788000
3 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;


-----Solution for the Total investment of each project-----

 Investment amount for BANK_2021: $0.00
 Investment amount for BANK_2022: $0.00
 Investment amount for BANK_2023: $0.00
 Investment amount for         A: $500000.00
 Investment amount for         B: $150000.00
 Investment amount for         C: $0.00
 Investment amount for         D: $500000.00
 Investment amount for         E: $545000.00

Without the rate of return,
the total investment for all projects: $1695000.00

The total investment with the return rate at end of 2022: $150000.00

The total investment with the return rate at end of 2023: $545000.00

The total investment with the return rate at end of 2024: $1788000.00

The total amount made from all the return rates at end of 2024: $2483000.00
```

b) As we used the shadow price to check on the rate of returns that could change the investment on either direction (up or down), the shadow price can be used to determine the hurdle rates consistent with Titan's available investments by doing the shadow price analysis on the maximum investment of the project. Since the committee is using the 10% hurdle rate and with IRR (Internal Rate of Return) values for each project (A, B, C, D, and E) presented in Figure 1: Shirazi's email, all of the projects (A, B, D, and E) that was chosen to maximize the portfolio does have IRR > Hurdle rate value, which means that each project is accepted. It can be confirmed by looking at the shadow prices of the maximum available investment for each project and how it is zeroes or close to zero.

```
--------part b: Hurdle rate--------
The shadow prices for the maximum investment of each project:
# $3 = projectMaxInvestment.down
:         projectMaxInvestment projectMaxInvestment.up  $3    :=
A               0.076                    0             0
B               0                        0             0
BANK_2021       0                        0             0
BANK_2022       0                        0             0
BANK_2023       0                        0             0
C               0                        0             0
D               0                        0             0
E               0                        0             0
;
```

c) We can be determined how sensitive the investment decision is to change in projects' final payouts by checking on the project's shadow prices. By looking at the shadow price for project D, unless the price change to be greater than or equal to 1.826 or less than or equal to 1.562, the payout for project D will stay the same. The same goes for project E. Since 1.34 is less than 1.9697 and greater than 1.33, the payout for project E will also stay the same. Since neither project is greater than the upper bound or less than the lower bound, the portfolio stays the same (The amount of investment for each project does not change; Project D still has $500,000 and Project E has $545,000 of investment). The only thing that would change is the amount of maximum return of investment due to the change of rate of returns.

```
--------part c: Project's Final Payout--------
Each project's investment, current rate of returns, shadow prices and reduce cost:
:              x    x.current      x.up          x.down          x.rc        :=
A           5e+05    0         1e+20            -0.076         0
B          150000    1             1.17091        0.746667     5.55112e-17
BANK_2021       0    0             1.6648    -1e+20           -1.6648
BANK_2022       0    0             1.336     -1e+20           -1.336
BANK_2023       0    0.06          1.4       -1e+20           -1.34
C               0    0             0.188     -1e+20           -0.188
D           5e+05    1.75          1.826          1.562        0
E          545000    1.4           1.9697         1.33028      0
;
```

d) Running the data from Table 4 and adding into the existing data, it clears that adding Project F into the portfolio is more beneficial since it increases the maximum total return while adding Project G does not. Since Project G was not used, adding both projects will be ineffective since the portfolio will only use Project F to improve its maximum total return value.

Below is the result when adding only **Project F**:

```
ampl: model Group11-HW2-p2d.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 1792753.623
4 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;


-----Solution for the Total investment of each project-----

 Investment amount for BANK_2021: $0.00
 Investment amount for BANK_2022: $0.00
 Investment amount for BANK_2023: $0.00
 Investment amount for        A: $500000.00
 Investment amount for        B: $387681.16
 Investment amount for        C: $0.00
 Investment amount for        D: $202898.55
 Investment amount for        E: $750000.00
 Investment amount for        F: $297101.45
```

Below is the result when adding only **Project G**:

