SIPPI

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November 20, 2014

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About

SIPPI is a Matlab toolbox (compatible with GNU Octave) that allows Sampling the solution of non-linear Inverse Problems with realistic a Priori Information.

In order to make use of SIPPI one has to

- Install and setup SIPPI
- Define the prior model, in form of the prior data structure
- Define the forward model, in form of the forward data structure, and the sippi_forward.m m-file
- Define the data and noise model, in form of the prior data structure
- Choose a method for sampling the a posteriori probability density (i.e., solution) of the inverse problem.

Details about the implementation and the methods implemented in SIPPI can be found in [HCM12], [CHM12], [HCLM13a], [HCLM13b] and, [HCM14].

This version of the documentation was compiled on Nov 20, 2014 , and refer to SIPPI version 1.10.

Chapter 1

Installation

1.1 SIPPI

Download the latest version of SIPPI from http://sippi.sourceforge.net.

Unpack ZIPPI.zip somewhere, for example to 'c:\Users\tmh\SIPPI'. Then setup the Matlab path to point to the appropriate SIPPI directories by typing:

```
addpath c:\Users\tmh\SIPPI
sippi_set_path
```

1.1.1 SGeMS (optional)

To make use of the SISIM and SNESIM type prior models, SGeMS needs to be available.

Currently only SGeMS version 2.1 (download) for Windows is supported by SIPPI.

Chapter 2

Setting up SIPPI

tomography example This section contains information about how to use and control SIPPI, which requires one to

- Define the prior model, in form of the prior data structure
- Define the forward model, in form of the forward data structure, and the sippi_forward.m m-file
- Define the data and noise model, in form of the prior data structure

[For examples of how to apply SIPPI for different problems, see the section with examples].

2.1 `prior`: The a priori model

A priori information is defined by the `prior` Matlab structure. Any number of different types of a priori models can be defined. For example a 1D uniform prior can be defined in `prior{1}`, and 2D Gaussian prior can be defined in `prior{2}`.

Once a prior data structure has been defined (see examples below), a realization from the prior model can be generated using

```
m=sippi_prior(prior);
```

The realization from the prior can be visualized using

```
sippi_plot_prior(prior,m);
```

A sample (many realizations) from the prior can be visualized using

```
m=sippi_plot_prior_sample(prior);
```

All a priori model types in SIPPI allow to generate a new model in the vicinity of a current model using

```
[m_new,prior]=sippi_prior(prior,m);
```

in such a way that the prior model will be sampled if the process is repeated (see Sequential Gibbs Sampling).

2.1.1 Types of a priori models

Six types of a priori models are available, and can be selected by setting the `type` in the `prior` structure using e.q. `prior{1}.type='gaussian'`.

The UNIFORM type prior specifies an uncorrelated ND uniform model.

The GAUSSIAN type prior specifies a 1D generalized Gaussian model.

The FFTMA type prior specifies a 1D-3D Gaussian type a prori model based on the FFT Moving Average method, whish is bery efficient for unconditional sampling, and for defining a prior Gaussian model with variable/uncertain mean, variance, ranges, and rotation.

The CHOLESKY type prior specifies a 1D-3D Gaussian tyype a priori model based on Cholesky decomposition of the covariance model,

The VISIM type prior model specifies 1D-3D Gaussian models, utilizing both sequential Gaussian simulation (SGSIM) and direct sequential simulation (DSSIM) that can be conditioned to data of both point- and volume support and linear average data.

The SNESIM type prior model specifies a 1D-3D multiple-point-based statistical prior model, which relies on training images from where the conditional dependencies of the spatial variables are obtained (i.e., learned). This type of prior model requires SGEMS to be installed.

The following section documents the properties of each type of prior model.

Examples of using different types of prior models or combining prior models can be found in the examples section.

2.1.1.1 Uniform distribtion

A uniform prior model can be specified using the 'uniform' type prior model

```
prior{1}.type='uniform';
```

The only parameters needed are the minimum (`min`) and maximum (`max`) values. A 1D uniform distribution between -1 and 1 can be specified as

```
prior{1}.type='uniform';
prior{1}.min=-1;
prior{1}.max=1;
```

By setting the `x`, `y`, and `z` parameter, a higher order prior (uncorrelated) can be set. For example 3 independent model parameters with a uniform prior distribution between 20 and 50, can be defined as

```
prior{1}.type='uniform';
prior{1}.x=[1 2 3];
prior{1}.min=20;
prior{1}.max=50;
```

Note that using the 'uniform' type priori model, is slightly more computational efficient than using a 'gaussian' type prior model with a high norm.

2.1.1.2 1D Generalized Gaussian

A 1D generalized Gaussian prior model can be specified using the 'gaussian' type prior model

```
prior{1}.type='gaussian';
```

A simple 1D Gaussian distribution with mean 10, and standard deviation 2, can be specified using

```
ip=1;
prior{ip}.type='gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;
```

The norm of a generalized Gaussian can be set using the 'norm' field. A generalized 1D Gaussian with mean 10, standard deviation of 2, and a norm of 70, can be specified using (The norm is equivalent to the beta factor referenced in Wikipedia:Generalized_normal_distribution)

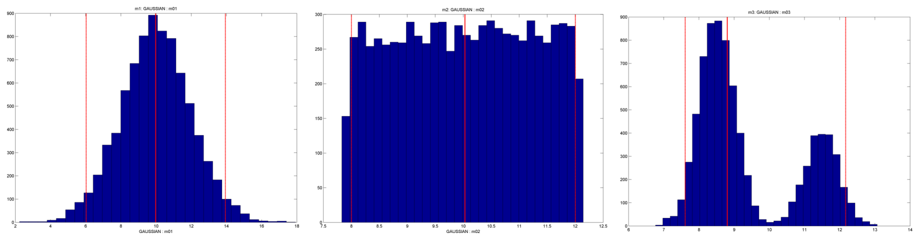
```
ip=2;
prior{ip}.type='gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=70;
```

A 1D distribution with an arbitrary shape can be defined by setting `d_target`, which must contain a sample of the distribution that one would like to replicate. For example, to generate a sample from a non-symmetric bimodal distribution, one can use e.g.

```
% Create target distribution
N=10000;
prob_chan=0.3;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];

% set the target distribution
ip=3;
prior{ip}.type='gaussian';
prior{ip}.d_target=d_target;
```

The following figure shows the 1D histogram of a sample, consisting of 8000 realizations, generated using

```
sippi_plot_prior_sample(prior,1:ip,8000);
```



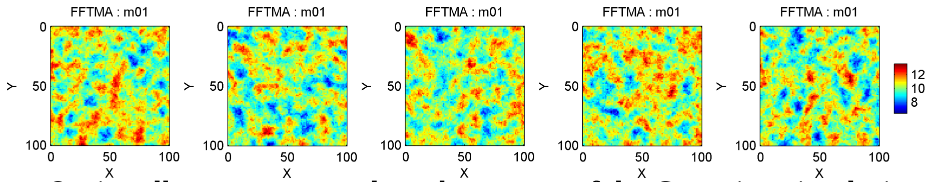
2.1.1.3 FFTMA - 3D Gaussian model

The FFT moving average method provides an efficient approach for computing unconditional realizations of a Gaussian random field.

The mean and the covariance model must be specified in the `m0` and `Cm` fields. The format for describing the covariance model follows 'gstat' notation, and is described in more details in the mGstat manual.

A 2D covariance model with mean 10, and a Spherical type covariance model can be defined in a 101x101 size grid (1 unit (e.g., meters) between the cells) using

```
im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
```

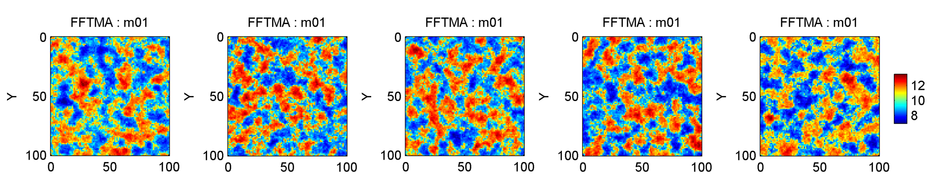


Optionally one can translate the output of the Gaussian simulation into an arbitrarily shaped 'target' distribution, using normal score transformation. Note that this transformation will ensure a certain 1D distribution of the model parameters to be reproduced, but will alter the assumed covariance model such that the properties of covariance model are not necessarily reproduced. To ensure that both the covariance model properties and the 1D distribution are reproduced, make use of the VISIM type prior model instead because it utilizes direct sequential simulation.

```
im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.Cm='1 Sph(10)';

% Create target distribution
N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;
prior{im}.m0=0; % to make sure no trend model is assumed.
```

Alternatively, the normal score transformation can be defined manually such that the tail behavior can be controlled:

```
N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
[d_nscore,o_nscore]=nscore(d_target,1,1,min(d_target),max(
    d_target),0);
prior{im}.o_nscore=o_nscore;
```



2.1.1.4 FFTMA - 3D Gaussian model wuth variable model properties

The FFTMA method also allows treating the parameters defining the Gaussian model, such as the mean, variance, ranges and angles of rotation as a priori model parameters (that can be inferred as part of inversion, see e.g. an example).

First a prior type defining the Gaussian model must be defined (exactly as listed above):

```
im=im+1;
prior{im}.type='FFTMA';
prior{im}.x=[0:.1:10]; % X array
prior{im}.y=[0:.1:20]; % Y array
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10,90,.25)';
```

Now, all parameter such as the mean, variance, ranges and angles of rotations, can be randomized by defining a 1D a priori model type ('uniform' or 'gaussian'), and with a specific 'name' indicating the parameter (see this example for a complete list of names), and by assigning the `prior_master` field that points the prior model id for which the parameters should randomized.

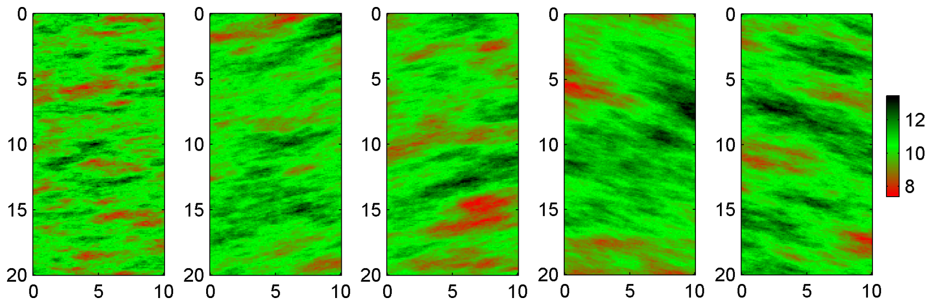
For example the range along the direction of maximum continuty can be randomized by defining a prior entry named 'range_1', and settting the `prior_master` to point to the prior with id 1:

```
im=2;
prior{im}.type='uniform';
prior{im}.name='range_1';
prior{im}.min=2;
prior{im}.max=14;
prior{im}.prior_master=1;
I this case the range is randomized following a uniform ↩
    distribution U[2,14].
```

Likewise, the first angle of rotation can be randomized using for example

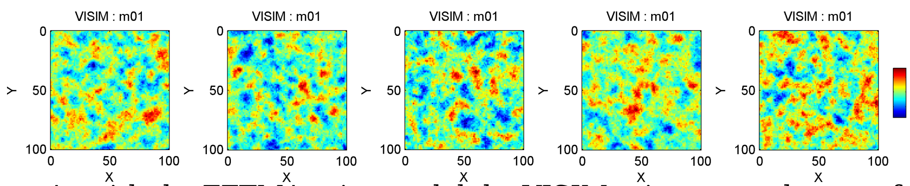
```
im=3;
prior{im}.type='gaussian';
prior{im}.name='ang_1';
prior{im}.m0=90;
prior{im}.std=10;
prior{im}.prior_master=1;
```

A sample from such a prior type model will thus show variability also in the range and angle of rotation, as seen here



2.1.1.5 VISIM

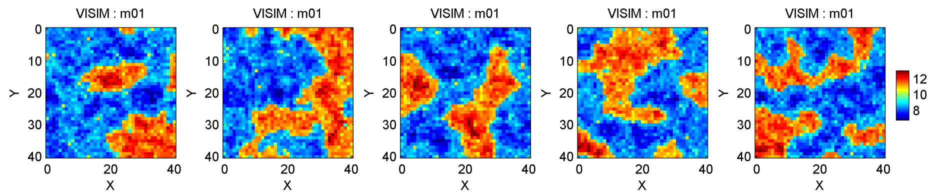
```
im=im+1;
prior{im}.type='VISIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
```



As with the FFTMA prior model the VISIM prior can make use of a target distribution. However, if a target distribution is set, the use of the VISIM prior model will utilize direct sequential simulation, which will ensure both histogram and covariance reproduction.

Using a target distribution together with the VISIM prior model is similar to that for the FFTMA prior model. Simply the `type` has to be changed from FFTMA to VISIM:

```
clear all;close all;
im=1;
prior{im}.type='VISIM';
prior{im}.x=[0:1:40];
prior{im}.y=[0:1:40];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

% Create target distribution
N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;
```

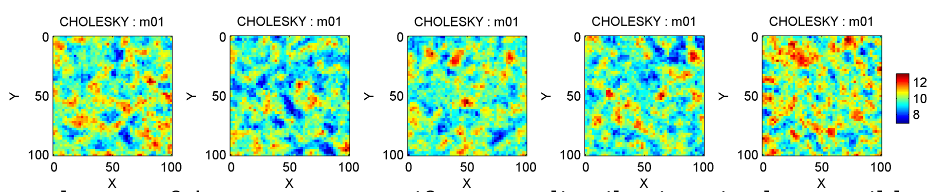


2.1.1.6 CHOLESKY - 3D Gaussian model

The CHOLESKY type prior utilizes Cholesky decompostion of the covariance in order to generate realizations of a Gaussian random field. The CHOLESKY type prior needs a full description of the covariance odel, which will be of size [nxyz*nxyz*nxyz], unlike using the FFTMA type prior model that only needs a specification of an isotropic covariance models of size [1,nxyz]. Hence, the CHOLEKSY type prior is much more demanding on memory, and CPU. However, the CHOLESKY type prior can be used to sample from any covariance model, also non-stationary covariance model.

The CHOLESKY model is can be defined almost identically to the FFTMA type prior model. As an example:

```
im=1;
prior{im}.type='CHOLESKY';
prior{im}.x=[0:2:100];
prior{im}.y=[0:2:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
```



the use of `d_target` to specify target distributions is also possible, using the same style as for the FFTMA type prior.

