$$f : X \to Y$$

$$A \subset X$$

$$f(A) = \{y \in Y : y = f(x)$$

$x \in A\}$

$M \subset Y.$

M

$$f^{-1}(M) = \{x \in X : f(x) \in M\}$$

$A, B \subset X.$

$$A \subset B \to f(A) \subset f(B)$$

$$y \in f(A) \Rightarrow y = f(x)$$

$x \in A$

$$A \subset B \Rightarrow x \in B \Rightarrow f(x) \in f(B)$$

$$f(A \cup B) = f(A) \cup f(B)$$

$\Rightarrow$

$$f(A \cup B) \in f(A) \cup f(B)$$

$$y \in f(A \cup B) \Rightarrow \exists x \in A \cup B$$

$$y = f(x)$$

$$x \in A \Rightarrow y = f(x) \in f(A) \Rightarrow y \in f(A) \cup f(B).$$

$\Leftarrow$

$x \in B$

$$A \subset (A \cup B) \Rightarrow f(A) \subset f(A \cup B)$$

$$f(B) \subset f(A \cup B) \Rightarrow f(A) \cup f(B) \subset$$

$$f(A \cap B) \subset f(A) \cap f(B)$$

$$A \cap B \subset A$$

$$A \cap B \subset B$$

$$f(A \cap B) \subset f(A) y f(A \cap B) \subset f(B)$$

$$M \subset Y, f^{-1}(M) = \{x \in X : f(x) \in M\}$$

$$F \subset G \Rightarrow f^{-1}(F) \subset f^{-1}(G)$$

$$x \in f^{-1}(F) \Rightarrow f(x) \in F \subset G \Rightarrow f(x) \in G \Rightarrow x \in f^{-1}(G).$$

$$f^{-1}(F \cup G) = f^{-1}(F) \cup f^{-1}(G)$$

$$x \in f^{-1}(F \cup G) \Rightarrow f(x) \in F \cup G \Rightarrow f(x) \in F \Rightarrow x \in f^{-1}(F) \Rightarrow x \in f^{-1}(F) \cup f^{-1}(G)$$

$$f^{-1}(F) \cup f^{-1}(G) \subset f^{-1}(F \cup G)$$

$$F \subset F \cup G \Rightarrow f^{-1}(F) \subset f^{-1}(F \cup G)$$

$$\Rightarrow G \subset F \cup G \Rightarrow f^{-1}(G) \subset f^{-1}(F \cup G) \Rightarrow f^{-1}(F) \cup f^{-1}(G) \subset f^{-1}(F \cup G)$$

$$f^{-1}(F \setminus G) = f^{-1}(F) - f^{-1}(G)$$

$$x \in f^{-1}(F \setminus G) \Rightarrow f(x) \in (F \setminus G) \Rightarrow f(x) \subset F$$

$$f(x) \notin G \Rightarrow x \in f^{-1}(F)$$

$$x \notin f^{-1}(G) \Rightarrow x \in f^{-1}(F) \setminus f^{-1}(G)$$

$$x \in f^{-1}(F) \setminus f^{-1}(G) \Rightarrow x \in F^{-1}(F)$$

$$x \notin f^{-1}(G) \Rightarrow f(x) \in F$$

$$f(x) \notin (G) \Rightarrow f(x) \in F \setminus G \Rightarrow x \in f^{-1}$$

L_1

L_2

$$\begin{cases} x = 1 + \lambda \\ y = -2 + 4\lambda \\ z = 1 + 7\lambda \end{cases}$$

$$\begin{cases} x = 4 + \mu \\ y = 1 + 2\mu \\ z = -1 + 5\mu \end{cases}$$

$$L_1 : (x, y, z) = \lambda \overline{u} + p_1$$

$$L_2 : (x, y, z) = r\overline{v} + p_2$$

$$L : (x, y, z) = t\overline{w}$$

$$\overline{w} = \overline{u} \times \overline{v}$$

$$\begin{cases} x = 4 + \mu \\ y = 1 + 2 + 4\mu \\ z = -1 + 5\mu \end{cases}$$

$$(1, 4, 7) = \overline{u}$$

$$p_1 = (a, b, c)$$

$$(x, y, z) = \lambda(u_1, u_2, u_3) + p$$

$$\begin{cases} x = \lambda u_1 + a \\ y = \lambda u_2 + b \\ z = \lambda u_3 + c \end{cases}$$

