Chapter 6. Force and motion Ⅱ.

- Drag Force $\vec{D}$: When there is a relative velocity between a fluid and a body, the body experiences a drag force. 阻力
  - Opposes the relative motion and points in the direction in which the fluid flows relative to the body.

$$D = \frac{1}{2} C\rho A v^2$$

  - C: drag coefficient; ρ: air density; A: **effective cross-sectional area** (perpendicular to the velocity); v: relative speed

The drag coefficient C: Typical values range from 0.4 to 1.0; and we use it as constant here.

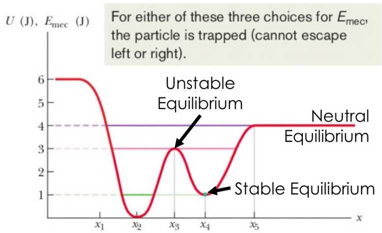
平衡态：

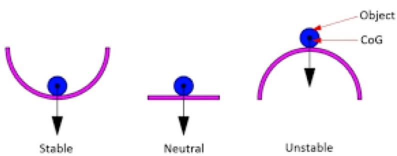
$D = G$

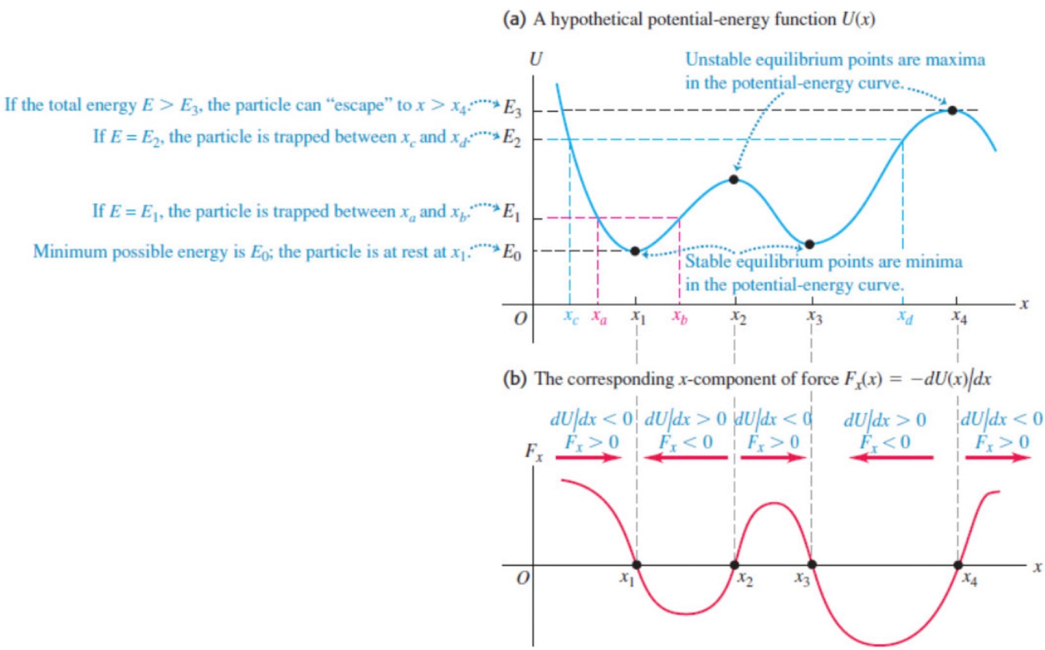
$v = \sqrt{\frac{2mg}{C\rho A}}$

Chapter 7. kinetic energy and work.

Chapter 8. potential energy and conservation of energy.

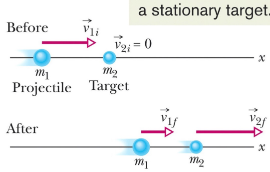






Chapter 9. Center of mass. Linear momentum.

Here is the generic (普通的) setup for an elastic collision with a stationary target.



$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$v_{2f} - v_{1f} = v_{1i} - v_{2i}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

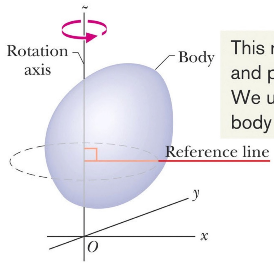
Here is the generic setup for an elastic collision with a moving target.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$

Chapter 10. Rotation

1、角加速度



This reference line is part of the body and perpendicular to the rotation axis. We use it to measure the rotation of the body relative to a fixed direction.

Pure rotation:

① Fixed axis.

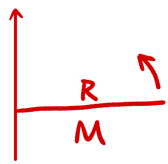
② same angle.

ω、α 根据右手定则定向，但 θ 不能视作矢量

区别 $a_t = \alpha r$，$a_r = \omega^2 r$. $|\vec{a}| = \sqrt{a_t^2 + a_r^2}$.

2. 转动惯量.

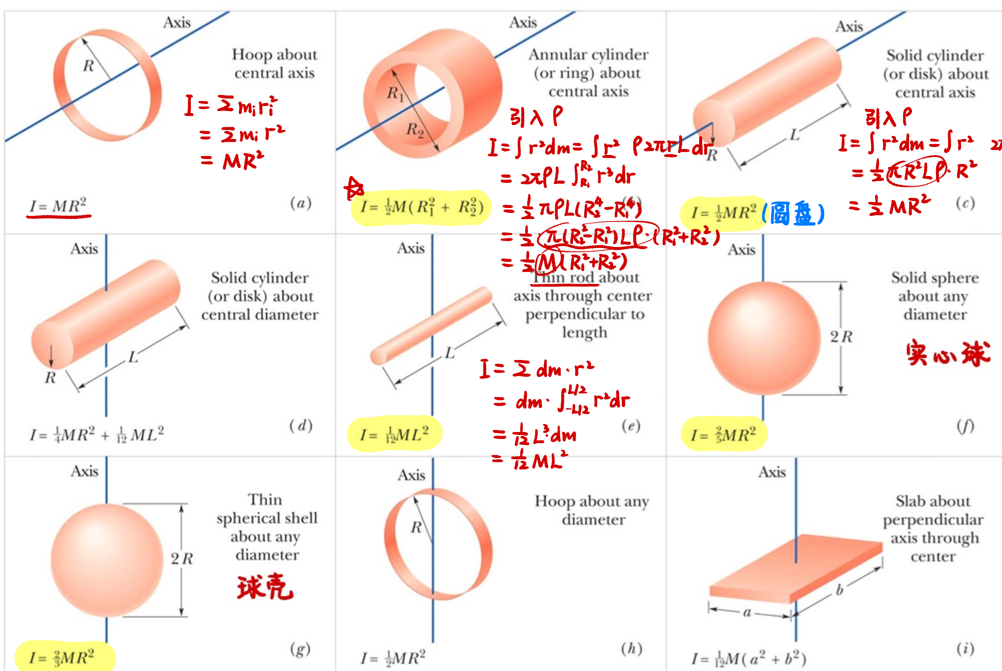
$$I = \int r^2 dm$$



$I = \frac{1}{3}MR^2$

$$I = \int r^2 dm = \int_0^R r^2 d\rho r = \rho \cdot \frac{1}{3}R^3 = \frac{1}{3}MR^2.$$

