

Homework 2

Please show all relevant work when you upload the assignment.

2. (Wealth independence) Suppose an investor has exponential utility function $U(x) = -e^{-ax}$ and an initial wealth level of W . The investor is faced with an opportunity to invest an amount $w \leq W$ and obtain a random payoff x . Show that his evaluation of this incremental investment is independent of W .

4. (Relative risk aversion) The Arrow–Pratt relative risk aversion coefficient is

$$\mu(x) = \frac{xU''(x)}{U'(x)}.$$

Show that the utility functions $U(x) = \ln x$ and $U(x) = \gamma x^\gamma$ have constant relative risk aversion coefficients

6. (HARA \diamond) The HARA (for hyperbolic absolute risk aversion) class of utility functions is defined by

$$U(x) = \frac{1-\gamma}{\gamma} \left(\frac{ax}{1-\gamma} + b \right)^\gamma, \quad b > 0.$$

The functions are defined for those values of x where the term in parentheses is nonnegative. Show how the parameters γ , a , and b can be chosen to obtain the following special cases (or an equivalent form).

- (a) Linear or risk neutral: $U(x) = x$
- (b) Quadratic: $U(x) = x - \frac{1}{2}cx^2$
- (c) Exponential: $U(x) = -e^{-ax}$ [Try $\gamma = -\infty$]
- (d) Power: $U(x) = cx^\gamma$
- (e) Logarithmic: $U(x) = \ln x$ [Try $U(x) = (1-\gamma)^{1-\gamma}((x^\gamma - 1)/\gamma)$]

Show that the Arrow–Pratt risk aversion coefficient is of the form $1/(cx + d)$