Be warned that the 'cholesky' type prior model is much more memory demanding than the 'fftma' and 'visim' type prior models, as a full nxyz*nxyz covariance model needs to setup (and inverted). Thus, the 'cholesky' type prior is mostly applicable when the number of model parameters (nx*ny*nx) is small.

2.1.1.7 SNESIM

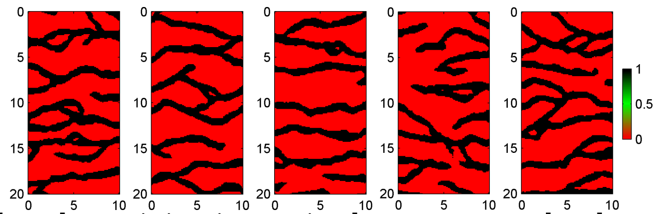
The 'SNESIM' type prior model utilizes the SNESIM algorithm, as implemented in SGeMS. As opposed to the Gaussian prior models defined above, the SNESIM prior model infer spatial statistics from a training image, which should be a 2D/3D stationary image.

By default a training image (channel structures) from Sebastian Strebelle's PhD theses is used (if no training image is specified). A simple 2D type SNESIM prior model can be defined using the following code:

```
ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array
```

and 5 realizations from this prior can be visualized using

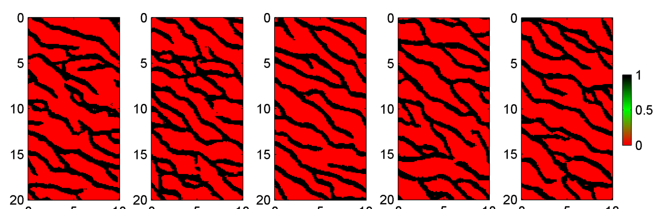
```
for i=1:5;
    m=sippi_prior(prior);
    subplot(1,5,i);
    imagesc(prior{1}.x,prior{1}.y,m{1});axis image
end
```



Note that the training image is always assumed to have the same units as the prior model, so in this case each pixel in the training image is assumed to be separated by a distance '0.1'.

Optionally 'scaling' and 'rotation' of the training image can be set. To scale the training image by 0.7 (i.e., structures will appear 30% smaller) and rotate the training 30 degrees from north use

```
ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array
prior{ip}.scaling=.7;
prior{ip}.rotation=30;
```



2.1.1.7.1 Custom training image A custom training image can be set using the `ti` field, which must be either a 2D or 3D matrix.

```
% create TI from image
EXAMPLE EXAMPLE

% setup the prior
ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array
prior{ip}.ti=ti;
```

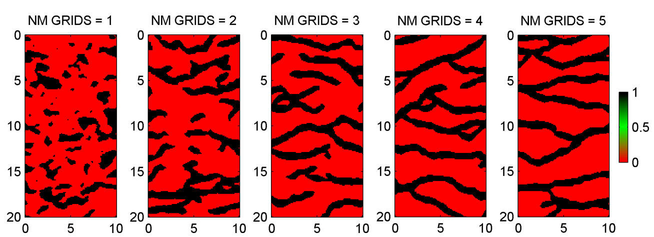
Note that the training image MUST consist of integer index values starting from 0 (i.e. '0', '1', '2', ...).

2.1.1.7.2 Complete customization If the `prior` structure is returned from sippi_prior using

```
[m,prior]=sippi_prior(prior);
```

then an XML structure `prior{1}.S.XML` will be available. This allows a complete customization of all settings available in SGeMS. For example, the different realizations, using 1, 2, and 3 multiple grids can be obtained using

```
ip=1;
prior{ip}.type='SNESIM';
prior{ip}.x=[0:.1:10]; % X array
prior{ip}.y=[0:.1:20]; % Y array
[m,prior]=sippi_prior(prior);
for i=1:5;
    prior{ip}.S.XML.parameters.Nb_Multigrids_ADVANCED.value= ↩
        i;
  subplot(1,3,5);
    imagesc(prior{1}.x,prior{1}.y,m{1});axis image
end
```



2.1.2 Sampling the prior

Once the prior data structure has been defined/modified, a sample from the prior distribution can be generated using

```
m=sippi_prior(prior);
```

'm' is a Matlab data structure of the same size as the 'prior' data structure. Thus, if two prior distributions have been defined in 'prior{1}' and 'prior{2}', then 'm{1}' will hold a realization of 'prior{1}', and 'm{2}' will hold a realization of 'prior{2}'.

Each time 'm=sippi_prior(prior)' is called, a new independent realization of the prior will be generated.

2.1.3 Sequential Gibbs sampling / Conditional Re-sampling

All the available types of prior models allow perturbing one realization of a prior into a new realization of the prior, where the degree of perturbation can be controlled (from a new independent realization to a very small change).

This means that a random walk, with an arbitrary 'step-length' can be performed for any of the a priori types available in SIPPI

For the a priori types 'FFTMA', 'VISIM', 'CHOLESKY', 'SISIM', 'SNESIM', sequential Gibbs sampling [HCM12] is applied. Sequential Gibbs is in essence a type of conditional re-simulation. From a current realization of a prior model, a number of model parameters are discarded and treated as unknown. The unknown model parameters are then re-simulated conditional to the known model parameters.

In order to generate a new realization 'm2' in the vicinity of the realization 'm1' use

```
[m1,prior]=sippi_prior(prior);
[m2,prior]=sippi_prior(prior,m1);
```

If this process is iterated, then a random walk in the space of a priori acceptable models will be perform. Moreover, the collection of realization obtained in this way will represent a sample from prior distribution.

Note that in order to use sequential Gibbs sampling `prior` must be given both as an input variable, and as an (possibly update) output variable.

2.1.3.1 Controlling sequential Gibbs sampling / Conditional Re-sampling

All properties related to sequential Gibbs sampling can be set in the 'seq_gibbs' structure (which will be avaiable the first time sippi_prior is called, or if sippi_prior_init is called), for the individual prior models.

The step-length (i.e. the degree of perturbation) is determined by the `prior{m}.seq_gibbs.step` parameter.

For the 'uniform' and 'gaussian' type a priori models a step-length closer to 0 zeros imples a 'shorter' step, while a step-length close to 1, implies a 'longer' step-length. A step length of 1, wil generate a new independent realization of the prior, while a step length of 0, will return the same realization of the prior

```
prior{m}.seq_gibbs.step=.1;
[m2,prior]=sippi_prior(prior,m1);
```

For the 'FFTMA', 'VISIM', 'CHOLESKY', 'SISIM', and 'SNESIM' type a priori models two types (defined in the `prior{m}.seq_gibbs.type` variable).

The default 'type' is 2, defined as

```
prior{m}.seq_gibbs.step=1;
prior{m}.seq_gibbs.type=2;
```

where the step length defines the percentage of the of model parameters (selected at random) defined in `prior{im}` is conditionally resampled. Thus, a step-length closer to 0 zeros imples a 'shorter' step, while a step-length close to 1, implies a 'longer' step-length.

If `prior{m}.seq_gibbs.step=1`, then `prior{m}.seq_gibbs.step` defines the size of a square rectangle/cube which is to be conditionally resimulated using sequential Gibbbs ssampling.

2.2 data: Data and data uncertainties/noise

`data` is a Matlab structure that defines any number of data and the associated uncertainty/noise model.

`data{1}` defines the first data set (which must always be defined), and any number of additional data sets can be defined in `data{2}`, `data{3}`, ...

This allows to consider for example seismic data in `data{1}`, and electromagnetic data in `data{2}`.

For each set of data, a Gaussian noise model (both correlated and uncorrelated) can be specified. The noise model for different data types (e.g. `data{1}` and `data{2}` are independent).

Once the noise model has been defined, the log-likelihood related to any model, `m`, with the corresponding forward response, `d`, can be computed using

```
[d,forward,prior,data]=sippi_forward(m,forward,prior,data)
logL=sippi_likelihood(data,d)
```

where `d` is the output of sippi_forward.

The specification of the noise model can be divided into a description of the measurement noise (mandatory) and the modeling error (optional).

2.2.1 Gaussian measurement noise

2.2.1.1 Uncorrelated Gaussian measurement noise

To define a set of observed data, [0,1,2], with an associated uncorrelated uncertainty defined by a Gaussian model with mean 0 and standard deviation 2, use

```
data{1}.d_obs=[0 1 2]';
data{1}.d_std=[2 2 2]';
```

which is equivalent to (as the noise model for each data is the same, and independent)

```
data{1}.d_obs=[0 1 2]';
data{1}.d_std=2;
```

One can also choose to define the uncertainty using a variance as opposed to the standard deviation

```
data{1}.d_obs=[0 1 2]';
data{1}.d_var=4;
```

2.2.1.2 Correlated Gaussian measurement noise

Correlated Gaussian measurement uncertainty can be specified using the `Cd` field, as for example

```
data{1}.Cd=[4 1 0 ; 1 4 1 ; 0 1 4];
```

Note that `data{1}.Cd` must be of size [NDxND], where ND is the the number of data in `data{1}.d_obs`.

2.2.2 Gaussian modeling error

The modeling error refers to errors caused by using for example an imperfect forward model, see [HCM14].

A Gaussian model of the modeling error can be specified by the mean, `dt`, and the covariance, `Ct`.

For example

```
data{1}.dt=[0 0 0];
data{1}.Ct=[4 4 4; 4 4 4; 4 4 4];
```

is equivalent to

```
data{1}.Ct=4
```

which implies a zero mean modeling error with a covariance model where all model parameters has a covariance of 4.

sippi_compute_modelization_forward_error can be used to estimate the modeling error related to using an approximate forward model. See the tomography example, for an example of accounting for correlated modeling errors, following [HCM14].

2.3 `forward`: The forward model

The specification of the `prior` and `data` is intended to be generic, applicable to any inverse problem considered. The forward problem, on the other hand, is typically specific for each different inverse problem.

In order to make use of SIPPI to sample the posterior distribution of an inverse problem, the solution to the forward problem must be embedded in a Matlab function with the following input and output arguments:

```
[d,forward,prior,data]=sippi_forward(m,forward,prior,data,id ↩
    )
```

`m` is a realization of the prior model, and `prior` and `data` are the Matlab structures defining the prior and the noise model (see Prior and Data)

`id` is optional, and can be used to compute the forward response of a subset of the different types of data available (i.e. `data{1}`, `data{2}`,...)

The `forward` variable is a Matlab structure that can contain any information needed to solve the forward problem. Thus, the parameters for the `forward` structure is problem dependent. One option, `forward.forward_function` is though generic, and point to the m-file that implements the forward problem.

The output variable `d` is a Matlab structure of the same size of `data`. Thus, if 4 types of data have been specified, then `d` must also be a structures of size 4.

```
length(data) == length(d);
```

Further, `d{i}` must refer to an array of the same size as `data{i}.d_obs`.

An example of an implementation of the forward problem related to a simple line fitting problem is:

```
function [d,forward,prior,data]=sippi_forward_linefit(m, ↩
    forward,prior,data);
```

```
    d{1}=forward.x*m{2}+m{1};
```

This implementation requires that the 'x'-locations, for which the y-values of the straight line is to be computed, is specified through `forward.x`. Say some y-data has been observed at locations x=[1,5,8], with the values [2,4,9], and a standard deviation of 1 specifying the uncertainty, the forward structure must be set as

```
forward.forward_function='sippi_forward_linefit';
forward.x=[1,5,8];
```

while the data structure will be

```
data{1}.d_obs=[2 4 9]
data{1}.d_std=1;
```

This implementation also requires that the prior model consists of two 1D prior types, such that

```
m=sippi_prior(prior)
```

returns the intercept in `m{1}` and the gradient in `m{2}`.

An example of computing the forward response using an intercept of 0, and a gradients of 2 is then

```
m{1}=0;
m{2}=2;
d=sippi_forward(m,forward)
```

and the corresponding log-likelihood of m, can be computed using

```
logL=sippi_likelihood(data,d);
```

[see more details and examples related to polynomial line fitting at polynomial line fitting].

The Examples section contains more example of implementation of different forward problems.

2.4 Validating `prior`, `data`, and `forward`

A simple way to test the validity of `prior`, `data`, and `forward` is to test if the following sequence can be evaluated without errors:

```
% Generate a realization, m, of the prior model
m=sippi_prior(prior);
% Compute the forward response
d=sippi_forward(m,forward,prior,data);
% Evaluate the log-likelihood of m
logL=sippi_likelihood(data,d);
```

Chapter 3

The a posteriori distribution

3.1 Sampling the a posteriori probability density

Once the `prior`, `data`, and `forward` data structures have been defined, the associated a posteriori probability can be sampled using the rejection sampler and the extended Metropolis sampler.

3.1.1 The rejection sampler

The rejection sampler provides a simples, and also in many cases inefficient, approach to sample the posterior distribtution.

At each iteration of the rejection sample an independent realization, m_pro, of the prior is generated, and the model is accepted as a realization of the posterior with probability Pacc = L(m_pro)/L_max. It can be initiated using

```
options.mcmc.nite=400000; % Number of iteration, defaults to ↩
    1000
options.mcmc.i_plot=500; % Number of iteration between  ↩
   visual updates, defaults to 500
options=sippi_rejection(data,prior,forward,options);
```

By default the rejection sampler is run assuming a maximum likelihood of 1 (i.e. L_max = 1). If L_max is known, then it can be set using in the `options.Lmax` or `options.logLmax` fields

```
options.mcmc.Lmax=1e-9;
options=sippi_rejection(data,prior,forward,options);
```

or

```
options.mcmc.logLmax=log(1e-9);
options=sippi_rejection(data,prior,forward,options);
```

Alternatively, L_max can be automatically adjusted to reflect the maximum likelihood found while running the rejection sampler using

```
options.mcmc.adaptive_rejection=1
options=sippi_rejection(data,prior,forward,options);
```

An alternative to rejection sampling, also utilizing independant realizations of the prior, that does not require one to set L_max is the independant extended metropolis sampler, which may be computatinoally superior to the rejection sampler,

3.1.2 The extended Metropolis sampler

The extended Metropolis algorithm is in general a mcuh more efficient algroirthm for sampling the a posteriori probability

The extended Metropolis sampler can be run using

```
options.mcmc.nite=40000;  % number of iterations, default ↩
    nite=30000
options.mcmc.i_sample=50; % save the current model for every ↩
     50 iterations, default, i_sample=500
options.mcmc.i_plot=1000;   % plot progress of the  ↩
    Metropolis sampler for every 100 iterations
                            % default i_plot=50;
options.txt='case_line_fit'; % descriptive name appended to  ↩
    output foldername, default txt='';

[options,data,prior,forward,m_current]=sippi_metropolis(data ↩
    ,prior,forward,options)
```

One can choose to accept all steps in the Metropolis sampler, which will result in an algorthm sampling the prior model, using

```
options.mcmc.accept_all=1; % default [0]
```

One can choose to accept models that lead to an improvement in the likelihood, which results in an optimization like algorithm using

```
options.mcmc.accept_only_improvements=1; % default [0]
```

See sippi_metropolis for more details.

