Lince Rumainum
DSA5303
HW#7

Problem 3

B-rated corporate bond] - same coupon, time to maturity, and price.
CAT bond] expected losses in each year of their life is the same.

I would advise a portfolio manager to buy CAT bond because CAT bond can be assumed to have zero systematic risk. Since CAT bond ~~does~~ is not exposed to market-related risk, ~~it can be calculated~~ the expected payoff & discount can be calculated at risk-free rate. Therefore, CAT bond would diversify the portfolio better than B-rated corporate bond would even though they both have the same price and time to maturity with the same expected losses.

Problem 4

show that, in traditional risk-neutral world:

commodity with constant volatility σ; $\ln S_T \sim \phi\left[\ln F(T) - \frac{\sigma^2 T}{2}, \sigma^2 T\right]$

S_T - the value of commodity at time T, $F(t)$ is the futures price at time 0 for a contract maturing at time t, and $\varphi(m,v)$ is a normal distribution with mean m and variance v.

$$d \ln S = \left(\mu(t) - \frac{\sigma^2}{2}\right) dt + \sigma\, dz^{\to 0}$$

$$= \ln S_0 + \int_0^T \mu(t)\, dt - \int_0^T \frac{\sigma^2}{2}\, dt$$

NOTE:

$\mu(t) = \frac{\partial}{\partial t}\left[\ln F(t)\right]$

$$= \ln S_0 + \ln F(T) - \ln F(0) - \frac{\sigma^2 T}{2}$$

$$= \ln S_0 + \ln F(T) - \ln S_0 - \frac{\sigma^2 T}{2}$$

$$= \ln F(T) - \frac{\sigma^2 T}{2}$$ and standard deviation $\sigma\sqrt{T}$

$$\therefore \ln S_T \sim \phi\left[\ln F(T) - \frac{\sigma^2 T}{2}, \sigma^2 T\right]$$

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Problem 5

$N(\mu, \sigma) = \phi(150, 50)$ insurance company's losses

Assume no difference between losses in a risk-neutral world and losses in the real-world

The 1-year risk-free rate is 5%.

a) A contract that will pay in 1 year's time 60% of the insurance company's losses on a pro-rata basis

$$\phi(150 \times 0.60, 50 \times 0.60)$$
$$\phi(90, 30)$$

$\therefore$ the cost of reinsurance μe^{-rt} $90 \times e^{-0.05 \times 1} = 85.611$ millions

b) A contract that pays \$100 million in 1 year's time if losses exceed \$200 million

$\mu = 150$ million

$P(Z > 1) = P(Z < -1) = 0.1587$

$\therefore$ cost of reinsurance $= 0.1587 \times 100 \times e^{-rt}$

$= 15.87 \times e^{-0.05 \times 1}$

$= 15.096$ million