SIPPI

**SIPPI** 

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SIPPI

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## **About**

SIPPI is a Matlab toolbox (compatible with GNU Octave) that allow sampling the solution of non-linear inverse problems with realistic a priori information.

In order to make use of SIPPI one has to

- Install and setup SIPPI
- Define the prior model, in form of the prior data structure
- Define the forward model, in form of the forward data structure, and the sippi\_forward.m m-file
- Define the data and noise model, in form of the prior data structure
- Choose a method for sampling the a posteriori probability density.

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# **Chapter 1**

# Introduction

The latest version of SIPPI is available from Sourceforge



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# **Chapter 2**

# Installation

## 2.1 SIPPI

Download the latest version of SIPPI from http://sippi.sourceforge.net.

Unpack ZIPPI\_1.0.zip somewhere, for example to 'c:\Users\tmh\SIPPI'. Then setup the Matlab path to point to the appropriate SIPPI directories:

addpath c:\Users\tmh\SIPPI
sippi\_set\_path

## 2.1.1 SGeMS (optional)

To make use of the SISIM and SNESIM type priori models SGeMS needs to be available.

Currently only SGeMS version 2.1 (download) for Windows is supported.

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## **Chapter 3**

# **Setting up SIPPI**

## 3.1 The a priori model

### 3.1.1 Types of a priori models

#### 3.1.1.1 1D Generalized Gaussian

A 1D generalized Gaussian prior model can be specified using the 'gaussian' type prior model

```
prior{1}.type='gaussian';
```

A simple 1D Gaussian distribution with mean 10, and standard deviation 2, can be specified using

```
prior{1}.type='gaussian';
prior{1}.m0=10;
prior{1}.std=2;
```

The norm of a generalized Gaussian can be set using the 'norm' field. A generalized 1D Gaussian with mean 10, standard devation of 2, and a norm of 70, can be specified using (The norm is equivelent of the beta factor referenced in Wikipedia: Generalized\_normal\_dist

```
prior{1}.type='gaussian';
prior{1}.m0=10;
prior{1}.std=2;
prior{1}.norm=70;
```

- 3.1.1.2 VISIM
- 3.1.1.3 FFTMA
- 3.1.1.4 SISIM
- 3.1.1.5 SNESIM

#### 3.1.2 Sampling the prior

Once the prior data structure has been defined a sample from the prior distribution can be generated using

```
m=sippi_prior(prior);
```

'm' is a Matlab data structure of the same size as the 'prior' data structure. Thus, if two prior distributions have been defined in 'prior{1}' and 'prior{2}', then 'm{1}' will hold a realization of 'prior{1}', and 'm{2}' will hold a realization of 'prior{2}'.

Each time 'm=sippi\_prior(prior)' is called, a new independant realization of the prior will be generated.

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#### 3.1.3 Sequential Gibbs sampling / Conditional Resampling

All the available a priori types available allow perturbing one realization of a prior into a new realization of prior, in the vicinity of the first one. To do this we make use of sequential Gibbs sampling [HCM12]. Sequential Gibbs in essence is a type of conditional resampling. From a current realization of a prior, a number of model parameters are discarded and treated as unknown, and the simulated conditional to the fixed values of the model parameters.

In order to generate a new realization 'm2' in the vicinity of the realization 'm1' use

```
m1=sippi_prior(prior);
[m2,prior]=sippi_prior(prior,m1);
```

If this process is iterated, then a random walk in the space of a priori acceptable models will be perform. And, the collection of realization obatined, will represent a sample from prior distribution.

#### 3.1.3.1 Controlling sequential Gibbs sampling / Conditional Resampling

All properties related to sequential Gibbs sampling can be set in the 'seq\_gibbs' data struture, for each prior type. The following two parameters determined how the a current model is perturbed

```
prior{m}.seq_gibbs.step=1;
prior{m}.seq_gibbs.type=2;
```

### 3.2 The data and the noise

## 3.3 The forward model

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## **Chapter 4**

# The a posteriori distribution

## 4.1 Sampling the a posteriori probability density

- 4.1.1 The rejection sampler
- 4.1.2 The extended Metropolis sampler
- 4.1.2.1 Controling the step length

One optionally, as part of running the extended Metropolis sampler, automatically update the 'step'-length of the sequential Gibbs sampler in order to ensure a specific approximate acceptance ratio of the Metropolios sampler. See [CHM12] for details.