3.1.2.1 Controling the step length

One optionally, as part of running the extended Metropolis sampler, automatically update the 'step'-length of the sequential Gibbs sampler in order to ensure a specific approximate acceptance ratio of the Metropolios sampler. See [CHM12] for details.

The default parameters for adjusting the step length, as given below, are set in the 'prior.seq_gibbs' structure. These parameters will be set the first time 'sippi_prior' is called with the 'prior' structure as output.The default parameters.

```
prior{m}.seq_gibbs.step_min=0;
prior{m}.seq_gibbs.step_min=1;
prior{m}.seq_gibbs.i_update_step=50
prior{m}.seq_gibbs.i_update_step_max=1000
prior{m}.seq_gibbs.n_update_history=50
prior{m}.seq_gibbs.P_target=0.3000
```

By default, adjustment of the step length, in order to achieve an acceptance ratio of 0.3 ('prior{m}.seq_gibbs.P_target'), will be performed for every 50 ('prior{m}.seq_gibbs.i_update_step') iterations, using the acceptance ratio observed in the last 50 ('prior{m}.seq_gibbs.i_update_history') iterations.

Adjustment of the step length will be performed only in the first 1000 ('prior{m}.seq_gibbs.i_update_step_max') iterations.

In order to disable automatiuc adjustment of the step length simply set

```
prior{m}.seq_gibbs.i_update_step_max=0; % disable automatic ↩
    step length
```

3.1.2.2 The independent extended Metropolis sampler

The 'independent' extended Metropolis sampler, in which each proposed model is independant of the previsouly visited model, can be chosen by forcing the 'step'-length to be 1 (i.e. leading to independant samples from the prior), using e.g.

```
% force indepedant prior sampling
for ip=1:length(prior);
  prior{ip}.seq_gibbs.step=1;
  prior{ip}.seq_gibbs.i_update_step_max=0;
end
% run 'independent' extended Metropolis sampling
[options,data,prior,forward,m_current]=sippi_metropolis(data ↩
    ,prior,forward,options)
```

3.1.2.3 Annealing schedule

Simulated annealing like behaviour can be controleld in the `options.mcmc.anneal` structure. By default annealing is disabled.

Annealing consist of multiplying the the noise level using an exponentially decerasing noise factor from `options.mcmc.anneal.fac_begin` to `options.mcmc.anneal.fac_end`, from iteration number `options.mcmc.anneal.i_begin` to `options.mcmc.anneal.i_end`.

The annealing schedule can be used start a Metropolis sampler that allow to explore more of the model space in the beginning. Recall though that the posterior is not sampled until (at least) the annealing has been ended at iteration, `options.mcmc.anneal.i_end`, if the `options.mcmc.anneal.fac_end=1`. This can potentially help not to get trapped in a local minima.

To use this type of annealing, where the annealing stops after 10000 iterations, after which the algorothm performs like a regular Metropolis sampler, use for example

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=10000; %  iteration number when annealing stops
```

which is equivalent to

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=10000; %  iteration number when annealing stops
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration i_begin
options.mcmc.anneal.fac_end=1; % default, noise is scaled by fac_end at iteration i_end
```

3.2 Simulated Annealing

Simulated annealing type optimization can be setup using an annealing schedule that is enable to the entire run og the Metropolis sampler, and that ends by a noise scaling factor less than 1. This can be obtained using e.g.

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=options.mcmc.nite; %  iteration number when annealing stops
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration i_begin
options.mcmc.anneal.fac_end=0.01; % 1/100 of the noise level
```

Chapter 4

Examples

SIPPI can be used as a convenient approach for unconditional an conditional simulation.

In order to use SIPPI to solve inverse problems, one must provide the solution to the forward problem. Essentially this amounts to implementing a Matlab function that solves the forward problem in using a specific input/output format. If a solution to the forward problem already exist, this can be quite easily done simply using a Matlab wrapper function.

A few implementations of solutions to forward problems are included as examples as part of SIPPI. These will be demonstrated in the following

4.1 Examples of A priori models

4.1.1 Multiple 1D Gaussian prior model

A prior model consisting of three independent 1D distributions (a Gaussian, Laplace, and Uniform distribution) can be defined using

```
ip=1;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;

ip=2;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Laplace';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=1;
```

```
ip=3;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Uniform';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=60;

m=sippi_prior(prior);

m =

    [14.3082]    [9.4436]    [10.8294]
```

1D histograms of a sample (consisting of 1000 realizations) of the prior models can be visualized using ...

```
sippi_plot_prior_sample(prior);
```

4.1.2 Multivariate Gaussian prior with unknown covariance model properties.

The FFT-MA type a priori model allow separation of properties of the covariance model (covariance parameters, such as range, and anisotropy ratio) and the random component of a Gaussian model. This allow one to define a Gaussian a priori model, where the covariance parameters can be treated as unknown variables.

In order to treat the covariance parameters as unknowns, one must define one a priori model of type `FFTMA`, and then a number of 1D `GAUSSIAN` type a priori models, one for each covariance parameter. Each gaussian type prior model must have a descriptive name, corresponding to the covariance parameter that is should describe:

```
prior{im}.type='gaussian';
prior{im}.name='m_0';     % to define a prior for the mean
prior{im}.name='sill';    % to define a prior for sill ( ↩
    variance)
prior{im}.name='range_1'; % to define a prior for the range  ↩
    parameter 1
prior{im}.name='range_2'; % to define a prior for the range  ↩
    parameter 2
prior{im}.name='range_3'; % to define a prior for the range  ↩
    parameter 3
prior{im}.name='ang_1';   % to define a prior for the first  ↩
    angle of rotation
prior{im}.name='ang_2';   % to define a prior for the second ↩
     angle of rotation
```

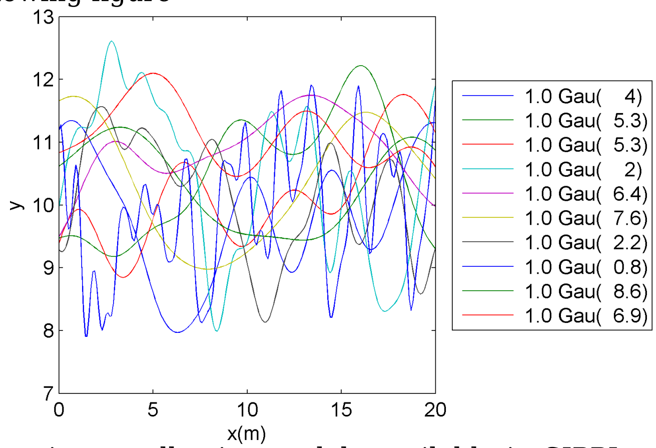
```
prior{im}.name='ang_3';   % to define a prior for the third ↩
    angle of rotation
prior{im}.name='nu';      % to define a prior for the shape ↩
    parameter, nu
         %   (only applies when the Mater type Covariance ↩
             model is used)
```

A very simple example of a prior model defining a 1D Spherical type covariance model with a range between 5 and 15 meters, can be defined using:

```
im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:.1:10]; % X array
prior{im}.m0=10;
prior{im}.Va='1 Sph(10)';
prior{im}.fftma_options.constant_C=0;


im=2;
prior{im}.type='gaussian';
prior{im}.name='range_1';
prior{im}.m0=10;
prior{im}.std=5
prior{im}.norm=80;
prior{im}.prior_master=1; % -- NOTE, set this to the FFT-MA ↩
    type prior for which this prior type
                          % should describe the range
```

Note that the the field `prior_master` must be set to point the to the FFT-MA type a priori model (through its id/number) for which it should define a covariance parameter (in this case the range).

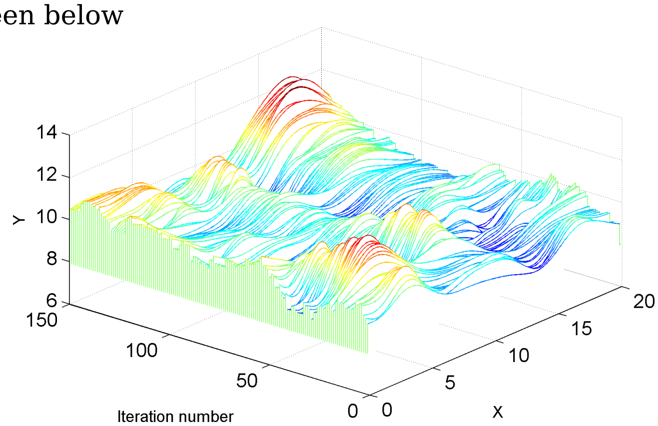
10 independent realizations of this type of a priori model are shown in the following figure



Such a prior, as all prior models available in SIPPI, works with sequential Gibbs sampling, allowing a random walk in the space of a prior acceptable models, that will sample the prior model. An example of such a random walk can be performed using

```
prior{1}.seq_gibbs.step=.005;
prior{2}.seq_gibbs.step=0.1;
clear m_real;
for i=1:150;
    [m,prior]=sippi_prior(prior,m);
    m_real(:,i)=m{1};
end
```

An example of such a set of 150 dependent realization of the prior can be seen below



4.2 Polynomial line fitting

Here follows simple polynomial (of order 0, 1 or 2) line-fitting is considered. Example m-files can be found in the `SIPPI/examples/case_linefit` folder.

First, the forward problem is defined. Then examples of stochastic inversion using SIPPI is demonstrated using a a synthetic data set.

4.2.1 The forward problem

The forward problem consists of computing the y-value as a function of the x-position of the data, and the polynomial coefficients determining the line. sippi_forward_linefit.m:

```
% sippi_forward_linefit Line fit forward solver for SIPPI
%
% [d,forward,prior,data]=sippi_forward_linefit(m,forward, ↩
    prior,data);
%
```

```
function [d,forward,prior,data]=sippi_forward_linefit(m, ↩
    forward,prior,data);

if length(m)==1;
    d{1}=forward.x*m{1};
elseif length(m)==2;
    d{1}=forward.x*m{1}+m{2};
else
    d{1}=forward.x.^2*m{1}+forward.x*m{2}+m{3};
end
```

the `forward.x` must be an array of the x-locations, for which the y-values of the corresponding line will be evaluated.

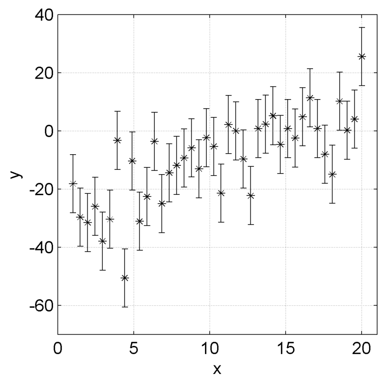
Note that the prior must be defined such that `prior{1}` refer to the intercept, `prior{2}` to the gradient, and `prior{3}` to the 2nd order polynomial coefficient.

If only one prior type is defined then the forward response will just be a constant, and if two prior types are defined, then the forward response will be a straight line.

4.2.2 Reference data, data, forward

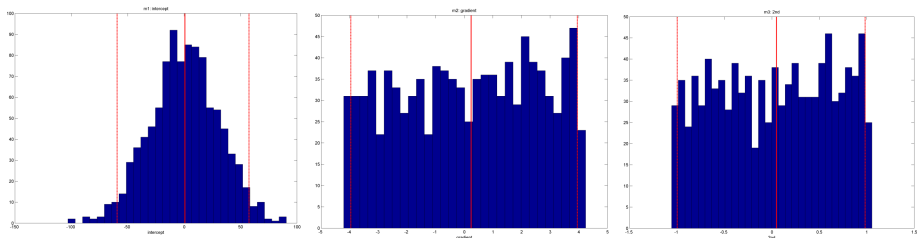
A reference data set can be computed using

```
clear all;close all;
rand('seed',1);randn('seed',1);

%% Select reference model
m_ref{1}=-30;
m_ref{2}=2;
m_ref{3}=0;

%% Setup the forward model in the 'forward' structure
nd=40;
forward.x=linspace(1,20,nd);
forward.forward_function='sippi_forward_linefit';

%% Compute a reference set of observed data
d=sippi_forward(m_ref,forward);
d_obs=d{1};
d_std=10;
d_obs=d_obs+randn(size(d_obs)).*d_std;

data{1}.d_obs=d_obs;
data{1}.d_std=d_std;
```



4.2.3 The prior model

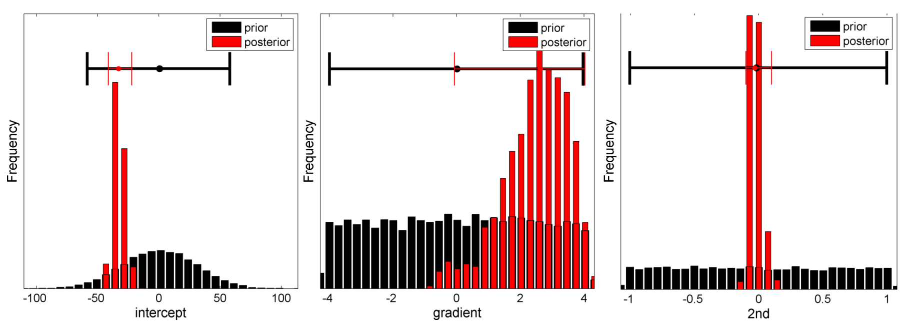
```
%% Setting up the prior model

% the intercept
im=1;
prior{im}.type='gaussian';
prior{im}.name='intercept';
prior{im}.m0=0;
prior{im}.std=30;
prior{im}.m_true=m_ref{1};

% 1st order, the gradient
im=2;
prior{im}.type='gaussian';
prior{im}.name='gradient';
prior{im}.m0=0;
prior{im}.std=4;
prior{im}.norm=80;
prior{im}.m_true=m_ref{2};

