

# SIPPI

Ed. version 1.0

---

# Contents

<b>1</b>	<b>Installation</b>	<b>1</b>
1.1	SIPPI	1
1.1.1	SGeMS (optional)	1
<b>2</b>	<b>Setting up SIPPI</b>	<b>2</b>
2.1	prior: The a priori model	2
2.1.1	Types of a priori models	3
2.1.1.1	Uniform distribtion	3
2.1.1.2	1D Generalized Gaussian	3
2.1.1.3	FFTMA - 3D Gaussian model	4
2.1.1.4	VISIM	5
2.1.1.5	CHOLESKY - 3D Gaussian model	6
2.1.1.6	SNESIM	7
2.1.1.6.1	Custom training image	8
2.1.1.6.2	Complete customization	8
2.1.2	Sampling the prior	9
2.1.3	Sequential Gibbs sampling / Conditional Re-sampling	9
2.1.3.1	Controlling sequential Gibbs sampling / Conditional Re-sampling	9
2.2	data: Data and data uncertainties/noise	9
2.2.1	Gaussian measurement noise	10
2.2.1.1	Uncorrelated Gaussian measurement noise	10
2.2.1.2	Correlated Gaussian measurement noise	10
2.2.2	Gaussian modeling error	10
2.3	forward: The forward model	11
2.4	Validating prior, data, and forward	12
<b>3</b>	<b>The a posteriori distribution</b>	<b>13</b>
3.1	Sampling the a posteriori probability density	13
3.1.1	The rejection sampler	13
3.1.2	The extended Metropolis sampler	14
3.1.2.1	Controlling the step length	14
3.1.2.2	The independent extended Metropolis sampler	15
3.1.2.3	Annealing schedule	15
3.2	Simulated Annealing	15

<b>4</b>	<b>Examples</b>	<b>16</b>
4.1	Examples of A priori models	16
4.1.1	Multiple 1D Gaussian prior model	16
4.1.2	Multivariate Gaussian prior with unknown covariance model properties.	17
4.2	Polynomial line fitting	19
4.2.1	The forward problem	19
4.2.2	Reference data, data, forward	19
4.2.3	The prior model	20
4.2.4	Setup and run the Metropolis sampler	21
4.2.5	Setup and run the rejection sampler	22
4.3	Cross hole tomography	22
4.3.1	Reference data set from Arrenæs	22
4.3.2	Travel delay computation: The forward problem	24
4.3.2.1	Ray type forward model (high frequency approximation)	24
4.3.2.2	Fat Ray type forward model (finite frequency approximation)	25
4.3.2.3	Born type forward model (finite frequency approximation)	25
4.3.2.4	The eikonal equation (high frequency approximation)	25
4.3.3	AM13 Gaussian: Inversion of cross hole GPR data from Arrenæs data with a Gaussian type a priori model	25
4.3.3.1	Setting up the data structure	26
4.3.3.2	Setting up the prior model	26
4.3.3.3	Setting up the forward structure	26
4.3.3.4	Testing the setup	27
4.3.3.5	Sampling the a posterior distribution using the extended Metropolis algorithm	28
4.3.3.5.1	Posterior statistics	28
4.3.4	AM13 Gaussian, accounting for modeling errors	29
4.3.5	AM13 Gaussian with bimodal velocity distribution	30
4.3.6	AM13 Gaussian with unknown Gaussian model parameters	32
4.4	Probabilistic covariance/semivariogram inference	34
4.4.1	Inferring a 2D covariance model from the Jura data set	34
<b>5</b>	<b>Bibliography</b>	<b>36</b>
<b>6</b>	<b>Reference</b>	<b>37</b>
6.1	SIPPI	37
6.1.1	getinunits	37
6.1.2	logdet	37
6.1.3	pathdef	38
6.1.4	plotboxpos	38
6.1.5	sippi_adjust_step_size	39

6.1.6	sippi_anneal_adjust_noise	39
6.1.7	sippi_anneal_factor	39
6.1.8	sippi_compute_acceptance_rate	40
6.1.9	sippi_compute_modelization_forward_error	40
6.1.10	sippi_forward	40
6.1.11	sippi_get_sample	40
6.1.12	sippi_get_sample_new	41
6.1.13	sippi_least_squares	41
6.1.14	sippi_likelihood	41
6.1.15	sippi_mcmc_init	42
6.1.16	sippi_metropolis	42
6.1.17	sippi_prior	43
6.1.18	sippi_prior_fftma	45
6.1.19	sippi_prior_init	46
6.1.20	sippi_prior_new	46
6.1.21	sippi_prior_old	47
6.1.22	sippi_prior_set_steplength	48
6.1.23	sippi_rejection	49
6.1.24	sippi_set_path	49
6.1.25	sippi_colormap	49
6.1.26	sippi_plot_current_model	50
6.1.27	sippi_plot_data	50
6.1.28	sippi_plot_defaults	50
6.1.29	sippi_plot_loglikelihood	50
6.1.30	sippi_plot_model	50
6.1.31	sippi_plot_movie	50
6.1.32	sippi_plot_posterior	51
6.1.33	sippi_plot_posterior_2d_marg	51
6.1.34	sippi_plot_posterior_data	51
6.1.35	sippi_plot_posterior_loglikelihood	51
6.1.36	sippi_plot_posterior_sample	52
6.1.37	sippi_plot_prior	52
6.1.38	sippi_plot_prior_sample	52
6.1.39	wiggle	53
6.2	SIPPI toolbox: Traveltime tomography	53
6.2.1	calc_Cd	53
6.2.2	eikonal	54
6.2.3	eikonal_raylength	54
6.2.4	eikonal_traveltime	54

---

6.2.5	kernel_buursink_2d . . . . .	55
6.2.6	kernel_finite_2d . . . . .	56
6.2.7	kernel_fresnel_2d . . . . .	56
6.2.8	kernel_fresnel_monochrome_2d . . . . .	56
6.2.9	kernel_multiple . . . . .	56
6.2.10	kernel_slowness_to_velocity . . . . .	57
6.2.11	mspectrum . . . . .	57
6.2.12	munk_fresnel_2d . . . . .	57
6.2.13	munk_fresnel_3d . . . . .	58
6.2.14	sippi_forward_traveltime . . . . .	58
6.2.15	tomography_kernel . . . . .	58

---

# List of Figures

4.1	Location of boreholes AM1, AM2, AM3, and AM4 at Arrenæs. . . . .	23
4.2	Ray coverage between wells left) AM1-AM3, middle) AM2-AM4, right) AM1-4. . . . .	23
4.3	AM13: One sample (15 realizations) of the prior (Gaussian) model. . . . .	27
4.4	AM13: Data response from one realization of the prior. . . . .	28
4.5	5 realizations from a FFTMA prior model type with top) Gaussian and b) Bimodal distribution . . . . .	31
4.6	Distribution of one realization using a Gaussian Bimodal target distribution . . . . .	31
4.7	A sample from a FFTMA type prior model with varying range_1, range_2, and ang_1. . . . .	33
4.8	Distribution of one sample of a 1D Gaussian distribution describing range_1, range_2, and ang_1 . . .	34

# About

SIPPI is a Matlab toolbox (compatible with GNU Octave) that allows Sampling the solution of non-linear Inverse Problems with realistic a Priori Information.

In order to make use of SIPPI one has to

- Install and setup SIPPI
- Define the **prior model**, in form of the prior data structure
- Define the **forward model**, in form of the forward data structure, and the `sippi_forward.m` m-file
- Define the **data and noise model**, in form of the prior data structure
- Choose a method for **sampling the a posteriori probability density (i.e., solution) of the inverse problem**.

Details about the implementation and the methods implemented in SIPPI can be found in [\[HCM12\]](#), [\[CHM12\]](#), [\[HCLM13a\]](#), [\[HCLM13b\]](#) and, [\[HCM14\]](#).

This version of the documentation was compiled on Oct 01, 2014 , and refer to SIPPI version 1.10.

# Chapter 1

## Installation

### 1.1 SIPPI

**Download** the latest version of SIPPI from <http://sippi.sourceforge.net>.

Unpack ZIPPI.zip somewhere, for example to 'c:\Users\tmh\SIPPI'. Then setup the Matlab path to point to the appropriate SIPPI directories by typing:

```
addpath c:\Users\tmh\SIPPI
sippi_set_path
```

#### 1.1.1 SGeMS (optional)

To make use of the SISIM and SNESIM type prior models, SGeMS needs to be available.

Currently only SGeMS version 2.1 (**download**) for Windows is supported by SIPPI.



## Chapter 2

# Setting up SIPPI

This section contains information about how to use and control SIPPI, which requires one to

- Define the **prior model**, in form of the prior data structure
- Define the **forward model**, in form of the forward data structure, and the sippi\_forward.m m-file
- Define the **data and noise model**, in form of the prior data structure

[For examples of how to apply SIPPI for different problems, see [the section with examples](#)].

### 2.1 prior: The a priori model

A priori information is defined by the prior Matlab structure. Any number of different types of a priori models can be defined. For example a 1D uniform prior can be defined in prior{1}, and 2D Gaussian prior can be defined in prior{2}.

Once a prior data structure has been defined, a realization from the prior model can be generated using

```
m=sippi_prior(prior);
```

The realization from the prior can be visualized using

```
sippi_plot_prior(prior);  
sippi_plot_prior(prior,m);
```

A sample (many realizations) from the prior can be visualized using

```
m=sippi_plot_prior_sample(prior);
```

Each prior type is defined by setting a number field in the prior Matlab structure. For example, an descriptive name (which is can be optionally set) describing the prior can be set in the name field, e.g.

```
prior{1}.name='My Prior';
```

## 2.1.1 Types of a priori models

Six types of a priori models are available, and can be selected by setting the `type` in the prior structure using e.g. `prior{1}.type='gaussian'`.

The **UNIFORM** type prior specifies an uncorrelated ND uniform model.

The **GAUSSIAN** type prior specifies a 1D generalized Gaussian model.

The **FFTMA** type prior specifies a 1D-3D Gaussian type a priori model based on the FFT Moving Average method, which is very efficient for unconditional sampling,

The **CHOLESKY** type prior specifies a 1D-3D Gaussian type a priori model based on Cholesky decomposition of the covariance model,

The **VISIM** type prior model specifies 1D-3D Gaussian models, utilizing both sequential Gaussian simulation (SGSIM) and direct sequential simulation (DSSIM) that can be conditioned to data of both point- and volume support and linear average data.

The **SNESIM** type prior model specifies a 1D-3D multiple-point-based statistical prior model, which relies on training images from where the conditional dependencies of the spatial variables are obtained (i.e., learned). This type of prior model requires **SGEMS** to be installed.

The following section documents the properties of each type of prior model.

Examples of using different types of prior models or combining prior models can be found in the [examples section](#).

### 2.1.1.1 Uniform distribution

A uniform prior model can be specified using the 'uniform' type prior model

```
prior{1}.type='uniform';
```

The only parameters needed are the minimum (`min`) and maximum (`max`) values. A 1D uniform distribution between -1 and 1 can be specified as

```
prior{1}.type='uniform';
prior{1}.min=-1;
prior{1}.max=1;
```

By setting the `x`, `y`, and `z` parameter, a higher order prior (uncorrelated) can be set. For example 3 independent model parameters with a uniform prior distribution between 20 and 50, can be defined as

```
prior{1}.type='uniform';
prior{1}.x=[1 2 3];
prior{1}.min=20;
prior{1}.max=50;
```

Note that using the 'uniform' type prior model, is slightly more computationally efficient than using a **'gaussian'** type prior model with a high norm.

### 2.1.1.2 1D Generalized Gaussian

A 1D generalized Gaussian prior model can be specified using the 'gaussian' type prior model

```
prior{1}.type='gaussian';
```

A simple 1D Gaussian distribution with mean 10, and standard deviation 2, can be specified using

```
ip=1;
prior{ip}.type='gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;
```

The norm of a generalized Gaussian can be set using the 'norm' field. A generalized 1D Gaussian with mean 10, standard deviation of 2, and a norm of 70, can be specified using (The norm is equivalent to the beta factor referenced in [Wikipedia:Generalized\\_normal\\_distribution](#))

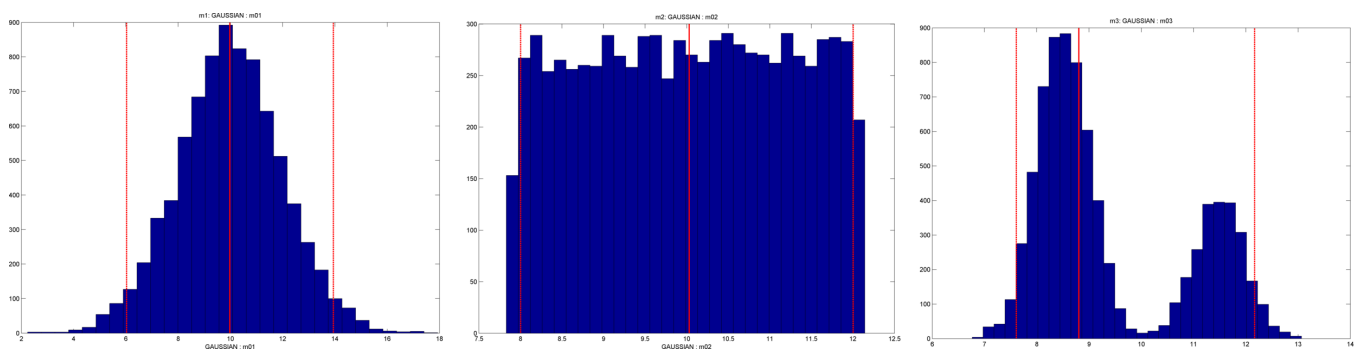
```
ip=2;
prior{ip}.type='gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=70;
```

A 1D distribution with an arbitrary shape can be defined by setting `d_target`, which must contain a sample of the distribution that one would like to replicate. For example, to generate a sample from a non-symmetric bimodal distribution, one can use e.g.

```
% Create target distribution
N=10000;
prob_chan=0.3;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];

% set the target distribution
ip=3;
prior{ip}.type='gaussian';
prior{ip}.d_target=d_target;
```

The following figure shows the 1D histogram of a sample, consisting of 8000 realizations, generated using `sippi_plot_prior_sample(prior,1:ip,8000);`



### 2.1.1.3 FFTMA - 3D Gaussian model

The FFT moving average method provides an efficient approach for computing unconditional realizations of a Gaussian random field.

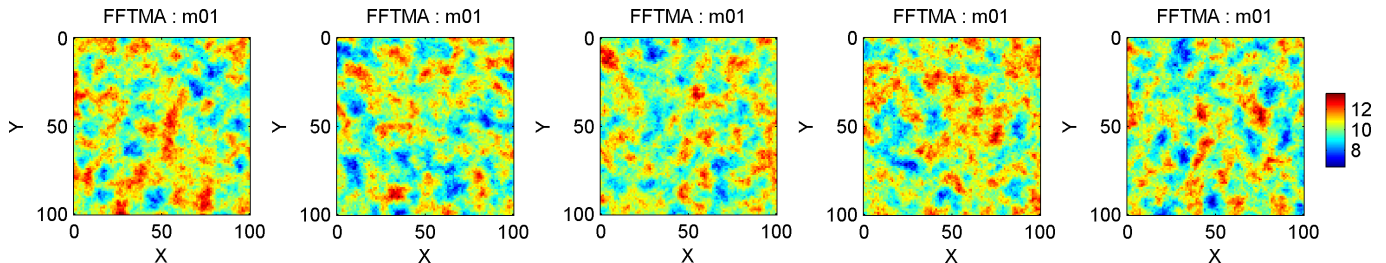
The mean and the covariance model must be specified in the `m0` and `Cm` fields. The format for describing the covariance model follows 'gstat' notation, and is described in more details in the [mGstat manual](#).