Table 10-2 Some Rotational Inertias



(a) $I = \sum m_i r_i^2 = \sum m_i r^2 = MR^2$

(b) 引入 ρ

$$I = \int r^2 dm = \int r^2 \rho 2\pi r L dr = 2\pi\rho L \int_{R_1}^{R_2} r^3 dr = \frac{1}{2}\pi\rho L(R_2^4 - R_1^4) = \frac{1}{2}\pi(R_2^2 - R_1^2)L\rho \cdot (R_1^2 + R_2^2) = \frac{1}{2}M(R_1^2 + R_2^2)$$

(c) 引入 ρ (圆盘)

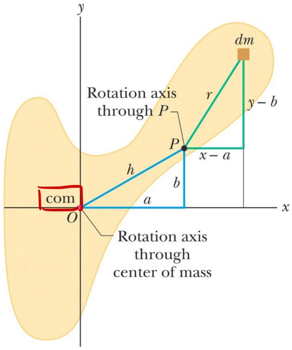
$$I = \int r^2 dm = \int r^2 2\pi r L\rho dr = \frac{1}{2}\pi R^2 L\rho \cdot R^2 = \frac{1}{2}MR^2$$

(e) $$I = \sum dm \cdot r^2 = dm \cdot \int_{-L/2}^{L/2} r^2 dr = \frac{1}{12}L^3 dm = \frac{1}{12}ML^2$$

(f) 实心球

(g) 球壳

平行轴定理：



$$I = I_{com} + Mh^2$$

3. Torque 力矩

$\tau = \vec{r} \times \vec{F}$.

牛顿第二定律.

$$\tau = F_t r = m\alpha r^2 = mr^2\alpha = I\alpha.$$

Problem

设 a 为转盘加速度（切向）= 物体加速度

$T = m(g-a)$

$a = \alpha r$

对转盘：$\tau = I\alpha = \frac{1}{2}MR^2\alpha = TR = \underline{m(g-a)R}$

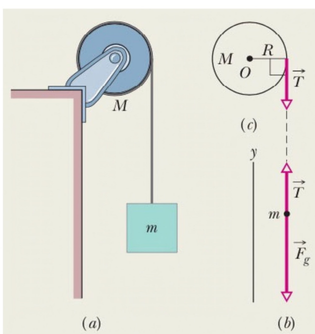
$\frac{1}{2} \times 2.5 \times 0.2 \times \alpha = 1.2 \times (9.8 - 0.2\alpha)$

$25\alpha = 12(98 - 2\alpha)$

$\alpha = \frac{12 \times 98}{49} = 24 \text{ rad/s}^2$

$a = \alpha r = 4.8 \text{ m/s}^2$

$T = m(g-a) = 6 \text{ N}$



√ A uniform disk, with mass $M = 2.5$ kg and radius $R = 20$ cm, mounted on a fixed horizontal axle. A block with mass $m = 1.2$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the **acceleration** of the falling block, the **angular acceleration** of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.

功 - 能关系. $W = \frac{1}{2}I\omega^2$, $W = F(x_f - x_i) = \tau(\theta_f - \theta_i)$.

constant

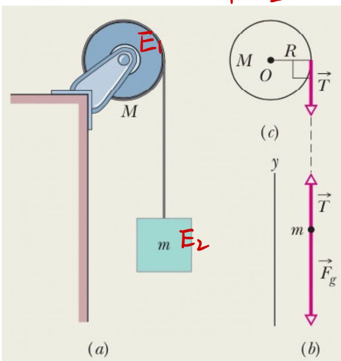
$P = \tau\omega$

Problem

$E_1 = \frac{\frac{1}{4}M}{\frac{1}{4}M + \frac{1}{2}m} mgS$

$E_2 = \frac{m}{\frac{1}{4}M + \frac{1}{2}m} mgS.$

$\underbrace{\frac{1}{2} \times \frac{1}{2} M R^2 \cdot \frac{v^2}{R^2}}_{E_1} + \underbrace{\frac{1}{2} m v^2}_{E_2} = mgS.$



A uniform disk, with mass ***M***, mounted on a fixed horizontal axle. A block with mass ***m*** hangs from a massless cord that is wrapped around the rim of the disk. Find the kinetic energy of M and m, respectively, after traveling for a distance **S**.

不是 constant force.

Solution:

$\begin{cases} \boxed{W = \frac{1}{2} I \omega^2} + \frac{1}{2} m v^2 = mgS. \\ \int a = \int \alpha r \Rightarrow v = \omega r. \end{cases}$

- Mechanical Energy Changed?

$$\Delta E_{mec} = 0$$

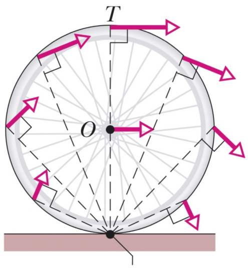
$$\Delta K + \Delta U = 0$$

$$(K_m - 0) + (K_M - 0) + (-mgS) = 0$$

$$mgS = K_m + K_M$$

Chapter 11. Rolling, torque and angular momentum.

Rolling = Translation + Rotation.



Rotation axis at P

$v_{top} = 2\omega R$

$v_{com} = \omega R$

$v_P = 0$

所有点关于P的角速度大小相等.

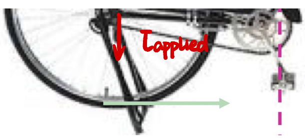
$\vec{v} = \vec{v}_{com} + \vec{v}_t$

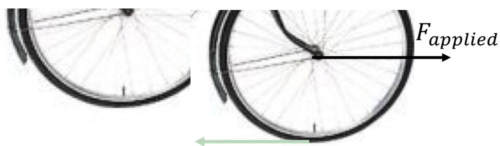
1、牛顿第二定律:

For translation $\rightarrow$: $a_{com} = \alpha R$

For rotation $\circlearrowleft$: $\tau_{net} = I_{com}\alpha = I_{com}\frac{a}{R}$

2、破烂自行车:





$$F_{net} = ma_{com}$$
$$f_s = F_{net} = ma_{com}$$
$$\tau_{net} = \tau_{applied} - f_sR = I\alpha = I\frac{a_{com}}{R}$$
$$a_{com} = \frac{\tau_{applied}}{mR}\frac{1}{(1+\frac{I}{mR^2})}$$

$$\tau_{applied} = I\frac{a_{com}}{R} + ma_{com}R$$
$$a_{com} = \frac{\tau_{applied}}{I/R + mR}$$

$$F_{net} = ma_{com}$$
$$F_{applied} - f_s = F_{net} = ma_{com}$$
$$\tau_{net} = f_sR = I\alpha = I\frac{a_{com}}{R}$$
$$a_{com} = \frac{F_{applied}}{m}\frac{1}{(1+\frac{I}{mR^2})}$$

$$F_1 = F_2 = F$$

$$F_{net} = ma_{com}$$
$$f_{s,back} - F = F_{net} = ma_{com} \quad (1)$$
$$\tau_{net} = \tau_{applied} - f_{s,back}R = I\alpha = I\frac{a_{com}}{R} \quad (2)$$

$$F_{net} = ma_{com}$$
$$F - f_{s,front} = F_{net} = ma_{com} \quad (3)$$
$$\tau_{net} = f_{s,front}R = I\alpha = I\frac{a_{com}}{R} \quad (4)$$

$$f_{s,back} - f_{s,front} = 2ma_{com} \quad (5)$$

$$a_{com} = \frac{\tau_{applied}}{2mR}\frac{1}{(1+\frac{I}{mR^2})} \quad (6)$$

$$\Delta K = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(I_{com}\omega^2 + Mh^2\omega^2\right) = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2$$

$$= \frac{1}{2} I_{com}(2\alpha \cdot \Delta\theta) + \frac{1}{2} M (2 a_{com} \Delta S)$$

$$= M a_{com} \Delta S \left(1 + \frac{I}{mR^2}\right)$$

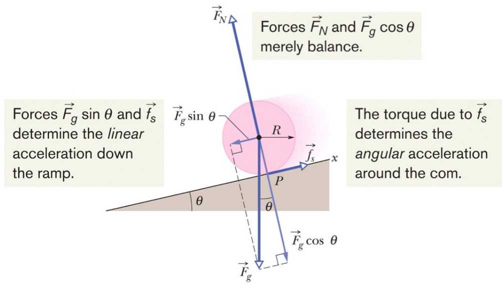
$$= \frac{\tau_{applied} \cdot \Delta S}{R} = \tau_{applied} \cdot \Delta\theta .$$

自行车动能.