% 2nd order
im=3;
prior{im}.type='gaussian';
prior{im}.name='2nd';
prior{im}.m0=0;
prior{im}.std=1;
prior{im}.norm=80;
prior{im}.m_true=m_ref{3};

sippi_plot_prior_sample(prior);
```

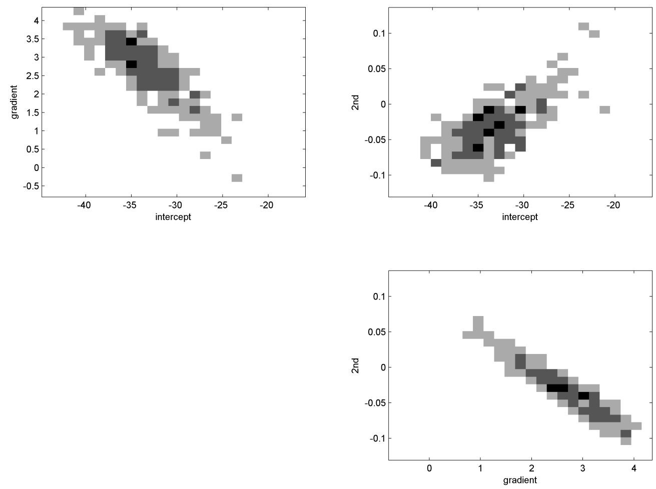


4.2.4 Setup and run the Metropolis sampler

Now, information about the model parameters can be inferred by running the extended Metropolis sampler using

```
options.mcmc.nite=40000;  % Run for 40000 iterations
options.mcmc.i_sample=50; % Save every 50th visited model to ↩
     disc
options.mcmc.i_plot=2500; % Plot the progress information  ↩
    for every 2500 iterations
options.txt='case_line_fit_2nd_order'; % descriptive name  ↩
    for the output folder

[options]=sippi_metropolis(data,prior,forward,options);

% plot posterior statistics, such as 1D and 2D marginals  ↩
    from the prior and posterior distributions
sippi_plot_prior_sample(options.txt);
sippi_plot_posterior(options.txt);
20140521 ↩
    _1644_sippi_metropolis_case_line_fit_2nd_order_m1_3_posterior_sample ↩
    .png
```





4.2.5 Setup and run the rejection sampler

In a similar manner the rejection sampler can be setup and run using

```
options.mcmc.adaptive_rejection=1; % automatically adjust the normalizing likelihood
options.mcmc.nite=100000;
options=sippi_rejection(data,prior,forward,options);
```

4.3 Cross hole tomography

SIPPI includes a reference cross hole GPR data from Arrenaes set is also available, and will be used here to demonstrate the use of SIPPI to solve cross hole tomographic inversion in a probabilistic framework.

SIPPI also includes the implementation of multiple methods for computing the travel time delay between a set of sources and receivers. This allows SIPPI to work on for example cross hole tomographic forward and inverse problems.

This section contains examples for setting up and running an cross hole tomographic inversion using SIPPI using the reference data from Arrenæs, different types of a priori and forward models.

Example Matlab scripts for the examples below, and more, are located in examples/case_tomography/.

Please see [HCLM13b] for more details on the example of using SIPPI to sample the posterior for cross hole tomographic inverse problems. See [LHC10] for more details on the data from Arrenæs.

4.3.1 Reference data set from Arrenæs

A 2D/3D data set of recorded travel time data from a cross hole Georadar experiment is available in the 'data/crosshole' folder.

4 Boreholes were drilled, AM1, AM2, AM3, and AM4 at the locations shown below

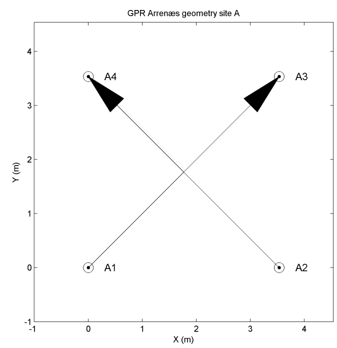


Figure 4.1: Location of boreholes AM1, AM2, AM3, and AM4 at Arrenæs.

Travel time data were collected between boreholes AM1 and AM3, and AM2 and AM4 respectively, in a depth interval between 1m and 12m. The travel times for each of the two 2D data sets are available in the AM13_data.mat and AM24_data.mat files. All the data have been combined in the 3D data set available in AM1234_data.mat.

All mat-files contains the following variable

```
S --> [ndata,ndim] each row contains the position of the ↩
    source
R --> [ndata,ndim] each row contains the position of the ↩
    receiver
d_obs --> [ndata,1] each row contains the observed travel ↩
    time in milliseconds
d_std --> [ndata,1] each row contains the standard deviation ↩
     of the uncertainty of the observed travel time in ↩
    milliseconds
```

All data are also available as ASCII formatted EAS files in `AM13_data.eas`, `AM24_data.eas`, and `AM1234_data.eas`.

The following 3 Figures show the ray coverage (using straight rays) for each of the AM13, AM24, and AM1234 data sets. The color of each ray indicates the average velocity along the ray computed using v_av = raylength/d_obs. AM13 ray coverageAM24 ray coverageAM1234 ray coverage.

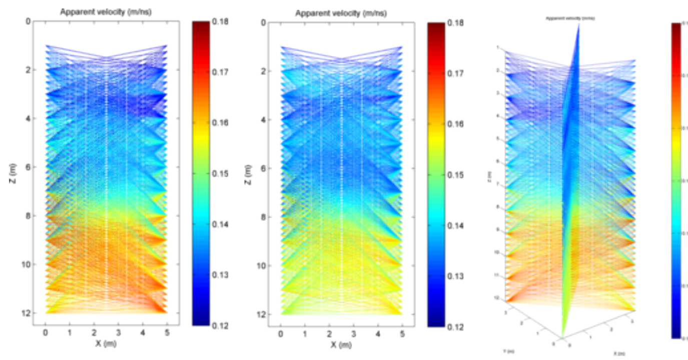


Figure 4.2: Ray coverage between wells left) AM1-AM3, middle) AM2-AM4, right) AM1-4.

4.3.2 Travel delay computation: The forward problem

A number of different methods for solving the problem of computing the first arrival travel time of a seismic or electromagnetic wave traveling between a source in one borehole and a receiver in another borehole has been implemented in the m-file 'sippi_forward_traveltime'.

```
[d,forward,prior,data]=sippi_forward_traveltime(m,forward, ↩
    prior,data,id,im)
```

In order to use this m-file to describe the forward problem specify the 'forward_function' field in the `forward` structure using

```
forward.forward_function='sippi_forward_traveltime';
```

In order to use `sippi_forward_traveltime`, the location of the sources and receivers must be specified in the `forward.S` and `forward.R`. The number of columns reflect the number of data, and the number of rows reflect whether data are 2D (2 columns) or 3D (3 columns):

```
forward.S %  [ndata,ndim]
forward.R %  [ndata,ndim]
```

Using for example the data from Arrenæs, the forward geometry can be set up using

```
D=load('AM13_data.mat');
forward.sources=D.S;
forward.receivers=D.R;
```

In addition the method used to compute the travel times must also be given (see below).

In order to use the geometry from the AM13 reference data, and the Eikonal solution to the wave-equation, the `forward` structure can be defined using

```
D=load('AM13_data.mat');
forward.forward_function='sippi_forward_traveltime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='eikonal';
```

4.3.2.1 Ray type forward model (high frequency approximation)

Ray type models are based on an assumption that the wave propagating between the source and the receiver has infinitely high frequency. Therefore the travel time delay is due to the velocity along a ray connecting the source and receiver.

The linear so-called straight ray approximation, which assumes that the travel time for a wave travelling between a source and a receiver is due to the travel time delay along a straight line connecting the source and receiver, can be chosen using

```
forward.type='ray';
forward.linear=1;
```

The corresponding so-called bended-ray approximation, where the travel time delay is due to the travel time delay along the fast ray path connecting a source and a receiver, can be chosen using

```
forward.type='ray';
forward.linear=0;
```

When sippi_forward_traveltime has been called once, the associated forward mapping operator is stored in 'forward.G' such the the forward problem can simply be solved by calling e.g. 'd{1}=forward.G*m{1}'

4.3.2.2 Fat Ray type forward model (finite frequency approximation)

Fat type model assume that the wave propagating between the source and the receiver has finite high frequency. This means that the travel time is sensitive to an area around the raypath, typically defined using the 1st Fresnel zone.

A linear fat ray kernel can be chosen using

```
forward.type='fat';
forward.linear=1;
forward.freq=0.1;
```

and the corresponding non-linear fat kernel using

```
bforward.type='fat';
forward.linear=0;
forward.freq=0.1;
```

Note that the center frequency of the propagating wave must also be provided in the 'forward.freq' field. The smaller the frequency, the 'fatter' the ray kernel.

For 'fat' type forward models we rely on the method described by Jensen, J. M., Jacobsen, B. H., and Christensen-Dalsgaard, J. (2000). Sensitivity kernels for time-distance inversion. Solar Physics, 192(1), 231-239

4.3.2.3 Born type forward model (finite frequency approximation)

Using the Born approximation, considering only first order scattering, can be chosen using

```
forward.type='born';
forward.linear=1;
forward.freq=0.1;
```

For a velocity field with small spatial variability one can compute 'born' type kernels (using 'forward.linear=0', but as the spatial variability increases this is not possible.

For the 'born' type forward model we make use if the method described by Buursink, M. L., Johnson, T. C., Routh, P. S., and Knoll, M. D. (2008). Crosshole radar velocity tomography with finite-frequency Fresnel volume sensitivities. Geophysical Journal International, 172(1), 1-17.

4.3.2.4 The eikonal equation (high frequency approximation)

The eikonal solution to the wave-equation is a high frequency approximation, such as the one given above.

However, it is computationally more efficient to solve the eikonal equation directly, that to used the 'forward.type='ray';' type forward model.

To choose the eikonal solver to compute travel times use

```
forward.type='eikonal';
```

The Accurate Fast Marching Matlab toolbox : http://www.mathworks.com/matlabcentral/filee
accurate-fast-marching is used to solve the Eikonal equation.

4.3.3 AM13 Gaussian: Inversion of cross hole GPR data from Arrenaes data with a Gaussian type a priori model

In the following a simple 2D Gaussian a priori model is defined, and SIPPI is used to sample the corresponding a posteriori distribution. (An example script is avalable at examples/case_tomography/-sippi_AM13_metropolis_gaussian.m).

4.3.3.1 Setting up the data structure

Initially we load the travel time data obtained at Arrenæs (See Arrenæs Data for more information)

```
D=load('AM13_data.mat');
```

This allow us to setup a SIPPI data structure defining the observed data as well as the associated model of uncertainty

```
%% SETUP DATA
id=1;
data{id}.d_obs=D.d_obs;
data{id}.d_std=D.d_std;
data{id}.dt=0; % Mean modelization error
data{id}.Ct=1; % Covariance describing modelization error
```

In the above example we define a Gaussian modelization error, N(dt,Ct). We do this because we will make use of a forward model, the eikonal solver, that we know will systematically provide faster travel times than can be obtained from the earth. In reality the wave travelling between bore holes never has infinitely high frequency as assumed by using the eikonal solver. The eikonal solver provides the fast travel time along a ray connecting the source and receiver. Therefore we introduce a modelization error, that will allow all the travel times to be biased with the same travel time.

4.3.3.2 Setting up the prior model

The a priori model is defined using the `prior` data structure. Here a 2D Gaussian type a priori model in a 7x13 m grid (grid cell size .25m) using the FFTMA type a priori model. The a priori mean is 0.145 m/ns, and the covariance function a Spherical type covariance model with a range of 6m, and a sill(variance) of 0.0003 mˆ2/nsˆ2.

```
%% SETUP PRIOR
im=1;
prior{im}.type='FFTMA';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
prior{im}.x=[-1:.15:6];
prior{im}.y=[0:.15:13];
```

One could make used of the VISIM type priori model simply by substituting 'FFTMA' with 'VISIM' above.

4.3.3.3 Setting up the forward structure

'sippi_forward_traveltime' require that the location of the sources an receivers are provided in 'forward' structure using the 'sources' and 'receivers' field names.

```
D=load('AM13_data.mat');
forward.forward_function='sippi_forward_traveltime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='eikonal';
```

Here the eikonal solution is chosen to solve the forward problem. See more detail about solving the forward problem related to cross hole first arrival travel time computation here.

4.3.3.4 Testing the setup

As the `prior`, `data`, and `forward` have been defined, one can in principle initiate an inversion. However, it is advised to perform a few test before applying the inversion.

First, one should check that independent realization of the prior model resemble the a priori knowledge. A sample from the prior model can be generated and visualized calling sippi_plot_prior_sample:

```
sippi_plot_prior_sample(prior);
```

which provides the following figure

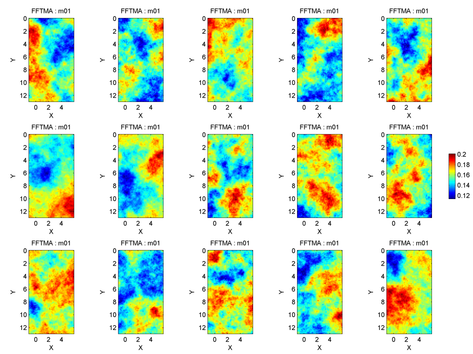


Figure 4.3: AM13: One sample (15 realizations) of the prior (Gaussian) model.

The one can check that the forward solver, and the computation of the likelihood wors as expected using

```
% generate a realization from the prior
m=sippi_prior(prior);
% Compute the forward response related to the realization of ↩
     the prior model generated above
[d]=sippi_forward(m,forward,prior,data);
% Compute the likelihood
[logL,L,data]=sippi_likelihood(d,data);
% plot the forward response and compare it to the observed  ↩
    data
sippi_plot_data(d,data);
```

which produce a figure similar to

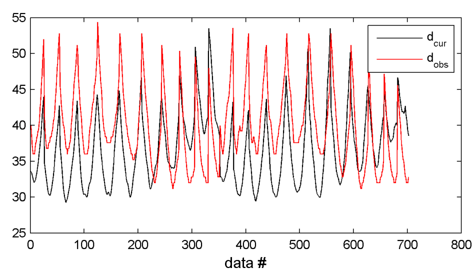


Figure 4.4: AM13: Data response from one realization of the prior.

4.3.3.5 Sampling the a posterior distribution using the extended Metropolis algorithm

The extended Metropolis sampler can now be run using sippi_metropolis.