A 2D covariance model with mean 10, and a Spherical type covariance model can be defined in a 101x101 size grid (1 unit (e.g., meters) between the cells) using

```

im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

```



Optionally one can translate the output of the Gaussian simulation into an arbitrarily shaped 'target' distribution, using normal score transformation. Note that this transformation will ensure a certain 1D distribution of the model parameters to be reproduced, but will alter the assumed covariance model such that the properties of covariance model are not necessarily reproduced. To ensure that both the covariance model properties and the 1D distribution are reproduced, make use of the VISIM type prior model instead because it utilizes direct sequential simulation.

```

im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.Cm='1 Sph(10)';

% Create target distribution
N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;
prior{im}.m0=0; % to make sure no trend model is assumed.

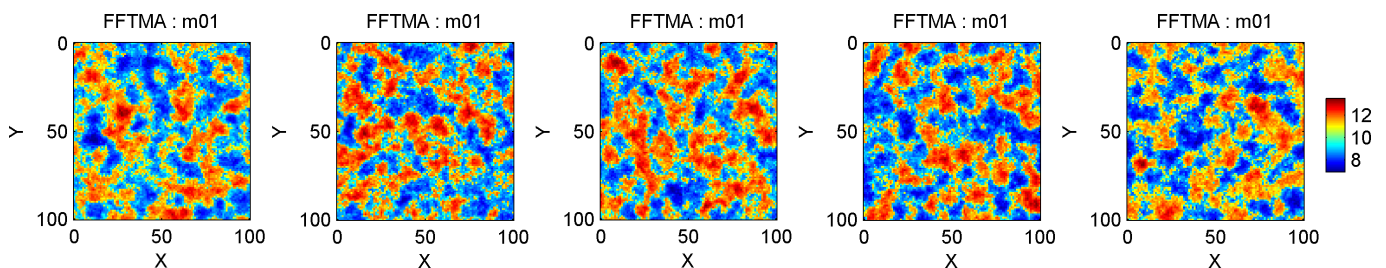
```

Alternatively, the normal score transformation can be defined manually such that the tail behavior can be controlled:

```

N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
[d_nscore,o_nscore]=nscore(d_target,1,1,min(d_target),max(d_target),0);
prior{im}.o_nscore=o_nscore;

```

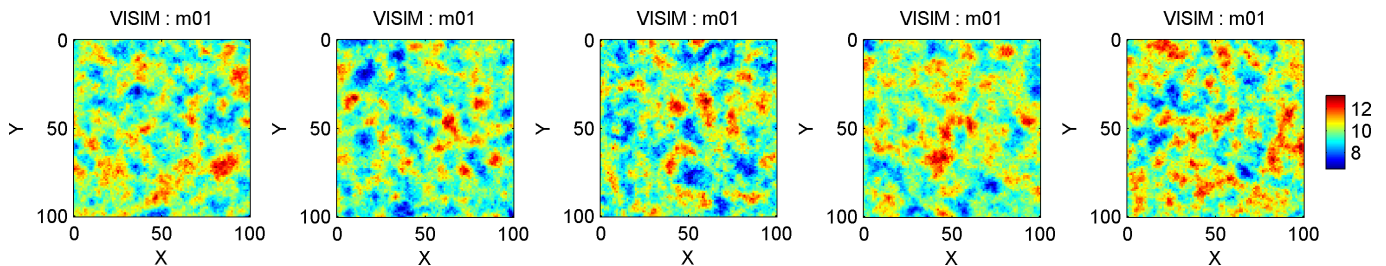


#### 2.1.1.4 VISIM

```

im=im+1;
prior{im}.type='VISIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

```



As with the FFTMA prior model the VISIM prior can make use of a target distribution. However, if a target distribution is set, the use of the VISIM prior model will utilize direct sequential simulation, which will ensure both histogram and covariance reproduction.

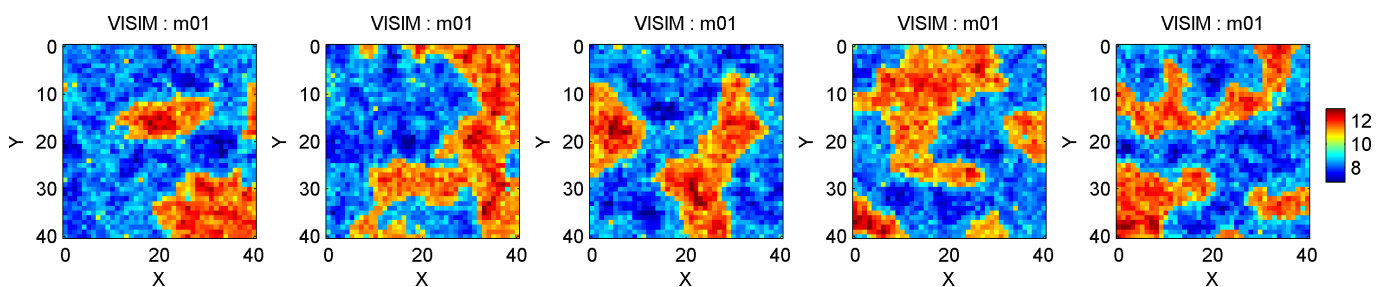
Using a target distribution together with the VISIM prior model is similar to that for the FFTMA prior model. Simply the type has to be changed from FFTMA to VISIM:

```

clear all;close all;
im=1;
prior{im}.type='VISIM';
prior{im}.x=[0:1:40];
prior{im}.y=[0:1:40];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

% Create target distribution
N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;

```



### 2.1.1.5 CHOLESKY - 3D Gaussian model

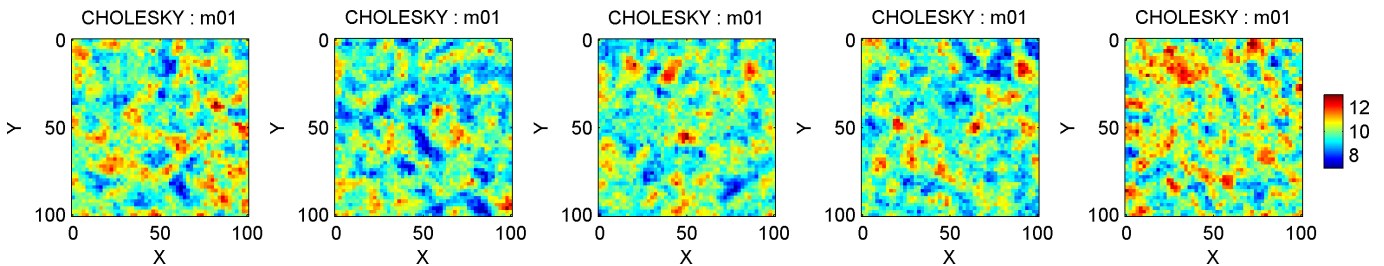
The CHOLESKY type prior utilizes Cholesky decomposition of the covariance in order to generate realizations of a Gaussian random field. The CHOLESKY type prior needs a full description of the covariance model, which will be of size  $[n_{xyz} \times n_{xyz} \times n_{xyz}]$ , unlike using the **FFTMA** type prior model that only needs a specification of an isotropic covariance models of size  $[1, n_{xyz}]$ . Hence, the CHOLESKY type prior is much more demanding on memory, and CPU. However, the CHOLESKY type prior can be used to sample from any covariance model, also non-stationary covariance model.

The CHOLESKY model is can be defined almost identically to the **FFTMA** type prior model. As an example:

```

im=1;
prior{im}.type='CHOLESKY';
prior{im}.x=[0:2:100];
prior{im}.y=[0:2:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

```



the use of `d_target` to specify target distributions is also possible, using the same style as for the **FFTMA** type prior.

Be warned that the 'cholesky' type prior model is much more memory demanding than the 'fftma' and 'visim' type prior models, as a full  $n_{xyz} \times n_{xyz}$  covariance model needs to setup (and inverted). Thus, the 'cholesky' type prior is mostly applicable when the number of model parameters ( $n_x \times n_y \times n_z$ ) is small.

### 2.1.1.6 SNESIM

The 'SNESIM' type prior model utilizes the SNESIM algorithm, as implemented in **SGeMS**. As opposed to the Gaussian prior models defined above, the SNESIM prior model infer spatial statistics from a training image, which should be a 2D/3D stationary image.

By default a training image (channel structures) from Sebastian Strebelle's PhD theses is used (if no training image is specified). A simple 2D type SNESIM prior model can be defined using the following code:

```

ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array

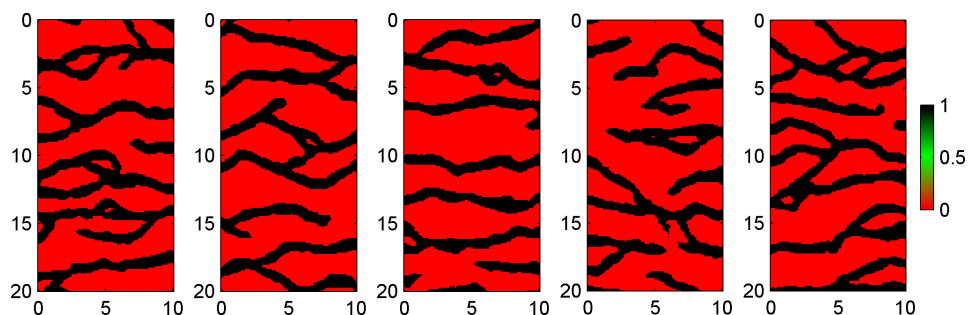
```

and 5 realizations from this prior can be visualized using

```

for i=1:5;
    m=sippi_prior(prior);
    subplot(1,5,i);
    imagesc(prior{1}.x,prior{1}.y,m{1});axis image
end

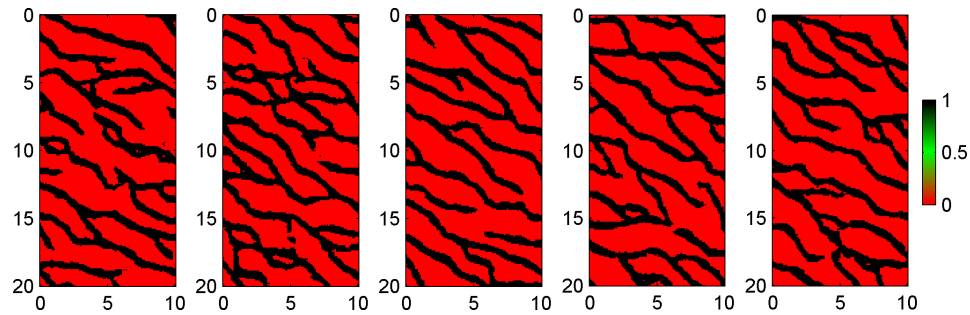
```



Note that the training image is always assumed to have the same units as the prior model, so in this case each pixel in the training image is assumed to be separated by a distance '0.1'.

Optionally 'scaling' and 'rotation' of the training image can be set. To scale the training image by 0.7 (i.e., structures will appear 30% smaller) and rotate the training 30 degrees from north use

```
ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array
prior{ip}.scaling=.7;
prior{ip}.rotation=30;
```



### 2.1.1.6.1 Custom training image

A custom training image can be set using the `ti` field, which must be either a 2D or 3D matrix.

```
% create TI from image
EXAMPLE EXAMPLE

% setup the prior
ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array
prior{ip}.ti=ti;
```

Note that the training image **MUST** consist of integer index values starting from 0 (i.e. '0', '1', '2', ...).

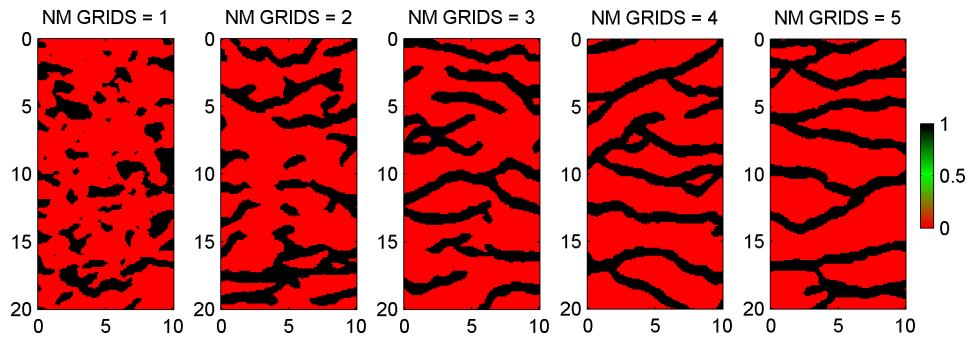
### 2.1.1.6.2 Complete customization

If the prior structure is returned from `sippi_prior` using

```
[m,prior]=sippi_prior(prior);
```

then an XML structure `prior{1}.S.XML` will be available. This allows a complete customization of all settings available in SGeMS. For example, the different realizations, using 1, 2, and 3 multiple grids can be obtained using

```
ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array
[m,prior]=sippi_prior(prior);
for i=1:5;
    prior{ip}.S.XML.parameters.Nb_Multigrids_ADVANCED.value=i;
    subplot(1,3,5);
    imagesc(prior{1}.x,prior{1}.y,m{1});axis image
end
```



## 2.1.2 Sampling the prior

Once the prior data structure has been defined/modified, a sample from the prior distribution can be generated using

```
m=sippi_prior(prior);
```

'm' is a Matlab data structure of the same size as the 'prior' data structure. Thus, if two prior distributions have been defined in 'prior{1}' and 'prior{2}', then 'm{1}' will hold a realization of 'prior{1}', and 'm{2}' will hold a realization of 'prior{2}'.

Each time 'm=sippi\_prior(prior)' is called, a new independent realization of the prior will be generated.

## 2.1.3 Sequential Gibbs sampling / Conditional Re-sampling

All the available types of prior models allow perturbing one realization of a prior into a new realization of prior, where the degree of perturbation can be controlled (from a new independent realization to a very small change). To do this, we make use of sequential Gibbs sampling [HCM12]. Sequential Gibbs is in essence a type of conditional re-simulation. From a current realization of a prior model, a number of model parameters are discarded and treated as unknown. The unknown model parameters are then re-simulated conditional to the known model parameters.

In order to generate a new realization 'm2' in the vicinity of the realization 'm1' use

```
m1=sippi_prior(prior);
[m2,prior]=sippi_prior(prior,m1);
```

If this process is iterated, then a random walk in the space of a priori acceptable models will be performed. Moreover, the collection of realization obtained in this way will represent a sample from prior distribution.

### 2.1.3.1 Controlling sequential Gibbs sampling / Conditional Re-sampling

All properties related to sequential Gibbs sampling can be set in the 'seq\_gibbs' structure, for the individual prior models. The following two parameters determine how the a current model is perturbed

```
prior{m}.seq_gibbs.step=1;
prior{m}.seq_gibbs.type=2;
```

## 2.2 data: Data and data uncertainties/noise

data is a Matlab structure that defines any number of data and the associated uncertainty/noise model.

data{1} defines the first data set (which must always be defined), and any number of additional data sets can be defined in data{2}, data{3}, ...



This allows to consider for example seismic data in `data{1}`, and electromagnetic data in `data{2}`.

For each set of data, a Gaussian noise model (both correlated and uncorrelated) can be specified. The noise model for different data types (e.g. `data{1}` and `data{2}` are independent).

Once the noise model has been defined, the log-likelihood related to any model,  $m$ , with the corresponding **forward response**,  $d$ , can be computed using

```
[d, forward, prior, data]=sippi_forward(m, forward, prior, data)
logL=sippi_likelihood(data, d)
```

where  $d$  is the output of **sippi\_forward**.

The specification of the noise model can be divided into a description of the **measurement noise** (mandatory) and the **modeling error** (optional).

## 2.2.1 Gaussian measurement noise

### 2.2.1.1 Uncorrelated Gaussian measurement noise

To define a set of observed data, `[0,1,2]`, with an associated uncorrelated uncertainty defined by a Gaussian model with mean 0 and standard deviation 2, use

```
data{1}.d_obs=[0 1 2]';
data{1}.d_std=[2 2 2]';
```

which is equivalent to (as the noise model for each data is the same, and independent)

```
data{1}.d_obs=[0 1 2]';
data{1}.d_std=2;
```

One can also choose to define the uncertainty using a variance as opposed to the standard deviation

```
data{1}.d_obs=[0 1 2]';
data{1}.d_var=4;
```

### 2.2.1.2 Correlated Gaussian measurement noise

Correlated Gaussian measurement uncertainty can be specified using the `Cd` field, as for example

```
data{1}.Cd=[4 1 0 ; 1 4 1 ; 0 1 4];
```

Note that `data{1}.Cd` must be of size  $[ND \times ND]$ , where  $ND$  is the the number of data in `data{1}.d_obs`.

