

Aufgabe 1

$$Ax = b \quad | \quad A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$|A| \leq \frac{1}{10} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad |Ab| \leq \frac{1}{10} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\tilde{x} = (0.3, 1.1)^T$$

$$\text{zu zeigen: } |\tilde{r}| \leq B|\tilde{x}| + c$$

$$\Rightarrow \tilde{r} = b - Ax = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0.3 \\ 1.1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 0.7 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0 \end{pmatrix}$$

$$\Rightarrow |\tilde{r}| = \begin{pmatrix} 0.3 \\ 0 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\Rightarrow B = \frac{1}{10} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad c = \frac{1}{10} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$B|\tilde{x}| + c = \frac{1}{10} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0.3 \\ 1.1 \end{pmatrix} + \frac{1}{10} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ \frac{1}{5} \end{pmatrix} + \begin{pmatrix} \frac{1}{10} \\ \frac{1}{10} \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\Rightarrow |\tilde{r}| = \frac{1}{10} \begin{pmatrix} 3 \\ 0 \end{pmatrix} \leq \frac{1}{10} \begin{pmatrix} 3 \\ 3 \end{pmatrix} = B|\tilde{x}| + c \quad \square$$

$\Rightarrow \tilde{x} \text{ approximiert}$

b)

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1.001 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |A| \leq 5 \cdot 10^{-4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = B, \quad |Ab| \leq 5 \cdot 10^{-4} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c, \quad \tilde{x} = (501, 500)^T$$

$$|\tilde{r}| \leq B|\tilde{x}| + c$$

$$\Leftrightarrow \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ -1 & 1001 \end{pmatrix} \begin{pmatrix} 501 \\ 500 \end{pmatrix} \right| \leq 5 \cdot 10^{-4} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 501 \\ 500 \end{pmatrix} + 5 \cdot 10^{-4} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Leftrightarrow \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -0.5 \end{pmatrix} \right| \leq 5 \cdot 10^{-4} \begin{pmatrix} 1001 \\ 1000 \end{pmatrix} + 5 \cdot 10^{-4} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Leftrightarrow \left| \begin{pmatrix} 0 \\ -0.5 \end{pmatrix} \right| \leq 5 \cdot 10^{-4} \begin{pmatrix} 1002 \\ 1002 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} \leq \begin{pmatrix} 0.501 \\ 0.501 \end{pmatrix} \quad \square$$

\tilde{x} als Resultat

Aufgabe 3

$$A = \begin{pmatrix} 1 & -5 & -20 \\ -4 & 11 & -1 \\ 8 & -4 & 2 \end{pmatrix}$$

$\boxed{j=1}$

$$1) A^{(1-1)} = A^{(0)} = A$$

$$2) v^{(1)} = a_1 + \text{sgn}(2) \cdot \|a_1\| \cdot e_1$$

$$\Leftrightarrow v^{(1)} = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix} + \text{sgn}(2) \cdot \sqrt{1+16+64} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ 8 \end{pmatrix}$$

$$3) H^{(0)} = E - \frac{2v^{(1)}v^{(1)\top}}{\sqrt{v^{(1)\top}v^{(1)}}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{2}{\sqrt{1+16+64}} \begin{pmatrix} 10 & -40 & 80 \\ -40 & 16 & -32 \\ 80 & -32 & 64 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{4}{3} & \frac{16}{3} \\ \frac{4}{3} & \frac{32}{3} & \frac{16}{3} \\ \frac{16}{3} & \frac{16}{3} & \frac{22}{3} \end{pmatrix}$$

$$4) H^{(1)} = H^{(0)} A^{(0)} = H^{(0)} A$$

$$5) A^{(0)} = \begin{pmatrix} \frac{-1}{3} & \frac{4}{3} & \frac{16}{3} \\ \frac{4}{3} & \frac{32}{3} & \frac{16}{3} \\ \frac{16}{3} & \frac{16}{3} & \frac{22}{3} \end{pmatrix} \cdot \begin{pmatrix} 1 & -5 & -20 \\ -4 & 11 & -1 \\ 8 & -4 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & \frac{3}{5} & 0 \\ 0 & \frac{27}{5} & -\frac{1}{5} \\ 0 & \frac{56}{5} & \frac{18}{5} \end{pmatrix}$$

$\boxed{j=2}$

$$1) A^{(1)} = \begin{pmatrix} \frac{27}{5} & -9 \\ \frac{36}{5} & 18 \end{pmatrix}$$

$$2) v_2 = a_2 + \sin(\varphi) \cdot \|a_2\| \cdot e$$

$$v_2 = \begin{pmatrix} \frac{27}{5} \\ \frac{36}{5} \end{pmatrix} + 1 \cdot \sqrt{81} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{54}{5} \\ \frac{36}{5} \end{pmatrix}$$

$$3) H^{(1)} = E - \frac{2}{\lambda^{(1)} \cdot v^{(1)}} v^{(1)} v^{(1)\top} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{2}{2 \cdot 27} \begin{pmatrix} \frac{27}{5}^2 & \frac{27 \cdot 9}{25} \\ \frac{27 \cdot 9}{25} & \frac{9}{5}^2 \end{pmatrix} = \begin{pmatrix} \frac{-13}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{13}{5} \end{pmatrix}$$

$$4) H^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$5) A^{(2)} = H^{(2)} A^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \cdot \begin{pmatrix} -9 & 9 & 0 \\ 0 & \frac{27}{5} & -1 \\ 0 & \frac{54}{5} & 18 \end{pmatrix} = \begin{pmatrix} -9 & 9 & 0 \\ 0 & -\frac{27}{5} & -3 \\ 0 & 0 & 18 \end{pmatrix} = R$$

$$Q^\top = H^{(2)} H^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{27}{5} & \frac{9}{5} \\ -\frac{3}{5} & \frac{27}{25} & \frac{16}{25} \\ -\frac{4}{5} & \frac{16}{25} & \frac{9}{25} \end{pmatrix} = \begin{pmatrix} 1 & \frac{27}{5} & \frac{9}{5} \\ -\frac{3}{5} & \frac{27}{25} & \frac{16}{25} \\ -\frac{4}{5} & \frac{16}{25} & \frac{9}{25} \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 1 & \frac{27}{5} & \frac{9}{5} \\ -\frac{3}{5} & \frac{27}{25} & \frac{16}{25} \\ -\frac{4}{5} & \frac{16}{25} & \frac{9}{25} \end{pmatrix}$$