$P(0,0,0)$

L_1

L_2

$$2x - 5y + z = 0$$

$$5x - 13y - 14z = -33$$

$$(x, y, z) = (165, 66, 0) + t(-83, -33, 1)$$

$$-83x - 33y + z + 365 = 0$$

$$z^2 + (2 - \sqrt{x^2 + y^2})^2 = 1$$

$$z^2 + (2 - \sqrt{x^2 + y^2})^2 = 2$$

$r = 2$

$r = 1$

$$r = 0$$

$$r = 0{,}5$$

$$r = 1{,}5$$

$r = 3$

$$x^2 + y^2 = 0$$

$$z = 1 - y$$

$$\alpha(t) = (cos\ t, sen\ t, 1 - sen\ t)$$

π

$P(2, -1, 4)$

$$Q(1, 2, 3)$$

$R(-2, 0, 5)$

$$\overline{QP} = P(2, -1, 4) - Q(1, 2, 3) = (1, -3, 1)$$

$$\overline{QR} = R(-2,0,5) - P(1,2,3) = (-3,-2,2)$$

$$\bar{u} = (1, -3, 1)$$

$$\bar{v} = (-3, -2, 2)$$

$$x_0, y_0, z_0 = (1, 2, 3)$$

∴

$$(x, y, z) = (1, 2, 3) + \lambda(1, -3, 1) + \mu(-3, -2, 2)$$

$$(x, y, z) = (x_0 + u_1\lambda + v_1\mu, y_0 + u_2\lambda + v_2\mu, z_0 + u_3\lambda + v_3\mu)$$

$$\begin{cases} x = x_0 + u_1\lambda + v_1\mu \\ y = y_0 + u_2\lambda + v_2\mu \\ z = z_0 + u_3\lambda + v_3\mu \end{cases}$$

$\Longrightarrow$

$$(x, y, z) = (1 + 1\lambda) + (-3)\mu), (2 - 3\lambda + (-2)\mu), (3 + 1\lambda + 2\mu) = (1 + \lambda - 3\mu), (2 - 3\lambda - 2\mu), (3 + \lambda + 2\mu)$$

∴

$$\begin{cases} x = 1 + \lambda - 3\mu \\ y = 2 - 3\lambda - 2\mu \\ z = 3 + \lambda + 2\mu \end{cases}$$

$$\overline{QP} \times \overline{QR}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ -3 & -2 & 2 \end{vmatrix}$$

$$[-3(2)-(-2)(1)]\hat{i}-[1(2)-(-3)(1)]\hat{j}+[1(-2)-(-3)(-3)]\hat{k}$$

$$(-6+2)\hat{i}-(2+3)\hat{j}+(-2-9)\hat{k}$$

$$-4\hat{i} - 5\hat{j} - 11\hat{k}$$

$\hat{n}$

$T(x, y, z)$

$$\overline{QT} = T(x, y, z) - P(1, 2, 3) = x - 1, y - 2, z - 3$$

$$\overline{QT} \cdot \overline{n} = 0$$

$$(x-1, y-2, z-3) \cdot (-4, -5, -11) = 0$$

$$(x-1)-4+(y-2)-5+(z-3)-11=0$$

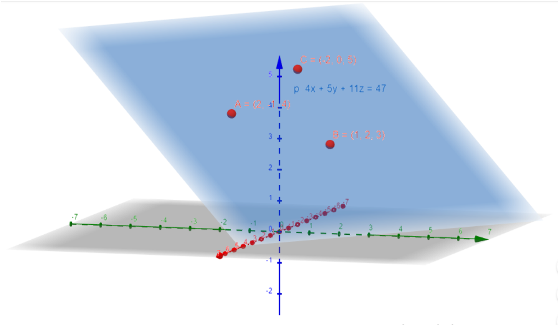
$$-4x + 4 - 5y + 10 - 11z + 33 = 0$$

$$-4x - 5y - 11z + 47 = 0$$

$$-4x - 5y - 11z = -47$$

∴

$$4x + 5y + 11z = 47$$



C = (-2, 0, 5)
p: 4x + 5y + 11z = 47
A = (2, 4, 4)
B = (1, 2, 3)

$$\begin{cases} x = 1 + \lambda \\ y = -2 + 4\lambda \\ z = 1 + 7\lambda \end{cases}$$

$$\begin{cases} x = 4 + \mu \\ y = 1 + 2\mu \\ z = -1 + 5\mu \end{cases}$$

$$\overline{b_1}$$

$\overline{b_2}$.

