Effects of fiscal stimulus policies on private consumption: a RS-DSGE approach for Japan *

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Abstract

In this paper, a New-Keynesian model with non-Ricardian households is used to analyze the impact of different fiscal policy measures on private consumption. The model is solved and estimated outside and at the Zero Lower Bound (ZLB) through a Markov-switching approach, based on Japanese data over the period 1980Q1-2020Q3, and also following two different fiscal rule specifications. Results show that consumption does not respond positively to fiscal stimulus measures, even after inclusion of rule-of-thumb consumers. This results from the prevalence of Ricardian behavior in the model, in addition to the decline in real wages that follows most measures. Inclusion of distortionary taxation alters the variables' responses through the impact of tax rate movements. Conversely, the presence of the ZLB is not found to affect the model substantially, except after a consumption or a wage income tax cut. A capital income tax cut is found the be the optimal fiscal stimulus measure. Its impact mainly results from the increase in investment which shifts the economy's productive capacity upward, thereby boosting consumption and real wages. Finally, a variance decomposition analysis shows that consumption is mainly driven by labor supply and technology shocks.

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1 Introduction

For a long time, monetary policy was considered as the primary tool to conduct stabilization policy. Then, the challenges it faced in the recent years brought the debate over the use of fiscal policy to the fore.[1] Yet, the understanding of fiscal policy effects on the economy is still insufficient. And although fiscal stimulus measures were undertaken during the Great Recession in many countries, fiscal policy effectiveness as an instrument for macroeconomic stabilization is still a matter of uncertainty.

Such an uncertainty is conspicuous in the disagreement prevailing in the literature on the fiscal multiplier, which extends to fiscal policy effects on GDP components such as private consumption. Nonetheless, recent developments of this literature give new hopes for a more efficient use of fiscal policy. More particularly, it has been recently found that fiscal multipliers can become large and above unity under some conditions. Usually this happens when a larger share of households has a non-optimizing behavior[2] or in exceptional circumstances that affect monetary policy or cause a situation of underutilization of resources (e.g., ZLB, long and deep recessions). A better understanding of the economy's behavior in these unusual cases has the potential to provide us with a better insight as to how to make stabilization policies more efficient.

In the present paper, I use a New-Keynesian model that includes rule-of-thumb consumers to analyze the effects of fiscal policy on private consumption at and outside the ZLB. The focus on private consumption is partly justified by the fact that it is usually the most significant component of output. Thereby, the disagreement over fiscal multipliers could be resulting from the varying effects on private consumption. This also makes it a critical element in devising successful fiscal stimulus measures or conducting fiscal consolidations. An additional reason to study consumption is the controversy around the direction and size of its reaction to fiscal policy in the theoretical and empirical literature. Lastly, consumers' behavior is often found to play a key role in the effectiveness of macroeconomic policy (e.g. Krusell and Smith (1996), Colciago (2011)).

[1] This view on monetary policy has not always been the predominant one. Blinder et al. (2004) points out that during the 1960s, discretionary fiscal policy was cast in the lead role while central bank policies were considered as playing more of an accommodating role for fiscal policy. But then several theoretical and empirical works called into question the ability of fiscal policy to accomplish countercyclical stabilization (e.g., Ricardian equivalence), in addition to some practical challenges. In particular, one strong argument against the use of fiscal policy is the fact that lags in the implementation of adequate measures are typically too long to be useful for combatting recessions (which is not the case for monetary policy). As a result, a paradigm shift occurred in the 1980s in the literature, leading to the belief in the preeminence of monetary policy as a macroeconomic stabilization tool.

[2] In that case, they are less prone to act following the Ricardian equivalence principle.

The model I use is based on the benchmark of Smets and Wouters (2003) and is estimated in two versions. The first version (called "baseline model") includes rule-of-thumb consumers with two regimes (at and outside the ZLB) and fiscal policy based on lump-sum taxation; the second one (called "extended model") also includes rule-of-thumb consumers and the two regimes but adds distortionary taxation (on consumption, wages and capital income). Estimation of the model is based on Japanese quarterly data from 1980Q1 to 2020Q3, using a Markov-switching regimes approach. Japan is an interesting case of study for this topic because it experiences a very long ZLB episode with near-zero interest rates since the mid-1990s. In addition, after the tax hikes of 2014 and 2019, devising optimal measures to boost demand has become an important stake.

The main contributions of this research are as follows. Even after inclusion of the rule-of-thumb consumers, consumption does not respond positively to fiscal stimulus measures. This is explained, first, by the fact that Ricardian behavior remains prevalent in the model, irrespective of the share of non-Ricardian households. And this behavior is closely linked to lump-sum taxes that are kept in both versions of the model. Second, although consumption also tracks income more closely compared to the benchmark model of Smets and Wouters (2003) that does not have hand-to-mouth consumers,[3] real wages decline after most measures, thereby contributing negatively to consumption's response.

The inclusion of distortionary taxation is found to alter the model's behavior after a fiscal shock through movements of the different tax rates. In particular, the model with distortionary taxes shows that measures that affect the capital income tax generate positive effects over the long-run, through an increase in capital and inflation (at the ZLB or interest rates outside the bound).

No significant difference is found across regimes (outside and at the ZLB) in the model's behavior after a government spending increase or a capital income tax cut. But a difference between the two regimes can be seen after a cut in the consumption tax or the wage income tax. Outside the ZLB, both measures have negative effects on the economy. At the ZLB, both policies have harmful effects in the short-run but the potential to yield better results over the long-run via an impact on investment through the expectations channel.

A comparison between the different fiscal policy measures indicates that a capital income tax cut is the most beneficial policy and also yields better results than a government spending increase. It has the disadvantage of increasing public debt more significantly. Finally, a variance decomposition

[3] And also to models for Japan that are based on Smets and Wouters (2003) such as Iiboshi et al. (2006).

analysis shows that labor supply and technology shocks are the most significant contributors to the variation of private consumption, whereas fiscal shocks play a small role.

The remainder of the paper is organized as follows. The next section presents an overview of the main literature on the relationship between fiscal policy, private consumption and output. The third section provides a description of the theoretical model. The fourth section shows a general depiction of the basic workings of the model based on a full calibration of its log-linearized version. The fifth section summarizes the main settings for the model's solution and estimation through a Markov-switching regimes approach, and discusses the main results. Finally, the last section summarizes the main points of the study.

2 Literature review

Although many macroeconomic models agree on a positive effect from government spending to output, different theoretical specifications give very different predictions of the magnitude of the fiscal multiplier.[4] Typically, analyses based on linearized models such as Smets and Wouters (2007), Cogan et al. (2010) tend to predict smaller multipliers while recent non-linear approaches yield higher multipliers under some conditions. Some examples include models that account for the Zero Lower Bound[5] (Eggertsson and Woodford (2003), Eggertsson and Krugman (2012), Christiano et al. (2011) and Woodford (2011)),[6]or changes in the business cycle reflected in the unemployment rate (Nakamura and Steinsson (2014)) or in financial frictions (Canzoneri et al. (2016), Fernández-Villaverde (2010)).

There is an even stronger disagreement in the literature regarding the impact on private consumption. Usually, negative or very small multipliers for consumption are found in analyses based on neoclassical models (Hall (2009)) and are considered to be the result of negative wealth effects. According to Galí et al. (2007), consumers in neoclassical models (especially RBC models) act in a Ricardian fashion, as opposed to the traditional IS-LM frameworks. Therefore, an increase in government spending lowers the present-value of after-tax income thereby inducing a negative wealth effect that decreases consumption. In contrast, Keynesian models usually imply high multipliers for both

[4]Usually defined as the percentage change in GDP in response to a change in government spending equal to one percent GDP.

[5]In that case, the non-linearity is accounted for through the monetary policy rule.

[6]Higher multipliers result from an increase in inflation expectations when interest rates are held constant.

output and consumption since the size of the multiplier mostly depends on the marginal propensity to consume. New Keynesian models on the other hand usually yield a negative response of consumption. Since they are constructed by adding rigidities to a neoclassical foundation, neoclassical effects tend to mute the Keynesian multiplier.

One way of increasing consumption's response in a New Keynesian model is the inclusion of rule-of-thumb (or non-Ricardian) consumers. That way, the marginal propensity to consume becomes much higher than it would be if consumers behaved optimally, and the Keynesian mechanism takes over. Rule-of-thumb consumers are individuals that do not borrow or save but consume their entire income. In practice, there are many factors that could explain this behavior; for instance: myopia, liquidity or borrowing constraints, lack of confidence in economic conditions, inability to form expectations in an uncertain context, etc. Some examples of models that yield higher multipliers after inclusion of non-Ricardian consumers in the literature include Galí et al. (2007) and Coenen and Straub (2005).[7] However, in both cases, the authors stress the fact that the share of rule-of-thumb consumers has to be very high in order to alter the model's behavior. In addition, Galí et al. (2007) state that it should be accompanied by a high value of the price stickiness parameter so that real wages increase and drive consumption up.

Still, the presence of rule-of-thumb consumers makes models more realistic especially that an extensive empirical and theoretical literature shows a strong dependence of consumption on current income (Hall (1978), Campbell and Mankiw (1989), Erceg et al. (2005), López Salido and Rabanal (2006)). In addition, a proportion of households has no access to bank credit and is therefore unable to smooth consumption over the business cycle. Therefore, I include the assumption of the presence of rule-of-thumb consumers in this study. In addition, I introduce two other assumptions taken from the fiscal multiplier literature. These two assumptions usually make fiscal policy more effective in stimulating the economy even without resorting to widespread non-optimizing behavior.

The first additional assumption is the inclusion of lump-sum taxes in the fiscal rule specification. Usually, models featuring expenditures that are financed through distortionary taxes lead to low output multipliers, in comparison with those in which expenditures are financed through current or future

[7] Perotti (1999) also uses a model that assumes the coexistence of credit-constrained individuals and individuals with free access to credit, albeit with an opposite outcome (negative correlation in bad times).

lump-sum taxes.[8] The second assumption is the Zero Lower Bound on interest rates.[9] As both assumptions often lead to higher fiscal multipliers, they have the potential to make consumption more responsive to fiscal shocks in the model.

The model used in this paper is estimated based on data for Japan over the period 1980Q1-2020Q3. A number of factors make Japan an interesting case for studying the economy's dynamics after fiscal shocks at the ZLB. One of these factors is the fact that Japan experiences a very long ZLB episode with interest rates being close to zero since the mid-1990s. Another one is that even within the low interest rates period, different patterns in interest rates movements can be found, resulting in separate episodes, each characterized by distinct macroeconomic conditions (e.g., different levels of inflation).

Evidence of fiscal policy effects in Japan in periods of low interest rates is mixed. For example, Ko and Morita (2013) found fiscal policy to be ineffective in stimulating consumption during the low interest rates period.[10] One explanation provided by the authors is a low expectations of future income increase by households, resulting from the sluggish economy.[11] In contrast, using the Jordà (2005) local projection method, Miyamoto et al. (2018) reached the conclusion that a government spending increase crowds out private consumption and investment in the normal period and crowds them in during the ZLB period. They also found CPI inflation and expected inflation to respond positively and significantly at the ZLB, compared to the normal period.

Finally, incorporating non-linearities arising from the ZLB in DSGE models has become increasingly widespread in macroeconomic modelling and can be done following different methods. In this

[8]Fiscal policy effects also tend to be smaller if the increase of expenses is perceived by consumers as transitory rather than permanent (see Hall (2009), Ramey (2011), Baxter and King (1993)). In the latter case, consumers work more after the fiscal stimulus which generates a positive wealth effect; whereas in the former case, consumers simply smooth their labor and consumption by investing less. In other words, higher government spending leads to a reduction in investment instead of stimulating output.

[9]As stated before, the size of the multiplier is often found to be higher when accounting for markets failures: cases of high unemployment (Lee et al. (2020)), the presence of borrowing constraints (Canzoneri et al. (2016)) or liquidity constraints caused by the zero lower bound as monetary policy cannot be used to stimulate the economy. These factors reflect conditions that often occur during recessions and many empirical studies based on non-linear estimation methods show higher multiplier effects during recessionary episodes (e.g., Auerbach and Gorodnichenko (2012, 2013), Batini et al. (2012), Mittnik and Semmler (2012)).

[10]They followed an empirical approach using a MS-VAR model. Results showed the presence of four major structural changes in Japanese fiscal policy between the 1970s and the recent period. These break-points occurred in the mid-1970s (oil shock), early 1990s (recession), late 1990s (liquidity trap and slight recovery) and late 2000s (global recession and recovery). But fiscal policy appeared to be effective in stimulating consumption only during the first, second and fifth regime.

[11]An investment crowding-out effect was also observed in the 2000s period. According to the authors this could be explained by the fact that a large share of public spending during this phase was made to bailout poorly productive, debt-ridden firms.

study, a regime-switching approach that builds on the contribution of Binning and Maih (2017) is adopted. Through this approach, perturbation methods can be applied to derive an approximated solution for the model.

3 Theoretical model

I consider a standard New-Keynesian DSGE model in the vein of Smets and Wouters (2003), augmented with rule-of-thumb consumers and an extended fiscal policy framework as in Coenen and Straub (2005), Galí et al. (2007) and Iwata (2009). However, as opposed to these papers, money is not removed from the Ricardian households utility function and a quadratic form of capital adjustment costs is assumed. Two versions of the model are studied. The first version (called "baseline model") includes rule-of-thumb consumers with two regimes (at and outside the ZLB) and fiscal policy based on lump-sum taxation; the second one (called "extended model") also includes rule-of-thumb consumers and the two regimes but adds distortionary taxation (on consumption, wages and capital).

3.1 Baseline model

3.1.1 Households

There is a continuum of households indexed by $h \in (0,1)$. This continuum includes two types of households: a share of $1-\mu$ of households corresponds to Ricardian consumers, with unrestricted access to financial markets where they can trade government bonds and physical capital. The remaining μ households have no access to financial markets. Ricardian households maximize their intertemporal utility as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^R \tag{1}$$

Where β is the discount factor and the utility function is expressed by

$$U_t^R = \varepsilon_t^B \left(\frac{1}{1-\sigma_c} \left(C_t^R - H_t \right)^{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(L_t^R \right)^{1+\sigma_l} + \frac{\varepsilon_t^M}{1-\sigma_m} \left(\frac{M_t^R}{P_t} \right)^{1-\sigma_m} \right) \tag{2}$$

The external habit stock H_t is proportional to aggregate past consumption $H_t = hC_{t-1}$. The term σ_c represents relative risk aversion, σ_l is the inverse of the elasticity of work effort with respect to real wage and σ_m is the inverse of the elasticity of money holding with respect to the interest rate.

The utility function includes three types of shocks that follow an AR(1) process: a preference shock $\ln\left(\varepsilon_t^B\right) = \rho_B \ln\left(\varepsilon_{t-1}^B\right) + \eta_t^B$, a shock to labor supply $\ln\left(\varepsilon_t^L\right) = \rho_L \ln\left(\varepsilon_{t-1}^L\right) + \eta_t^L$, and a money demand shock $\ln\left(\varepsilon_t^M\right) = \rho_M \ln\left(\varepsilon_{t-1}^M\right) + \eta_t^M$. Ricardian households intertemporal budget constraint is expressed by

$$\frac{M_t^R}{P_t} + \frac{B_t^R/R_t}{P_t} = \frac{M_{t-1}^R}{P_t} + \frac{B_{t-1}^R}{P_t} + Y_t^R - T_t^R - C_t^R - I_t^R \tag{3}$$

T_t^R are lump-sum taxes and I_t^R investment. As in Ireland (2001), it is assumed that in order to generate new units of capital, households must pay an adjustment cost, measured through the quadratic term $\frac{\Theta_k}{2}\left(\frac{K_{t+1}^R}{K_t^R} - 1\right)^2 K_t^R$. Households' real income is then given by

$$Y_t^R = w_t L_t^R + r_t^k K_t^R - \frac{\Theta_k}{2}\left(\frac{K_{t+1}^R}{K_t^R} - 1\right)^2 K_t^R + Div_t^R \tag{4}$$

Where $r_t^k K_t^R$ represent total payments from supplying units of capital and Div_t^R dividends from firms of intermediate goods. The capital accumulation process is given by

$$K_{t+1}^R = (1-\delta) K_t^R + x_t I_t^R \tag{5}$$

Where δ is the rate of depreciation of capital with $0 < \delta < 1$ and x_t represents a shock to the marginal efficiency of investment and is expressed as $\ln(x_t) = \rho_x \ln(x_{t-1}) + \eta_t^x$.

On the other hand, Non-Ricardian households do not optimize their consumption decision and fully consume their labor income as follows

$$C_t^{NR} = w_t L_t^{NR} - T_t^{NR} \tag{6}$$

3.1.2 Labor markets

Wages are determined in the labor market by a continuum of unions acting as wage setters. The probability that a particular union changes its nominal wage in period t is constant and equal to $1 - \xi_w$. The households that cannot reoptimize their wages adjust based on past inflation

$$W_{h,t} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} W_{h,\,t-1} \tag{7}$$

Where γ_w is the degree of wage indexation. The demand for labor is determined by

$$L_{h,t} = \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{8}$$

$\lambda_{w,t}$ a stochastic parameter representing the wage mark-up with $\lambda_{w,t} = \lambda_w + \eta_t^w$ and the shock η_t^w is assumed to be i.i.d. normal. Aggregate labor demand L_t and aggregate nominal wage W_t are given by Dixit-Stiglitz type aggregator functions as follows

$$L_t = \left[\int_0^1 \left(L_{h,t} \right)^{\frac{1}{1+\lambda_{w,t}}} d\tau \right]^{1+\lambda_{w,t}} \tag{9}$$

$$W_t = \left[\int_0^1 \left(W_{h,t} \right)^{\frac{-1}{\lambda_{w,t}}} d\tau \right]^{-\lambda_{w,t}} \tag{10}$$

The optimization problem leads to the following mark-up equation for the optimized wage (see Appendix A for more details)[12]

$$\frac{W_{h,t}^*}{P_t} E_t \sum_{i=0}^{\infty} \left(\beta \xi_w \right)^i \left[\Lambda_{t+i} \frac{\left({}^{P_{t+i-1}}/_{P_{t-1}} \right)^{\gamma_w}}{{}^{P_{t+i}}/_{P_t}} L_{h,t+i} \right] = E_t \sum_{i=0}^{\infty} \left(\beta \xi_w \right)^i \left(1 + \lambda_{w,t+i} \right) \varepsilon_{t+i}^B \varepsilon_{t+i}^L (L_{h,t+i})^{1+\sigma_l} \tag{11}$$

The aggregate nominal wage is given by

$$W_t = \left[\left(1 - \xi_w \right) \left(W_{h,t}^* \right)^{-\frac{1}{\lambda_{w,t}}} + \xi_w \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \left(W_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}} \tag{12}$$

Aggregate consumption is given by

$$C_t = (1 - \mu) C_t^R + \mu C_t^{NR} \tag{13}$$

And aggregate labor

$$L_t = (1 - \mu) L_t^R + \mu L_t^{NR} \tag{14}$$

With $L_t = L_t^R = L_t^{NR}$ at equilibrium. Aggregate lump-sum taxes are

$$T_t = (1 - \mu) T_t^R + \mu T_t^{NR} \tag{15}$$

[12] Λ_{t+i} can be interpreted as the Lagrange multiplier equivalent to the marginal utility of consumption.

