

# **SIPPI**

Ed. version 1.0

---

# Contents

<b>1</b>	<b>Installation</b>	<b>1</b>
1.1	SIPPI	1
1.1.1	SGeMS (optional)	1
<b>2</b>	<b>Setting up SIPPI</b>	<b>2</b>
2.1	prior: The a priori model	2
2.1.1	Types of a priori models	3
2.1.1.1	1D Generalized Gaussian	3
2.1.1.2	FFTMA - 3D Gaussian model	4
2.1.1.3	VISIM	5
2.1.1.4	SNESIM	6
2.1.2	Sampling the prior	6
2.1.3	Sequential Gibbs sampling / Conditional Resampling	6
2.1.3.1	Controlling sequential Gibbs sampling / Conditional Resampling	6
2.2	data: The data and the noise	6
2.2.1	Gaussian measurement noise	7
2.2.1.1	Uncorrelated Gaussian measurement noise	7
2.2.1.2	correlated Gaussian measurement noise	7
2.2.2	Gaussian modeling error	7
2.3	forward: The forward model	8
2.4	Validating prior, data, and forward	9
<b>3</b>	<b>The a posteriori distribution</b>	<b>10</b>
3.1	Sampling the a posteriori probability density	10
3.1.1	The rejection sampler	10
3.1.2	The extended Metropolis sampler	11
3.1.2.1	Controlling the step length	11
3.1.2.2	The independent extended Metropolis sampler	12
3.1.2.3	Annealing schedule	12
3.2	Simulated Annealing	12

<b>4</b>	<b>Examples</b>	<b>13</b>
4.1	Examples of A priori models . . . . .	13
4.1.1	Multiple 1D Gaussian prior model . . . . .	13
4.1.2	Multivariate Gaussian prior with unknown covariance model properties. . . . .	14
4.2	Polynomial line fitting . . . . .	16
4.2.1	The forward problem . . . . .	16
4.2.2	Reference data, data, forward . . . . .	16
4.2.3	The prior model . . . . .	17
4.2.4	Setup and run the Metropolis sampler . . . . .	18
4.2.5	Setup and run the rejection sampler . . . . .	19
4.3	Cross hole tomography . . . . .	19
<b>5</b>	<b>Bibliography</b>	<b>20</b>
<b>6</b>	<b>Reference</b>	<b>21</b>
6.1	SIPPI . . . . .	21
6.1.1	getinunits . . . . .	21
6.1.2	logdet . . . . .	21
6.1.3	pathdef . . . . .	22
6.1.4	plotboxpos . . . . .	22
6.1.5	sippi_adjust_step_size . . . . .	23
6.1.6	sippi_anneal_adjust_noise . . . . .	23
6.1.7	sippi_anneal_factor . . . . .	23
6.1.8	sippi_colormap . . . . .	24
6.1.9	sippi_compute_acceptance_rate . . . . .	24
6.1.10	sippi_forward . . . . .	24
6.1.11	sippi_forward_traveltime . . . . .	24
6.1.12	sippi_get_sample . . . . .	24
6.1.13	sippi_least_squares . . . . .	25
6.1.14	sippi_likelihood . . . . .	25
6.1.15	sippi_mcmc_init . . . . .	26
6.1.16	sippi_metropolis . . . . .	26
6.1.17	sippi_plot_current_model . . . . .	26
6.1.18	sippi_plot_data . . . . .	27
6.1.19	sippi_plot_loglikelihood . . . . .	27
6.1.20	sippi_plot_model . . . . .	27
6.1.21	sippi_plot_movie . . . . .	27
6.1.22	sippi_plot_posterior . . . . .	28
6.1.23	sippi_plot_prior . . . . .	28

---

6.1.24 sippi_prior . . . . .	28
6.1.25 sippi_prior_fftma . . . . .	30
6.1.26 sippi_prior_init . . . . .	31
6.1.27 sippi_prior_new . . . . .	31
6.1.28 sippi_prior_old . . . . .	32
6.1.29 sippi_prior_set_steplength . . . . .	34
6.1.30 sippi_rejection . . . . .	34
6.1.31 sippi_set_path . . . . .	34
6.2 SIPPI toolbox: Traveltime tomography . . . . .	34
6.2.1 calc_Cd . . . . .	34
6.2.2 eikonal . . . . .	35
6.2.3 eikonal_raylength . . . . .	35
6.2.4 eikonal_traveltime . . . . .	35
6.2.5 kernel_buursink_2d . . . . .	36
6.2.6 kernel_finite_2d . . . . .	37
6.2.7 kernel_fresnel_2d . . . . .	37
6.2.8 kernel_fresnel_monochrome_2d . . . . .	37
6.2.9 kernel_multiple . . . . .	37
6.2.10 kernel_slowness_to_velocity . . . . .	38
6.2.11 mspectrum . . . . .	38
6.2.12 munk_fresnel_2d . . . . .	38
6.2.13 munk_fresnel_3d . . . . .	39
6.2.14 tomography_kernel . . . . .	39

---

# About

SIPPI is a Matlab toolbox (compatible with GNU Octave) that allow sampling the solution of non-linear inverse problems with realistic a priori information.

In order to make use of SIPPI one has to

- Install and setup SIPPI
- Define the **prior model**, in form of the prior data structure
- Define the **forward model**, in form of the forward data structure, and the sippi\_forward.m m-file
- Define the **data and noise model**, in form of the prior data structure
- Choose a method for **sampling the a posteriori probability density**.

Details about the implentation and the methods implemented in SIPPI can be found in [[HCM12](#)], [[CHM12](#)], [[HCLM13a](#)], [[HCLM13b](#)] and, [[HCM14](#)].

---

# Chapter 1

## Installation

### 1.1 SIPPI

**Download** the latest version of SIPPI from <http://sippi.sourceforge.net>.

Unpack ZIPPI.zip somewhere, for example to 'c:\Users\tmh\SIPPI'. Then setup the Matlab path to point to the appropriate SIPPI directories:

```
addpath c:\Users\tmh\SIPPI
sippi_set_path
```

#### 1.1.1 SGeMS (optional)

To make use of the SISIM and SNESIM type priori models SGeMS needs to be available.

Currently only SGeMS version 2.1 (**download**) for Windows is supported.

## Chapter 2

# Setting up SIPPI

This section contains information about how to use and control SIPPI, which requires one to

- Define the **prior model**, in form of the prior data structure
- Define the **forward model**, in form of the forward data structure, and the sippi\_forward.m m-file
- Define the **data and noise model**, in form of the prior data structure

[For examples of how to apply SIPPI for different problems, see [the section with examples](#)].

### 2.1 prior: The a priori model

A priori information is defined by the prior Matlab structure. Any number of types of a priori models can be defined. For example a 1D uniform prior can be defined in prior{1}, and 2D Gaussian prior can be defined in prior{2}.

Once a prior data structure has been defined, a realization from the prior model can be generated using

```
m=sippi_prior(prior);
```

The realization from the prior can be visualized using

```
sippi_plot_prior(prior);  
sippi_plot_prior(prior,m);
```

A sample from the prior can be visualized using

```
m=sippi_plot_prior_sample(prior);
```

Each prior type is defined by setting a number field in the prior Matlab structure. For example, an descriptive name (which is can be optionally set) describing the prior can be set in the name field, e.g.

```
prior{1}.name='My Prior';
```

## 2.1.1 Types of a priori models

5 types of a priori models are available, and can be selected by setting the type in the prior structure using e.g. `prior{1}.type='gaussian'`.

The **GAUSSIAN** type prior specifies a 1D generalized Gaussian model.

The **FFTMA** specifies 1D-3D Gaussian Gaussian model using efficient unconditional sampling,

The **VISIM** type prior model specifies a 1D-3D Gaussian Gaussian model, utilizing both sequential Gaussian simulation and direct sequential simulation, and conditioning the data of point support and linear average data.

The **SNESIM** type prior model specifies a 1D-3D multiple point statistical model, relying on training images to infer a model multiple point statistics. This type of prior requires **SGEMS** to be installed.

The following section documents the properties of each type of prior model.

Examples of different types of (combinations of) a priori model can be found in the [examples section](#).

### 2.1.1.1 1D Generalized Gaussian

A 1D generalized Gaussian prior model can be specified using the 'gaussian' type prior model

```
prior{1}.type='gaussian';
```

A simple 1D Gaussian distribution with mean 10, and standard deviation 2, can be specified using

```
ip=1;
prior{ip}.type='gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;
```

The norm of a generalized Gaussian can be set using the 'norm' field. A generalized 1D Gaussian with mean 10, standard deviation of 2, and a norm of 70, can be specified using (The norm is equivalent to the beta factor referenced in [Wikipedia:Generalized\\_normal\\_distribution](#))

```
ip=2;
prior{ip}.type='gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=70;
```

A 1D distribution with an arbitrary distribution shape, can be defined by setting `d_target`, which must contain a sample of the distribution that one would like to replicate. For example, to generate a sample from a non-symmetric bimodal distribution, one can use e.g.