$$L_2(1, 2, 5) = \overline{v}$$

$$\overline{w} = \overline{u} \times \overline{v} = (6, 2, -2)$$

$$L : (x, y, z) = t(6, 2, -2), t \in \mathbb{R}$$

$$\begin{cases} x = 6t \\ y = 2t \\ z = -2t \end{cases}$$

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{x}\,y^2}{x+y^3}$$

$$x = y$$

$$\frac{\sqrt[3]{x}x^2}{x+x^3} = \frac{x^{2/3}}{x+x^3}$$

$$x = y^3$$

$$\frac{\sqrt[3]{y^3}y^2}{y^3+y^3} = \frac{y^3}{2y^3} = \frac{1}{2}$$

$$x = t^6$$

$$y = -t$$

$$\frac{\sqrt[3]{t^6}t^2}{t^6-t^3}=\frac{t^4}{t^6-t^3}=\frac{t^3}{t^3}=\frac{t}{t^3-1}$$

$t \to 0$

$$\Rightarrow 0$$

$$f(x, y, z) = z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1$$

$$\mathbb{R}^4$$

$$(x, y, z)/z^2 + (\sqrt{x^2 + y^2 - 2})^2 - 1 = C$$

$$C = -1$$

$$C = 0$$

$$C = 1$$

$$x = 0$$

(YZ)

$$\sqrt{y^2} = |y|$$

$$z^2 + (|y| - 2)^2 - 1 = C$$

$$z^2 + (|y| - 2)^2 = C + 1 > 0$$

$$y = 0$$

(XZ)

$$z^2 + (x-2)^2 = C + 1$$

$$z = 0$$

$$\sqrt{x^2 + y^2 - 2)^2} = C + 1$$

$$x^2 + y^2 - 4\sqrt{x^2 + y^2 + 4} = C + 1$$

$$\cup_{\alpha\in I}A_\alpha)\cap B=\cup_{\alpha\in I}(A_\alpha\cap B)$$

$$x \in \left(\cup_{\alpha \in I} A_\alpha\right) \cap B$$

$x \in \cup_{\alpha \in I}(A_\alpha \cap B)$

$x \in \cup_{\alpha \in I} A_\alpha$

$x \in A_{\alpha 0}$

$\alpha_0 \in I$

$x \in A_{\alpha 0} \cup B$

$\cap_{\alpha\in I}(\cup_{\alpha\in I}(A_\alpha \cap B).$

$$x \in \cup_{\alpha \in I}(A_\alpha \cap B) \Rightarrow \exists \alpha_0 \in I$$

$$x \in A_{\alpha 0} \cap B$$

$x \in \cup_{\alpha \in I}$

$$F = \{A_\alpha : \alpha \in I\}$$

$\cup_{\alpha \in I} A_\alpha)^c = \cap_{\alpha \in I} A_\alpha^c$

$$\cap_{\alpha\in I}A_\alpha)^c = \cup_{\alpha\in I}A_\alpha^c$$

$$x \in (\cap_{\alpha \in I} A_\alpha)^c$$

$$x \notin A_\alpha \forall \alpha \in I$$

$\alpha \in I, x \in A_\alpha^c$

$x \in \cup_{\alpha \in I} A_\alpha^c$

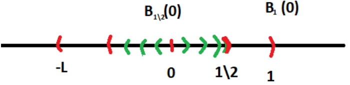
$x \in \bigcap_{\alpha \in I} A_\alpha^c$

$x \in A_\alpha^c \forall \alpha \in I$

$\notin A_\alpha \forall \alpha \in I$

$x \notin \cup_{\alpha \in I} A_\alpha$

$$\cap_{k=1}^{\infty} B_{\frac{1}{K}}(0) = \{0\}$$



B1\2(0)
B1 (0)
-L
0
1\2
1

$$\cap_{k\in\mathbb{N}}(-\frac{1}{k},\frac{1}{k})=\{0\}$$

$$\cap\left(\frac{1}{k}, \frac{1}{k}\right) \neq \{0\}$$

$\exists \epsilon > 0$

$$0 < \epsilon < \frac{1}{k} \forall k \in \mathbb{N}$$

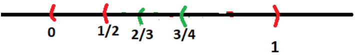
$k < \frac{1}{\epsilon} \forall k \in \mathbb{N}!$

