# HW1

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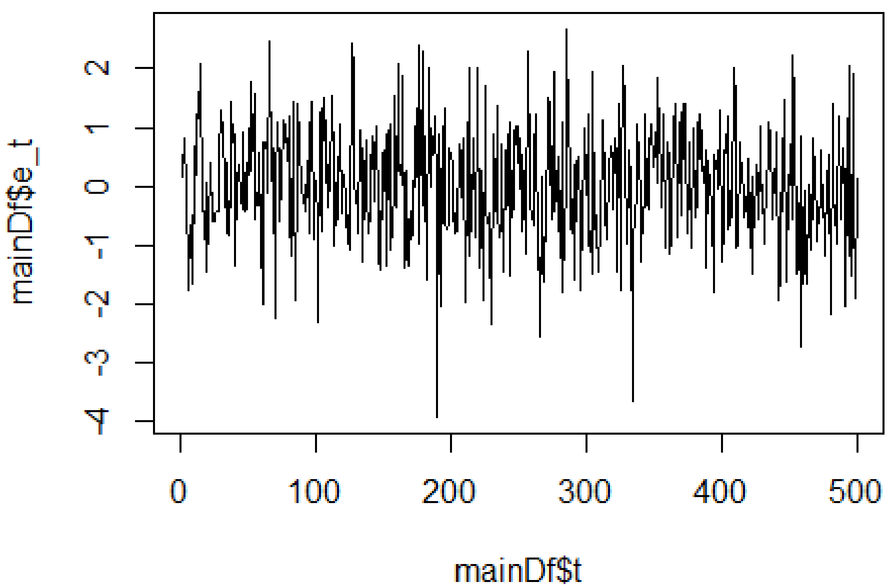
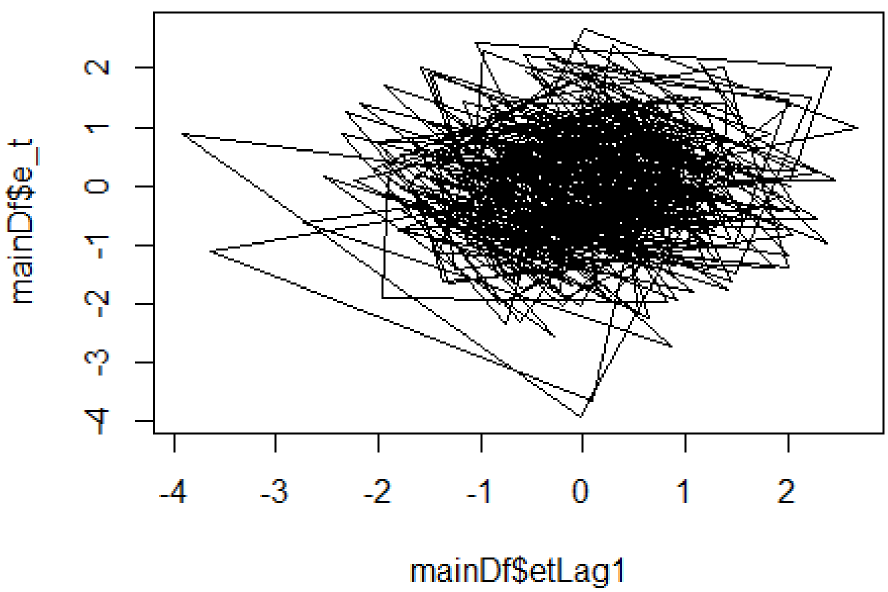
## R Markdown

```r
# Lince Rumainum
# Time Series Analysis
# HW1
library(DataCombine)

# Create time sequences from 1 to 500
mainDf <- data.frame(t = seq(1, 500, by = 1))

# Create main data frame for iid epsilon_t values (500 samples)
mainDf$e_t <- rnorm(500, mean = 0, sd = 1)

# Line plot epsilon_t vs t
plot(mainDf$t, mainDf$e_t, type = "l")
```