```
ampl: model Group11-HW2-p2d.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 1788000
3 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;


-----Solution for the Total investment of each project-----

 Investment amount for BANK_2021: $0.00
 Investment amount for BANK_2022: $0.00
 Investment amount for BANK_2023: $0.00
 Investment amount for         A: $500000.00
 Investment amount for         B: $150000.00
 Investment amount for         C: $0.00
 Investment amount for         D: $500000.00
 Investment amount for         E: $545000.00
 Investment amount for         G: $0.00
```

Below is the result when adding both projects (**Project F & Project G**):

```
ampl: model Group11-HW2-p2d.mod;
CPLEX 12.9.0.0: sensitivity
CPLEX 12.9.0.0: optimal solution; objective 1792753.623
4 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;


-----Solution for the Total investment of each project-----

 Investment amount for BANK_2021: $0.00
 Investment amount for BANK_2022: $0.00
 Investment amount for BANK_2023: $0.00
 Investment amount for         A: $500000.00
 Investment amount for         B: $387681.16
 Investment amount for         C: $0.00
 Investment amount for         D: $202898.55
 Investment amount for         E: $750000.00
 Investment amount for         F: $297101.45
 Investment amount for         G: $0.00
```

e) By assuming that Project F is available and original data of Project D and Project E, we get the result below:

```
CPLEX 12.9.0.0: optimal solution; objective 1792753.623
4 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

-----Solution for the Total investment of each project-----

 Investment amount for BANK_2021: $0.00
 Investment amount for BANK_2022: $0.00
 Investment amount for BANK_2023: $0.00
 Investment amount for         A: $500000.00
 Investment amount for         B: $387681.16
 Investment amount for         C: $0.00
 Investment amount for         D: $202898.55
 Investment amount for         E: $750000.00
 Investment amount for         F: $297101.45

Without the rate of return,
the total investment for all projects: $2137681.16

The total investment with the return rate at end of 2022: $387681.16

The total investment with the return rate at end of 2023: $750000.00

The total investment with the return rate at end of 2024: $1792753.62

The total amount made from all the return rates at end of 2024: $2930434.78

Each project's investment, current rate of returns, shadow prices and reduce cost:
:              x    x.current       x.up            x.down           x.rc
 :=
A           5e+05    0       1e+20              -0.0507246    0
B          387681    1          1.05263          0.98          5.55112e-17
BANK_2021       0    0          1.66522     -1e+20            -1.66522
BANK_2022       0    0          1.33043     -1e+20            -1.33043
BANK_2023       0    0.06       1.37681     -1e+20            -1.31681
C               0    0          0.195652    -1e+20            -0.195652
D          202899    1.75       1.766            1.6625        0
E          750000    1.4     1e+20               1.37681       0
F          297101    0          0.0321101       -0.016        -2.22045e-16
```

When we change Project E to pay only $1.34 per dollar invested, the portfolio changed. It does that because now project E is not using its maximum investment limit, which make the shadow price of project E to be much more sensitive than when it was at $1.40 per dollar invested (at $1.40 – x.up = 1+e20 while at $1.34 x.up = 1.37681). It makes sense why investment project F ended up being shifted to create better maximum return value for the portfolio. Even though the pay is lowered for project E, 1.34 return is still higher than 1.25 return from Project F. The result is shown below:

```
CPLEX 12.9.0.0: optimal solution; objective 1755300
3 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;


-----Solution for the Total investment of each project-----

 Investment amount for BANK_2021: $0.00
 Investment amount for BANK_2022: $0.00
 Investment amount for BANK_2023: $0.00
 Investment amount for         A: $500000.00
 Investment amount for         B: $150000.00
 Investment amount for         C: $0.00
 Investment amount for         D: $500000.00
 Investment amount for         E: $545000.00
 Investment amount for         F: $0.00