$$\overline{b_1} = (1, 4, 7)$$

$$\overline{b_2} = (1, 2, 5)$$

$$P_0 = (0, 0, 0)$$

$$a \perp b$$

$\Leftrightarrow$

$$\overline{a} \cdot \overline{b} = 0$$

$$\overline{n} = (a, b, c)$$

$$\overline{n} \cdot \overline{b_1} = 0$$

$$(a, b, c) \cdot (1, 4, 7) = 0$$

$$a + 4b + 7c = 0$$

$$\overline{n} \cdot \overline{b_2} = 0$$

$$(a, b, c) \cdot (1, 2, 5) = 0$$

$$a + 2b + 5c = 0$$

a

$$2b + 2c = 0$$

$$2b = -2c$$

$$b = -c$$

$$a + (-4c) + 7c = 0$$

$$a + 3c = 0$$

$$a = -3c$$

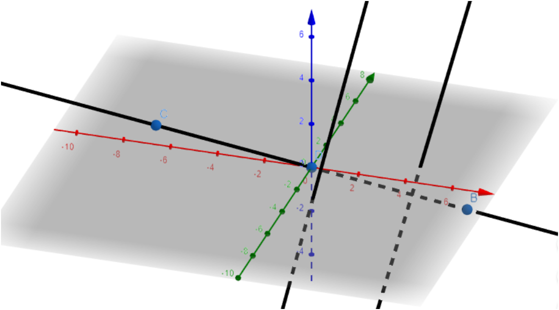
$$\overline{n} = (-3c, -c, c) = c(-3, -1, 1)$$

$$L = \{(0, 0, 0) + \lambda(-3, -1, 1)\}$$

$$(x, y, z) = (0, 0, 0) + (-3\lambda, -\lambda, \lambda)$$

∴

$$\begin{cases} x = -3\lambda \\ y = -\lambda \\ z = \lambda \end{cases}$$


C
B
-10
-8
-6
-4
-2
0
2
4
6
8

$$P(1, -1, 4)$$

$$\left\{\begin{array}{r} x = 2 + 3\lambda \\ y = -1 + \lambda \\ z = 7 + 4\lambda \end{array}\right.$$

$$\frac{x-6}{5} = \frac{y-4}{2} = \frac{z-7}{6}$$

$$\overline{u} = (3, 1, 4)$$

$$\overline{v} = (5, 2, 6)$$

$\overline{n}$

$\overline{v}$

$$\overline{n} \cdot \overline{v} = (a, b, c) \cdot (5, 2, 6) = 5a + 2b + 6c = 0$$

$$\overline{n} \cdot \overline{u} = (a, b, c) \cdot (3, 1, 4) = 3a + b + 4c = 0.$$

$$2a + b + 2c = 0$$

$$b = -2c - 2a.$$

b

$$5a + 2(-2c - 2a) + 6c = 0$$

$$a + 2c = 0$$

$$a = -2c$$

$$b = -2c - 2(-2c)$$

$$b = 2c$$

$$\overline{n} = (-2c, 2c, c) = c(-2, 2, 1)$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