题型1:

1. Smooth Rolling (No sliding)

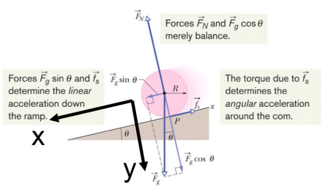
- Translational motion determined by the net force
- Rotational motion determined by the net torque, created by the static frictional force f_s



- Along x-axis, net force creates translation

$$F_g sin\theta - f_s = M a_{com,x}$$

The f_s creates the rotation:



$$\tau_{net} = R f_s = I_{com}\alpha = I_{com}\frac{a_{com,x}}{R}$$

$$f_s = I_{com}\frac{a_{com,x}}{R^2}$$

$$a_{com,x} = \frac{g sin\theta}{1 + I_{com}/MR^2}$$

- Apply to find the linear acceleration of any body rolling down smoothly along an incline of angle θ with the horizontal

2. $f_s \le \mu_s F_N$, if exceed, not smooth rolling down

$$mgh = \Delta K = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} m v_{com}^2$$

$$\Rightarrow \quad h = \frac{(\omega_0 R)^2}{2g}\left(1 + \frac{I_{com}}{mR^2}\right)$$

if the ramp is frictionless.

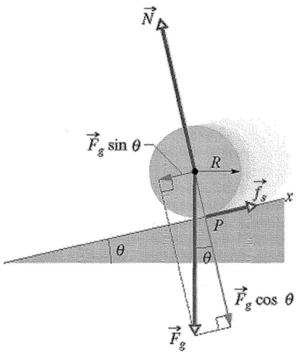
$$h' = \frac{(\omega_0 R)^2}{2g} .$$

Problem

(a) $Mgh = \frac{1}{2} \underbrace{I_{com} \omega^2}_{\text{转动}} + \underbrace{\frac{1}{2} M v^2}_{\text{平动}}$

$Mgh = \frac{1}{2} \times \frac{2}{5} MR^2 \times (\frac{v}{R})^2 + \frac{1}{2} M v^2$

$v = (9.8 \times 1.2 \times \frac{10}{7})^{\frac{1}{2}} = 4.10\ m/s$.



A uniform solid ball of mass M = 6.00 kg and radius R, rolls smoothly from rest down a ramp at angle $\theta = 30.0°$

(a) The ball descends a vertical height h =1.20 m to reach the bottom of the ramp. What is its speed at the bottom?

(b) What are the magnitude and direction of the friction force f_s on the ball as it rolls down the ramp?

(b) upward.

$\begin{cases} Mg\sin\theta - f_s = Ma. \\ \tau = R f_s = I\alpha = I \frac{a}{R} \end{cases}$

$\Rightarrow f_s + Ma = Mg\sin\theta$

$f_s (1 + \frac{R^2}{I} M) = Mg\sin\theta$

$f_s = \frac{Mg\sin\theta}{1 + MR^2/(\frac{2}{5}MR^2)} = 8.4N$

题型二：

- Rolling down smoothly
- Along y-axis

$$F_g - T = Ma_{com}$$

The torque creates the rotation:

$$\tau_{net} = TR_0 = I_{com}\alpha = I_{com}\frac{a_{com}}{R_0}$$

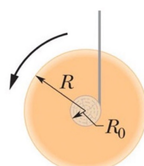
$$T = I_{com}\frac{a_{com}}{R_0^2}$$

$$a_{com,x} = \frac{g}{1 + I_{com}/MR_0^2}$$

- The yo-yo has the same downward acceleration when it is climbing back up
- Careful about which R to be applied!

$$a_{com} = \frac{dv_{com}}{dt};$$

$$v_{com} = \frac{ds}{dt} = \frac{d\theta}{dt}R_0 \rightarrow a_{com} = \alpha R_0$$



(rolling with slipping)

3. 角动量

For single particles

- Translational Motion

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

- Angular Form

$$\vec{\tau}_{net} = \frac{d\vec{l}}{dt}$$

proof: $\frac{d\vec{l}}{dt} = m\frac{d(\vec{r}\times\vec{v})}{dt}$

$= m\left(\frac{d\vec{r}}{dt}\times\vec{v} + \vec{r}\times\frac{d\vec{v}}{dt}\right)$

$= m(\vec{v}\times\vec{v} + \vec{r}\times\vec{a})$

$= \vec{r}\times(m\vec{a})$

$= \vec{r}\times\vec{F}$

$= \tau.$

τ的三种写法：① TR，② $I\alpha$，③ $\frac{d\vec{l}}{dt}$.

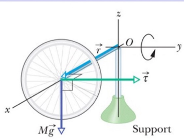
$L = \sum_{i=1}^{n} l_i = \sum_{i=1}^{n} m_i v_{i\perp} r_i$

$= \sum_{i=1}^{n} m_i r_i^2 \omega$

$= I\omega$

4. 陀螺仪的进动.

Precession of a Gyroscope

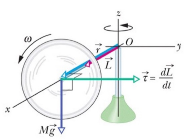


(a)

- No spinning: The torque created by the gravitational force

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\tau = Mgrsin90° = Mgr$$



(b)

- Spinning: Precession, rotate horizontally about O

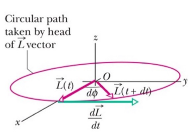
$$L = I\omega$$

dL=L $d\emptyset$

$$\tau = \frac{dL}{dt} = Mgr$$

$$dL = Mgrdt$$

$$d\emptyset = \frac{dL}{L} = \frac{Mgrdt}{I\omega}$$



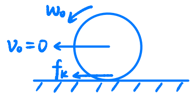
(c)

Procession rate: $\Omega = \frac{d\emptyset}{dt} = \frac{Mgr}{I\omega}$

Chapter 12. Equilibrium and elasticity 平衡与弹力

- On a frictional surface, it is not smooth rolling

$$f_k = \mu_k mg = ma_{com}$$
$$f_k R = \mu_k mgR = I_{com}\alpha$$



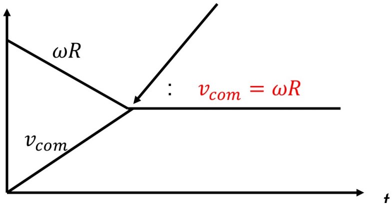
At time goes on

$$v_{com} = a_{com}t$$

$$\omega = \omega_0 - \alpha t$$

not smooth rolling → smooth rolling

- Smooth Rolling: $v_{com} = \omega R$; $t = \dfrac{\omega_0 R}{\mu_k g(1+\frac{mR^2}{I_{com}})}$



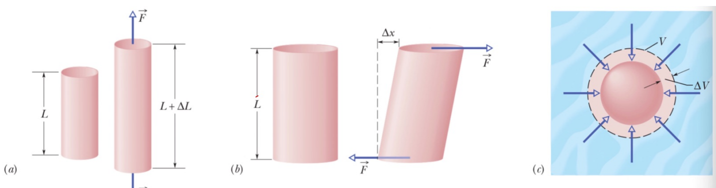


Figure 12-11 (*a*) A cylinder subject to *tensile stress* stretches by an amount ΔL. (*b*) A cylinder subject to *shearing stress* deforms by an amount Δx, somewhat like a pack of playing cards would. (*c*) A solid sphere subject to uniform *hydraulic stress* from a fluid shrinks in volume by an amount ΔV. All the deformations shown are greatly exaggerated.