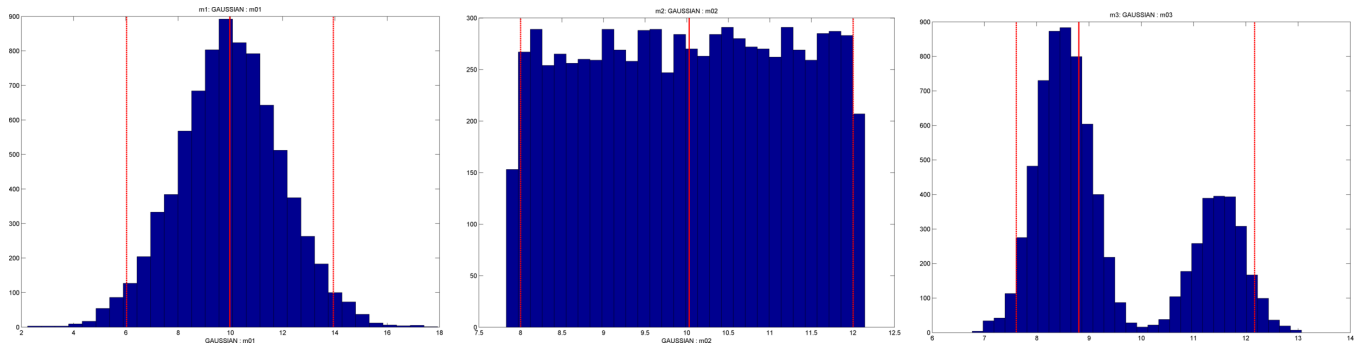
```
% Create target distribution
N=10000;
prob_chan=0.3;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];

% set the target distribution
ip=3;
prior{ip}.type='gaussian';
prior{ip}.d_target=d_target;
```

The following figure shows the 1D histogram of a sample, consisting of 8000 realizations, generated using

```
sippi_plot_prior_sample(prior,1:ip,8000);
```





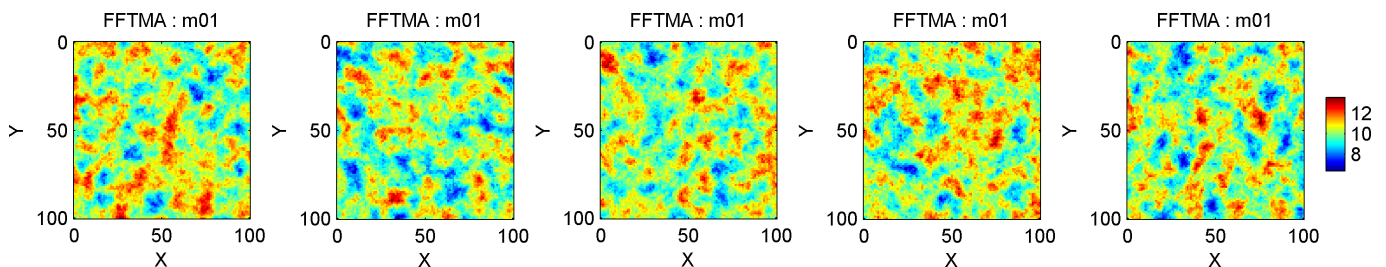
### 2.1.1.2 FFTMA - 3D Gaussian model

The FFT moving average method provides an efficient approach for computing unconditional realizations of a Gaussian random field.

The mean and the covariance model must be specified in the `m0` and `Cm` fields. The format for describing the covariance model follows 'gstat'-type notation, and is described in more details in the [mGstat manual](#).

A 2D covariance model with mean 10, and a Spherical type covariance model can be defined in a 101x101 size grid (1m between cells) using

```
im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
```



Optionally one can translate the output of the Gaussian simulation into an arbitrarily shaped 'target' distribution, using normal score transformation. Note that this transformation will ensure a certain distribution, but will alter the assumed covariance model, such the covariance model properties are no longer ensured. To ensure the covariance model properties are honored, make use of the VISIM type prior model.

```
im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

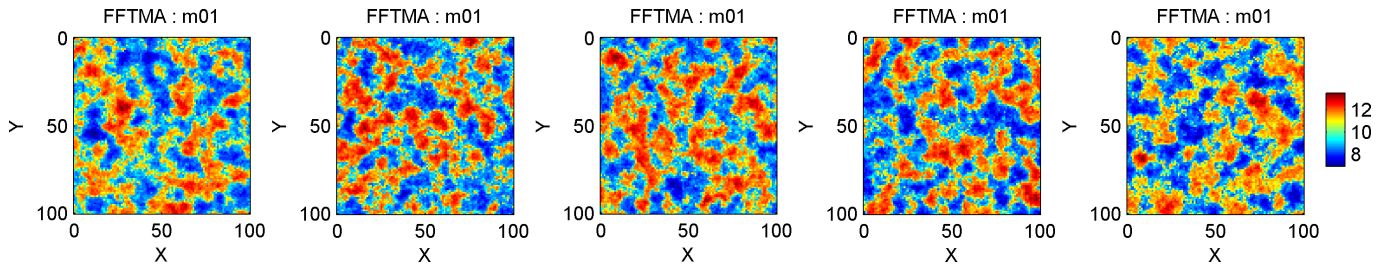
% Create target distribution
N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;
```

Alternatively, the normal score transformation can be defined manually to control tail behaviour using

```

N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
[d_nscore,o_nscore]=nscore(d_target,1,1,min(d_target),max(d_target),0);
prior{im}.o_nscore=o_nscore;

```

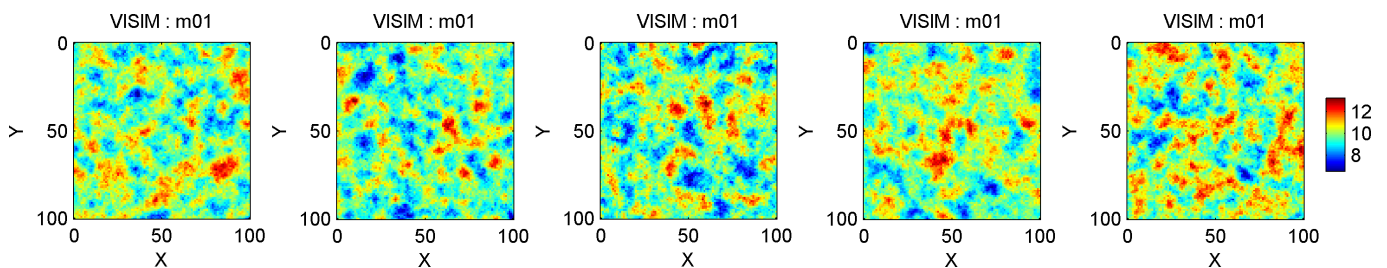


### 2.1.1.3 VISIM

```

im=im+1;
prior{im}.type='VISIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

```



As with the FFTMA type prior the VISIM type prior can make use of a target distribution. However, if a target distribution is set, the use of the VISIM type prior will utilize direct sequential simulation, which will ensure both histogram and covariance reproduction.

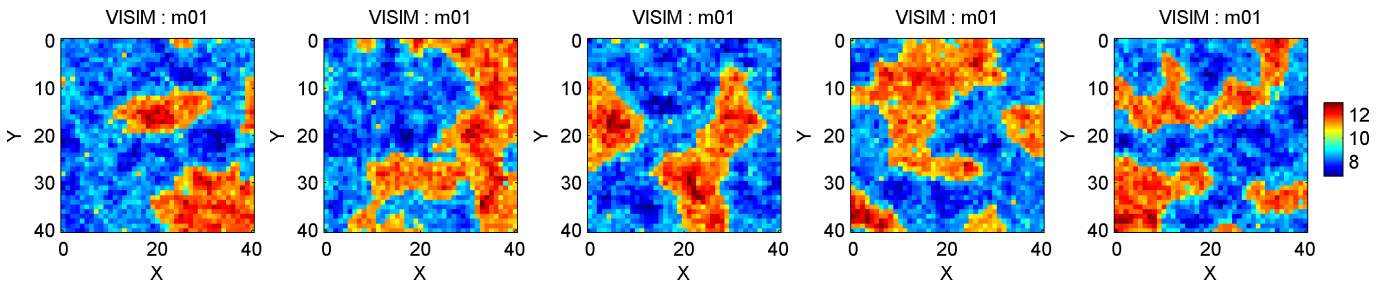
Except for the type the use of a target distribution is similar to that of the FFTMA type prior

```

clear all;close all;
im=1;
prior{im}.type='VISIM';
prior{im}.x=[0:1:40];
prior{im}.y=[0:1:40];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';

% Create target distribution
N=10000;
prob_chan=0.5;
d1=randn(1,ceil(N*(1-prob_chan)))*.5+8.5;
d2=randn(1,ceil(N*(prob_chan)))*.5+11.5;
d_target=[d1(:);d2(:)];
prior{im}.d_target=d_target;

```



#### 2.1.1.4 SNESIM

### 2.1.2 Sampling the prior

Once the prior data structure has been defined a sample from the prior distribution can be generated using

```
m=sippi_prior(prior);
```

'm' is a Matlab data structure of the same size as the 'prior' data structure. Thus, if two prior distributions have been defined in 'prior{1}' and 'prior{2}', then 'm{1}' will hold a realization of 'prior{1}', and 'm{2}' will hold a realization of 'prior{2}'.

Each time 'm=sippi\_prior(prior)' is called, a new independant realization of the prior will be generated.

### 2.1.3 Sequential Gibbs sampling / Conditional Resampling

All the available a priori types available allow perturbing one realization of a prior into a new realization of prior, in the vicinity of the first one. To do this we make use of sequential Gibbs sampling [HCM12]. Sequential Gibbs in essence is a type of conditional resampling. From a current realization of a prior, a number of model parameters are discarded and treated as unknown, and the simulated conditional to the fixed values of the model parameters.

In order to generate a new realization 'm2' in the vicinity of the realization 'm1' use

```
m1=sippi_prior(prior);
[m2,prior]=sippi_prior(prior,m1);
```

If this process is iterated, then a random walk in the space of a priori acceptable models will be perform. And, the collection of realization obatined, will represent a sample from prior distribution.

#### 2.1.3.1 Controlling sequential Gibbs sampling / Conditional Resampling

All properties related to sequential Gibbs sampling can be set in the 'seq\_gibbs' data struture, for each prior type. The follwing two parameters determined how the a current model is perturbed

```
prior{m}.seq_gibbs.step=1;
prior{m}.seq_gibbs.type=2;
```

## 2.2 data: The data and the noise

data is Matlab structure that defines any number of data and a corresponding noise model.

data{1} defines the first data set (which must always be defined), and ny number of additional data sets can be defined in data{2}, data{3}, ...

This allow to consider for example seismic data in `data{1}`, and electromagnetic data in `data{2}`.

For each set of data, a Gaussian noise model (both correlated and uncorrelated) can be specified). The noise model for different data types (e.g. `data{1}` and `data{2}` are independent).

Once the noise model has been defined the log-likelihood related to any model, `m`, with the corressponding **forward response**, `d`, can be computed using

```
d=[d,forward,prior,data]=sippi_forward(m,forward,prior,data,
logL=sippi_likelihood(data,d)
```

where `d` is the output of **sippi\_forward**.

The specification of the noise model can be divided into a description of the **measurement noise** (mandatory) and the **modeling error** (optional).