Without the rate of return,
the total investment for all projects: $1695000.00

The total investment with the return rate at end of 2022: $150000.00

The total investment with the return rate at end of 2023: $545000.00

The total investment with the return rate at end of 2024: $1755300.00

The total amount made from all the return rates at end of 2024: $2450300.00

Each project's investment, current rate of returns, shadow prices and reduce cost:
:             x    x.current      x.up          x.down          x.rc         :=
A           5e+05    0         1e+20            -0.0106       0
B          150000    1            1.03175         0.964667   -1.11022e-16
BANK_2021       0    0            1.66588    -1e+20           -1.66588
BANK_2022       0    0            1.3216     -1e+20           -1.3216
BANK_2023       0    0.06         1.34       -1e+20           -1.28
C               0    0            0.2078     -1e+20           -0.2078
D           5e+05    1.75         1.7606          1.7246      0
E          545000    1.34         1.37681         1.33028     0
F               0    0            0.0254     -1e+20           -0.0254
```

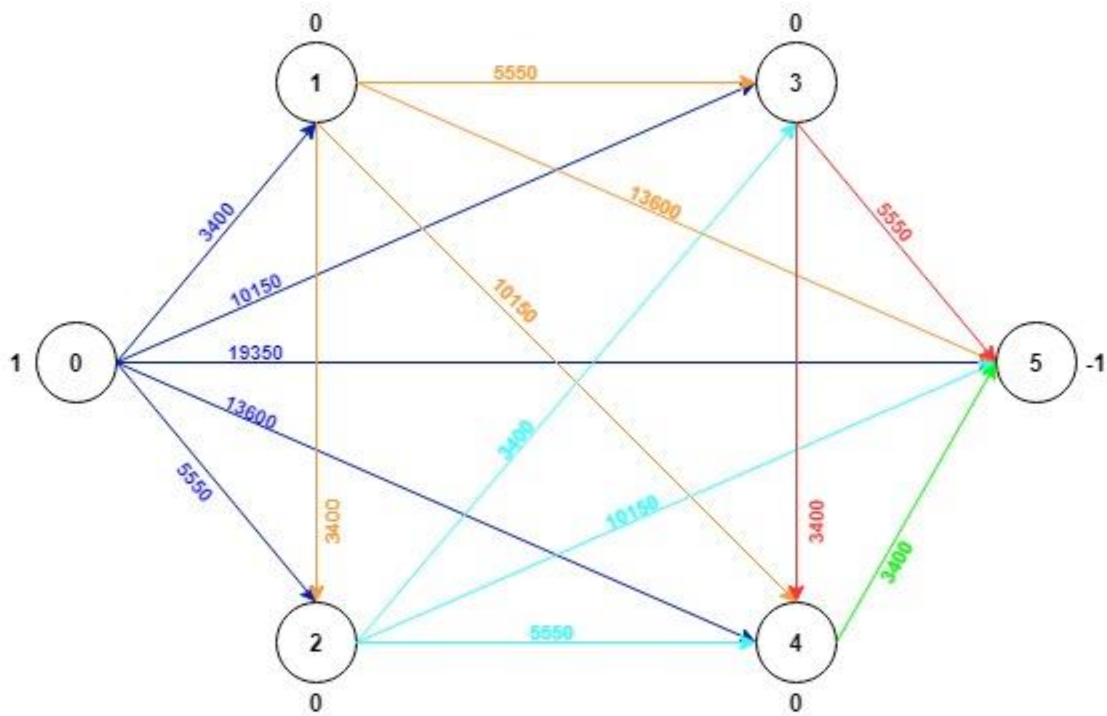
On the other hand, when we change Project D to pay only $1.70 per dollar invested, the portfolio does not change. Even though Project F was added, the shadow price of Project D remains the same and create insignificant shift of investment. The result is shown below:

```
CPLEX 12.9.0.0: optimal solution; objective 1782608.696
4 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

