

Aufgabe 1.

$$A = \begin{pmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 4 & 4 & 0 \end{pmatrix}$$

a) $b = (6, 2, 4, 12)^T$

$$Ab = \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 2 & 1 & 0 & 0 & 4 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) \quad | \cdot (-2)I$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & -5 & -2 & -2 & -8 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) \quad \begin{array}{l} | \cdot 5II \\ | \cdot (-4)II \end{array}$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & -2 & 8 & 2 \\ 0 & 0 & 4 & -8 & 4 \end{array} \right) \quad | \cdot 2IV$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & -2 & 8 & 2 \\ 0 & 0 & 0 & 8 & 8 \end{array} \right)$$

$$\Rightarrow 8d = 8 \Rightarrow d = 1$$

$$\hookrightarrow -2c + 8 = 2 \Rightarrow c = 3$$

$$\hookrightarrow b + 0 + 2 = 2 \Rightarrow b = 0$$

$$\hookrightarrow a + 3b + 3 + 1 = 6 \Rightarrow a = 2$$

$$\Leftrightarrow Ab = (2, 0, 3, 1)^T$$

b) $b = (8, 7, 1, 12)^T$

$$Ab = \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 2 & 1 & 0 & 0 & 1 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) \quad | \cdot (-2)I$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & -5 & -2 & -2 & -15 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) \quad \begin{array}{l} | \cdot 5II \\ | \cdot (-4)II \end{array}$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & -2 & 8 & 20 \\ 0 & 0 & 4 & -8 & -16 \end{array} \right) \quad | \cdot 2III$$

$$\Leftrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & -2 & 8 & 20 \\ 0 & 0 & 0 & 8 & 24 \end{array} \right)$$

$$\Rightarrow 8d = 24 \Rightarrow d = 3$$

$$\hookrightarrow -2c + 8(3) = 20 \Rightarrow c = 2$$

$$\hookrightarrow b + 0 + 2(3) = 7 \Rightarrow b = 1$$

$$\hookrightarrow a + 3 + 2 + 3 = 8 \Rightarrow a = 0$$

$$\Leftrightarrow Ab = (0, 1, 2, 3)^T$$