```
options=sippi_metropolis(data,prior,forward);
```

In practice the user will have to set a few options, controlling the behavior of the algorithm. In the following example the number of iterations is set to 500000; the current model is saved to disc for every 500 iterations; the log-likelihood and current model is shown for every 1000 iterations:

```
options.mcmc.nite=500000; % optional, default:nite=30000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);
```

By default no annealing schedule is used. By default the 'step'-length for sequential Gibbs sampling is adjusted (to obtain an average acceptance ratio of 30%) for every 50 iterations until iteration number 1000.

An output folder will be generated with a filename formatted using 'YYYYMMDD-HHMM', followed by a automatic description. In the above case the output folder could be name '20140701_1450_sippi_metropolis_eikonal'. The actual folder name is return in `options.txt`.

One can define a description for the folder name by setting `options.txt` before running sippi_metropolis.

The folder contains one mat file, with the same name as the folder name, and N ASCII files (where N=length(prior); one for each a priori type) which contains the models saved to disc. They also have the same name as the folder name, appended with '_m1.asc', '_m2.asc', and so forth.

4.3.3.5.1 Posterior statistics

The function sippi_plot_posterior can be called when sippi_metropolis (or sippi_rejection) and will plot the progress of the log-likelihood curve, a sample of the posterior, data response from a sample of the posterior, and (if applicable) 1D and 2D marginal posterior distributions.

Located in the output folder of the inversion use

```
sippi_plot_posterior;
```

If the location of the folder with the output is known (such as options.txt) one can call

```
sippi_plot_posterior(options.txt)
```

4.3.4 AM13 Gaussian, accounting for modeling errors

[A Matlab script for the following example is avalable at examples/-case_tomography/sippi_AM13_metropolis_modeling_error.m.]

The use of any of the forward models defined above, will be approximation to solving the perfect forward problem. This leads to a 'modeling' error as demonstrated by [HCM14]. If one has access to an optimal (but perhaps computational inefficient) forward model, and a faster (less accurate) forward model, then a Gaussian model of the modeling error caused by using the approximate, as opposed to the optima, forward model can be estimated using sippi_compute_modelization_forward_error. sippi_compute_modelization_forward_error.

SIPPI allows accounting for such modeling error through the `dt` and `Ct` fields for the data structure.

The setup of the data, prior and forward structures is identical to the one described in the previous example.

```
%% Load the travel time data set from ARRENAES
clear all;close all
D=load('AM13_data.mat');
options.txt='AM13';

%% SETUP DATA
id=1;
data{id}.d_obs=D.d_obs;
data{id}.d_std=D.d_std;
data{id}.Ct=D.Ct+1; % Covariance describing modeling error

%% SETUP PRIOR
im=1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
dx=0.15;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];

% SETUP THE FORWARD MODEL USED IN INVERSION
forward.forward_function='sippi_forward_traveltime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='fat';forward.linear=1;forward.freq=0.1;
```

In order to compute the modeling with respect to using the 'Born' type forward model, one can define a new forward structure, here `forward_full`, and estimate a Gaussian model for the modeling error using

```
% SETUP THE 'OPTIMAL' FORWARD MODEL
forward_full.forward_function='sippi_forward_traveltime';
forward_full.sources=D.S;
forward_full.receivers=D.R;
forward_full.type='Born';forward_full.linear=1;forward_full. ↩
    freq=0.1;

% COMPUTE MODELING ERROR DUE TO USE OF forward AS OPPOSED TO ↩
     forward_full
N=100;
[Ct,dt,dd]=sippi_compute_modelization_forward_error( ↩
    forward_full,forward,prior,data,N);

% ASSIGN MODELING ERROR TO DATA
data{1}.dt=dt{1 };
data{1}.Ct=Ct{1};
```

Sampling of the posterior can proceed exactly as for the previous example, using

```
options.mcmc.nite=500000; % optional, default:nite=30000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);

% plot posterior statistics
sippi_plot_posterior(options.txt);
```

4.3.5 AM13 Gaussian with bimodal velocity distribution

[A Matlab script for the following example is avalable at examples/-case_tomography/sippi_AM13_metropolis_bimodal.m.]

The GAUSSIAN and FFTMAa prior types implicitly assume a normal distribution of the model parameter.

It is however possible to change the Gaussian distribution to any shaped distribution, using a normal score transform. Note that when this is done the given semivariogram model for the FFTMA a priori model will not be reproduced. If this is a concern, then the VISIM type a priori model should be used.

The data and forward structures is identical to the one described in the previous example.

```
%% Load the travel time data set from ARRENAES
clear all;close all
D=load('AM13_data.mat');
```

```
options.txt='AM13';

%% SETUP DATA
id=1;
data{id}.d_obs=D.d_obs;
data{id}.d_std=D.d_std;
data{id}.Ct=D.Ct+1; % Covariance describing modeling error

% SETUP THE FORWARD MODEL USED IN INVERSION
forward.forward_function='sippi_forward_traveltime';
forward.sources=D.S;
forward.receivers=D.R;
forward.type='fat';forward.linear=1;forward.freq=0.1;
```

The desired distribution (the 'target' distribution) must be provided as a sample of the target distribution, in the `data{id}.d_target` distribution.

```
%% SETUP PRIOR
im=1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
dx=0.15;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];

% SET TARGET
N=1000;
prob_chan=0.5;
dd=.014*2;
d1=randn(1,ceil(N*(1-prob_chan)))*.01+0.145-dd;  %0.1125;
d2=randn(1,ceil(N*(prob_chan)))*.01+0.145+dd; %0.155;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;
```

5 realizations from the corresponding a priori model looks like

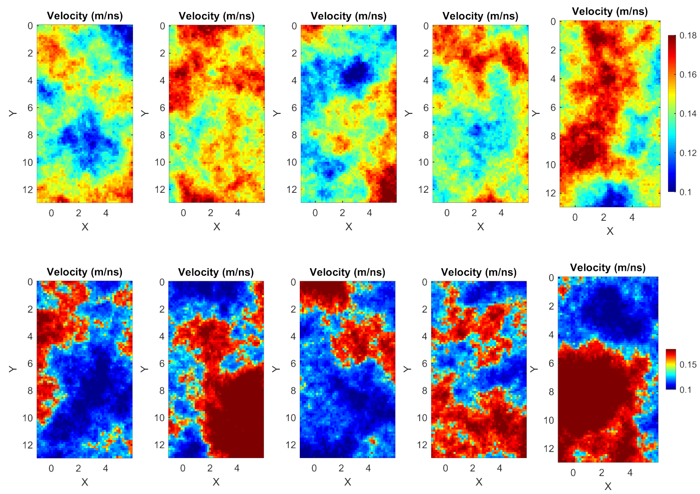


Figure 4.5: 5 realizations from a FFTMA prior model type with top) Gaussian and b) Bimodal distribution

Figure Figure 4.6 compares the distribution from one realization of both prior models considered above.

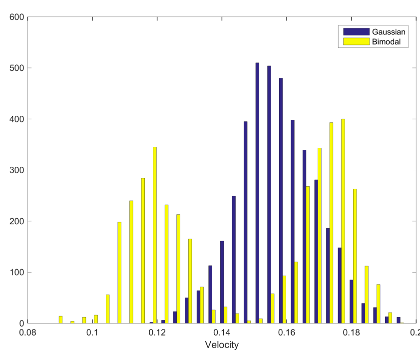


Figure 4.6: Distribution of one realization using a Gaussian Bimodal target distribution

As for the examples above, the a posteriori distribution can be samples using e.g.

```
options.mcmc.nite=500000; % optional, default:nite=30000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);
```

```
% plot posterior statistics
sippi_plot_posterior(options.txt);
```

4.3.6 AM13 Gaussian with unknown Gaussian model parameters

[A Matlab script for the following example is avalable at examples/-case_tomography/sippi_AM13_metropolis_gaussian_covariance_inference.m.]

One of the most intriguing benefits (in addition to the computational efficiency) of using the FFTMA type a priori model, is that it allows separation of the random component and the covariance model parameters. See [HCLM13a].

This means that one can in SIPPI define an inverse problem, where the a priori model is Gaussian, but where the properties of the Gaussian model (such as the mean, range, anisotropy) can be treated as unknown model parameters

Each property the Gaussian prior model that should be treated as an unknown model parameter, must be defined as a separate 1D type GAUSSIAN type prior model, with a specific name (identifying the covariance model property it describes), and it mist point to the prior model type number for which it describes a covariance model property-

The example below describes a 2D FFTMA type e a priori model (prior with id 1) with an unknown range (prior with id 2) with an a priori distribution described by a close to uniform distribution between 1.5m and 10.5m:

```
im=1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
dx=0.25;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];

i_master=im;

% range - horizontal
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='range_1'; % the name covariance model ↩
    property to define
prior{im}.m0=6;
prior{im}.min=1.5;
prior{im}.max=10.5;
```

```
prior{im}.norm=50;
prior{im}.prior_master=i_master; % point to the id of the ↩
    prior it describes
```

Any combination of the following parameters can be set:

```
prior{im}.name='range_1; % Range, along direction of angle_1
prior{im}.name='range_2; % Range, along direction of angle_2
prior{im}.name='range_3; % Range, along direction of angle_3
prior{im}.name='ang_1;   % Angle 1, degrees from North
prior{im}.name='ang_2;   % Angle 2
prior{im}.name='ang_3;   % Angle 3
prior{im}.name='sill;    % sill,
prior{im}.name='nu;      % the 'nu' parameter, only applies ↩
    when using the Matern covariance model type.
prior{im}.name='m0;      % A priori mean
```

As an example consider case where the two ranges, and the angle of anisotropy fro a 2D Gaussian(FFTMA) a priori type model is treated as model parameters:

```
im=0;
% velocity field
im=im+1;
prior{im}.type='FFTMA';
prior{im}.name='Velocity (m/ns)';
prior{im}.m0=0.145;
prior{im}.Va='.0003 Sph(6)';
dx=0.25;
prior{im}.x=[-1:dx:6];
prior{im}.y=[0:dx:13];
prior{im}.cax=[.1 .18];
i_master=im;

% range - horizontal
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='range_1';
prior{im}.min=1.5;
prior{im}.max=10.5;
prior{im}.norm=50;
prior{im}.prior_master=i_master;

% range - horizontal
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='range_2';
prior{im}.min=1.5;
prior{im}.max=5.5;
prior{im}.norm=50;
prior{im}.prior_master=i_master;
```

```
% rotation
im=im+1;
prior{im}.type='gaussian';
prior{im}.name='ang_1';
prior{im}.m0=90;
prior{im}.std=20;
prior{im}.norm=2;
prior{im}.prior_master=i_master;
```

A sample from the corresponding a priori model (FFTMA type) is shown below:

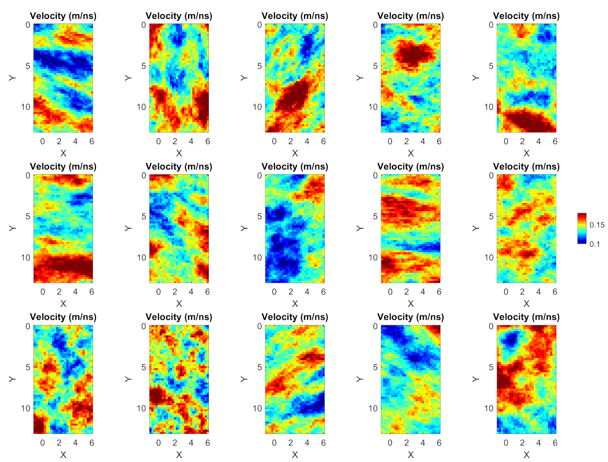


Figure 4.7: A sample from a FFTMA type prior model with varying range_1, range_2, and ang_1.

Samples of the a priori distributions for range_1, range_2, and ang_1 are shown here:

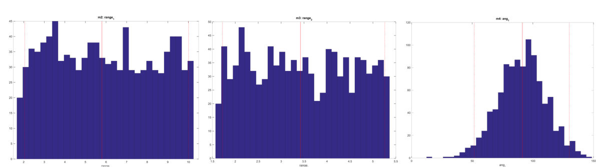


Figure 4.8: Distribution of one sample of a 1D Gaussian distribution describing range_1, range_2, and ang_1

As for the examples above, the a posteriori distribution can be samples using e.g.

```
options.mcmc.nite=500000; % optional, default:nite=30000
options.mcmc.i_sample=500; % optional, default:i_sample=500;
options.mcmc.i_plot=1000; % optional, default:i_plot=50;
options=sippi_metropolis(data,prior,forward,options);

% plot posterior statistics
sippi_plot_posterior(options.txt);
```

4.4 Probilistic covariance/semivariogram indeference

This chapter documents how to use SIPPI to infer properties of a covariance/semivariogram model from noisy data (both data of point support and linear average data can be considered)

To perform probabilistic inference of covariance model parameters one must

1. define the data and associated uncertainty (if any),
2. define a prior model of the covariance model parameters that one wish to infer information about, and
3. define the linear forward operator (only applicable if data are not of point support).

The methodology is published in [HCM15].

4.4.1 Specification of covariance model parameters

The following covariance model properties can be defined, that allow defining an isotropic or an-isotropic covariance model in 1D, 2D, or 3D:

```
type                % covariance model type (1->Sph, 2->Exp, ↩
     3->Gau)
m0                  % the mean
sill                % the variance
nugget_fraction     % percentage of the variance assigned to ↩
     a Nugget
range_1             % range in primary direction
range_2             % range in secondary direction
range_3             % range in tertiary direction
ang_1               % first angle of rotation
ang_2               % second angle of rotation
ang_3               % third angle of rotation
```

Inference of a full 1D covariance model requires defining [type,sill,nugget_fraction,range_1].
Inference of a full 2D covariance model requires defining [type,sill,nugget_fraction,range_1,ra
Inference of a full 3D covariance model requires defining [type,sill,nugget_fraction,range_1,ra

In order to define which of the covariance model parameters to infer information about, simply define a prior structure for any of these parameters, as 1D type SIPPI prior model.

For example, to simple infer information about the range in the primary direction, with a priori distribution of the range as U[0,3] use

```
forward.Cm='1 Sph(10)';

im=1;
prior{im}.type='uniform';
prior{im}.name='range_1'; % the 'name' field is used to identify the covariance model parameter!
prior{im}.min=0;
prior{im}.max=3;
```

In this case an `range_1` refers to the isotropic range in the covariance model defined in the `forward.Cm` field

If, instead

```
forward.Cm='1 Sph(10,90,.25)';
```

then `range_1` would refer to the range in the direction of maximum continuity (90 degrees from North). `range_2` will in this case be fixed.

As described above, the covariance model type can be considered as a unknown parameter, that can be inferred during inversion. This may pose some problems as discussed in [HCM15].

To infer the covariance model type, a prior 1D structure should be defined as e.g.

