Lince Rumarnum
Algorithm Analysis

HW#8

1. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

Multiplying A and B matrix will give us matrix C, which is:

$C = AB$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

To get C_{11}, C_{12}, C_{21}, and C_{22} we need to compute the following first:

$X_1 = (a_{11} + a_{22}) \times (b_{11} + b_{22})$

$X_2 = (a_{21} + a_{22}) \times b_{11}$

$X_3 = a_{11} \times (b_{12} - b_{22})$

$X_4 = a_{22} \times (b_{21} - b_{11})$

$X_5 = (a_{11} + a_{12}) \times b_{22}$

$X_6 = (a_{21} - a_{11}) \times (b_{11} + b_{12})$

$X_7 = (a_{12} - a_{22}) \times (b_{21} + b_{22})$

And then we get $C_{11}, C_{12}, C_{21}, C_{22}$ and check if the result is the same as standard method:

$C_{11} = X_1 + X_4 - X_5 + X_7$ (Strassen's method)

$= [(a_{11} + a_{22}) \times (b_{11} + b_{22})] + [a_{22} \times (b_{21} - b_{11})] - [(a_{11} + a_{12}) \times b_{22}] + [(a_{12} - a_{22}) \times (b_{21} + b_{22})]$

$= a_{11}b_{11} + a_{11}b_{22} + a_{22}b_{11} + a_{22}b_{22} + a_{22}b_{21} - a_{22}b_{11} - a_{11}b_{22} - a_{12}b_{22} + a_{12}b_{21} + a_{12}b_{22} - a_{22}b_{21} - a_{22}b_{22}$

$C_{11} = a_{11}b_{11} + a_{12}b_{21}$ (standard method)

$C_{12} = X_3 + X_5$ (Strassen's method)

$= [a_{11} \times (b_{12} - b_{22})] + [(a_{11} + a_{12}) \times b_{22}] = a_{11}b_{12} - a_{11}b_{22} + a_{11}b_{22} + a_{12}b_{22}$

$C_{12} = a_{11}b_{12} + a_{12}b_{22}$ (Standard method

$C_{21} = X_2 + X_4$ (Strassen's method)

$= [(a_{21} + a_{22}) \times b_{11}] + [a_{22} \times (b_{21} - b_{11})] = a_{21}b_{11} + a_{22}b_{11} + a_{22}b_{21} - a_{22}b_{11}$

$C_{21} = a_{21}b_{11} + a_{22}b_{21}$ (standard method

(Strassen's method

$C_{22} = X_1 + X_3 - X_2 + X_6 = [(a_{11} + a_{22}) \times (b_{11} + b_{22})] + [a_{11} \times (b_{12} - b_{22})] - [(a_{21} + a_{22}) \times b_{11}] + [(a_{21} - a_{11}) \times (b_{11} + b_{12})]$

$= a_{11}b_{11} + a_{11}b_{22} + a_{22}b_{11} + a_{22}b_{22} + a_{11}b_{12} - a_{11}b_{22} - a_{21}b_{11} - a_{22}b_{11} + a_{21}b_{11} + a_{21}b_{12} - a_{11}b_{11} - a_{11}b_{12}$

$C_{22} = a_{21}b_{12} + a_{22}b_{22}$ (standard method

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2. $T(n) = 7T(n/2) + n^2$ when $n = 2^k$, $T(1) = 1$ ... ①

$T(\frac{n}{2}) = 7T(n/4) + \frac{n^2}{4}$ ... ② plug in ② into ①

$T(n) = 7(7T(n/4) + \frac{n^2}{4}) + n^2$ ... ③

$T(\frac{n}{4}) = 7T(n/8) + \frac{n^2}{16}$ ... ④ plug in ④ to ③

$T(n) = 7[7(7T(n/8) + \frac{n^2}{16}) + \frac{n^2}{4}] + n^2$

$= 7[7^2 T(n/8) + 7\frac{n^2}{16} + \frac{n^2}{4}] + n^2$

$= 7^3 T(\frac{n}{2^3}) + 7^2 \frac{n^2}{16} + 7\frac{n^2}{4} + n^2$

Note: $\sum_{i=1}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}$ if $|x| > 1$

$= 7^k T(\frac{n}{2^k}) + n^2 \sum_{i=0}^{k-1} (\frac{7}{4})^i$

$= 7^k T(\frac{n}{2^k}) + n^2 \frac{(\frac{7}{4})^{k-1+1} - 1}{\frac{7}{4} - 1}$

$= 7^k T(\frac{n}{2^k}) + \frac{4}{3} n^2 [(\frac{7}{4})^k - 1]$

we know $n = 2^k$ and $T(1) = 1$

$= 7^k T(\frac{2^k}{2^k}) + \frac{4}{3}(2^k)^2(\frac{7}{4})^k - \frac{4}{3} n^2$

$= 7^k T(1) + \frac{4}{3}(2^{2k})(2^{-2k})7^k - \frac{4}{3} n^2$

$= 7^k (1) + \frac{4}{3}(7^k - n^2)$; let $x = 7^k$

$= (\frac{7}{3}) x - \frac{4}{3} n^2$; we know $n = 2^k \rightarrow k = \log_2 n$

$= \frac{7}{3} n^{\log_2 7} - \frac{4}{3} n^2$; $x = 7^{\log_2 n} = n^{\log_2 7}$

$\therefore, T(n) = O(n^{\log_2 7}) = O(n^{2.81})$

3. $M(n) = 7M(n/2)$ where $M(1) = 1$ ... ①

$M(n/2) = 7M(n/4)$ ... ② plug in ② into ①

$M(n) = 7(7M(n/4))$ ... ③

$M(n/4) = 7M(n/8)$ ... ④ plug in ④ into ③

$M(n) = 7[7(7M(n/2^3))]$

$= 7^3 M(\frac{n}{2^3})$

$= 7^k M(\frac{n}{2^k})$

we know $M(1) = 1$, if we have $n = 2^k$

$= 7^k M(\frac{2^k}{2^k}) = 7^k M(1)$; Since $n = 2^k$

$= 7^k (1) = 7^k$; $k = \log_2 n$

$= 7^{\log_2 n}$

$= n^{\log_2 7}$

$M(n) = O(n^{\log_2 7}) = O(n^{2.81})$