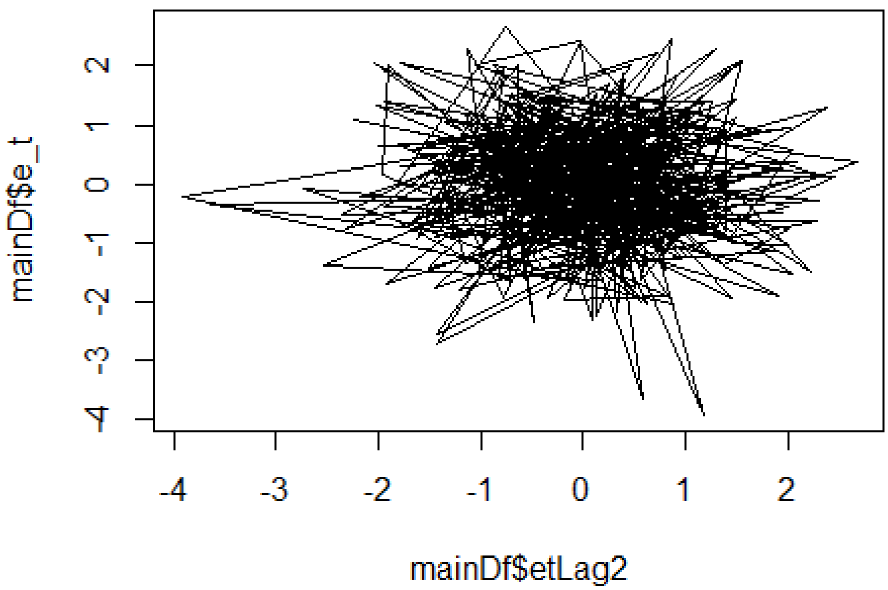



```r
# Create epsilon_t-1 and epsilon_t-2
mainDf <- slide(mainDf,"e_t", "t", NewVar="etLag1", slideBy = -1)
```

```
##
## Lagging e_t by 1 time units.

mainDf <- slide(mainDf,"e_t","t",NewVar="etLag2", slideBy = -2)

##
## Lagging e_t by 2 time units.

# Line plot epsilon_t vs epsilon_t-1
plot(mainDf$etLag1, mainDf$e_t, type = "l")
```

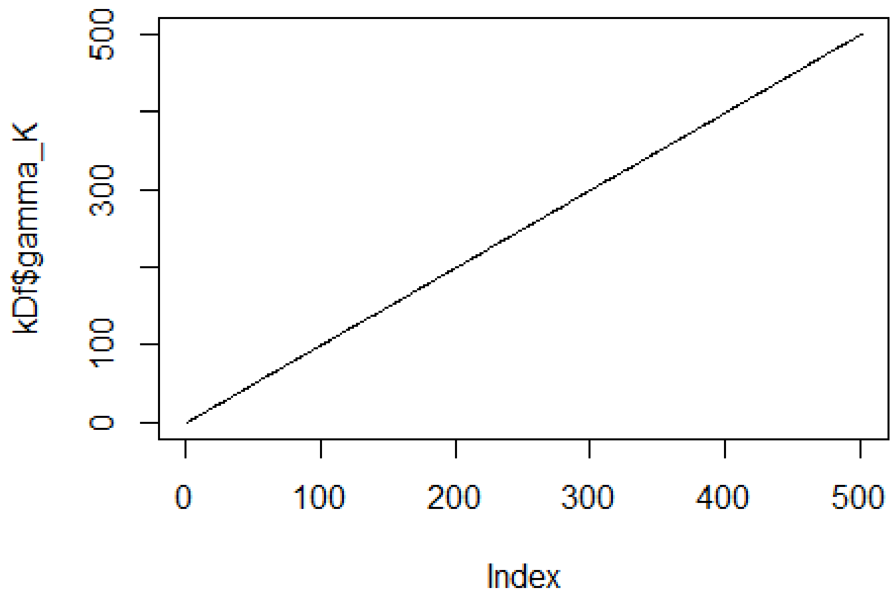



```
# Line plot epsilon_t vs epsilon_t-2
plot(mainDf$etLag2, mainDf$e_t, type = "l")
```