The fact that only Ricardian households hold government bonds, accumulate capital, invest and receive dividends implies the following aggregate expressions

$$B_t = (1-\mu)\, B_t^R \tag{16}$$

$$K_{t+1} = (1-\mu)\, K_{t+1}^R \tag{17}$$

$$I_t = (1-\mu)\, I_t^R \tag{18}$$

$$Div_t = (1-\mu)\, Div_t^R \tag{19}$$

3.1.3 Firms

Final goods sector

Firms produce the final good from bundled intermediate goods such that

$$Y_t = \left[\int_0^1 \left(y_t^j \right)^{\frac{1}{\left(1+\lambda_{p,t}\right)}} dj \right]^{1+\lambda_{p,t}} \tag{20}$$

Where y_t^j is the quantity of domestic intermediate goods j and $\lambda_{p,t}$ a stochastic parameter representing mark-up in goods market with $\lambda_{p,t} = \lambda_p + \eta_t^p$ and the shock η_t^p is assumed to be i.i.d. normal. Using the previous relation in the firms' profit maximization expression yields the following equation

$$y_t^j = \left(\frac{p_t^j}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{21}$$

Replacing in the expression of the nominal value of the final good $p_t y_t = \int_0^1 p_t^j y_t^j dj$ leads to

$$P_t = \left[\int_0^1 \left(p_t^j \right)^{-\frac{1}{\lambda_{p,t}}} dj \right]^{-\lambda_{p,t}} \tag{22}$$

Intermediate goods

Intermediate goods' producers use the following production technology based on labor and capital services rent by households.

$$y_t^j = \varepsilon_t^a K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi \tag{23}$$

Φ are fixed costs of production and the technology shock is $\ln\left(\varepsilon_t^a\right) = \rho_a \ln\left(\varepsilon_{t-1}^a\right) + \eta_t^a$ Minimizing the total cost $W_t L_{j,t} + R_t^k K_{j,t}$ subject to the production technology yields the following expression

$$\frac{W_t L_{j,t}}{R_t^k K_{j,t}} = \frac{1-\alpha}{\alpha} \tag{24}$$

The firms' marginal costs correspond to

$$MC_t = \frac{{W_t}^{1-\alpha} R_t^{k\alpha}}{\varepsilon_t^a \alpha^\alpha (1-\alpha)^{(1-\alpha)}} \tag{25}$$

Nominal profits are expressed as

$$\Pi_t^j = \left(p_t^j - MC_t\right)\left(\frac{p_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} (Y_t) - MC_t \Phi \tag{26}$$

Based on a Calvo price setting model with partial indexation, a share of intermediate goods firms can re-optimize their price with a probability $1-\xi_p$ to a new level $\tilde{p}_t^j$. The remaining share of firms index the price to the previous period's inflation with a probability ξ_p. The former category of firms chooses their price by solving the problem of maximization of future profits that will be generated by this optimal price.

$$E_t \sum_{i=0}^{\infty} \beta^i \rho_{t+i} \xi_p^i \left(p_{t+i}^j - MC_{t+i}\right) y_{t+i}^j = 0 \tag{27}$$

The expected profits are discounted using the rate $\beta\rho_t$ with $\rho_{t+i} = \frac{\lambda_{t+i}}{\lambda_t}\frac{1}{P_{t+i}}$

The first order condition is given by

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} y_{t+i} \left(\frac{\tilde{p}_t^j}{P_t} \frac{\left(P_{t-1+i}/P_{t-1}\right)^{\gamma_p}}{P_{t+i}/P_t} - \left(1+\lambda_{p,t+i}\right) mc_{t+i}\right) = 0 \tag{28}$$

And the aggregate price index is given by

$$(P_t)^{-\frac{1}{\lambda_{p,t}}} = \xi_p \left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p}\right)^{-\frac{1}{\lambda_{p,t}}} + \left(1-\xi_p\right)\left(\tilde{p}_t^j\right)^{-\frac{1}{\lambda_{p,t}}} \tag{29}$$

3.1.4 Monetary policy

Monetary policy sets the gross nominal interest rate based on the following rule

$$R_t = \max\left\{1, R_{t-1}{}^{\phi_r}\left[\frac{\pi}{\beta}\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{(1-\phi_r)}\right\} \tag{30}$$

Where ϕ_r captures the degree of interest rate smoothing, ϕ_π measures the response of monetary policy to inflation and ϕ_Y to output (both expressed as deviations from the steady state).

3.1.5 Fiscal policy

The government budget constraint is defined as

$$\frac{B_t/R_t}{P_t} + T_t = \frac{B_{t-1}}{P_t} + G_t \tag{31}$$

As in Gali et al. (2007), the fiscal policy rule is assumed to be as follows

$$t_t = \phi_b b_t + \phi_g g_t \tag{32}$$

With $t_t \equiv (T_t - T)/Y$, $g_t \equiv (G_t - G)/Y$, $b_t \equiv ((B_t/P_{t+1}) - (B/P))/Y$ and the parameters ϕ_b and ϕ_g are positive coefficients. Government spending is assumed to follow an exogenously given AR(1) process

$$g_t = \rho_g g_{t-1} + \eta_t^g \tag{33}$$

3.1.6 Market equilibrium

The final goods market will be in equilibrium if production equals demand by households and the government such that

$$Y_t = C_t + G_t + I_t + \frac{\Theta_k}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{34}$$

3.2 Extended model with an alternative fiscal rule

To assess the importance of the fiscal rule specification, I extend the baseline model by including distortionary taxation and additional rules in a subsequent step. In that case, the Ricardian household's budget constraint becomes

$$\frac{M_t^R}{P_t} + \frac{B_t^R/R_t}{P_t} = \frac{M_{t-1}^R}{P_t} + \frac{B_{t-1}^R}{P_t} + (1-\tau_{w,t})\, w_t L_t^R + (1-\tau_{k,t})\, r_t^k K_t^R - \frac{\Theta_k}{2}\left(\frac{K_{t+1}^R}{K_t^R} - 1\right)^2 K_t^R$$
$$+ (1-\tau_{k,t})\, Div_t^R + \tau_{k,t}\delta K_t^R - T_t^R - (1+\tau_{c,t})\, C_t^R - \frac{K_{t+1}^R - (1-\delta)\, K_t^R}{x_t} \tag{35}$$

Where $\tau_{c,t}$ $\tau_{w,t}$ and $\tau_{k,t}$ are consumption, labor and capital income taxes, respectively. Non-Ricardian households' budget constraint is

$$(1+\tau_{c,t})\, C_t^{NR} = (1-\tau_{w,t})\, w_t L_t^{NR} - T_t^{NR} \tag{36}$$

The government budget constraint becomes

$$\frac{B_t/R_t}{P_t} + T_t + \tau_{c,t}C_t + \tau_{k,t}Div_t + \tau_{w,t}w_tL_t - \tau_{k,t}\delta K_t + \tau_{k,t}r_t^kK_t = \frac{B_{t-1}}{P_t} + G_t \tag{37}$$

The fiscal rule is also changed. As in Kliem and Kriwoluzky (2014), government spending, transfers and consumption tax rate are assumed to respond to deviations of output and debt from their steady states. Labor and capital tax rates are assumed to respond to deviations of labor hours and investment, respectively, in addition to debt.

$$t_t = \rho_{tr}t_{t-1} + (1-\rho_{tr})\left(\rho_{try}\hat{Y}_t + \rho_{trb}b_{t-1}\right) + \eta_t^{tr} \tag{38}$$

$$\hat{\tau}_{c,t} = \rho_{ct}\hat{\tau}_{c,t-1} + (1-\rho_{ct})\left(\rho_{cty}\hat{Y}_t + \rho_{ctb}b_{t-1}\right) + \eta_t^{ct} \tag{39}$$

$$\hat{\tau}_{w,t} = \rho_{wt}\hat{\tau}_{w,t-1} + (1-\rho_{wt})\left(\rho_{wtl}\hat{L}_t + \rho_{wtb}b_{t-1}\right) + \eta_t^{wt} \tag{40}$$

$$\hat{\tau}_{k,t} = \rho_{kt}\hat{\tau}_{k,t-1} + (1-\rho_{kt})\left(\rho_{ktI}\hat{I}_t + \rho_{ktb}b_{t-1}\right) + \eta_t^{kt} \tag{41}$$

$$g_t = \rho_g g_{t-1} + \left(1-\rho_g\right)\left(\rho_{gy}\hat{Y}_t + \rho_{gb}b_{t-1}\right) + \eta_t^{g} \tag{42}$$

4 Model preliminary simulation with no binding constraint

4.1 Log-linearization of the baseline model around the steady state

In a first step, the model is log-linearized and the Ricardian and non-Ricardian consumption equations are combined (more details are provided in Appendix B). The resulting equations are as follows.

$$\Theta_k \hat{K}_t = (1+\beta)\,\Theta_k E_t \hat{K}_{t+1} - \beta \Theta_k E_t \hat{K}_{t+2} - \beta r^k E_t \hat{r}^k_{t+1} + \hat{R}_t - E_t \hat{\pi}_{t+1} + \beta(1-\delta) E_t \hat{x}_{t+1} - \hat{x}_t \tag{43}$$

$$\begin{aligned} \frac{C}{Y}\hat{C}_t = (1-\mu)\left[\frac{m}{Y\sigma_m}\left[\hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1} - \left(\hat{\varepsilon}^B_t - \hat{\varepsilon}^B_{t-1}\right) - \left(\hat{\varepsilon}^M_t - \hat{\varepsilon}^M_{t-1}\right)\right]\right. \\ \left. + \left[\hat{Y}_t - \frac{I}{Y}\hat{I}^R_t\right]\right] + \mu\left[\frac{\theta(1-\alpha)}{1+\lambda_p}\left(\hat{L}_t + \hat{w}_t\right) + \frac{b_t}{R} - b_{t-1}\right] - g_t \end{aligned} \tag{44}$$

$$\frac{K}{Y}E_t\hat{K}_{t+1} = (1-\delta)\frac{K}{Y}\hat{K}_t + \frac{I}{Y}\hat{x}_t + \frac{I}{Y}\hat{I}_t \tag{45}$$

$$\hat{L}_t = -\hat{w}_t + \hat{r}^k_t + \hat{K}_t \tag{46}$$

$$\hat{Y}_t = \theta\left[\hat{\varepsilon}^a_t + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t\right] \tag{47}$$

$$\hat{R}_t = \phi_r \hat{R}_{t-1} + (1-\phi_r)\left(\phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t\right) \tag{48}$$

$$\frac{b_t}{R} = g_t\left(1-\phi_g\right) + b_{t-1}\left(1-\phi_b\right) \tag{49}$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + g_t + \frac{I}{Y}\hat{I}_t \tag{50}$$

$$\hat{\pi}_t = \frac{\beta}{(1+\beta\gamma_p)}E_t\hat{\pi}_{t+1} + \frac{\gamma_p}{(1+\beta\gamma_p)}\hat{\pi}_{t-1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)\left[\eta^p_t + \alpha\hat{r}^k_t + (1-\alpha)\hat{w}_t - \hat{\varepsilon}^a_t\right]}{\xi_p\left(1+\beta\gamma_p\right)} \tag{51}$$

$$\begin{aligned} \hat{w}_t = \frac{\beta}{(1+\beta)}E_t\hat{w}_{t+1} + \frac{1}{(1+\beta)}\hat{w}_{t-1} + \frac{\beta}{(1+\beta)}E_t\hat{\pi}_{t+1} - \frac{(1+\beta\gamma_w)}{(1+\beta)}\hat{\pi}_t + \frac{\gamma_w}{(1+\beta)}\hat{\pi}_{t-1} \\ + \left[\hat{\varepsilon}^L_t + \sigma_l\hat{L}_t + \frac{\sigma_c}{(1-h)}\left(\hat{C}_t - h\hat{C}_{t-1}\right) + \eta^w_t - \hat{w}_t\right]\frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\beta)(1+\sigma_l)\left(\frac{1+\lambda_w}{\lambda_w}\right)} \end{aligned} \tag{52}$$

$$\hat{\varepsilon}^B_t = \rho_B\hat{\varepsilon}^B_{t-1} + \hat{\eta}^B_t \tag{53}$$

$$\hat{\varepsilon}^L_t = \rho_L\hat{\varepsilon}^L_{t-1} + \hat{\eta}^L_t \tag{54}$$

$$\hat{\varepsilon}^M_t = \rho_M\hat{\varepsilon}^M_{t-1} + \hat{\eta}^M_t \tag{55}$$

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \hat{\eta}^x_t \tag{56}$$

$$\hat{\varepsilon}^a_t = \rho_a \hat{\varepsilon}^a_{t-1} + \hat{\eta}^a_t \tag{57}$$

$$g_t = \rho_g g_{t-1} + \eta^g_t \tag{58}$$

$$\lambda_{p,t} = \lambda_p + \eta^p_t \tag{59}$$

$$\lambda_{w,t} = \lambda_w + \eta^w_t \tag{60}$$

4.2 Baseline model calibration, simulation and basic mechanisms

I proceed with a preliminary simulation of the baseline model without any binding constraint to shed some light on the model's underlying mechanisms. This preliminary analysis is also useful to distinguish between results that are explained by the model specification and those that are explained by the data in later steps. The calibrated parameters are taken from the existing literature. The discount factor β is set to the value of 0.99, the elasticity of output with respect to capital α to 0.3, the depreciation rate of capital δ to 0.025. The steady-state ratios of consumption, investment and government spending over output are assumed to be respectively equal to 0.57, 0.2, 0.22 (corresponding to the average ratios calculated from collected data on Japan[13]). The steady-state ratio of money to output is taken as 0.9 (calculated based on average M2 to GDP over the period) and the ratio of capital to output is set to 2.2 as in previous similar studies.[14]

λ_w is set to the value of 0.5 and the share of non-Ricardian households to 0.35 (as in Iwata (2009)). The share of fixed costs over total production is considered equal to 0.45[15] and λ_p to 0.45.[16]. The capital adjustment cost parameter Θ_k is considered to be equal to 10 as in Ireland (2001) and the inverse of the elasticity of money holding with respect to the interest rate σ_M is set to 2 (Kuo and Miyamoto (2016)) To calibrate the remaining parameters, the posterior means obtained by Iiboshi et al. (2006) from Japanese data are used (see Table 1). Finally, parameters of the fiscal rule are taken from Galí et al. (2007) and those of the monetary policy rule from Smets and Wouters (2003).

Impulse response functions to a one standard deviation positive shock to government spending

[13] See following section.

[14] See Watanabe (2009), Iwata (2009), Iiboshi et al. (2006), Smets and Wouters (2003).

[15] As in Iiboshi et al. (2006), Coenen and Straub (2005), Smets and Wouters (2003).

[16] Such that $1 + \lambda_p = 1 + \frac{\Phi}{Y}$, implying zero profits at the steady state.

are shown on Figure 1. Based on zero steady state values, these preliminary results indicate that an increase in government spending induces a decline in consumption, investment, inflation, capital and real wages. At the same time, the shock generates a hike in labor supply and output and raises the public debt level. Most of these outcomes are in line with the model of Smets and Wouters (2003) and Iiboshi et al. (2006) (for Japan).

From these responses and equation (44), it appears that the decline in private consumption is a direct consequence of a government spending crowding-out effect which manifests through higher taxes and higher public debt (both of which affect households' consumption through the budget constraint). The reduction in inflation (explained by lower wages) also negatively affects consumption through a positive change in money holdings, but this impact remains relatively weak in magnitude, even if the share of Ricardian households is increased. The positive effect of higher labor is offset by the decline in wages, resulting in an overall negative impact. This impact is also relatively small compared to that of government spending. But it can be increased if the parameters of the elasticity of output with respect to capital or the price markup are smaller or if the share of fixed costs in production is higher, since these parameters affect the steady state ratio of labor income over output. The effect of wages and labor also slightly increases with the share of non-Ricardian households (because consumption tracks income more closely).

The decline in consumption is strongly cushioned by the increase in output which enters the Ricardian households consumers' equation through dividends. And this increase in output is also a direct effect of higher government spending. In other words, this model shows that the increase in government spending only stimulates the consumption of households that own firms, through higher profits generated by the higher government demand. On the other hand, the decrease in investment is favorable for consumption since more income is available for Ricardian consumers, but it plays a limited role due to the small magnitude of the response.

Estimation of consumption's impulse response functions for different values of the share of rule-of-thumb consumers is shown on Figure 2. No matter the value of this parameter, response of consumption does not get positive. On the contrary, the decline in consumption is more marked when the parameter increases. This is unsurprising since the stimulating effect on the economy in this model mainly affects Ricardian households through firms' profits.

4.3 Basic workings of the extended model with distortionary taxation

I use the same calibration described previously to simulate the extended model with distortionary taxation. In addition, the steady state tax rates are set to the values $\tau_w = 0.32$, $\tau_c = 0.08$, $\tau_k = 0.61$.[17] At this stage, autoregressive parameters in the tax rules are set to 0.8[18] and the remaining parameters of the rules to 0.1.[19] The changes to the log-linearized model are included in Appendix C.

Figure 3 shows a comparison between the impulse responses in the extended version of the model and those of the baseline model. On impact, consumption decreases more strongly in the baseline model, but the opposite becomes true after two quarters. The primary reason for this is that the decline in the baseline model (as opposed to the second model) mainly results from the direct impact of government spending (through the fiscal rule). Therefore, this response weakens as spending converges to the steady state. Conversely, based on the aggregate consumption equation of the second model (equation (179) in Appendix C), consumption's decline is mainly driven by the declining wages (a direct effect on income and to a lesser extent an indirect effect through lower inflation).

As before, the increase in output in the extended version is mainly a result of the increase in government spending since both consumption and investment decline (the same shock is applied in both models). The slight difference in output response results from a higher response of labor in the extended model. Another reason is that government spending converges more rapidly to the steady state in the baseline model compared to the extended model. Finally, lump-sum taxes and tax rates gradually increase after the spending shock. They all contribute equivalently to the debt repayment except the capital income tax rate which increases with a smaller amount compared to the other taxes. The divergence of capital in the baseline model, although not observed in the extended model, is consistent with the response in the benchmark model of Smets and Wouters (2003) and also in Iiboshi et al. (2006).

5 Markov-Switching Bayesian approach

I then proceed with the model estimation through Bayesian methods using Japanese data from 1980 Q1 to 2020 Q3. Since Japan experiences a long ZLB episode during the period of study (nominal

[17] Taken from Iwata (2009).
[18] **?**, Iwata (2009), Kliem and Kriwoluzky (2014).
[19] Prior means in Iwata (2009).

interest rates have been close to zero since 1995 Q3), relying on non-linear solution and estimation techniques is necessary.

5.1 Solution methodology

Binning and Maih (2016) provide a brief description of the main solution approaches to solve non-linear DSGE models and which include: extended path solutions, piecewise-linear solutions, anticipated shocks, projection methods and regime-switching methods. In the present study, the regime-switching model approach is adopted. Based on Maih (2015), the regime-switching dynamic stochastic general equilibrium model (RS-DSGE) model can be expressed as follows.

$$E_t \sum_{r_{(t+1)}=1}^{h} \pi_{r_t,r_{t+1}}(\Omega_t)\tilde{d}_{r_t}(\nu) = 0 \tag{61}$$

$\tilde{d}_{r_t} : \mathbb{R}^{(n_\nu)} \rightarrow \mathbb{R}^{(n_d)}$ is a $n_d x1$ vector of possibly non-linear functions of their argument ν

The variable r_t corresponds to the regime at time t

$\pi_{r_t,r_{t+1}}(\Omega_t)$ is the transition probability for going from regime r_t at time t to regime r_{t+1} in the following period. It depends on the information set at time t represented by Ω_t. The special case where transition probabilities are constant is referred to as the Markov-Switching DSGE model.

The argument ν represents a n_ν vector that includes the stacked vectors of the following categories of variables: static variables (s_t), forward-looking variables (f_t), predetermined variables (p_t), variables that are both predetermined and forward looking (b_t), shocks (ε_t) and switching parameters appearing with a lead in the model ($\theta_{r_{t+1}}$).

For $d_{r_t,r_{t+1}}$, a $n_d x1$ vector such that $d_{r_t,r_{t+1}} \equiv \pi_{r_t,r_{t+1}}(\Omega_t)\,\tilde{d}_{r_t}$, the objective function becomes

$$E_t \sum_{r_{t+1}=1}^{h} d_{r_t,r_{t+1}}(\nu) = 0 \tag{62}$$

It is assumed that if agents have information for all or some of the shocks k periods ahead of time, then the state variables vector z_t can be expressed as a $n_z x1$ vector that contains the variables (p_{t-1}), (b_{t-1}), a perturbation parameter σ, the present shock and k future shocks, with $n_z = n_p + n_b + (k+1)\,n_\varepsilon + 1$. This is an important distinction with the traditional approach based on "news shocks", usually included in the system by adding additional terms to the law of motion of the shock process.