- Three ways a solid might change its dimensions when forces act on it.

a) Tensile Stress 拉伸应力

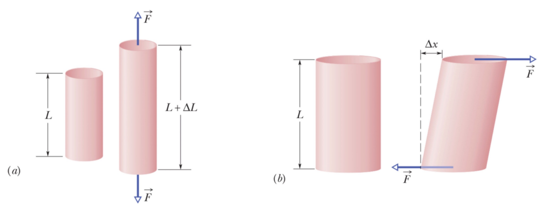
b) Shearing stress 切应力

c) Hydraulic stress 水应力

> 应力 压力
> Stress and Strain are proportional to each other →
> **Modulus of Elasticity**
> $Stress = Modulus \times strain$

- Young's Modulus: E

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

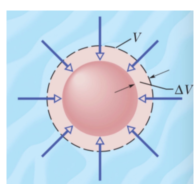


- Shear Modulus: G

$$\frac{F}{A} = G\frac{\Delta x}{L}$$

- Bulk Modulus: B

$$p = B\frac{\Delta V}{V}$$



- Translational Equilibrium: Balance of Forces

$$\vec{P} = constant$$ 动量为常数.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

- Rotational Equilibrium: Balance of Torques

$$\vec{L} = constant$$ 角动量为常数.

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

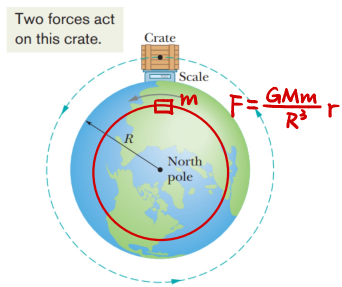
Balance of forces	Balace of torques
$F_{\text{net},x} = 0$	$\tau_{\text{net},x} = 0$
$F_{\text{net},y} = 0$	$\tau_{\text{net},y} = 0$
$F_{\text{net},z} = 0$	$\tau_{\text{net},z} = 0$

Chapter 13. Gravitation

$$F = G\frac{m_1 m_2}{r^2} \quad \text{(Newton's law of gravitation).}$$

G is the **gravitational constant:**

$$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$$
$$= 6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2.$$



$$g = a_g - \omega^2 R$$

测量值

We may take the gravitational potential energy of the two-particle system to be

$$U = -\frac{GMm}{r} \quad \text{(gravitational potential energy).}$$

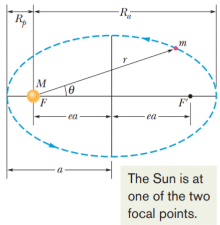
For the three-particle system

算逃逸速度.

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$

开普勒定律 kepler's laws

1. THE LAW OF ORBITS: All planets move in elliptical orbits with Sun at one focus



2. THE LAW OF AREAS: A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate dA/dt at which it sweeps out area A is constant.

The instantaneous rate at which area is being swept out is

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega,$$

The angular momentum is

$$L = rp_\perp = (r)(mv_\perp) = (r)(m\omega r)$$

$$= mr^2\omega,$$

Based on the above two equations

$$\frac{dA}{dt} = \frac{L}{2m}.$$

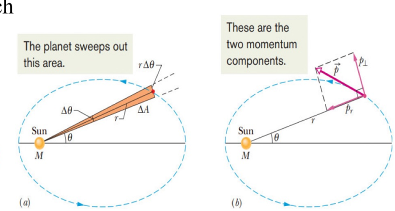


Figure 13-13 (*a*) In time Δt, the line r connecting the planet to the Sun moves through an angle $\Delta\theta$, sweeping out an area ΔA (shaded). (*b*) The linear momentum $\vec{p}$ of the planet and the components of $\vec{p}$.

角动量守恒.

Law of conservation of angular momentum

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3. THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3$$

For a circular motion,

$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$

As we know,

$$T = 2\pi/\omega$$

We have

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \quad \text{(law of periods)}.$$

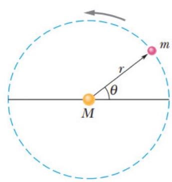


Figure 13-14 A planet of mass m moving around the Sun in a circular orbit of radius r.

13

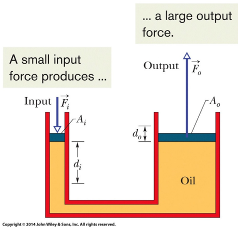
$$E = -\frac{GMm}{2a} \quad \text{(elliptical orbit)}.$$

系统机械能

Chapter 14. Fluid 流体.

静止流体只承受垂直表面的压力，不承受任何剪切力.

帕斯卡定理： $\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o}$



$A_i d_i = A_o d_o$

$\Rightarrow W = F_i d_i = \frac{A_i}{A_o} F_o \cdot \frac{A_o}{A_i} d_o = F_o d_o$

流体类型：

Four Assumptions about ideal fluid:

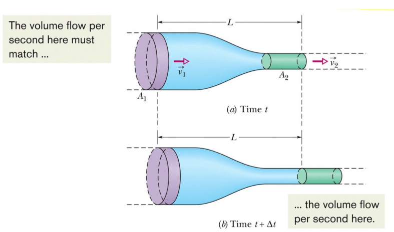
稳流 • **Steady Flow**: the $\boxed{\text{velocity}}$ of the moving fluid at any fixed point doesn't change with time

不可压缩流 • **Incompressible Flow**: as for fluids at rest, ideal fluid is incompressible; that is, its $\boxed{\text{density}}$ has a constant, uniform value.

非粘滞流 • **Nonviscous Flow**: Viscosity of a fluid is a measure of how resistive the fluid is to flow. No viscous drag force – that is, no resistive force due to viscosity.