## 2.2.2 Gaussian modeling error

The modeling error refers to errors caused by using for example an imperfect forward model, see [\[HCM14\]](#).

A Gaussian model of the modeling error can be specified by the mean,  $dt$ , and the covariance,  $Ct$ .

For example

```
data{1}.dt=[0 0 0];
data{1}.Ct=[4 4 4; 4 4 4; 4 4 4];
```

is equivalent to

```
data{1}.Ct=4
```

which implies a zero mean modeling error with a covariance model where all model parameters has a covariance of 4.

`sippi_compute_modelization_forward_error` can be used to estimate the modeling error related to using an approximate forward model. See the [tomography example](#), for an [example of accounting for correlated modeling errors](#), following [\[HCM14\]](#).

## 2.3 forward: The forward model

The specification of the prior and data is intended to be generic, applicable to any inverse problem considered. The forward problem, on the other hand, is typically specific for each different inverse problem.

In order to make use of SIPPI to sample the posterior distribution of an inverse problem, the solution to the forward problem must be embedded in a Matlab function with the following input and output arguments:

```
[d, forward, prior, data] = sippi_forward(m, forward, prior, data, id)
```

`m` is a realization of the prior model, and `prior` and `data` are the Matlab structures defining the prior and the noise model (see [Prior](#) and [Data](#))

`id` is optional, and can be used to compute the forward response of a subset of the different types of data available (i.e. `data{1}`, `data{2}`, ...) )

The forward variable is a Matlab structure that can contain any information needed to solve the forward problem. Thus, the parameters for the forward structure is problem dependent. One option, `forward.forward_function` is though generic, and point to the m-file that implements the forward problem.

The output variable `d` is a Matlab structure of the same size of `data`. Thus, if 4 types of data have been specified, then `d` must also be a structures of size 4.

```
length(data) == length(d);
```

Further, `d{i}` must refer to an array of the same size as `data{i}.d_obs`.

An example of an implementation of the forward problem related to a simple line fitting problem is:

```
function [d, forward, prior, data] = sippi_forward_linefit(m, forward, prior, data);
    d{1} = forward.x*m{2} + m{1};
```

This implementation requires that the 'x'-locations, for which the y-values of the straight line is to be computed, is specified through `forward.x`. Say some y-data has been observed at locations `x=[1,5,8]`, with the values `[2,4,9]`, and a standard deviation of 1 specifying the uncertainty, the forward structure must be set as

```
forward.forward_function = 'sippi_forward_linefit';
forward.x = [1, 5, 8];
```

while the data structure will be

```
data{1}.d_obs = [2 4 9]
data{1}.d_std = 1;
```

This implementation also requires that the prior model consists of two 1D prior types, such that

```
m = sippi_prior(prior)
```

returns the intercept in `m{1}` and the gradient in `m{2}`.

An example of computing the forward response using an intercept of 0, and a gradients of 2 is then

```
m{1} = 0;
m{2} = 2;
d = sippi_forward(m, forward)
```

and the corresponding log-likelihood of  $m$ , can be computed using

```
logL=sippi_likelihood(data,d);
```

[see more details and examples related to polynomial line fitting at [polynomial line fitting](#)].

The [Examples](#) section contains more example of implementation of different forward problems.

## 2.4 Validating prior, data, and forward

A simple way to test the validity of prior, data, and forward is to test if the following sequence can be evaluated without errors:

```
% Generate a realization, m, of the prior model
m=sippi_prior(prior);
% Compute the forward response
d=sippi_forward(m,forward,prior,data);
% Evaluate the log-likelihood of m
logL=sippi_likelihood(data,d);
```

## Chapter 3

# The a posteriori distribution

### 3.1 Sampling the a posteriori probability density

Once the prior, data, and forward data structures have been defined, the associated a posteriori probability can be sampled using the rejection sampler and the extended Metropolis sampler.

#### 3.1.1 The rejection sampler

The rejection sampler provides a simple, and also in many cases inefficient, approach to sample the posterior distribution.

At each iteration of the rejection sample an independent realization,  $m_{\text{pro}}$ , of the prior is generated, and the model is accepted as a realization of the posterior with probability  $P_{\text{acc}} = L(m_{\text{pro}})/L_{\text{max}}$ . It can be initiated using

```
options.mcmc.nite=400000; % Number of iteration, defaults to 1000
options.mcmc.i_plot=500; % Number of iteration between visual updates, defaults to 500
options=sippi_rejection(data,prior,forward,options);
```

By default the rejection sampler is run assuming a maximum likelihood of 1 (i.e.  $L_{\text{max}} = 1$ ). If  $L_{\text{max}}$  is known, then it can be set using in the options.Lmax or options.logLmax fields

```
options.mcmc.Lmax=1e-9;
options=sippi_rejection(data,prior,forward,options);
```

or

```
options.mcmc.logLmax=log(1e-9);
options=sippi_rejection(data,prior,forward,options);
```

Alternatively,  $L_{\text{max}}$  can be automatically adjusted to reflect the maximum likelihood found while running the rejection sampler using

```
options.mcmc.adaptive_rejection=1
options=sippi_rejection(data,prior,forward,options);
```

An alternative to rejection sampling, also utilizing independent realizations of the prior, that does not require one to set  $L_{\text{max}}$  is the **independent extended metropolis sampler**, which may be computationally superior to the rejection sampler,

### 3.1.2 The extended Metropolis sampler

The extended Metropolis algorithm is in general a much more efficient algorithm for sampling the a posteriori probability

The extended Metropolis sampler can be run using

```
options.mcmc.nite=40000; % number of iterations, default nite=30000
options.mcmc.i_sample=50; % save the current model for every 50 iterations, default, ↵
    i_sample=500
options.mcmc.i_plot=1000; % plot progress of the Metropolis sampler for every 100 ↵
    iterations
                        % default i_plot=50;
options.txt='case_line_fit'; % descriptive name appended to output foldername, default txt ↵
    ='';

[options,data,prior,forward,m_current]=sippi_metropolis(data,prior,forward,options)
```

One can choose to accept all steps in the Metropolis sampler, which will result in an algorithm sampling the prior model, using

```
options.mcmc.accept_all=1; % default [0]
```

One can choose to accept models that lead to an improvement in the likelihood, which results in an optimization like algorithm using

```
options.mcmc.accept_only_improvements=1; % default [0]
```

See [sippi\\_metropolis](#) for more details.

#### 3.1.2.1 Controlling the step length

One optionally, as part of running the [extended Metropolis sampler](#), automatically update the 'step'-length of the [sequential Gibbs sampler](#) in order to ensure a specific approximate acceptance ratio of the Metropolis sampler. See [CHM12] for details.

The default parameters for adjusting the step length, as given below, are set in the '[prior.seq\\_gibbs](#)' structure. These parameters will be set the first time 'sippi\_prior' is called with the 'prior' structure as output. The default parameters.

```
prior{m}.seq_gibbs.step_min=0;
prior{m}.seq_gibbs.step_min=1;
prior{m}.seq_gibbs.i_update_step=50
prior{m}.seq_gibbs.i_update_step_max=1000
prior{m}.seq_gibbs.n_update_history=50
prior{m}.seq_gibbs.P_target=0.3000
```

By default, adjustment of the step length, in order to achieve an acceptance ratio of 0.3 ('prior{m}.seq\_gibbs.P\_target'), will be performed for every 50 ('prior{m}.seq\_gibbs.i\_update\_step') iterations, using the acceptance ratio observed in the last 50 ('prior{m}.seq\_gibbs.i\_update\_history') iterations.

Adjustment of the step length will be performed only in the first 1000 ('prior{m}.seq\_gibbs.i\_update\_step\_max') iterations.

In order to disable automatic adjustment of the step length simply set

```
prior{m}.seq_gibbs.i_update_step_max=0; % disable automatic step length
```

### 3.1.2.2 The independent extended Metropolis sampler

The 'independent' extended Metropolis sampler, in which each proposed model is independent of the previously visited model, can be chosen by forcing the 'step'-length to be 1 (i.e. leading to independent samples from the prior), using e.g.

```
% force independent prior sampling
for ip=1:length(prior);
    prior{ip}.seq_gibbs.step=1;
    prior{ip}.seq_gibbs.i_update_step_max=0;
end
% run 'independent' extended Metropolis sampling
[options,data,prior,forward,m_current]=sippi_metropolis(data,prior,forward,options)
```

### 3.1.2.3 Annealing schedule

Simulated annealing like behaviour can be controlled in the `options.mcmc.anneal` structure. By default annealing is disabled.

Annealing consists of multiplying the noise level using an exponentially decreasing noise factor from `options.mcmc.anneal.fac_begin` to `options.mcmc.anneal.fac_end`, from iteration number `options.mcmc.anneal.i_begin` to `options.mcmc.anneal.i_end`.

The annealing schedule can be used to start a Metropolis sampler that allows to explore more of the model space in the beginning. Recall though that the posterior is not sampled until (at least) the annealing has been ended at iteration `options.mcmc.anneal.i_end`, if the `options.mcmc.anneal.fac_end=1`. This can potentially help not to get trapped in a local minima.

To use this type of annealing, where the annealing stops after 10000 iterations, after which the algorithm performs like a regular Metropolis sampler, use for example

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=10000; % iteration number when annealing stops
```

which is equivalent to

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=10000; % iteration number when annealing stops
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration i_begin ↔
options.mcmc.anneal.fac_end=1; % default, noise is scaled by fac_end at iteration i_end
```

## 3.2 Simulated Annealing

Simulated annealing type optimization can be setup using an **annealing schedule** that is enabled for the entire run of the Metropolis sampler, and that ends by a noise scaling factor less than 1. This can be obtained using e.g.

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=options.mcmc.nite; % iteration number when annealing stops
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration i_begin ↔
options.mcmc.anneal.fac_end=0.01; % 1/100 of the noise level
```

# Chapter 4

## Examples

SIPPI can be used as a convenient approach for unconditional and conditional simulation.

In order to use SIPPI to solve inverse problems, one must provide the solution to the forward problem. Essentially this amounts to implementing a Matlab function that solves the **forward problem** in using a specific input/output format. If a solution to the forward problem already exists, this can be quite easily done simply using a Matlab wrapper function.

A few implementations of solutions to forward problems are included as examples as part of SIPPI. These will be demonstrated in the following

### 4.1 Examples of A priori models

#### 4.1.1 Multiple 1D Gaussian prior model

A prior model consisting of three independent 1D distributions (a Gaussian, Laplace, and Uniform distribution) can be defined using

```
ip=1;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;

ip=2;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Laplace';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=1;

ip=3;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Uniform';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=60;

m=sippi_prior(prior);

m =

    [14.3082]    [ 9.4436]    [10.8294]
```

1D histograms of a sample (consisting of 1000 realizations) of the prior models can be visualized using ...

```
sippi_plot_prior_sample(prior);
```

#### 4.1.2 Multivariate Gaussian prior with unknown covariance model properties.

The **FFT-MA** type a priori model allow separation of properties of the covariance model (covariance parameters, such as range, and anisotropy ratio) and the random component of a Gaussian model. This allow one to define a Gaussian a priori model, where the covariance parameters can be treated as unknown variables.

In order to treat the covariance parameters as unknowns, one must define one a priori model of type FFTMA, and then a number of 1D GAUSSIAN type a priori models, one for each covariance parameter. Each gaussian type prior model must have a descriptive name, corresponding to the covariance parameter that it should describe:

```
prior{im}.type='gaussian';
prior{im}.name='m_0';      % to define a prior for the mean
prior{im}.name='sill';     % to define a prior for sill (variance)
prior{im}.name='range_1'; % to define a prior for the range parameter 1
prior{im}.name='range_2'; % to define a prior for the range parameter 2
prior{im}.name='range_3'; % to define a prior for the range parameter 3
prior{im}.name='ang_1';   % to define a prior for the first angle of rotation
prior{im}.name='ang_2';   % to define a prior for the second angle of rotation
prior{im}.name='ang_3';   % to define a prior for the third angle of rotation
prior{im}.name='nu';      % to define a prior for the shape parameter, nu
                        % (only applies when the Mater type Covariance model is used)
```

A very simple example of a prior model defining a 1D Spherical type covariance model with a range between 5 and 15 meters, can be defined using:

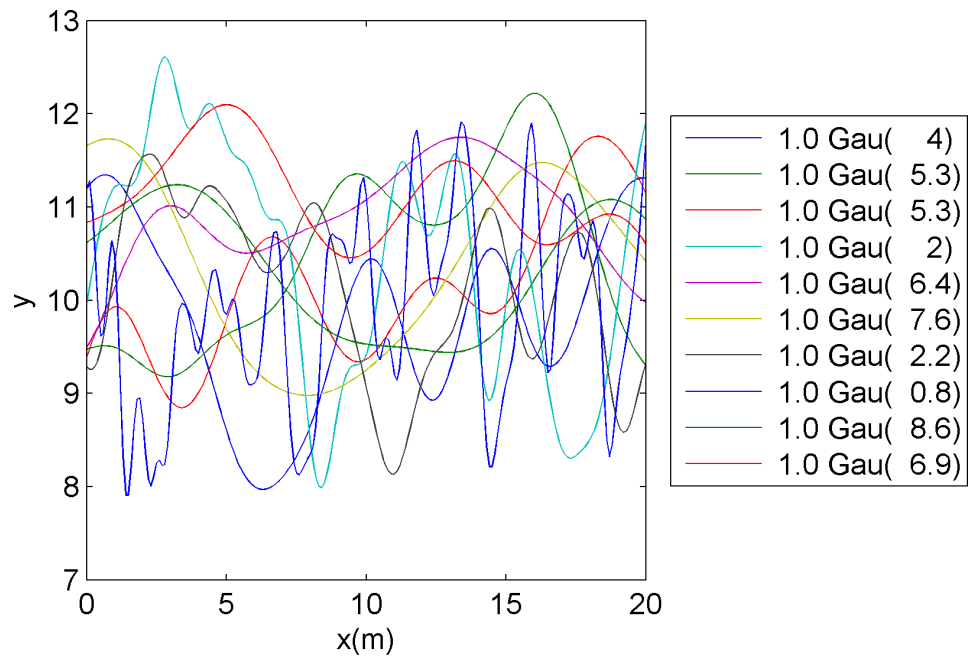
```
im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:.1:10]; % X array
prior{im}.m0=10;
prior{im}.Va='1 Sph(10)';
prior{im}.fftma_options.constant_C=0;

im=2;
prior{im}.type='gaussian';
prior{im}.name='range_1';
prior{im}.m0=10;
prior{im}.std=5;
prior{im}.norm=80;
prior{im}.prior_master=1; % -- NOTE, set this to the FFT-MA type prior for which this prior ←
    type
                        % should describe the range
```

Note that the the field `prior_master` must be set to point the to the FFT-MA type a priori model (through its id/number) for which it should define a covariance parameter (in this case the range).

10 independent realizations of this type of a priori model are shown in the following figure

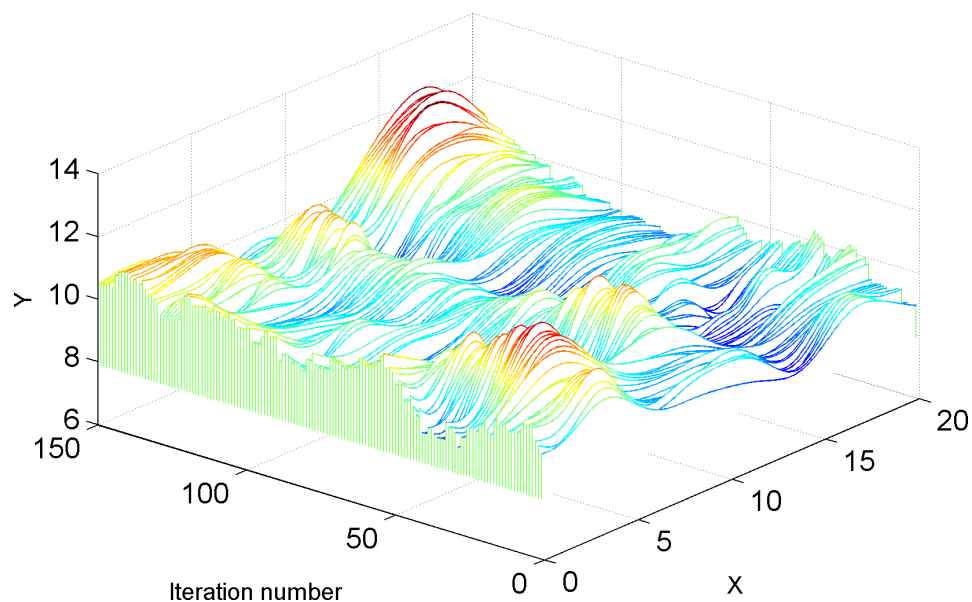




Such a prior, as all prior models available in SIPPI, works with **sequential Gibbs sampling**, allowing a random walk in the space of a prior acceptable models, that will sample the prior model. An example of such a random walk can be performed using