Another difference is that anticipated shocks are considered as structural shocks and therefore can be related to other parts of the system.

For $y_t\left(r_t\right)$ being the $n_y x1$ vector of all endogenous variables with $n_y = n_s + n_p + n_b + n_f$, the solution would be expressed as a function of the state variables, in the following form

$$y_t\left(r_t\right) \equiv \begin{bmatrix} s_t\left(r_t\right) \\ p_t\left(r_t\right) \\ b_t\left(r_t\right) \\ f_t\left(r_t\right) \end{bmatrix} = \Im^{r_t}\left(z_t\right) \equiv \begin{bmatrix} S^{r_t}\left(z_t\right) \\ P^{r_t}\left(z_t\right) \\ B^{r_t}\left(z_t\right) \\ F^{r_t}\left(z_t\right) \end{bmatrix} \tag{63}$$

To solve the model, a perturbation method is used to approximate the decision rules above. To do so, the vector y_t is decomposed to isolate the predetermined from the forward-looking components. Similarly the vector z_t is also partitioned so that all variables of the system can be expressed in terms of the state variables vector z_t. Thus, the vector ν becomes

$$\nu = \begin{pmatrix} \lambda_{bf}\Im^{r_{t+1}}\left(z_{t+1}\left(z_t\right)\right) \\ \Im^{r_t}\left(z_t\right) \\ m_p z_t \\ m_b z_t \\ m_{\varepsilon,0} z_t \\ \theta_{r_{t+1}} m_\sigma z_t \end{pmatrix} \tag{64}$$

Where λ_{bf} is the matrix that selects the variables b and f and m_g refers to a $n_g x n_z$ matrix that selects the g type variables from the state variatbles vector z_t. The objective function therefore becomes

$$E_t \sum_{r_{t+1}=1}^{h} d_{r_t,r_{t+1}}\left(\nu\left(z_t, u\right)\right) = 0 \tag{65}$$

Using this expression, successive Taylor approximations around an approximation point can be performed up to higher orders to find the perturbation solutions. Several algorithms to solve the approximated problem are discussed in Maih (2015) and Farmer et al. (2011). In the present study, the model is solved using a functional iteration algorithm that offers the advantage of converging fast when a solution is found.[20]

5.2 Calibration, prior specification and data

Bayesian estimation of the model is based on the following quarterly macroeconomic series: output, private consumption, private investment, nominal interest rate, labor hours and real hourly wage. Data for the first three series are obtained from the database of the Cabinet Office and divided by the labor force. Labor hours are calculated based on the series of aggregate weekly labor hours for non-agricultural industries obtained from the Portal Site of Official Statistics of Japan. Real wages are calculated based on real wage indices from the monthly labor survey of the Ministry of Health, Labor and Welfare and the value of real hourly wage at 2020Q3 taken from statistics of the Japan Institute for Labor Policy and Training. The short-term nominal interest rates correspond to the overnight call rates taken from the Bank of Japan database. All variables are transformed to logarithms and detrended using a one-sided Hodrick-Prescott filter,[21] except the interest rate, that is detrended without log-transformation.

As is usual in the literature, the parameter values of β, α, δ, λ_p, λ_w, Θ_k and the ratios over output are fixed, using the calibration values of the previous section. The prior distributions for estimated parameters are reported on Table 3. In setting these values, I follow similar studies based on variants of Smets and Wouters (2003) for Japan such as Iiboshi et al. (2006), Iwata (2009) and Watanabe (2009). All standard deviations of shocks are assumed to follow an inverted gamma distribution with degrees of freedom equal to two. The distribution of autoregressive parameters is on the other hand assumed to follow a beta distribution with mean 0.8 and standard error 0.1. Prior of the inverse of the elasticity of money holding with respect to the interest rate σ_M is set based on Kuo and Miyamoto (2016) and the standard error of money demand shock ${\eta_t}^M$ from Kano (2019). Priors for the fiscal rule parameters are taken from Coenen and Straub (2005) in the case of the baseline model, and from Iwata (2009) in the case of the extended model.

Priors for the monetary policy rule in the first regime are taken from Smets and Wouters (2003). At the ZLB, parameters of the monetary rule are more difficult to obtain. I therefore decide to keep the same type of probability distribution as in the first regime but set estimates of the mean and

[20] The mfi algorithm implemented in the Rationality In Switching Environments (RISE) Toolbox.

[21] Non-causal filters such as the two-sided HP or Baxter-King filters are not used since they take future values to construct present data.

standard deviation based on a regression model. This necessitates a definition of the ZLB period. Looking at interest rates data, it can be noted that interest rates have been almost steadily declining since 1991Q3.[22] But despite this decline, interest rates were not close to zero in the first quarters of the 1991Q3-2020Q3 period. Within this period, one major drop occurred in 1995Q3 when interest rates went below 1%. They then stayed below this level for the remaining part of the sample period. For this reason, many studies define the ZLB period as the entire period that follows this drop, and which is characterized by interest rates that are within the unit interval (in percentage). For example, all the post-1995Q4 period is chosen as the ZLB period in Miyamoto et al. (2018), Borağan Aruoba et al. (2018) and Adjemian and Juillard (2010). On the other hand, Hayashi and Koeda (2014) follow a different approach and define the ZLB as the period where interest rates are below the critical rate of 0.05%.[23]

In the present study, the chosen period to estimate the model is 1995Q3-2016Q1 (the remaining quarters are excluded because interest rates are negative). One difficulty that arises from the choice of this period is the changing behavior of inflation (see Figure 4). More specifically, the long deflationary period imposes a negative relationship between interest rates and inflation, which in reality does not apply to the whole low interest rates period. To overcome this problem, a threshold regression approach is adopted. That way, it is possible to distinguish between different patterns of relationships between the interest rate and the monetary rule variables depending on the interest rate level.[24] Results of this regression are reported on Table 2. Prior means of the monetary rule parameters are set based on an average of these coefficients. Finally, the Markov process is defined based on constant transition probabilities for which the prior is defined as on Table 3.

[22]By applying a Zivot-Andrews unit root test over the whole data series, I also find one breakpoint at 1992Q1.

[23]This corresponds to the rate below which bank reserves in Japan are greater than required reserves (and often several times greater). Based on this definition, the ZLB corresponds to the following episodes: March 1999 - July 2000, March 2001 - June 2006, and December 2008 to the end of the period of study (2014).

[24]The most common approach in estimating the monetary policy rule is an IV tobit regression (e.g., Kiesel and Wolters (2014), Kim and Mizen (2010), Kato and Nishiyama (2005)) since the variable of short-term interest rates is left-censored. However, the threshold regression approach is chosen in this study; first because no significant differences are obtained in the estimated coefficients compared to a tobit regression for the selected short time span, and second because the relationship between variables may differ across the different periods of low interest rates.

5.3 Results

5.3.1 Estimation settings and smoothed probabilities

Estimation of the posterior distribution is based on a stochastic optimizer with 1000 iterations. After multiple runs of the optimization procedure, the estimation result with the highest log marginal data density (Laplace approximation) is selected. This corresponds to a value of -1465 for the baseline model and -1346 for the extended model. In a subsequent step, the Metropolis-Hastings algorithm is applied with 1000 parameter draws from the distribution (in addition to a ratio of 0.1 burnin). Finally, impulse response functions are constructed based on random parameter draws (300 replications) from the MCMC simulation.

Then, the smoothed probabilities are calculated to detect the timing of regime shifts. These probabilities are plotted on Figure 5 along with the time series of short-term nominal interest rates. It can be noted that regime 1 switches abruptly at the following dates: 2001Q3-2006Q1, 2009Q3-2011Q2, 2011Q4-2015Q4 and 2016Q3-2018Q2. These shifts suggest that the second regime represents all periods with interest rates below 0.10. There are however two main exceptions. First, the model does not capture the period 1999Q2-2000Q2 in which interest rates are also below the 0.1 threshold. Second, there is a regime change that occurs during the low interest-rates period, at 2016Q2, when interest rates turn negative. Conversely, the model perfectly captures the period 2001Q3-2006Q1 where interest rates are in most quarters close to or below 0.001, with probabilities of being in regime 1 that are close to zero for this entire interval. To conclude, the timing of these shifts suggests that regime 1 corresponds to a period where interest rates are unbounded whereas regime 2 is the ZLB period.

5.3.2 Parameters' posterior mode estimates

Estimated posterior modes are provided on Tables 3 and 4. The estimated parameter of rule-of-thumb consumers appears as lower than the prior mean in the baseline model and higher in the extended model. The parameter of Calvo price stickiness has a small value in both models,[25] suggesting a higher price flexibility than expected. In the extended model, estimates for the fiscal rules are close to the prior in the case of consumption, wage income and capital income tax rates. Conversely, the parameters of response of government spending to deviations in output and public debt are extremely

[25]This parameter is considered by Galí et al. (2007) as an important factor to generate an increase in real wages (if it is sufficiently high) and thereby a positive response in consumption.

small. This implies that budget adjustments made after an increase in public debt are mainly made through tax rates and not government spending. Response of lump-sum taxes to public debt is also relatively smaller than the prior mean. Finally, among fiscal rule variables, lump-sum taxes show the least persistence.

5.3.3 Impact of a government spending shock

I then apply a positive shock to government spending. Although a shock of the same magnitude is applied in both the baseline and extended models (+0.34), there are some differences in terms of responses between both. For example, there is almost no difference between variables' reaction at and outside the ZLB in the baseline model (Figure 6), but a slight difference can be seen between both regimes in the extended model (Figure 7). Nonetheless, the response of consumption on impact is negative in all cases. Even at the ZLB, the model does not produce the crowding-in effect of consumption that has been discussed in some previous studies. It can however be noted that in the extended model, consumption's reaction gets positive over the long-run (after 6 quarters outside the ZLB and 5 at the ZLB). The shape of consumption's response in this case suggests an impact from tax rate adjustments that follow the spending increase (see Figure 8).

The aggregate consumption equations[26] are used to uncover the main contributors to consumption's response. In the baseline model, the decline in consumption (about -0.11), both at and outside the ZLB, is found to be mainly a direct crowding-out effect of government spending (through higher taxes and public debt). This decline is mitigated by a higher output and an increase in labor that offsets the effect of falling real wages (overall positive effect of labor income). In the extended model, consumption drops by 0.12 after the shock in both regimes. This effect results essentially from the increase in lump-sum taxes (negative transfers) and a decline in wages. In particular, it can be seen that the path of consumption closely tracks real wages' response.

Turning to output, a positive reaction is observed in all cases (+0.269 outside the ZLB and +0.266 at the ZLB in the baseline model and +0.31 outside the ZLB and +0.35 at the ZLB in the extended version). The fiscal multiplier on impact is relatively higher in the extended model, especially at the ZLB (where it slighlty exceeds unity with a value of 1.03). This positive response is mostly explained by the government spending increase itself which raises aggregate demand. The higher aggregate

[26]Equation (44) in section 3.4 for the baseline model and equation (179) in Appendix C for the extended model.

demand also induces higher labor, which plays a significant role in explaining output through the production function.

One important distinction between the baseline and the extended model is a different response of capital, investment, inflation and interest rates. Inflation (at the ZLB) and interest rates (outside the bound) respond positively in the extended model as opposed to the baseline model. At the same time, there is almost no difference between the two models in the responses of the rental rate of capital and real wages. This implies that the difference in the reaction of inflation at the ZLB is not explained by the marginal cost and can only be attributed to different inflation's expectations or parameter estimates. It is true that the estimated parameter of the degree of partial indexation of price γ_p is lower in the extended model while Calvo price stickiness ξ_p is slightly higher but this difference is not sufficient in explaining the opposite sign of inflation's response. Therefore, different inflation's response is mainly due to different expectations. The positive response of capital in the second model is also affected by higher expectations, in addition to a higher steady state of the rental rate of capital[27] and more particularly the impact of the capital income tax.[28] The difference between capital's evolution at and outside the bound is explained by the difference in the evolution of the tax rate in both regimes. On the other hand, the path of investment is mainly explained by capital.

Based on these results, I conclude that the fiscal rule specification plays an important role in determining the model's response to fiscal shocks. In a model that does not account for distortionary taxation, a crowding-out effect of investment is obtained, which is consistent with the main findings of Smets and Wouters (2003) and Iiboshi et al. (2006). But the presence of taxes alters this impact by affecting the path of the model's variables. The increase in capital in the extended model has a positive effect on the economy since it eventually leads to an increase in consumption and wages although their immediate response to the spending increase is negative.

Overall, the way consumption reacts to government spending in both the baseline and extended model suggests that Ricardian behavior remains important even after inclusion of rule-of-thumb consumers. Such a conclusion is corroborated by the model's response to a contractionary fiscal policy (a reduction of government spending) as shown on Figure 10. It can be seen from the figure that private consumption responds positively to a cut in government spending. And the shape of this response

[27] Impact of the steady state capital income tax rate.

[28] Capital is a predetermined variable in the model and therefore positively affected by the tax rate at time t.

reveals its relationship with the movement of lump-sum taxes (Figure 9). To sum up, consumption appears to be driven by two key factors in both models: lump-sum taxes and real wages. Real wages play a negative role in this model because they decline after the positive government spending shock, but they can be increased in the long-run if there is a positive accumulation of capital, in which case they contribute to boosting consumption. Finally, output's response is strongly linked to labor.

5.3.4 Effects of tax cut policies

I then simulate different tax cuts and compare their outcomes. In case of a consumption income tax cut (see Figure 11), there is a slight difference between the model's behavior outside and at the ZLB. In the first regime, consumption increases on impact. On the other hand, there is a small drop in output and interest rates. In the second regime, consumption also increases on impact but declines after two quarters to go up again before the tenth quarter. Response of output is negative and more acute than in the first regime. Finally, as interest rates are bounded, a deflationary response is caused by the shock, which further depresses the economy.

A similarity can be observed between consumption's response and the response of real wages. On impact, there is a slight increase of wages and consumption in both regimes resulting from the lower consumption tax. Then the path of both variables is negatively affected by the increase in other taxes. Both variables rise again as taxes converge to their steady states. Therefore, movements in both variables correspond to the inverse of those of taxes which follow a hump-shaped pattern.

Overall, it appears that this policy is ineffective. First, it causes a drop in labor (and thereby in output). Second, even the immediate positive response of consumption is extremely small. This can be justified by the parameterization of the model. The parameters that determine the impact of a consumption tax are small (especially compared to other taxes). The consumption tax rate steady state value is 0.08 (compared to 0.61 for the capital income tax). The steady state ratio of consumption over output is 0.57 (compared to 2.2 for capital). The estimated posterior mode for the standard deviations of shocks is also smaller compared to other taxes. Third, the immediate positive effect of the tax cut on consumption is offset by the negative effect of other tax increases. Finally, at the ZLB, this policy is even more detrimental in the short-run because of the negative reaction of inflation.

Although a consumption tax cut is inefficient in stimulating the economy, it is not as disadvantageous in the short-run as a wage income tax cut (Figure 12). A wage income tax cut causes a drop in consumption and output both at and outside the ZLB. The behavior of interest rates and inflation is the same as in the consumption tax cut. There is a negative response of labor, capital and investment on impact, causing the decline in output.

Nevertheless, in the long-run, at the ZLB, the impact of both consumption tax and wage income tax cuts becomes positive. This can be justified by an increase in investment after the second quarter which increases the capital stock and inverts the response of inflation, real wages, consumption and output. The higher investment is linked to the increase in capital income tax rate which is more marked at the ZLB. The model specification implies that a future increase in the capital income tax rate has a negative effect on present capital, but a present increase has a positive effect because of the expected decrease of the tax.

The most advantageous policy in all tax cut policies is a capital income tax cut (Figure 13). Although it causes a drop in consumption on impact, consumption tends to increase over time both at and outside the ZLB. The response of output is positive over the whole simulation period. One main driver of these dynamics is the jump in investment that follows the policy, which is accompanied by higher labor. Over time, there is a positive accumulation of capital, which induces a progressive decline in the rental rate of capital. As opposed to the previous tax cuts, inflation's response is positive at the ZLB. Still, this inflationary effect does not significantly alter the model's behavior which stays almost the same across both regimes.

Finally, one can remark that successful policies are those that increase inflation (at the ZLB or interest rates outside the bound) and generate higher investment. Capital accumulation plays an important role by shifting wages and production upward.

5.3.5 Comparison between policies of a government spending increase and a capital income tax cut

I compare between the effect of a government spending increase and a capital income tax cut. Plots are reported on Figure 14. On impact, many responses are almost at the same level for both measures (consumption, output, labor, rental rate of capital, real wages and interest rates). However, after a few quarters, a capital income tax cut has a better stimulating effect on the economy than a government spending increase. This mainly results from the higher increase in investment and the ensuing capital accumulation which shifts the economy's productive potential upward, resulting in a higher level of output. This process also leads to a substantial increase in real wages over the long run, which eventually decreases labor. However, one negative aspect of this measure is that it generates a higher level of debt compared to a government spending increase. As a result, tax rates also increase more significantly to repay the debt, as shown on Figure 15.

5.3.6 Variance decomposition of private consumption

Based on the extended model, I calculate the variance decomposition of private consumption, output, investment and real wages to detect what structural shocks drive these variables. Results are shown in Table 5 with various horizons (short-run: one quarter, medium-run: 10 quarters and long-run: 20 quarters).

One significant difference between the obtained results and those of the benchmark model of Smets and Wouters (2003) is that preference shocks are not found to be the main drivers of consumption. Conversely, labor supply shocks and technology shocks contribute significantly to the variance of this variable in both regimes, especially in the first period. In addition, the decomposition also shows that the respective shares of shocks to the marginal efficiency of investment and capital income tax shocks progressively grow over time, in both regimes. The main difference between both regimes lies in the fact that lump-sum taxes play a much more important role in the variation of private consumption at the ZLB than outside it. Finally, Sugo and Ueda (2007) that used a model close to Smets and Wouters (2003) for Japan[29] also found consumption to be mainly driven by technology shocks, but the weight of preference shocks was not as insignificant in their model as in the present model.

Overall, fiscal shocks do not affect private consumption significantly. In comparison, they contribute more notably in variations of output and investment. As shown in Table 5, output is significantly affected by wage income tax rate fluctuations, in addition to government spending (in the short-run), capital income tax rate shocks (over the long-run), and also lump-sum taxes at the ZLB. In the case of the investment variable, there is a significant contribution of the capital income tax shock (in addition to lump-sum taxes at the ZLB). Finally, the variance decomposition of consumption is closer to the one of real wages. Real wages' variation is mainly explained by a technology shock, a shock to the mark-up in goods market, a shock to the marginal efficiency of investment (over the

[29] Sugo and Ueda (2007) made two main modifications: they used actual capital utilization rate data for estimation, and they incorporated a negative correlation between capital utilization rates and rental costs of capital.

long-run) and lump-sum taxes at the ZLB.

Figure 16 shows the response of private consumption to the shocks that contribute the most to its variance. It can be seen that an exogenous positive technology shock generates a positive and high response in consumption.[30] On the other hand, a labor supply shock generates a small negative response, which is consistent with Iiboshi et al. (2006). It is important to note that this shock is accompanied by a decrease in labor hours.[31] A shock to the marginal efficiency of investment has a positive and progressively growing impact on consumption (hump-shaped). Finally, response of consumption to a negative shock to lump-sum taxes is negative at the ZLB.

6 Conclusion

In this paper, I study the effects of fiscal shocks on private consumption through a standard new Keynesian model that includes rule-of-thumb consumers and the ZLB on interest rates. A Markov-switching approach is used for the estimation. In addition, to assess the importance of the fiscal rule specification, findings from a baseline model with lump-sum taxes are compared with those from an extended model with distortionary taxation.