$\mathbb{N}$

$$\{ \frac{n}{n+1} : n \in \mathbb{N} \}$$

$$F_r(A)$$

A°


0
1/2
2/3
3/4
1

$$B_r(\tfrac{2}{3})|\tfrac{2}{3}) \cap A = \emptyset$$

A

$$(B_\epsilon(1)|\{1\}) \cap A \neq \emptyset$$

$$\frac{n}{n+1} \leq 1 - \epsilon \forall n \in \mathbb{N}$$

$$n \leq (n+1)(1-\epsilon) \forall n \in \mathbb{N}$$

$$n \leq n - n\epsilon + 1 - \epsilon \forall n \in \mathbb{N}$$

$$n \leq \frac{1-\epsilon}{\epsilon} \forall n \in \mathbb{N}!$$

$$f : U \subset \mathbb{R}^n \to \mathbb{R}^m$$

$$g : V \subset \mathbb{R}^m \to \mathbb{R}^p$$

U

$\mathbb{R}^n$

V

$\mathbb{R}^m$

f

$x_0 \in U$

$g \circ f : U \subset \mathbb{R}^p$

x_0

$\epsilon > 0$

g

$$f(x_0)$$

$n > 0$

$$\|y - f(x_0)\| < n$$

$$\|g(y) - g(f(x))\| < \epsilon$$

$\delta > 0$

$$\|x - x_0\| < \delta$$

$$\|f(x) - f()x_0\| < n$$

$$\|g(f(x_0))\| < \epsilon$$

$$g \circ f$$

$$(\cos t, \sin t, 1 - \sin t)$$

$$\begin{cases} x = \cos t \\ y = \sin t \\ z = 1 - \sin t \end{cases}$$

$$\cos^2 t + \sin^2 t = 1$$

$$x^2 + y^2 = 1$$

$$z = 1 - y$$

$$0x + y + z - 1 = 0$$

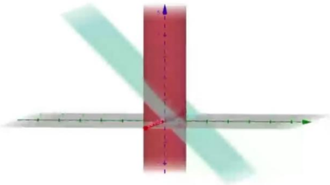
$$(0, 1, 1) = \overline{n}$$

$$(x, y, z) = t\overline{u} + \overline{v}, t \in \mathbb{R}$$

$$\gamma : \mathbb{R} \to \mathbb{R}^3$$

$$\gamma(t) = t\overline{u} + \overline{v}$$

$$t \to t\overline{u} + \overline{v}$$



$$f : \mathbb{R}^3 \to \mathbb{R}$$

$$C_1 = -1$$

$$C_2 = 0$$

$$C_3 = 1$$

$$z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1 = -1$$

$$z^2 + (\sqrt{x^2 + y^2} - 2)^2 = 0$$

$\mathbb{R}^3$

$$z^2 + (|y| - 2)^2 = 1$$

$$y = 2$$

$$y = -2$$

$$\{0, 2, 0\}, \{0, -2, 0\}$$

$$(ZX)$$

$$z^2 + (|x| - 2)^2 = 1$$

$$(\sqrt{x^2 + y^2} - 2)^2 = 0$$

$$\sqrt{x^2 + y^2} - 2 = 0$$

$$x^2 + y^2 = 4$$

$$(\sqrt{x^2 + y^2} - 2) = 1$$

$$x^2 + y^2 = 9$$

3

$$z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1$$

$$z^2 = -(\sqrt{x^2 + y^2} - 2)^2 + 1$$

$$z = \pm\sqrt{1 - (\sqrt{x^2 + y^2} - 2)^2}$$

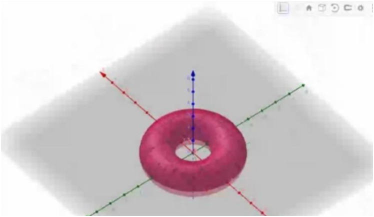
(x, y)

$$1 - (\sqrt{x^2 + y^2} - 2)^2 \geq 0$$

$$1 \geq (\sqrt{x^2 + y^2} - 2)^2$$

$$1 = \sqrt{x^2 + y^2} - 2$$

$$3^2 = x^2 + y^2$$



$$\left\{\begin{array}{r} x=-1+2\lambda+3\mu \\ y=4\lambda-\mu \\ z=2-3\lambda+2\mu \end{array}\right.$$

$$2x - 5y + z = 0 : \pi_2$$

$$\pi : (x, y, z) = t\overline{u} + s\overline{v} + P$$

$$\overline{u} \times \overline{v} = \overline{n} = \Pi \cap \Pi$$

$$< (x, y, z) - P, \overline{n} >= 0$$

$$\Pi_1 = \lambda\overline{\alpha} + \mu\overline{\beta} + \overline{\gamma} = (x, y, z)$$

$$\lambda(2,4,-3)+\mu(3,\tfrac{-1}{\beta},2)+(-1,0,2)$$

$$\overline{\alpha} \times \overline{\beta} = \overline{n_1}$$

$$\Pi_1 :< \overline{x} - \overline{\Gamma}, \overline{n_1} = 0$$