```
prior{1}.seq_gibbs.step=.005;
prior{2}.seq_gibbs.step=0.1;
clear m_real;
for i=1:150;
    [m,prior]=sippi_prior(prior,m);
    m_real(:,i)=m{1};
end
```

An example of such a set of 150 dependent realization of the prior can be seen below



## 4.2 Polynomial line fitting

Here follows simple polynomial (of order 0, 1 or 2) line-fitting is considered. Example m-files can be found in the SIPPI/examples/case\_linefit folder.

First, the forward problem is defined. Then examples of stochastic inversion using SIPPI is demonstrated using a synthetic data set.

### 4.2.1 The forward problem

The forward problem consists of computing the y-value as a function of the x-position of the data, and the polynomial coefficients determining the line. [sippi\\_forward\\_linefit.m](#):

```
% sippi_forward_linefit Line fit forward solver for SIPPI
%
% [d,forward,prior,data]=sippi_forward_linefit(m,forward,prior,data);
%
function [d,forward,prior,data]=sippi_forward_linefit(m,forward,prior,data);

if length(m)==1;
    d{1}=forward.x*m{1};
elseif length(m)==2;
    d{1}=forward.x*m{1}+m{2};
else
    d{1}=forward.x.^2*m{1}+forward.x*m{2}+m{3};
end
```

the forward.x must be an array of the x-locations, for which the y-values of the corresponding line will be evaluated. Note that the prior must be defined such that prior{1} refer to the intercept, prior{2} to the gradient, and prior{3} to the 2nd order polynomial coefficient.

If only one prior type is defined then the forward response will just be a constant, and if two prior types are defined, then the forward response will be a straight line.

### 4.2.2 Reference data, data, forward

A reference data set can be computed using

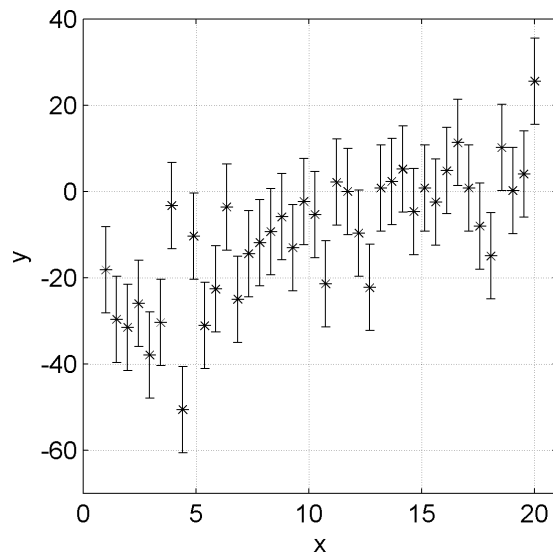
```
clear all;close all;
rand('seed',1);randn('seed',1);

%% Select reference model
m_ref{1}=-30;
m_ref{2}=2;
m_ref{3}=0;

%% Setup the forward model in the 'forward' structure
nd=40;
forward.x=linspace(1,20,nd);
forward.forward_function='sippi_forward_linefit';

%% Compute a reference set of observed data
d=sippi_forward(m_ref,forward);
d_obs=d{1};
d_std=10;
d_obs=d_obs+randn(size(d_obs)).*d_std;

data{1}.d_obs=d_obs;
data{1}.d_std=d_std;
```



### 4.2.3 The prior model

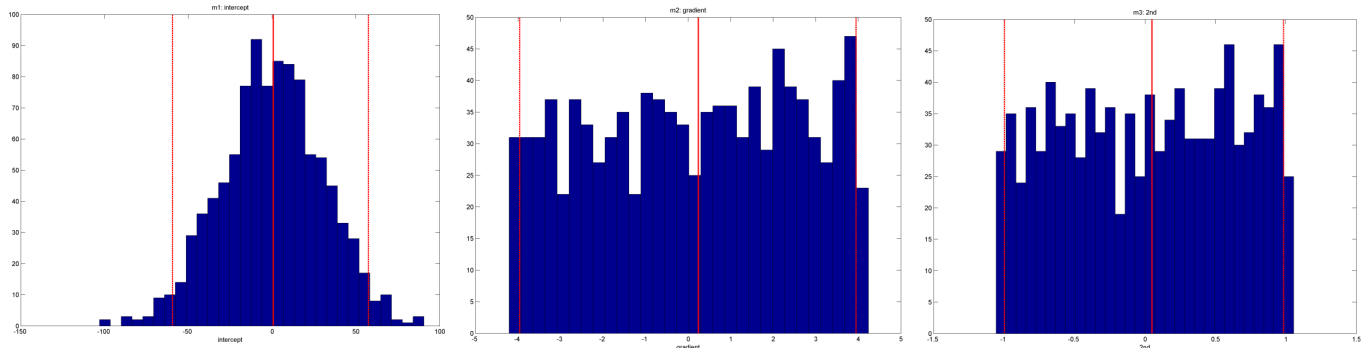
```
%% Setting up the prior model

% the intercept
im=1;
prior{im}.type='gaussian';
prior{im}.name='intercept';
prior{im}.m0=0;
prior{im}.std=30;
prior{im}.m_true=m_ref{1};

% 1st order, the gradient
im=2;
prior{im}.type='gaussian';
prior{im}.name='gradient';
prior{im}.m0=0;
prior{im}.std=4;
prior{im}.norm=80;
prior{im}.m_true=m_ref{2};

% 2nd order
im=3;
prior{im}.type='gaussian';
prior{im}.name='2nd';
prior{im}.m0=0;
prior{im}.std=1;
prior{im}.norm=80;
prior{im}.m_true=m_ref{3};

sippi_plot_prior_sample(prior);
```



#### 4.2.4 Setup and run the Metropolis sampler

Now, information about the model parameters can be inferred by running the **extended Metropolis sampler** using

```
options.mcmc.nite=40000; % Run for 40000 iterations
options.mcmc.i_sample=50; % Save every 50th visited model to disc
options.mcmc.i_plot=2500; % Plot the progress information for every 2500 iterations
options.txt='case_line_fit_2nd_order'; % descriptive name for the output folder
```

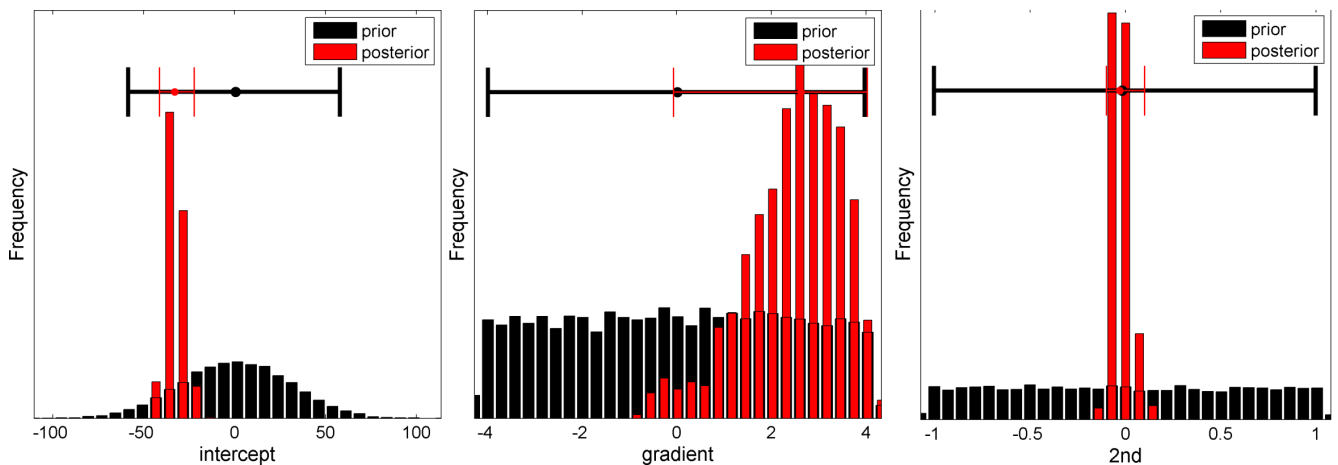
```
[options]=sippi_metropolis(data,prior,forward,options);
```

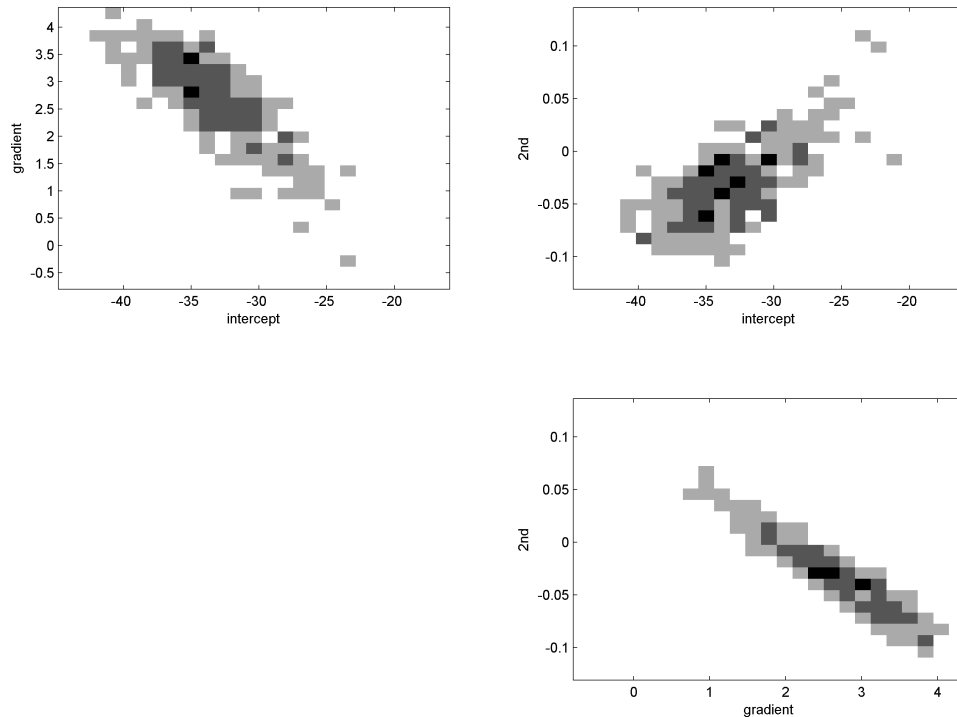
```
% plot posterior statistics, such as 1D and 2D marginals from the prior and posterior ↔
distributions
```

```
sippi_plot_prior_sample(options.txt);
```

```
sippi_plot_posterior(options.txt);
```

```
20140521_1644_sippi_metropolis_case_line_fit_2nd_order_m1_3_posterior_sample.png
```





### 4.2.5 Setup and run the rejection sampler

In a similar manner the **rejection sampler** can be setup and run using

```
options.mcmc.adaptive_rejection=1; % automatically adjust the normalizing likelihood
options.mcmc.nite=100000;
options=sippi_rejection(data,prior,forward,options);
```

## 4.3 Cross hole tomography

SIPPI includes a **reference cross hole GPR data from Arrenæs** set is also available, and will be used here to demonstrate the use of SIPPI to solve cross hole tomographic inversion in a probabilistic framework.

SIPPI also includes the implementation of multiple methods for **computing the travel time delay between a set of sources and receivers**. This allows SIPPI to work on for example cross hole tomographic forward and inverse problems.

This section contains examples for setting up and running an cross hole tomographic inversion using SIPPI using the **reference data from Arrenæs**, different types of a priori and **forward models**.

Example Matlab scripts for the examples below, and more, are located in [examples/case\\_tomography/](#).

Please see [\[HCLM13b\]](#) for more details on the example of using SIPPI to sample the posterior for cross hole tomographic inverse problems. See [\[LHC10\]](#) for more details on the data from Arrenæs.

### 4.3.1 Reference data set from Arrenæs

A 2D/3D data set of recorded travel time data from a cross hole Georadar experiment is available in the 'data/cross-hole' folder.

4 Boreholes were drilled, AM1, AM2, AM3, and AM4 at the locations shown below

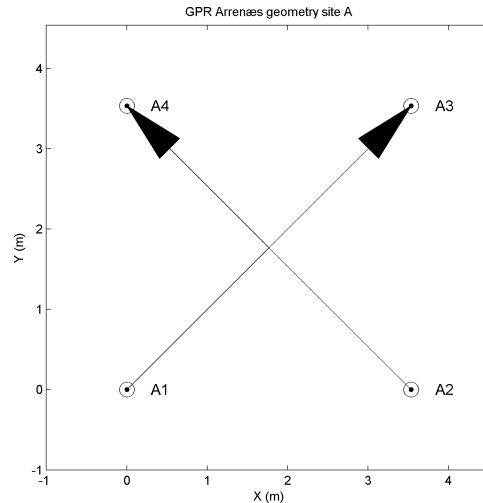


Figure 4.1: Location of boreholes AM1, AM2, AM3, and AM4 at Arrenæs.

Travel time data were collected between boreholes AM1 and AM3, and AM2 and AM4 respectively, in a depth interval between 1m and 12m. The travel times for each of the two 2D data sets are available in the [AM13\\_data.mat](#) and [AM24\\_data.mat](#) files. All the data have been combined in the 3D data set available in [AM1234\\_data.mat](#).

All mat-files contains the following variable

```
S --> [ndata,ndim] each row contains the position of the source
R --> [ndata,ndim] each row contains the position of the receiver
d_obs --> [ndata,1] each row contains the observed travel time in milliseconds
d_std --> [ndata,1] each row contains the standard deviation of the uncertainty of the ←
           observed travel time in milliseconds
```

All data are also available as ASCII formatted EAS files in [AM13\\_data.eas](#), [AM24\\_data.eas](#), and [AM1234\\_data.eas](#).

The following 3 Figures show the ray coverage (using straight rays) for each of the AM13, AM24, and AM1234 data sets. The color of each ray indicates the average velocity along the ray computed using  $v_{av} = \text{raylength}/d_{obs}$ . AM13 ray coverage AM24 ray coverage AM1234 ray coverage.

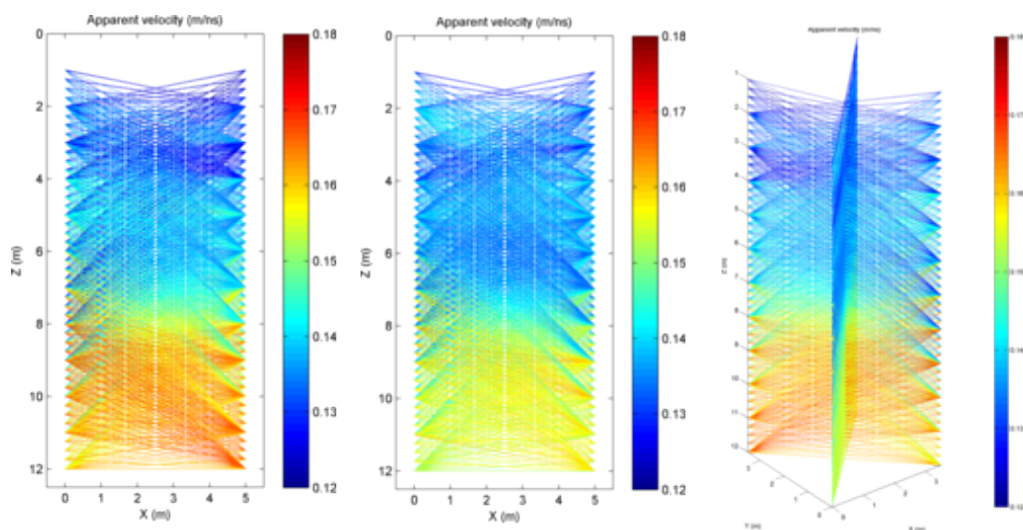


Figure 4.2: Ray coverage between wells (left) AM1-AM3, (middle) AM2-AM4, (right) AM1-4.

### 4.3.2 Travel delay computation: The forward problem

A number of different methods for solving the problem of computing the first arrival travel time of a seismic or electromagnetic wave traveling between a source in one borehole and a receiver in another borehole has been implemented in the m-file 'sippi\_forward\_travelttime'.