-----Solution for the Total investment of each project-----

 Investment amount for BANK_2021: $0.00
 Investment amount for BANK_2022: $0.00
 Investment amount for BANK_2023: $0.00
 Investment amount for         A: $500000.00
 Investment amount for         B: $387681.16
 Investment amount for         C: $0.00
 Investment amount for         D: $202898.55
 Investment amount for         E: $750000.00
 Investment amount for         F: $297101.45

Without the rate of return,
the total investment for all projects: $2137681.16

The total investment with the return rate at end of 2022: $387681.16

The total investment with the return rate at end of 2023: $750000.00

The total investment with the return rate at end of 2024: $1782608.70

The total amount made from all the return rates at end of 2024: $2920289.86

Each project's investment, current rate of returns, shadow prices and reduce cost:
:             x    x.current       x.up             x.down            x.rc
 :=
A           5e+05    0         1e+20               -0.0217391    0
B          387681    1            1.02256            0.9175      -1.11022e-16
BANK_2021       0    0            1.61652      -1e+20            -1.61652
BANK_2022       0    0            1.31304      -1e+20            -1.31304
BANK_2023       0    0.06         1.30435      -1e+20            -1.24435
C               0    0            0.169565     -1e+20            -0.169565
D          202899    1.7          1.766              1.6625      0
E          750000    1.4       1e+20                 1.30435     0
F          297101    0            0.0137615    -0.066            -1.11022e-16
```

Code files:
Group11-HW2-p2a.dat, Group11-HW2-p2a.mod,
Group11-HW2-p2d.dat, Group11-HW2-p2d.mod, and
Group11-HW2-p2e.dat, Group11-HW2-p2e.mod.

Problem 3 – Outdoor Grilling

Using the shortest-path problem, we have:



Each denote the age of the grill. The start node, Node 0, is the age of the grill when the grill is less than a year old (the first year when the grill was purchased). Since this is the shortest-path problem, we use b(0) as our supply/source node with a value of 1 and b(5) as our demand/terminal node with a value of -1. The other nodes in this case are transshipment nodes, which carry value of zeroes.

Each arc for this problem has costs depending on the net cost of the grill from time A to time B. From ***Table 5***, these net costs can be calculated as such:

Keeping the grill for one year:

cost of the grill + maintenance costs up to 0 year – eBay value at 1 year of age =

$7,600 + $800 - $5,000 = $3,400

Keeping the grill for two years:

cost of the grill + maintenance costs up to 1 year – eBay value at 2 year of age =

$7,600 + ($800 + $1,250) - $4,100 = $5,500

Keeping the grill for three years:

cost of the grill + maintenance costs up to 2 year – eBay value at 3 year of age =

$7,600 + ($800 + $1,250 + $2,000) - $1,500 = $10,150

Keeping the grill for four years:

cost of the grill + maintenance costs up to 3 year – eBay value at 4 year of age =

$7,600 + ($800 + $1,250 + $2,000 + $2,900) - $950 = $13,600

Keeping the grill for five years:

cost of the grill + maintenance costs up to 4 year – eBay value at 5 year of age =

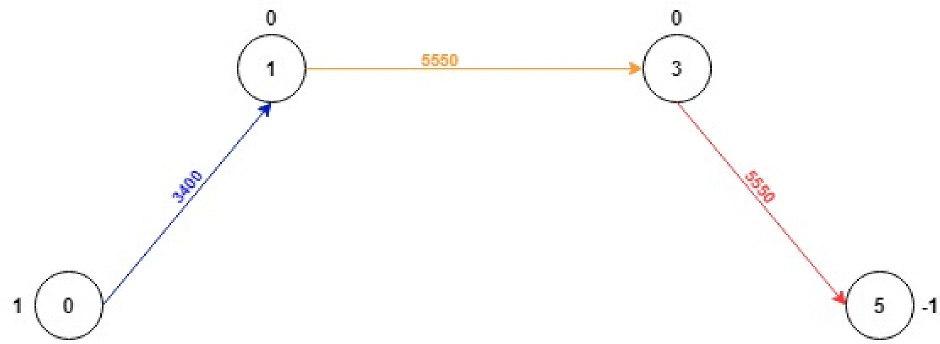
$7,600 + ($800 + $1,250 + $2,000 + $2,900 + $4,800) - $0 = $19,350

Using the data file below and the mcnfp model file,