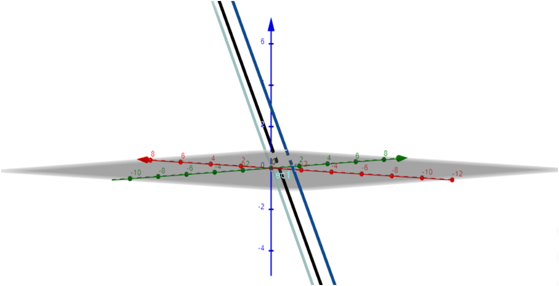
$$-2(x-1)+2(y-(-1)+1(z-4)=0$$

$$-2x + 2 + 2y + 2 + z - 4 = 0$$

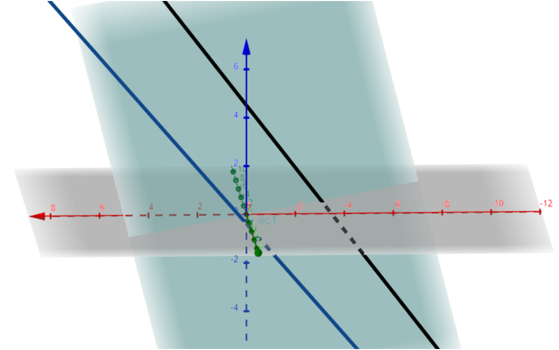
$$-2x + 2y + z = -2 - 2 + 4$$

$$-2x + 2y + z = 0$$

∴



6
2
-2
-4
8
6
4
2
0
-2
-4
-6
-8
-10
-12
2
4
6
8
-10
-8
-6
-4
-2



6
4
2
-2
-4
8
6
4
2
-2
-4
-6
-8
-10
-12
Vec1

$P(2, 6, -1)$

$$\begin{cases} x = -1 + 2\lambda + 3\mu \\ y = 4\lambda - \mu \\ z = 2 - 3\lambda + 2\mu \end{cases} \qquad \text{y} \quad \text{2x-5y+z=0}$$

$$(x, y, z) = (-1, 0, 2) + \lambda(2, 4, -3) + \mu(3, -1, 2)$$

$$(x+1, y, z-2) = \lambda(2, 4, -3) + \mu(3, -1, 2)$$

$\mathbb{R}^3$

$$\begin{vmatrix} x+1 & y & z-2 \\ 2 & 4 & -3 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$$\therefore (x+1)(8-3) - y(4+9) + (z-2)(-2-12) = 0$$

$$\Rightarrow (x+1)(5) - y(13) + (z-2)(-14) = 0$$

$$\Rightarrow 5x + 5 - 13y - 14z + 28 = 0$$

$$\Rightarrow 5x - 13y - 14z = -33$$

$$\begin{cases} 2x - 5y + z = 0 \\ 5x - 13y - 14z = -33 \end{cases}$$

$$\left(\begin{array}{ccc|c} 2 & -5 & 1 & 0 \\ 5 & -13 & -14 & -33 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -5 & 1 & 0 \\ 0 & -1 & -33 & -66 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 166 & 330 \\ 0 & -1 & -33 & -66 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 83 & 165 \\ 0 & -1 & -33 & -66 \end{array}\right)$$

$$\therefore \begin{cases} x + 83z = 165 \\ -y - 33z = -66 \end{cases}$$

$$\begin{cases} x = 165 - 83t \\ y = -33t + 66 \quad , \text{ t} \in \mathbb{R} \\ z = t \end{cases}$$

$$\vec{v} = (-83, -33, 1)$$

$P(2, -6, 1)$

$$\pi : -83x - 33y + z + D = 0$$

P

D

$$-83(2) - 33(6) - 1 + D = 0 \Rightarrow -365 = -D \Rightarrow D = 365$$

$$\therefore -83x - 33y + z + 365 = 0$$

$$\cap_{k=1}^{\infty} B_{1/k}(0) = \{0\}$$

$$\cap_{k=1}^{\infty} B_{1/k}(0) = \lim_{n\to\infty} \cap_{k=1}^{n} B_{1/k}(0) = \lim_{n\to\infty} B_{1/n}(0) = B_0(0) = \{0\}$$

$$B_1(0) \supset B_{1/k}(0) \supset B_{1/(k+1)}(0) \supset B_{1/n}(0) \supset B_0(0) = \{0\}$$