```
[d,forward,prior,data]=sippi_forward_travelttime(m,forward,prior,data,id,im)
```

In order to use this m-file to describe the forward problem specify the 'forward\_function' field in the forward structure using

```
forward.forward_function='sippi_forward_travelttime';
```

In order to use sippi\_forward\_travelttime, the location of the sources and receivers must be specified in the forward.S and forward.R. The number of columns reflect the number of data, and the number of rows reflect whether data are 2D (2 columns) or 3D (3 columns):

```
forward.S % [ndata,ndim]
forward.R % [ndata,ndim]
```

Using for example the data from Arrenæs, the forward geometry can be set up using

```
D=load('AM13_data.mat');
forward.sources=D.S;
forward.receivers=D.R;
```

In addition the method used to compute the travel times must also be given (see below).

In order to use the geometry from the AM13 reference data, and the Eikonal solution to the wave-equation, the forward structure can be defined using

```
D=load('AM13_data.mat');
forward.forward_function='sippi_forward_travelttime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='eikonal';
```

#### 4.3.2.1 Ray type forward model (high frequency approximation)

Ray type models are based on an assumption that the wave propagating between the source and the receiver has infinitely high frequency. Therefore the travel time delay is due to the velocity along a ray connecting the source and receiver.

The linear so-called straight ray approximation, which assumes that the travel time for a wave travelling between a source and a receiver is due to the travel time delay along a straight line connecting the source and receiver, can be chosen using

```
forward.type='ray';
forward.linear=1;
```

The corresponding so-called bended-ray approximation, where the travel time delay is due to the travel time delay along the fast ray path connecting a source and a receiver, can be chosen using

```
forward.type='ray';
forward.linear=0;
```

When sippi\_forward\_travelttime has been called once, the associated forward mapping operator is stored in 'forward.G' such the the forward problem can simply be solved by calling e.g. 'd{1}=forward.G\*m{1}'

#### 4.3.2.2 Fat Ray type forward model (finite frequency approximation)

Fat type model assume that the wave propagating between the source and the receiver has finite high frequency. This means that the travel time is sensitive to an area around the raypath, typically defined using the 1st Fresnel zone.

A linear fat ray kernel can be chosen using

```
forward.type='fat';
forward.linear=1;
forward.freq=0.1;
```

and the corresponding non-linear fat kernel using

```
bforward.type='fat';
forward.linear=0;
forward.freq=0.1;
```

Note that the center frequency of the propagating wave must also be provided in the 'forward.freq' field. The smaller the frequency, the 'fatter' the ray kernel.

For 'fat' type forward models we rely on the method described by Jensen, J. M., Jacobsen, B. H., and Christensen-Dalsgaard, J. (2000). Sensitivity kernels for time-distance inversion. *Solar Physics*, 192(1), 231-239

#### 4.3.2.3 Born type forward model (finite frequency approximation)

Using the Born approximation, considering only first order scattering, can be chosen using

```
forward.type='born';
forward.linear=1;
forward.freq=0.1;
```

For a velocity field with small spatial variability one can compute 'born' type kernels (using 'forward.linear=0', but as the spatial variability increases this is not possible.

For the 'born' type forward model we make use of the method described by Buursink, M. L., Johnson, T. C., Routh, P. S., and Knoll, M. D. (2008). Crosshole radar velocity tomography with finite frequency Fresnel volume sensitivities. *Geophysical Journal International*, 172(1), 1-17.

#### 4.3.2.4 The eikonal equation (high frequency approximation)

The eikonal solution to the wave-equation is a high frequency approximation, such as the one given above.

However, it is computationally more efficient to solve the eikonal equation directly, that to used the 'forward.type='ray';' type forward model.

To choose the eikonal solver to compute travel times use

```
forward.type='eikonal';
```

The Accurate Fast Marching Matlab toolbox : <http://www.mathworks.com/matlabcentral/fileexchange/24531-accurate-fast-marching> is used to solve the Eikonal equation.

### 4.3.3 AM13 Gaussian: Inversion of cross hole GPR data from Arrenaes data with a Gaussian type a priori model

In the following a simple 2D Gaussian a priori model is defined, and SIPPI is used to sample the corresponding a posteriori distribution. (An example script is available at [examples/case\\_tomography/sippi\\_AM13\\_metropolis\\_gaussian.m](#)).



#### 4.3.3.1 Setting up the data structure

Initially we load the travel time data obtained at Arrenæs (See Arrenæs Data for more information)

```
D=load('AM13_data.mat');
```

This allow us to setup a SIPPI data structure defining the observed data as well as the associated model of uncertainty

```
%% SETUP DATA
id=1;
data{id}.d_obs=D.d_obs;
data{id}.d_std=D.d_std;
data{id}.dt=0; % Mean modelization error
data{id}.Ct=1; % Covariance describing modelization error
```

In the above example we define a Gaussian modelization error,  $N(dt, Ct)$ . We do this because we will make use of a forward model, the eikonal solver, that we know will systematically provide faster travel times than can be obtained from the earth. In reality the wave travelling between bore holes never has infinitely high frequency as assumed by using the eikonal solver. The eikonal solver provides the fast travel time along a ray connecting the source and receiver. Therefore we introduce a modelization error, that will allow all the travel times to be biased with the same travel time.

#### 4.3.3.2 Setting up the prior model

The a priori model is defined using the prior data structure. Here a 2D Gaussian type a priori model in a 7x13 m grid (grid cell size .25m) using the **FFTMA** type a priori model. The a priori mean is 0.145 m/ns, and the covariance function a Spherical type covariance model with a range of 6m, and a sill(variance) of 0.0003 m<sup>2</sup>/ns<sup>2</sup>.

```
%% SETUP PRIOR
im=1;
prior{im}.type='FFTMA';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
prior{im}.x=[-1:.15:6];
prior{im}.y=[0:.15:13];
```

One could make used of the **VISIM** type priori model simply by substituting 'FFTMA' with 'VISIM' above.

#### 4.3.3.3 Setting up the forward structure

'**sippi\_forward\_traveltime**' require that the location of the sources an receivers are provided in 'forward' structure using the 'sources' and 'receivers' field names.

```
D=load('AM13_data.mat');
forward.forward_function='sippi_forward_traveltime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='eikonal';
```

Here the eikonal solution is chosen to solve the forward problem. See more detail about solving the forward problem related to cross hole first arrival travel time computation [here](#).

#### 4.3.3.4 Testing the setup

As the prior, data, and forward have been defined, one can in principle initiate an inversion. However, it is advised to perform a few test before applying the inversion.

First, one should check that independent realization of the prior model resemble the a priori knowledge. A sample from the prior model can be generated and visualized calling `sippi_plot_prior_sample`:

```
sippi_plot_prior_sample(prior);
```

which provides the following figure

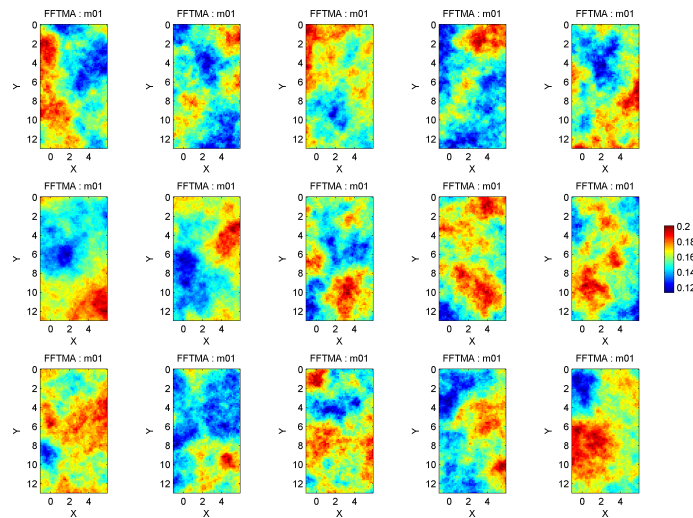


Figure 4.3: AM13: One sample (15 realizations) of the prior (Gaussian) model.

The one can check that the forward solver, and the computation of the likelihood works as expected using

```
% generate a realization from the prior
m=sippi_prior(prior);
% Compute the forward response related to the realization of the prior model generated  ←
  above
[d]=sippi_forward(m,forward,prior,data);
% Compute the likelihood
[logL,L,data]=sippi_likelihood(d,data);
% plot the forward response and compare it to the observed data
sippi_plot_data(d,data);
```

which produce a figure similar to

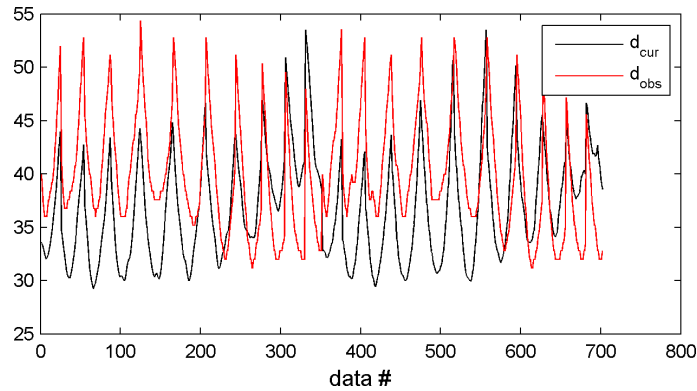


Figure 4.4: AM13: Data response from one realization of the prior.

#### 4.3.3.5 Sampling the a posterior distribution using the extended Metropolis algorithm

The **extended Metropolis sampler** can now be run using **sippi\_metropolis**.

```
options=sippi_metropolis(data,prior,forward);
```

In practice the user will have to set a few options, controlling the behavior of the algorithm. In the following example the number of iterations is set to 500000; the current model is saved to disc for every 500 iterations; the log-likelihood and current model is shown for every 1000 iterations:

```
options.mcmc.nite=500000; % optional, default:nite=300000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);
```

By default no **annealing schedule** is used. By default the **'step'-length** for sequential Gibbs sampling is adjusted (to obtain an average acceptance ratio of 30%) for every 50 iterations until iteration number 1000.

An output folder will be generated with a filename formatted using 'YYYYMMDD-HHMM', followed by a automatic description. In the above case the output folder could be name '20140701\_1450\_sippi\_metropolis\_eikonal'. The actual folder name is return in options.txt.

One can define a description for the folder name by setting options.txt before running sippi\_metropolis.

The folder contains one mat file, with the same name as the folder name, and N ASCII files (where N=length(prior); one for each a priori type) which contains the models saved to disc. They also have the same name as the folder name, appended with '\_m1.asc', '\_m2.asc', and so forth.

##### 4.3.3.5.1 Posterior statistics

The function **sippi\_plot\_posterior** can be called when **sippi\_metropolis** (or **sippi\_rejection**) and will plot the progress of the log-likelihood curve, a sample of the posterior, data response from a sample of the posterior, and (if applicable) 1D and 2D marginal posterior distributions.

Located in the output folder of the inversion use

```
sippi_plot_posterior;
```

If the location of the folder with the output is known (such as options.txt) one can call

```
sippi_plot_posterior(options.txt)
```

### 4.3.4 AM13 Gaussian, accounting for modeling errors

[A Matlab script for the following example is available at [examples/case\\_tomography/sippi\\_AM13\\_metropolis\\_modeling\\_e](#)

The use of any of the **forward models** defined above, will be approximation to solving the perfect forward problem. This leads to a 'modeling' error as demonstrated by [HCM14]. If one has access to an optimal (but perhaps computational inefficient) forward model, and a faster (less accurate) forward model, then a Gaussian model of the modeling error caused by using the approximate, as opposed to the optima, forward model can be estimated using **sippi\_compute\_modelization\_forward\_error**. `sippi_compute_modelization_forward_error`.

SIPPI allows accounting for such modeling error through the `dt` and `Ct` fields for the **data** structure.

The setup of the data, prior and forward structures is identical to the one described in the **previous** example.

```
%% Load the travel time data set from ARRENAES
clear all; close all
D=load('AM13_data.mat');
options.txt='AM13';

%% SETUP DATA
id=1;
data{id}.d_obs=D.d_obs;
data{id}.d_std=D.d_std;
data{id}.Ct=D.Ct+1; % Covariance describing modeling error

%% SETUP PRIOR
im=1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
dx=0.15;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];

% SETUP THE FORWARD MODEL USED IN INVERSION
forward.forward_function='sippi_forward_traveltime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='fat'; forward.linear=1; forward.freq=0.1;
```

In order to compute the modeling with respect to using the **'Born'** type forward model, one can define a new forward structure, here `forward_full`, and estimate a Gaussian model for the modeling error using

```
% SETUP THE 'OPTIMAL' FORWARD MODEL
forward_full.forward_function='sippi_forward_traveltime';
forward_full.sources=D.S;
forward_full.receivers=D.R;
forward_full.type='Born'; forward_full.linear=1; forward_full.freq=0.1;

% COMPUTE MODELING ERROR DUE TO USE OF forward AS OPPOSED TO forward_full
N=100;
[Ct,dt,dd]=sippi_compute_modelization_forward_error(forward_full,forward,prior,data,N);

% ASSIGN MODELING ERROR TO DATA
data{1}.dt=dt{1};
data{1}.Ct=Ct{1};
```

Sampling of the posterior can proceed exactly as for the previous example, using

```
options.mcmc.nite=500000; % optional, default:nite=30000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
```

```
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);

% plot posterior statistics
sippi_plot_posterior(options.txt);
```

### 4.3.5 AM13 Gaussian with bimodal velocity distribution

[A Matlab script for the following example is available at [examples/case\\_tomography/sippi\\_AM13\\_metropolis\\_bimodal.m](#).

The **GAUSSIAN** and **FFTMA** a priori types implicitly assume a normal distribution of the model parameter.

It is however possible to change the Gaussian distribution to any shaped distribution, using a normal score transform. Note that when this is done the given semivariogram model for the **FFTMA** a priori model will not be reproduced. If this is a concern, then the **VISIM** type a priori model should be used.

The data and forward structures is identical to the one described in the [previous](#) example.

```
%% Load the travel time data set from ARRENAES
clear all;close all
D=load('AM13_data.mat');
options.txt='AM13';

%% SETUP DATA
id=1;
data{id}.d_obs=D.d_obs;
data{id}.d_std=D.d_std;
data{id}.Ct=D.Ct+1; % Covariance describing modeling error

% SETUP THE FORWARD MODEL USED IN INVERSION
forward.forward_function='sippi_forward_traveltime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='fat'; forward.linear=1; forward.freq=0.1;
```

The desired distribution (the 'target' distribution) must be provided as a sample of the target distribution, in the `data{id}.d_target` distribution.