Results indicate that the presence of Non-Ricardian consumers is not sufficient to reduce the weight of Ricardian behavior which still plays a major role in shaping consumption's response. The presence of Non-Ricardian consumers does however strengthen the dependence of consumption on wage fluctuations. The inclusion of distortionary taxation also alters the model's behavior through movements in the tax rates.

It is particularly noted that successful fiscal policies are those that increase investment and inflation (at the ZLB) or interest rates (outside the bound). Investment plays a fundamental role in the model as it affects the economy's production level through the capital accumulation process.

There is no significant difference between the model's behavior at and outside the ZLB after a government spending increase or a capital income tax cut. But a difference across regimes can be observed after a consumption tax or a wage income tax cut. Outside the ZLB, both measures have a negative impact on the economy. At the ZLB, both policies have harmful effects in the short-run but the potential to yield better results over the long-run. A comparison between the different policies

[30] A technology shock also induces a positive response in output and a negative response in public debt.
[31] It affects the model through the wage equation.

indicates that a capital income tax cut is the most beneficial measure because of its stimulating impact on investment. Nonetheless, it generates a much higher level of public debt which causes future tax hikes.

Finally, a variance decomposition of private consumption reveals that it is mainly driven by shocks to the labor supply and technology shocks. At the ZLB, shocks to lump-sum taxes are also found to play an important role. Among these shocks, a technology shock is the one that generates the highest positive response in consumption.

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Tables

TABLE 1: Calibration of baseline model

Parameter		Calibration	Smets and Wouters (2003)*
Structural parameters			
Habit of consumption	h	0.795	0.592
Inverse of elasticity of work effort	σ_L	2.077	2.503
Relative risk aversion of Ricardian households	σ_c	1.912	1.391
Inverse of the elasticity of money holding with respect to the interest rate	σ_M	2	____
Share of fixed cost in production +1 $\left(1+\frac{\Phi}{Y}\right)$	θ	1.45	1.417
Share of non-Ricardian consumers	μ	0.35	____
Degree of partial indexation of price	γ_p	0.579	0.477
Degree of partial ind. of wage	γ_w	0.581	0.728
Calvo price stickiness	ξ_p	0.791	0.905
Calvo wage stickiness	ξ_w	0.275	0.742
Policy Parameters			
Response of monetary policy to inflation	ϕ_π	1.70	1.688
Autoregressive coefficient of interest rate	ϕ_r	0.80	0.956
Response of monetary policy to output gap	ϕ_Y	0.125	0.098
Fiscal rule parameter with respect to debt	ϕ_b	0.33	____
Fiscal rule par. with respect to government spending	ϕ_g	0.10	____
Shock persistence			
Autoregressive parameter of preference shock	ρ_B	0.214	0.838
Autoregressive parameter of labor supply shock	ρ_L	0.406	0.881
Autoregressive parameter of shock to the marginal efficiency of investment	ρ_x	0.933	0.913
Autoregressive parameter of technology shock	ρ_a	0.818	0.811
Autoregressive parameter of government spending shock	ρ_g	0.793	0.943
Autoregressive parameter of a money demand shock	ρ_M	0.800	____

* Posterior mean

TABLE 2: Threshold regression estimation of the monetary policy rule using Japanese data over the period 1995Q3-2016Q1

IR threshold	**<0.001**			**<0.01**			**<0.1**			**Average**
	Coeff	S.E.	P-value	Coeff	S.E.	P-value	Coeff	S.E.	P-value	Coeff
ϕ_r	0.28***	$5x10^{-15}$	0	0.33***	0.02	0	0.58**	0.18	0.0021	0.40
ϕ_π	-0.001***	$3x10^{-18}$	0	-0.001***	0.0003	0	0.012	0.008	0.13	0.003
ϕ_y	0.00003***	$6x10^{-18}$	0	0.0013*	0.0005	0.017	0.015*	0.006	0.02	0.005

*** p<0.001, **p<0.01, *p<0.05

TABLE 3: Prior distributions and posterior mode of parameters for the Markov-switching baseline model estimation

Parameter	Distribution	Prior mean	Standard deviation	Posterior Mode	Mode SD
Structural parameters					
h	beta	0.795	0.1	0.624	0.082
σ_l	gamma	2	0.375	2.251	0.947
σ_c	gamma	1.5	0.2	2.078	0.708
σ_M	gamma	2	0.5	1.452	0.939
θ	gamma	1.45	0.25	1.613	0.471
μ	beta	0.2	0.05	0.100	0.041
γ_p	beta	0.75	0.15	0.819	0.083
γ_w	beta	0.75	0.15	0.823	0.189
ξ_p	beta	0.75	0.15	0.011	0.069
ξ_w	beta	0.75	0.15	0.018	0.122
Fiscal Policy Parameters					
ϕ_b	Inverse gamma	0.1	2	0.034	0.179
ϕ_g	normal	0.1	0.05	0.099	0.075
Shock persistence					
ρ_B	beta	0.8	0.1	0.998	0.270
ρ_L	beta	0.8	0.1	0.295	0.201
ρ_x	beta	0.8	0.1	0.990	0.054
ρ_a	beta	0.8	0.1	0.846	0.186
ρ_g	beta	0.8	0.1	0.745	0.132
ρ_M	beta	0.8	0.1	0.931	0.067
Standard deviation of shocks					
η_t^B	Inverse gamma	0.2	2	10.000	0.791
η_t^L	Inverse gamma	1	2	6.962	1.155
η_t^x	Inverse gamma	0.1	2	8.545	2.960
η_t^a	Inverse gamma	0.4	2	0.888	0.868
η_t^g	Inverse gamma	0.3	2	0.339	0.099
η_t^p	Inverse gamma	0.15	2	0.050	3.149
η_t^w	Inverse gamma	0.25	2	0.084	0.470
η_t^M	Inverse gamma	0.5	2	10.000	0.543
Switching Monetary Policy Parameters					
Regime 1					
ϕ_π	normal	1.7	0.1	1.790	0.201
ϕ_r	beta	0.8	0.1	0.571	0.111
ϕ_Y	normal	0.125	0.05	0.120	0.097
Regime 2					
ϕ_π	normal	0.003	0.001	0.006	0.001
ϕ_r	beta	0.4	0.1	0.857	0.224
ϕ_Y	normal	0.005	0.001	0.005	0.002
Transition probabilities					
TP (1 to 2)	beta	0.005	0.15	0.030	0.161
TP (2 to 1)	beta	0.005	0.15	0.247	0.239

TABLE 4: Prior distributions and posterior mode of parameters for the Markov-switching extended model estimation

Parameter	Distribution	Prior mean	Standard deviation	Posterior Mode	Mode SD
Structural parameters					
h	beta	0.795	0.1	0.490	0.111
σ_l	gamma	2	0.375	1.602	0.348
σ_c	gamma	1.5	0.2	1.817	0.236
σ_M	gamma	2	0.5	4.563	0.552
θ	gamma	1.45	0.25	1.999	0.151
μ	beta	0.2	0.05	0.422	0.036
γ_p	beta	0.75	0.15	0.488	0.202
γ_w	beta	0.75	0.15	0.564	0.183
ξ_p	beta	0.75	0.15	0.036	0.076
ξ_w	beta	0.75	0.15	0.011	0.079
Fiscal Policy Parameters					
ρ_{tr}	beta	0.8	0.1	0.217	0.126
ρ_{ct}	beta	0.8	0.1	0.853	0.172
ρ_{wt}	beta	0.8	0.1	0.771	0.130
ρ_{kt}	beta	0.8	0.1	0.847	0.146
ρ_{try}	normal	0.1	0.05	0.102	0.048
ρ_{cty}	normal	0.1	0.05	0.100	0.105
ρ_{wtl}	normal	0.1	0.05	0.102	0.047
ρ_{ktI}	normal	0.1	0.05	0.098	0.081
ρ_{gy}	normal	0.1	0.05	-0.026	0.047
ρ_{trb}	normal	0.1	0.05	0.068	0.106
ρ_{ctb}	normal	0.1	0.05	0.092	0.051
ρ_{wtb}	normal	0.1	0.05	0.110	0.100
ρ_{ktb}	normal	0.1	0.05	0.096	0.063
ρ_{gb}	normal	0.1	0.05	0.005	0.076
Shock persistence					
ρ_B	beta	0.8	0.1	0.846	0.153
ρ_L	beta	0.8	0.1	0.333	0.196
ρ_x	beta	0.8	0.1	0.874	0.088
ρ_a	beta	0.8	0.1	0.357	0.130
ρ_g	beta	0.8	0.1	0.704	0.064
ρ_M	beta	0.8	0.1	0.917	0.174
Standard deviation of shocks					
η_t^B	Inverse gamma	0.2	2	0.067	0.886
η_t^L	Inverse gamma	1	2	4.523	1.879
η_t^x	Inverse gamma	0.1	2	6.084	1.089
η_t^a	Inverse gamma	0.4	2	0.687	1.083
η_t^g	Inverse gamma	0.3	2	0.338	1.007
η_t^p	Inverse gamma	0.15	2	0.049	2.938
η_t^w	Inverse gamma	0.25	2	0.083	2.414
η_t^M	Inverse gamma	0.5	2	6.749	1.906
η_t^{tr}	Inverse gamma	0.1	2	6.405	0.472
η_t^{ct}	Inverse gamma	0.1	2	0.033	0.808
η_t^{wt}	Inverse gamma	0.1	2	0.057	3.370
η_t^{kt}	Inverse gamma	0.4	2	6.024	1.197
Switching Monetary Policy Parameters					
Regime 1					
ϕ_π	normal	1.7	0.1	1.694	0.073
ϕ_r	beta	0.8	0.1	0.694	0.106
ϕ_Y	normal	0.125	0.05	0.102	0.057
Regime 2					
ϕ_π	normal	0.003	0.001	0.003	0.003
ϕ_r	beta	0.4	0.1	0.799	0.178
ϕ_Y	normal	0.005	0.001	0.005	0.001
Transition probabilities					
TP (1 to 2)	beta	0.005	0.15	0.057	0.209
TP (2 to 1)	beta	0.005	0.15	0.217	0.058

TABLE 5: Forecast error variance decomposition of consumption, output, investment and real wages at various horizons (extended model)

Private consumption

		Regime 1			Regime 2 (ZLB)		
		t=1	t=10	t=20	t=1	t=10	t=20
Labor supply shock	η_t^L	0.65	0.31	0.19	0.44	0.25	0.14
Technology shock	η_t^a	0.25	0.12	0.08	0.14	0.08	0.04
Shock to mark-up in goods markets	η_t^p	0.05	0.01	0.01	0.01	0.00	0.00
Wage mark-up shock	η_t^w	0.03	0.01	0.01	0.02	0.01	0.00
Shock to the marg. effic. of inv.	η_t^x	0.01	0.49	0.48	0.02	0.29	0.25
Government spending shock	η_t^g	0.01	0.01	0.01	0.01	0.01	0.01
Capital income tax shock	η_t^{kt}	0.00	0.04	0.21	0.00	0.06	0.19
Lump-sum taxes shock	η_t^{tr}	0.00	0.01	0.02	0.31	0.29	0.35
Wage-income tax shock	η_t^{wt}	0.00	0.00	0.00	0.00	0.00	0.00
Money demand shock	η_t^M	0.00	0.00	0.00	0.04	0.02	0.01
Consumption tax shock	η_t^{ct}	0.00	0.00	0.00	0.00	0.00	0.00
Preference shock	η_t^B	0.00	0.00	0.00	0.00	0.00	0.00

Output

		Regime 1			Regime 2 (ZLB)		
		t=1	t=10	t=20	t=1	t=10	t=20
Wage-income tax shock	η_t^{wt}	0.32	0.17	0.10	0.29	0.14	0.11
Technology shock	η_t^a	0.28	0.18	0.12	0.09	0.07	0.05
Shock to the marg. effic. of inv.	η_t^x	0.16	0.13	0.12	0.09	0.07	0.07
Government spending shock	η_t^g	0.09	0.06	0.04	0.04	0.03	0.03
Capital income tax shock	η_t^{kt}	0.07	0.38	0.56	0.04	0.20	0.31
Shock to mark-up in goods markets	η_t^p	0.05	0.02	0.01	0.01	0.01	0.00
Wage mark-up shock	η_t^w	0.04	0.02	0.01	0.02	0.01	0.01
Labor supply shock	η_t^L	0.01	0.01	0.01	0.00	0.01	0.00
Lump-sum taxes shock	η_t^{tr}	0.01	0.03	0.02	0.35	0.40	0.36
Money demand shock	η_t^M	0.00	0.00	0.00	0.04	0.03	0.03
Preference shock	η_t^B	0.00	0.00	0.00	0.02	0.04	0.03
Consumption tax shock	η_t^{ct}	0.00	0.00	0.00	0.00	0.00	0.00

Investment

		Regime 1			Regime 2 (ZLB)		
		t=1	t=10	t=20	t=1	t=10	t=20
Shock to the marg. effic. of inv.	η_t^x	0.65	0.82	0.89	0.43	0.60	0.70
Capital income tax shock	η_t^{kt}	0.30	0.16	0.10	0.18	0.12	0.09
Lump-sum taxes shock	η_t^{tr}	0.03	0.01	0.00	0.26	0.22	0.16
Technology shock	η_t^a	0.01	0.01	0.00	0.02	0.00	0.00
Government spending shock	η_t^g	0.00	0.00	0.00	0.00	0.00	0.00
Money demand shock	η_t^M	0.00	0.00	0.00	0.02	0.01	0.01
Preference shock	η_t^B	0.00	0.00	0.00	0.08	0.03	0.02
Shock to mark-up in goods markets	η_t^p	0.00	0.00	0.00	0.00	0.00	0.00
Labor supply shock	η_t^L	0.00	0.00	0.00	0.00	0.00	0.00
Wage mark-up shock	η_t^w	0.00	0.00	0.00	0.00	0.00	0.00
Wage-income tax shock	η_t^{wt}	0.00	0.00	0.00	0.00	0.02	0.02
Consumption tax shock	η_t^{ct}	0.00	0.00	0.00	0.00	0.00	0.00

Real wages

		Regime 1			Regime 2 (ZLB)		
		t=1	t=10	t=20	t=1	t=10	t=20
Technology shock	η_t^a	0.51	0.22	0.12	0.32	0.16	0.09
Shock to mark-up in goods markets	η_t^p	0.46	0.17	0.09	0.27	0.11	0.06
Wage-income tax shock	η_t^{wt}	0.01	0.01	0.00	0.00	0.01	0.02
Shock to the marg. effic. of inv.	η_t^x	0.01	0.54	0.62	0.00	0.37	0.45
Government spending shock	η_t^g	0.00	0.00	0.00	0.00	0.00	0.00
Capital income tax shock	η_t^{kt}	0.00	0.07	0.16	0.00	0.06	0.14
Wage mark-up shock	η_t^w	0.00	0.00	0.00	0.00	0.00	0.00
Lump-sum taxes shock	η_t^{tr}	0.00	0.00	0.00	0.16	0.13	0.14
Labor supply shock	η_t^L	0.00	0.00	0.00	0.00	0.00	0.00
Preference shock	η_t^B	0.00	0.00	0.00	0.13	0.08	0.05
Money demand shock	η_t^M	0.00	0.00	0.00	0.11	0.07	0.04
Consumption tax shock	η_t^{ct}	0.00	0.00	0.00	0.00	0.00	0.00

Figures

FIGURE 1: Baseline model's response to a government spending positive shock in the absence of a binding constraint

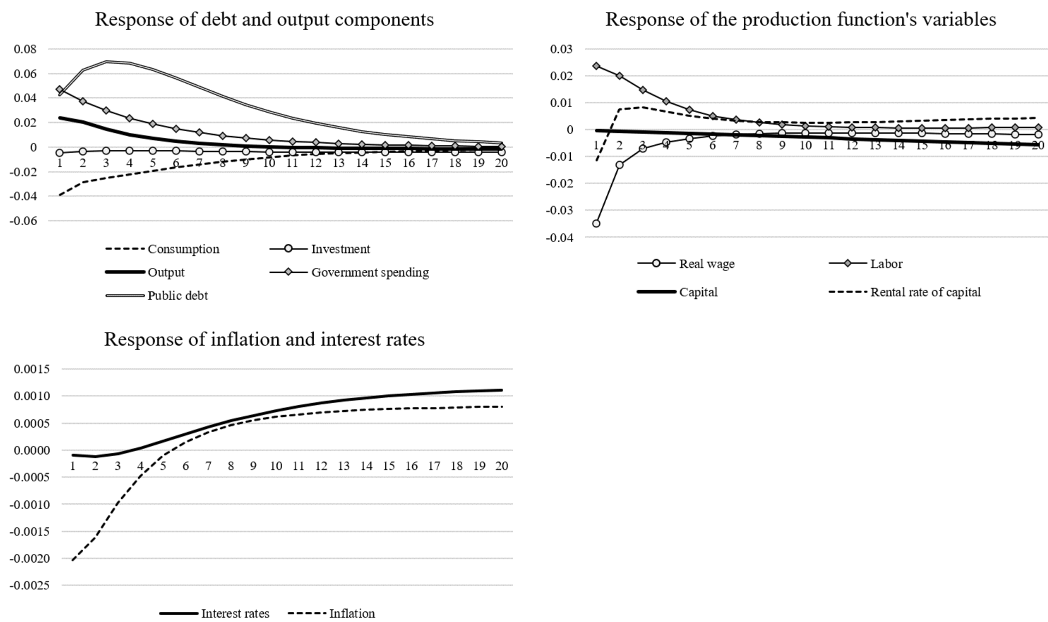


FIGURE 2: Response of consumption to a government spending shock for different values of the share of rule-of-thumb consumers (calibrated baseline model)

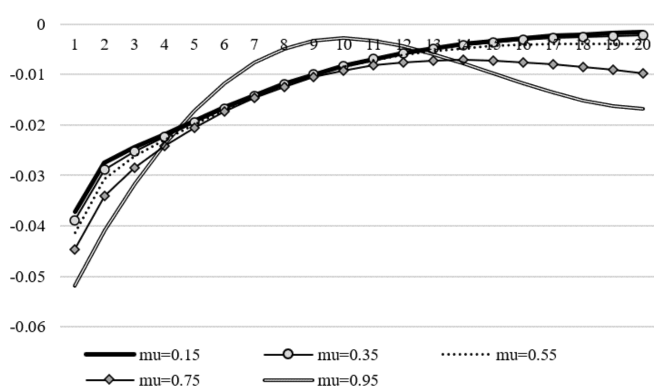


FIGURE 3: Response to a positive government spending shock in the baseline and extended model (fully calibrated model with no binding constraint)

