DSA 5303
Summer 2018

Homework 1

Please show all relevant work when you upload the assignment.

1. (Shorting with margin) Suppose that to short a stock you are required to deposit an amount equal to the initial price X_0 of the stock. At the end of 1 year the stock price is X_1 and you liquidate your position. You receive your profit from shorting equal to $X_0 - X_1$ and you recover your original deposit. If R is the total return of the stock, what is the total return on your short?

4. (Two stocks) Two stocks are available. The corresponding expected rates of return are $\bar{r}_1$ and $\bar{r}_2$; the corresponding variances and covariances are σ_1^2, σ_2^2, and σ_{12}. What percentages of total investment should be invested in each of the two stocks to minimize the total variance of the rate of return of the resulting portfolio? What is the mean rate of return of this portfolio?

8. (Tracking) Suppose that it is impractical to use all the assets that are incorporated into a specified portfolio (such as a given efficient portfolio). One alternative is to find the portfolio, made up of a given set of n stocks, that tracks the specified portfolio most closely—in the sense of minimizing the variance of the difference in returns.

Specifically, suppose that the target portfolio has (random) rate of return r_M. Suppose that there are n assets with (random) rates of return $r_1, r_2, \ldots, r_n$. We wish to find the portfolio rate of return

$$r = \alpha_1 r_1 + \alpha_2 r_2 + \quad + \alpha_n r_n$$

(with $\sum_{i=1}^{n} \alpha_i = 1$) minimizing $\text{var}(r - r_M)$.

(*a*) Find a set of equations for the α_i's.

(*b*) Although this portfolio tracks the desired portfolio most closely in terms of variance, it may sacrifice the mean. Hence a logical approach is to minimize the variance of the tracking error subject to achieving a given mean return. As the mean is varied, this results in a family of portfolios that are efficient in a new sense—say, tracking efficient. Find the equation for the α_i's that are tracking efficient.

6. (Simpleland) In Simpleland there are only two risky stocks, A and B, whose details are listed in Table 7.4.

TABLE 7.4
Details of Stocks A and B

	Number of shares outstanding	Price per share	Expected rate of return	Standard deviation of return
Stock A	100	\$1.50	15%	15%
Stock B	150	\$2.00	12%	9%

Furthermore, the correlation coefficient between the returns of stocks A and B is $\rho_{AB} = \frac{1}{3}$. There is also a risk-free asset, and Simpleland satisfies the CAPM exactly.

(*a*) What is the expected rate of return of the market portfolio?
(*b*) What is the standard deviation of the market portfolio?
(*c*) What is the beta of stock A?
(*d*) What is the risk-free rate in Simpleland?