```
#parameters and sets-----------------------------------------------
#Nodes for every age of the grill
#age 0 is for the first year when the grill was purchased
set NODES := 0 1 2 3 4 5;

#Arcs for each possible route each node can take
set ARCS :=
    (0,1) (0,2) (0,3) (0,4) (0,5)
    (1,2) (1,3) (1,4) (1,5)
    (2,3) (2,4) (2,5)
    (3,4) (3,5)
    (4,5);

#using the formulation of the shortest path problem
#age 0 as the source node (b = 1),
#age 5 as the terminal node (b = -1), and
#all other nodes are transshipment nodes (b = 0, which is the default value),
#so we have:
param b :=
    0    1
    5   -1;

#for the shortest path problem lower and upper bounds are set to default
#using table 5 information and calculating the cost
#where net cost,
#c = purchasing costs + maintenance costs - money received from eBay
#we have:
param c :=
        [0, *] 1 3400   2 5550   3 10150 4 13600 5 19350
        [1, *] 2 3400   3 5550   4 10150 5 13600
        [2, *] 3 3400   4 5550   5 10150
        [3, *] 4 3400   5 5550
        [4, *] 5 3400;

#cost of keeping the grill for:
#1  year  - $ 3,400
#2  years - $ 5,550
#3  years - $10,150
#4  years - $13,600
