Übung 9

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Aufgabe 1

$$B_i^n(x) = \binom{n}{i} (1-x)^{n-i} \cdot x^i \qquad 0 \leq i \leq n$$

a) | $x=0$ ist i-fache Nullstelle |

$$B_i^n(x) = \binom{n}{i} (1-x)^{n-i} \cdot x^i = B_i^n(x) = f(x) \cdot x^i$$

$$\Rightarrow f(x) = \binom{n}{i}(1-x)^{n-i}$$

$$f(0) = \binom{n}{i}(1)^{n-i}$$

$$\Leftrightarrow f(0) = \binom{n}{i} \neq 0 \Rightarrow i\text{-fache Nullstelle}$$

b) | $x=1$ ist $n-i$-fache Nullstelle |

$$B_i^n(x) = \binom{n}{i} (1-x)^{n-i} \cdot x^i = B_i^n(x) = f(x) \cdot (1-x)^{n-i}$$

$$\Rightarrow f(x) = \binom{n}{i} x^i$$

$$f(0) = \binom{n}{i} (0)^i$$

$$\Leftrightarrow f(0) = \binom{n}{i} \neq 0 \Rightarrow i\text{-fache Nullstelle}$$

c) $\left| B_i^n(x) = B_{n-i}^n(1-x) \right|$

$$B_i^n(x) = \binom{n}{i}(1-x)^{n-i} \cdot x^i$$

$$B_{n-i}^n(1-x) = \binom{n}{n-i}\left(1-(1-x)\right)^{n-(n-i)} \cdot (1-x)^{(n-i)}$$

$$= \binom{n}{n-i}\underline{(x)^i} \cdot \underline{(1-x)^{(n-i)}}$$

$$= \binom{n}{i}(1-x)^{n-i} \cdot x^i$$

$\square$

d.1) $\left| (1-x)\, B_0^n(x) = B_0^{n+1}(x) \right|$

$$(1-x)\, B_0^n(x) = (1-x)\binom{n}{0}(1-x)^{n-0} \cdot x^0 = (1-x)^{n+1} \cdot x^0$$

$$B_0^{n+1}(x) = \binom{n+1}{0}(1-x)^{n+1-0} \cdot x^0 = (1-x)^{n+1} \cdot x^0$$

$\Big| = \square$

d.2) $\left| x\, B_n^n(x) = B_{n+1}^{n+1}(x) \right|$

$$x\, B_n^n(x) = x\binom{n}{n}(1-x)^{n-n}\, x^n = x^{n+1}$$

$$B_{n+1}^{n+1}(x) = \binom{n+1}{n+1}(1-x)^{(n+1)-(n+1)}\, x^{n+1} = x^{n+1}$$

$\Big| = \square$

e) $\left| B_i^n \geq 0 \ \forall x \in [0,1] \right|$

$$B_i^n(x) = \underbrace{\binom{n}{i}}_{>0} \underbrace{(1-x)^{n-i}}_{\substack{\text{für } x=1:\ =0 \\ \text{sonst } >0}} \underbrace{x^i}_{\substack{\text{für } x=1:\ =1 \\ \text{für } x=0:\ =0 \\ \Rightarrow \text{ zwischen } 0 \text{ und } 1}}$$

$\Rightarrow \geq 0 \ \forall x \in [0;1]$

$\square$

f) $\left| B_i^n(x) = x B_{i-1}^{n-1}(x) + (1-x) B_i^{n-1}(x) \right|$

$$\begin{aligned}
B_i^n(x) &= \binom{n}{i}(1-x)^{n-i} x^i \\
x B_{i-1}^{n-1}(x) + (1-x) B_i^{n-1}(x) &= x \binom{n-1}{i-1}(1-x)^{(n-1)-(i-1)} \cdot x^{i-1} + (1-x)\binom{n-1}{i}(1-x)^{(n-1)-i} \cdot x^i \\
&= \binom{n-1}{i-1}(1-x)^{n-i} \cdot x^i + \binom{n-1}{i}(1-x)^{n-i} \cdot x^i \\
&= \left(\binom{n-1}{i-1} + \binom{n-1}{i}\right)\left((1-x)^{n-i} \cdot x^i\right) \\
&= \binom{n}{i} \cdot (1-x)^{n-i} \cdot x^i \\
&= B_i^n(x)
\end{aligned}$$

$\square$

g) $\left| 1 = \sum_{i=0}^{n} B_i^n \right|$

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

$$\sum_{i=0}^{n} B_i^n(x) = \sum_{i=0}^{n} \binom{n}{i} \underbrace{(1-x)^{n-i}}_{a} \cdot \underbrace{x^i}_{b}$$

$$= (1-x+x)^n = 1^n = 1 \quad \square$$