## 2.2.1 Gaussian measurement noise

### 2.2.1.1 Uncorrelated Gaussian measurement noise

To define a set of observed data, `[0,1,2]`, with an associated uncertainty defined by a Gaussian model with mean 0 and standard deviation 2, use

```
data{1}.d_obs=[0 1 2]';
data{1}.d_std=[2 2 2]';
```

which is equivalent to (as the noise model for each data the same, and independent)

```
data{1}.d_obs=[0 1 2]';
data{1}.d_std=2;
```

One can also choose to define the uncertainty using a variance as opposed to the standard deviation

```
data{1}.d_obs=[0 1 2]';
data{1}.d_var=4;
```

### 2.2.1.2 correlated Gaussian measurement noise

Correlated Gaussian measurement uncertainty can be specified using the `Cd` field, as for example

```
data{1}.Cd=[4 1 0 ; 1 4 1 ; 0 1 4];
```

Note that `data{1}.Cd` must be of size `[NDxND]`, where `ND` is the the number of data in `data{1}.d_obs`.

## 2.2.2 Gaussian modeling error

The modeling error refer to errors caused by using for example an imperfect forward model, see [\[HCM14\]](#).

A Gaussian model of the modeling error can is specified by the mean, `dt`, and the covariance, `Ct`.

For example

```
data{1}.dt=[0 0 0];
data{1}.Ct=[4 4 4; 4 4 4; 4 4 4];
```

is equivalent to

```
data{1}.Ct=4
```

which implies a zero mean modeling error with a coavraince model where all model paremeters has a covariace of 4.

See the **tomography example**, for an example of accounting for correlated modeling errors.

## 2.3 forward: The forward model

The specification of the prior and data is intended to be generic, applicable to any inverse problem considered. The forward problem, on the other hand, is typically specific for each different inverse problem.

In order to make use of SIPPI to sample the posterior distribution, the solution to the forward problem, must be embedded in a Matlab function with the following input and output arguments:

```
[d, forward, prior, data]=sippi_forward(m, forward, prior, data, id)
```

$m$  is a realization of the prior model, and `prior` and `data` are the Matlab structures defining the prior and the noise model (see [Prior](#) and [Data](#))

`id` is optional, and can be used to compute the forward response of a subset of the different types of data available (i.e. `data{1}`, `data{2}`, ...) )

The forward variable is a Matlab structure that can contain any information needed to solve the forward problem. Thus, the parameters for the forward structure is problem dependant. One option, `forward.forward_function` is though generic, and point to the m-file that implements the forward problem.

The output variable `d` is a Matlab structure of the same size of `data`. Thus, if 4 types of data have been specified, then `d` must also be a structures of size 4.

```
length(data) == length(d);
```

Further, `d{i}` must refer to an array of the same size as `data{i}.d_obs`.

An example of an implementation of the forward problem related to a simple line fitting problem can be:

```
function [d, forward, prior, data]=sippi_forward_linefit(m, forward, prior, data);
    d{1}=forward.x*m{2}+m{1};
```

This implementation requires that the 'x'-locations, for which the y-values of the straight line is to be computed, is specified through `forward.x`. Say some y-data has been observed at locations `x=[1,5,8]`, with the values `[2,4,9]`, and a standard deviation of 1 specifying the uncertainty, the forward structure must be set as

```
forward.forward_function='sippi_forward_linefit';
forward.x=[1,5,8];
```

while the data structure will be

```
data{1}.d_obs=[2 4 9]
data{1}.d_std=1;
```

This implementation also requires that the prior model consists of two 1D prior types, such that

```
m=sippi_prior(prior)
```

returns the intercept in `m{1}` and the gradient in `m{2}`.

An example of computing the forward response using an intercept of 0, and a gradients of 2 is then

```
m{1}=0;
m{2}=2;
d=sippi_forward(m, forward)
```

and the corresponding log-likelihood of `m`, can be computed using

```
logL=sippi_likelihood(data, d);
```

[see more details and example related to polynomial line fitting at [polynomial line fitting](#)].

The [Examples](#) section contains more example of implementation of different forward problems.

## 2.4 Validating prior, data, and forward

A simple way to test the validity of prior, data, and forward is test if the following sequence can be evaluated without errors:

```
% Generate a realization, m, of the prior model
m=sippi_prior(prior);
% Compute the forward response
d=sippi_forward(m,forward,prior,data);
% Evaluate the log-likelihood of m
logL=sippi_likelihood(data,d);
```

## Chapter 3

# The a posteriori distribution

### 3.1 Sampling the a posteriori probability density

Once the prior, data, and forward data structures have been defined, the associated a posteriori probability can be sampled using the rejection sampler and the extended Metropolis sampler.

#### 3.1.1 The rejection sampler

The rejection sampler provides a simple, and also in many cases inefficient, approach to sample the posterior distribution.

At each iteration of the rejection sample an independent realization,  $m_{\text{pro}}$ , of the prior is generated, and the model is accepted as a realization of the posterior with probability  $P_{\text{acc}} = L(m_{\text{pro}})/L_{\text{max}}$ . It can be initiated using

```
options.mcmc.nite=400000; % Number of iteration, defaults to 1000
options.mcmc.i_plot=500; % Number of iteration between visual updates, defaults to 500
options=sippi_rejection(data,prior,forward,options);
```

By default the rejection sampler is run assuming a maximum likelihood of 1 (i.e.  $L_{\text{max}} = 1$ ). If  $L_{\text{max}}$  is known, then it can be set using in the options.Lmax or options.logLmax fields

```
options.mcmc.Lmax=1e-9;
options=sippi_rejection(data,prior,forward,options);
```

or

```
options.mcmc.logLmax=log(1e-9);
options=sippi_rejection(data,prior,forward,options);
```

Alternatively,  $L_{\text{max}}$  can be automatically adjusted to reflect the maximum likelihood found while running the rejection sampler using

```
options.mcmc.adaptive_rejection=1
options=sippi_rejection(data,prior,forward,options);
```

An alternative to rejection sampling, also utilizing independent realizations of the prior, that does not require one to set  $L_{\text{max}}$  is the **independent extended metropolis sampler**, which may be computationally superior to the rejection sampler,

### 3.1.2 The extended Metropolis sampler

The extended Metropolis algorithm is in general a much more efficient algorithm for sampling the a posteriori probability

The extended Metropolis sampler can be run using

```
options.mcmc.nite=40000; % number of iterations, default nite=30000
options.mcmc.i_sample=50; % save the current model for every 50 iterations, default, ↵
    i_sample=500
options.mcmc.i_plot=1000; % plot progress of the Metropolis sampler for every 100 ↵
    iterations
                        % default i_plot=50;
options.txt='case_line_fit'; % descriptive name appended to output foldername, default txt ↵
    ='';

[options,data,prior,forward,m_current]=sippi_metropolis(data,prior,forward,options)
```

One can choose to accept all steps in the Metropolis sampler, which will result in an algorithm sampling the prior model, using

```
options.mcmc.accept_all=1; % default [0]
```

One can choose to accept models that lead to an improvement in the likelihood, which results in an optimization like algorithm using

```
options.mcmc.accept_only_improvements=1; % default [0]
```

See [sippi\\_metropolis](#) for more details.

#### 3.1.2.1 Controlling the step length

One optionally, as part of running the [extended Metropolis sampler](#), automatically update the 'step'-length of the [sequential Gibbs sampler](#) in order to ensure a specific approximate acceptance ratio of the Metropolis sampler. See [CHM12] for details.

The default parameters for adjusting the step length, as given below, are set in the '[prior.seq\\_gibbs](#)' structure. These parameters will be set the first time 'sippi\_prior' is called with the 'prior' structure as output. The default parameters.

```
prior{m}.seq_gibbs.step_min=0;
prior{m}.seq_gibbs.step_min=1;
prior{m}.seq_gibbs.i_update_step=50
prior{m}.seq_gibbs.i_update_step_max=1000
prior{m}.seq_gibbs.n_update_history=50
prior{m}.seq_gibbs.P_target=0.3000
```

By default, adjustment of the step length, in order to achieve an acceptance ratio of 0.3 ('prior{m}.seq\_gibbs.P\_target'), will be performed for every 50 ('prior{m}.seq\_gibbs.i\_update\_step') iterations, using the acceptance ratio observed in the last 50 ('prior{m}.seq\_gibbs.i\_update\_history') iterations.

Adjustment of the step length will be performed only in the first 1000 ('prior{m}.seq\_gibbs.i\_update\_step\_max') iterations.

In order to disable automatic adjustment of the step length simply set

```
prior{m}.seq_gibbs.i_update_step_max=0; % disable automatic step length
```



### 3.1.2.2 The independent extended Metropolis sampler

The 'independent' extended Metropolis sampler, in which each proposed model is independent of the previously visited model, can be chosen by forcing the 'step'-length to be 1 (i.e. leading to independent samples from the prior), using e.g.

```
% force independent prior sampling
for ip=1:length(prior);
    prior{ip}.seq_gibbs.step=1;
    prior{ip}.seq_gibbs.i_update_step_max=0;
end
% run 'independent' extended Metropolis sampling
[options,data,prior,forward,m_current]=sippi_metropolis(data,prior,forward,options)
```

### 3.1.2.3 Annealing schedule

Simulated annealing like behaviour can be controlled in the `options.mcmc.anneal` structure. By default annealing is disabled.

Annealing consists of multiplying the noise level using an exponentially decreasing noise factor from `options.mcmc.anneal.fac_begin` to `options.mcmc.anneal.fac_end`, from iteration number `options.mcmc.anneal.i_begin` to `options.mcmc.anneal.i_end`.

The annealing schedule can be used to start a Metropolis sampler that allows to explore more of the model space in the beginning. Recall though that the posterior is not sampled until (at least) the annealing has been ended at iteration `options.mcmc.anneal.i_end`, if the `options.mcmc.anneal.fac_end=1`. This can potentially help not to get trapped in a local minima.

To use this type of annealing, where the annealing stops after 10000 iterations, after which the algorithm performs like a regular Metropolis sampler, use for example

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=10000; % iteration number when annealing stops
```

which is equivalent to

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=10000; % iteration number when annealing stops
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration i_begin ↔
options.mcmc.anneal.fac_end=1; % default, noise is scaled by fac_end at iteration i_end
```

## 3.2 Simulated Annealing

Simulated annealing type optimization can be setup using an **annealing schedule** that is enabled for the entire run of the Metropolis sampler, and that ends by a noise scaling factor less than 1. This can be obtained using e.g.