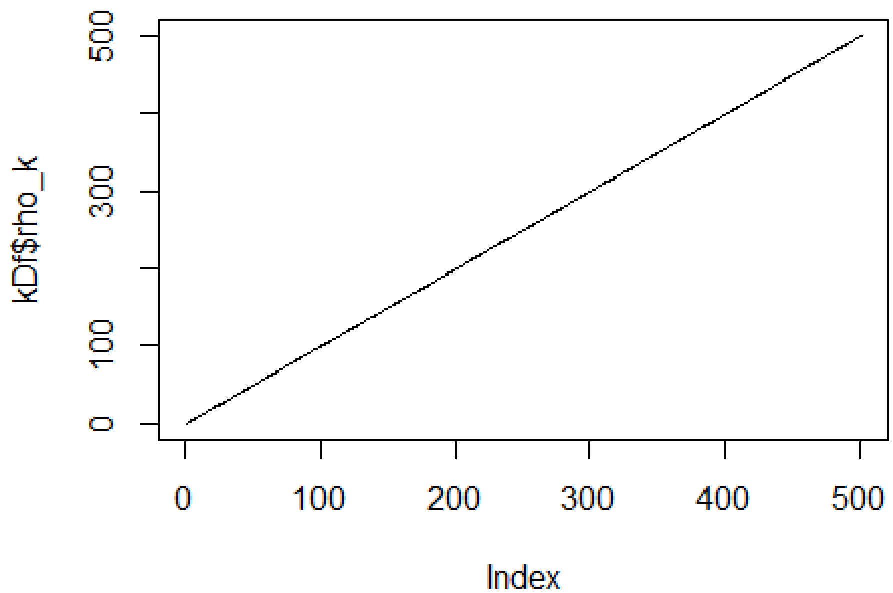



```r
# Calculate mean and variance of epsilon_t
mean(mainDf$e_t)

## [1] 0.008256479

var(mainDf$e_t)

## [1] 1.010508

# Create the rest of the table for epsilon_t-k
for(k in 3:500){
  # Create new variable name
  varName <- paste ("etLag", k, sep ="")
  # Create new column and data of epsilon_t-k
  mainDf <- slide(mainDf,"e_t","t",NewVar = varName, slideBy = -k)
}

# Create new data frame with only epsilon_t-k
newDf <- mainDf[,2:(length(t)+1)]

# Create new data frame for k, gamma_k, and rho_k
kDf <- data.frame(k = seq(0, 500, by = 1))

#Calculate gamma_0 for rho_k calculation
gamma_0 <- cov(mainDf$e_t,mainDf$e_t)
j <- 1 # current row for kDf
```

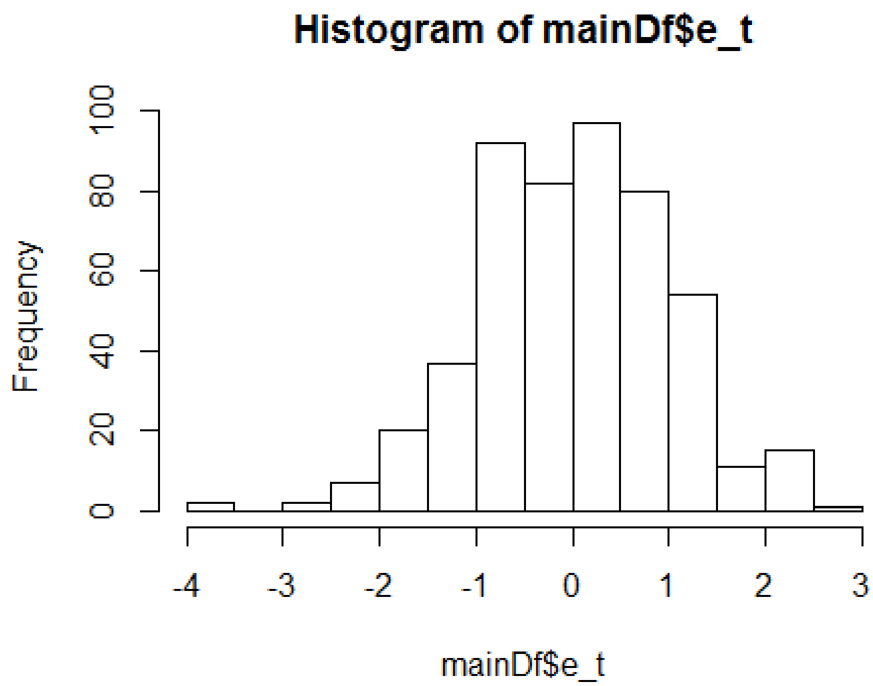
```r
for(i in colnames(newDf)){
  # Calculate gamma_k and rho_k
  kDf$gamma_k[j] <- cov(newDf[j:500,i],newDf$e_t[j:500])
  kDf$rho_k[j] <-  kDf$gamma_k[j]/gamma_0
  j <- j+1 # increment j for next row
}

# Line plot of gamma_k vs k
plot(kDf$k,kDf$gamma_K,type="l")
```