The default parameters for adjusting the step length, as given below, are set in the 'prior.seq\_gibbs' structure. These parameters will be set the first time 'sippi\_prior' is called with the 'prior' structure as output. The default parameters.

```
prior{m}.seq_gibbs.step_min=0;
prior{m}.seq_gibbs.step_min=1;
prior{m}.seq_gibbs.i_update_step=50
prior{m}.seq_gibbs.i_update_step_max=1000
prior{m}.seq_gibbs.n_update_history=50
prior{m}.seq_gibbs.P_target=0.3000
```

By default, adjustment of the step length, in order to achieve an acceptance ratio of 0.3 ('prior{m}.seq\_gibbs.P\_target'), will be performed for every 50 ('prior{m}.seq\_gibbs.i\_update\_step') iterations, using the acceptance ratio observed in the last 50 ('prior{m}.seq\_gibbs.i\_update\_history') iterations.

Adjustment of the step length will be performed only in the first 1000 ('prior{m}.seq\_gibbs.i\_update\_step\_max') iterations.

In order to disable automatiuc adjustment of the step length simply set

```
prior{m}.seq_gibbs.i_update_step_max=0; % disable automatic step length
```

- 4.1.2.2 The extended independent Metropolis sampler
- 4.1.2.3 Annealing schedule
- 4.1.3 linear least squares

## 4.2 Simulated Annealing

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## **Chapter 5**

# **Examples**

SIPPI can be used as a convenient approach for unconditional an conditional simulation.

In ordet to use SIPPI to solve inverse problems, one must provide the solution to the forward problem. Essentially this amounts to implementing a Matlab function that solves the forward problem in using a specific input/output format. If a solution to the forward problem allready exist, this can be quite easily done simply using a Matlab wrapper function.

A few implementations of solutions to forward problems are included as examples as part of SIPPI. These will be demonstrated in the following

## 5.1 Polynomial line fitting

The forward problem consists of computing the y-value as a function of the x-position of the data, and the polynomial coefficients determining the line. sippi\_forward\_linefit.m:

```
% sippi_forward_linefit Line fit forward solver for SIPPI
%
% [d,forward,prior,data]=sippi_forward_linefit(m,forward,prior,data);
%
function [d,forward,prior,data]=sippi_forward_linefit(m,forward,prior,data);
if length(m)==1;
    d{1}=forward.x*m{1};
elseif length(m)==2;
    d{1}=forward.x*m{1}+m{2};
else
    d{1}=forward.x.^2*m{1}+forward.x*m{2}+m{3};
end
```

## 5.2 Covariance model inference

## 5.3 Cross hole tomography

## 5.4 Reflection seismic inversion

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## **Chapter 6**

# **Bibliography**

[CHM12] K. S. Cordua, T. M. Hansen, and K. Mosegaard, Monte Carlo full waveform inversion of crosshole GPR data using multiple-point geostatistical a priori information, H19--H31.
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Accepted for publication in Geophysics, xx, 2014.

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## **Chapter 7**

## Reference

### 7.1 SIPPI

## 7.1.1 getinunits

```
Get object properties in specified units
GETINUNITS
  V = GETINUNITS(H, PROP, UNITS) returns the object property
  in the specified UNITS. It will leave the 'Units' and 'FontUnits'
  property unchanged afterwards.
  H is the handle of the object. If it is an M-element array of handles,
  the function will return an M-by-1 cell array. PROP can be a string or
  a cell array of strings. If it is a 1-by-N or N-by-1 cell array, the
  function will return an M-by-N cell array of values. UNITS can be a
  string or a cell array. If it is a cell array, then PROP must also be a
  cell array with the same size as UNITS, and each cell element of UNITS
  corresponds to a cell element of PROP.
  V = GETINUNITS(H, PROP) is the same as GET(H, PROP)
  Examples:
    V = GETINUNITS(H, 'Position', 'Pixels')
    V = GETINUNITS(H, {'FontSize', 'Position'}, 'Normalized')
    V = GETINUNITS(H, {'FontSize', 'Position'}, {'Points', 'Pixels'})
  See also GET, SET
```

### **7.1.2** logdet

```
LOGDET Computation of logarithm of determinant of a matrix

v = logdet(A);
   computes the logarithm of determinant of A.

Here, A should be a square matrix of double or single class.
   If A is singular, it will returns -inf.

Theoretically, this function should be functionally equivalent to log(det(A)). However, it avoids the overflow/underflow problems that are likely to happen when applying det to large matrices.
```