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
options.mcmc.anneal.i_end=options.mcmc.nite; % iteration number when annealing stops
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration i_begin ↔
options.mcmc.anneal.fac_end=0.01; % 1/100 of the noise level
```

# Chapter 4

## Examples

SIPPI can be used as a convenient approach for unconditional and conditional simulation.

In order to use SIPPI to solve inverse problems, one must provide the solution to the forward problem. Essentially this amounts to implementing a Matlab function that solves the **forward problem** in using a specific input/output format. If a solution to the forward problem already exists, this can be quite easily done simply using a Matlab wrapper function.

A few implementations of solutions to forward problems are included as examples as part of SIPPI. These will be demonstrated in the following

### 4.1 Examples of A priori models

#### 4.1.1 Multiple 1D Gaussian prior model

A prior model consisting of three independent 1D distributions (a Gaussian, Laplace, and Uniform distribution) can be defined using

```
ip=1;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Gaussian';
prior{ip}.m0=10;
prior{ip}.std=2;

ip=2;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Laplace';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=1;

ip=3;
prior{ip}.type='GAUSSIAN';
prior{ip}.name='Uniform';
prior{ip}.m0=10;
prior{ip}.std=2;
prior{ip}.norm=60;

m=sippi_prior(prior);

m =

    [14.3082]    [9.4436]    [10.8294]
```

1D histograms of a sample (consisting of 1000 realizations) of the prior models can be visualized using ...

```
sippi_plot_prior_sample(prior);
```

#### 4.1.2 Multivariate Gaussian prior with unknown covariance model properties.

The **FFT-MA** type a priori model allow separation of properties of the covariance model (covariance parameters, such as range, and anisotropy ratio) and the random component of a Gaussian model. This allow one to define a Gaussian a priori model, where the covariance parameters can be treated as unknown variables.

In order to treat the covariance parameters as unknowns, one must define one a priori model of type FFTMA, and then a number of 1D GAUSSIAN type a priori models, one for each covariance parameter. Each gaussian type prior model must have a descriptive name, corresponding to the covariance parameter that it should describe:

```
prior{im}.type='gaussian';
prior{im}.name='m_0';      % to define a prior for the mean
prior{im}.name='sill';     % to define a prior for sill (variance)
prior{im}.name='range_1'; % to define a prior for the range parameter 1
prior{im}.name='range_2'; % to define a prior for the range parameter 2
prior{im}.name='range_3'; % to define a prior for the range parameter 3
prior{im}.name='ang_1';   % to define a prior for the first angle of rotation
prior{im}.name='ang_2';   % to define a prior for the second angle of rotation
prior{im}.name='ang_3';   % to define a prior for the third angle of rotation
prior{im}.name='nu';      % to define a prior for the shape parameter, nu
                        % (only applies when the Matérn type Covariance model is used)
```

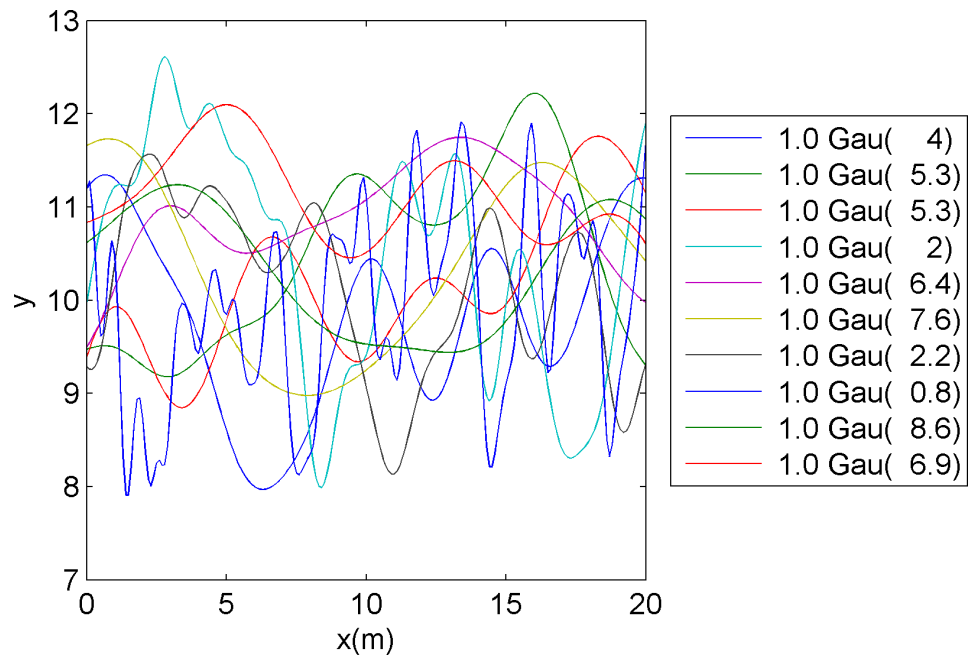
A very simple example of a prior model defining a 1D Spherical type covariance model with a range between 5 and 15 meters, can be defined using:

```
im=1;
prior{im}.type='FFTMA';
prior{im}.x=[0:.1:10]; % X array
prior{im}.m0=10;
prior{im}.Va='1 Sph(10)';
prior{im}.fftma_options.constant_C=0;

im=2;
prior{im}.type='gaussian';
prior{im}.name='range_1';
prior{im}.m0=10;
prior{im}.std=5;
prior{im}.norm=80;
prior{im}.prior_master=1; % -- NOTE, set this to the FFT-MA type prior for which this prior ←
    type
                        % should describe the range
```

Note that the the field `prior_master` must be set to point to the FFT-MA type a priori model (through its id/number) for which it should define a covariance parameter (in this case the range).

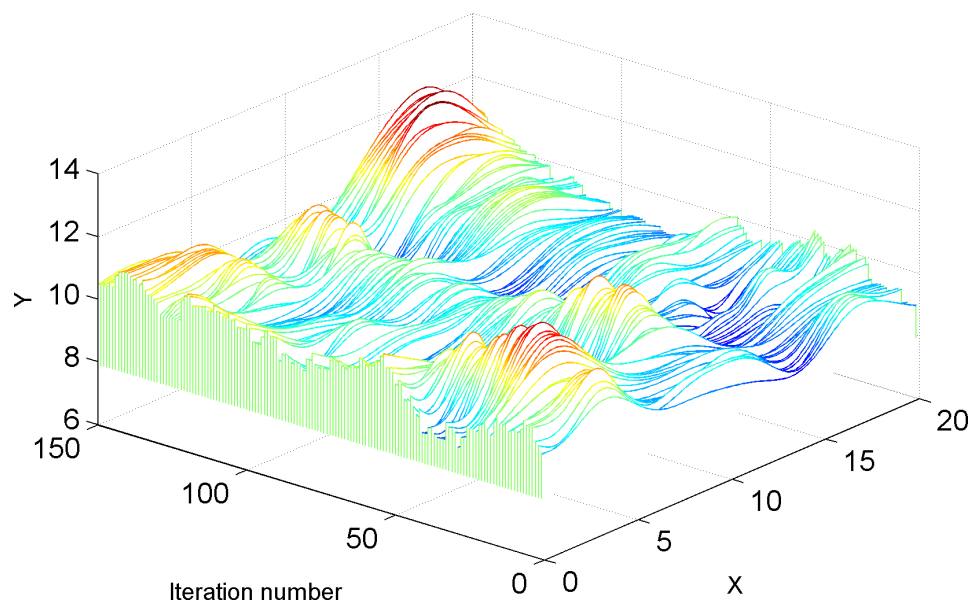
10 independent realizations of this type of a priori model are shown in the following figure



Such a prior, as all prior models available in SIPPI, works with **sequential Gibbs sampling**, allowing a random walk in the space of a prior acceptable models, that will sample the prior model. An example of such a random walk can be performed using

```
prior{1}.seq_gibbs.step=.005;
prior{2}.seq_gibbs.step=0.1;
clear m_real;
for i=1:150;
    [m,prior]=sippi_prior(prior,m);
    m_real(:,i)=m{1};
end
```

An example of such a set of 150 dependent realization of the prior can be seen below



## 4.2 Polynomial line fitting

Here follows simple polynomial (of order 0, 1 or 2) line-fitting is considered. Example m-files can be found in the SIPPI/examples/case\_linefit folder.

First, the forward problem is defined. Then examples of stochastic inversion using SIPPI is demonstrated using a synthetic data set.

### 4.2.1 The forward problem

The forward problem consists of computing the y-value as a function of the x-position of the data, and the polynomial coefficients determining the line. [sippi\\_forward\\_linefit.m](#):

```
% sippi_forward_linefit Line fit forward solver for SIPPI
%
% [d,forward,prior,data]=sippi_forward_linefit(m,forward,prior,data);
%
function [d,forward,prior,data]=sippi_forward_linefit(m,forward,prior,data);

if length(m)==1;
    d{1}=forward.x*m{1};
elseif length(m)==2;
    d{1}=forward.x*m{1}+m{2};
else
    d{1}=forward.x.^2*m{1}+forward.x*m{2}+m{3};
end
```

the forward.x must be an array of the x-locations, for which the y-values of the corresponding line will be evaluated. Note that the prior must be defined such that prior{1} refer to the intercept, prior{2} to the gradient, and prior{3} to the 2nd order polynomial coefficient.

If only one prior type is defined then the forward response will just be a constant, and if two prior types are defined, then the forward response will be a straight line.

### 4.2.2 Reference data, data, forward

A reference data set can be computed using

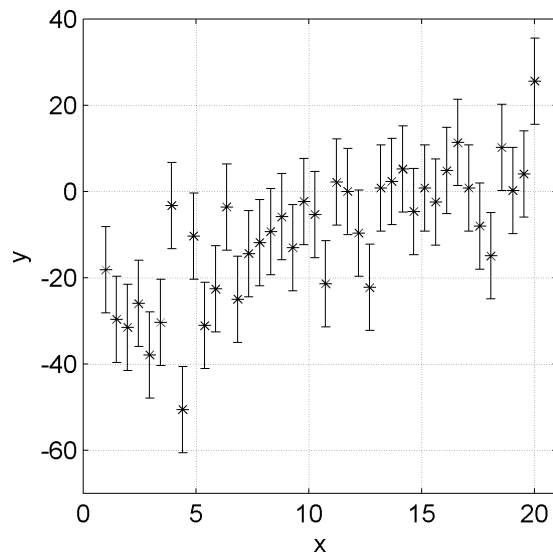
```
clear all;close all;
rand('seed',1);randn('seed',1);