$$\{y \in X : d(x, y) < r\}$$

$$\{z \in X : z \in \overline{B} - B^\circ\}$$

$$\{z \in X : z \in \overline{B} \ \ \& \ \ z \notin B^\circ\}$$

$$\{z \in X : d(x,z) \leq r\} \cap \{z \in X : d(x,z) \geq r\}$$

$$\{z \in X : d(x, z) = r\}$$

$$\{a \in X : d(a, y) = r\}$$

$$\{y \in X : d(x, y) = r\}$$

$$\{y \in X : y \in \overline{S} - S^\circ\}$$

$$\{y \in X : y \in \overline{S} \ \ \& \ \ y \notin S^\circ\}$$

O

$\emptyset$

$$\{y \in X : y \in \overline{S}\}$$

$$\mathbb{R}^n$$

$p \in R^n$

R^n

p

$x \in R^n$

E

$B(x, r) \subset \mathbb{R}^n$

p

∞

$$A = (0, \infty) \times (0, \infty)$$

A

$$\mathbb{Q} \times \mathbb{Q}$$

$\mathbb{R}^2$

$$Fr(\mathbb{Q} \times \mathbb{Q}) = \mathbb{R}^2$$

$$Fr(\mathbb{Q} \times \mathbb{Q}) \subseteq \mathbb{R}^2$$

$$Fr(\mathbb{Q} \times \mathbb{Q}) \supseteq \mathbb{R}^2$$

$r > 0$

$(x_0, y_0) \in \mathbb{R}^2$

$B_r(x_0, y_0)$

x

$r/2$

$\overline{p} \in \mathbb{Q}$

$\overline{p} \in (x_0, x_0 + r/2)$

y

$\overline{q} \in \mathbb{Q}$

$\overline{q} \in (y_0, y_0 + r/2)$

$$x_0 < \bar{p} < x_0 + r/2$$

$\Longrightarrow$

$$|\overline{p} - x_0| < |x_0 + r/2 - x_0| = r/2$$

$$y_0 < \overline{q} < y_0 + r/2$$

$$|\overline{q} - y_0| < |y_0 + r/2 - y_0| = r/2$$

$$||(\overline{p}, \overline{q}) - (x_0, y_0)|| \leq |\overline{p} - x_0| + |\overline{q} - x_0| < r$$

$$(\overline{p}, \overline{q}) \in B_r(x_0, y_0)$$

$(\overline{p}, \overline{q}) \in \mathbb{Q} \times \mathbb{Q}$

$$B_r(x_0, y_0) \cap (\mathbb{Q} \times \mathbb{Q}) \neq \emptyset$$

$\exists \overline{p}' \in \mathbb{Q}^c$

$$\overline{p}' \in (x_0, x_0 + r/2)$$

$\exists \bar{q}' \in \mathbb{Q}^c$

$\overline{q}' \in (y_0, y_0 + r/2)$

$$||(\overline{p}', \overline{q}') - (x_0, y_0)|| \leq |\overline{p}' - x_0| + |\overline{q}' - x_0| < r$$

$$(\overline{p}', \overline{q}') \in B_r(x_0, y_0)$$

$$(\overline{p}', \overline{q}') \in \mathbb{Q}^c \times \mathbb{Q}^c$$

$$B_r(x_0, y_0) \cap (\mathbb{Q}^c \times \mathbb{Q}^c) \neq \emptyset$$

$$(x_0, y_0) \in Fr(\mathbb{Q} \times \mathbb{Q})$$

$$\mathbb{R}^2 \subseteq Fr(\mathbb{Q} \times \mathbb{Q})$$

∴

$(\mathbb{R}^2$

$\mathbb{Q} \times \mathbb{Q})$

$$\mathbb{Q} \subset \mathbb{R}$$

$$(\mathbb{Q} \times \mathbb{Q})^c = (\mathbb{Q} \times \mathbb{R}) \bigcup (\mathbb{R} \times \mathbb{Q})$$

$$Fr(A) = Fr(A^c)$$

$$Fr((\mathbb{Q} \times \mathbb{Q})^c) = Fr(\mathbb{Q} \times \mathbb{Q}) = \mathbb{R}^2$$