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```
The key idea is based on the mathematical fact that
    the determinant of a triangular matrix equals the
    product of its diagonal elements. Hence, the matrix's
    log-determinant is equal to the sum of their logarithm
    values. By keeping all computations in log-scale, the
    problem of underflow/overflow caused by product of
    many numbers can be effectively circumvented.
    The implementation is based on LU factorization.
v = logdet(A, 'chol');
    If A is positive definite, you can tell the function
    to use Cholesky factorization to accomplish the task
    using this syntax, which is substantially more efficient
    for positive definite matrix.
Remarks
   logarithm of determinant of a matrix widely occurs in the
    context of multivariate statistics. The log-pdf, entropy,
    and divergence of Gaussian distribution typically comprises
    a term in form of log-determinant. This function might be
    useful there, especially in a high-dimensional space.
    Theoretially, LU, QR can both do the job. However, LU
    factorization is substantially faster. So, for generic
   matrix, LU factorization is adopted.
    For positive definite matrices, such as covariance matrices,
    Cholesky factorization is typically more efficient. And it
    is STRONGLY RECOMMENDED that you use the chol (2nd syntax above)
    when you are sure that you are dealing with a positive definite
    matrix.
Examples
    % compute the log-determinant of a generic matrix
    A = rand(1000);
    v = logdet(A);
    \mbox{\ensuremath{\$}} compute the log-determinant of a positive-definite matrix
    A = rand(1000);
```

#### 7.1.3 pathdef

C = A \* A';

v = logdet(C, 'chol');

```
PATHDEF Search path defaults.

PATHDEF returns a string that can be used as input to MATLABPATH

in order to set the path.
```

% this makes C positive definite

#### 7.1.4 plotboxpos

```
PLOTBOXPOS Returns the position of the plotted axis region

pos = plotboxpos(h)
```

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```
This function returns the position of the plotted region of an axis, which may differ from the actual axis position, depending on the axis limits, data aspect ratio, and plot box aspect ratio. The position is returned in the same units as the those used to define the axis itself. This function can only be used for a 2D plot.

Input variables:

h: axis handle of a 2D axis (if ommitted, current axis is used).

Output variables:

pos: four-element position vector, in same units as h
```

## 7.1.5 sippi\_adjust\_step\_size

```
sippi_adjust_step_size Adjust step length length for Metropolis sampler in SIPPI

Call:
    step=sippi_adjust_step_size(step,P_average,P_target);

step : current step
P_current : Current acceptance ratio
P_target : preferred acceptance ratio (def=0.3);

See also sippi_compute_acceptance_rate, sippi_prior_set_steplength
```

#### 7.1.6 sippi\_anneal\_adjust\_noise

```
sippi_anneal_adjust_noise : Adjust noise level in annealing schedul

Call:
    [data_adjust,mcmc]=sippi_anneal_adjust_noise(data,i,mcmc,prior);

See also: sippi_metropolis, sippi_anneal_factor
```

#### 7.1.7 sippi\_anneal\_factor

```
sippi_anneal_factor : compute simple noise multiplication factor for
annealing type sampling
See also sippi_metropolis, sippi_anneal_adjust_noise
```

## 7.1.8 sippi\_colormap

```
sippi_colormap Default colormap for sippi
Call :
   sippi_colormap; % the same as sippi_colormap(3);
```

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```
or:
    sippi_colormap(1) - Red Green Black
    sippi_colormap(2) - Red Green Blue Black
    sippi_colormap(3) - Jet
```

## 7.1.9 sippi\_compute\_acceptance\_rate

```
sippi_compute_acceptance_rate Computes acceptance rate for the Metropolis sampler in 
        SIPPI

Call:
    P_acc=sippi_compute_acceptance_rate(acc,n_update_history);
```

## 7.1.10 sippi\_forward

```
sippi_forward Simple forward wrapper for SIPPI

Assumes that the actual forward solver has been defined by forward.forward_function

Call:
   [d, forward, prior, data] = sippi_forward(m, forward, prior, data, id, im)
```

## 7.1.11 sippi\_forward\_traveltime

```
call :
    [d, forward, prior, data] = sippi_forward_traveltime (m, forward, prior, data, id, im)

forward.type determines the method used to compute travel times
forward.type='ray';
forward.type='fat';
forward.type='eikonal';
forward.type='born';
```

### 7.1.12 sippi\_get\_sample

```
sippi_get_sample Get a posterior sample

Call :
   [reals,etype_mean,etype_var]=sippi_get_sample(data,prior,id,im,n_reals,options);
```

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#### 7.1.13 sippi\_least\_squares

#### 7.1.14 sippi\_likelihood