%% Select reference model
m_ref{1}=-30;
m_ref{2}=2;
m_ref{3}=0;

%% Setup the forward model in the 'forward' structure
nd=40;
forward.x=linspace(1,20,nd);
forward.forward_function='sippi_forward_linefit';

%% Compute a reference set of observed data
d=sippi_forward(m_ref,forward);
d_obs=d{1};
d_std=10;
d_obs=d_obs+randn(size(d_obs)).*d_std;

data{1}.d_obs=d_obs;
data{1}.d_std=d_std;
```



### 4.2.3 The prior model

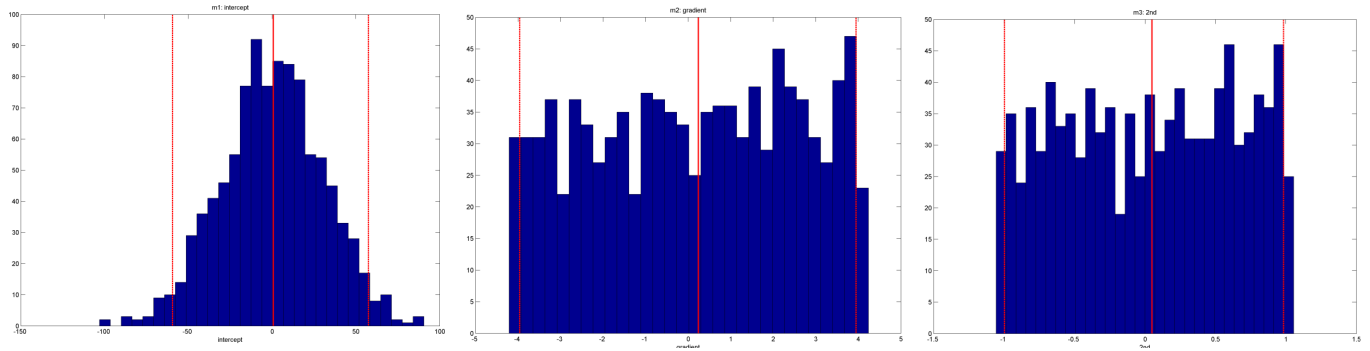
```
%% Setting up the prior model

% the intercept
im=1;
prior{im}.type='gaussian';
prior{im}.name='intercept';
prior{im}.m0=0;
prior{im}.std=30;
prior{im}.m_true=m_ref{1};

% 1st order, the gradient
im=2;
prior{im}.type='gaussian';
prior{im}.name='gradient';
prior{im}.m0=0;
prior{im}.std=4;
prior{im}.norm=80;
prior{im}.m_true=m_ref{2};

% 2nd order
im=3;
prior{im}.type='gaussian';
prior{im}.name='2nd';
prior{im}.m0=0;
prior{im}.std=1;
prior{im}.norm=80;
prior{im}.m_true=m_ref{3};

sippi_plot_prior_sample(prior);
```



#### 4.2.4 Setup and run the Metropolis sampler

Now, information about the model parameters can be inferred by running the **extended Metropolis sampler** using

```
options.mcmc.nite=40000; % Run for 40000 iterations
options.mcmc.i_sample=50; % Save every 50th visited model to disc
options.mcmc.i_plot=2500; % Plot the progress information for every 2500 iterations
options.txt='case_line_fit_2nd_order'; % descriptive name for the output folder
```

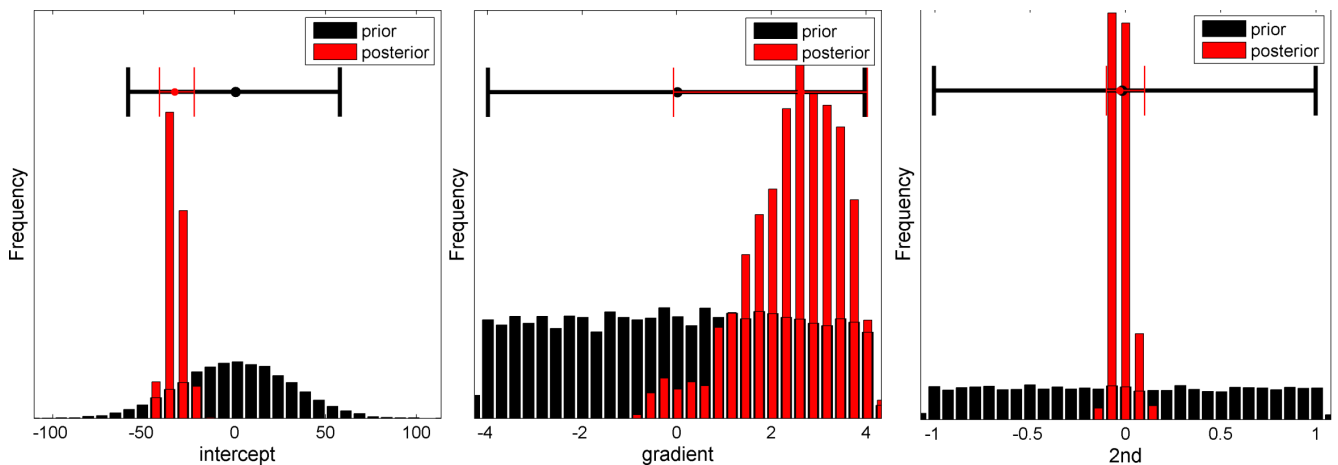
```
[options]=sippi_metropolis(data,prior,forward,options);
```

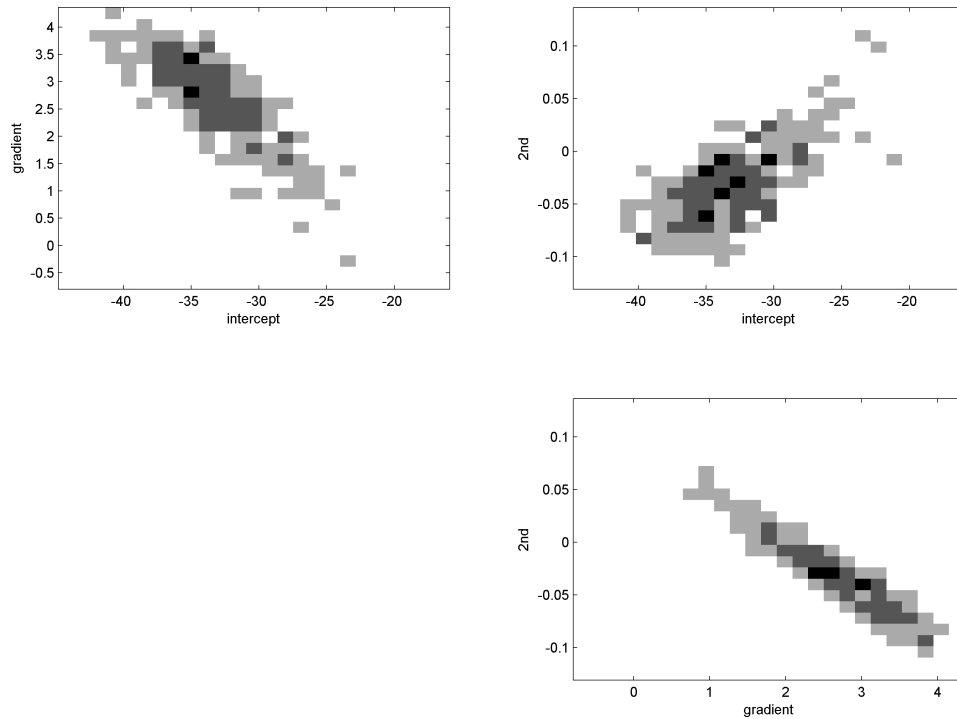
```
% plot posterior statistics, such as 1D and 2D marginals from the prior and posterior ↔
distributions
```

```
sippi_plot_prior_sample(options.txt);
```

```
sippi_plot_posterior(options.txt);
```

```
20140521_1644_sippi_metropolis_case_line_fit_2nd_order_m1_3_posterior_sample.png
```





### 4.2.5 Setup and run the rejection sampler

In a similar manner the **rejection sampler** can be setup and run using

```
options.mcmc.adaptive_rejection=1; % automatically adjust the normalizing likelihood
options.mcmc.nite=100000;
options=sippi_rejection(data,prior,forward,options);
```

## 4.3 Cross hole tomography

For now, please see [[HCLM13b](#)] for example of using SIPPI to sample the posterior for cross hole tomographic inverse problems.



## Chapter 5

# Bibliography

- [CHM12] K. S. Cordua, T. M. Hansen, and K. Mosegaard, Monte Carlo full waveform inversion of crosshole GPR data using multiple-point geostatistical a priori information, H19--H31. *Geophysics*, 77, 2012.
- [HCLM13a] T.M. Hansen, K.S. Cordua, M.C. Looms, and K. Mosegaard, SIPPI: a Matlab toolbox for sampling the solution to inverse problems with complex prior information: Part 1, methodology, 470--480. *Computers & Geosciences*, 52, 03 2013.
- [HCLM13b] T.M. Hansen, K.S. Cordua, M.C. Looms, and K. Mosegaard, SIPPI: a Matlab toolbox for sampling the solution to inverse problems with complex prior information: Part 2, Application to cross hole GPR tomography, 481--492. *Computers & Geosciences*, 52, 03 2013.
- [HCM12] T. M. Hansen, K. C. Cordua, and K. Mosegaard, Inverse problems with non-trivial priors - efficient solution through sequential Gibbs sampling, 593--611. *Computational Geosciences*, 16, 2012.
- [HCM14] T. M. Hansen, K. S. Cordua, B. H. Jacobsen, and K. Mosegaard, Accounting for imperfect forward modeling in geophysical inverse problems - exemplified for cross hole tomography, xx. Accepted for publication in *Geophysics*, xx, 2014.
-

## Chapter 6

# Reference

### 6.1 SIPPI

#### 6.1.1 getinunits

**GETINUNITS** Get object properties in specified units  
V = GETINUNITS(H, PROP, UNITS) returns the object property in the specified UNITS. It will leave the 'Units' and 'FontUnits' property unchanged afterwards.

H is the handle of the object. If it is an M-element array of handles, the function will return an M-by-1 cell array. PROP can be a string or a cell array of strings. If it is a 1-by-N or N-by-1 cell array, the function will return an M-by-N cell array of values. UNITS can be a string or a cell array. If it is a cell array, then PROP must also be a cell array with the same size as UNITS, and each cell element of UNITS corresponds to a cell element of PROP.

V = GETINUNITS(H, PROP) is the same as GET(H, PROP)

Examples:

```
V = GETINUNITS(H, 'Position', 'Pixels')
V = GETINUNITS(H, {'FontSize', 'Position'}, 'Normalized')
V = GETINUNITS(H, {'FontSize', 'Position'}, {'Points', 'Pixels'})
```

See also GET, SET

#### 6.1.2 logdet

**LOGDET** Computation of logarithm of determinant of a matrix

```
v = logdet(A);
    computes the logarithm of determinant of A.
```

Here, A should be a square matrix of double or single class. If A is singular, it will returns -inf.

Theoretically, this function should be functionally equivalent to log(det(A)). However, it avoids the overflow/underflow problems that are likely to

happen when applying `det` to large matrices.

The key idea is based on the mathematical fact that the determinant of a triangular matrix equals the product of its diagonal elements. Hence, the matrix's log-determinant is equal to the sum of their logarithm values. By keeping all computations in log-scale, the problem of underflow/overflow caused by product of many numbers can be effectively circumvented.

The implementation is based on LU factorization.

```
v = logdet(A, 'chol');
```

If `A` is positive definite, you can tell the function to use Cholesky factorization to accomplish the task using this syntax, which is substantially more efficient for positive definite matrix.

#### Remarks

-----

logarithm of determinant of a matrix widely occurs in the context of multivariate statistics. The log-pdf, entropy, and divergence of Gaussian distribution typically comprises a term in form of log-determinant. This function might be useful there, especially in a high-dimensional space.

Theoretically, LU, QR can both do the job. However, LU factorization is substantially faster. So, for generic matrix, LU factorization is adopted.

For positive definite matrices, such as covariance matrices, Cholesky factorization is typically more efficient. And it is **STRONGLY RECOMMENDED** that you use the `chol` (2nd syntax above) when you are sure that you are dealing with a positive definite matrix.

#### Examples

-----

```
% compute the log-determinant of a generic matrix
A = rand(1000);
v = logdet(A);

% compute the log-determinant of a positive-definite matrix
A = rand(1000);
C = A * A';      % this makes C positive definite
v = logdet(C, 'chol');
```

### 6.1.3 pathdef

**PATHDEF** Search path defaults.

**PATHDEF** returns a string that can be used as input to **MATLABPATH** in order to set the path.

### 6.1.4 plotboxpos

**PLOTBOXPOS** Returns the position of the plotted axis region

```
pos = plotboxpos(h)
```

This function returns the position of the plotted region of an axis, which may differ from the actual axis position, depending on the axis limits, data aspect ratio, and plot box aspect ratio. The position is returned in the same units as the those used to define the axis itself. This function can only be used for a 2D plot.

Input variables:

h: axis handle of a 2D axis (if omitted, current axis is used).

Output variables:

pos: four-element position vector, in same units as h

### 6.1.5 sippi\_adjust\_step\_size

**sippi\_adjust\_step\_size** Adjust step length length for Metropolis sampler in SIPPI

Call :

```
step=sippi_adjust_step_size(step,P_average,P_target);
```

step : current step

P\_current : Current acceptance ratio

P\_target : preferred acceptance ratio (def=0.3);

See also `sippi_compute_acceptance_rate`, `sippi_prior_set_steplength`

### 6.1.6 sippi\_anneal\_adjust\_noise

**sippi\_anneal\_adjust\_noise** : Adjust noise level in annealing schedul

Call:

```
[data_adjust,mcnc]=sippi_anneal_adjust_noise(data,i,mcnc,prior);
```

See also: `sippi_metropolis`, `sippi_anneal_factor`

### 6.1.7 sippi\_anneal\_factor

**sippi\_anneal\_factor** : compute simple noise multiplication factor for annealing type sampling

See also `sippi_metropolis`, `sippi_anneal_adjust_noise`

### 6.1.8 sippi\_colormap

```
sippi_colormap Default colormap for sippi

Call :
    sippi_colormap; % the same as sippi_colormap(3);

or :
    sippi_colormap(1) - Red Green Black
    sippi_colormap(2) - Red Green Blue Black
    sippi_colormap(3) - Jet
```

### 6.1.9 sippi\_compute\_acceptance\_rate

```
sippi_compute_acceptance_rate Computes acceptance rate for the Metropolis sampler in SIPPI ↔

Call:
    P_acc=sippi_compute_acceptance_rate(acc,n_update_history);
```

### 6.1.10 sippi\_forward

```
sippi_forward Simple forward wrapper for SIPPI

Assumes that the actual forward solver has been defined by
forward.forward_function

Call:
    [d,forward,prior,data]=sippi_forward(m,forward,prior,data,id,im)
```

### 6.1.11 sippi\_forward\_traveltime

```
sippi_forward_traveltime Traveltime computation in SIPPI

Call :
    [d,forward,prior,data]=sippi_forward_traveltime(m,forward,prior,data,id,im)

forward.type determines the method used to compute travel times
forward.type='ray';
forward.type='fat';
forward.type='eikonal';
forward.type='born';
```

### 6.1.12 sippi\_get\_sample

`sippi_get_sample` Get a posterior sample

Call :

```
[reals, etype_mean, etype_var]=sippi_get_sample(data, prior, id, im, n_reals, options);
```

### 6.1.13 sippi\_least\_squares

`sippi_least_squares` Least squares type inversion for SIPPI

Call :

```
[m_reals, m_est, Cm_est]=sippi_least_squares(data, prior, forward, n_reals, lsq_type, id, im);
```

```
lsq_type : 'lsq' (def), classical least squares
           'error_sim', simulation through error simulation
           'visim', simulation through SGSIM of DSSIM
```

### 6.1.14 sippi\_likelihood

`sippi_likelihood` Compute likelihood given an observed dataset

Call

```
[logL, L, data]=sippi_likelihood(d, data);
```

```
data{1}.d_obs [N_data,1] N_data data observations
data{1}.d_std [N_data,1] N_data uncorrelated Gaussian STD
data{1}.d_var [N_data,1] N_data uncorrelated Gaussian variances
```

Gaussian modelization error,  $N(dt, Ct)$ , is specified as  
`data{1}.dt [N_data,1]` : Bias/mean of modelization error  
`data{1}.Ct [N_data, N_data]` : Covariance of modelization error

```
data{1}.Ct [1,1] : Constant Covariance of modelization error
                  implies data{1}.Ct=ones(N_data.N_data)*data{1}.Ct;
```

`data{id}.recomputeCD [default=0]`, if '1' then `data{1}.iCD` is recomputed each time `sippi_likelihood` is called. This should be used if the noise model changes between each call to `sippi_likelihood`.

```
data{id}.full_likelihood [default=]0; if '1' the the full likelihood
(including the determinant) is computed. This not needed if the data
civariance is constant, but if it changes, then use
data{id}.full_likelihood=1;
```

### 6.1.15 sippi\_mcmc\_init

`sippi_mcmc_init` Initialize MCMC options for Metropolis and rejection sampling in SIPPI

Call:

```
options=sippi_mcmc_init(options,prior);
```

### 6.1.16 sippi\_metropolis

`sippi_metropolis` Extended Metropolis sampling in SIPPI

Metropolis sampling.

See e.g. Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012.

Inverse problems with non-trivial priors - Efficient solution through Sequential ↩  
Gibbs Sampling.

Computational Geosciences. doi:10.1007/s10596-011-9271-1.

Call :

```
[options,data,prior,forward,m_current]=sippi_metropolis(data,prior,forward,options)
```

Input :

```
data : sippi data structure
prior : sippi prior structure
forward : sippi forward structure
```

options :

```
options.txt [string] : string to be used as part of all output files
```

```
options.mcmc.nite [1] : Number of iterations
```

```
options.mcmc.i_plot [1]: Number of iterations between updating plots
```

```
options.mcmc.i_sample=: Number of iterations between saving model to disk
```

```
options.mcmc.m_init : Manually chosen starting model
```

```
options.mcmc.m_ref : Reference known target model
```

```
options_mcmc.accept_only_improvements [0] : Optimization
```

```
%% PERTUBATION STRATEGY
```

```
options.mcmc.pert_strategy.perturb_all=1; % Perturb all priors in each
                                         % iteration. def =[0]
```

```
%% SIMULATED ANNEALING
```

```
options.mcmc.anneal.i_begin=1; % default, iteration number when annealing begins
```

```
options.mcmc.anneal.i_end=100000; % iteration number when annealing stops
```

```
options.mcmc.anneal.fac_begin=20; % default, noise is scaled by fac_begin at iteration ↩
i_begin
```

```
options.mcmc.anneal.fac_end=1; % default, noise is scaled by fac_end at iteration ↩
i_end
```

See also `sippi_rejection`

### 6.1.17 sippi\_plot\_current\_model

`sippi_plot_current_model` Plots the current model during Metropolis sampling

Call :

```
sippi_plot_current_model(mcmc,data,d,m_current,prior);
```

### 6.1.18 sippi\_plot\_data

sippi\_plot\_data plot data in SIPPI

Call.  
sippi\_plot\_data(d,data);

### 6.1.19 sippi\_plot\_loglikelihood

sippi\_plot\_loglikelihood Plot loglikelihood time series

Call :  
acc=sippi\_plot\_loglikelihood(logL,i\_acc,N,itext)

### 6.1.20 sippi\_plot\_model

sippi\_plot\_model Plot a 'model', i.e. a realization of the prior model

Call :  
sippi\_plot\_model(prior,m,im\_array);  
  
prior : Matlab structure for SIPPI prior model  
m : Matlab structure for SIPPI realization  
im\_array : integer array of type of models to plot (typically 1)

Example