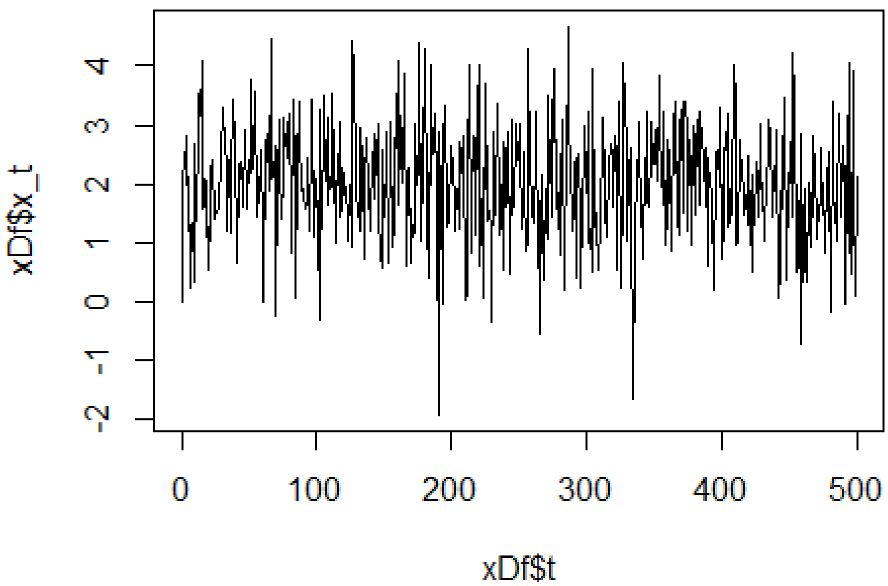

```r
# Line plot of rho_k vs k
plot(kDf$k,kDf$rho_k,type="l")
```



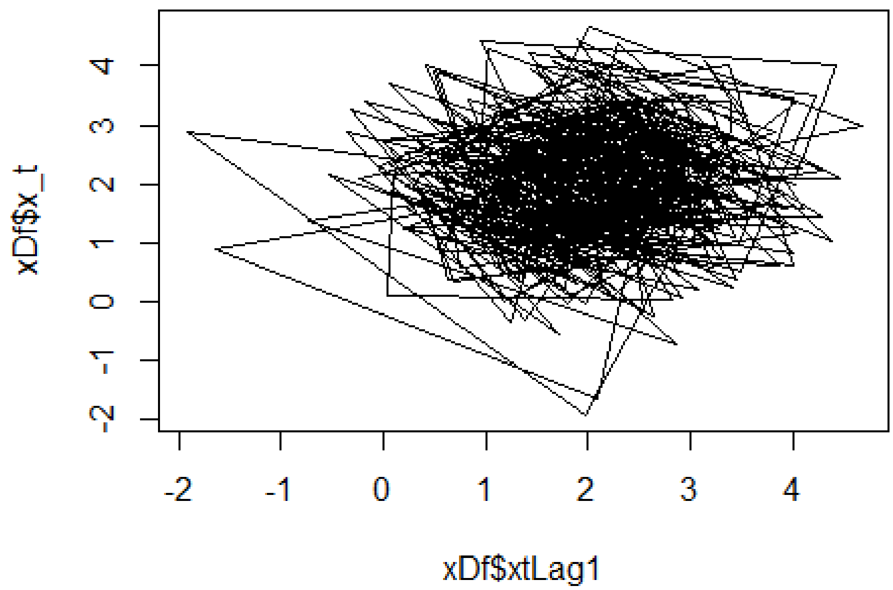

```
# Compute the min, max and plot the histogram
min(mainDf$e_t)

## [1] -3.921132

max(mainDf$e_t)

## [1] 2.671768

hist(mainDf$e_t)
```