```
sippi_likelihood Compute likelihood given an observed dataset
 [logL, L, data] = sippi_likelihood(d, data);
data{1}.d_obs [N_data,1] N_data data observations
data{1}.d_std [N_data,1] N_data uncorrelated Gaussian STD
data{1}.d_var [N_data,1] N_data uncorrelated Gaussian variances
Gaussian modelization error, N(dt,Ct), is specified as
data{1}.dt [N_data,1] : Bias/mean of modelization error
\verb|data{1}.Ct [N_data,N_data]| : Covariance of modelization error|\\
data{1}.Ct [1,1] : Constant Covariance of modelization error
                    imples data{1}.Ct=ones(N_data.N_data)*data{1}.Ct;
data{id}.recomputeCD [default=0], if '1' then data{1}.iCD is recomputed
each time sippi_likelihood is called. This should be used if the noise model
changes between each call to sippi_likelihood.
data{id}.full_likelihood [default=]0; if '1' the the full likelihood
 (including the determinant) is computed. This not needed if the data
 civariance is constant, but if it changes, then use
 data{id}.full_likelihood=1;
```

## 7.1.15 sippi\_mcmc\_init

```
sippi_mcmc_init Initialize McMC options for Metropolis and rejection sampling in SIPPI
Call:
    options=sippi_mcmc_init(options,prior);
```

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#### 7.1.16 sippi\_metropolis

```
sippi_metropolis Extended Metropolis sampling in SIPPI
Metropolis sampling.
 See e.g. Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012.
   Gibbs Sampling.
   Computational Geosciences. doi:10.1007/s10596-011-9271-1.
Call :
  [options, data, prior, forward, m_current] = sippi_metropolis (data, prior, forward, options)
Input :
  data : sippi data structure
  prior : sippi prior structure
  forward : sippi forward structure
options :
  options.txt [string] : string to be used as part of all output files
   options.mcmc.nite [1] : Number if iterations
   options.mcmc.i_plot [1]: Number of iterations between updating plots
   options.mcmc.i_sample=: Number of iterations between saving model to disk
   options.mcmc.m_init : Manually chosen starting model
   options.mcmc.m_ref : Reference known target model
  options_mcmc.accept_only_improvements [0] : Optimization
  %% PERTUBATION STRATEGY
  options.mcmc.pert_strategy.perturb_all=1; % Perturb all priors in each
                                           % iteration. def = [0]
  %% SIMULATED ANNEALING
  options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
  options.mcmc.anneal.i_end=100000; % iteration number when annealing stops
  options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration \leftarrow
  options.mcmc.anneal.fac_end=1; % default, noise is scaled by fac_end at iteration \leftrightarrow
      i end
See also sippi_rejection
```

## 7.1.17 sippi\_plot\_current\_model

```
sippi_plot_current_model Plots the current model during Metropolis sampling
Call :
    sippi_plot_current_model(mcmc, data, d, m_current, prior);
```

#### 7.1.18 sippi\_plot\_data

```
sippi_plot_data plot data in SIPPI
Call.
    sippi_plot_data(d,data);
```

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## 7.1.19 sippi\_plot\_loglikelihood

```
sippi_plot_loglikelihood Plot loglikelihood time series

Call :
    acc=sippi_plot_loglikelihood(logL,i_acc,N,itext)
```

## 7.1.20 sippi\_plot\_model

```
Call :
    sippi_plot_model (prior, m, im_array);

prior : Matlab structure for SIPPI prior model
    m : Matlab structure for SIPPI realization
    im_array : integer array of type of models to plot (typically 1)

Example
    m=sippi_prior(prior);
    sippi_plot_model(prior, m);

m=sippi_prior(prior);
    sippi_plot_model(prior, m, 2);

See also sippi_plot_prior
```

### 7.1.21 sippi\_plot\_movie

```
sippi_plot_movie plot movie of prior and posterior realizations

Call :
    sippi_plot_movie(fname);
    sippi_plot_movie(fname,im_array,n_frames,skip_burnin);
        fname : name of folder with results (e.g. options.txt)
        im_array : array of indexes of model parameters to make into movies
        n_frames [200] : number of frames in movie
        skip_burnin [200] : start movie after burn_in;

Ex:
    sippi_plot_movie('20130812_Metropolis');
    sippi_plot_movie(options.txt);

%% 1000 realization including burn-in, for prior number 1
    sippi_plot_movie('20130812_Metropolis',1,1000,0);
```

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## 7.1.22 sippi\_plot\_posterior

```
sippi_plot_posterior Plot statistics from posterior sample

Call :
    sippi_plot_posterior(fname,im_arr,prior,options,n_reals);

See also sippi_plot_prior
```

## 7.1.23 sippi\_plot\_prior

```
sippi_plot_prior Plot a sample of the prior in SIPPI

Call :
    sippi_plot_prior(prior,ip,n_reals,cax,supt);