```
m=sippi_prior(prior);  
sippi_plot_model(prior,m);  
  
m=sippi_prior(prior);  
sippi_plot_model(prior,m,2);
```

See also sippi\_plot\_prior

### 6.1.21 sippi\_plot\_movie

sippi\_plot\_movie plot movie of prior and posterior realizations

Call :  
sippi\_plot\_movie(fname);  
sippi\_plot\_movie(fname,im\_array,n\_frames,skip\_burnin);  
fname : name of folder with results (e.g. options.txt)  
im\_array : array of indexes of model parameters to make into movies  
n\_frames [200] : number of frames in movie  
skip\_burnin [200] : start movie after burn\_in;



```
Ex:
sippi_plot_movie('20130812_Metropolis');
sippi_plot_movie(options.txt);

%% 1000 realization including burn-in, for prior number 1
sippi_plot_movie('20130812_Metropolis',1,1000,0);
```

### 6.1.22 sippi\_plot\_posterior

```
sippi_plot_posterior Plot statistics from posterior sample

Call :
    sippi_plot_posterior(fname,im_arr,prior,options,n_reals);

See also sippi_plot_prior
```

### 6.1.23 sippi\_plot\_prior

```
sippi_plot_prior Plot a sample of the prior in SIPPI

Call :
    sippi_plot_prior(prior,ip,n_reals,cax,supt);

See also sippi_plot_posterior, sippi_plot_model
```

### 6.1.24 sippi\_prior

```
sippi_prior A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:
[m_propose,prior]=sippi_prior(prior);

To generate a realization of the prior model defined by the prior structure,
in the vicinity of a current model (using sequential Gibbs sampling) use:
[m_propose,prior]=sippi_prior(prior,m_current);

The following types of a priori models can be used
SNESIM [1D-3D] : based on a multiple point statistical model inferref from a training ←
                images. Relies in the SNESIM algorithm
SISIM [1D-3D] : based on Sequential indicator SIMULATION
VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
GAUSSIAN [1D] : 1D generalized gaussian model

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaissian based prior model based on the
% FFT-MA method, can be defined using
im=1;
prior{im}.type='FFTMA';
```

```

prior{im}.name='A SIMPLE PRIOR';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Va='1 Sph(10)';
prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
m=sippi_prior(prior);
% This realization can now be plotted using
sippi_plot_prior(m,prior);
% or
imagesc(prior{1}.x,prior{1}.y,m{1})

%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

im=1;
prior{im}.type='GAUSSIAN';
prior{im}.m0=100;
prior{im}.std=50;
prior{im}.norm=100;
im=2;
prior{im}.type='FFTMA';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
im=3;
prior{im}.type='SISIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];
prior{im}.m0=10;
prior{im}.Cm='1 Sph(10)';
im=4;
prior{im}.type='SNESIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];

sippi_plot_model(prior);

%% Sequential Gibbs sampling

All a priori model types can be perturbed, such that a new realization
is generated in the vicinity of a current model.
To do this Sequential Gibbs Sampling is used.
For more information, see <a href="http://dx.doi.org/10.1007/s10596-011-9271-1">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse
problems with non-trivial priors - Efficient solution through Sequential Gibbs
Sampling. Computational Geosciences</a>.
The type of sequential Gibbs sampling can be controlled in the
'seq_gibbs' structures, e.g. prior{1}.seq_gibbs

im=1;
prior{im}.type='SNESIM';
prior{im}.x=[0:1:100];
prior{im}.y=[0:1:100];

[m,prior]=sippi_prior(prior);
prior{1}.seq_gibbs.step=1; % Large step--> independant realizations
prior{1}.seq_gibbs.step=.1; % Smaller step--> Dependant realizations
for i=1:30;
    [m,prior]=sippi_prior(prior,m); % One iteration of Sequential Gibbs
    sippi_plot_model(prior,m);

```

end

See also: sippi\_prior\_init, sippi\_plot\_prior, sippi\_prior\_set\_steplength.m

TMH/2012

### 6.1.25 sippi\_prior\_fftma

sippi\_prior A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:  
[m\_propose,prior]=sippi\_prior(prior);

To generate a realization of the prior model defined by the prior structure,  
in the vicinity of a current model (using sequential Gibbs sampling) use:  
[m\_propose,prior]=sippi\_prior(prior,m\_current);

The following types of a priori models can be used

SNESIM [1D-3D] : based on a multiple point statistical model inferred from a training images. Relies in the SNESIM algorithm ←  
SISIM [1D-3D] : based on Sequential indicator SIMULATION  
VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation  
FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)  
GAUSSIAN [1D] : 1D generalized gaussian model

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaussian based prior model based on the  
% FFT-MA method, can be defined using

```
id=1;
prior{id}.type='FFTMA';
prior{id}.name='A SIMPLE PRIOR';
prior{id}.x=[0:1:100];
prior{id}.y=[0:1:100];
prior{id}.m0=10;
prior{id}.Va='1 Sph(10)';
prior=sippi_prior_init(prior);
```

% A realization from this prior model can be generated using  
m=sippi\_prior(prior);

% This realization can now be plotted using  
sippi\_plot\_prior(m,prior);

% or  
imagesc(prior{1}.x,prior{1}.y,m{1})

%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

```
id=1;
prior{id}.type='FFTMA';
prior{id}.x=[0:1:100];
prior{id}.y=[0:1:100];
prior{id}.m0=10;
prior{id}.Cm='1 Sph(10)';
id=2;
prior{id}.type='SISIM';
prior{id}.x=[0:1:100];
prior{id}.y=[0:1:100];
prior{id}.m0=10;
prior{id}.Cm='1 Sph(10)';
```

```

id=3;
prior{id}.type='GAUSSIAN';
prior{id}.m0=100;
prior{id}.std=50;
prior{id}.norm=100;
prior=sippi_prior_init(prior);

sippi_plot_model(prior);

%% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596-011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems
  with non-trivial priors - Efficient solution through Sequential Gibbs Sampling.
  Computational Geosciences</a>.

See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m

TMH/2012

```

### 6.1.26 sippi\_prior\_init

sippi\_prior\_init Initialize PRIOR structure for SIPPI

Call

```
prior=sippi_prior_init(prior);
```

See also sippi\_prior

### 6.1.27 sippi\_prior\_new

sippi\_prior A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:

```
[m_propose,prior]=sippi_prior(prior);
```

To generate a realization of the prior model defined by the prior structure, in the vicinity of a current model (using sequential Gibbs sampling) use:

```
[m_propose,prior]=sippi_prior(prior,m_current);
```

The following types of a priori models can be used

SNESIM [1D-3D] : based on a multiple point statistical model inferred from a training images. Relies in the SNESIM algorithm

SISIM [1D-3D] : based on Sequential indicator SIMULATION

VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation

FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)

GAUSSIAN [1D] : 1D generalized gaussian model

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaussian based prior model based on the

% FFT-MA method, can be defined using

```
id=1;
```

```
prior{id}.type='FFTMA';
```

```

    prior{id}.name='A SIMPLE PRIOR';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Va='1 Sph(10)';
    prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
    m=sippi_prior(prior);
% This realization can now be plotted using
    sippi_plot_prior(m,prior);
% or
    imagesc(prior{1}.x,prior{1}.y,m{1})

%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

    id=1;
    prior{id}.type='FFTMA';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=2;
    prior{id}.type='SISIM';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=3;
    prior{id}.type='GAUSSIAN';
    prior{id}.m0=100;
    prior{id}.std=50;
    prior{id}.norm=100;
    prior=sippi_prior_init(prior);

    sippi_plot_model(prior);

%% Sequential Gibbs sampling
% For more information, see <a href="http://dx.doi.org/10.1007/s10596-011-9271-1">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems
    with non-trivial priors - Efficient solution through Sequential Gibbs Sampling. Computational Geosciences</a>.

See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m

TMH/2012

```

## 6.1.28 sippi\_prior\_old

sippi\_prior A priori models for SIPPI

To generate a realization of the prior model defined by the prior structure use:  
 [m\_propose,prior]=sippi\_prior(prior);

To generate a realization of the prior model defined by the prior structure, in the vicinity of a current model (using sequential Gibbs sampling) use:  
 [m\_propose,prior]=sippi\_prior(prior,m\_current);

The following types of a priori models can be used

```

    SNESIM [1D-3D] : based on a multiple point statistical model inferred from a training ←
                     images. Relies in the SNESIM algorithm
    SISIM [1D-3D] : based on Sequential indicator SIMULATION
    VISIM [1D-3D] : based on Sequential Gaussian and Direct Sequential simulation
    FFTMA [1D-3D] : based on the FFT-MA method (Multivariate Gaussian)
    GAUSSIAN [1D] : 1D generalized gaussian model

%%% SIMPLE EXAMPLE %%%

% A simple 2D multivariate Gaussian based prior model based on the
% FFT-MA method, can be defined using
    id=1;
    prior{id}.type='FFTMA';
    prior{id}.name='A SIMPLE PRIOR';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Va='1 Sph(10)';
    prior=sippi_prior_init(prior);
% A realization from this prior model can be generated using
    m=sippi_prior(prior);
% This realization can now be plotted using
    sippi_plot_prior(m,prior);
% or
    imagesc(prior{1}.x,prior{1}.y,m{1})

%%% A PRIOR MODEL WITH SEVERAL 'TYPES OF A PRIORI MODEL'

    id=1;
    prior{id}.type='FFTMA';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=2;
    prior{id}.type='SISIM';
    prior{id}.x=[0:1:100];
    prior{id}.y=[0:1:100];
    prior{id}.m0=10;
    prior{id}.Cm='1 Sph(10)';
    id=3;
    prior{id}.type='GAUSSIAN';
    prior{id}.m0=100;
    prior{id}.std=50;
    prior{id}.norm=100;
    prior=sippi_prior_init(prior);

    sippi_plot_model(prior);

%% Sequential Gibbs sampling
% For more information, see <a href="matlab:web('http://dx.doi.org/10.1007/s10596 ←
-011-9271-1')">Hansen, T. M., Cordua, K. S., and Mosegaard, K., 2012. Inverse problems ←
with non-trivial priors - Efficient solution through Sequential Gibbs Sampling. ←
Computational Geosciences</a>.