#5  years - $19,350
```

The minimum cost for having the grill for the next five years is $14,500. This can be done by buying the grill after a year at first and then buying a new grill every two years after that.

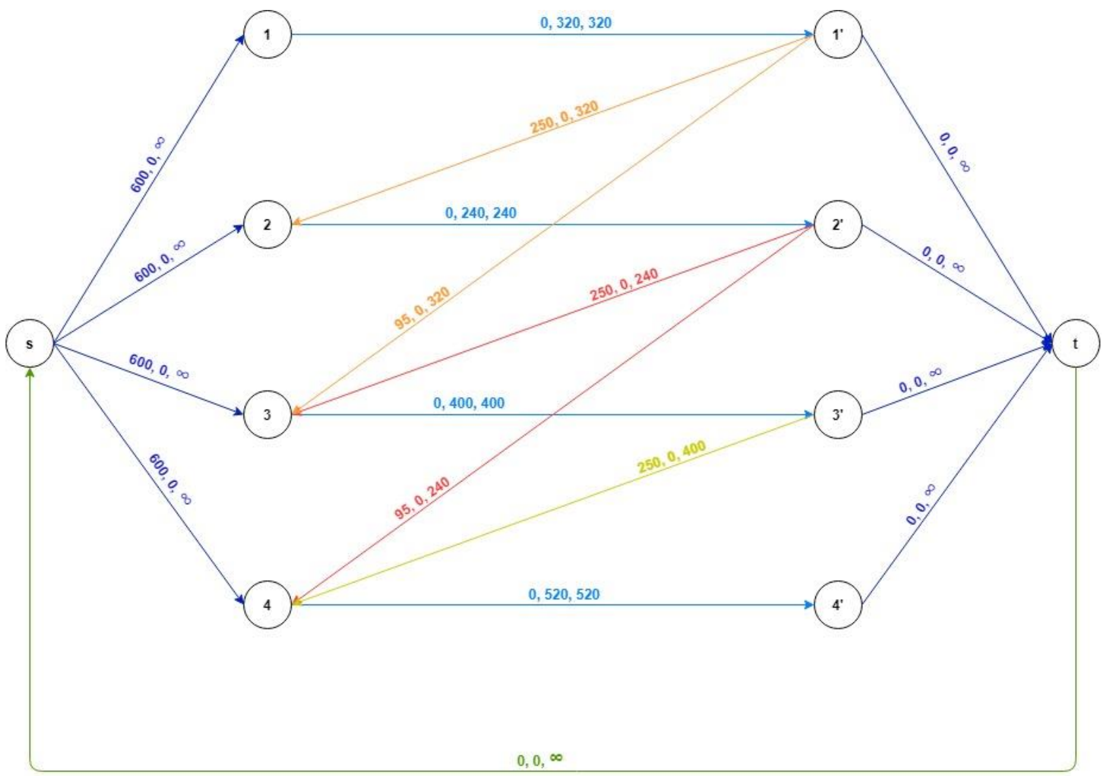
```
The minimum cost for the grill is $14500.00
The path taken to minimize the cost:
x :=
0 1   1
0 2   0
0 3   0
0 4   0
0 5   0
1 2   0
1 3   1
1 4   0
1 5   0
2 3   0
2 4   0
2 5   0
3 4   0
3 5   1
4 5   0
;
```



Code files:
Group11-HW2-p3.dat, mcnfp.txt.

Problem 4 – Racecar Tires

Using the circulation formulation, we have:



Assumptions:

- Each race day can use the tires from previous day but only when using the reshaping services.
- The tires that were reshaped after day 1 can be reshaped again after day 2 or day 3, the same goes for the tires that are used on other days of the race.
- The number of new tires that can be purchased each day is at most the total number needed for the race, which is 320, 240, 400, and 520 for day 1, 2, 3, and 4, respectively.
- No inventory of new tires from previous day.

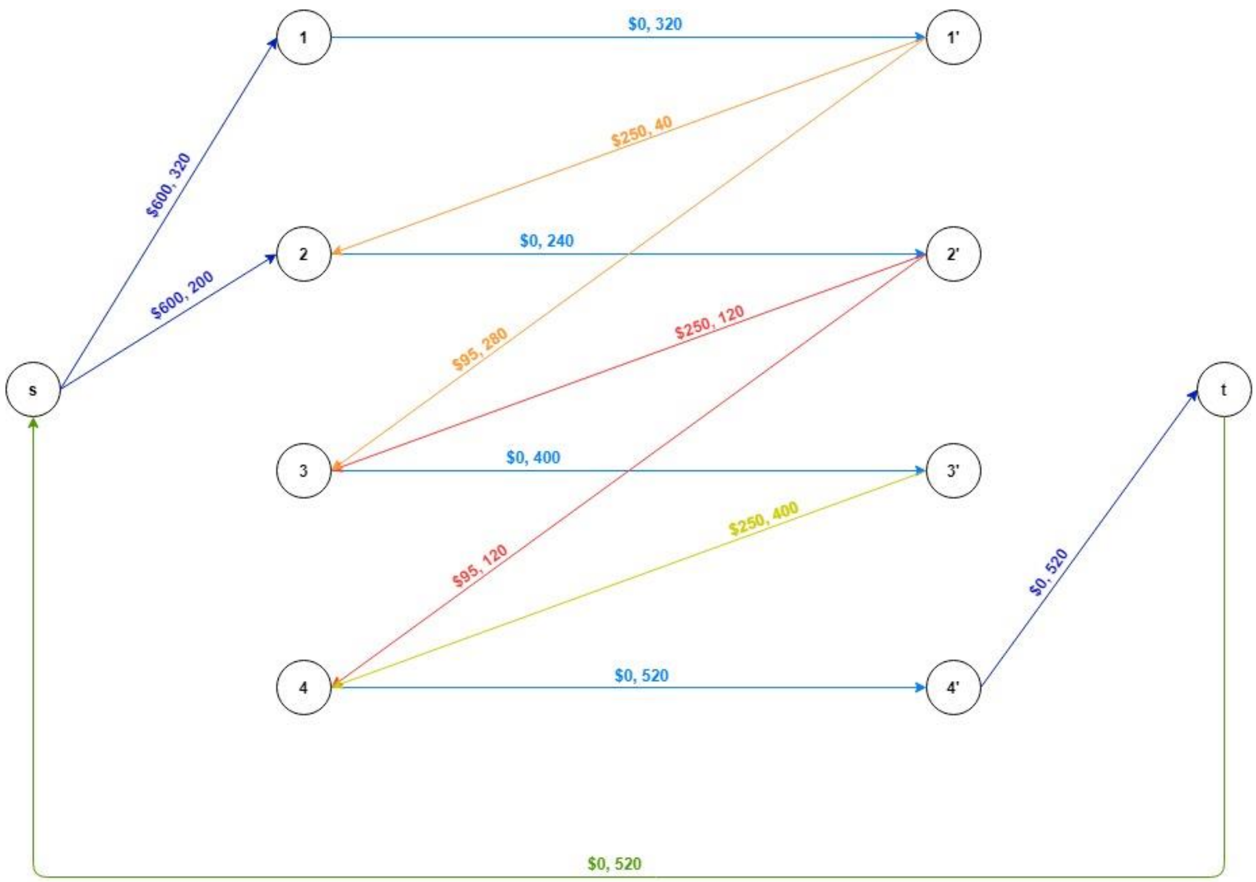
Using the data file below and the mcnfp model file,

