Lince Rumainum
Algorithm Analysis

HW#4

1. a) $T(n) = 3T(n/2) + 5n$, $n = 2^k$, $T(1) = 1$ ... ①

$T(\frac{n}{2}) = 3T(\frac{n}{4}) + 5(\frac{n}{2})$ ... ② plug in ② into ①

$T(n) = 3\left[3T(\frac{n}{2^2}) + 5(\frac{n}{2})\right] + 5n$ ...... ③

$T(\frac{n}{4}) = 3T(\frac{n}{8}) + 5(\frac{n}{4})$ ... ④ plug in ④ into ③

$$T(n) = 3\left[3\left[3T(\frac{n}{2^3}) + 5(\frac{n}{2^2})\right] + 5(\frac{n}{2})\right] + 5n$$

$$= 3\left[3^2T(\frac{n}{2^3}) + 3\cdot5(\frac{n}{2^2}) + 5(\frac{n}{2})\right] + 5n$$

$$= 3^3T(\frac{n}{2^3}) + 3^2\cdot5(\frac{n}{2^2}) + 3.5(\frac{n}{2}) + 5n$$

$$= 3^3T(\frac{n}{2^3}) + 5n(\frac{3^2}{2^2} + \frac{3}{2} + 1)$$

$$\therefore T(n) = 3^kT(\frac{n}{2^k}) + 5n\cdot\sum_{i=0}^{k-1}(\frac{3}{2})^i$$

NOTE $\sum_{i=1}^{n} x^i = \frac{x^{n+1}-1}{x-1}$ if $|x| > 1$

$$T(n) = 3^kT(\frac{n}{2^k}) + 5n\cdot\frac{(\frac{3}{2})^{(k-1)+1} - 1}{(\frac{3}{2}) - 1}$$

$$= 3^kT(\frac{n}{2^k}) + 5n\cdot2((\frac{3}{2})^k - 1)$$

We know $n = 2^k$, $T(1) = 1$

$$= 3^kT(\frac{2^k}{2^k}) + 5n\cdot2((\frac{3}{2})^k - 1)$$

$$= 3^k(1) + (10)(2^k)(2^{-k})(3^k) - 10n$$

$$= 11x - 10n$$

$$= 11n^{\log_2 3} - 10n$$

$\therefore T(n) = O(n^{\log_2 3})$

let $x = 3^k$

$n = 2^k \rightarrow \therefore k = \log_2 n$

$x = 3^{\log_2 n}$

$x = n^{\log_2 3}$

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1 b) $T(n) = 4T\left(\frac{n}{2}\right) + n \quad , n = 2^k \quad , T(1) = 1 \quad \cdots ①$

$T\left(\frac{n}{2}\right) = 4T\left(\frac{n}{4}\right) + \frac{n}{2} \quad \cdots ②$ plug in ② to ①

$T(n) = 4\left[4T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n \cdots ③$

$T\left(\frac{n}{4}\right) = 4\left[T\left(\frac{n}{8}\right) + \frac{n}{4}\right] \cdots ④$ plug in ④ to ③

$T(n) = 4\left[4\left[4T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + \frac{n}{2}\right] + n$

$= 4\left[4^2 T\left(\frac{n}{2^3}\right) + 4\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$

$= 4^3 T\left(\frac{n}{2^3}\right) + 4^2\left(\frac{n}{2^2}\right) + 4\left(\frac{n}{2}\right) + n$

$= 4^3 T\left(\frac{n}{2^3}\right) + n \sum_{i=0}^{k-1} \left(\frac{4}{2}\right)^i$

$\therefore,\ T(n) = 4^k T\left(\frac{n}{2^k}\right) + n \sum_{i=0}^{k-1} 2^i$ NOTE: $\sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}$ if $|x| > 1$

$= 4^k T\left(\frac{n}{2^k}\right) + n\left(\frac{2^{k-1+1} - 1}{2 - 1}\right)$

$= 4^k T\left(\frac{n}{2^k}\right) + n\left(2^k - 1\right)$

We know $n = 2^k$ and $T(1) = 1$

$= 4^k T\left(\frac{2^k}{2^k}\right) + n \cdot 2^k - n$

$= 4^k(1) + n(n) - n = (2^k)^2 + n^2 - n$

$= n^2 + n^2 - n$

$\therefore,\ T(n) = O(n^2)$

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1. c) $T(n) = T(\sqrt{n}) + 1$, $n = 2^{2^k}$, $T(2) = 1$ ...... ①

$T(\sqrt{n}) = T((n^{1/2})^{1/2}) + 1$

$= T(n^{1/4}) + 1$ ...... ② plug in ② into ①

$T(n) = T(n^{1/4}) + 2$ ...... ③

$T(n^{1/4}) = T(n^{1/8}) + 1$ ...... ④ plug in ④ into ③

$T(n) = T(n^{1/8}) + 3$ $\quad 8 = 2^3$

$\therefore T(n) = T(n^{1/2^k}) + k$

We know that $n = 2^{2^k}$ and $T(2) = 1$

$T(n) = T\left(\left(2^{2^k}\right)^{\frac{1}{2^k}}\right) + k$

$= T(2) + k = 1 + k$

$= 1 + \log_2(\log_2 n)$

$\therefore T(n) = O(\log_2^{(2)} n)$

find k:

$n = 2^{2^k}$

$\log_2 n = 2^k$

$k = \log_2(\log_2 n)$

2. a) If $10 = 2^x$

$\log_2 10 = \log_2 2^x$

$x = \log_2 10$

$x = 3.32$

$Y = 2^{2^{2^{2^2}}} = (2^{2^2})^4$

$= (2^2)^{16}$

$Y = 2^{65536}$

b) $Y = 2^{2^{2^{2^2}}}$

$\log_2 Y = 2^{2^{2^2}}$

$\log_2^{(2)} Y = 2^{2^2}$

$\log_2^{(3)} Y = 2^2$

$\log_2^{(4)} Y = 2$

$\log_2^{(5)} Y = 1$ $\quad \therefore k = 5$

c) $\log_2^* n$ where $n = 10^{12}$

$\log_2 n = 12 \log_2 10 = 39.84$

$\log_2^{(2)} n = \log_2(39.84) = 5.32$

$\log_2^{(3)} n = \log_2(5.32) = 2.41$

$\to \log_2^{(4)} n = \log_2(2.41) = 1.27$

$\log_2^{(5)} n = \log_2(1.27) = 0.35$

$\therefore * = 5$