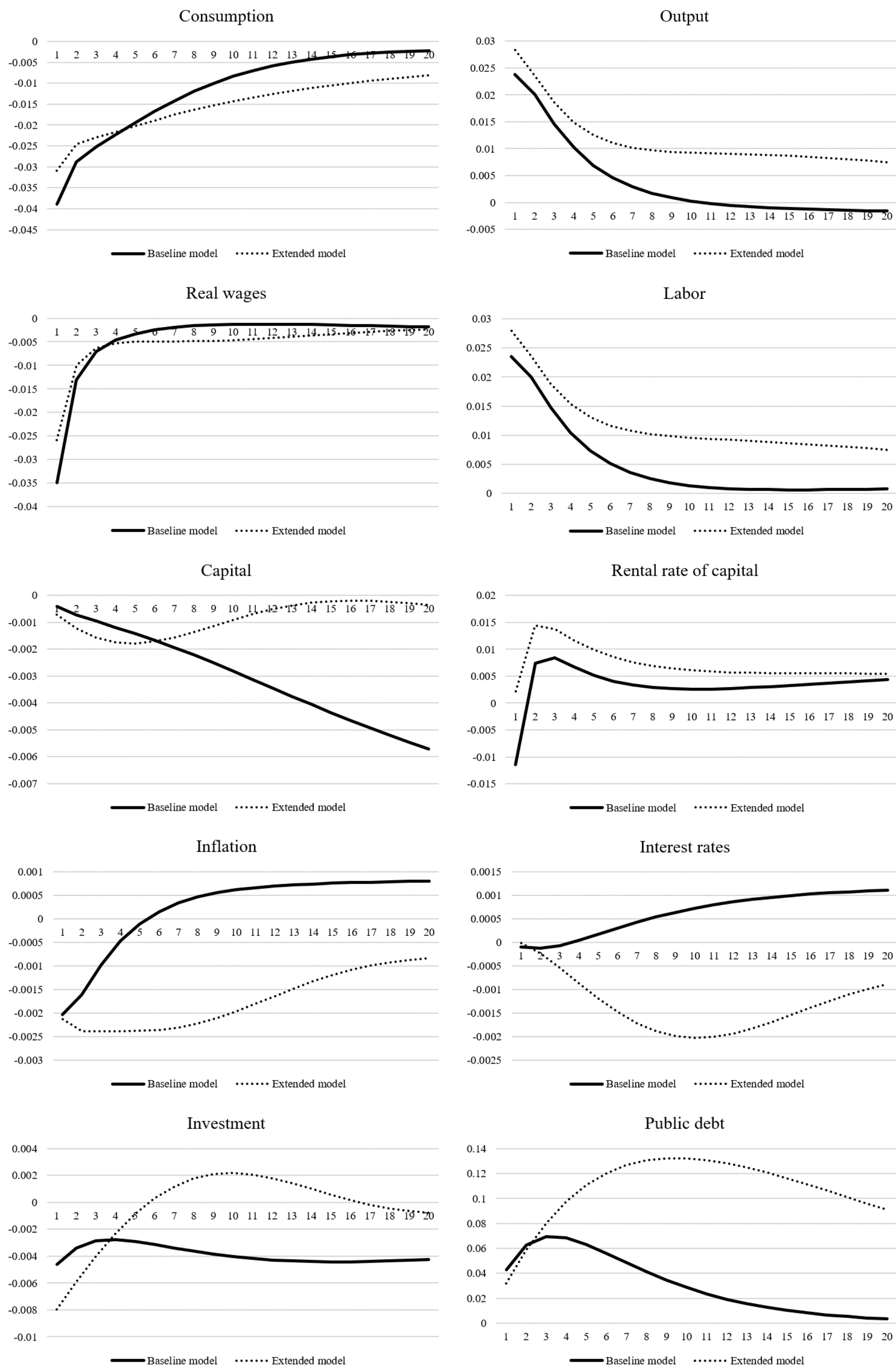


FIGURE 4: CPI inflation and interest rates over the period of study

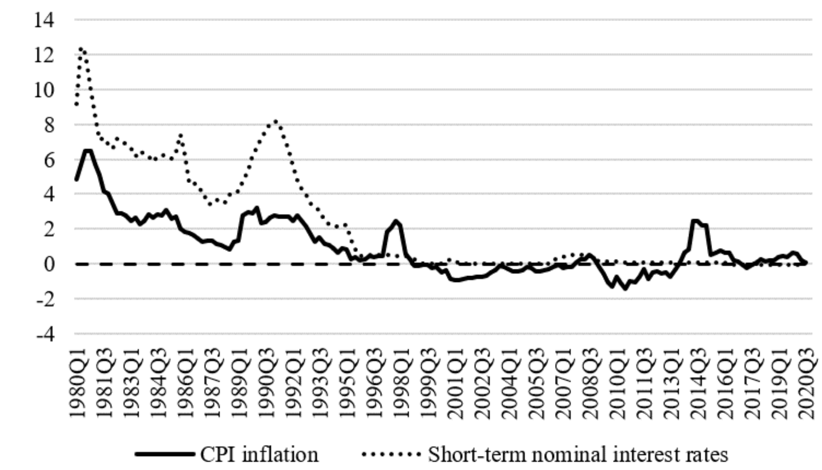


FIGURE 5: Interest rates Vs smoothed probabilities of an unbounded interest rates regime

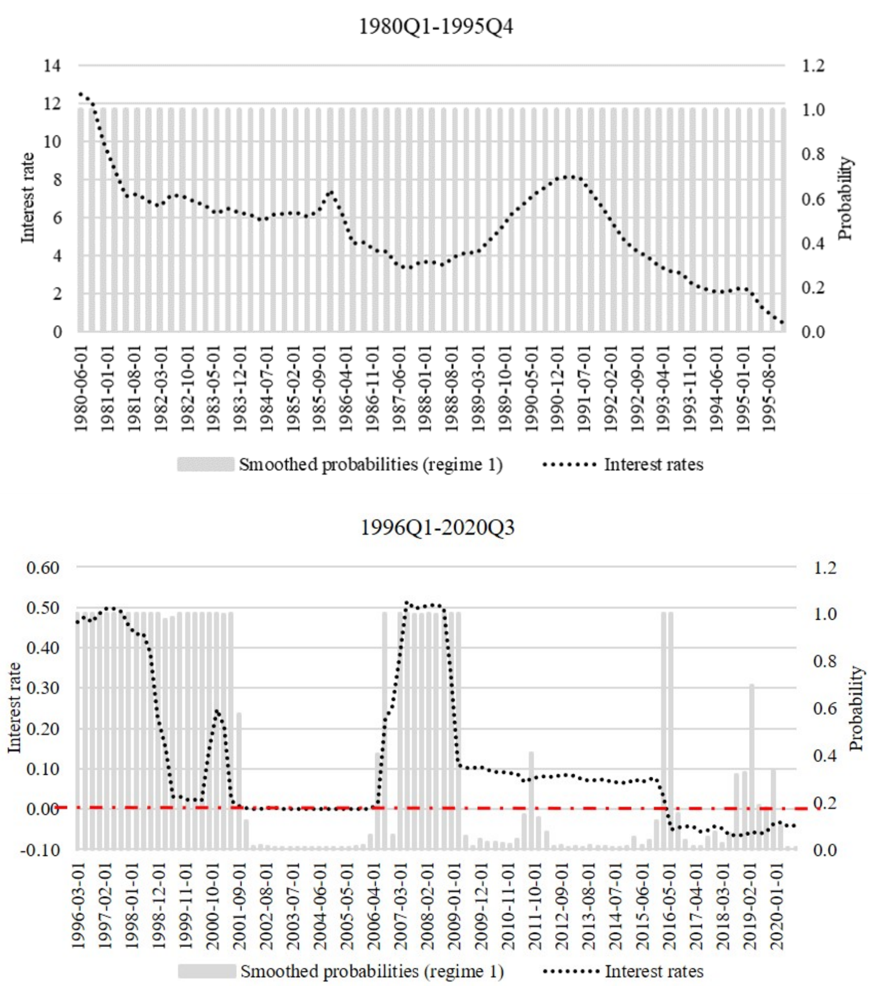


FIGURE 6: Response to a positive government spending shock in the baseline model (regime-switching)

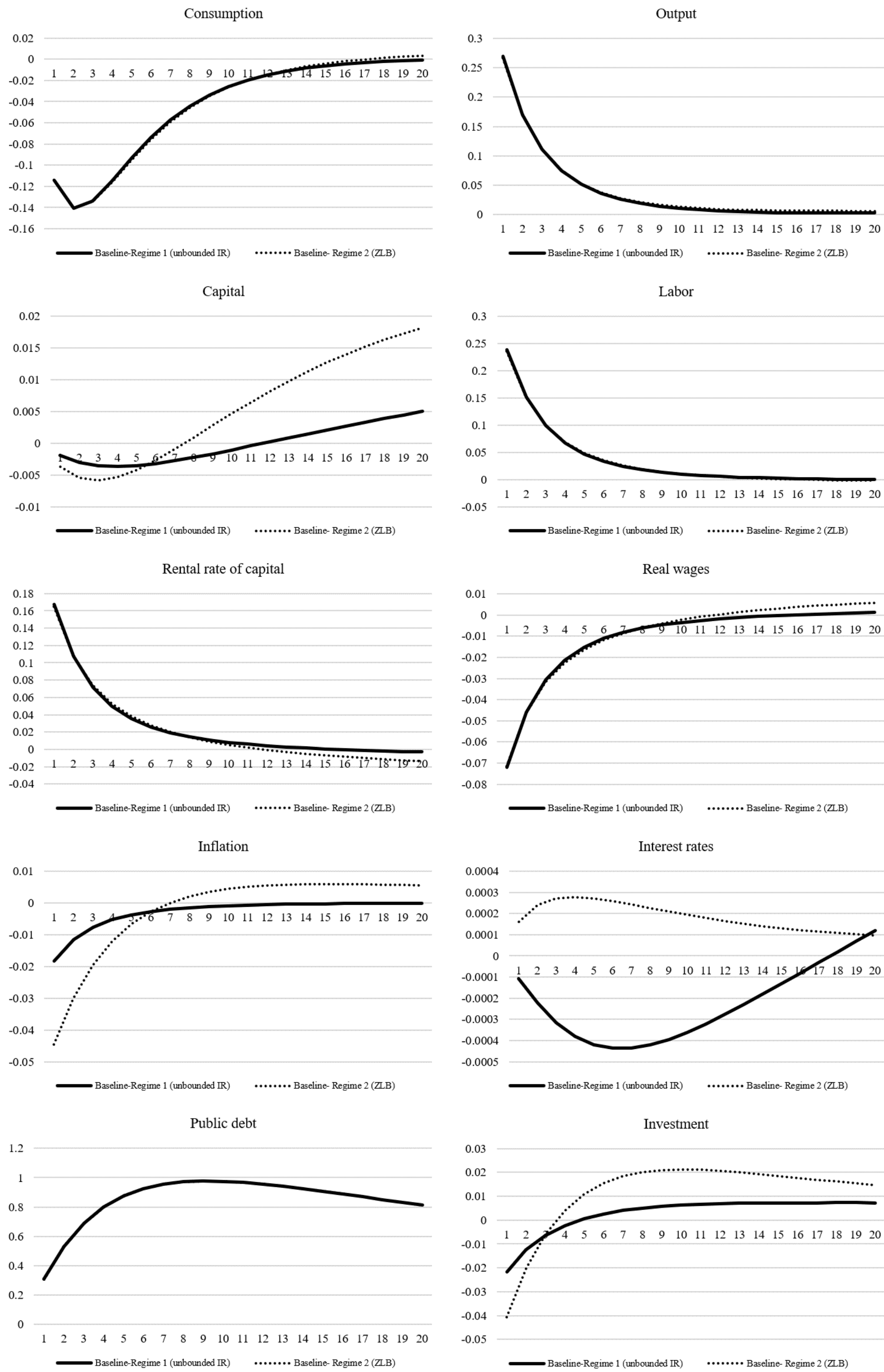


FIGURE 7: Response to a positive government spending shock in the extended model (regime-switching)

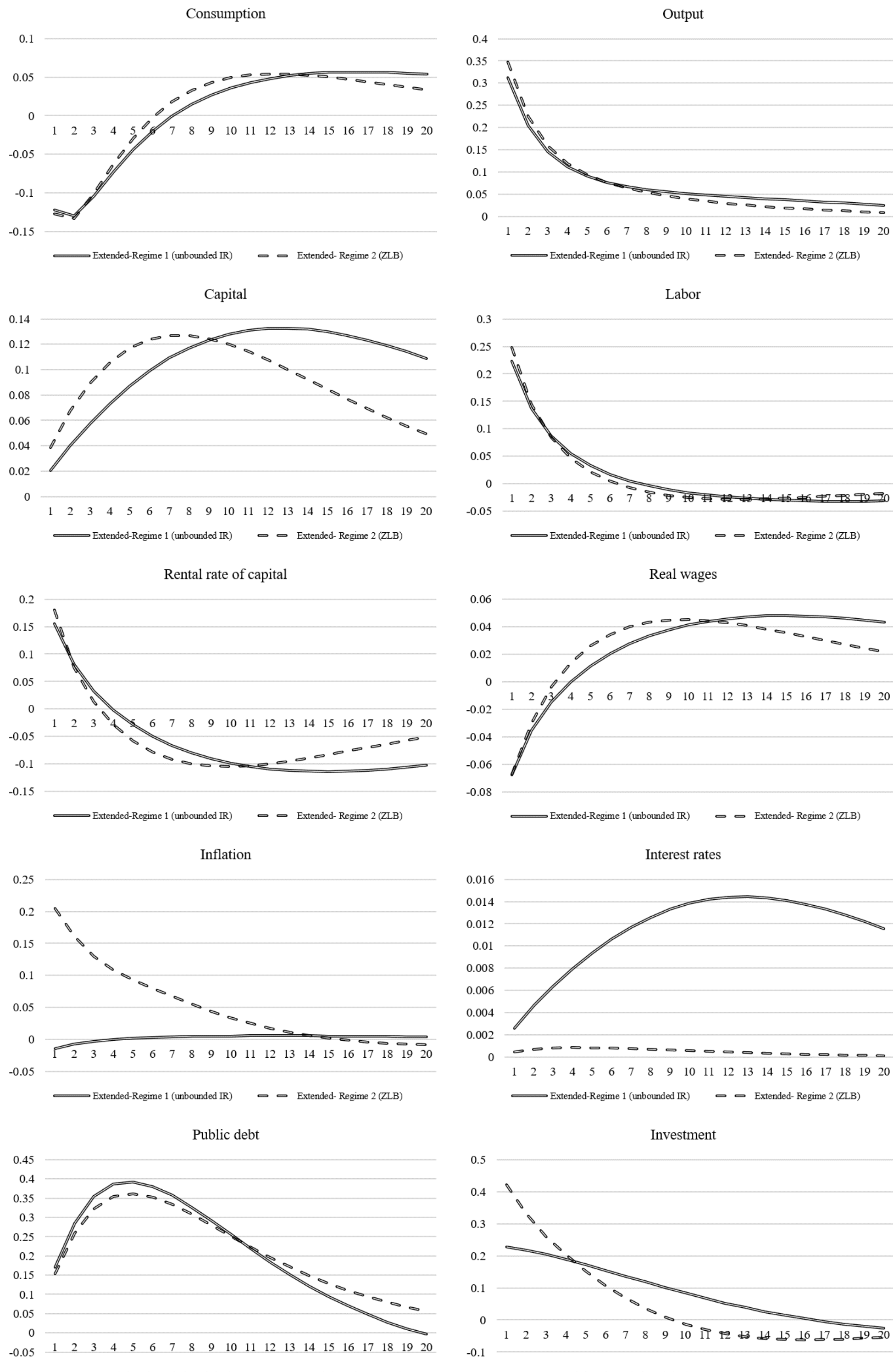


FIGURE 8: Lump-sum taxes and tax rate adjustments after a positive government spending shock in the extended model (regime-switching)

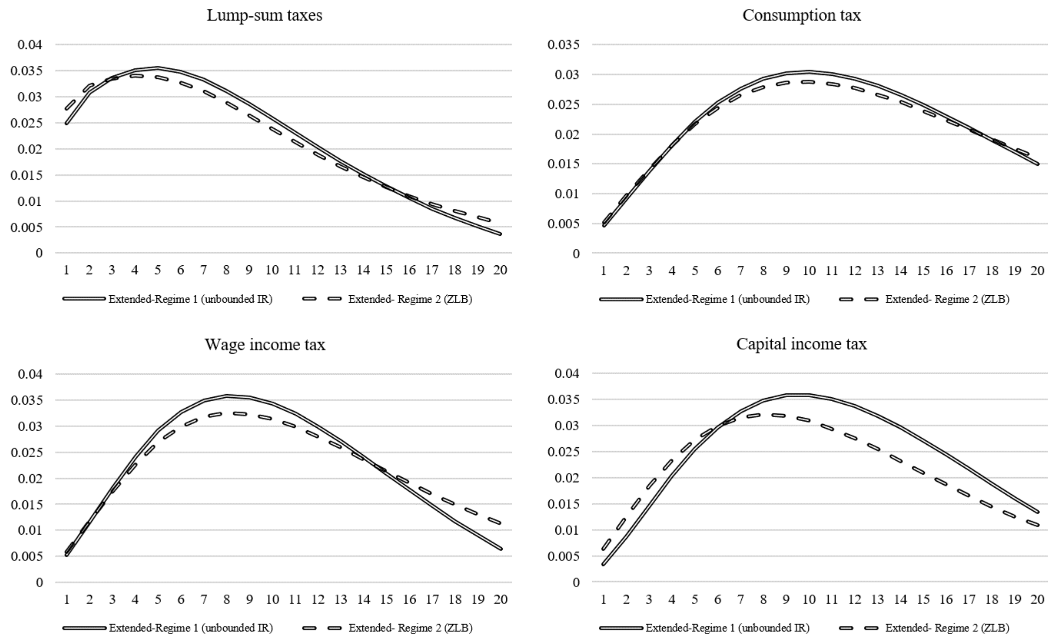


FIGURE 9: Lump-sum taxes and tax rate adjustments after a negative government spending shock in the extended model (regime-switching)

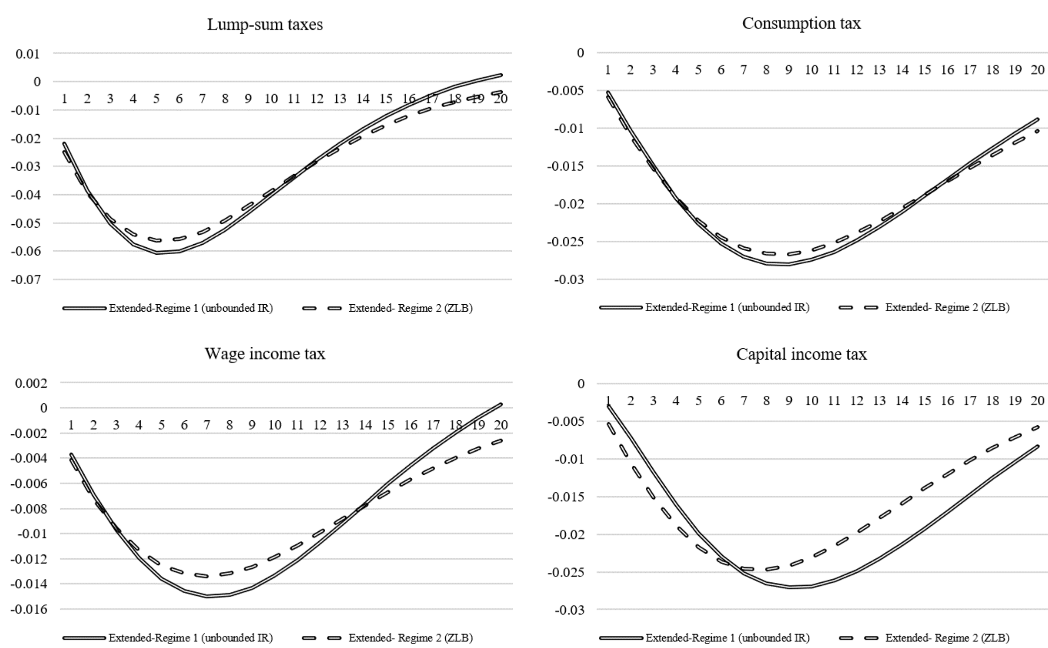


FIGURE 10: Response to a negative shock to government spending (regime-switching)