```
#parameters and setssets---------------------------------------------
#Using the circulation formulation
#source node is where it started
#terminal node is the sink node to close the loop back to source node
#node, such as, 1p, is node split of NODE 1
#1p is for 1', 2p is for 2', and so on
#1 is for day 1, 2 is for day 2, and so on
#the prime days are used to get the demand and
#node to calculate reshaping services
set NODES:=  source 1 2 3 4 1p 2p 3p 4p  terminal;

# Arcs
set ARCS:=  (source, *) 1 2 3 4 #virtual source node
            #node split
            (1, 1p)
            (2, 2p)
            (3, 3p)
            (4, 4p)
            #quick and normal service
            (1p, *) 2 3 terminal
            (2p, *) 3 4 terminal
            (3p, *) 4 terminal
            #closed loop
            (4p, terminal)
            (terminal , source);

# c: new tire cost or normal or quick reshaping costs
param:  c l u :=
#source node
#buying new tires for each day of the race
[source,1]  600 . .
[source,2]  600 . .
[source,3]  600 . .
[source,4]  600 . .

#for node 1 and node 1'
[1,1p]    .  320 320 #given number of tires needed
[1p,2]  250   .  320 #for quick service
[1p,3]   95   .  320 #for normal service

#for node 2 and node 2'
[2,2p]    .  240 240 #given number of tires needed
[2p,3]  250   .  240 #for quick service
[2p,4]   95   .  240 #for normal service

#for node 3 and node 3'
[3,3p]    .  400 400 #given number of tires needed
[3p,4]  250   .  400 #for quick service

