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HW 6

Chapter 5, Problem 5

1. $X_1 = (2.5, 1)^T$, $X_2 = (3.5, 4)^T$, $X_3 = (2, 2.1)^T$

a) $\sigma^2 = 5$ $\quad K(x_i, x_j) = \exp\left\{-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right\}$

$K(X_1, X_1) = \exp(0) = 1.0$

$K(X_1, X_2) = \exp\left\{-\frac{\|(-1.0, -3)^T\|^2}{2(5)}\right\} = \exp\left(-\frac{1^2+3^2}{10}\right) = \cancel{2.718}\ 0.3679$

$K(X_1, X_3) = \exp\left\{-\frac{\|(0.5, -1.1)^T\|^2}{2(5)}\right\} = \exp\left(-\frac{0.5^2+1.1^2}{10}\right) = 0.8779$

$K(X_2, X_1) = K(X_1, X_2) = 0.3679$

$K(X_2, X_2) = \exp(0) = 1.0$

$K(X_2, X_3) = \exp\left\{-\frac{\|(1.5, 1.9)^T\|^2}{2(5)}\right\} = \exp\left(-\frac{1.5^2+1.9^2}{10}\right) = 0.5565$

$K(X_3, X_1) = K(X_1, X_3) = 0.8779$

$K(X_3, X_2) = K(X_2, X_3) = 0.5565$

$K(X_3, X_3) = \exp(0) = 1.0$

$$K = \begin{bmatrix} 1.0000 & 0.3679 & 0.8779 \\ 0.3679 & 1.0000 & 0.5565 \\ 0.8779 & 0.5565 & 1.0000 \end{bmatrix}$$

b) the mean, $\|\mu_\phi\|^2 = \frac{1}{n^2} \sum_{i=1}^{3} \sum_{j=1}^{3} K(x_i, x_j)$

$$= \frac{6.6046}{9} = 0.7338$$

1. (continue)

The distance of the point $\phi(x_1)$ from the mean in Feature Space

$$\| \phi(x_1) - \mu_\phi \|^2 = K(x_1, x_1) - \frac{2}{3} \sum_{j=1}^{3} K(x_1, x_j) + \| \mu_\phi \|^2$$

$$= 1.0000 - \frac{2}{3}(1 + 0.3679 + 0.8779) + 0.7338$$

$$= 0.2336$$

c.) $\det(K - I\lambda) = \det \begin{pmatrix} 1-\lambda & 0.3679 & 0.8779 \\ 0.3679 & 1-\lambda & 0.5565 \\ 0.8779 & 0.5565 & 1-\lambda \end{pmatrix}$

$$= (1-\lambda) \det \begin{pmatrix} 1-\lambda & 0.5565 \\ 0.5565 & 1-\lambda \end{pmatrix} - (0.3679) \begin{pmatrix} 0.3679 & 0.5565 \\ 0.8779 & 1-\lambda \end{pmatrix} + (0.8779) \begin{pmatrix} 0.3679 & 1-\lambda \\ 0.8779 & 0.5565 \end{pmatrix}$$

$$= (1-\lambda)(\lambda^2 - 2\lambda + 0.6903) - (0.3679)(-0.3679\lambda - 0.12065) + (0.8779)(0.8779\lambda - 0.67316)$$

$$= -\lambda^3 + 3\lambda^2 + 1.7842\lambda + 0.1437$$

$\therefore$, $\lambda = 2.228$, $\lambda = 0.6765$, $\lambda = 0.09535$

2. Chapter 21, Problem 1

a) Equations for the two hyper planes h_1 and h_2:

$h(x) = w^T x + b = w_1 x_1 + w_2 x_2 + b = 0$

$p_1 = (4,4)$ $\qquad$ $p_2 = (2,0)$

$q_1 = (2,8)$ $\qquad$ $q_2 = (5,5)$

$\text{slope}_1 = -\frac{w_1}{w_2} = \frac{8-4}{2-4} = -2$ $\qquad$ $\text{slope}_2 = -\frac{w_1}{w_2} = \frac{5-0}{5-2} = +\frac{5}{3}$

$b_1 = -4x_1 - 2x_2$ $\qquad$ $b_2 = -5x_1 - 3x_2$

$b_2 = -5(2) - 3(0) = -5(2) - 0 = -10$

$b_1 = -4(4) - 2(4) = -24$

$h_1(x) = (4 \quad 2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - 24 = 0$ $\qquad$ $h_2(x) = (5 \quad 3)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - 10 = 0$

b) Support vectors for h_1 and h_2

For h_1: $\begin{pmatrix} 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

for h_2: $\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 7 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5/2 \end{pmatrix}$

c) $\delta^* = \min_{x_i} \left\{ \frac{y^* (w^T x^* + b)}{\|w\|} \right\}$

$\delta_1^* = \frac{(-1)(4 \ 2)\begin{pmatrix} 2 \\ 6 \end{pmatrix} - 24}{\sqrt{4^2 + 2^2}}$ $\qquad$ $\delta_2^* = \frac{(-1)(5 \ 3)\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 10}{\sqrt{5^2 + 3^2}}$

$\delta_1^* = \frac{-(8+12) - 24}{\sqrt{20}} = -9.8388 \leq 0$ $\qquad$ $\delta_2^* = \frac{-(5+3) - 10}{\sqrt{34}} = -3.0870 \leq 0$

if $y_i^* = 1$ $\qquad$ if $y_i^* = 1$

$\delta_1^* = \frac{20 - 24}{\sqrt{20}} = -0.89$ $\qquad$ $\delta_2^* = \frac{8-10}{\sqrt{34}} = 0.34$

$\therefore$, h_1 with $y^* = 1$