```
%% SETUP PRIOR
im=1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
dx=0.15;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];

% SET TARGET
N=1000;
prob_chan=0.5;
dd=.014*2;
d1=randn(1,ceil(N*(1-prob_chan)))*.01+0.145-dd; %0.1125;
d2=randn(1,ceil(N*(prob_chan)))*.01+0.145+dd; %0.155;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;
```

5 realizations from the corresponding a priori model looks like

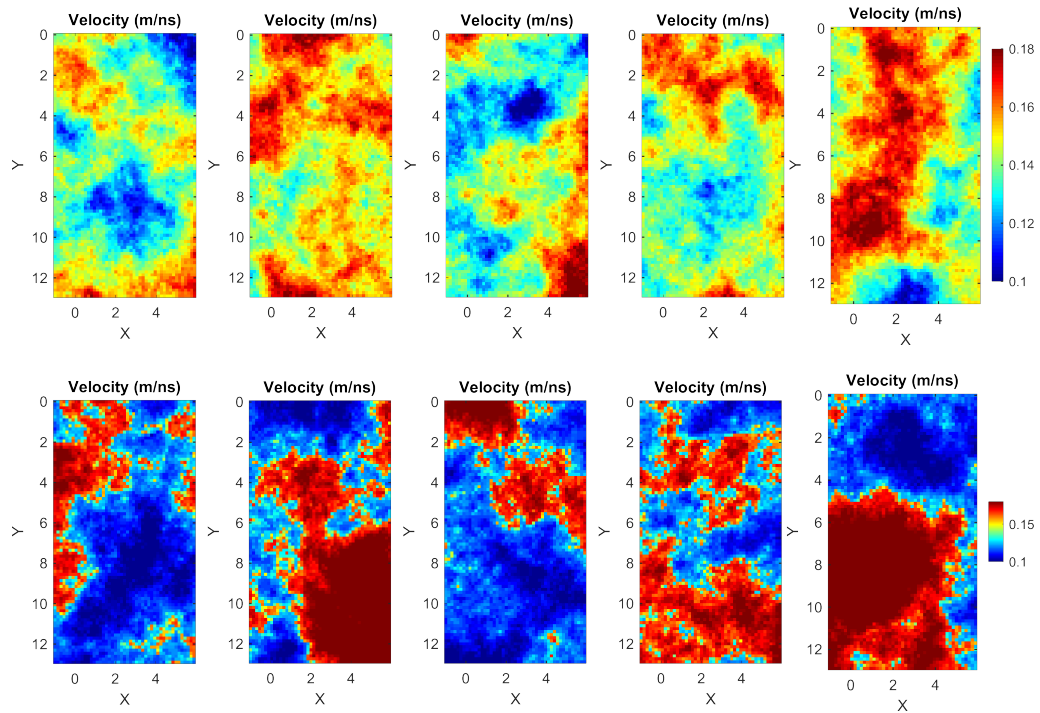


Figure 4.5: 5 realizations from a FFTMA prior model type with top) Gaussian and b) Bimodal distribution

Figure Figure 4.6 compares the distribution from one realization of both prior models considered above.

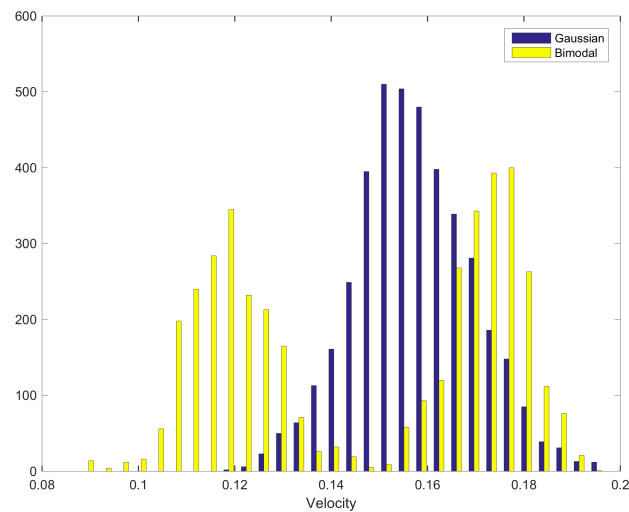


Figure 4.6: Distribution of one realization using a Gaussian Bimodal target distribution

As for the examples above, the a posteriori distribution can be samples using e.g.

```
options.mcmc.nite=500000; % optional, default:nite=30000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);

% plot posterior statistics
sippi_plot_posterior(options.txt);
```

### 4.3.6 AM13 Gaussian with unknown Gaussian model parameters

[A Matlab script for the following example is available at [examples/case\\_tomography/sippi\\_AM13\\_metropolis\\_gaussian\\_co](#)

One of the most intriguing benefits (in addition to the computational efficiency) of using the **FFTMA** type a priori model, is that it allows separation of the random component and the covariance model parameters. See [HCLM13a].

This means that one can in SIPPI define an inverse problem, where the a priori model is Gaussian, but where the properties of the Gaussian model (such as the mean, range, anisotropy) can be treated as unknown model parameters

Each property the Gaussian prior model that should be treated as an unknown model parameter, must be defined as a separate 1D type **GAUSSIAN** type prior model, with a specific name (identifying the covariance model property it describes), and it must point to the prior model type number for which it describes a covariance model property-

The example below describes a 2D FFTMA type e a priori model (prior with id 1) with an unknown range (prior with id 2) with an a priori distribution described by a close to uniform distribution between 1.5m and 10.5m:

```
im=1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
dx=0.25;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];

i_master=im;

% range - horizontal
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='range_1'; % the name covariance model property to define
prior{im}.m0=6;
prior{im}.min=1.5;
prior{im}.max=10.5;
prior{im}.norm=50;
prior{im}.prior_master=i_master; % point to the id of the prior it describes
```

Any combination of the following parameters can be set:

```
prior{im}.name='range_1'; % Range, along direction of angle_1
prior{im}.name='range_2'; % Range, along direction of angle_2
prior{im}.name='range_3'; % Range, along direction of angle_3
prior{im}.name='ang_1';   % Angle 1, degrees from North
prior{im}.name='ang_2';   % Angle 2
prior{im}.name='ang_3';   % Angle 3
prior{im}.name='sill';     % sill,
prior{im}.name='nu';       % the 'nu' parameter, only applies when using the Matern ↔
                           covariance model type.
prior{im}.name='m0';       % A priori mean
```

As an example consider case where the two ranges, and the angle of anisotropy for a 2D Gaussian(FFTMA) a priori type model is treated as model parameters:

```
im=0;
% velocity field
im=im+1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
```

```

dx=0.25;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];
i_master=im;

% range - horizontal
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='range_1';
prior{im}.min=1.5;
prior{im}.max=10.5;
prior{im}.norm=50;
prior{im}.prior_master=i_master;

% range - horizontal
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='range_2';
prior{im}.min=1.5;
prior{im}.max=5.5;
prior{im}.norm=50;
prior{im}.prior_master=i_master;

% rotation
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='ang_1';
prior{im}.m0=90;
prior{im}.std=20;
prior{im}.norm=2;
prior{im}.prior_master=i_master;

```

A sample from the corresponding a priori model (FFTMA type) is shown below:

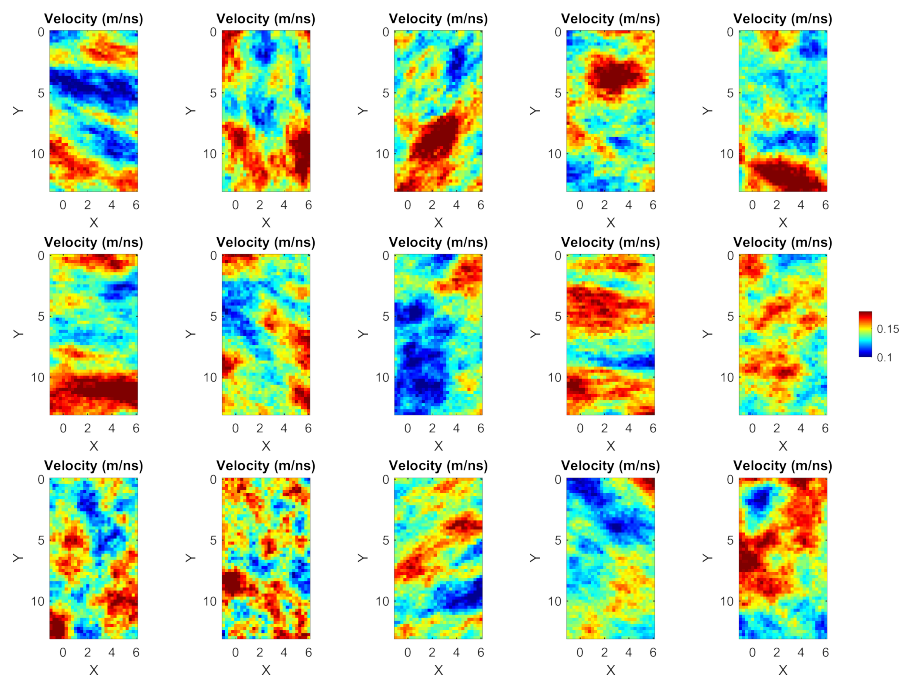


Figure 4.7: A sample from a FFTMA type prior model with varying range\_1, range\_2, and ang\_1.



Samples of the a priori distributions for range\_1, range\_2, and ang\_1 are shown here:

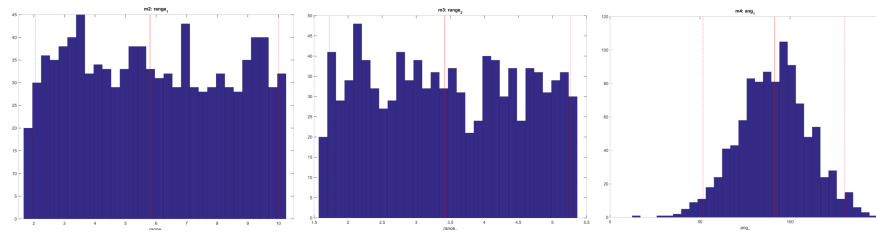


Figure 4.8: Distribution of one sample of a 1D Gaussian distribution describing range\_1, range\_2, and ang\_1

As for the examples above, the a posteriori distribution can be samples using e.g.

```
options.mcmc.nite=500000; % optional, default:nite=30000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);

% plot posterior statistics
sippi_plot_posterior(options.txt);
```

## 4.4 Probilistic covariance/semivariogram inference

This chapter documnets how to use SIPPI to infer properties of a covariance/semivariogram model from noisy data (both data of point support and linear average data can be considered)

To apply covariance indeference one must 1) define the data and associated uncertainty (if any), 2) define a prior model describing covariance model parameters, and 3) define the linear forward operate (only applicable if data are not of point support).

### 4.4.1 Inferring a 2D covariance model from the Jura data set

```
% jura_covariance_inference
%
% Example of inferring properties of a Gaussian model from point data
%

%% LOAD THE JURA DATA
clear all;close all
[d_prediction,d_transect,d_validation,h_prediction,h_transect,h_validation,x,y,pos_est]= ←
    jura;
ix=1;
iy=2;
id=6;

% get the position of the data
pos_known=[d_prediction(:,[ix iy])];

% perform normal score transformation of tha original data
[d,o_nscore]=nscore(d_prediction(:,id));
h_tit=h_prediction{id};

%% SETUP A PRIORI MODEL / ONLY RANGE --> ISTROPIC COVARIANCE MODEL
im=0;
```

```
% A close to uniform distribution of the range, U[0;3].
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='range_1';
prior{im}.min=0.01;
prior{im}.max=3;
prior{im}.norm=100;

%% DATA
data{1}.d_obs=d; % observed data
data{1}.d_std=0;.5; % uncertainty of observed data (in form of standard deviation of the ←
    noise)
%data{1}.i_use=1:1:30;

%% FORWARD
forward.forward_function='sippi_forward_covariance_inference';
forward.point_support=1;
forward.pos_known=pos_known;
forward.stabilize=0;
% initial choice of  $N(m_0, C_m)$ , mean and sill are 0, and 1, due
% due to normal score
forward.m0=0;
forward.Cm='1 Sph(2)';

%% METROPOLIS SAMPLING
options.mcmc.nite=5000;
options.mcmc.i_plot=100;
options.mcmc.i_sample=10;
[options,data,prior,forward,m_current]=sippi_metropolis(data,prior,forward,options)

sippi_plot_posterior(options.txt);
```

## Chapter 5

# Bibliography

- [CHM12] K. S. Cordua, T. M. Hansen, and K. Mosegaard, Monte Carlo full waveform inversion of crosshole GPR data using multiple-point geostatistical a priori information [PDF](#), H19--H31. Geophysics, 77, 2012.
- [HCLM13a] T.M. Hansen, K.S. Cordua, M.C. Looms, and K. Mosegaard, SIPPI: a Matlab toolbox for sampling the solution to inverse problems with complex prior information: Part 1, methodology [PDF](#), 470--480. Computers & Geosciences, 52, 03 2013.
- [HCLM13b] T.M. Hansen, K.S. Cordua, M.C. Looms, and K. Mosegaard, SIPPI: a Matlab toolbox for sampling the solution to inverse problems with complex prior information: Part 2, Application to cross hole GPR tomography [PDF](#), 481--492. Computers & Geosciences, 52, 03 2013.
- [HCM12] T. M. Hansen, K. C. Cordua, and K. Mosegaard, Inverse problems with non-trivial priors - efficient solution through sequential Gibbs sampling [PDF](#), 593--611. Computational Geosciences, 16, 2012.
- [HCM14] T. M. Hansen, K. S. Cordua, B. H. Jacobsen, and K. Mosegaard, Accounting for imperfect forward modeling in geophysical inverse problems - exemplified for cross hole tomography [PDF](#), H1-H21. Geophysics, 39, 2014.
- [LHC10] M. C. Looms, T. M. Hansen, K. S. Cordua, L. Nielsen, K. H. Jensen, and A. Binley, Geostatistical inference using crosshole ground-penetrating radar : Geostatistical inference using GPR [PDF](#), J29--J41. Geophysics, 75, 2010.
-

## Chapter 6

# Reference

### 6.1 SIPPI

#### 6.1.1 getinunits

**GETINUNITS** Get object properties in specified units  
V = GETINUNITS(H, PROP, UNITS) returns the object property in the specified UNITS. It will leave the 'Units' and 'FontUnits' property unchanged afterwards.

H is the handle of the object. If it is an M-element array of handles, the function will return an M-by-1 cell array. PROP can be a string or a cell array of strings. If it is a 1-by-N or N-by-1 cell array, the function will return an M-by-N cell array of values. UNITS can be a string or a cell array. If it is a cell array, then PROP must also be a cell array with the same size as UNITS, and each cell element of UNITS corresponds to a cell element of PROP.

V = GETINUNITS(H, PROP) is the same as GET(H, PROP)

Examples:

```
V = GETINUNITS(H, 'Position', 'Pixels')
V = GETINUNITS(H, {'FontSize', 'Position'}, 'Normalized')
V = GETINUNITS(H, {'FontSize', 'Position'}, {'Points', 'Pixels'})
```

See also GET, SET

#### 6.1.2 logdet

**LOGDET** Computation of logarithm of determinant of a matrix

```
v = logdet(A);
    computes the logarithm of determinant of A.
```

Here, A should be a square matrix of double or single class. If A is singular, it will returns -inf.

Theoretically, this function should be functionally equivalent to log(det(A)). However, it avoids the overflow/underflow problems that are likely to

happen when applying `det` to large matrices.

The key idea is based on the mathematical fact that the determinant of a triangular matrix equals the product of its diagonal elements. Hence, the matrix's log-determinant is equal to the sum of their logarithm values. By keeping all computations in log-scale, the problem of underflow/overflow caused by product of many numbers can be effectively circumvented.

The implementation is based on LU factorization.