See also sippi_plot_posterior, sippi_plot_model
```

## 7.1.24 sippi\_prior

```
sippi_prior A priori models for SIPPI
To generate a realization of the prior model defined by the prior structure use:
  [m_propose, prior] = sippi_prior(prior);
To generate a realization of the prior model defined by the prior structure,
in the vicinity of a current model (using sequential Gibbs sampling) use:
  [m_propose, prior] = sippi_prior(prior, m_current);
The following types of a priori models can be used
  images. Relies in the SNESIM algorithm
         [1D-3D] : based on Sequential indicator SIMULATION
         [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
  VISIM
          [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
            [1D] : 1D generalized gaussian model
%%% SIMPLE EXAMPLE %%%
\mbox{\ensuremath{\$}} A simple 2D multivariate Gaissian based prior model based on the
% FFT-MA method, can be defined using
  im=1:
  prior(im).type='FFTMA';
  prior(im).name='A SIMPLE PRIOR';
  prior{im}.x=[0:1:100];
  prior{im}.y=[0:1:100];
  prior(im).m0=10;
  prior{im}.Va='1 Sph(10)';
  prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
  m=sippi_prior(prior);
% This realization can now be plotted using
  sippi_plot_prior(m,prior);
 imagesc(prior{1}.x,prior{1}.y,m{1})
```

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```
%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'
  im=1;
  prior{im}.type='GAUSSIAN';
   prior(im).m0=100;
   prior(im).std=50;
   prior(im).norm=100;
  im=2:
  prior(im).type='FFTMA';
  prior{im}.x=[0:1:100];
  prior{im}.y=[0:1:100];
   prior(im).m0=10;
   prior{im}.Cm='1 Sph(10)';
   im=3;
   prior(im).type='SISIM';
   prior{im}.x=[0:1:100];
   prior{im}.y=[0:1:100];
  prior(im).m0=10;
  prior{im}.Cm='1 Sph(10)';
  im=4:
  prior(im).type='SNESIM';
  prior{im}.x=[0:1:100];
  prior{im}.y=[0:1:100];
   sippi_plot_model(prior);
%% Sequential Gibbs sampling
  All a priori model types can be perturbed, such that a new realization
   is generated in the vicinity of a current model.
   To do this Sequential Gibbs Sampling is used.
   For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596 ↔
       -011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse
      problems with non-trivial priors - Efficient solution through Sequential Gibbs \ \leftrightarrow
      Sampling. Computational Geosciences</a>.
   The type of sequential Gibbs sampling can be controlled in the
   'seq_gibbs' structures, e.g. prior{1}.seq_gibbs
  im=1:
  prior{im}.type='SNESIM';
  prior{im}.x=[0:1:100];
  prior{im}.y=[0:1:100];
   [m, prior] = sippi_prior(prior);
  prior{1}.seq_gibbs.step=1; % Large step--> independant realizations
   prior{1}.seq_gibbs.step=.1; % Smaller step--> Dependant realizations
   for i=1:30;
      [m,prior]=sippi_prior(prior,m); % One iteration of Sequential Gibbs
      sippi_plot_model(prior,m);
   end
 See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m
 TMH/2012
```

#### 7.1.25 sippi\_prior\_fftma

```
sippi_prior A priori models for SIPPI
```

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```
To generate a realization of the prior model defined by the prior structure use:
   [m_propose, prior] = sippi_prior(prior);
To generate a realization of the prior model defined by the prior structure,
 in the vicinity of a current model (using sequential Gibbs sampling) use:
   [m_propose, prior] = sippi_prior(prior, m_current);
 The following types of a priori models can be used
   SNESIM [1D-3D] : based on a multiple point statistical model inferref from a training \leftrightarrow
      images. Relies in the SNESIM algorithm
   SISIM
           [1D-3D] : based on Sequential indicator SIMULATION
           [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
           [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
   GAUSSIAN [1D] : 1D generalized gaussian model
%%% SIMPLE EXAMPLE %%%
% A simple 2D multivariate Gaissian based prior model based on the
% FFT-MA method, can be defined using
  id=1;
  prior{id}.type='FFTMA';
  prior{id}.name='A SIMPLE PRIOR';
  prior{id}.x=[0:1:100];
  prior{id}.y=[0:1:100];
  prior{id}.m0=10;
   prior{id}.Va='1 Sph(10)';
  prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
  m=sippi_prior(prior);
% This realization can now be plotted using
  sippi_plot_prior(m,prior);
   imagesc(prior{1}.x,prior{1}.y,m{1})
%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'
   id=1;
   prior{id}.type='FFTMA';
   prior{id}.x=[0:1:100];
  prior{id}.y=[0:1:100];
  prior{id}.m0=10;
  prior{id}.Cm='1 Sph(10)';
   id=2:
   prior{id}.type='SISIM';
   prior{id}.x=[0:1:100];
   prior{id}.y=[0:1:100];
   prior{id}.m0=10;
   prior{id}.Cm='1 Sph(10)';
   id=3;
   prior{id}.type='GAUSSIAN';
   prior(id).m0=100;
   prior{id}.std=50;
   prior{id}.norm=100;
   prior=sippi_prior_init(prior);
   sippi_plot_model(prior);
%% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596 \leftrightarrow
   -011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems \leftrightarrow
```

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```
with non-trivial priors - Efficient solution through Sequential Gibbs Sampling. ←
   Computational Geosciences</a>.

See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m

TMH/2012
```