```
im=1;
prior{im}.type='uniform';
prior{im}.name='type'; %
prior{im}.min=0;
prior{im}.max=3;
```

Any value between 0 and 1 defines a spherical type covariance. Any value between 1 and 2 defines an exponential type covariance. Any value between 3 and 3 defines a Gaussian type covariance.

Thus no prior should be defined for the 'type' prior that can provide values below 0, and above 3. In the case above, all three covariance model types has the sane a priori probability.

A detailed description of how to parameterize the inverse covariance model parameter problem, can be found in sippi_forward_covariance_inference.

4.4.2 Inferring a 2D covariance model from the Jura data set - Example of point support

The Jura data set (see Goovaerts, 1997) contains a number observations of different properties in 2D. Below is an example of how to infer properties of a 2D covariance model from this data set.

A Matlab script implementing the steps below can be found here: jura_covariance_inference.m

4.4.2.1 Load the Jura data

Firs the Jura data is loaded.

```
% jura_covariance_inference
%
% Example of inferring properties of a Gaussian model from ↩
    point data
%

%% LOAD THE JURA DATA
clear all;close all
[d_prediction,d_transect,d_validation,h_prediction, ↩
    h_transect,h_validation,x,y,pos_est]=jura;
ix=1;
iy=2;
id=6;

% get the position of the data
pos_known=[d_prediction(:,[ix iy])];

% perform normal score transformation of tha original data
[d,o_nscore]=nscore(d_prediction(:,id));
h_tit=h_prediction{id};
```

4.4.2.2 Setting up SIPPI for covariance parameter inference

First a SIPPI 'prior' data structure is setup do infer covariance model parameters for a 2D an-isotropic covariance model. That is, the `range_1`, `range_2`, `ang_1`, and `nugget_fraction` are defined using

```
im=0;
% A close to uniform distribution of the range, U[0;3].
im=im+1;
prior{im}.type='uniform';
prior{im}.name='range_1';
prior{im}.min=0;
prior{im}.max=3;
```

```
im=im+1;
prior{im}.type='uniform';
prior{im}.name='range_2';
prior{im}.min=0;
prior{im}.max=3;

im=im+1;
prior{im}.type='uniform';
prior{im}.name='ang_1';
prior{im}.min=0;
prior{im}.max=90;

im=im+1;
prior{im}.type='uniform';
prior{im}.name='nugget_fraction';
prior{im}.min=0;
prior{im}.max=1;
```

Thus the a priori information consists of uniform distributions of ranges between 0 and 3, rotation between 0 and 90, and a nugget fraction between 0 and 1 is.

Then the data structure is set up, using the Jura data selected above, while assuming a Gaussian measurement uncertainty with a standard deviation of 0.1 times the standard deviation of the data:

```
%% DATA
data{1}.d_obs=d; % observed data
data{1}.d_std=0.1*std(d);.4; % uncertainty of observed data ↩
    (in form of standard deviation of the noise)
```

Finally the forward structure is setup such that `sippi_forward_covariance_inference` allow inference of covariance model parameters.

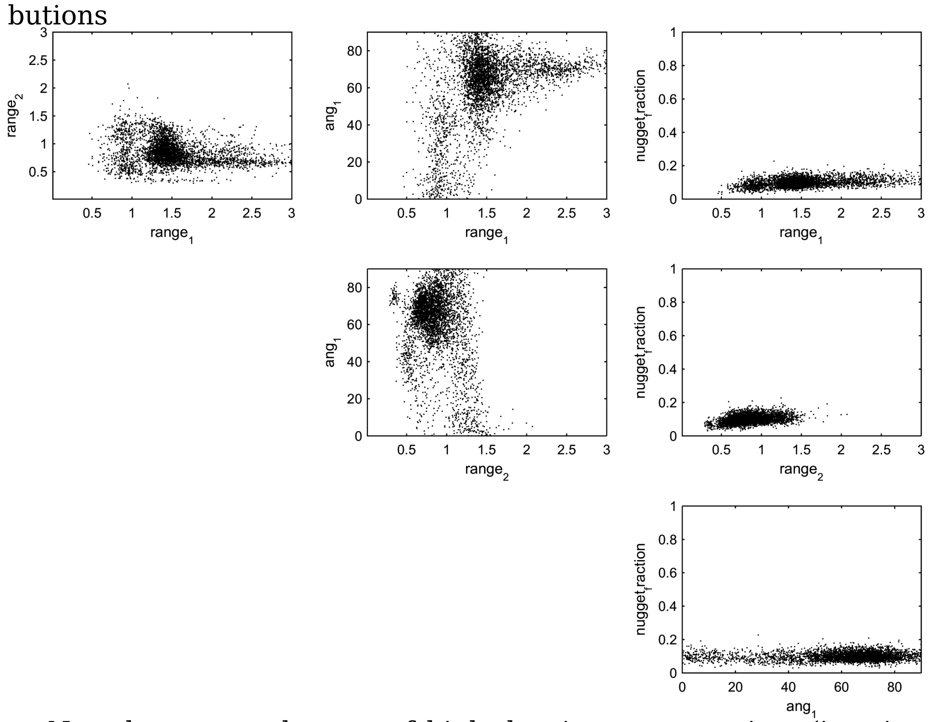
In the `forward` structure the location of the point data needs to be given in the `pos_known` field, and the initial mean and covariance needs to be set. Also, the name of the forward function used (in this case sippi_forward_covariance_inference) must be set. Use e.g.:

```
%% FORWARD
forward.forward_function='sippi_forward_covariance_inference ↩
    ';
forward.point_support=1;
forward.pos_known=pos_known;

% initial choice of N(m0,Cm), mean and sill are 0, and 1, ↩
    due
% due to normal score
forward.m0=mean(d);
forward.Cm=sprintf('%3.1f Sph(2)',var(d));
```

Now, SIPPI is set up for inference of covariance model parameters. Use for example the Metropolis sampler to sample the a posterior distribution over the covariance model parameters using:

```
options.mcmc.nite=100000;
options.mcmc.i_plot=1000;
options.mcmc.i_sample=25;
options.txt=name;
[options,data,prior,forward,m_current]=sippi_metropolis(data ↩
    ,prior,forward,options)

sippi_plot_prior(options.txt);
sippi_plot_posterior(options.txt);
```

Sampling the posterior provides the following 2D marginal distributions



Note how several areas of high density scatter points (i.e. i.e. area with high posterior probability) can be found.

Chapter 5

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Chapter 6

Reference

6.1 SIPPI

6.1.1 sippi_adjust_step_size

```
sippi_adjust_step_size Adjust step length length for  ↩
    Metropolis sampler in SIPPI

Call :
  step=sippi_adjust_step_size(step,P_average,P_target);

step : current step
P_current : Current acceptance ratio
P_target  : preferred acceptance ratio (def=0.3);

See also sippi_compute_acceptance_rate,  ↩
    sippi_prior_set_steplength
```

6.1.2 sippi_anneal_adjust_noise

```
sippi_anneal_adjust_noise : Adjust noise level in  ↩
    annealing schedul

Call:
   [data_adjust,mcmc]=sippi_anneal_adjust_noise(data,i, ↩
       mcmc,prior);

See also: sippi_metropolis, sippi_anneal_factor
```

6.1.3 sippi_anneal_factor

```
sippi_anneal_factor : compute simple noise multiplication ↩
    factor for
annealing type sampling

See also sippi_metropolis, sippi_anneal_adjust_noise
```

6.1.4 sippi_compute_acceptance_rate

```
sippi_compute_acceptance_rate Computes acceptance rate for ↩
     the Metropolis sampler in SIPPI

Call:
  P_acc=sippi_compute_acceptance_rate(acc,n_update_history ↩
      );
```

6.1.5 sippi_compute_modelization_forward_error

```
sippi_compute_modelization_forward_error Computes an ↩
    estimate of the modelization erro

Computes and estimate of the Gaussian modelization error, ↩
    N(dt,Ct)
caused by the use of an imperfect forward kernel

If called with only one output '[Ct]=sippi..]' then the ↩
    Gaussian model is
assumed by centered around 0, (dt{1}=0).

Call
  [Ct,dt,dd]=sippi_compute_modelization_forward_error( ↩
      forward_full,forward_app,prior,data,N);


For details see:
 Hansen, T.M., Cordua, K. S., Jacobsen, B. J., and ↩
     Mosegaard, K. (2014)
 Accounting for imperfect forward modeling in geophysical ↩
     inverse problems - exemplified for cross hole ↩
     tomography.
 Geophsyics, 79(3) H1-H21, 2014. doi:10.1190/geo2013 ↩
     -0215.1
```

6.1.6 sippi_forward

```
sippi_forward Simple forward wrapper for SIPPI

Assumes that the actual forward solver has been defined by
forward.forward_function

Call:
  [d,forward,prior,data]=sippi_forward(m,forward)

Optional:
  [d,forward,prior,data]=sippi_forward(m,forward,prior)
  [d,forward,prior,data]=sippi_forward(m,forward,prior, ↩
     data)
  [d,forward,prior,data]=sippi_forward(m,forward,prior, ↩
     data,options)
```

6.1.7 sippi_get_sample

```
sippi_get_sample: Get a posterior sample

Call :
 [reals,etype_mean,etype_var,reals_all,reals_ite]= ↩
    sippi_get_sample(wordking_directory,im,n_reals, ↩
    skip_seq_gibbs);

   im: A priori model type
   n_reals: Number of realizations to return
   skip_seq_gibbs [1] Skip all realization where  ↩
       sequential gibbs is enabled
                  [0] Use all realization
   data: SIPPI data structure
   prior: SIPPI prior structure
   options: options structure when running  ↩
       sippi_metropolis


If located in a SIPPI output folder one can simple use :
   [reals,etype_mean,etype_var,reals_all,reals_ite]= ↩
       sippi_get_sample(im,n_reals);
or
   skip_seq_gibbs=0;
   [reals,etype_mean,etype_var,reals_all,reals_ite]= ↩
       sippi_get_sample(im,n_reals,skip_seq_gibbs);
```

6.1.8 sippi_least_squares

```
sippi_least_squares Least squares type inversion for SIPPI

Call :
   [m_reals,m_est,Cm_est]=sippi_least_squares(data,prior, ↩
       forward,n_reals,lsq_type,id,im);



  lsq_type : 'lsq' (def), classical least squares
             'error_sim', simulation through error  ↩
                 simulation
             'visim', simulation through SGSIM of DSSIM
```

6.1.9 sippi_likelihood

```
sippi_likelihood Compute likelihood given an observed  ↩
    dataset

Call
  [logL,L,data]=sippi_likelihood(d,data);


 data{1}.d_obs [N_data,1] N_data data observations
 data{1}.d_std [N_data,1] N_data uncorrelated Gaussian STD

 data{1}.d_var [N_data,1] N_data uncorrelated Gaussian  ↩
     variances


Gaussian modelization error, N(dt,Ct), is specified as
 data{1}.dt [N_data,1] : Bias/mean of modelization error
 data{1}.Ct [N_data,N_data] : Covariance of modelization  ↩
     error

 data{1}.Ct [1,1] : Constant Covariance of modelization  ↩
     error
                    imples data{1}.Ct=ones(N_data.N_data)* ↩
                        data{1}.Ct;




data{id}.recomputeCD [default=0], if '1' then data{1}.iCD  ↩
    is recomputed
```

```
each time sippi_likelihood is called. This should be used ↩
    if the noise model
changes between each call to sippi_likelihood.

 data{id}.full_likelihood [default=]0; if '1' the the full ↩
      likelihood
 (including the determinant) is computed. This not needed ↩
     if the data
 civariance is constant, but if it changes, then use
 data{id}.full_likelihood=1;
```

6.1.10 sippi_mcmc_init

```
sippi_mcmc_init Initialize McMC options for Metropolis and ↩
     rejection sampling in SIPPI

Call:
   options=sippi_mcmc_init(options,prior);
```

6.1.11 sippi_metropolis

```
sippi_metropolis Extended Metropolis sampling in SIPPI

Metropolis sampling.
  See e.g. Hansen, T. M., Cordua, K. S., and Mosegaard, K ↩
      ., 2012.
    Inverse problems with non-trivial priors - Efficient ↩
        solution through Sequential Gibbs Sampling.
    Computational Geosciences. doi:10.1007/s10596 ↩
        -011-9271-1.