$$\overline{A} = A^{\circ} \cup Fr(A)$$

$$x \in \overline{A}$$

$\longrightarrow$

$$x \in A^{\circ} \cup Fr(A)$$

$x \in A^\circ$

$x \in Fr(A)$

$$x \in A^\circ \subset \overline{A}$$

$$x \notin A^\circ \subset \overline{A}$$

$$x \notin A^{\circ}$$

$$x \in \overline{A} - A^{\circ} = Fr(A)$$

$$x \in Fr(A) \cup A^{\circ}$$

$$\therefore \overline{A} \subseteq A^\circ \cup Fr(A)$$

A°

$$Fr(A)$$

$$x \in A^{\circ} \subset A \subset \overline{A}$$

$$\{y \in X : y \in A \cap (A^{\circ})^{C}\}$$

$$x \in A \subset \overline{A}$$

$$\therefore \overline{A} \supseteq A^\circ \cup Fr(A)$$

$$\therefore \overline{A} = A^\circ \cup Fr(A)$$

$$A = \{\, \frac{n}{n+1} \,:\, n \in \mathbb{N} \}$$

n

$$\frac{n}{n+1}$$

r

$$r_k \in \mathbb{R} \leq r$$

$$\frac{n}{n+1} + r_k \neq \frac{m}{m+1}, m \in \mathbb{N}$$

$$n \to \infty$$

ϵ

$$\lim_{n\to\infty} \frac{n}{n+1} = 1$$

$\epsilon > 0$

c

$$\left|\frac{n}{n+1}-1\right|<\epsilon$$

$$n > c$$

$$A^d = \{x \in \mathbb{R} : \forall r (B_r(x) - \{x\}) \cap A \neq \emptyset\} = \{1\}$$

$$x = \frac{n}{n+1} \in A$$

$$A \cap B_r(x) \neq \emptyset$$

$$B_\epsilon(1) \supset \{\tfrac{n}{n+1} : n > c\}$$

$B_\epsilon(1) \cap A \neq \emptyset$

$$\bar{A} = A \cup \{1\}$$

$$Fr(A) = \overline{A} \cap \overline{A^c}$$

$$\overline{A^c}$$

$$\overline{A^c} = \{x \in \mathbb{R} : \forall r B_r(x) \cap A^c \neq \emptyset\}$$

$$A^c$$

$$\overline{A^c}$$

$B_r(y)$

$$y \in A^c$$

$$B_r(y) \cap A^c \neq \emptyset$$

$a \in A$

$$B_r(a) \cap A^c \neq \emptyset$$

$$\overline{A^c} = A \cup A^c = \mathbb{R}$$

$$Fr(A) = \overline{A} \cap \overline{A^c} = \overline{A} \cap \mathbb{R} = \overline{A}$$

$$Fr(A) = \overline{A} - A^{\circ}$$

$$Fr(A) = \overline{A}$$

$$\overline{A} = \overline{A} - A^\circ$$

$$A^\circ = \emptyset$$

$$\overline{A} = A \cup A^a$$

$$\overline{A} \subseteq A \cup A^a$$

$x \in A^a$

$$x \in A \cup A^a$$

$$x \notin A^a$$

$$x \notin A^a \quad \& \quad x \in \overline{A}$$

$$(\forall r : B_r^\circ(x) \cap A = \emptyset) \quad \& \quad (\forall r : B_r(x) \cap A \neq \emptyset)$$

$$\{x\} \cap A \neq \emptyset$$

$$x \in A$$

∴

$$A \cup A^a \subseteq \overline{A}$$

$$A \subset \overline{A}$$

$$A^a \subset \overline{A}$$

∴

$$f : R^n \to R^m$$

R^m

R^n

$V \subset R^m$

$x \in f^{-1}(V)$

$$f(x) \in V$$

V

$\epsilon_x > 0$

$$B_{\epsilon_x}(f(x)) \subset V$$

f

$\delta_x > 0$

$$B_{\delta_x}(x) \cap R^n \subset f^{-1}(B_{\epsilon_x}(f(x))$$

$(\bigcup_{x\in f^{-1}(V)} B_{\delta_x}(x)) \cap R^n$

$$\bigcup_{x\in f^{-1}(V)}(B_{\delta_x}(x)\cap R^n)\subset\bigcup_{x\in f^{-1}(V)}f^{-1}(B_{\epsilon_x}(f(x)))\subset f^{-1}(V)\subset(\bigcup_{x\in f^{-1}(V)}B_{\delta_x}(x))\cap R^n$$