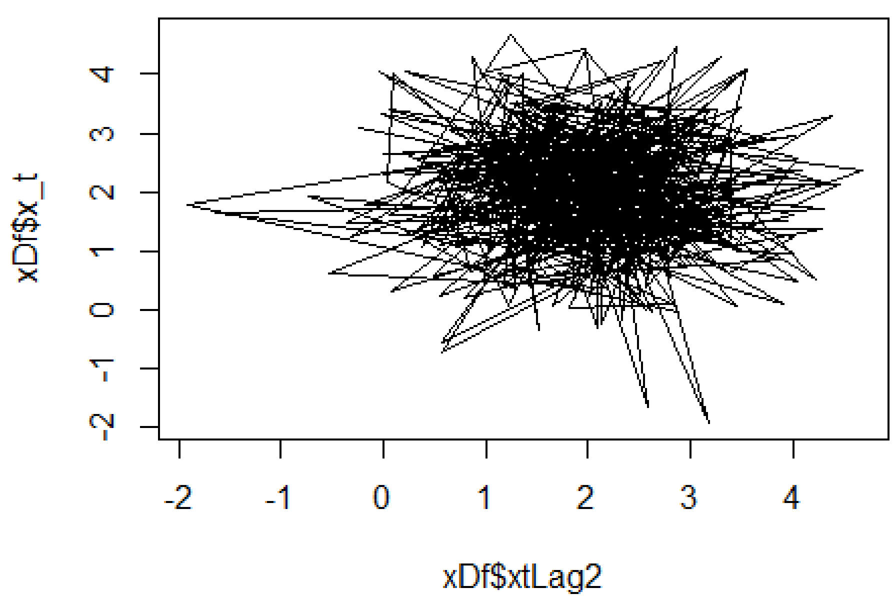

```r
# Problem #2 x_t = 2.0 + epsilon_t, x_0 = 0

# Create time sequences from 0 to 500
xDf <- data.frame(t = seq(0, 500, by = 1))

# Set x_t values from the given equation
xDf$x_t[1] <- 0

# Create the rest of the table for x_t
for(i in 2:501){
  # Calculate data of x_t
  xDf$x_t[i] <- 2.0 + mainDf$e_t[i-1]
}

# Line plot epsilon_t vs t
plot(xDf$t, xDf$x_t, type = "l")
```



