

Aufgabe 1.

$$A = \begin{pmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 4 & 4 & 0 \end{pmatrix}$$

a) $b = (6, 2, 4, 12)^T$

$$Ab = \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 2 & 1 & 0 & 0 & 4 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) | \cdot (-2)I$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & -5 & -2 & -2 & -8 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) | \cdot 5II \quad | \cdot (-4)II$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & -2 & 8 & 2 \\ 0 & 0 & 4 & -8 & 4 \end{array} \right) | \cdot 2III$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & -2 & 8 & 2 \\ 0 & 0 & 0 & 8 & 2 \end{array} \right)$$

$\Rightarrow 8d = 8 \Rightarrow d = 1$

$\hookrightarrow -2c + 8 = 2 \Rightarrow c = 3$

$\hookrightarrow b + 0 + 2 = 2 \Rightarrow b = 0$

$\hookrightarrow a + 3(c) + 3 + 1 = 6 \Rightarrow a = 2$

$\Leftrightarrow Ab = (2, 0, 3, 1)^T$

b) $b = (8, 7, 1, 12)^T$

$$Ab = \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 2 & 1 & 0 & 0 & 1 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) | \cdot (-2)I$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & -5 & -2 & -2 & -15 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) | \cdot 5II \quad | \cdot (-4)II$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & -2 & 8 & 20 \\ 0 & 0 & 4 & -8 & -16 \end{array} \right) | \cdot 2III$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & -2 & 8 & 20 \\ 0 & 0 & 0 & 8 & 24 \end{array} \right)$$

$\Rightarrow 8d = 24 \Rightarrow d = 3$

$\hookrightarrow -2c + 8(3) = 20 \Rightarrow c = 2$

$\hookrightarrow b + 0 + 2(3) = 7 \Rightarrow b = 1$

$\hookrightarrow a + 3(c) + 2 + 3 = 8 \Rightarrow a = 0$

$\Leftrightarrow Ab = (0, 1, 2, 3)^T$

A \tilde{x} = b

a)

$$A\tilde{x} = \tilde{b}$$

Für $n=4$

$$\Rightarrow \left(\begin{array}{ccc|c} -2 & 0 & 0 & 1 \\ -2 & 1 & 0 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -2 & 1 \end{array} \right) \xrightarrow[-2]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ -2 & 1 & 0 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ -2 & 1 & 0 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -2 & 1 \end{array} \right] \xrightarrow[-3]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -2 & 1 \end{array} \right] \xrightarrow[-4]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 0 & 0 & -4 & 6 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 0 & 0 & -4 & 6 \\ 0 & 0 & 0 & -5 \end{array} \right]$$

$$\Rightarrow -5d = 10 \Rightarrow d = -2$$

$$\swarrow -4c + 3(-2) = 6 \Rightarrow c = -3$$

$$\swarrow -3b + 2(-3) = 3 \Rightarrow b = -3$$

$$\swarrow -2a + (-3) = 1 \Rightarrow a = -2$$

$$\Rightarrow A\tilde{x} \cdot \tilde{b} = (-2, -3, -3, -2)$$

A-für bc 4

$$A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 5 & -5 & 5 \\ 1 & -5 & 4 & -4 \\ -1 & 5 & -4 & 30 \end{pmatrix} = L \cdot L^T = \begin{pmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{pmatrix} \cdot \begin{pmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ 0 & l_{22} & l_{23} & l_{24} \\ 0 & 0 & l_{33} & l_{34} \\ 0 & 0 & 0 & l_{44} \end{pmatrix} = \begin{pmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} & l_{11}l_{41} \\ l_{11}l_{21} & l_{21}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} & l_{21}l_{41} + l_{22}l_{42} + l_{33}l_{43} \\ l_{11}l_{31} & l_{21}l_{31} + l_{22}l_{32} & l_{31}^2 + l_{32}^2 + l_{33}^2 & l_{31}l_{41} + l_{32}l_{42} + l_{33}l_{43} \\ l_{11}l_{41} & l_{21}l_{41} + l_{22}l_{42} & l_{31}l_{41} + l_{32}l_{42} + l_{33}l_{43} & l_{41}^2 + l_{42}^2 + l_{43}^2 + l_{44}^2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 5 & -5 & 5 \\ 1 & -5 & 4 & -4 \\ -1 & 5 & -4 & 30 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{pmatrix} \cdot \begin{pmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ 0 & l_{22} & l_{23} & l_{24} \\ 0 & 0 & l_{33} & l_{34} \\ 0 & 0 & 0 & l_{44} \end{pmatrix}$$

$$\Rightarrow 1 = l_{11}^2 \Rightarrow l_{11} = \sqrt{1} = 1$$

$$\Rightarrow -1 = l_{11} \cdot l_{21} \Rightarrow l_{21} = -\frac{1}{1} = -1$$

$$\Rightarrow 1 = l_{11} \cdot l_{31} \Rightarrow l_{31} = 1$$

$$\Rightarrow -1 = l_{11} \cdot l_{41} \Rightarrow l_{41} = -1$$

$$\Rightarrow 5 = l_{21}^2 + l_{22}^2 \Rightarrow l_{22} = \sqrt{4} = 2$$

$$\Rightarrow -5 = l_{11} l_{31} + l_{21} l_{32} \Rightarrow -1 + 2 l_{32} = -5 \Rightarrow l_{32} = -2$$

$$\Rightarrow 5 = l_{21} l_{41} + l_{22} l_{42} \Rightarrow -1 + 2 l_{42} = 5 \Rightarrow l_{42} = 2$$

$$\Rightarrow 4 = l_{31}^2 + l_{32}^2 + l_{33}^2 \Rightarrow 1 + 4 + l_{33}^2 = 4 \Rightarrow l_{33} = \sqrt{-1} \Rightarrow \text{ist nicht positiv definit}$$

↳ keine Cholesky-Zerlegung möglich

$$\left. \begin{aligned} \Rightarrow -4 &= l_{31} l_{41} + l_{32} l_{42} + l_{33} l_{43} \Rightarrow 0 + 0 + l_{43} = -4 \Rightarrow l_{43} \\ \Rightarrow 30 &= l_{41}^2 + l_{42}^2 + l_{43}^2 + l_{44}^2 \Rightarrow -1 + 2 + l_{44} = 30 \Rightarrow l_{44} = \end{aligned} \right\}$$