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HW5

Problem 3

Current stock price, $S_0 = 50$

expected return, $\mu = 0.12$

volatility, $\sigma = 0.30$

probability, p, that $S_T > 80$ in two years?

Hint: $S_T > 80$ when $\ln S_T > \ln 80$

$\therefore$, $\ln S_T > 4.3820$

$T = 2$, $K = 80$

$$p = 1 - N\left(\frac{\ln K - (\ln S_T + (\mu - \sigma^2/2)T)}{\sigma\sqrt{T}}\right)$$

$$= 1 - N\left(\frac{\ln 80 - (\ln 50 + (0.12 - 0.30^2/2)2)}{0.30\sqrt{2}}\right)$$

$$= 1 - N(0.754)$$

$$= 1 - 0.7746$$

$$p = 0.2254$$

Find $N(0.754)$:

$$\frac{0.754 - 0.750}{x - 0.7734} = \frac{0.760 - 0.754}{0.7764 - x}$$

$$0.004(0.7764 - x) = 0.006(x - 0.7734)$$

$$-0.01x = -0.007746$$

$$x = 0.7746$$

Problem 4

Consider a variable S, that follows the process

$$dS = \mu\, dt + \sigma\, dz$$

The first three years, $\mu = 2$ and $\sigma = 3$, the probability distribution of the value of the variable,

$$\begin{aligned}\frac{\Delta S}{S} &\approx \phi\left(\mu \Delta t, \sigma^2 \Delta t\right) \\ &\approx \phi\left(2(3), 3^2(3)\right) \\ &\approx \phi\left(6, 27\right)\end{aligned}$$

For the next three years, $\mu = 3$ and $\sigma = 4$, the probability distribution of the value of the variable,

$$\begin{aligned}\frac{\Delta S}{S} &\approx \phi\left(\mu \Delta t, \sigma^2 \Delta t\right) \\ &\approx \phi\left(3(3), 4^2(3)\right) \\ &\approx \phi\left(9, 48\right)\end{aligned}$$

Initial value of the variable is 5, the probability distribution of the value of the variable at the end of year six:

$$\begin{aligned}\frac{\Delta S}{S} &\approx \phi\left(5+6+9, 27+48\right) \\ &\approx \phi\left(20, 75\right)\end{aligned}$$

Problem 5

expected return, $\mu = 0.16 = r$

volatility, $\sigma = 0.35$

current price, $S_0 = 38$

a) $\ln S_T > \ln 40 \therefore$, $\ln S_T > 3.6889$ for European call option probability

$T = 0.5$, $K = 40$

$$p = N(d_2) = N\left(\frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$= N\left(\frac{\ln(38/40) + (0.16 - 0.35^2/2)0.5}{0.35(\sqrt{0.5})}\right)$$

$$= N\left(\frac{-0.001918}{0.24749}\right)$$

$$= N(-0.007750)$$

$$p = 0.4969$$

b) For European put option probability, it would be 1 - the probability of the European call option

$$\therefore p = 1 - 0.4969$$

$$= 0.5031$$

Find $N(-0.007750)$:

$$\frac{-0.007750 - 0.00}{x - 0.500} = \frac{-0.01 - (-0.007750)}{0.4960 - x}$$

$$-0.007750(0.4960 - x) = -0.00225(x - 0.500)$$

$$x = 0.4969$$