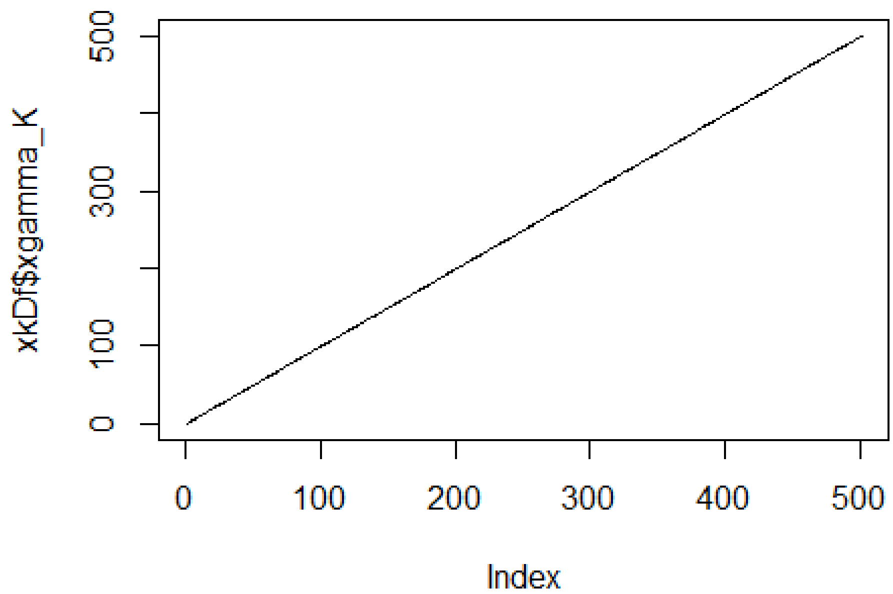

```
# Create epsilon_t-1 and epsilon_t-2
xDf <- slide(xDf,"x_t", "t", NewVar="xtLag1", slideBy = -1)

##
## Lagging x_t by 1 time units.

xDf <- slide(xDf,"x_t","t",NewVar="xtLag2", slideBy = -2)

##
## Lagging x_t by 2 time units.

# Line plot epsilon_t vs epsilon_t-1
plot(xDf$xtLag1, xDf$x_t, type = "l")
```



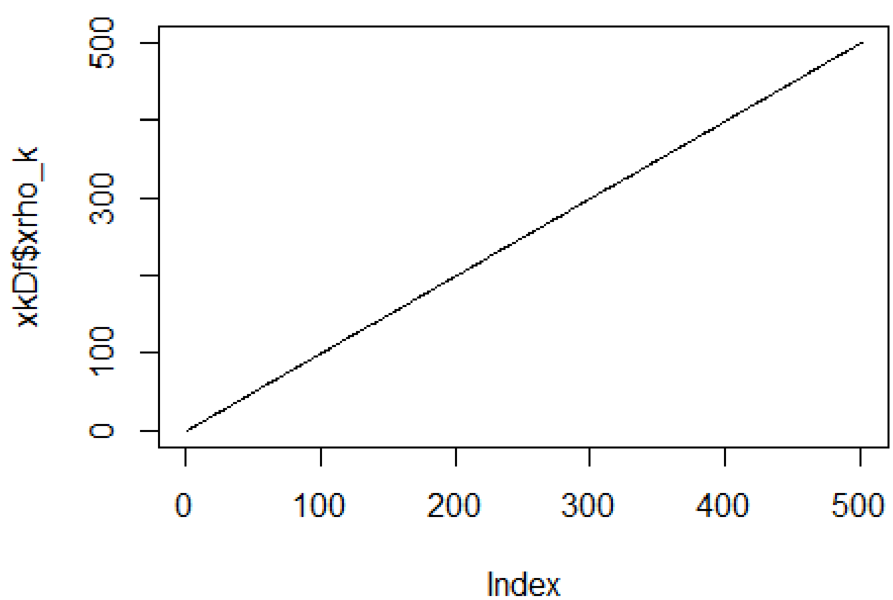

```r
# Line plot epsilon_t vs epsilon_t-2
plot(xDf$xtLag2, xDf$x_t, type = "l")
```