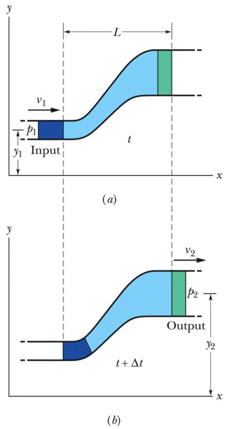
无旋流 • **Irrotational Flow**: the test body may (or may not) move in a circular path, in irrotational flow the test body will not rotate about its COM. (like Ferris Wheel)

流体连续性方程：



$R_V = A_1 v_1 = A_2 v_2$

伯努利方程： $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$



Proof

- Work done on the system

重力势能

$$W = W_g + W_p = \Delta K$$

系统做功

- Work done by Gravitational Force

$$W_g = -\Delta m g(y_2 - y_1) = -\rho \Delta V g(y_2 - y_1)$$

- Work done by the system

$$F\Delta x = pA\Delta x = p\Delta V$$

$$W_p = p_1\Delta V + (-p_2\Delta V) = -\Delta V(p_2 - p_1)$$

- K.E gained:

$$\Delta K = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$$

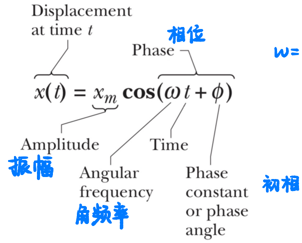
- Thus,

$$-\rho\cancel{\Delta V}g(y_2 - y_1) - \cancel{\Delta V}(p_2 - p_1) = \frac{1}{2}\rho\cancel{\Delta V}(v_2^2 - v_1^2)$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Chapter 15. Oscillation

简谐运动 simple harmonic motion



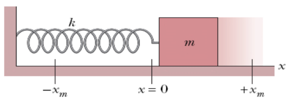
$w = 2\pi f$.

$$x(t) = x_m \cos(wt + \phi)$$ 通常用 cos

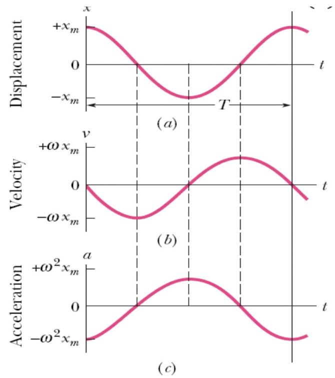
$$v(t) = -\omega x_m \sin(\omega t + \emptyset) = \omega x_m \cos(\omega t + \emptyset + \pi/2)$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \emptyset) = -\omega^2 x(t)$$

$$\Leftrightarrow \quad \frac{d^2x}{dt^2} + w^2 x = 0$$



与弹簧振子联系



$$F = ma = -mw^2 x = -kx$$

$$\Rightarrow \quad k = mw^2$$

$$w = \sqrt{\frac{k}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

能量分布：

- Potential Energy:

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 cos^2(\omega t + \emptyset)$$

- Kinetic Energy:

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_m^2 sin^2(\omega t + \emptyset)$$

$$= \frac{1}{2}kx_m^2 sin^2(\omega t + \emptyset)$$

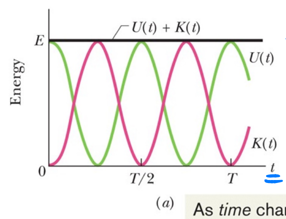
- Mechanical Energy

$$E = U(t) + K(t)$$

$$= \frac{1}{2}kx_m^2[cos^2(\omega t + \emptyset) + sin^2(\omega t + \emptyset)]$$

$$E = U + K = \frac{1}{2}kx_m^2$$

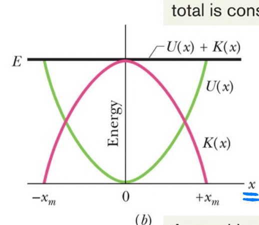
- Only conservative forces present, hence the total energy is conserved.



(a)

能量守恒.

As *time* changes, the energy shifts between the two types, but the total is constant.



(b)

As *position* changes, the energy shifts between the two types, but the total is constant.

角简谐运动 Angular simple harmonic motion.

- Restoring Torque

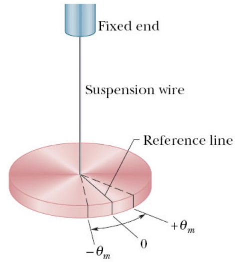
$$\tau = -\kappa\theta$$

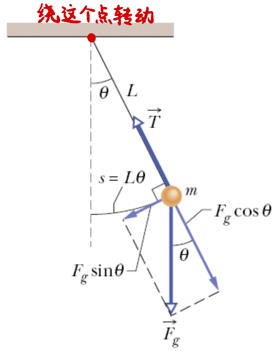
- Torsion constant: κ (kappa), depends on the length, diameter, and material.

Angular Form of Hooke's Law

扭秤

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$





- The restoring torque acting on the mass when its angular displacement is θ, is:

$$\tau = -L(F_g sin\theta) = I\alpha$$

角速度 $$\alpha = -\frac{mgL}{I}sin\theta \approx -\frac{mgL}{I}\theta$$

- SHM in angular form

需要掌握推导过程.

$$\alpha(t) = -\omega^2\theta(t)$$

- Therefore, it is a SHM if it swings through only a small angle.

- Angular Frequency

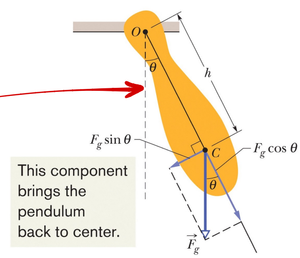
$$\alpha(t) = -\omega^2\theta(t) = -\frac{mgL}{I}\theta(t)$$

$$\omega = \sqrt{\frac{mgL}{\mathrm{I}}}$$

- Period

通解.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgL}}$$



- For Simple Pendulum

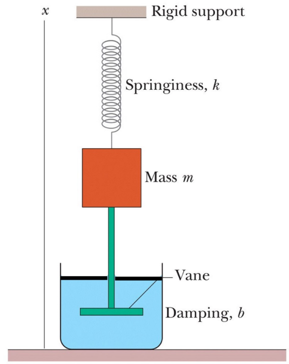
$$I = mL^2$$

- Thus,

单摆

$$T = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$

Damped simple harmonic motion 阻尼简谐运动



- Mechanical Energy transfers into thermal energy 热能
- Damping Force $\vec{F}_d$: (Opposite direction with v)

$$F_d = -bv$$

b 阻尼系数

- b: damping constant → Depends on the characteristics of both the vane and the liquid
- SI-unit: kg/s
- Newton's Second Law

$$F_{net} = F_d + F_{spring} = ma$$

$$-bv - kx = ma$$

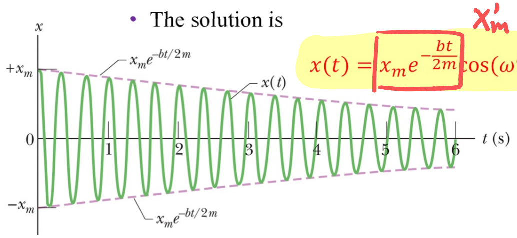
$$-b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$m\frac{dx^2}{dt^2} + b\frac{dx}{dt} + kx = 0$$

- The solution is

x'_m

$$x(t) = \boxed{x_m e^{-\frac{bt}{2m}}}\cos(\omega' t + \emptyset)$$



- The Angular Frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- If $b = 0$, $\omega' = \omega = \sqrt{\frac{k}{m}}$

$$x(t) = x_m \cos(\omega t + \emptyset)$$

- If $b \ll \sqrt{km}$, $\omega' \approx \omega = \sqrt{\frac{k}{m}}$
- Damped Energy: not a constant, decreases with time
- If the damping is small, we can replace $\boldsymbol{x_m}$ with $\boldsymbol{x_m e^{-\frac{bt}{2m}}}$

$$E(t) = U(t) + K(t)$$

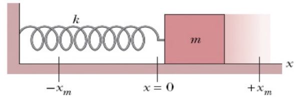
$$E(t) \approx \frac{1}{2}kx_m^2 e^{-\frac{bt}{m}}$$

Forced Oscillation and Resonance

受迫振荡 谐振。

- Free Oscillation:

$$\omega = \sqrt{\frac{k}{m}}$$



- Forced (or Driven) Oscillation: Oscillations are subject to a periodic applied force.
- The oscillator will oscillate with the same (angular) $\boldsymbol{\omega_d}$ frequency as the driving force.