```
v = logdet(A, 'chol');
```

If `A` is positive definite, you can tell the function to use Cholesky factorization to accomplish the task using this syntax, which is substantially more efficient for positive definite matrix.

#### Remarks

-----

logarithm of determinant of a matrix widely occurs in the context of multivariate statistics. The log-pdf, entropy, and divergence of Gaussian distribution typically comprises a term in form of log-determinant. This function might be useful there, especially in a high-dimensional space.

Theoretically, LU, QR can both do the job. However, LU factorization is substantially faster. So, for generic matrix, LU factorization is adopted.

For positive definite matrices, such as covariance matrices, Cholesky factorization is typically more efficient. And it is **STRONGLY RECOMMENDED** that you use the `chol` (2nd syntax above) when you are sure that you are dealing with a positive definite matrix.

#### Examples

-----

```
% compute the log-determinant of a generic matrix
A = rand(1000);
v = logdet(A);

% compute the log-determinant of a positive-definite matrix
A = rand(1000);
C = A * A';      % this makes C positive definite
v = logdet(C, 'chol');
```

### 6.1.3 pathdef

**PATHDEF** Search path defaults.

**PATHDEF** returns a string that can be used as input to **MATLABPATH** in order to set the path.

### 6.1.4 plotboxpos

**PLOTBOXPOS** Returns the position of the plotted axis region

```
pos = plotboxpos(h)
```

This function returns the position of the plotted region of an axis, which may differ from the actual axis position, depending on the axis limits, data aspect ratio, and plot box aspect ratio. The position is returned in the same units as the those used to define the axis itself. This function can only be used for a 2D plot.

Input variables:

h: axis handle of a 2D axis (if omitted, current axis is used).

Output variables:

pos: four-element position vector, in same units as h

### 6.1.5 sippi\_adjust\_step\_size

**sippi\_adjust\_step\_size** Adjust step length for Metropolis sampler in SIPPI

Call :

```
step=sippi_adjust_step_size(step,P_average,P_target);
```

step : current step

P\_current : Current acceptance ratio

P\_target : preferred acceptance ratio (def=0.3);

See also sippi\_compute\_acceptance\_rate, sippi\_prior\_set\_steplength

### 6.1.6 sippi\_anneal\_adjust\_noise

**sippi\_anneal\_adjust\_noise** : Adjust noise level in annealing schedul

Call:

```
[data_adjust,mcnc]=sippi_anneal_adjust_noise(data,i,mcnc,prior);
```

See also: sippi\_metropolis, sippi\_anneal\_factor

### 6.1.7 sippi\_anneal\_factor

**sippi\_anneal\_factor** : compute simple noise multiplication factor for annealing type sampling

See also sippi\_metropolis, sippi\_anneal\_adjust\_noise

### 6.1.8 sippi\_compute\_acceptance\_rate

`sippi_compute_acceptance_rate` Computes acceptance rate for the Metropolis sampler in SIPPI ↔

Call:

```
P_acc=sippi_compute_acceptance_rate(acc,n_update_history);
```

### 6.1.9 sippi\_compute\_modelization\_forward\_error

`sippi_compute_modelization_forward_error` Computes an estimate of the modelization error

Computes and estimate of the Gaussian modelization error,  $N(dt,Ct)$  caused by the use of an imperfect forward kernel

If called with only one output '`[Ct]=sippi..`' then the Gaussian model is assumed by centered around 0, ( $dt\{1\}=0$ ).

Call

```
[Ct,dt,dd]=sippi_compute_modelization_forward_error(forward_full,forward_app,prior,data ↔
,N);
```

### 6.1.10 sippi\_forward

`sippi_forward` Simple forward wrapper for SIPPI

Assumes that the actual forward solver has been defined by `forward.forward_function`

Call:

```
[d,forward,prior,data]=sippi_forward(m,forward,prior,data,id,im)
```

### 6.1.11 sippi\_get\_sample

`sippi_get_sample`: Get a posterior sample

Call :

```
[reals,etype_mean,etype_var,reals_all,reals_ite]=sippi_get_sample(im,n_reals, ↔
skip_seq_gibbs,data,prior,options);
```

`im`: A priori model type

`n_reals`: Number of realizations to return

`skip_seq_gibbs` [1] Skip all realization where sequential gibbs is enabled  
[0] Use all realization

`data`: SIPPI data structure

`prior`: SIPPI prior structure

`options`: options structure when running `sippi_metropolis`

If located in a SIPPI output folder one can simple use :

```
[reals, etype_mean, etype_var, reals_all, reals_ite]=sippi_get_sample(im, n_reals);
or
skip_seq_gibbs=0;
[reals, etype_mean, etype_var, reals_all, reals_ite]=sippi_get_sample(im, n_reals, ←
    skip_seq_gibbs);
```

### 6.1.12 sippi\_get\_sample\_new

sippi\_get\_sample: Get a posterior sample

Call :

```
[reals, etype_mean, etype_var, reals_all, reals_ite]=sippi_get_sample(im, n_reals, ←
    skip_seq_gibbs, data, prior, options);
```

im: A priori model type

n\_reals: Number of realizations to return

skip\_seq\_gibbs [1] Skip all realization where sequential gibbs is enabled  
 [0] Use all realization

data: SIPPI data structure

prior: SIPPI prior structure

options: options structure when running sippi\_metropolis

If located in a SIPPI output folder one can simple use :

```
[reals, etype_mean, etype_var, reals_all, reals_ite]=sippi_get_sample(im, n_reals);
or
skip_seq_gibbs=0;
[reals, etype_mean, etype_var, reals_all, reals_ite]=sippi_get_sample(im, n_reals, ←
    skip_seq_gibbs);
```

### 6.1.13 sippi\_least\_squares

sippi\_least\_squares Least squares type inversion for SIPPI

Call :

```
[m_reals, m_est, Cm_est]=sippi_least_squares(data, prior, forward, n_reals, lsq_type, id, im);
```

lsq\_type : 'lsq' (def), classical least squares  
 'error\_sim', simulation through error simulation  
 'visim', simulation through SGSIM of DSSIM

### 6.1.14 sippi\_likelihood

sippi\_likelihood Compute likelihood given an observed dataset

Call

```
[logL, L, data]=sippi_likelihood(d, data);
```



```

data{1}.d_obs [N_data,1] N_data data observations
data{1}.d_std [N_data,1] N_data uncorrelated Gaussian STD

data{1}.d_var [N_data,1] N_data uncorrelated Gaussian variances

Gaussian modelization error, N(dt,Ct), is specified as
data{1}.dt [N_data,1] : Bias/mean of modelization error
data{1}.Ct [N_data,N_data] : Covariance of modelization error

data{1}.Ct [1,1] : Constant Covariance of modelization error
                    implies data{1}.Ct=ones(N_data.N_data)*data{1}.Ct;

```

data{id}.recomputeCD [default=0], if '1' then data{1}.iCD is recomputed each time sippi\_likelihood is called. This should be used if the noise model changes between each call to sippi\_likelihood.

```

data{id}.full_likelihood [default=]0; if '1' the the full likelihood
(including the determinant) is computed. This not needed if the data
civariance is constant, but if it changes, then use
data{id}.full_likelihood=1;

```

### 6.1.15 sippi\_mcmc\_init

sippi\_mcmc\_init Initialize MCMC options for Metropolis and rejection sampling in SIPPI

Call:

```
options=sippi_mcmc_init(options,prior);
```

### 6.1.16 sippi\_metropolis

sippi\_metropolis Extended Metropolis sampling in SIPPI

Metropolis sampling.

See e.g. Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012.

Inverse problems with non-trivial priors - Efficient solution through Sequential ↔  
Gibbs Sampling.

Computational Geosciences. doi:10.1007/s10596-011-9271-1.

Call :

```
[options,data,prior,forward,m_current]=sippi_metropolis(data,prior,forward,options)
```

Input :

```

data : sippi data structure
prior : sippi prior structure
forward : sippi forward structure

```

options :

```
options.txt [string] : string to be used as part of all output files
```

```
options.mcmc.nite [1] : Number if iterations
```

```
options.mcmc.i_plot [1]: Number of iterations between updating plots
```

```

options.mcmc.i_sample=: Number of iterations between saving model to disk

options.mcmc.m_init : Manually chosen starting model
options.mcmc.m_ref  : Reference known target model

options_mcmc.accept_only_improvements [0] : Optimization

%% PERTUBATION STRATEGY
options.mcmc.pert_strategy.perturb_all=1; % Perturb all priors in each
                                     % iteration. def =[0]

%% SIMULATED ANNEALING
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=100000; % iteration number when annealing stops
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration ←
    i_begin
options.mcmc.anneal.fac_end=1; % default, noise is scaled by fac_end at iteration ←
    i_end

```

See also `sippi_rejection`

### 6.1.17 sippi\_prior

`sippi_prior` A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:

```
[m_propose,prior]=sippi_prior(prior);
```

To generate a realization of the prior model defined by the prior structure, in the vicinity of a current model (using sequential Gibbs sampling) use:

```
[m_propose,prior]=sippi_prior(prior,m_current);
```

The following types of a priori models can be used

```

SNESIM [1D-3D] : based on a multiple point statistical model inferref from a training ←
    images. Relies in the SNESIM algorithm
SISIM   [1D-3D] : based on Sequential indicator SIMULATION
VISIM   [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
FFTMA   [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
GAUSSIAN [1D]   : 1D generalized gaussian model

```

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaussian based prior model based on the  
% FFT-MA method, can be defined using

```

im=1;
prior{im}.type='FFTMA';
prior{im}.name='A SIMPLE PRIOR';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Va='1 Sph(10)';
prior=sippi_prior_init(prior);

```

% A realization from this prior model can be generated using

```
m=sippi_prior(prior);
```

% This realization can now be plotted using

```
sippi_plot_prior(m,prior);
```

% or

```
imagesc(prior{1}.x,prior{1}.y,m{1})
```

```

%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

im=1;
prior{im}.type='GAUSSIAN';
prior{im}.m0=100;
prior{im}.std=50;
prior{im}.norm=100;
im=im+1;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
im=im+1;
prior{im}.type='VISIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
im=im+1;
prior{im}.type='SISIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
im=im+1;
prior{im}.type='SNESIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];

sippi_plot_prior(prior);

%%% Sequential Gibbs sampling

All a priori model types can be perturbed, such that a new realization
is generated in the vicinity of a current model.
To do this Sequential Gibbs Sampling is used.
For more information, see <a href="http://dx.doi.org/10.1007/s10596-011-9271-1">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse
problems with non-trivial priors - Efficient solution through Sequential Gibbs
Sampling. Computational Geosciences</a>.
The type of sequential Gibbs sampling can be controlled in the
'seq_gibbs' structures, e.g. prior{1}.seq_gibbs

im=1;
prior{im}.type='SNESIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];

[m,prior]=sippi_prior(prior);
prior{1}.seq_gibbs.step=1; % Large step--> independant realizations
prior{1}.seq_gibbs.step=.1; % Smaller step--> Dependant realizations
for i=1:30;
    [m,prior]=sippi_prior(prior,m); % One iteration of Sequential Gibbs
    sippi_plot_prior(prior,m);
end

See also: sippi_prior_init, sippi_plot_prior, sippi_plot_prior_sample,
sippi_prior_set_steplength.m

```

### 6.1.18 sippi\_prior\_fftma

sippi\_prior A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:

```
[m_propose,prior]=sippi_prior(prior);
```

To generate a realization of the prior model defined by the prior structure, in the vicinity of a current model (using sequential Gibbs sampling) use:

```
[m_propose,prior]=sippi_prior(prior,m_current);
```

The following types of a priori models can be used

SNESIM [1D-3D] : based on a multiple point statistical model inferred from a training images. Relies in the SNESIM algorithm ↵

SISIM [1D-3D] : based on Sequential indicator SIMULATION

VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation

FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)

GAUSSIAN [1D] : 1D generalized gaussian model

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaussian based prior model based on the

% FFT-MA method, can be defined using

```
id=1;
prior{id}.type='FFTMA';
prior{id}.name='A SIMPLE PRIOR';
prior{id}.x=[0:1:100];
prior{id}.y=[0:1:100];
prior{id}.m0=10;
prior{id}.Va='1 Sph(10)';
prior=sippi_prior_init(prior);
```

% A realization from this prior model can be generated using

```
m=sippi_prior(prior);
```

% This realization can now be plotted using

```
sippi_plot_prior(m,prior);
```

% or

```
imagesc(prior{1}.x,prior{1}.y,m{1})
```

%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

```
id=1;
prior{id}.type='FFTMA';
prior{id}.x=[0:1:100];
prior{id}.y=[0:1:100];
prior{id}.m0=10;
prior{id}.Cm='1 Sph(10)';
id=2;
prior{id}.type='SISIM';
prior{id}.x=[0:1:100];
prior{id}.y=[0:1:100];
prior{id}.m0=10;
prior{id}.Cm='1 Sph(10)';
id=3;
prior{id}.type='GAUSSIAN';
prior{id}.m0=100;
prior{id}.std=50;
prior{id}.norm=100;
```

```
prior=sippi_prior_init(prior);

sippi_plot_model(prior);

%% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596
-011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems
with non-trivial priors - Efficient solution through Sequential Gibbs Sampling.
Computational Geosciences</a>.

See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m

TMH/2012
```

### 6.1.19 sippi\_prior\_init

```
sippi_prior_init Initialize PRIOR structure for SIPPI

Call
    prior=sippi_prior_init(prior);

See also sippi_prior
```

### 6.1.20 sippi\_prior\_new

```
sippi_prior A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:
    [m_propose,prior]=sippi_prior(prior);

To generate a realization of the prior model defined by the prior structure,
in the vicinity of a current model (using sequential Gibbs sampling) use:
    [m_propose,prior]=sippi_prior(prior,m_current);

The following types of a priori models can be used
    SNESIM [1D-3D] : based on a multiple point statistical model inferref from a training
        images. Relies in the SNESIM algorithm
    SISIM [1D-3D] : based on Sequential indicator SIMULATION
    VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
    FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
    GAUSSIAN [1D] : 1D generalized gaussian model

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaissian based prior model based on the
% FFT-MA method, can be defined using
    id=1;
    prior{id}.type='FFTMA';
    prior{id}.name='A SIMPLE PRIOR';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Va='1 Sph(10)';
```

```

    prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
    m=sippi_prior(prior);
% This realization can now be plotted using
    sippi_plot_prior(m,prior);
% or
    imagesc(prior{1}.x,prior{1}.y,m{1})

%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

    id=1;
    prior{id}.type='FFTMA';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=2;
    prior{id}.type='SISIM';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=3;
    prior{id}.type='GAUSSIAN';
    prior{id}.m0=100;
    prior{id}.std=50;
    prior{id}.norm=100;
    prior=sippi_prior_init(prior);

    sippi_plot_model(prior);

% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596-
-011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems
with non-trivial priors - Efficient solution through Sequential Gibbs Sampling.
Computational Geosciences</a>.

See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m

TMH/2012

```

### 6.1.21 sippi\_prior\_old

sippi\_prior A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:  
 [m\_propose,prior]=sippi\_prior(prior);

To generate a realization of the prior model defined by the prior structure,  
 in the vicinity of a current model (using sequential Gibbs sampling) use:  
 [m\_propose,prior]=sippi\_prior(prior,m\_current);

The following types of a priori models can be used

SNESIM [1D-3D] : based on a multiple point statistical model inferred from a training  
 images. Relies in the SNESIM algorithm ↵  
 SISIM [1D-3D] : based on Sequential indicator SIMULATION  
 VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation  
 FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)

```

GAUSSIAN    [1D] : 1D generalized gaussian model

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaussian based prior model based on the
% FFT-MA method, can be defined using
    id=1;
    prior{id}.type='FFTMA';
    prior{id}.name='A SIMPLE PRIOR';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Va='1 Sph(10)';
    prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
    m=sippi_prior(prior);
% This realization can now be plotted using
    sippi_plot_prior(m,prior);
% or
    imagesc(prior{1}.x,prior{1}.y,m{1})

%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

    id=1;
    prior{id}.type='FFTMA';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=2;
    prior{id}.type='SISIM';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=3;
    prior{id}.type='GAUSSIAN';
    prior{id}.m0=100;
    prior{id}.std=50;
    prior{id}.norm=100;
    prior=sippi_prior_init(prior);

    sippi_plot_model(prior);

%% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596-011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems with non-trivial priors - Efficient solution through Sequential Gibbs Sampling. Computational Geosciences</a>.