## 7.1.26 sippi prior init

```
sippi_prior_init Initialize PRIOR structure for SIPPI

Call
    prior=sippi_prior_init(prior);

See also sippi_prior
```

## 7.1.27 sippi\_prior\_new

```
sippi_prior A priori models for SIPPI
To generate a realization of the prior model defined by the prior structure use:
  [m_propose,prior] = sippi_prior(prior);
To generate a realization of the prior model defined by the prior structure,
in the vicinity of a current model (using sequential Gibbs sampling) use:
  [m_propose, prior] = sippi_prior(prior, m_current);
The following types of a priori models can be used
  images. Relies in the SNESIM algorithm
  SISIM [1D-3D] : based on Sequential indicator SIMULATION
        [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
  VISIM
          [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
           [1D] : 1D generalized gaussian model
%%% SIMPLE EXAMPLE %%%
% A simple 2D multivariate Gaissian based prior model based on the
% FFT-MA method, can be defined using
  id=1;
  prior{id}.type='FFTMA';
  prior{id}.name='A SIMPLE PRIOR';
  prior{id}.x=[0:1:100];
  prior{id}.y=[0:1:100];
  prior{id}.m0=10;
  prior{id}.Va='1 Sph(10)';
  prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
  m=sippi_prior(prior);
% This realization can now be plotted using
  sippi_plot_prior(m,prior);
  imagesc(prior{1}.x,prior{1}.y,m{1})
```

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```
%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'
   prior{id}.type='FFTMA';
   prior{id}.x=[0:1:100];
   prior{id}.y=[0:1:100];
   prior{id}.m0=10;
   prior{id}.Cm='1 Sph(10)';
   id=2:
   prior{id}.type='SISIM';
   prior{id}.x=[0:1:100];
   prior{id}.y=[0:1:100];
   prior{id}.m0=10;
   prior{id}.Cm='1 Sph(10)';
   id=3;
   prior{id}.type='GAUSSIAN';
   prior{id}.m0=100;
   prior{id}.std=50;
   prior{id}.norm=100;
  prior=sippi_prior_init(prior);
   sippi_plot_model(prior);
%% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596 \leftrightarrow
   -011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems
   with non-trivial priors - Efficient solution through Sequential Gibbs Sampling. \leftrightarrow
   Computational Geosciences</a>.
 See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m
 TMH/2012
```

### 7.1.28 sippi\_prior\_old

```
sippi_prior A priori models for SIPPI
To generate a realization of the prior model defined by the prior structure use:
   [m_propose, prior] = sippi_prior(prior);
To generate a realization of the prior model defined by the prior structure,
in the vicinity of a current model (using sequential Gibbs sampling) use:
   [m_propose, prior] = sippi_prior(prior, m_current);
The following types of a priori models can be used
   SNESIM [1D-3D]: based on a multiple point statistical model inferref from a training \leftrightarrow
      images. Relies in the SNESIM algorithm
  SISIM [1D-3D] : based on Sequential indicator SIMULATION
  VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
  FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
  GAUSSIAN [1D] : 1D generalized gaussian model
%%% SIMPLE EXAMPLE %%%
% A simple 2D multivariate Gaissian based prior model based on the
% FFT-MA method, can be defined using
 id=1;
```

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```
prior{id}.type='FFTMA';
  prior{id}.name='A SIMPLE PRIOR';
  prior{id}.x=[0:1:100];
  prior{id}.y=[0:1:100];
  prior{id}.m0=10;
  prior{id}.Va='1 Sph(10)';
  prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
  m=sippi_prior(prior);
% This realization can now be plotted using
  sippi_plot_prior(m,prior);
% or
  imagesc(prior{1}.x,prior{1}.y,m{1})
%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'
   id=1:
   prior{id}.type='FFTMA';
  prior{id}.x=[0:1:100];
  prior{id}.y=[0:1:100];
  prior{id}.m0=10;
  prior{id}.Cm='1 Sph(10)';
  id=2;
  prior{id}.type='SISIM';
  prior{id}.x=[0:1:100];
  prior{id}.y=[0:1:100];
  prior{id}.m0=10;
  prior{id}.Cm='1 Sph(10)';
  id=3;
  prior{id}.type='GAUSSIAN';
  prior{id}.m0=100;
  prior{id}.std=50;
  prior{id}.norm=100;
  prior=sippi_prior_init(prior);
   sippi_plot_model(prior);
%% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596 \leftrightarrow
   -011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems
   with non-trivial priors - Efficient solution through Sequential Gibbs Sampling. \leftrightarrow
   Computational Geosciences</a>.
 See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m
 TMH/2012
```