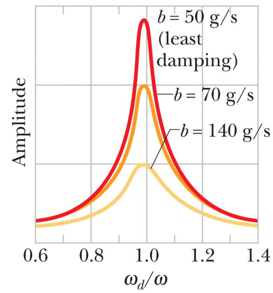
$$x(t) = x_m \cos(\omega_d t + \emptyset)$$

- When

$$\omega_d = \omega$$

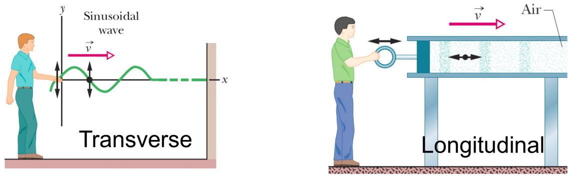
Resonance

- The displacement amplitude x_m (approximately) and velocity amplitude v_m are the greatest values.



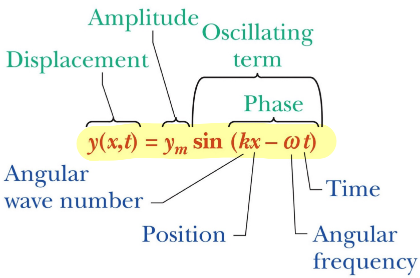
Chapter 16. Waves 波

Mechanical waves exsist only within material media.



The oscillation is perpendicular to the direction of the wave's travel.

The oscillation is parallel to the direction of the wave's travel.



统一形式

$k = \frac{2\pi}{\lambda}$ 角波数.

$v = \frac{\omega}{k} = \lambda f$.

(波速)

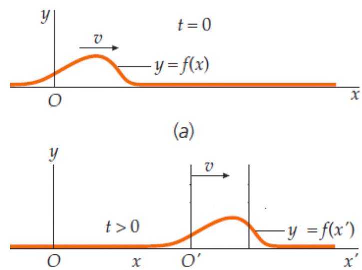
$u = \frac{\partial h}{\partial t}$

(质点速度)

- To illustrate a wave on a string, we have a function to describe the wave.
- Describe the motion of any element along its length

$$y = h(x,t)$$

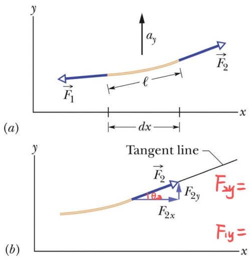
- y: the transverse displacement of any string element
- x: position of the element along the string at time t.



- Therefore, $h(x,t) = f(x \pm vt)$ 与波的理论位置关联.
- Then the displacement fit the differential equation:

$$\frac{1}{v^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2}$$

推导波速的决定式 tension & linear density



- We can find the wave speed through the wave equation, that governs the ***travel of waves of any type.***

$$F_{net,y} = ma_y$$

$$F_{2y} - F_{1y} = dma_y$$

取极小的一段

$$dm = \mu dx;\ a_y = \frac{d^2y}{dt^2}$$

μ = ρ

- The force

$$\tau = \sqrt{{F_{2x}}^2 + {F_{2y}}^2}$$

$F_{2y} = \tau \cdot \tan\theta_2 = \tau S_2$

- Assume the slope is $S = \frac{dy}{dx}$

$F_{1y} = \tau \cdot \tan\theta_1 = \tau S_1$

$$F_{2y} = \tau S_2;\ F_{1y} = \tau S_1$$

$F_{2y} - F_{1y} = \tau(S_2 - S_1) = \mu dx \times \frac{d^2y}{dt^2}$

$$\underbrace{\tau S_2 - \tau S_1}_{d(\frac{dy}{dx})} = \mu dx \times \frac{d^2y}{dt^2}$$

- Continued

$$\tau(S_2 - S_1) = \mu dx \times \frac{d^2y}{dt^2}$$

$$\frac{S_2 - S_1}{dx} = \frac{\mu}{\tau} \times \frac{d^2y}{dt^2}$$

- The difference between S_2 and S_1 can be a differential amount dS

$$dS = d\left(\frac{dy}{dx}\right)$$

$$\frac{dS}{dx} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{\mu}{\tau} \times \frac{d^2y}{dt^2}$$

$h(x,t) = f(x - vt)$

- Since the wave equation for all traveling wave is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\frac{1}{v^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2}$$

- Therefore, the wave speed is

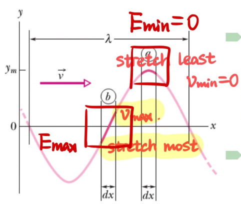
$$\boxed{v = \sqrt{\frac{\tau}{\mu}}}$$

$\sqrt{\frac{张力}{密度}}$

3

波中一点的能量分布

- Kinetic Energy: Maximum at b; zero at a.
- Potential Energy: the wave must necessarily stretch the string. → Maximum at b; zero at a.



- The region of the string at its maximum displacement has no energy, and the regions at zero displacement has maximum energy.
- The wave *transports* the energy along the string.

- The K.E. dK of an element dm:

$$dK = \frac{1}{2}dm \times u^2$$

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

$$dm = \mu \times dx = \mu \times v \times dt$$

- Therefore,

$$dK = \frac{1}{2}dmu^2 = \frac{1}{2} \times [-\omega y_m \cos(kx - \omega t)]^2 \times \mu \times v \times dt$$

$$dK = \frac{1}{2}\mu\omega^2 y_m^2 cos^2(kx - \omega t) \times v \times dt$$

$$\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 cos^2(kx - \omega t)$$

- Including the Potential Energy

$$P_{avg} = 2 \times \frac{1}{2}\mu v \omega^2 y_m^2 [cos^2(kx - \omega t)]_{avg}$$

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2$$

Additional Information

- For **Transverse Sinusoidal Wave**:

$$y(x,t) = y_m sin(kx - \omega t)$$

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m cos(kx - \omega t) \text{ and } \frac{\partial y}{\partial x} = k y_m cos(kx - \omega t)$$

$$\boldsymbol{dK} = \frac{1}{2}dmu^2 = \frac{1}{2} \times \mu dx \times [-\omega y_m \cos(kx - \omega t)]^2$$

$$= \frac{1}{2}\mu\omega^2 y_m^2 cos^2(kx - \omega t)dx$$

$(1+\alpha)^{\frac{1}{2}} \approx 1 + \frac{1}{2}\alpha.$

- **For potential energy**: The amount of stretch is

$$\sqrt{dx^2 + dy^2} - dx = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2} - dx$$

$$\approx dx\left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 + \cdots\right] - dx \approx \frac{dx}{2}\left(\frac{\partial y}{\partial x}\right)^2$$

$$P.E. = dU = tension \times amount\ of\ stretch = \tau \times \frac{dx}{2} \times \left(\frac{\partial y}{\partial x}\right)^2$$

$$\boldsymbol{dU} = \frac{1}{2}\mu v^2 k^2 y_m^2 cos^2(kx - \omega t)dx = \frac{1}{2}\mu\omega^2 y_m^2 cos^2(kx - \omega t)dx = \boldsymbol{dK}$$

9

- The average potential and kinetic power are the same
- The average of cosine squared over a wavelength is 1/2

Interference of Waves

- The phenomenon of combining waves: **Interference**
- If two sinusoidal waves of the same amplitude and wavelength travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

正弦曲线

- Wave 1: $y_1(x,t) = y_m sin(kx - \omega t)$
- Wave 2: $y_2(x,t) = y_m sin(kx - \omega t + \emptyset)$

$$y' = y_1(x,t) + y_2(x,t)$$
$$= y_m sin(kx - \omega t) + y_m sin(kx - \omega t + \emptyset)$$
$$y' = \left[2y_m \cos\left(\frac{1}{2}\emptyset\right)\right] sin\left(kx - \omega t + \frac{1}{2}\emptyset\right)$$

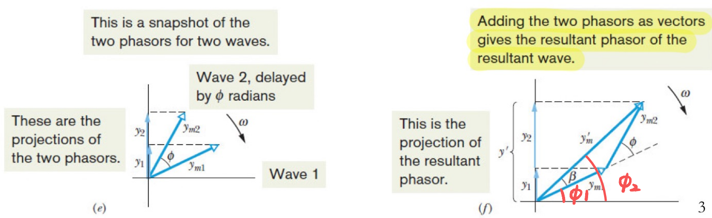
Phasor

- Phasors can be used to combine waves same k and ω even if their amplitudes are different.