See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m

TMH/2012

```

## 6.1.22 sippi\_prior\_set\_steplength

```
sippi_prior_set_steplength Set step length for Metropolis sampler in SIPPI
```

Call

```
prior=sippi_prior_set_steplength(prior,mcmc,im);
```

### 6.1.23 sippi\_rejection

```
sippi_rejection Rejection sampling
```

Call :

```
options=sippi_rejection(data,prior,forward,options)
```

input arguments

```
options.mcmc.i_plot
```

```
options.mcmc.nite      % maximum number of iterations
```

```
options.mcmc.logLmax
```

```
options.mcmc.rejection_normalize_log = log(options.mcmc.Lmax)
```

```
options.mcmc.adaptive_rejection=1, adaptive setting of maximum likelihood
```

```
(def=[0])
```

```
At each iteration Lmax will be set if log(L(m_cur))>=options.mcmc.logLmax
```

```
options.mcmc.max_run_time_hours = 1; % maximum runtime in hours
```

```
% (overrides options.mcmc.nite if needed)
```

See also sippi\_metropolis

### 6.1.24 sippi\_set\_path

```
sippi_set_path Set paths for running sippi
```

### 6.1.25 sippi\_colormap

```
sippi_colormap Default colormap for sippi
```

Call :

```
sippi_colormap; % the same as sippi_colormap(3);
```

or :

```
sippi_colormap(1) - Red Green Black
```

```
sippi_colormap(2) - Red Green Blue Black
```

```
sippi_colormap(3) - Jet
```



### 6.1.26 sippi\_plot\_current\_model

`sippi_plot_current_model` Plots the current model during Metropolis sampling

Call :

```
sippi_plot_current_model(mcmc,data,d,m_current,prior);
```

### 6.1.27 sippi\_plot\_data

`sippi_plot_data` plot data in SIPPI

Call.

```
sippi_plot_data(d,data);
```

### 6.1.28 sippi\_plot\_defaults

`sippi_plot_defaults`: Sets default options for plotting (such as fontsize)

Call :

```
options==sippi_plot_defaults(options);
```

```
% ALWAYS USE DEFAULT SETTING (overrules options.axis)
```

```
overrule=1; % {default overrule=0}
```

```
options==sippi_plot_defaults(options,overrule);
```

See also: `sippi_plot_posterior`, `sippi_plot_posterior_2d_marg`

### 6.1.29 sippi\_plot\_loglikelihood

`sippi_plot_loglikelihood` Plot loglikelihood time series

Call :

```
acc=sippi_plot_loglikelihood(logL,i_acc,N,itext)
```

### 6.1.30 sippi\_plot\_model

### 6.1.31 sippi\_plot\_movie

`sippi_plot_movie` plot movie of prior and posterior realizations

Call :

```
sippi_plot_movie(fname);
```

```
sippi_plot_movie(fname,im_array,n_frames,skip_burnin);
```

```

    fname : name of folder with results (e.g. options.txt)
    im_array : array of indexes of model parameters to make into movies
    n_frames [200] : number of frames in movie
    skip_burnin [200] : start movie after burn_in;

Ex:
sippi_plot_movie('20130812_Metropolis');
sippi_plot_movie(options.txt);

%% 1000 realization including burn-in, for prior number 1
sippi_plot_movie('20130812_Metropolis',1,1000,0);

Using options.plot.skip_seq_gibbs=1, (set in sippi_plot_defaults)
removes realizations obtained using sequential Gibbs sampling
(equivalent to setting skip_burnin=1)

```

### 6.1.32 sippi\_plot\_posterior

```

sippi_plot_posterior Plot statistics from posterior sample

Call :
    sippi_plot_posterior(fname,im_arr,prior,options,n_reals);

See also sippi_plot_prior

```

### 6.1.33 sippi\_plot\_posterior\_2d\_marg

```

sippi_plot_posterior_2d_marg: plots 2D posterior marginal distributions

Call:
    [options,reals_all]=sippi_plot_posterior_2d_marg(options,prior,data,fname);

See also: sippi_plot_posterior

```

### 6.1.34 sippi\_plot\_posterior\_data

```

sippi_plot_posterior_data: plots posterior data and noise realizations

Call
    [options]=sippi_plot_posterior_data(options,prior,data,forward);

See also: sippi_plot_posterior

```

### 6.1.35 sippi\_plot\_posterior\_loglikelihood

```
sippi_plot_posterior_loglikelihood : plots log(L) and autorrecreation of log(L)

Call:
  sippi_plot_posterior_loglikelihood; % when located in an output folder
                                     % generated by SIPPI

  sippi_plot_posterior_loglikelihood(foldername); % Where 'foldername'
                                               % is a folder generated by SIPPI

  sippi_plot_posterior_loglikelihood(options); % where options is the
                                               % output of sippi_rejection or sippi_metropolis

  options=sippi_plot_posterior_loglikelihood(options,prior,data,mcmc,fname);

See also: sippi_plot_posterior
```

### 6.1.36 sippi\_plot\_posterior\_sample

```
sippi_plot_posterior_sample: plots posterior sample statistics

Call
  [options]=sippi_plot_posterior_sample(options,prior,data,forward);

See also: sippi_plot_posterior
```

### 6.1.37 sippi\_plot\_prior

```
sippi_plot_prior Plot a 'model', i.e. a realization of the prior model

Call :
  sippi_plot_prior(prior,m,im_array);

  prior : Matlab structure for SIPPI prior model
  m : Matlab structure for SIPPI realization
  im_array : integer array of type of models to plot (typically 1)

Example
  m=sippi_prior(prior);
  sippi_plot_prior(prior,m);

  m=sippi_prior(prior);
  sippi_plot_prior(prior,m,2);

See also sippi_plot_prior
```

### 6.1.38 sippi\_plot\_prior\_sample

```
sippi_plot_prior Plot a sample of the prior in SIPPI

Call :
    sippi_plot_prior_sample(prior,im_array,n_reals,cax);

See also sippi_plot_posterior, sippi_plot_prior
```

### 6.1.39 wiggle

```
wiggle : plot wiggle/VA/image plot

Call
    wiggle(Data); % wiggle plot
    wiggle(Data,scale); % scaled wiggle plot
    wiggle(x,t,Data); % wiggle plt
    wiggle(x,t,Data,'VA') % variable Area (pos->black;neg->transp)
    wiggle(x,t,Data,'VA2') % variable Area (pos->black;neg->red)
    wiggle(x,t,Data,'wiggle',scale); % Scaled wiggle
    wiggle(x,t,Data,'wiggle',scale,showmax); % Scaled wiggle and max
                                         showmax traces.
    wiggle(x,t,Data,'wiggle',scale,showmax,plimage); % wiggle + image
    wiggle(x,t,Data,'wiggle',scale,showmax,plimage,caxis); % wiggle +
                                                         scaled image

Data : [nt,ntraces]
x : [1:ntraces] X axis (ex [SegyTraceheaders.offset])
t : [1:nt] Y axis
style : ['VA'] : Variable Area
        ['wiggle'] : Wiggle plot
scale : scaling factor, can be left empty as []
showmax [scalar] : max number of traces to show on display [def=100]
plimage [0/1] : Show image beneath wiggles [def=0];
caxis [min max]/[scalar] : amplitude range for colorscale

MAKE IT WORK FOR ANY X-AXIS !!!
```

## 6.2 SIPPI toolbox: Traveltime tomography

### 6.2.1 calc\_Cd

```
Calc_cd Setup a covariance model to account for borehole imperfections

Call: Cd=calc_Cd(ant_pos,var_uncor,var_cor1,var_cor2,L)
This function sets up a data covariance matrix that accounts for static
(i.e. correlated) data errors.

Inputs:
* ant_pos: A N x 4 array that contains N combinations of transmitter/source
and receiver positions. The first two columns are the x- and y-coordinates
of the transmitter/source position. The last two columns are the x- and
y-coordinates of the receiver position.
```

```

* var_uncor: The variance of the uncorrelated data errors.
* var_cor1: The variance of the correlated data errors
related to the transmitter/source positions.
* var_cor2: The variance of the correlated data errors
related to the receiver positions.
* L: The correlation length for the correlation between the individual
transmitter/source or receiver positions using an exponential covariance
function. For typical static errors the correlation length is set to a
small number (e.g. 10^-6).

```

For more details and practical examples see:  
 Cordua et al., 2008 in Vadose zone journal.  
 Cordua et al., 2009 in Journal of applied geophysics.

Knud S. Cordua (2012)

## 6.2.2 eikonal

eikonal Traveltime computation by solving the eikonal equation

```
tmap=eikonal(x,y,z,V,Sources,type);
```

```

x,y,z : arrays defining the x, y, and z axis
V: velocity field, with size (length(y),length(x),length(z));
Sources [ndata,ndim] : Source positions
type (optional): type of eikonal solver: [1]:Fast Marching(default), [2]:FD

tmap [size(V)]: travel times computed everywhere in the velocity grid

```

```

%Example (2D):
x=[1:1:100];
y=1:1:100;
z=1;
V=ones(100,100);V(:,1:50)=2;
Sources = [10 50;75 50];
t=eikonal(x,y,z,V,Sources);
subplot(1,2,1);imagesc(x,y,t(:,:,1,1));axis image;colorbar
subplot(1,2,2);imagesc(x,y,t(:,:,1,2));axis image;colorbar

```

See also eikonal\_traveltime

## 6.2.3 eikonal\_raylength

eikonal\_raylength : Computes the raylength from S to R using the eikonal equation

```

Call:
raylength=eikonal_raylength(x,y,v,S,R,tS,doPlot)

```

## 6.2.4 eikonal\_traveltime

eikonal\_traveltime Computes traveltime between sources and receivers by solving the eikonal equation ↩

```
t=eikonal_traveltime(x,y,z,V,Sources,Receivers,iuse,type);

x,y,z : arrays defining the x, y, and z axis
V: velocity field, with size (length(y),length(x),length(z));
Sources [ndata,ndim] : Source positions
Receivers [ndata,ndim] : Receiver positions
iuse (optional): optionally only use subset of data. eg.g i_use=[1 2 4];
type (optional): type of eikonal solver: [1]:Fast Marching(default), [2]:FD

tmap [size(V)]: travel times computed everywhere in the velocity grid
```

%Example (2%

Example 2d traveltime computation

```
Example (2D):
x=1:1:100;
y=1:1:100;
z=1;
V=ones(100,100);V(:,1:50)=2;
S=[50 50 1;50 50 1];
R=[90 90 1; 90 80 1];
t=eikonal_traveltime(x,y,z,V,S,R)
```

```
Example (3D):
nx=50;ny=50;nz=50;
x=1:1:nx;
y=1:1:ny;
z=1:1:nz;
V=ones(ny,nx,nz);V(:,1:50,:)=2;
S=[10 10 1;10 10 1;10 9 1];
R=[40 40 40; 40 39 40; 40 40 40];
t=eikonal_traveltime(x,y,z,V,S,R)
```

See also eikonal

## 6.2.5 kernel\_buursink\_2d

kernel\_buursink\_2k Computes 2D Sensitivity kernel based on 1st order EM scattering theory

See

Buursink et al. 2008. Crosshole radar velocity tomography  
with finite-frequency Fresnel. Geophys J. Int.  
(172) 117;

CALL :

```
% specify a source trace (dt, wf_trace):
[kernel,L,L1_all,L2_all]=kernel_buursink_2d(model,x,z,S,R,dt,wf_trace);
% Use a ricker wavelet with center frequency 'f0'
[kernel,L,L1_all,L2_all]=kernel_buursink_2d(model,x,z,S,R,f0));
```

Knud Cordua, 2009,  
Thomas Mejer Hansen (small edits, 2009)

## 6.2.6 kernel\_finite\_2d

kernel\_finite\_2d 2D sensitivity kernels

Call:

```
[Knorm,K,dt,options]=kernel_finite_2d(v_ref,x,y,S,R,freq,options);
```

## 6.2.7 kernel\_fresnel\_2d

kernel\_fresnel\_2d Sensitivity kernel for amplitude and first arrival

Call:

```
[kernel_t,kernel_a,P_omega,omega]=kernel_fresnel_2d(v,x,y,S,R,omega,P_omega);
```

Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity kernels for seismic Fresnel volume Tomography, Geophysics, 75(5), U35-U46

See also kernel\_fresnel\_monochrome\_2d

Run with no argument for an example.

## 6.2.8 kernel\_fresnel\_monochrome\_2d

kernel\_fresnel\_monochrome\_2d 2D monochrome kernel for amplitude and first arrival

Call:

```
[kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S,R,omega);
```

or

```
[kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S,R,omega,L,L1,L2);
```

Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity kernels for seismic Fresnel volume Tomography, Geophysics, 75(5), U35-U46

See also, kernel\_fresnel\_2d

## 6.2.9 kernel\_multiple

kernel\_multiple Computes the sensitivity kernel for a wave traveling from S to R.

CALL :

```
[K,RAY,Gk,Gray,timeS,timeR,raypath]=kernel_multiple(Vel,x,y,z,S,R,T,alpha,Knorm);
```

IN :

Vel [ny,nx] : Velocity field

x [1:nx] :

y [1:ny] :

z [1:nz] :

S [1,3] : Location of Source

R [1,3] : Location of Receiver

```

T : Dominant period
alpha: controls exponential decay away ray path
Knorm [1] : normalization of K [0]:none, K:[1]:vertical

OUT :
K : Sensitivity kernel
R : Ray sensitivity kernel (High Frequency approx)
timeS : travel computed form Source
timeR : travel computed form Receiver
raypath [nraydata,ndim] : the center of the raypath

The sensitivity is the length travelled in each cell.

See also : fast_fd_2d

TMH/2006

```

### 6.2.10 kernel\_slowness\_to\_velocity

```

kernel_slowness_to_velocity Converts from slowness to velocity parameterizations

G : kernel [1,nkernels]
V : Velocity field (

CALL:
  G_vel=kernel_slowness_to_velocity(G,V);
or
  [G_vel,v_obs]=kernel_slowness_to_velocity(G,V,t);
or
  [G_vel,v_obs,Cd_v]=kernel_slowness_to_velocity(G,V,t,Cd);

```

### 6.2.11 mspectrum

```

mspectrum : Amplitude and Power spectrum
Call :
  function [A,P,smoothP,kx]=mspectrum(x,dx)

1D (A)mplitude and (P)owerspectrum of x-series with spacing dx

```

### 6.2.12 munk\_fresnel\_2d

```

2D frechet kernel, First Fresnel Zone

See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar Physics 192.

Call :
S=munk_fresnel_2d(T,dt,alpha,As,Ar,K);

T : dominant period

```



```

dt :
alpha : degree of cancellation
As : Amplitude fo the wavefield propagating from the source
Ar : Amplitude fo the wavefield propagating from the receiver
K : normalization factor

```

### 6.2.13 munk\_fresnel\_3d

3D frechet kernel, First Fresnel Zone

See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar Physics 192.

Call :

### 6.2.14 sippi\_forward\_travelttime

sippi\_forward\_travelttime Travelttime computation in SIPPI

```

Call :
[d,forward,prior,data]=sippi_forward_travelttime(m,forward,prior,data,id,im)

forward.type determines the method used to compute travel times
forward.type='ray';
forward.type='fat';
forward.type='eikonal';
forward.type='born';

```

### 6.2.15 tomography\_kernel

tomography\_kernel Computes the sensitivity kernel for a wave traveling from S to R.

```

CALL :
[K,RAY,Gk,Gray,timeS,timeR,raypath]=tomography_kernel(Vel,x,y,z,S,R,T,alpha,Knorm);

```

```

IN :
Vel [ny,nx] : Velocity field
x [1:nx] :
y [1:ny] :
z [1:nz] :
S [1,3] : Location of Source
R [1,3] : Location of Receiver
T : Donminant period
alpha: controls exponential decay away ray path
Knorm [1] : normaliztion of K [0]:none, K:[1]:vertical

```

```

OUT :
K : Sensitivity kernel
R : Ray sensitivity kernel (High Frequency approx)
timeS : travel computed form Source
timeR : travel computed form Receiver
raypath [nraydata,ndim] : the center of the raypath

```

The sensitivity is the length travelled in each cell.