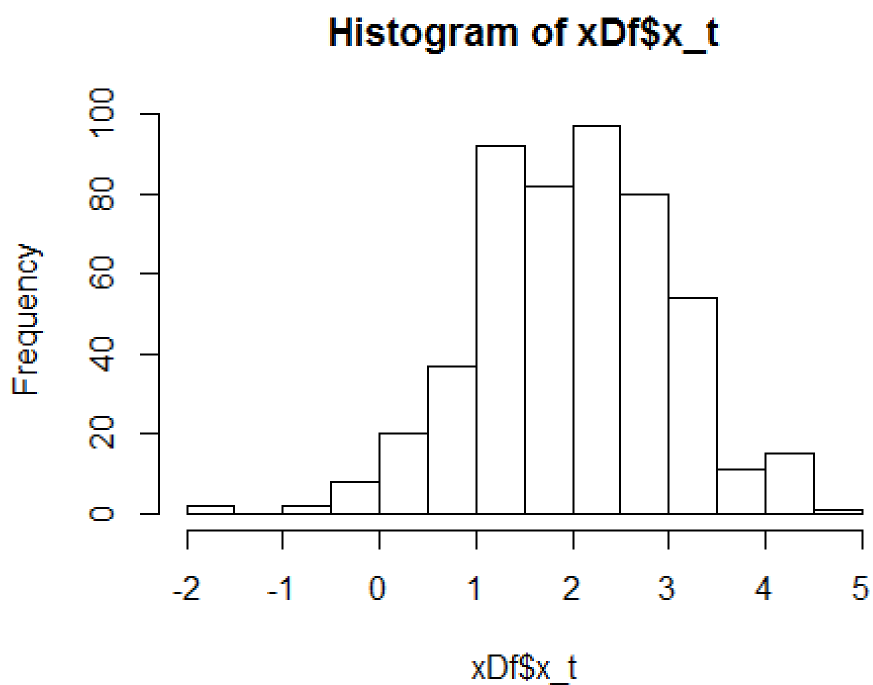
```r
# Calculate mean and variance of epsilon_t
mean(xDf$x_t)

## [1] 2.004248

var(xDf$x_t)

## [1] 1.016537

# Create the rest of the table for epsilon_t-k
for(k in 3:500){
  # Create new variable name
  varName <- paste ("xtLag", k, sep ="")
  # Create new column and data of epsilon_t-k
  xDf <- slide(xDf,"x_t","t",NewVar = varName, slideBy = -k)
}

# Create new data frame with only epsilon_t-k
newXDf <- xDf[,2:(length(t)+1)]

# Create new data frame for k, gamma_k, and rho_k
xkDf <- data.frame(k = seq(0, 500, by = 1))

#Calculate gamma_0 for rho_k calculation
xgamma_0 <- cov(xDf$x_t,xDf$x_t)
j <- 1 # current row for kDf

for(i in colnames(newXDf)){
  # Calculate gamma_k and rho_k
  xkDf$xgamma_k[j] <- cov(newXDf[j:500,i],newXDf$x_t[j:500])
  xkDf$xrho_k[j] <-  xkDf$xgamma_k[j]/xgamma_0
  j <- j+1 # increment j for next row
}

# Line plot of gamma_k vs k
plot(xkDf$k,xkDf$xgamma_K,type="l")
```




```r
# Line plot of rho_k vs k
plot(xkDf$k,xkDf$xrho_k,type="l")
```



```r
# Compute the min, max and plot the histogram
min(xDf$x_t)

## [1] -1.921132

max(xDf$x_t)

## [1] 4.671768

hist(xDf$x_t)
```



3. Covariance function $f(i,j) = \Gamma(i,j) = \gamma(|i-j|)$ and by definition the auto-covariance function is non-negative definite.

If we step back to the beginning and we let:

$\{x_t\}$ – weakly stationary process

$\gamma(k)$ – $\{x_t\}$'s auto correlation function

$z_i = x_i - \mathbf{E}(x_t),\ 1 \le i \le n\ and\ \boldsymbol{z} = (z_1, z_2, \ldots z_n)^T \in \mathbb{R}^n$, where $z$ is the centered random variables

Then,

$0 \le \mathbf{Var}(a^t z) = \mathbf{E}(a^t z)^2 = \mathbf{E}[a^t\ z\ z^t\ a] = a^T \Gamma \mathrm{a} = \sum_{i,j=1}^{n} a_i\ \Gamma(\mathrm{i},\mathrm{j})\ a_j$, where $\Gamma = \Gamma(\mathrm{i},\mathrm{j}) = \mathbf{E}(z_i z_j)$

$\Gamma$ – $n$ x $n$ covariance matrix.

Since it is shown that $\sum_{i,j=1}^{n} a_i\ \Gamma(\mathrm{i},\mathrm{j})\ a_j \ge 0$, the auto-covariance function $\Gamma(i,j) = \gamma(|i-j|)$ function is non-negative definite.