Übung 7

Donnerstag, 12. Mai 2022 09:00

Aufgabe 1

a)

$$\min_{i=1}^{n} |\lambda - \lambda_i| \leq \frac{\|d\|_2}{\|x\|_2}$$

$\mathbb{R}^{n \times n}\ \mathbb{R}^n \quad \mathbb{R}^n$
$Ax = a$

$$\Leftrightarrow \quad \min_{i=1}^{n} |\lambda - \lambda_i| \leq \frac{\|Ax - \lambda x\|_2}{\|x\|_2}$$

$$\Leftrightarrow \quad \min_{i=1}^{n} |\lambda - \lambda_i| \leq \frac{\|(a_1 - \lambda x_1, \ldots, a_n - \lambda x_n)^T\|_2}{\|(x_1, \ldots, x_n)^T\|_2}$$

$$\Leftrightarrow \quad \min_{i=1}^{n} |\lambda - \lambda_i| \leq \frac{\sqrt{(a_1 - \lambda x_1)^2 + \ldots + (a_n - \lambda x_n)^2}}{\sqrt{(x_1)^2 + \ldots + (x_n)^2}}$$

$\sqrt{(!!)}$

b)

$$\begin{pmatrix} 6 & 4 & 3 \\ 4 & 6 & 3 \\ 3 & 3 & 7 \end{pmatrix} \quad \text{mit } EW_{a_1} = 2,\ c_2 = 4,\ c_3 = 13 \quad \Big| \ \lambda = 12 \quad x = (0{,}9,\ 1,\ 1{,}1)^T$$

$$\min_{i=1}^{n} |12 - \lambda_i| \leq \frac{\left\| \begin{pmatrix} 12{,}7 \\ 12{,}9 \\ 13{,}4 \end{pmatrix} - 12 \begin{pmatrix} 0{,}9 \\ 1 \\ 1{,}1 \end{pmatrix} \right\|_2}{\|(0{,}9,\ 1,\ 1{,}1)^T\|_2}$$

$|12 - 13|$

$$\Leftrightarrow \quad 1 \leq \frac{\|(1{,}3,\ 0{,}3,\ 0{,}2)\|_2}{\|(0{,}9,\ 1,\ 1{,}1)^T\|_2}$$

$$\Leftrightarrow \quad 1 \leq \frac{\sqrt{(1{,}3)^2 + (0{,}3)^2 + (0{,}2)^2}}{\sqrt{(0{,}9)^2 + (1)^2 + (1{,}1)^2}}$$

$$\Leftrightarrow \quad 1 \leq \frac{\frac{\sqrt{446}}{10}}{\frac{\sqrt{302}}{10}}$$

$$\Leftrightarrow \quad 1 \leq 1{,}21521\ldots$$