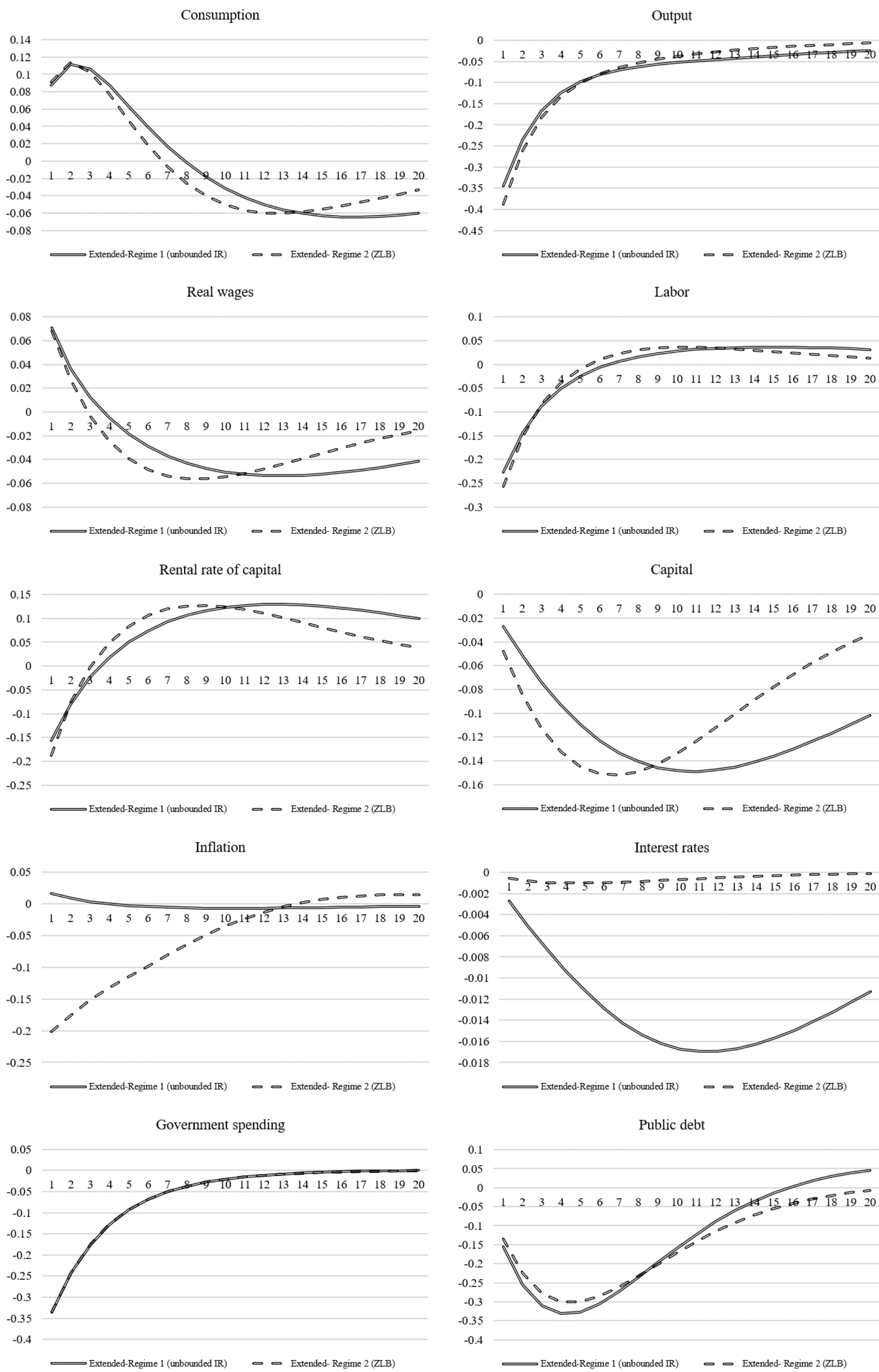


FIGURE 11: Response to a negative shock to the consumption tax rate (regime-switching)

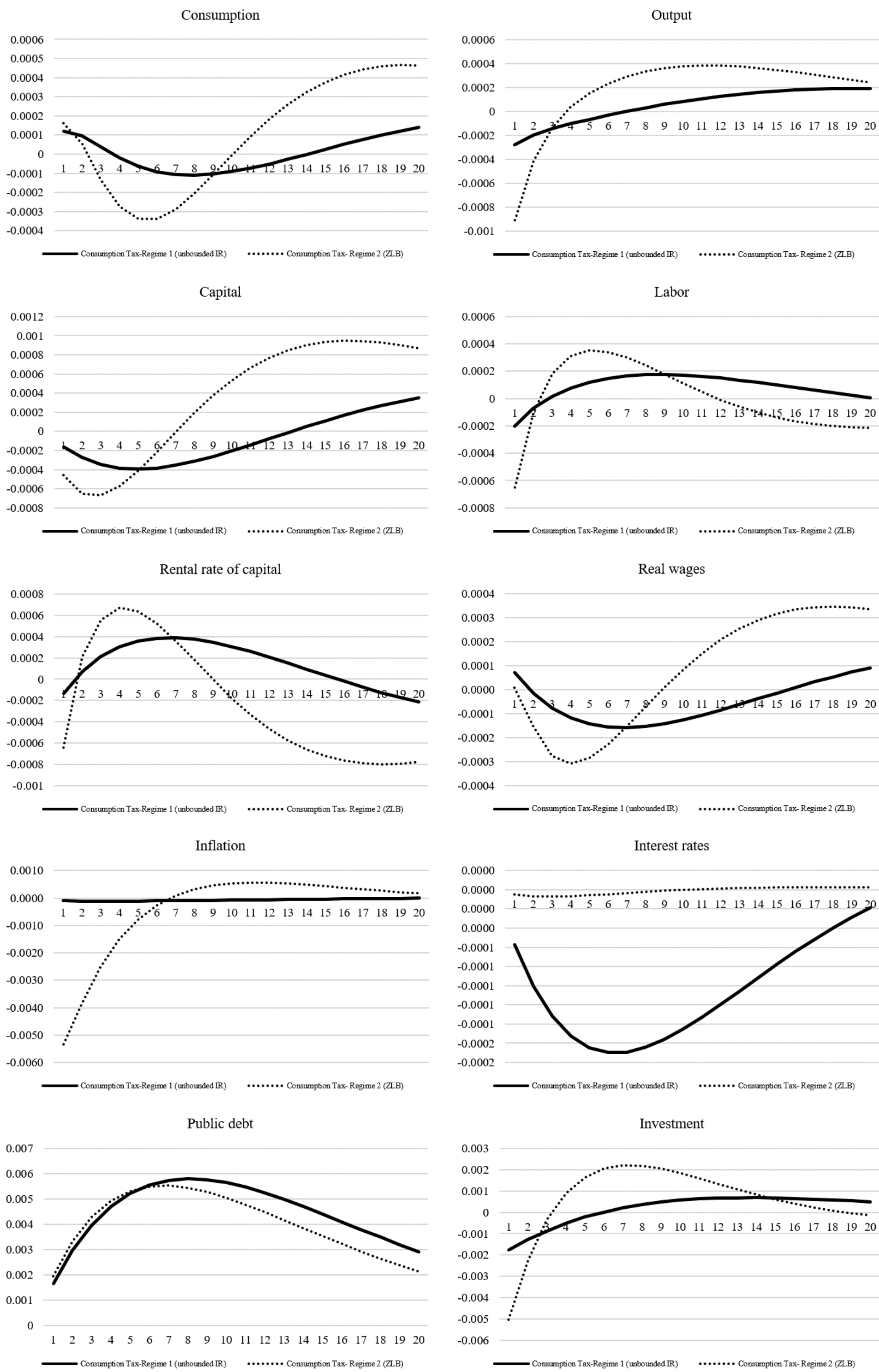


FIGURE 12: Response to a negative shock to the wage income tax rate (regime-switching)

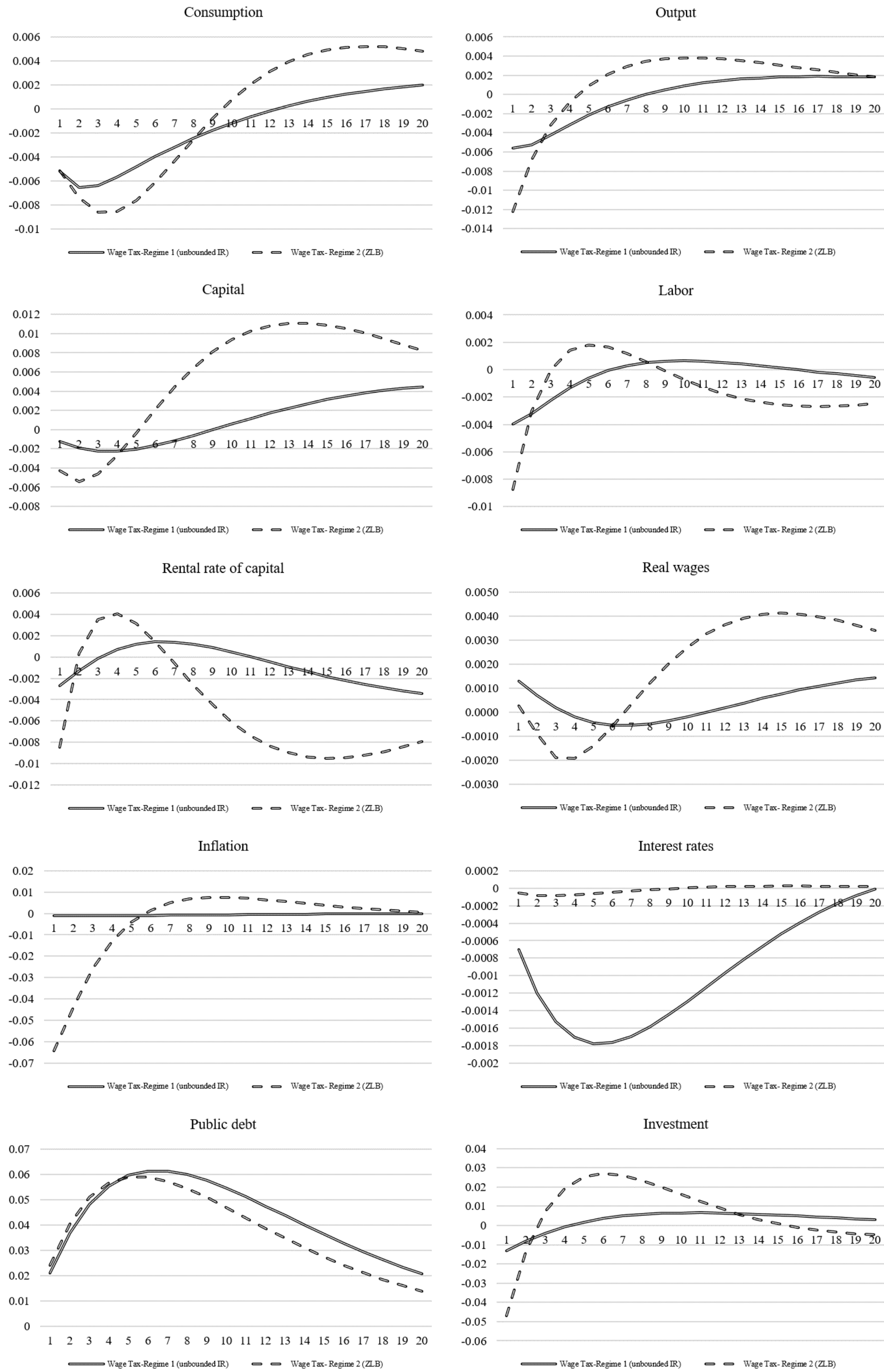


FIGURE 13: Response to a negative shock to the capital income tax rate (regime-switching)

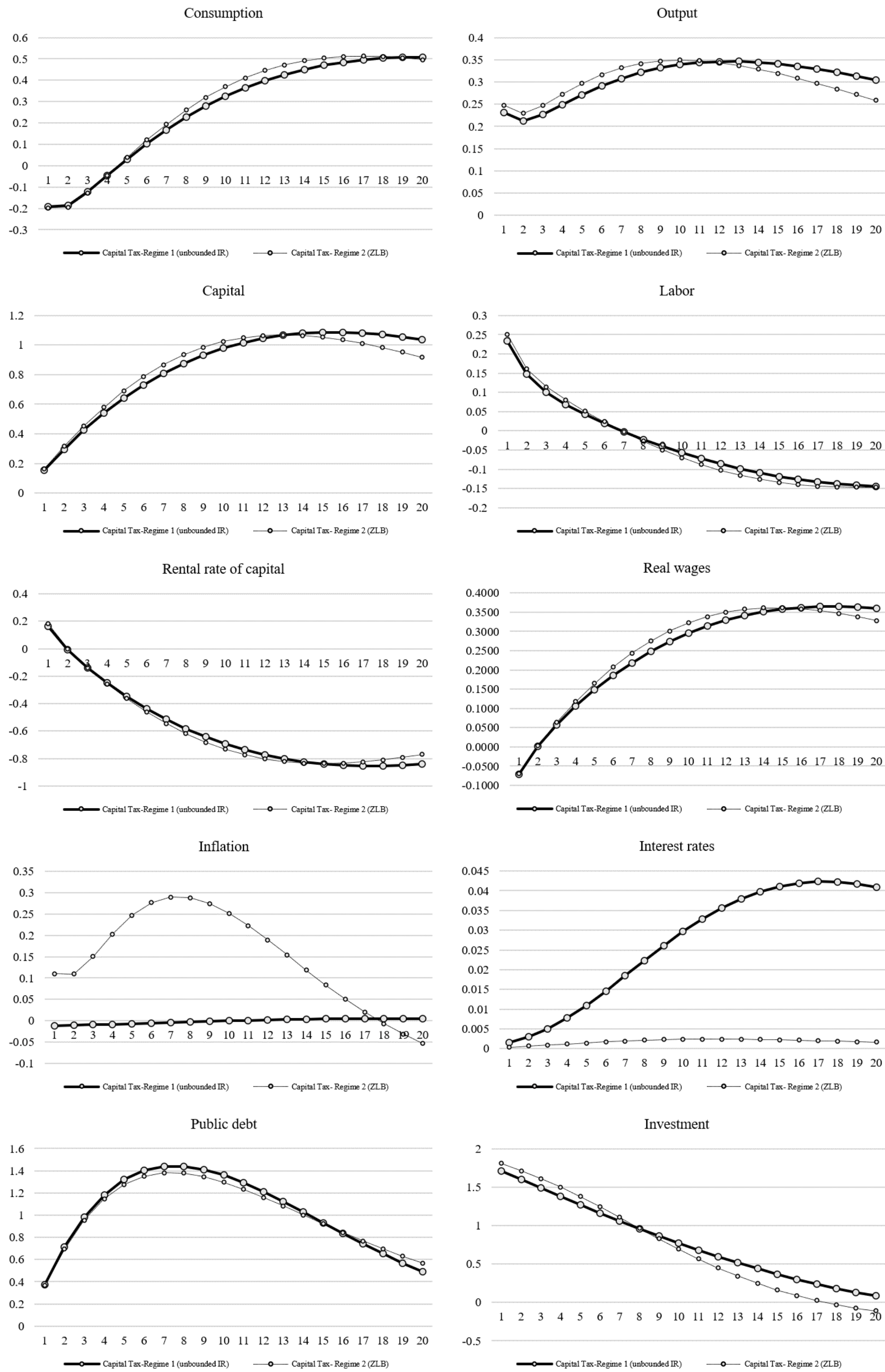


FIGURE 14: Comparison between a government spending increase measure and a capital income tax cut

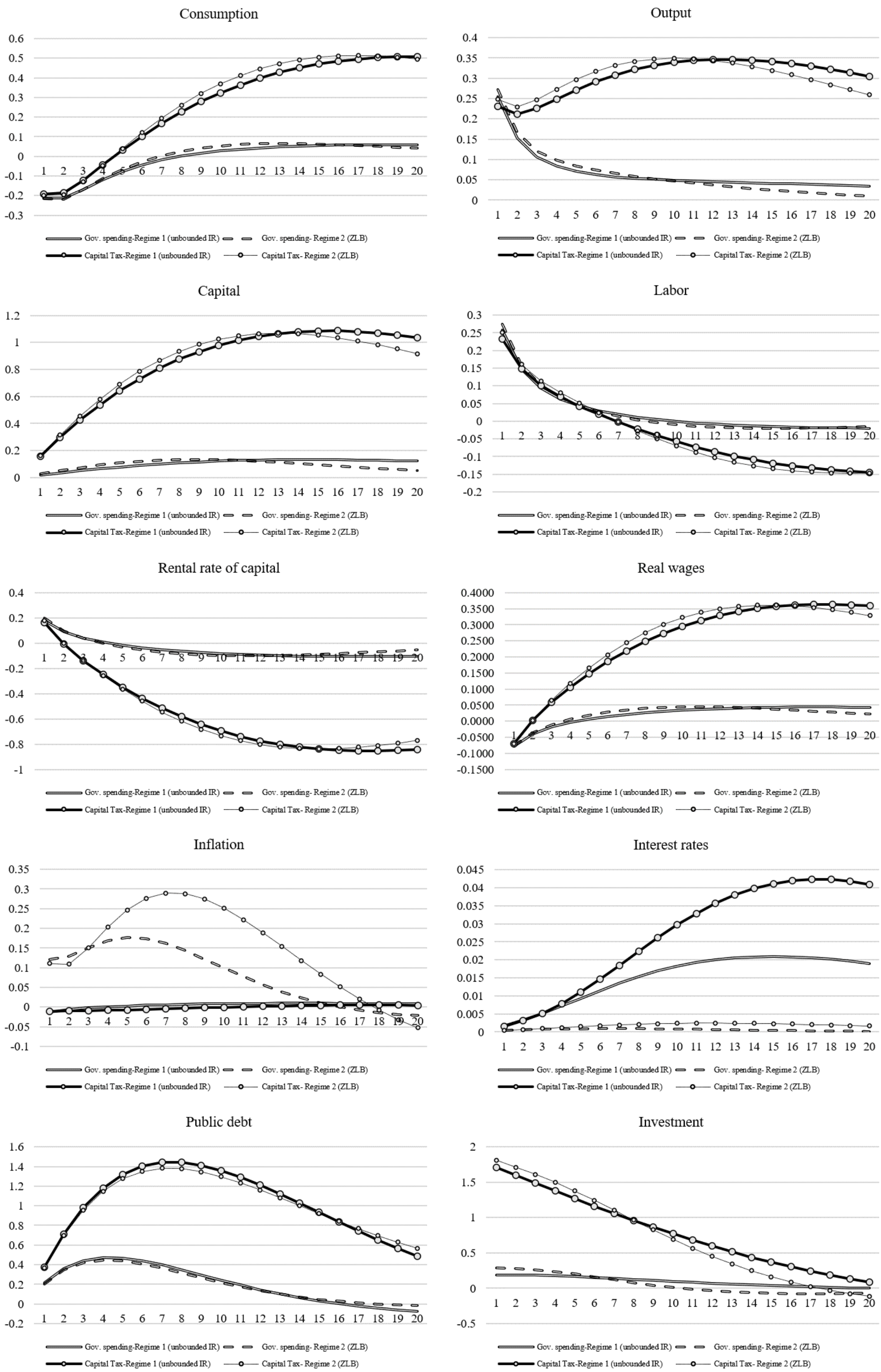


FIGURE 15: Spending and tax adjustments after a government spending increase and a capital income tax cut

