Algorithm Analysis

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HW#3
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$$\int_{x}^{n} \log_{e} x \, dx = \int (u \, dv - v \, du) = u \int dv - v \int du$$

$$u = \log_{e} x$$

$$du = \frac{1}{x} \, dx$$

$$= (\log_{e} x \cdot x) \cdot (x \cdot x)$$

$$= (\log_{e} x \cdot x) \cdot (x \cdot x)$$

$$= n \cdot \log_{e} n - \log_{e} 1 - (n - 1)$$

$$dv = dx$$

$$= n \cdot \log_{e} n - n + 1$$

$$\frac{2}{2} = \frac{1}{2} = \frac{1}$$

$$\frac{1}{2} = \frac{1}{2} = \int_{1}^{n} \frac{dx}{x^{2}} \rightarrow \int_{1}^{n} \frac{1}{x^{2}} dx = -\frac{1}{x} \int_{1}^{n} \frac{1}{x^{2}} dx = -\frac{1}{n} - \frac{1}{n} - \frac{1}{n} = 1 - \frac{1}{n}$$

$$= 1 - \frac{1}{n} = 1$$
i., As $n \rightarrow \infty$, $1 - \frac{1}{\infty} = 1$

4. a)
$$T(n) = T(n-1) + (n-1)$$
 where $T(2) = 1$ -- (r)

$$T(n-1) = T(n-1-1) + (n-1-1)$$

$$T(n-1) = T(n-2) + (n-2)$$

$$T(n-2) = T(n-2-1) + (n-2-1)$$

= $T(n-3) + (n-3)$

:, Plug into eq. (2)
$$T(n) = [[T(n-3)+(n-3)]+(n-2)] + (n-1)$$

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$$T(n) = T(n-3) + (n-3) + (n-2) + (n-1)$$

= $T(n-3) + \sum_{i=1}^{n-3} (n-i)$
 $T(n) = T(n-k) + \sum_{i=1}^{k} (n-i)$

We know
$$T(2) = 1$$

$$n - k = 2 \rightarrow k = n - 2$$

$$T(n) = T(n - (n-2)) + \sum_{i=1}^{n-2} (n-i)$$

$$= T(2) + (n-1) + (n-2) + ... + (n-(n-2))$$

$$= 1 + (n-1) + (n-2) + ... + 2$$

$$\vdots$$

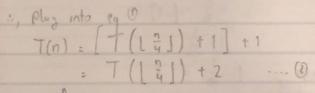
$$\vdots$$

$$i = (n-1) + (n-2) + ... + 1 = \frac{n(n-1)}{2}$$

b)
$$T(n) = T\left(\lfloor \frac{n}{2} \rfloor\right) + 1$$
 where $T(1) = 1$ and $n = 2^k - 1 - 0$

on $= \frac{n}{2}$

$$T\left(\frac{n}{2}\right) = T\left(\lfloor \frac{n}{4} \rfloor\right) + 1$$



$$Q n = \frac{1}{4}$$

$$T(\frac{n}{4}) = T(\lfloor \frac{n}{8} \rfloor) + 1$$

.., plug into eq (3)
$$T(n) = [T(\lfloor \frac{n}{8} \rfloor) + 1] + 2$$

$$= T(\lfloor \frac{n}{8} \rfloor) + 3 - - 3$$
We know $T(1) = 1$ and $n = 2^{k} - 1$

From
$$\mathfrak{C}$$
, \mathfrak{D} , \mathfrak{D} we get $T(n) = T\left(\left\lfloor \frac{n}{2^k} \right\rfloor\right) + k$

$$T(2^k) = T(\frac{2^k}{2^k}) + k = T(1) + k = 1 + k$$

where
$$k = log_2 n + 1$$
 = 1 + log_2 n $k \approx log_2 n$