

1. Demuestra que R^3 es un espacio vectorial.

Prueba: Será necesario verificar los 10 axiomas del EV. Consideramos tres elementos $u=(a, b, c)$, $v = (d, e, f)$, $w=(g,h,i) \in R^3$; y los escalares α y $\beta \in R$

1. $u+v=(a, b, c)+(d, e, f)=(a+d,b+e,c+f) \in R^3$
2. $u+v=(a, b, c)+(d, e, f)=(a+d,b+e,c+f)=(d+a,e+b,f+c)=(d, e, f)+(a, b, c)$
3. $u+(v+w)=(a, b, c)+((d, e, f)+(g,h,i))=(a, b, c)+(d+g,e+h,f+i)=(a+d+g, b+e+f, c+f+i)=(a+d, b+e, c+f)+(g,h,i)=(u+v)+w$
4. Sea 0 del espacio, entonces $u+0=(a,b,c)+(0,0,0)=(a+0,b+0,c+0)=(a,b,c)$
5. Sea el inverso aditivo -u, entonces $u+(-u)=(a,b,c)+(-a,-b,-c)=(a-a, b-b, c-c)=(0,0,0)$ que es el 0 del espacio.
6. $\alpha u=\alpha(a,b,c)=(\alpha a,\alpha b,\alpha c) \in R^3$
7. $\alpha(u+v)=\alpha((a, b, c)+(d, e, f))=\alpha(a+d,b+e,c+f)=(\alpha(a+d),\alpha(b+e),\alpha(c+f))$
8. $(\alpha + \beta)u=(\alpha + \beta)(a, b, c)=(\alpha + \beta a, \alpha + \beta b, \alpha + \beta c)$
9. $\alpha(\beta v)=\alpha(\beta(a, b, c))=\alpha(\beta a, \beta b, \beta c)=(\alpha \beta a, \alpha \beta b, \alpha \beta c) = (\alpha \beta(a, b, c))=\alpha \beta(a, b, c)=\alpha \beta u$
10. $1 u = 1(a, b, c) = (1a,1b,1c)=(a,b,c)=u$

- b) Demuestra el Teorema 2. Considerando los vectores $v, w, u \in R^n$ y $\alpha \in R$ entonces:

- a. $\vec{v} \cdot \vec{0}=0$

$$\vec{v} \cdot \vec{0} = \sum \vec{v} \vec{0} = v_1 0 + v_2 0 + \dots + v_n 0 \in R$$

$$= 0(v_1 + \dots + v_n) = 0$$

- b. $\vec{v} \cdot \vec{w} = \sum \vec{v} \vec{w} = v_1 w_1 + v_2 w_2 + \dots + v_n w_n \in R$

$$= w_1 v_1 + \dots + w_n v_n = \sum \vec{w} \vec{v} = \vec{w} \cdot \vec{v}.$$

- c. $\vec{u} \cdot (\vec{v} + \vec{w}) = \sum \vec{u} \cdot (\vec{v} + \vec{w}) = \sum u(v_1 + w_1, \dots, v_n + w_n)$

$$u_1(v_1 + w_1) + \dots + u_n(v_n + w_n) = u_1 v_1 + u_1 w_1 + \dots + u_n v_n + u_n w_n =$$

$$(u_1 v_1 + \dots + u_n v_n) + (u_1 w_1 + \dots + u_n w_n) = \sum \vec{u} \vec{v} + \sum \vec{u} \vec{w} = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$