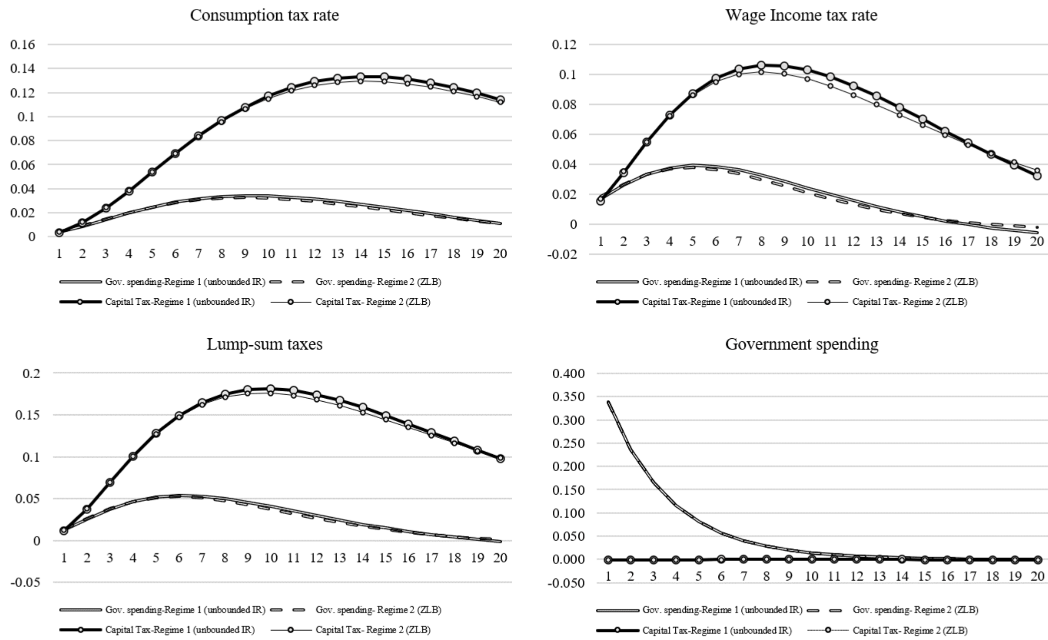
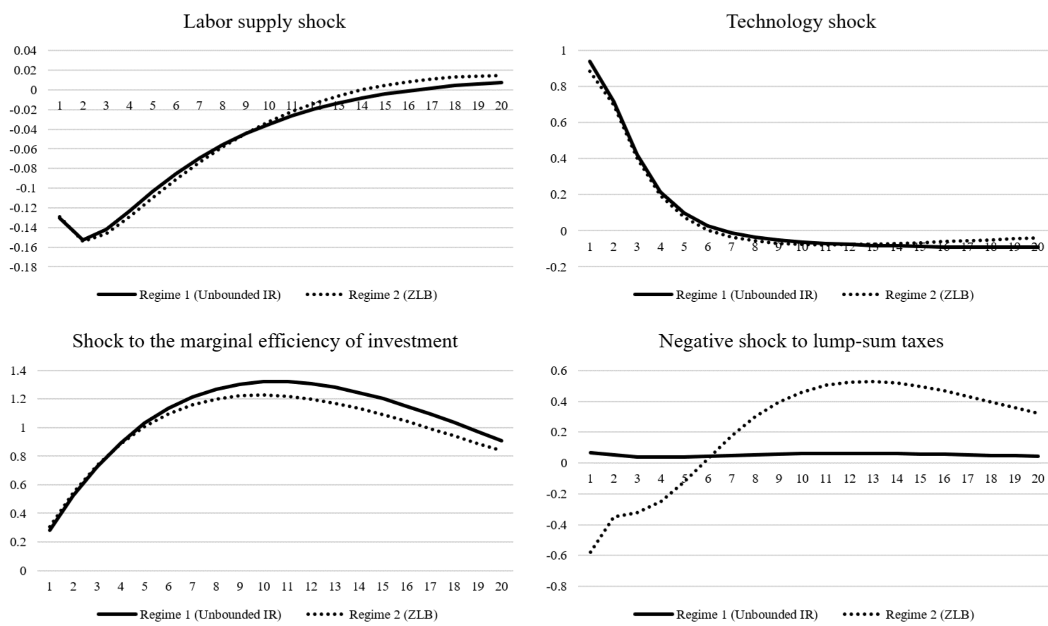


FIGURE 16: Response of private consumption to various shocks (extended model)



Appendix A: Baseline model summary

Households

The households' optimization problem is as follows

Maximizing utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t^B \left(\frac{1}{1-\sigma_c} \left(C_t^R - H_t \right)^{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(L_t^R \right)^{1+\sigma_l} + \frac{\varepsilon_t^M}{1-\sigma_m} \left(\frac{M_t^R}{P_t} \right)^{1-\sigma_m} \right) \right] \tag{66}$$

s.c.

$$\frac{M_t^R}{P_t} + \frac{B_t^R / R_t}{P_t} = \frac{M_{t-1}^R}{P_t} + \frac{B_{t-1}^R}{P_t} + w_t L_t^R + r_t^k K_t^R - \frac{\Theta_k}{2} \left(\frac{K_{t+1}^R}{K_t^R} - 1 \right)^2 K_t^R + Div_t^R - T_t^R - C_t^R - \frac{K_{t+1}^R - (1-\delta) K_t^R}{x_t} \tag{67}$$

Setting the Lagrangien as

$$\begin{aligned} L = E_t \sum_{t=0}^{\infty} \beta^t \Bigg[& \varepsilon_t^B \left(\frac{1}{1-\sigma_c} \left(C_t^R - H_t \right)^{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(L_t^R \right)^{1+\sigma_l} + \frac{\varepsilon_t^M}{1-\sigma_m} \left(\frac{M_t^R}{P_t} \right)^{1-\sigma_m} \right) \\ & -\lambda_t \left(\left[\frac{M_t^R}{P_t} + \frac{B_t^R}{R_t P_t} + C_t^R + \frac{\Theta_k}{2} \left(\frac{K_{t+1}^R}{K_t^R} - 1 \right)^2 K_t^R + \frac{K_{t+1}^R - (1-\delta) K_t^R}{x_t} \right] \right. \\ & \left. - \left[\frac{M_{t-1}^R}{P_t} + \frac{B_{t-1}^R}{P_t} + w_t L_t^R + r_t^k K_t^R + Div_t^R - T_t^R \right] \right) \Bigg] \end{aligned} \tag{68}$$

The following first order conditions are obtained

$$\frac{dL}{dC_t^R} : \lambda_t = \frac{\varepsilon_t^B}{\left(C_t^R - H_t \right)^{\sigma_c}} \tag{69}$$

$$\frac{dL}{dM_t^R} : \left(\frac{M_t^R}{P_t} \right)^{\sigma_m} \left[\lambda_t - \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} \right] - \varepsilon_t^B \varepsilon_t^M = 0 \tag{70}$$

$$\frac{dL}{dB_t^R} : \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} = \frac{1}{R_t} \tag{71}$$

$$\frac{dL}{dK_{t+1}^R} : \lambda_t \left[\frac{1}{x_t} + \Theta_k \left(\frac{K_{t+1}^R}{K_t^R} - 1 \right) \right] = \beta E_t \lambda_{t+1} \left[\frac{(1-\delta)}{x_{t+1}} + \frac{\Theta_k}{2} \left(\left(\frac{K_{t+2}^R}{K_{t+1}^R} \right)^2 - 1 \right) + r_{t+1}^k \right] \tag{72}$$

$$\frac{dL}{d\lambda_t}: \frac{M_t^R}{P_t} + \frac{B_t^R/R_t}{P_t} + C_t^R + \frac{\Theta_k}{2}\left(\frac{K_{t+1}^R}{K_t^R} - 1\right)^2 K_t^R + \frac{K_{t+1}^R - (1-\delta)K_t^R}{x_t} = \frac{M_{t-1}^R}{P_t} + \frac{B_{t-1}^R}{P_t} + w_t L_t^R + r_t^k K_t^R + Div_t^R - T_t^R \tag{73}$$

Combining conditions 69 to 71

$$\left[\left(C_t^R - H_t\right)^{\sigma_c} \varepsilon_t^M\right]^{1/\sigma_m} = \frac{M_t^R}{P_t}\left(1 - \frac{1}{R_t}\right)^{1/\sigma_m} \tag{74}$$

As stated in the model description, the capital accumulation process is given by

$$K_{t+1}^R = (1-\delta) K_t^R + x_t I_t^R \tag{75}$$

And shocks are expressed as

$$\ln\left(\varepsilon_t^B\right) = \rho_B \ln\left(\varepsilon_{t-1}^B\right) + \eta_t^B \tag{76}$$

$$\ln\left(\varepsilon_t^L\right) = \rho_L \ln\left(\varepsilon_{t-1}^L\right) + \eta_t^L \tag{77}$$

$$\ln\left(\varepsilon_t^M\right) = \rho_M \ln\left(\varepsilon_{t-1}^M\right) + \eta_t^M \tag{78}$$

$$\ln(x_t) = \rho_x \ln(x_{t-1}) + \eta_t^x \tag{79}$$

The non-ricardian consumers budget constraint is given by

$$C_t^{NR} = w_t L_t^{NR} - T_t^{NR} \tag{80}$$

Aggregate variables

$$C_t = (1-\mu) C_t^R + \mu C_t^{NR} \tag{81}$$

$$L_t = (1-\mu) L_t^R + \mu L_t^{NR} \tag{82}$$

With $L_t = L_t^R = L_t^{NR}$ at equilibrium.

$$T_t = (1-\mu) T_t^R + \mu T_t^{NR} \tag{83}$$

$$B_t = (1-\mu) B_t^R \tag{84}$$

$$K_{t+1} = (1-\mu) K_{t+1}^R \tag{85}$$

$$I_t = (1-\mu) I_t^R \tag{86}$$

$$Div_t = (1-\mu) Div_t^R \tag{87}$$

Wage equation

Unions choose the wage that maximizes the utility of future periods as follows

$$\max E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[\Lambda_{t+i} \frac{W_{h,t+i}}{P_{t+i}} L_{h,t+i} - \frac{\varepsilon^B_{t+i} \varepsilon^L_{t+i}}{1+\sigma_l} (L_{h,t+i})^{1+\sigma_l} \right] \tag{88}$$

Where Λ_{t+i} can be interpreted as the Lagrange multiplier equivalent to the marginal utility of consumption. In other words, the expression shows the gain in utility of consumption based on labor with wage $W_{h,t+i}$minus the disutility of labor, provided that the chosen wage is kept for infinity. Since

$$W_{h,t} = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{h,\, t-1} \tag{89}$$

Then

$$W_{h,t+i} = \prod_{s=1}^{i} \left(\frac{P_{t+s-1}}{P_{t+s-2}} \right)^{\gamma_w} W_{h,\, t} \tag{90}$$

Where γ_w is the degree of wage indexation. The demand for labor is

$$L_{h,t+i} = \left(\frac{W_{h,t+i}}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_{t+i} \tag{91}$$

Replacing in 88 and deriving the first order condition with respect to wage yields the expression

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[\Lambda_{t+i} \frac{\prod_{s=1}^{i} \left(\frac{P_{t+s-1}}{P_{t+s-2}} \right)^{\gamma_w}}{P_{t+i}} L_{h,t+i} - \frac{\varepsilon^B_{t+i} \varepsilon^L_{t+i} \left(1+\lambda_{w,t+i}\right)}{W^*_{h,t}} (L_{h,t+i})^{1+\sigma_l} \right] = 0 \tag{92}$$

Thus

$$\frac{W^*_{h,t}}{P_t} E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[\Lambda_{t+i} \frac{\left({P_{t+i-1}}/{P_{t-1}} \right)^{\gamma_w}}{{P_{t+i}}/{P_t}} L_{h,t+i} \right] = E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left(1+\lambda_{w,t+i}\right) \varepsilon^B_{t+i} \varepsilon^L_{t+i} (L_{h,t+i})^{1+\sigma_l} \tag{93}$$

Or

$$\frac{W^*_{h,t}}{P_t} E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[U'_c \frac{\left({P_{t+i-1}}/{P_{t-1}} \right)^{\gamma_w}}{{P_{t+i}}/{P_t}} L_{h,t+i} \right] = E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left(1+\lambda_{w,t+i}\right) L_{h,t+i} U'_L \tag{94}$$

Where U'_c is the marginal utility of an additional unit of consumption and U'_L the marginal disutility of labor. The aggregate nominal wage is given by

$$W_t = \left[(1-\xi_w) \left(W^*_{h,t} \right)^{-\frac{1}{\lambda_{w,t}}} + \xi_w \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} (W_{\,t-1})^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}} \tag{95}$$

Firms

Final goods profits maximization problem leads to the relation

$$y_t^j = \left(\frac{p_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{96}$$

Replacing in the expression of the nominal value of the final good $p_t y_t = \int_0^1 p_t^j y_t^j dj$, leads to

$$P_t = \left[\int_0^1 \left(p_t^j\right)^{-\frac{1}{\lambda_{p,t}}} dj\right]^{-\lambda_{p,t}} \tag{97}$$

The production technology for intermediate goods firms is

$$y_t^j = \varepsilon_t^a K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi \tag{98}$$

Φ are fixed costs of production and the technology shock is $\ln\left(\varepsilon_t^a\right) = \rho_a \ln\left(\varepsilon_{t-1}^a\right) + \eta_t^a$

Minimizing the total cost $W_t L_{j,t} + R_t^k K_{j,t}$ subject to the production technology yields the following expression

$$\frac{W_t L_{j,t}}{R_t^k K_{j,t}} = \frac{1-\alpha}{\alpha} \tag{99}$$

The firms' marginal costs correspond to

$$MC_t = \frac{W_t{}^{1-\alpha} R_t^{k\,\alpha}}{\varepsilon_t^a \alpha^{\alpha} (1-\alpha)^{(1-\alpha)}} \tag{100}$$

Profits are expressed as

$$\Pi_t^j = \left(p_t^j - MC_t\right)\left(\frac{p_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} (Y_t) - MC_t \Phi \tag{101}$$

Price setting equation

The first order condition is given by

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} y_{t+i} \left(\frac{\tilde{p}_t^j}{P_t} \frac{\left(P_{t-1+i}/P_{t-1}\right)^{\gamma_p}}{P_{t+i}/P_t} - \left(1+\lambda_{p,t+i}\right) mc_{t+i}\right) = 0 \tag{102}$$

And the aggregate price index is given by

$$(P_t)^{-\frac{1}{\lambda_{p,t}}} = \xi_p \left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p}\right)^{-\frac{1}{\lambda_{p,t}}} + \left(1-\xi_p\right)\left(\tilde{p}_t^j\right)^{-\frac{1}{\lambda_{p,t}}} \tag{103}$$

Monetary and fiscal authorities

Monetary policy sets the gross nominal interest rate based on

$$R_t = \max\left\{1, {R_{t-1}}^{\phi_r}\left[\frac{\pi}{\beta}\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{(1-\phi_r)}\right\} \tag{104}$$

The government budget constraint

$$\frac{B_t/R_t}{P_t} + T_t = \frac{B_{t-1}}{P_t} + G_t \tag{105}$$

The fiscal policy rule

$$t_t = \phi_b b_t + \phi_g g_t \tag{106}$$

The government spending process

$$g_t = \rho_g g_{t-1} + \eta_t^g \tag{107}$$

Goods market equilibrium

The final goods market equilibrium condition

$$Y_t = C_t + G_t + I_t + \frac{\Theta_k}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{108}$$

Appendix B: log-linearization of the baseline model with lump-sum taxes

Steady States

In the absence of shocks, the economy converges to the steady state. Then $\eta_t^B = \eta_t^L = \eta_t^M = \eta_t^x = \eta_t^a = 0$, implying that $\varepsilon^B = \varepsilon^L = \varepsilon^M = x = \varepsilon^a = 1$

Also

$$R = \frac{1}{\beta} \tag{109}$$

$$r^k = \frac{1}{\beta} - 1 + \delta \tag{110}$$

It is assumed that the debt level is null at the steady state and that the budget is balanced

$$B = 0 \tag{111}$$

$$T = G \tag{112}$$

The real marginal cost is equivalent to

$$mc = \frac{1}{1+\lambda_p} \tag{113}$$

This leads to

$$\frac{wL}{Y} = \frac{(1-\alpha)}{1+\lambda_p}\left(1+\frac{\Phi}{Y}\right) = \frac{(1-\alpha)}{1+\lambda_p}\theta \tag{114}$$

Where $\theta = 1 + \frac{\Phi}{Y}$

Real dividends relative to output at the steady state can be expressed as

$$\frac{Div}{Y} = 1 - mc\theta = \frac{1+\lambda_p-\theta}{1+\lambda_p} \tag{115}$$

As in Gali et al (2007), steady state consumption is assumed to be the same across all households

$$C = C^R = C^{NR} \tag{116}$$

And as stated before

$$L = L^R = L^{NR} \tag{117}$$

Log-linearized model excluding prices and wages equations

Letters with a hat represent the log-linearized variables around the steady state, i.e.$\hat{X}_t = \ln(X_t) - \ln(X) \approx \frac{X_t - X}{X}$

The log-linearization of 69 and 71 give the following relations

$$\hat{\lambda}_t = \hat{\varepsilon}_t^B - \frac{\sigma_c}{(1-h)}\left(\hat{C}_t^R - h\hat{C}_{t-1}^R\right) \tag{118}$$

$$\hat{R}_t = E_t\hat{\pi}_{t+1} + \hat{\lambda}_t - \hat{\lambda}_{t+1} \tag{119}$$

Combining the above relations yields the following expression for log-linear consumption of Ricardian households

$$\hat{C}_t^R = \frac{h\hat{C}_{t-1}^R}{(1+h)} + \frac{E_t\hat{C}_{t+1}^R}{(1+h)} - \frac{(1-h)}{\sigma_c(1+h)}\left(\hat{R}_t - E_t\hat{\pi}_{t+1} + E_t\hat{\varepsilon}_{t+1}^B - \hat{\varepsilon}_t^B\right) \tag{120}$$

The same expression can also be obtained by combining the log-linearized 70 and 74

$$\sigma_m\hat{m}_t + \frac{1}{(1-\beta)}\left(\hat{\lambda}_t - \beta E_t\hat{\lambda}_{t+1} + \beta E_t\hat{\pi}_{t+1}\right) = \hat{\varepsilon}_t^B + \hat{\varepsilon}_t^M \tag{121}$$

$$\frac{\sigma_c}{\sigma_m(1-h)}\left(\hat{C}_t^R - h\hat{C}_{t-1}^R\right) + \frac{1}{\sigma_m}\hat{\varepsilon}_t^M = \hat{m}_t + \frac{\hat{R}_t}{\sigma_m(R-1)} \tag{122}$$

The expression of consumption for non-Ricardian consumers is

$$\hat{C}_t^{NR} = \frac{L^{NR}w}{C^{NR}}\left(\hat{w}_t + \hat{L}_t^{NR}\right) - \frac{Y}{C^{NR}}t_t^{NR} \tag{123}$$

Or

$$\hat{C}_t^{NR} = \frac{(1-\alpha)}{1+\lambda_p}\theta\frac{Y}{C}\left(\hat{w}_t + \hat{L}_t^{NR}\right) - \frac{Y}{C}t_t^{NR} \tag{124}$$

Where $t_t^{NR} = \frac{T_t^{NR} - T^{NR}}{Y}$. Aggregate consumption is given by

$$\hat{C}_t = (1-\mu)\,\hat{C}_t^R + \mu\hat{C}_t^{NR} \tag{125}$$

Log-linearization of 72

$$\Theta_k\hat{K}_t = (1+\beta)\,\Theta_k E_t\hat{K}_{t+1} - \beta\Theta_k E_t\hat{K}_{t+2} - \ \beta r^k E_t\hat{r}_{t+1}^k + \hat{R}_t - E_t\hat{\pi}_{t+1} + \beta\,(1-\delta)\,E_t\hat{x}_{t+1} - \hat{x}_t \tag{126}$$

Log-linearization of 73

$$\begin{aligned} &\frac{m}{Y}\left[\hat{m}_t^R - \hat{m}_{t-1}^R\right] + \frac{b_t}{R} - b_{t-1} + \frac{C}{Y}\hat{C}_t^R + \frac{I}{Y}\hat{I}_t^R + t_t^R - \frac{Div}{Y}\widehat{Div}_t \\ &= \left(\frac{C}{Y} + \frac{I}{Y} + \frac{T^R}{Y} - \frac{Div}{Y}\right)\frac{1}{wL^R + r^kK}\left[wL^R\left(\hat{w}_t + \hat{L}_t^R\right) + r^kK\left(\hat{r}_t^k + \hat{K}_t\right)\right] \end{aligned} \tag{127}$$

Dividends are equivalent to real profits from firms of intermediate goods, therefore

$$Div_{j,\,t} = \left(1 - mc_{j,t}\right)Y_{j,t} - mc_{j,t}\Phi \tag{128}$$

This leads to

$$\widehat{Div}_t = \frac{\lambda_p}{1+\lambda_p-\theta}\hat{Y}_t - \frac{\theta}{1+\lambda_p-\theta}\widehat{mc}_t \tag{129}$$

