DSA 5303
Summer 2018

Homework 3

Please show all relevant work when you upload the assignment.

8. (General tilting ◦) A general model for information about expected returns can be expressed in vector–matrix form as

$$\mathbf{p} = \mathbf{P}\bar{\mathbf{r}} + \mathbf{e}.$$

In the model $\mathbf{P}$ is an $m \times n$ matrix, $\bar{\mathbf{r}}$ is an n-dimensional vector, and $\mathbf{p}$ and $\mathbf{e}$ are m-dimensional vectors. The vector $\mathbf{p}$ is a set of observation values and $\mathbf{e}$ is a vector of errors having zero mean. The error vector has a covariance matrix $\mathbf{Q}$. The best (minimum-variance) estimate of $\bar{\mathbf{r}}$ is

$$\hat{\bar{\mathbf{r}}} = (\mathbf{P}^T\mathbf{Q}^{-1}\mathbf{P})^{-1}\mathbf{P}^T\mathbf{Q}^{-1}\mathbf{p}. \tag{8.12}$$

(*a*) Suppose there is a single asset and just one measurement of the form $p = \bar{r} + e$. Show that according to (8.12), we have $\hat{\bar{r}} = p$.

(*b*) Suppose there are two uncorrelated measurements with values p_1 and p_2, having variances σ_1^2 and σ_2^2. Show that

$$\hat{\bar{r}} = \left(\frac{p_1}{\sigma_1^2} + \frac{p_2}{\sigma_2^2}\right)\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^{-1}$$

2. (APT factors) Two stocks are believed to satisfy the two-factor model

$$r_1 = a_1 + 2f_1 + f_2$$
$$r_2 = a_2 + 3f_1 + 4f_2$$

In addition, there is a risk-free asset with a rate of return of 10%. It is known that $\bar{r}_1 = 15\%$ and $\bar{r}_2 = 20\%$. What are the values of λ_0, λ_1, and λ_2 for this model?

4. (Variance estimate) Let r_i, for $i = 1, 2, \ldots, n$, be independent samples of a return r of mean $\bar{r}$ and variance σ^2. Define the estimates

$$\hat{\bar{r}} = \frac{1}{n}\sum_{i=1}^{n} r_i$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \hat{\bar{r}})^2.$$

Show that $\mathrm{E}(s^2) = \sigma^2$.