### 7.1.29 sippi prior set steplength

```
sippi_prior_set_steplength Set step length for Metropolis sampler in SIPPI

Call
    prior=sippi_prior_set_steplength(prior, mcmc, im);
```

#### 7.1.30 sippi\_rejection

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## 7.1.31 sippi\_set\_path

sippi\_set\_path Set paths for running sippi

## 7.2 SIPPI toolbox: Traveltime tomography

## 7.2.1 calc\_Cd

```
Calc_cd Setup a covariance model to account for borehole imperfections
Call: Cd=calc_Cd(ant_pos, var_uncor, var_cor1, var_cor2, L)
This function sets up a data covariance matrix that accounts for static
(i.e. correlated) data errors.
Inputs:
\star ant_pos: A N x 4 array that contains N combinations of transmitter/source
and receiver positions. The first two columns are the x- and y-coordinates
of the transmitter/source position. The last two columns are the x- and
y-coordiantes of the receiver position.
* var_uncor: The variance of the uncorrelated data errors.
* var_corl: The variance of the correlated data errors
related to the transmitter/source positions.
* var_cor2: The variance of the correlated data errors
related to the receiver positions.
* L: The correlation length for the correlation between the individual
transmitter/source or receiver positions using an exponential covariance
function. For typical static errors the correlation length is set to a
small number (e.g. 10^-6).
For more details and practical examples see:
Cordua et al., 2008 in Vadose zone journal.
```

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```
Cordua et al., 2009 in Journal of applied geophysics.

Knud S. Cordua (2012)
```

#### 7.2.2 eikonal

```
eikonal Traveltime computation by solving the eikonal equation
tmap=eikonal(x,y,z,V,Sources,type);
 x,y,z: arrays defining the x, y, and z axis
 V: velocity field, with size (length(y),length(x),length(z));
 Sources [ndata,ndim] : Source positions
 type (optional): type of eikonal solver: [1]:Fast Marching(default), [2]:FD
 tmap [size(V)]: travel times computed everywhere in the velocity grid
%Example (2D):
  x=[1:1:100];
  y=1:1:100;
  z=1;
  V=ones(100,100); V(:,1:50)=2;
  Sources = [10 50;75 50];
  t=eikonal(x,y,z,V,Sources);
  subplot(1,2,1); imagesc(x,y,t(:,:,1,1)); axis image; colorbar
  subplot(1,2,2); imagesc(x,y,t(:,:,1,2)); axis image; colorbar
See also eikonal_traveltime
```

#### 7.2.3 eikonal\_raylength

```
eikonal_raylength : Computes the raylength from S to R using the eikonal equaiton
Call:
    raylength=eikonal_raylength(x,y,v,S,R,tS,doPlot)
```

#### 7.2.4 eikonal\_traveltime

```
eikonal_traveltime Computes traveltime between sources and receivers by solving the
eikonal equation

t=eikonal_traveltime(x,y,z,V,Sources,Receivers,iuse,type);

x,y,z: arrays defining the x, y, and z axis
V: velocity field, with size (length(y),length(x),length(z));
Sources [ndata,ndim]: Source positions
Receivers [ndata,ndim]: Receiver positions
iuse (optional): optionally only use subset of data. eg.g i_use=[1 2 4];
type (optional): type of eikonal solver: [1]:Fast Marching(default), [2]:FD

tmap [size(V)]: travel times computed everywhere in the velocity grid

%Example (2%
```

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```
Example 2d traveltime compuation
Example (2D):
 x=[1:1:100];
 y=1:1:100;
  z=1;
  V=ones(100,100); V(:,1:50)=2;
  S=[50 50 1;50 50 1];
  R=[90 90 1; 90 80 1];
  t = eikonal\_traveltime (x, y, z, V, S, R)
Example (3D):
  nx=50; ny=50; nz=50;
  x=1:1:nx;
  y=1:1:ny;
  z=1:1:nz;
  V=ones(ny,nx,nz);V(:,1:50,:)=2;
  S=[10 10 1;10 10 1;10 9 1];
  R=[40 40 40; 40 39 40; 40 40 40];
  t=eikonal_traveltime(x,y,z,V,S,R)
See also eikonal
```