$$f^{-1}(V)$$

$$U = \bigcup_{x \in f^{-1}(V)} B_{\delta_x}(x)$$

U

V^c

$$f^{-1}(V^c) = f^{-1}(R^m - V) = R^n \cap (R^n - f^{-1}(V)) = R^n - f^{-1}(V)$$

$x \in R^n$

$$x \in f^{-1}(V^c) \Rightarrow x \in f^{-1}(R^m - V) \Rightarrow f(x) \in R^m - V \Rightarrow f(x) \notin V \Rightarrow x \notin f^{-1}(V) \Rightarrow x \in R^n - f^{-1}(V)$$

$$f^{-1}(V^c)$$

$$f^{-1}(V^c) = F \cap R^n$$

$F \subset R^n$

$$f^{-1}(V) = R^n - (R^n - f^{-1}(V)) = R^n - f^{-1}(V^c) = R^n - (R^n \cap F) = R^n \cap (R^n - F)$$

F

F^c

$$f^{-1}(V) = R^n \cap F^c$$

$f : \mathbb{R}^n \to \mathbb{R}^m$

$y \in \mathbb{R}^m$

$$\{x \in \mathbb{R}^m : f(x) = y\}$$

$$\{x \in \mathbb{R}^m : f(x) = y\} = f^{-1}(\mathbb{R}^m)$$

$\mathbb{R}^m$

$$g : \mathbb{R}^n \to \mathbb{R}^m$$

$$\{x \in \mathbb{R}^n : f(x) = g(x)\}$$

$f : \mathbb{R}^3 \to \mathbb{R}$

$$f(x, y, z) = z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1.$$

$$z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1 = c$$

$$z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1 = -1$$

$$z^2 + (2 - \sqrt{x^2 + y^2})^2 = 0$$

$$z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1 = 0$$

1

$$z^2 + (\sqrt{x^2 + y^2} - 2)^2 - 1 = 1$$

$$\sqrt{2}$$

$$r^2 + (2 - \sqrt{x^2 + y^2})^2 = c + 1$$

$$z^2 + (2 - \sqrt{x^2 + r^2})^2 = c + 1$$

$$z^2 + (2 - \sqrt{r^2 + y^2})^2 = c + 1$$

$$\alpha(t) = (\cos\ t, \operatorname{sen}\ t, 1 - \operatorname{sen}\ t).$$

$$x = \cos\ t$$

$$y = \text{sen } t$$

$$z = 1 - \operatorname{sen}\ t$$

$$\operatorname{sen}^2 t + \cos^2 t = 1$$

$$\alpha(t)$$

$$\pi := z = 1 - y$$

$$\forall \vec{x} \in \alpha(t)$$

$$\vec{x} \in \pi$$

$$y + z - 1 = 0$$

Z

$$x = \lambda$$

$$z = \mu$$

$$y = 1 - \mu$$

$$\begin{cases} x = \lambda \\ y = 1 - \mu \qquad \lambda, \mu \in \mathbb{R} \\ z = \mu \end{cases}$$

$$P = (0, 1, 0)$$

$$\vec{v} = (1, 0, 0)$$

$$\vec{u} = (0, -1, 1)$$

$$(x, y, z) = (0, 1, 0) + \lambda(1, 0, 0) + \mu(0, -1, 1)$$

$$f(x,y) = c\sqrt{1 - \frac{x2}{a^2} + \frac{y^2}{b^2}}$$

$$\frac{x2}{a^2} - \frac{y^2}{b^2} \leq 1$$

R^2

$\{(x, y) \in R^2$

$$\frac{x2}{a^2} - \frac{y^2}{b^2} \leq 1\}$$

$$f(x, y) = z$$

$$\frac{z^2}{c^2} + \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

xy

yz

$$z = 0$$

$$x = 0$$

$$y = 0$$

xz

$$a = b = c = 1$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{\sqrt{x^2+y^2}}$$