振幅

- Wave 1: $y_1(x,t) = y_{m1} sin(kx - \omega t)$
- Wave 2: $y_2(x,t) = y_{m2} sin(kx - \omega t + \emptyset)$
- Thus, the resultant wave should be

$$y'(x,t) = y'_m sin(kx - \omega t + \beta)$$

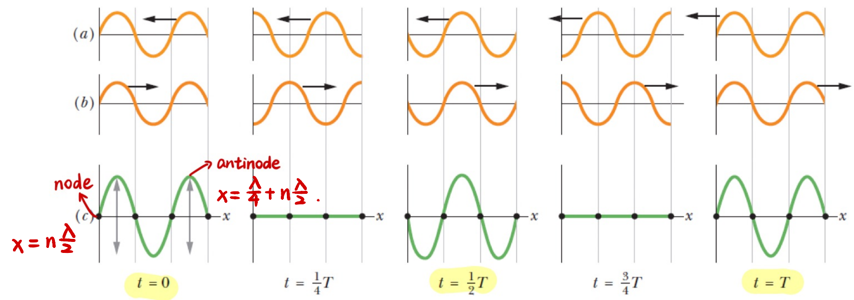


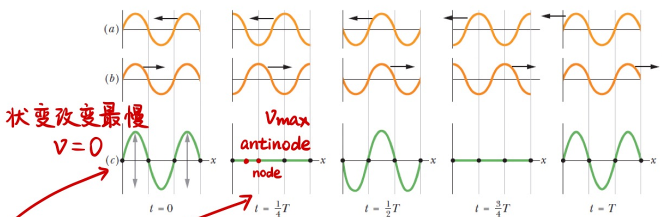
- $y_1(x,t) = y_{m1}\, sin(kx - \omega t + \emptyset_1) = y_{m1}[sin(kx - \omega t) \cos \emptyset_1 + \cos(kx - \omega t)\, sin\emptyset_1]$
- $y_2(x,t) = y_{m2} sin(kx - \omega t + \emptyset_2) = y_{m2}[sin(kx - \omega t)\ \cos\emptyset_2 + \cos(kx - \omega t)\, sin\,\emptyset_2]$
- $y'(x,t) = y_1 + y_2 = (y_{m1} \cos \emptyset_1 + y_{m2} \cos \emptyset_2) sin(kx - \omega t) + (y_{m1} \sin \emptyset_1 + y_{m2} \sin \emptyset_2) \cos(kx - \omega t)$
- Let $A_1 = y_{m1} \cos \emptyset_1 + y_{m2} \cos \emptyset_2$ and $A_2 = y_{m1} \sin \emptyset_1 + y_{m2} \sin \emptyset_2$
- Set $y'_m = \sqrt{A_1^2 + A_2^2}$, $cos\beta = \frac{A_1}{y'_m}$ and $sin\beta = \frac{A_2}{y'_m}$ $\qquad tan\beta = \frac{A_2}{A_1}$
- Thus,

$$y'(x,t) = y'_m cos\beta\ sin(kx - \omega t) + y'_m sin\beta\ cos(kx - \omega t)$$
$$= y'_m sin(kx - \omega t + \beta)$$

Standing Waves 驻波

- If two sinusoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a **standing wave**.
- Some points (**nodes**) never move and some (**antinodes**) move the most.





The resultant wave is no longer a traveling wave!!!

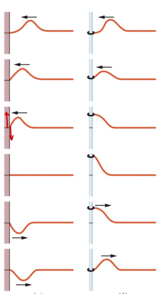
$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \sin kx]}_{\text{Magnitude gives amplitude at position } x} \underbrace{\cos \omega t}_{\text{Oscillating term}}$$

- When the string is at maximum curvature, there is no K.E. At this moment, the nodes have maximum energy, in the form of P.E. (they stretch the most)
- When the string is straight, there is minimum P.E. At this moment, the antinodes have maximum energy, in the form of K.E. (largest speed)
- The mechanical energy flows back and forth from a node to an antinode as the string goes from maximum curvature to straight, and does not propagate along the string.

V_{max}

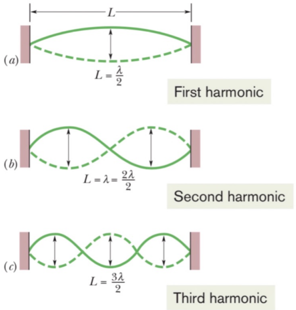
传播

驻波能量不是波传播，是在 node 和 antinode 之间转化.



- A wave reflects at the boundary and travels in the opposite direction to its incident direction.
- At a "Hard" boundary (a): the displacement is fixed to be zero. So the boundary is a node.
- At a "free" boundary(b): the displacement can be the maximum. So the boundary can be an antinode.

- For a standing Wave between two Hard boundaries, Each boundary is a node point.



- The distance L between two boundaries:

$$L = n\frac{\lambda}{2}$$

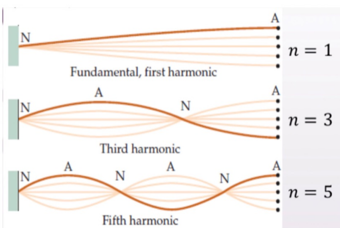
(for $n = 1, 2, 3, \ldots$)

- The resonance frequencies f:

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

(for $n = 1, 2, 3, \ldots$)

- Resonant Mode: one Hard and one Free boundaries
- The distance between two boundaries must be an odd integer of quarter wavelength



- The distance L between two boundaries:

$$L = n\frac{\lambda}{4}$$

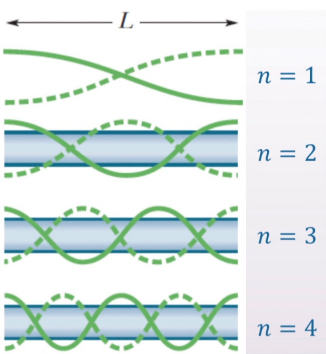
(for $n = 1, 3, 5, \ldots$)

- The resonance frequencies f:

$$f = \frac{v}{\lambda} = n\frac{v}{4L}$$

(for $n = 1, 3, 5, \ldots$)

- Resonant Mode: two Free boundaries, Each boundary is an antinode point.