Call :
   [options,data,prior,forward,m_current]=sippi_metropolis ↩
       (data,prior,forward,options)
Input :
   data : sippi data structure
   prior : sippi prior structure
   forward : sippi forward structure

options :
   options.txt [string] : string to be used as part of all ↩
        output files

   options.mcmc.nite=30000;   % [1] : Number if iterations
```

```
    options.mcmc.i_sample=100; % : Number of iterations  ↩
        between saving model to disk
    options.mcmc.i_plot=50;  % [1]: Number of iterations  ↩
        between updating plots
    options.mcmc.i_save_workspace=10000;  % [1]: Number of  ↩
        iterations between
                                        saving the  ↩
                                            complete  ↩
                                            workspace
    options.mcmc.i_sample=100; % : Number of iterations  ↩
        between saving model to disk

    options.mcmc.m_init : Manually chosen starting model
    options.mcmc.m_ref  : Reference known target model

    options_mcmc.accept_only_improvements [0] :  ↩
        Optimization

   %% PERTUBATION STRATEGY
   options.mcmc.pert_strategy.perturb_all=1; % Perturb all  ↩
       priors in each
                                          % iteration.  ↩
                                              def =[0]
    %% SIMULATED ANNEALING
    options.mcmc.anneal.i_begin=1; % default, iteration  ↩
        number when annealing begins
    options.mcmc.anneal.i_end=100000; %  iteration number  ↩
        when annealing stops
    options.mcmc.anneal.fac_begin=20; % default, noise is  ↩
        scaled by fac_begin at iteration i_begin
    options.mcmc.anneal.fac_end=1; % default, noise is  ↩
        scaled by fac_end at iteration i_end


  See also sippi_rejection
```

6.1.12 sippi_prior

```
  sippi_prior A priori models for SIPPI

  To generate a realization of the prior model defined by  ↩
      the prior structure use:
    [m_propose,prior]=sippi_prior(prior);

  To generate a realization of the prior model defined by  ↩
      the prior structure,
```

```
 in the vicinity of a current model (using sequential Gibbs ↩
      sampling) use:
   [m_propose,prior]=sippi_prior(prior,m_current);

 The following types of a priori models can be used
   % two point statistics bases
   GAUSSIAN   [1D] : 1D generalized gaussian model
   UNIFORM [1D-3D] : 1D-3D uncorrelated uniform  ↩
       distribution
   CHOLESKY[1D-3D] : based on Cholesky decomposition
   FFTMA   [1D-3D] : based on the FFT-MA method ( ↩
       Multivariate Gaussian)
   VISIM   [1D-3D] : based on Sequential Gaussian and  ↩
       Direct Sequential simulation
   SISIM   [1D-3D] : based on Sequential indicator  ↩
       SIMULATION
   % multiple point based statistics
   SNESIM  [1D-3D] : based on a multiple point statistical  ↩
       model inferref from a training images. Relies in the  ↩
       SNESIM algorithm


%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaissian based prior model based ↩
     on the
% FFT-MA method, can be defined using
   im=1;
   prior{im}.type='FFTMA';
   prior{im}.name='A SIMPLE PRIOR';
   prior{im}.x=[0:1:100];
   prior{im}.y=[0:1:100];
   prior{im}.m0=10;
   prior{im}.Va='1 Sph(10)';
   prior=sippi_prior_init(prior);
% A realization from this prior model can be generated  ↩
    using
   m=sippi_prior(prior);
% This realization can now be plotted using
   sippi_plot_prior(m,prior);
% or
   imagesc(prior{1}.x,prior{1}.y,m{1})

%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

   im=1;
   prior{im}.type='GAUSSIAN';
   prior{im}.m0=100;
   prior{im}.std=50;
   prior{im}.norm=100;
```

```
    im=im+1;
    prior{im}.type='FFTMA';
    prior{im}.x=[0:1:100];
    prior{im}.y=[0:1:100];
    prior{im}.m0=10;
    prior{im}.Cm='1 Sph(10)';
    im=im+1;
    prior{im}.type='VISIM';
    prior{im}.x=[0:1:100];
    prior{im}.y=[0:1:100];
    prior{im}.m0=10;
    prior{im}.Cm='1 Sph(10)';
    im=im+1;
    prior{im}.type='SISIM';
    prior{im}.x=[0:1:100];
    prior{im}.y=[0:1:100];
    prior{im}.m0=10;
    prior{im}.Cm='1 Sph(10)';
    im=im+1;
    prior{im}.type='SNESIM';
    prior{im}.x=[0:1:100];
    prior{im}.y=[0:1:100];

    sippi_plot_prior(prior);

%% Sequential Gibbs sampling

    All a priori model types can be perturbed, such that a new realization
    is generated in the vicinity of a current model.
    To do this Sequential Gibbs Sampling is used.
    For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596-011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems with non-trivial priors - Efficient solution through Sequential Gibbs Sampling. Computational Geosciences</a>.
    The type of sequential Gibbs sampling can be controlled in the
    'seq_gibbs' structures, e.g. prior{1}.seq_gibbs

    im=1;
    prior{im}.type='SNESIM';
    prior{im}.x=[0:1:100];
    prior{im}.y=[0:1:100];

    [m,prior]=sippi_prior(prior);
    prior{1}.seq_gibbs.step=1; % Large step--> independant realizations
```

```
    prior{1}.seq_gibbs.step=.1; % Smaller step--> Dependant  ↩
        realizations
    for i=1:30;
       [m,prior]=sippi_prior(prior,m); % One iteration of  ↩
           Sequential Gibbs
       sippi_plot_prior(prior,m);
    end

  See also: sippi_prior_init, sippi_plot_prior,  ↩
      sippi_plot_prior_sample, sippi_prior_set_steplength.m

  TMH/2012
```

6.1.13 sippi_prior_cholesky

```
  sippi_prior_cholesky : Cholesky type Gaussian prior for  ↩
      SIPPI

 % Example:
    ip=1;
    prior{ip}.type='cholesky';
    prior{ip}.m0=10;
    prior{ip}.Cm='.001 Nug(0) + 1 Gau(10)';
    prior{ip}.x=0:1:100;linspace(0,100,20);
    prior{ip}.y=0:1:50;linspace(0,33,30);
    [m,prior]=sippi_prior_cholesky(prior);
    sippi_plot_prior(prior,m);

 % Sequential Gibbs sampling
    prior{1}.seq_gibbs.step=.1;
    for i=1:100;
        [m,prior]=sippi_prior_cholesky(prior,m);
        sippi_plot_prior(prior,m);
        caxis([8 12]);drawnow;
    end

 % Prior covariance model
  The prior covarince model can be setup using
    prior{ip}.m0=10;
    prior{ip}.Cm='.001 Nug(0) + 1 Gau(10)';
  or
    prior{ip}.m0=10;
   and the 'Cmat' variable 'prior{ip}.Cmat' which much the  ↩
       contain a full
   nd X nd size covariance matrix.
   (it is computed the first the sippi_prior_cholesky is  ↩
       called)
```

```
See also: gaussian_simulation_cholesky
```

6.1.14 sippi_prior_dsim

```
sippi_prior_dsim : Direct simulation in SIPPI


TMH/2014

See also: sippi_prior_init, sippi_prior
```

6.1.15 sippi_prior_init

```
sippi_prior_init Initialize PRIOR structure for SIPPI

Call
  prior=sippi_prior_init(prior);

See also sippi_prior
```

6.1.16 sippi_prior_set_steplength

```
sippi_prior_set_steplength Set step length for Metropolis ↩
    sampler in SIPPI

Call
  prior=sippi_prior_set_steplength(prior,mcmc,im);
```

6.1.17 sippi_prior_sisim

```
sippi_prior_sisim: SISIM (SGeMS) type prior for SIPPI

% Example:
    ip=1;
    prior{ip}.type='sisim';
    prior{ip}.x=1:1:80;
    prior{ip}.y=1:1:80;
```

```
    prior{ip}.Cm='1 Sph(60)';
    prior{ip}.marginal_prob=[.1 .4 .5];
    m=sippi_prior(prior);
    sippi_plot_prior(prior,m)

    % optionally a specific random seed can be set using
    prior{ip}.seed=1;

% Sequential Gibbs sampling type 1 (box selection of pixels ↩
    )
    prior{ip}.seq_gibbs.type=1;%
    prior{ip}.seq_gibbs.step=10; % resim data in 10x10  ↩
        pixel grids
    [m,prior]=sippi_prior(prior);
    for i=1:10;
       [m,prior]=sippi_prior(prior,m);
       sippi_plot_prior(prior,m);
       drawnow;
    end

% Sequential Gibbs sampling type 2 (random pixels)
    prior{ip}.seq_gibbs.type=2;%
    prior{ip}.seq_gibbs.step=.6; % Resim 60% of data
    [m,prior]=sippi_prior(prior);
    for i=1:10;
       [m,prior]=sippi_prior(prior,m);
       sippi_plot_prior(prior,m);
       drawnow;
    end

 See also: sippi_prior, sgems
```

6.1.18 sippi_prior_snesim

```
 sippi_prior_snesim : SNESIM type Gaussian prior for SIPPI

% Example:
    ip=1;
    prior{ip}.type='snesim';
    prior{ip}.x=1:1:80;
    prior{ip}.y=1:1:80;
    prior{ip}.ti=channels;
    % prior{ip}.ti=maze;
    m=sippi_prior(prior);
    sippi_plot_prior(prior,m)
    figure(1);imagesc(prior{ip}.ti);axis image
```

```
% Sequential Gibbs sampling type 1 (box selection of pixels ↩
    )
    prior{ip}.seq_gibbs.type=1;%
    prior{ip}.seq_gibbs.step=10; % resim data in 10x10  ↩
        pixel grids
    [m,prior]=sippi_prior(prior);
    for i=1:10;
       [m,prior]=sippi_prior(prior,m);
       sippi_plot_prior(prior,m);
       drawnow;
    end

% Sequential Gibbs sampling type 2 (random pixels)
    prior{ip}.seq_gibbs.type=2;%
    prior{ip}.seq_gibbs.step=.6; % Resim 60% of data
    [m,prior]=sippi_prior(prior);
    for i=1:10;
       [m,prior]=sippi_prior(prior,m);
       sippi_plot_prior(prior,m);
       drawnow;
    end

 See also: sippi_prior, ti
```

6.1.19 sippi_prior_uniform

```
 sippi_prior_uniform : Uniform prior for SIPPI


% Example 1D uniform
   ip=1;
   prior{ip}.type='uniform';
   prior{ip}.min=10;
   prior{ip}.max=25;
   [m,prior]=sippi_prior_uniform(prior);
   sippi_plot_prior_sample(prior);

% Example 10D uniform
   ip=1;
   prior{ip}.type='uniform';
   prior{ip}.x=1:1:10; % As dimensions are uncorrelated,  ↩
       only the lehgth
                    % of prior{ip}.x matters, not its  ↩
                        actual values.
   prior{ip}.min=10;
   prior{ip}.max=25;
   [m,prior]=sippi_prior_uniform(prior);
```

```
   sippi_plot_prior_sample(prior);

% Sequential Gibbs sampling
   prior{1}.seq_gibbs.step=.1;
   for i=1:1000;
       [m,prior]=sippi_prior(prior,m);
       mm(i)=m{1};
   end
   subplot(1,2,1);plot(mm);
   subplot(1,2,2);hist(mm);

 TMH/2014

 See also: sippi_prior_init, sippi_prior
```

6.1.20 sippi_prior_visim

```
 sippi_prior_visim : VISIM type Gaussian prior for SIPPI

% Example:
    ip=1;
    prior{ip}.type='visim';
    prior{ip}.x=1:1:80;
    prior{ip}.y=1:1:80;
    prior{ip}.Cm='1 Sph(60)';
    m=sippi_prior(prior);
    sippi_plot_prior(prior,m)

    % optionally a specific random seed can be set using
    prior{ip}.seed=1;

% Sequential Gibbs sampling type 1 (box selection of pixels ↩
    )
    prior{ip}.seq_gibbs.type=1;%
    prior{ip}.seq_gibbs.step=10; % resim data in 10x10  ↩
        pixel grids
    [m,prior]=sippi_prior(prior);
    for i=1:10;
       [m,prior]=sippi_prior(prior,m);
       sippi_plot_prior(prior,m);
       drawnow;
    end

% Sequential Gibbs sampling type 2 (random pixels)
    prior{ip}.seq_gibbs.type=2;%
    prior{ip}.seq_gibbs.step=.6; % Resim 60% of data
    [m,prior]=sippi_prior(prior);
```

```
    for i=1:10;
       [m,prior]=sippi_prior(prior,m);
       sippi_plot_prior(prior,m);
       drawnow;
    end

 See also: sippi_prior, visim
```

6.1.21 sippi_rejection

```
 sippi_rejection Rejection sampling

 Call :
     options=sippi_rejection(data,prior,forward,options)

 input arguments

   options.mcmc.i_plot
   options.mcmc.nite     % maximum number of iterations
   options.mcmc.logLmax

   options.mcmc.rejection_normalize_log = log(options.mcmc. ↩
       Lmax)

   options.mcmc.adaptive_rejection=1, adaptive setting of  ↩
       maxiumum likelihood
                 (def=[0])
                 At each iteration Lmax will be set if log ↩
                     (L(m_cur)=>options.mcmc.logLmax


   options.mcmc.max_run_time_hours = 1; % maximum runtime  ↩
       in hours
                                     % (overrides  ↩
                                         options.mcmc. ↩
                                         nite if needed)

 See also sippi_metropolis
```

6.1.22 sippi_set_path

```
 sippi_set_path Set paths for running sippi
```

6.2 SIPPI plotting commands

6.2.1 sippi_colormap

```
sippi_colormap Default colormap for sippi

Call :
  sippi_colormap; % the same as sippi_colormap(3);

or :
  sippi_colormap(1) - Red Green Black
  sippi_colormap(2) - Red Green Blue Black
  sippi_colormap(3) - Jet
```

6.2.2 sippi_plot_current_model

```
sippi_plot_current_model Plots the current model during  ↩
    Metropolis sampling

Call :
  sippi_plot_current_model(mcmc,data,d,m_current,prior);
```

6.2.3 sippi_plot_data

```
sippi_plot_data plot data in SIPPI

Call.
   sippi_plot_data(d,data);
```

6.2.4 sippi_plot_defaults

```
sippi_plot_defaults: Sets default options for ploting ( ↩
    such as fontsize)

Call :
  options==sippi_plot_defaults(options);

  % ALWAYS USE DEFULT SETTING (overrules options.axis)
  overrule=1; % {default overrule=0)
  options==sippi_plot_defaults(options,overrule);
```

```
See also: sippi_plot_posterior, ↩
    sippi_plot_posterior_2d_marg
```

6.2.5 sippi_plot_loglikelihood

```
sippi_plot_loglikelihood Plot loglikelihood time series

Call :
   acc=sippi_plot_loglikelihood(logL,i_acc,N,itext)
```

6.2.6 sippi_plot_model

```

```

6.2.7 sippi_plot_movie

```
sippi_plot_movie plot movie of prior and posterior ↩
    realizations

Call :
  sippi_plot_movie(fname);
  sippi_plot_movie(fname,im_array,n_frames,skip_burnin);
     fname : name of folder with results (e.g. options.txt ↩
         )
     im_array : array of indexes of model parameters to ↩
         make into movies
     n_frames [200] : number of frames in movie
     skip_burnin [200] : start movie after burn_in;

Ex:
sippi_plot_movie('20130812_Metropolis');
sippi_plot_movie(options.txt);

%% 1000 realization including burn-in, for prior number 1
sippi_plot_movie('20130812_Metropolis',1,1000,0);

Using options.plot.skip_seq_gibbs=1, (set in ↩
    sippi_plot_defaults)
removes realizations obtained using sequential Gibbs ↩
    sampling
(equivalent to setting skip_burnin=1)
```

```
See also: sippi_plot_defaults
```

6.2.8 sippi_plot_posterior

```
sippi_plot_posterior Plot statistics from posterior sample

Call :
   sippi_plot_posterior(fname,im_arr,prior,options,n_reals ↩
       );

See also sippi_plot_prior
```

6.2.9 sippi_plot_posterior_2d_marg

```
sippi_plot_posterior_2d_marg: plots 2D posterior marginal ↩
    distributions

Call:
   [options,reals_all]=sippi_plot_posterior_2d_marg( ↩
       options,prior,data,fname);