$\forall \epsilon > 0$

$\exists \delta > 0$

$$0 < \sqrt{x^2 + y^2} < \delta$$

$$\left|\frac{x^2}{\sqrt{x^2+y^2}} - 0\right| < \epsilon$$

$$0 < x^2 \leq x^2 + y^2$$

$$x^2 \leq x^2 + y^2 \Leftrightarrow \frac{1}{x^2+y^2} \leq \frac{1}{x^2} \Leftrightarrow \frac{1}{\sqrt{x^2+y^2}} \leq \frac{1}{\sqrt{x^2}} \Leftrightarrow \frac{x^2}{\sqrt{x^2+y^2}} \leq \frac{x^2}{\sqrt{x^2}} = \frac{|x^2|}{|x|} = \frac{|x|^2}{|x|} = |x|$$

$$|x| < \sqrt{x^2 + y^2} < \delta$$

$$\delta = \epsilon$$

$$|x| < \epsilon$$

$$\left|\frac{x^2}{\sqrt{x^2+y^2}} - 0\right| = \left|\frac{x^2}{\sqrt{x^2+y^2}}\right| = \frac{x^2}{\sqrt{x^2+y^2}} \leq \frac{x^2}{\sqrt{x^2}} = |x| < \epsilon$$

$$(\epsilon - \delta)$$

$$(0,0)$$

$\delta > 0$

$$||(x,y) - 0|| < \delta \implies ||f(x,y) - f(0,0)|| < \epsilon$$

$$f(x,y) = \begin{cases} \frac{x^2-y^2}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

$$\left|\frac{x^2-y^2}{\sqrt{x^2+y^2}}-0\right| = \left|\frac{x^2-y^2}{\sqrt{x^2+y^2}}\right| = \frac{\left|x^2-y^2\right|}{\sqrt{x^2+y^2}} = \frac{\left|x^2+(-y^2)\right|}{\sqrt{x^2+y^2}} \leq \frac{|x^2|+|(-y^2)|}{\sqrt{x^2+y^2}} = \frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2}$$

$$||(x,y)-0|| = \sqrt{x^2+y^2} < \delta \implies ||f(x,y)-f(0,0)|| = \left|\frac{x^2-y^2}{\sqrt{x^2+y^2}} - 0\right| < \epsilon$$

$$f(x,y)=\begin{cases}\frac{x^4-y^4}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

$$\left|\frac{x^4-y^4}{x^2+y^2}-0\right| = \left|\frac{x^4-y^4+2x^2y^2}{x^2+y^2}\right| = \left|\frac{x^4+(-y^4)+2x^2y^2}{x^2+y^2}\right| \leq \frac{|x^4+2x^2y^2|+|-y^4|}{x^2+y^2}$$

$$= \frac{|x^4 + 2x^2y^2|+y^4}{x^2+y^2} \leq \frac{|x^4| + |2x^2y^2| + y^4}{x^2+y^2} = \frac{x^4 + 2x^2y^2 + y^4}{x^2+y^2} = \frac{(x^2+y^2)^2}{x^2+y^2} = x^2 + y^2$$

$$= \sqrt{x^2 + y^2}\sqrt{x^2 + y^2}$$

$$\sqrt{x^2 + y^2} < \delta$$

$$\sqrt{x^2+y^2}\sqrt{x^2+y^2} < \delta\sqrt{x^2+y^2} < \delta^2$$

$$x^2 + y^2 < \delta^2$$

$$\delta = \sqrt{\epsilon}$$

$$||f(x,y) - f(0,0)|| = \left| \frac{x^4 - y^4}{x^2 + y^2} - 0 \right| < x^2 + y^2 < \delta^2 = \epsilon$$