$\mathbb{R}^n$

$$A^{\mathrm{o}} \cap B^{\mathrm{o}} = (A \cap B)^{\mathrm{o}}$$

$$A^{\mathrm{o}} \cap B^{\mathrm{o}} \subseteq (A \cap B)^{\mathrm{o}}$$

$$
\begin{aligned}
x \in (A^{\mathrm{o}} \cap B^{\mathrm{o}}) &\Rightarrow (x \in A^{\mathrm{o}}) \wedge (x \in B^{\mathrm{o}}) \Rightarrow (\exists B_r(x) \subset A) \wedge (\exists B_r(x) \subset B) \Rightarrow \exists B_r(x) \subset (A \cap B) \\
&\Rightarrow x \in (A \cap B)^{\mathrm{o}} \\
&\therefore A^{\mathrm{o}} \cap B^{\mathrm{o}} \subseteq (A \cap B)^{\mathrm{o}}
\end{aligned}
$$

$$(A \cap B)^{\mathrm{o}} \subseteq A^{\mathrm{o}} \cap B^{\mathrm{o}}$$

$$
\begin{aligned}
x \in (A \cap B)^{\mathrm{o}} &\Rightarrow \exists B_r(x) \subset (A \cap B) \Rightarrow (\exists B_r(x) \subset A) \land (\exists B_r(x) \subset B) \Rightarrow (x \in A^{\mathrm{o}}) \land (x \in B^{\mathrm{o}}) \\
&\Rightarrow x \in (A \cap B)^{\mathrm{o}} \\
&\therefore (A \cap B)^{\mathrm{o}} \subseteq A^{\mathrm{o}} \cap B^{\mathrm{o}}
\end{aligned}
$$

$$\therefore A^{\mathrm{o}} \cap B^{\mathrm{o}} = (A \cap B)^{\mathrm{o}}$$

$$\overline{A} \cup \overline{B} = \overline{A \cup B}$$

$$\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$$

$$
\begin{aligned}
x \in (\overline{A} \cup \overline{B}) &\Rightarrow (x \in \overline{A}) \vee (x \in \overline{B}) \Rightarrow (\forall B_r(x), B_r(x) \cap A \neq \emptyset) \vee (\forall B_r(x), B_r(x) \cap B \neq \emptyset) \\
&\Rightarrow (\forall B_r(x), B_r(x) \cap (A \cup B) \neq \emptyset) \Rightarrow x \in (\overline{A \cup B}) \\
&\therefore \overline{A} \cup \overline{B} \subseteq \overline{A \cup B}
\end{aligned}
$$

$$\overline{A \cup B} \subseteq \overline{A} \cup \overline{B}$$

$$
\begin{aligned}
x \in (\overline{A \cup B}) &\Rightarrow \forall B_r(x), B_r(x) \cap (A \cup B) \neq \emptyset \Rightarrow (\forall B_r(x), B_r(x) \cap A \neq \emptyset) \cup (\forall B_r(x), B_r(x) \cap B \neq \emptyset) \\
&\Rightarrow (\forall B_r(x), B_r(x) \cap A \neq \emptyset) \vee (\forall B_r(x), B_r(x) \cap B \neq \emptyset) \\
&\Rightarrow (x \in \overline{A}) \vee (x \in \overline{B}) \Rightarrow x \in (\overline{A} \cup \overline{B}) \\
&\therefore \overline{A \cup B} \subseteq \overline{A} \cup \overline{B}
\end{aligned}
$$

$$\therefore \overline{A} \cup \overline{B} = \overline{A \cup B}$$

$$\overline{A \cap B} \subset \overline{A} \cap \overline{B}$$

$$A \subset \overline{A}$$

$$B \subset \overline{B}$$

$$A \cap B \subset \overline{A} \cap \overline{B}$$

$$\overline{A} \cap \overline{B}$$

$\Longrightarrow$

$A = (0, 1)$

$$B = (1, 2)$$

$$\overline{A \cap B} = \emptyset$$

$$\overline{A} = [0, 1]$$

$$\overline{B} = [1, 2]$$

$$\overline{A} \cap \overline{B} = 1$$