- The distance L between two boundaries:

$$L = n\frac{\lambda}{2}$$

(for $n = 1, 2, 3, \ldots$)

- The resonance frequencies f:

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

(for $n = 1, 2, 3, ..$

- Resonance in 2D:

- The Resonance Frequencies f:
  - Two Hard Boundaries:

    $f = \frac{v}{\lambda} = n\frac{v}{2L}$ (for $n = 1, 2, 3, \ldots$)

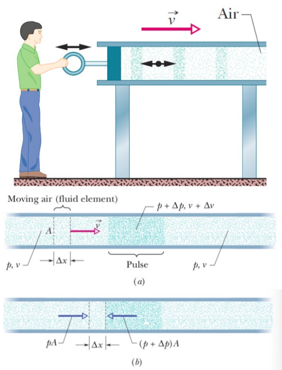
  - One Hard and one Free Boundaries:

    $f = \frac{v}{\lambda} = n\frac{v}{4L}$ (for $n = 1, 3, 5, \ldots$)

  - Two Free Boundaries:

    $f = \frac{v}{\lambda} = n\frac{v}{2L}$ (for $n = 1, 2, 3, \ldots$)

Derivation of Sound Speed



- For the moving air element: Δm at speed v

$$\Delta m = \rho A \Delta x = \rho A v \Delta t$$

- When it enters the compression region, it will decrease with Δv

$$a = \frac{\Delta v}{\Delta t}$$

- The net force acting on the element when it enters compression (shown in fig (b))

$$F = (p + \Delta p)A - pA = \Delta p A$$

- Based on Newton's 2nd Law

$$F = \Delta m a$$

$$\Delta p A = \rho A v \Delta t \times \frac{\Delta v}{\Delta t}$$

$$\Delta p = \rho v \Delta v$$

- The air is compressed with volume change ΔV

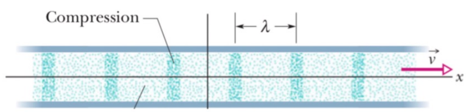
$$\Delta p = B\frac{\Delta V}{V} = B\frac{\Delta v \Delta t A}{v \Delta t A} = B\frac{\Delta v}{v}$$

- Therefore,

$$\Delta p = \rho v \Delta v = B\frac{\Delta v}{v}$$

$$v^2 = \frac{B}{\rho}$$

$$v = \sqrt{\frac{B}{\rho}}$$



- For elastic deformation

$$\Delta p = -B\frac{\Delta V}{V}$$

$$V = A\Delta x;$$

$\Delta V = A\Delta s$ (Vol. change caused by oscillation)

$$\Delta p = -B\frac{A\Delta s}{A\Delta x} = -B\frac{\partial s}{\partial x}$$

$$\frac{\partial s}{\partial x} = -ks_m \sin(kx - \omega t)$$

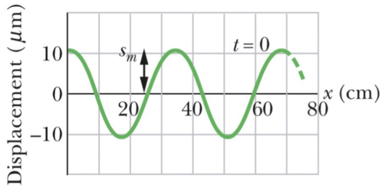
$$\Delta p = Bks_m \sin(kx - \omega t)$$

- Thus,

$$\Delta p_m = Bks_m = v^2 \rho k s_m$$

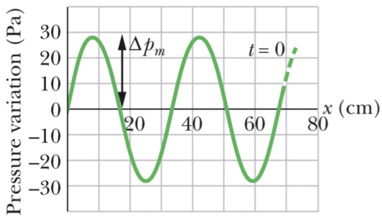
$$v = \frac{\omega}{k}$$

$$\Delta p_m = v^2 \rho k s_m = v \rho \omega s_m$$



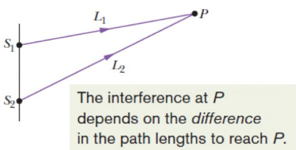
(a)

$$s(x,t) = s_m \cos(kx - \omega t)$$



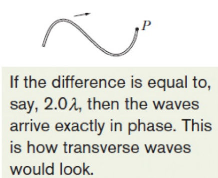
(b)

- Zero displacement → either Compression or Expansion
- $u(x,t) = \frac{\partial s}{\partial t} = \omega s_m \sin(kx - \omega t)$
- $a(x,t) = \frac{\partial u}{\partial t} = -\omega^2 s_m \cos(kx - \omega t)$

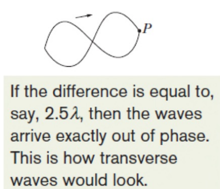


(a)

- Two point sources shown in (a)
- If in phase (b), fully constructive interference; if out of phase as (c), fully destructive interference.



(b)



(c)

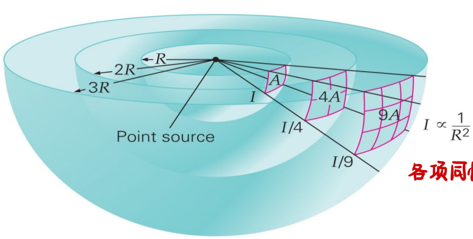
$$\Delta L = n\lambda$$
$(n = 0,1,2 \dots)$
(Fully constructive)

$$\Delta L = \left(n + \tfrac{1}{2}\right)\lambda$$
$(n = 0,1,2 \dots)$
(Fully destructive)

- To find the phase difference $\emptyset$:

$$\frac{\emptyset}{2\pi} = \frac{\Delta L}{\lambda}$$

$$\emptyset = 2\pi \frac{\Delta L}{\lambda}$$



- For an isotropic point source with power P_s

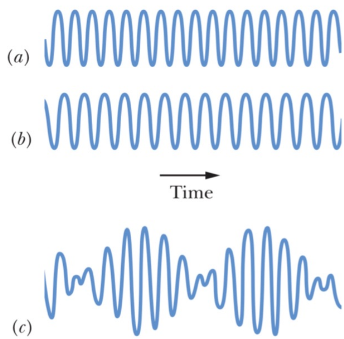
$$I = \frac{P_s}{4\pi r^2}$$

$$I = \frac{1}{2}\rho v \omega^2 s_m^2$$

- The intensity of sound from an 各项同性的 isotropic point source decreases with the square of the distance r from the source.

$$\beta = (10\ dB)\log \frac{I}{I_0}$$

Beats



- Wave (a)

$$s_1 = s_m \cos(kx + \omega_1 t)$$

- Wave (b)

$$s_2 = s_m \cos(kx + \omega_2 t)$$

- The net wave

$$s' = s_1 + s_2$$

- Since

$$\cos\alpha + \cos\beta = 2\cos[\frac{1}{2}(\alpha-\beta)]\cos[\frac{1}{2}(\alpha+\beta)]$$

- Thus,

$$s' = 2s_m \cos\omega' t \cos(kx + \omega t)$$

- Where $\omega' = \frac{1}{2}(\omega_1 - \omega_2)$ and $\omega = \frac{1}{2}(\omega_1 + \omega_2)$

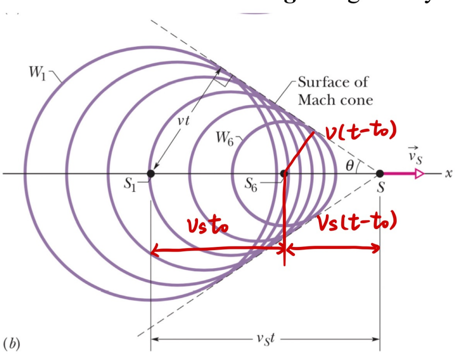
The Doppler Effect

- The general equation:

$$f' = f\,\frac{v \pm v_D}{v \pm v_s}$$

- Where, v is the speed of sound through the air, v_D is the detector's speed relative to air, and v_s is the source's speed relative to the air.
- When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from each other, the sign on its speed must give a downward shift in frequency.

- The ***Mach cone angle*** is given by



- The ***Mach number***

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$