## 7.2.5 kernel buursink 2d

#### 7.2.6 kernel\_finite\_2d

```
kernel_finite_2d 2D sensitivity kernels

Call:
   [Knorm, K, dt, options] = kernel_finite_2d(v_ref, x, y, S, R, freq, options);
```

#### 7.2.7 kernel\_fresnel\_2d

```
kernel_fresnel_2d Sensitivity kernel for amplitude and first arrival
Call:
```

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```
[kernel_t,kernel_a,P_omega,omega]=kernel_fresnel_2d(v,x,y,S,R,omega,P_omega);
Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity kernels for seismic Fresenl volume Tomography, Geophysics, 75(5), U35-U46
See also kernel_fresnel_monochrome_2d
Run with no argument for an example.
```

#### 7.2.8 kernel\_fresnel\_monochrome\_2d

```
kernel_fresnel_monochrome_2d 2D monchrome kernel for amplitude and first arrival

Call:
    [kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S,R,omega);
or
    [kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S,R,omega,L,L1,L2);

Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity kernels for seismic Fresenl volume Tomography, Geophysics, 75(5), U35-U46

See also, kernel_fresnel_2d
```

## 7.2.9 kernel\_multiple

```
kernel_multiple Computes the sensitivity kernel for a wave traveling
from S to R.
CALL :
  [K,RAY,Gk,Gray,timeS,timeR,raypath]=kernel_multiple(Vel,x,y,z,S,R,T,alpha,Knorm);
IN :
  Vel [ny,nx] : Velocity field
  x [1:nx] :
  y [1:ny] :
   z [1:nz] :
  S [1,3] : Location of Source
  R [1,3] : Location of Receiver
  T : Donminant period
  alpha: controls exponential decay away ray path
  Knorm [1] : normaliztion of K [0]:none, K:[1]:vertical
OUT :
  K : Sensitivity kernel
  R : Ray sensitivity kernel (High Frequency approx)
  timeS : travel computed form Source
  timeR : travel computed form Receiver
  raypath [nraydata,ndim] : the center of the raypath
The sensitivity is the length travelled in each cell.
See also : fast_fd_2d
```

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TMH/2006

### 7.2.10 kernel\_slowness\_to\_velocity

```
kernel_slowness_to_velocity Converts from slowness to velocity parameterizations

G : kernel [1,nkernels]

V : Velocity field (

CALL:
    G_vel=kernel_slowness_to_velocity(G,V);
or
    [G_vel,v_obs]=kernel_slowness_to_velocity(G,V,t);
or
    [G_vel,v_obs,Cd_v]=kernel_slowness_to_velocity(G,V,t,Cd);
```

### 7.2.11 mspectrum

```
mspectrum : Amplitude and Power spectrum
Call :
    function [A,P,smoothP,kx]=mspectrum(x,dx)

1D (A)mplitude and (P)owerspectrum of x-series with spacing dx
```

## 7.2.12 munk\_fresnel\_2d

```
2D frechet kernel, First Fresnel Zone

See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar Physics 192.

Call:
S=munk_fresnel_2d(T,dt,alpha,As,Ar,K);

T : dominant period
dt :
alpha : degree of cancellation
As : Amplitude fo the wavefield propagating from the source
Ar : Amplitude fo the wavefield propagating from the receiver
K : normalization factor
```

#### 7.2.13 munk\_fresnel\_3d

```
3D frechet kernel, First Fresnel Zone

See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar Physics 192.

Call:
```

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## 7.2.14 tomography\_kernel

```
tomography_kernel Computes the sensitivity kernel for a wave traveling from S to R.
  [K,RAY,Gk,Gray,timeS,timeR,raypath]=tomography_kernel(Vel,x,y,z,S,R,T,alpha,Knorm);
IN :
  Vel [ny,nx] : Velocity field
  x [1:nx] :
  y [1:ny] :
  z [1:nz] :
  S [1,3] : Location of Source
  R [1,3] : Location of Receiver
  T : Donminant period
  alpha: controls exponential decay away ray path
  Knorm [1] : normaliztion of K [0]:none, K:[1]:vertical
  K : Sensitivity kernel
  R : Ray sensitivity kernel (High Frequency approx)
  timeS : travel computed form Source
   timeR : travel computed form Receiver
   raypath [nraydata,ndim] : the center of the raypath
The sensitivity is the length travelled in each cell.
```