See also: sippi_prior_init, sippi_plot_prior, sippi_prior_set_steplength.m

TMH/2012

```

### 6.1.29 sippi\_prior\_set\_steplength

`sippi_prior_set_steplength` Set step length for Metropolis sampler in SIPPI

Call

```
prior=sippi_prior_set_steplength(prior,mcmc,im);
```

### 6.1.30 sippi\_rejection

`sippi_rejection` Rejection sampling

Call :

```
options=sippi_rejection(data,prior,forward,options)
```

input arguments

```
options.mcmc.i_plot
```

```
options.mcmc.nite      % maximum number of iterations
```

```
options.mcmc.logLmax
```

```
options.mcmc.rejection_normalize_log = log(options.mcmc.Lmax)
```

```
options.mcmc.adaptive_rejection=1, adaptive setting of maximum likelihood
```

```
(def=[0])
```

```
At each iteration Lmax will be set if log(L(m_cur))>=options.mcmc.logLmax
```

```
options.mcmc.max_run_time_hours = 1; % maximum runtime in hours
```

```
% (overrides options.mcmc.nite if needed)
```

See also `sippi_metropolis`

### 6.1.31 sippi\_set\_path

`sippi_set_path` Set paths for running sippi

## 6.2 SIPPI toolbox: Traveltime tomography

### 6.2.1 calc\_Cd

`Calc_cd` Setup a covariance model to account for borehole imperfections

Call: `Cd=calc_Cd(ant_pos,var_uncor,var_cor1,var_cor2,L)`

This function sets up a data covariance matrix that accounts for static (i.e. correlated) data errors.

Inputs:

\* `ant_pos`: A  $N \times 4$  array that contains  $N$  combinations of transmitter/source and receiver positions. The first two columns are the x- and y-coordinates of the transmitter/source position. The last two columns are the x- and

y-coordinates of the receiver position.  
 \* var\_uncor: The variance of the uncorrelated data errors.  
 \* var\_cor1: The variance of the correlated data errors related to the transmitter/source positions.  
 \* var\_cor2: The variance of the correlated data errors related to the receiver positions.  
 \* L: The correlation length for the correlation between the individual transmitter/source or receiver positions using an exponential covariance function. For typical static errors the correlation length is set to a small number (e.g.  $10^{-6}$ ).

For more details and practical examples see:  
 Cordua et al., 2008 in Vadose zone journal.  
 Cordua et al., 2009 in Journal of applied geophysics.

Knud S. Cordua (2012)

## 6.2.2 eikonal

eikonal Traveltime computation by solving the eikonal equation

```
tmap=eikonal(x,y,z,V,Sources,type);
```

x,y,z : arrays defining the x, y, and z axis  
 V: velocity field, with size (length(y),length(x),length(z));  
 Sources [ndata,ndim] : Source positions  
 type (optional): type of eikonal solver: [1]:Fast Marching(default), [2]:FD

tmap [size(V)]: travel times computed everywhere in the velocity grid

```
%Example (2D):
x=[1:1:100];
y=1:1:100;
z=1;
V=ones(100,100);V(:,1:50)=2;
Sources = [10 50;75 50];
t=eikonal(x,y,z,V,Sources);
subplot(1,2,1);imagesc(x,y,t(:,:,1,1));axis image;colorbar
subplot(1,2,2);imagesc(x,y,t(:,:,1,2));axis image;colorbar
```

See also eikonal\_traveltime

## 6.2.3 eikonal\_raylength

eikonal\_raylength : Computes the raylength from S to R using the eikonal equation

Call:

```
raylength=eikonal_raylength(x,y,v,S,R,tS,doPlot)
```

## 6.2.4 eikonal\_traveltime



`eikonal_travelttime` Computes travelttime between sources and receivers by solving the eikonal equation ↔

```
t=eikonal_travelttime(x,y,z,V,Sources,Receivers,iuse,type);
```

`x,y,z` : arrays defining the `x`, `y`, and `z` axis

`V`: velocity field, with size `(length(y),length(x),length(z))`;

`Sources [ndata,ndim]` : Source positions

`Receivers [ndata,ndim]` : Receiver positions

`iuse (optional)`: optionally only use subset of data. eg.g `i_use=[1 2 4]`;

`type (optional)`: type of eikonal solver: `[1]:Fast Marching(default)`, `[2]:FD`

`tmap [size(V)]`: travel times computed everywhere in the velocity grid

%Example (2%

Example 2d travelttime computation

Example (2D):

```
x=[1:1:100];
```

```
y=1:1:100;
```

```
z=1;
```

```
V=ones(100,100);V(:,1:50)=2;
```

```
S=[50 50 1;50 50 1];
```

```
R=[90 90 1; 90 80 1];
```

```
t=eikonal_travelttime(x,y,z,V,S,R)
```

Example (3D):

```
nx=50;ny=50;nz=50;
```

```
x=1:1:nx;
```

```
y=1:1:ny;
```

```
z=1:1:nz;
```

```
V=ones(ny,nx,nz);V(:,1:50,:)=2;
```

```
S=[10 10 1;10 10 1;10 9 1];
```

```
R=[40 40 40; 40 39 40; 40 40 40];
```

```
t=eikonal_travelttime(x,y,z,V,S,R)
```

See also `eikonal`

## 6.2.5 kernel\_buursink\_2d

`kernel_buursink_2k` Computes 2D Sensitivity kernel based on 1st order EM scattering theory

See

Buursink et al. 2008. Crosshole radar velocity tomography  
with finite-frequency Fresnel. *Geophys J. Int.*  
(172) 117;

CALL :

```
% specify a source trace (dt, wf_trace):
```

```
[kernel,L,L1_all,L2_all]=kernel_buursink_2d(model,x,z,S,R,dt,wf_trace);
```

```
% Use a ricker wavelet with center frequency 'f0'
```

```
[kernel,L,L1_all,L2_all]=kernel_buursink_2d(model,x,z,S,R,f0));
```

Knud Cordua, 2009,

Thomas Mejer Hansen (small edits, 2009)

## 6.2.6 kernel\_finite\_2d

kernel\_finite\_2d 2D sensitivity kernels

Call:  
`[Knorm,K,dt,options]=kernel_finite_2d(v_ref,x,y,S,R,freq,options);`

## 6.2.7 kernel\_fresnel\_2d

kernel\_fresnel\_2d Sensitivity kernel for amplitude and first arrival

Call:  
`[kernel_t,kernel_a,P_omega,omega]=kernel_fresnel_2d(v,x,y,S,R,omega,P_omega);`

Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity kernels for seismic Fresnel volume Tomography, Geophysics, 75(5), U35-U46

See also kernel\_fresnel\_monochrome\_2d

Run with no argument for an example.

## 6.2.8 kernel\_fresnel\_monochrome\_2d

kernel\_fresnel\_monochrome\_2d 2D monochrome kernel for amplitude and first arrival

Call:  
`[kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S,R,omega);`  
 or  
`[kernel_t,kernel_a]=kernel_fresnel_monochrome_2d(v,x,y,S,R,omega,L,L1,L2);`

Based on Liu, Dong, Wang, Zhu and Ma, 2009, Sensitivity kernels for seismic Fresnel volume Tomography, Geophysics, 75(5), U35-U46

See also, kernel\_fresnel\_2d

## 6.2.9 kernel\_multiple

kernel\_multiple Computes the sensitivity kernel for a wave traveling from S to R.

CALL :  
`[K,RAY,Gk,Gray,timeS,timeR,raypath]=kernel_multiple(Vel,x,y,z,S,R,T,alpha,Knorm);`

IN :  
 Vel [ny,nx] : Velocity field  
 x [1:nx] :  
 y [1:ny] :  
 z [1:nz] :  
 S [1,3] : Location of Source  
 R [1,3] : Location of Receiver

```

T : Dominant period
alpha: controls exponential decay away ray path
Knorm [1] : normalization of K [0]:none, K:[1]:vertical

OUT :
K : Sensitivity kernel
R : Ray sensitivity kernel (High Frequency approx)
timeS : travel computed from Source
timeR : travel computed from Receiver
raypath [nraydata,ndim] : the center of the raypath

The sensitivity is the length travelled in each cell.

See also : fast_fd_2d

TMH/2006

```

### 6.2.10 kernel\_slowness\_to\_velocity

```

kernel_slowness_to_velocity Converts from slowness to velocity parameterizations

G : kernel [1,nkernels]
V : Velocity field (

CALL:
  G_vel=kernel_slowness_to_velocity(G,V);
or
  [G_vel,v_obs]=kernel_slowness_to_velocity(G,V,t);
or
  [G_vel,v_obs,Cd_v]=kernel_slowness_to_velocity(G,V,t,Cd);

```

### 6.2.11 mspectrum

```

mspectrum : Amplitude and Power spectrum
Call :
  function [A,P,smoothP,kx]=mspectrum(x,dx)

1D (A)mplitude and (P)owerspectrum of x-series with spacing dx

```

### 6.2.12 munk\_fresnel\_2d

```

2D frechet kernel, First Fresnel Zone

See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar Physics 192.

Call :
S=munk_fresnel_2d(T,dt,alpha,As,Ar,K);

T : dominant period

```

```

dt :
alpha : degree of cancellation
As : Amplitude fo the wavefield propagating from the source
Ar : Amplitude fo the wavefield propagating from the receiver
K : normalization factor

```

### 6.2.13 munk\_fresnel\_3d

3D frechet kernel, First Fresnel Zone

See Jensen, Jacobsen, Christensen-Dalsgaard (2000) Solar Physics 192.

Call :

### 6.2.14 tomography\_kernel

tomography\_kernel Computes the sensitivity kernel for a wave traveling from S to R.

```

CALL :
[K,RAY,Gk,Gray,timeS,timeR,raypath]=tomography_kernel(Vel,x,y,z,S,R,T,alpha,Knorm);

```

```

IN :
Vel [ny,nx] : Velocity field
x [1:nx] :
y [1:ny] :
z [1:nz] :
S [1,3] : Location of Source
R [1,3] : Location of Receiver
T : Donminant period
alpha: controls exponential decay away ray path
Knorm [1] : normaliztion of K [0]:none, K:[1]:vertical

```

```

OUT :
K : Sensitivity kernel
R : Ray sensitivity kernel (High Frequency approx)
timeS : travel computed form Source
timeR : travel computed form Receiver
raypath [nraydata,ndim] : the center of the raypath

```

The sensitivity is the length travelled in each cell.