#for node 4
[4,4p]    .  520 520 #given number of tires needed

#close the loop
[terminal,source]  . . . ;
```

From the above formulation, the minimum cost for the race car tires comes out to be $490,000.

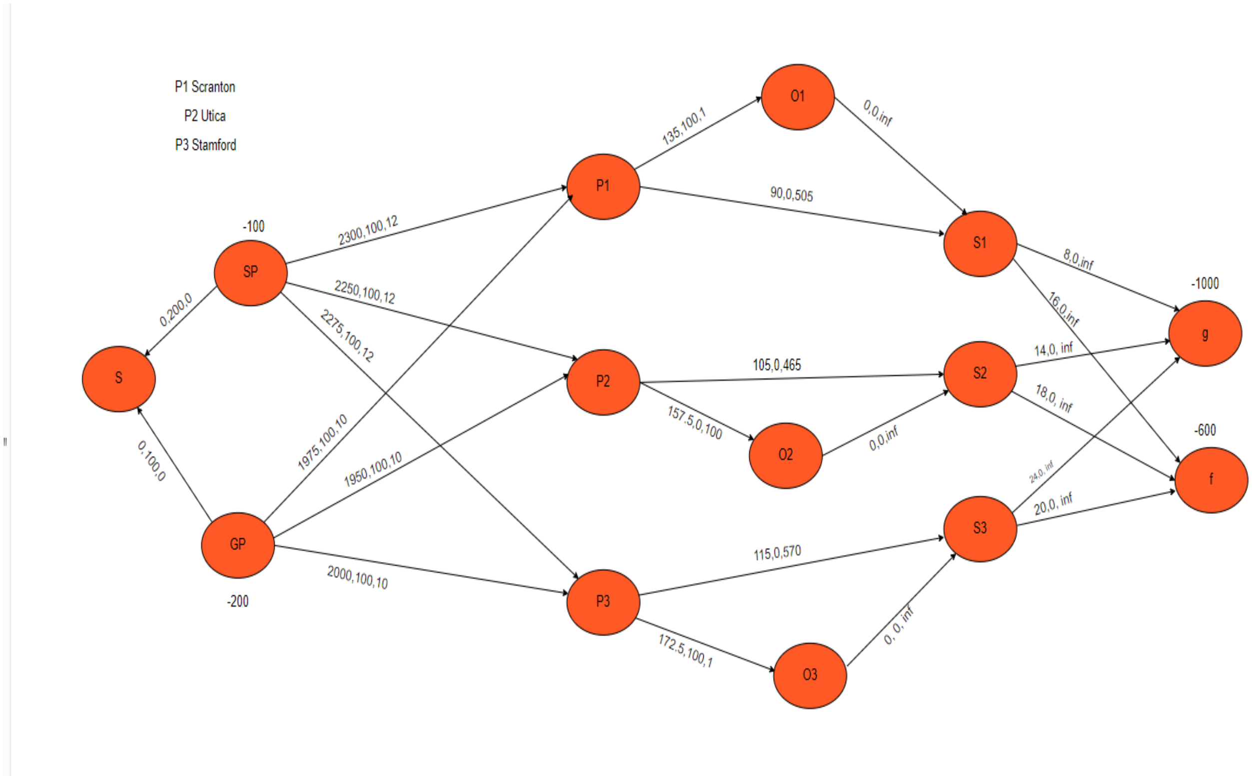
The association will need 320 new tires on day 1, where the 40 of those will be used for the second day of the race using the quick service and the 280 of those are used on the third day after the normal reshaping service. For the second day, 200 new tires need to be purchased. From the total of 240 tires used on that day, 120 will need to be quick service for day 3 and the other 120 will use normal service so it can be used on day 4. The 400 tires needed for day 3 will come from day 1 – 280 reshaped tires and day 2 – 120 reshaped tires. All those tires will then need to be reshaped again using quick service to be used on the final day of the race. On that final day, the 520 tires will all come from the reshaping service from day 3 (400 tires) and day 2 (120 tires).

```
ampl: model mcnfp.txt;
CPLEX 12.9.0.0: optimal solution; objective 490000
5 dual simplex iterations (0 in phase I)

The minimum cost for the race car tires: $490000.00
The path taken to minimize the cost:
x :=
1        1p        320
2        2p        240
3        3p        400
4        4p        520
1p       2          40
1p       3         280
1p       terminal    0
2p       3         120
2p       4         120
2p       terminal    0
3p       4         400
3p       terminal    0
4p       terminal  520
source   1         320
source   2         200
source   3           0
source   4           0
terminal source    520
;
```



Code files:
Group11-HW2-p4.dat, mcnfp.txt.

Problem 5 – Dunder Mifflin Paper Company



Code files:

Group11-HW2-p5.dat, gmcnfp.txt.