See also: sippi_plot_posterior
```

6.2.10 sippi_plot_posterior_data

```
sippi_plot_posterior_data: plots posterior data and noise ↩
    relaizations

Call
   [options]=sippi_plot_posterior_data(options,prior,data, ↩
       forward);

See also: sippi_plot_posterior
```

6.2.11 sippi_plot_posterior_loglikelihood

```
sippi_plot_posterior_loglikelihod : plots log(L) and ↩
    autorreation of log(L)

Call:
    sippi_plot_posterior_loglikelihood; % when located in ↩
        an output folder
                                        % generated by ↩
                                            SIPPI

    sippi_plot_posterior_loglikelihood(foldername); % ↩
        Where 'foldername'
                                        % is a folder ↩
                                            generated by ↩
                                            SIPPI


    sippi_plot_posterior_loglikelihood(options); % where ↩
        options is the
                       % output of sippi_rejection or ↩
                           sippi_metropolis


    options=sippi_plot_posterior_loglikelihood(options, ↩
        prior,data,mcmc,fname);


See also: sippi_plot_posterior
```

6.2.12 sippi_plot_posterior_sample

```
sippi_plot_posterior_sample: plots posterior sample ↩
    statistics

Call
   [options]=sippi_plot_posterior_sample(options,prior, ↩
       data,forward);

See also: sippi_plot_posterior
```

6.2.13 sippi_plot_prior

```
sippi_plot_prior Plot a 'model', i.e. a realization of the ↩
     prior model
```

```
Call :
  sippi_plot_prior(prior,m,im_array);

  prior : Matlab structure for SIPPI prior model
  m : Matlab structure for SIPPI realization
  im_array : integer array of type of models to plot ( ↩
     typically 1)


Example
  m=sippi_prior(prior);
  sippi_plot_prior(prior,m);

  m=sippi_prior(prior);
  sippi_plot_prior(prior,m,2);

See also sippi_plot_prior, sippi_prior
```

6.2.14 sippi_plot_prior_sample

```
sippi_plot_prior Plot a sample of the prior in SIPPI

Call :
   sippi_plot_prior_sample(prior,im_array,n_reals,cax, ↩
       options);

 See also sippi_plot_posterior, sippi_plot_prior,  ↩
     sippi_prior
```

6.2.15 sippi_plot_set_axis

```
sippi_plot_set_axis
see also sippi_plot_defaults
```

6.2.16 wiggle

```
wiggle : plot wiggle/VA/image plot

Call
   wiggle(Data); % wiggle plot
   wiggle(Data,scale); % scaled wiggle plot
```

```
    wiggle(x,t,Data); % wiggle plt
    wiggle(x,t,Data,'VA') % variable Area (pos->black;neg-> ↩
        transp)
    wiggle(x,t,Data,'VA2') % variable Area (pos->black;neg ↩
        ->red)
    wiggle(x,t,Data,'wiggle',scale); % Scaled wiggle
    wiggle(x,t,Data,'wiggle',scale,showmax); % Scaled  ↩
        wiggle and max
                                              showmax  ↩
                                                  traces.
    wiggle(x,t,Data,'wiggle',scale,showmax,plimage); %  ↩
        wiggle + image
    wiggle(x,t,Data,'wiggle',scale,showmax,plimage,caxis);  ↩
        % wiggle +
                                                           scaled ↩
                                                                 ↩
                                                               image ↩




 Data : [nt,ntraces]
 x : [1:ntraces] X axis (ex [SegyTraceheaders.offset])
 t : [1:nt] Y axis
 style : ['VA'] : Variable Area
         ['wiggle'] : Wiggle plot
 scale : scaling factor, can be left empty as []
 showmax [scalar] : max number of traces to show on display ↩
      [def=100]
 plimage [0/1] : Show image beneath wiggles [def=0];
 caxis [min max]/[scalar] : amplitude range for colorscale


 MAKE IT WORK FOR ANY X-AXIS !!!
```

6.3 SIPPI toolbox: Traveltime tomography

6.3.1 calc_Cd

```
 Calc_cd Setup a covariance model to account for borehole  ↩
     imperfections

 Call: Cd=calc_Cd(ant_pos,var_uncor,var_cor1,var_cor2,L)
 This function sets up a data covariance matrix that  ↩
     accounts for static
 (i.e. correlated) data errors.
```

```
Inputs:
* ant_pos: A N x 4 array that contains N combinations of ↩
    transmitter/source
and receiver positions. The first two columns are the x- ↩
    and y-coordinates
of the transmitter/source position. The last two columns ↩
    are the x- and
y-coordiantes of the receiver position.
* var_uncor: The variance of the uncorrelated data errors.
* var_cor1: The variance of the correlated data errors
related to the transmitter/source positions.
* var_cor2: The variance of the correlated data errors
related to the receiver positions.
* L: The correlation length for the correlation between ↩
    the individual
transmitter/source or receiver positions using an ↩
    exponential covariance
function. For typical static errors the correlation length ↩
     is set to a
small number (e.g. 10^-6).

For more details and practical examples see:
Cordua et al., 2008 in Vadose zone journal.
Cordua et al., 2009 in Journal of applied geophysics.

Knud S. Cordua (2012)
```

6.3.2 eikonal

```
eikonal Traveltime computation by solving the eikonal ↩
    equation

tmap=eikonal(x,y,z,V,Sources,type);

 x,y,z : arrays defining the x, y, and z axis
 V: velocity field, with size (length(y),length(x),length( ↩
     z));
 Sources [ndata,ndim] : Source positions
 type (optional): type of eikonal solver: [1]:Fast ↩
     Marching(default), [2]:FD

 tmap [size(V)]: travel times computed everywhere in the ↩
     velocity grid

%Example (2D):
   x=[1:1:100];
   y=1:1:100;
   z=1;
```

```
    V=ones(100,100);V(:,1:50)=2;
    Sources = [10 50;75 50];
    t=eikonal(x,y,z,V,Sources);
    subplot(1,2,1);imagesc(x,y,t(:,:,1,1));axis image; ↩
       colorbar
    subplot(1,2,2);imagesc(x,y,t(:,:,1,2));axis image; ↩
       colorbar

  See also eikonal_traveltime
```

6.3.3 eikonal_raylength

```
  eikonal_raylength : Computes the raylength from S to R  ↩
     using the eikonal equaiton

  Call:
    raylength=eikonal_raylength(x,y,v,S,R,tS,doPlot)
```

6.3.4 eikonal_traveltime

```
  eikonal_traveltime Computes traveltime between sources and ↩
      receivers by solving the eikonal equation

  t=eikonal_traveltime(x,y,z,V,Sources,Receivers,iuse,type);

   x,y,z : arrays defining the x, y, and z axis
   V: velocity field, with size (length(y),length(x),length( ↩
      z));
   Sources [ndata,ndim] : Source positions
   Receivers [ndata,ndim] : Receiver positions
   iuse (optional): optionally only use subset of data. eg.g ↩
       i_use=[1 2 4];
   type (optional): type of eikonal solver: [1]:Fast  ↩
      Marching(default), [2]:FD

   tmap [size(V)]: travel times computed everywhere in the  ↩
      velocity grid

 %Example (2%

  Example 2d traveltime compuation

  Example (2D):
    x=[1:1:100];
```

```
  y=1:1:100;
  z=1;
  V=ones(100,100);V(:,1:50)=2;
  S=[50 50 1;50 50 1];
  R=[90 90 1; 90 80 1];
  t=eikonal_traveltime(x,y,z,V,S,R)

Example (3D):
  nx=50;ny=50;nz=50;
  x=1:1:nx;
  y=1:1:ny;
  z=1:1:nz;
  V=ones(ny,nx,nz);V(:,1:50,:)=2;
  S=[10 10 1;10 10 1;10 9 1];
  R=[40 40 40; 40 39 40; 40 40 40];
  t=eikonal_traveltime(x,y,z,V,S,R)


See also eikonal
```

6.3.5 kernel_buursink_2d

```
kernel_buursink_2k Computes 2D Sensitivity kernel based on ↩
     1st order EM scattering theory

See
  Buursink et al. 2008. Crosshole radar velocity  ↩
     tomography
                        with finite-frequency Fresnel.  ↩
                            Geophys J. Int.
                        (172) 117;

 CALL :
    % specify a source trace (dt, wf_trace):
    [kernel,L,L1_all,L2_all]=kernel_buursink_2d(model,x,z, ↩
        S,R,dt,wf_trace);
    % Use a ricker wavelet with center frequency 'f0'
    [kernel,L,L1_all,L2_all]=kernel_buursink_2d(model,x,z, ↩
        S,R,f0));


Knud Cordua, 2009,
Thomas Mejer Hansen (small edits, 2009)
```

6.3.6 kernel_finite_2d

```
kernel_finite_2d 2D sensitivity kernels

 Call:
   [Knorm,K,dt,options]=kernel_finite_2d(v_ref,x,y,S,R, ↩
       freq,options);
```

6.3.7 kernel_fresnel_2d

```
kernel_fresnel_2d Sensitivity kernel for amplitude and  ↩
    first arrival

Call:
  [kernel_t,kernel_a,P_omega,omega]=kernel_fresnel_2d(v,x, ↩
      y,S,R,omega,P_omega);


Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity  ↩
    kernels for
seismic Fresenl volume Tomography, Geophysics, 75(5), U35- ↩
    U46

See also kernel_fresnel_monochrome_2d

Run with no argument for an example.
```

6.3.8 kernel_fresnel_monochrome_2d

```
kernel_fresnel_monochrome_2d 2D monchrome kernel for  ↩
    amplitude and first arrival

Call:
  [kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S ↩
      ,R,omega);
or
  [kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S ↩
      ,R,omega,L,L1,L2);

Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity  ↩
    kernels for
seismic Fresenl volume Tomography, Geophysics, 75(5), U35- ↩
    U46

See also, kernel_fresnel_2d
```

6.3.9 kernel_multiple

```
kernel_multiple Computes the sensitivity kernel for a wave ↩
     traveling
from S to R.

CALL :
   [K,RAY,Gk,Gray,timeS,timeR,raypath]=kernel_multiple(Vel ↩
       ,x,y,z,S,R,T,alpha,Knorm);

IN :
   Vel [ny,nx] : Velocity field
   x [1:nx] :
   y [1:ny] :
   z [1:nz] :
   S [1,3] : Location of Source
   R [1,3] : Location of Receiver
   T : Donminant period
   alpha: controls exponential decay away ray path
   Knorm [1] : normaliztion of K [0]:none, K:[1]:vertical

OUT :
   K : Sensitivity kernel
   R : Ray sensitivity kernel (High Frequency approx)
   timeS : travel computed form Source
   timeR : travel computed form Receiver
   raypath [nraydata,ndim] : the center of the raypath

The sensitivity is the length travelled in each cell.


See also : fast_fd_2d

TMH/2006
```

6.3.10 kernel_slowness_to_velocity

```
kernel_slowness_to_velocity Converts from slowness to  ↩
    velocity parameterizations

G : kernel [1,nkernels]
V : Velocity field (
```

```
CALL:
  G_vel=kernel_slowness_to_velocity(G,V);
or
  [G_vel,v_obs]=kernel_slowness_to_velocity(G,V,t);
or
  [G_vel,v_obs,Cd_v]=kernel_slowness_to_velocity(G,V,t,Cd) ↩
      ;
```

6.3.11 mspectrum

```
mspectrum : Amplitude and Power spectrum
Call  :
    function [A,P,smoothP,kx]=mspectrum(x,dx)

1D (A)mplitude and (P)owerspectrum of x-series with  ↩
    spacing dx
```

6.3.12 munk_fresnel_2d

```
2D frechet kernel, First Fresnel Zone

See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar  ↩
    Physics 192.

Call :
S=munk_fresnel_2d(T,dt,alpha,As,Ar,K);

T : dominant period
dt :
alpha : degree of cancellation
As : Amplitude fo the wavefield propagating from the  ↩
    source
Ar : Amplitude fo the wavefield propagating from the  ↩
    receiver
K : normalization factor
```

6.3.13 munk_fresnel_3d

```
3D frechet kernel, First Fresnel Zone
```

```
See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar ↩
    Physics 192.

Call :
```

6.3.14 sippi_forward_traveltime

```
sippi_forward_traveltime Traveltime computation in SIPPI

Call :
  [d,forward,prior,data]=sippi_forward_traveltime(m, ↩
      forward,prior,data,id,im)

  forward.type determines the method used to compute ↩
      travel times
  forward.type='ray';
  forward.type='fat';
  forward.type='eikonal';
  forward.type='born';
```

6.3.15 tomography_kernel

```
tomography_kernel Computes the sensitivity kernel for a ↩
    wave traveling from S to R.

CALL :
   [K,RAY,Gk,Gray,timeS,timeR,raypath]=tomography_kernel( ↩
       Vel,x,y,z,S,R,T,alpha,Knorm);

IN :
   Vel [ny,nx] : Velocity field
   x [1:nx] :
   y [1:ny] :
   z [1:nz] :
   S [1,3] : Location of Source
   R [1,3] : Location of Receiver
   T : Donminant period
   alpha: controls exponential decay away ray path
   Knorm [1] : normaliztion of K [0]:none, K:[1]:vertical

OUT :
   K : Sensitivity kernel
   R : Ray sensitivity kernel (High Frequency approx)
   timeS : travel computed form Source
   timeR : travel computed form Receiver
```

```
   raypath [nraydata,ndim] : the center of the raypath

The sensitivity is the length travelled in each cell.
```

6.4 SIPPI toolbox: Covariance inference

6.4.1 sippi_forward_covariance_inference

```
sippi_forward_covariance_inference : Probabilitsic ↩
    covariance inference

Call :
 [d,forward,prior,data]=sippi_forward_covariance_inference ↩
     (m,forward,prior,data,id,im)


 forward.pos_known : [x' y' z'],  [ndata,ndim] with ↩
     position of observed data
 forward.G : Forward operator

 Prior covariance model, N(m0,Cm) is chosen as
 forward.m0 : initial mean model
 forward.Cm/forward.Va : inital covariance model

 prior{im}.m0
 prior{im}.type (round(type) is itype)
 prior{im}.range_1
 prior{im}.range_2
 prior{im}.range_3
 prior{im}.ang_1
 prior{im}.ang_2
 prior{im}.ang_3
 prior{im}.sill
 prior{im}.nugget_fraction

See Hansen et al. (201X), A general probabilistic approach ↩
     for inference of Gaussian model parameters from noisy ↩
    data of point and volume support. Submitted to ↩
    Mathematical Geosciences, 2014.

See also: sippi_forward
```