Replacing in the above expression yields

$$\begin{aligned} &\frac{m}{Y\sigma_m}\left[\frac{\sigma_c}{1-h}\left(\hat{C}_t^R - (1+h)\,\hat{C}_{t-1}^R + h\hat{C}_{t-2}^R\right) + \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right) - \frac{\beta}{1-\beta}\left(\hat{R}_t - \hat{R}_{t-1}\right)\right] \\ &\qquad + \frac{b_t}{R} - b_{t-1} + \frac{C}{Y}\hat{C}_t^R + \frac{I}{Y}\hat{I}_t^R + t_t^R - \left[\frac{\lambda_p}{1+\lambda_p}\hat{Y}_t - \frac{\theta}{1+\lambda_p}\widehat{mc}_t\right] \\ &\qquad = \left(\frac{\theta}{1+\lambda_p}\right)\left[(1-\alpha)\left(\hat{w}_t + \hat{L}_t^R\right) + \alpha\left(\hat{r}_t^k + \hat{K}_t\right)\right] \end{aligned} \tag{130}$$

With

$$\widehat{mc}_t = (1-\alpha)\,\hat{w}_t + \alpha\hat{r}_t^k - \hat{\varepsilon}_t^a \tag{131}$$

Simplifying

$$\begin{aligned}\frac{C}{Y}\hat{C}_t^R = \frac{m}{Y\sigma_m}\left[\left(-\hat{\varepsilon}_t^B + \hat{\varepsilon}_{t-1}^B\right) - \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right) + \hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1}\right] \\ + \left[-\frac{b_t}{R} + b_{t-1} - \frac{I}{Y}\hat{I}_t^R - t_t^R\right] + \left[\frac{\lambda_p}{1+\lambda_p}\hat{Y}_t - \frac{\theta}{1+\lambda_p}\widehat{mc}_t\right] \\ + \left(\frac{\theta}{1+\lambda_p}\right)\left[(1-\alpha)\left(\hat{w}_t + \hat{L}_t^R\right) + \alpha\left(\hat{r}_t^k + \hat{K}_t\right)\right]\end{aligned} \tag{132}$$

Or

$$\begin{aligned}\frac{C}{Y}\hat{C}_t^R = \frac{m}{Y\sigma_m}\left[\left(-\hat{\varepsilon}_t^B + \hat{\varepsilon}_{t-1}^B\right) - \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right) + \hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1}\right] \\ + \left[-\frac{b_t}{R} + b_{t-1} - \frac{I}{Y}\hat{I}_t^R - t_t^R\right] + \left[\frac{\lambda_p}{1+\lambda_p}\hat{Y}_t\right] + \left(\frac{\theta}{1+\lambda_p}\right)\left[(1-\alpha)\hat{L}_t^R + \alpha\hat{K}_t + \hat{\varepsilon}_t^a\right]\end{aligned} \tag{133}$$

Replacing in the aggregate consumption equation

$$\begin{aligned}\frac{C}{Y}\hat{C}_t = (1-\mu)\left[\frac{m}{Y\sigma_m}\left[\left(-\hat{\varepsilon}_t^B + \hat{\varepsilon}_{t-1}^B\right) - \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right) + \hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1}\right]\right. \\ \left. + \left[-\frac{b_t}{R} + b_{t-1} - \frac{I}{Y}\hat{I}_t^R - t_t^R\right] + \left[\frac{\lambda_p}{1+\lambda_p}\hat{Y}_t\right] + \left(\frac{\theta}{1+\lambda_p}\right)\left[(1-\alpha)\hat{L}_t^R + \alpha\hat{K}_t + \hat{\varepsilon}_t^a\right]\right] \\ +\mu\left[\frac{(1-\alpha)}{1+\lambda_p}\theta\left(\hat{w}_t + \hat{L}_t^{NR}\right) - t_t^{NR}\right]\end{aligned} \tag{134}$$

Using the fiscal rule

$$\begin{aligned}\frac{C}{Y}\hat{C}_t = (1-\mu)\left[\frac{m}{Y\sigma_m}\left[\left(-\hat{\varepsilon}_t^B + \hat{\varepsilon}_{t-1}^B\right) - \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right) + \hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1}\right]\right. \\ \left. + \left[\frac{\lambda_p}{1+\lambda_p}\hat{Y}_t - \frac{I}{Y}\hat{I}_t^R\right] + \left(\frac{\theta}{1+\lambda_p}\right)\left[(1-\alpha)\hat{L}_t^R + \alpha\hat{K}_t + \hat{\varepsilon}_t^a\right]\right] \\ +\mu\left[\frac{\theta(1-\alpha)}{1+\lambda_p}\left(\hat{w}_t + \hat{L}_t^{NR}\right) + \frac{b_t}{R} - b_{t-1}\right] - g_t\end{aligned} \tag{135}$$

Aggregate labor is given by

$$\hat{L}_t = (1-\mu)\hat{L}_t^R + \mu\hat{L}_t^{NR} \tag{136}$$

Replacing in the expression

$$\begin{aligned}\frac{C}{Y}\hat{C}_t = (1-\mu)\left[\frac{m}{Y\sigma_m}\left[\hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1} - \left(\hat{\varepsilon}_t^B - \hat{\varepsilon}_{t-1}^B\right) - \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right)\right]\right. \\ \left. + \left[\frac{\lambda_p}{1+\lambda_p}\hat{Y}_t - \frac{I}{Y}\hat{I}_t^R\right] + \left(\frac{\theta}{1+\lambda_p}\right)\left[\alpha\hat{K}_t + \hat{\varepsilon}_t^a\right]\right] \\ +\mu\left[\frac{\theta(1-\alpha)}{1+\lambda_p}\hat{w}_t + \frac{b_t}{R} - b_{t-1}\right] + \frac{\theta(1-\alpha)}{1+\lambda_p}\hat{L}_t - g_t\end{aligned} \tag{137}$$

The following equation for aggregate consumption is obtained

$$\frac{C}{Y}\hat{C}_t = (1-\mu)\left[\frac{m}{Y\sigma_m}\left[\hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1} - \left(\hat{\varepsilon}_t^B - \hat{\varepsilon}_{t-1}^B\right) - \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right)\right] + \left[\hat{Y}_t - \frac{I}{Y}\hat{I}_t^R\right]\right] + \mu\left[\frac{\theta(1-\alpha)}{1+\lambda_p}\left(\hat{L}_t + \hat{w}_t\right) + \frac{b_t}{R} - b_{t-1}\right] - g_t \tag{138}$$

Log-linearization of the capital accumulation equation 75, the labor demand equation 99 and firms production function gives

$$\frac{K}{Y}\hat{K}_{t+1} = (1-\delta)\frac{K}{Y}\hat{K}_t + \frac{I}{Y}\hat{x}_t + \frac{I}{Y}\hat{I}_t \tag{139}$$

$$\hat{L}_t = -\hat{w}_t + \hat{r}_t^k + \hat{K}_t \tag{140}$$

$$\hat{Y}_t = \theta\left[\hat{\varepsilon}_t^a + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t\right] \tag{141}$$

The monetary policy equation when the ZLB is not binding

$$\hat{R}_t = \phi_r\hat{R}_{t-1} + (1-\phi_r)\left(\phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t\right) \tag{142}$$

Based on a zero debt level at the steady state and a balanced budget, the following expression holds

$$b_t = R\left(g_t + b_{t-1} - t_t\right) \tag{143}$$

Using the fiscal rule 106 leads to

$$\frac{b_t}{R} = g_t\left(1-\phi_g\right) + b_{t-1}\left(1-\phi_b\right) \tag{144}$$

The condition for a non-explosive debt path is $R(1-\phi_b) < 1$ leading to $\phi_b > 1 - \frac{1}{R}$

The final goods market equilibrium condition is given by

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + g_t + \frac{I}{Y}\hat{I}_t \tag{145}$$

Log-linearized inflation equation

From log-linearization of

$$\left(P_t\right)^{-\frac{1}{\lambda_{p,t}}} = \xi_p\left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p}\right)^{-\frac{1}{\lambda_{p,t}}} + \left(1-\xi_p\right)\left(\tilde{p}_t^j\right)^{-\frac{1}{\lambda_{p,t}}} \tag{146}$$

The following expression is obtained

$$\hat{P}_t = \xi_p\left(\hat{P}_{t-1} + \gamma_p\hat{\pi}_{t-1}\right) + \left(1-\xi_p\right)\tilde{p}_t^j \tag{147}$$

Thus

$$\tilde{p}_t^j = \frac{\hat{P}_t - \xi_p \left(\hat{P}_{t-1} + \gamma_p \hat{\pi}_{t-1}\right)}{\left(1 - \xi_p\right)} \tag{148}$$

Log-linearization of 102 yields

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left(\frac{\lambda_p}{\left(1+\lambda_p\right)} \hat{\lambda}_{p,t+i} + \widehat{mc}_{t+i} + \hat{P}_{t+i} - \gamma_p \hat{P}_{t+i-1} \right) = E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left(\tilde{p}_t^j - \gamma_p \hat{P}_{t-1} \right) \tag{149}$$

Or

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left(\frac{\lambda_p}{\left(1+\lambda_p\right)} \hat{\lambda}_{p,t+i} + \widehat{mc}_{t+i} + \hat{P}_{t+i} - \gamma_p \hat{P}_{t+i-1} \right) = \frac{1}{1-\beta\xi_p} \left(\tilde{p}_t^j - \gamma_p \hat{P}_{t-1} \right) \tag{150}$$

The difference between this expression at time t+i and t+i+1 gives

$$\frac{\lambda_p}{\left(1+\lambda_p\right)} \hat{\lambda}_{p,t} + \widehat{mc}_t + \hat{P}_t - \gamma_p \hat{P}_{t-1} = \frac{1}{1-\beta\xi_p} \left[\left(\tilde{p}_t^j - \gamma_p \hat{P}_{t-1} \right) - \beta\xi_p \left(\tilde{p}_{t+1}^j - \gamma_p \hat{P}_t \right) \right] \tag{151}$$

Replacing with the expression of $\tilde{p}_t^j$ and simplifying yields

$$\frac{\lambda_p}{\left(1+\lambda_p\right)} \hat{\lambda}_{p,t} + \widehat{mc}_t = \frac{1}{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)} \left[\hat{\pi}_t \left(\xi_p + \beta\xi_p\gamma_p\right) - \beta\xi_p E_t \hat{\pi}_{t+1} - \xi_p \gamma_p \hat{\pi}_{t-1} \right] \tag{152}$$

Replacing with the expression of $\widehat{mc}_t$ and simplifying, the inflation equation is as follows

$$\hat{\pi}_t = \frac{\beta}{\left(1+\beta\gamma_p\right)} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{\left(1+\beta\gamma_p\right)} \hat{\pi}_{t-1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)\left[\eta_t^p + \alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a\right]}{\xi_p \left(1+\beta\gamma_p\right)} \tag{153}$$

Log-linearized wage equation

From

$$\frac{W_{h,t}^*}{P_t} E_t \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \left[\Lambda_{t+i} \frac{\left(P_{t+i-1}/P_{t-1}\right)^{\gamma_w}}{P_{t+i}/P_t} L_{h,t+i} \right] = E_t \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \left(1+\lambda_{w,t+i}\right) \varepsilon_{t+i}^B \varepsilon_{t+i}^L \left(L_{h,t+i}\right)^{1+\sigma_l} \tag{154}$$

The linearized version is

$$\begin{aligned} E_t \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \left[\sigma_l \hat{L}_{h,t+i} - \hat{\Lambda}_{t+i} + \hat{P}_{t+i} - \gamma_w \hat{P}_{t+i-1} + \frac{\lambda_w}{1+\lambda_w} \hat{\lambda}_{w,t+i} + \hat{\varepsilon}_{t+i}^B + \hat{\varepsilon}_{t+i}^L \right] \\ = E_t \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \left[\hat{W}_{h,t}^* - \gamma_w \hat{P}_{t-1} \right] \end{aligned} \tag{155}$$

Since

$$\hat{\Lambda}_t = \hat{\varepsilon}_t^B - \frac{\sigma_c}{(1-h)} \left(\hat{C}_t - h\hat{C}_{t-1} \right) \tag{156}$$

And

$$\hat{L}_{h,t+i} = -\left(\frac{1+\lambda_w}{\lambda_w}\right)\left(\hat{W}^*_{h,t+i} - \hat{W}_{t+i}\right) + \hat{L}_{t+i} \tag{157}$$

And

$$\hat{W}_{h,t+i} = \gamma_w\left(\hat{P}_{t+i-1} - \hat{P}_{t-1}\right) + \hat{W}_{h,t} \tag{158}$$

By replacing in the linearized relation and calculating the difference between this expression at time t+i and time t+i+1, the following equation is obtained

$$\begin{aligned}\left[\hat{\varepsilon}^L_t + \sigma_l\hat{L}_t + \frac{\sigma_c}{(1-h)}\left(\hat{C}_t - h\hat{C}_{t-1}\right) + \left(\frac{1+\lambda_w}{\lambda_w}\right)\hat{\lambda}_{w,t} + \sigma_l\left(\frac{1+\lambda_w}{\lambda_w}\right)\hat{W}_t\right]\frac{(1-\beta\xi_w)}{1+\sigma_l\left(\frac{1+\lambda_w}{\lambda_w}\right)} \\ = \hat{W}^*_{h,t} - \beta\xi_w\hat{W}^*_{h,t+1} - \gamma_w\hat{P}_{t-1} + \gamma_w\hat{P}_t + \gamma_w(1-\beta\xi_w)\hat{P}_{t-1} - \frac{(1-\beta\xi_w)}{(1+\sigma_l)\left(\frac{1+\lambda_w}{\lambda_w}\right)}\hat{P}_t\end{aligned} \tag{159}$$

Also (from 95)

$$\hat{W}^*_{h,t} = \frac{\hat{W}_t - \xi_w\left(\hat{W}_{t-1} + \gamma_w\hat{\pi}_{t-1}\right)}{1-\xi_w} \tag{160}$$

Replacing in the above expression and simplifying, the real wage equation is

$$\begin{aligned}\hat{w}_t = \frac{\beta}{(1+\beta)}E_t\hat{w}_{t+1} + \frac{1}{(1+\beta)}\hat{w}_{t-1} + \frac{\beta}{(1+\beta)}E_t\hat{\pi}_{t+1} - \frac{(1+\beta\gamma_w)}{(1+\beta)}\hat{\pi}_t + \frac{\gamma_w}{(1+\beta)}\hat{\pi}_{t-1} \\ + \left[\hat{\varepsilon}^L_t + \sigma_l\hat{L}_t + \frac{\sigma_c}{(1-h)}\left(\hat{C}_t - h\hat{C}_{t-1}\right) + \eta^w_t - \hat{w}_t\right]\frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\beta)(1+\sigma_l)\left(\frac{1+\lambda_w}{\lambda_w}\right)}\end{aligned} \tag{161}$$

The dynamic system also includes

$$\hat{\varepsilon}^B_t = \rho_B\hat{\varepsilon}^B_{t-1} + \hat{\eta}^B_t \tag{162}$$

$$\hat{\varepsilon}^L_t = \rho_L\hat{\varepsilon}^L_{t-1} + \hat{\eta}^L_t \tag{163}$$

$$\hat{\varepsilon}^M_t = \rho_M\hat{\varepsilon}^M_{t-1} + \hat{\eta}^M_t \tag{164}$$

$$\hat{x}_t = \rho_x\hat{x}_{t-1} + \hat{\eta}^x_t \tag{165}$$

$$\hat{\varepsilon}^a_t = \rho_a\hat{\varepsilon}^a_{t-1} + \hat{\eta}^a_t \tag{166}$$

$$g_t = \rho_g g_{t-1} + \eta^g_t \tag{167}$$

$$\lambda_{p,t} = \lambda_p + \eta^p_t \tag{168}$$

$$\lambda_{w,t} = \lambda_w + \eta^w_t \tag{169}$$

Appendix C: log-linearization of the extended model with distortionary taxation

Based on the modified Ricardian households budget constraint

$$\frac{M_t^R}{P_t}+\frac{B_t^R/R_t}{P_t}=\frac{M_{t-1}^R}{P_t}+\frac{B_{t-1}^R}{P_t}+(1-\tau_{w,t})\,w_tL_t^R+(1-\tau_{k,t})\,r_t^kK_t^R-\frac{\Theta_k}{2}\left(\frac{K_{t+1}^R}{K_t^R}-1\right)^2K_t^R$$
$$+(1-\tau_{k,t})\,Div_t^R+\tau_{k,t}\delta K_t^R-T_t^R-(1+\tau_{c,t})\,C_t^R-\frac{K_{t+1}^R-(1-\delta)\,K_t^R}{x_t} \tag{170}$$

Equations 69, 72, 73 and 74 become

$$\frac{\mathrm{dL}}{\mathrm{dC_t^R}}:\lambda_\mathrm{t}=\frac{\varepsilon_\mathrm{t}^\mathrm{B}}{(1+\tau_{c,t})\left(\mathrm{C_t^R}-\mathrm{H_t}\right)^{\sigma_c}} \tag{171}$$

$$\frac{dL}{dK_{t+1}^R}:\ \lambda_t\left[\frac{1}{x_t}+\Theta_k\left(\frac{K_{t+1}^R}{K_t^R}-1\right)\right]=\beta E_t\lambda_{t+1}\left[\frac{(1-\delta)}{x_{t+1}}+\frac{\Theta_k}{2}\left(\left(\frac{K_{t+2}^R}{K_{t+1}^R}\right)^2-1\right)\right.$$
$$\left.+(1-E_t\tau_{k,t+1})\,E_tr_{t+1}^k+E_t\tau_{k,t+1}\delta\right] \tag{172}$$

$$\frac{dL}{d\lambda_t}:\frac{M_t^R}{P_t}+\frac{B_t^R/R_t}{P_t}+T_t^R+(1+\tau_{c,t})\,C_t^R+\frac{K_{t+1}^R-(1-\delta)\,K_t^R}{x_t}+\frac{\Theta_k}{2}\left(\frac{K_{t+1}^R}{K_t^R}-1\right)^2K_t^R$$
$$=\frac{M_{t-1}^R}{P_t}+\frac{B_{t-1}^R}{P_t}+(1-\tau_{w,t})\,w_tL_t^R+(1-\tau_{k,t})\,r_t^kK_t^R\ +(1-\tau_{k,t})\,Div_t^R+\tau_{k,t}\delta K_t^R \tag{173}$$

$$\left[\left(C_t^R-H_t\right)^{\sigma_c}\varepsilon_t^M\right]^{1/\sigma_m}=\frac{M_t^R}{P_t}\left(\frac{R_t-1}{R_t\,(1+\tau_{c,t})}\right)^{1/\sigma_m} \tag{174}$$

The steady state rental rate of capital is

$$r^k=\frac{1-\beta}{\beta\,(1-\tau_k)}+\delta \tag{175}$$

As the government budget is balanced at the steady state, then

$$\frac{G}{Y}=\frac{T}{Y}+\tau_c\frac{C}{Y}+\tau_k\left(1-\frac{\theta}{(1+\lambda_p)}+\frac{K\,(1-\beta)}{Y\beta\,(1-\tau_k)}\right)+\tau_w\frac{\theta\,(1-\alpha)}{(1+\lambda_p)} \tag{176}$$

The log-linearized Ricardian households consumption equation becomes

$$\hat{C}_t^R=\frac{h\hat{C}_{t-1}^R}{(1+h)}+\frac{E_t\hat{C}_{t+1}^R}{(1+h)}-\frac{(1-h)}{\sigma_c\,(1+h)}\left(\hat{R}_t-E_t\hat{\pi}_{t+1}+E_t\hat{\varepsilon}_{t+1}^B-\hat{\varepsilon}_t^B+\frac{\tau_c}{1+\tau_c}\,(\hat{\tau}_{c,t}+E_t\hat{\tau}_{c,t+1})\right) \tag{177}$$

For non-Ricardian households

$$\hat{C}_t^{NR} = \frac{(1-\tau_w)(1-\alpha)\theta Y}{(1+\tau_c)(1+\lambda_p)C^{NR}}\left(\hat{w}_t + \hat{L}_t^{NR}\right) - \frac{\tau_w(1-\alpha)\theta Y}{(1+\tau_c)(1+\lambda_p)C^{NR}}\hat{\tau}_{w,t} - \frac{Y}{(1+\tau_c)C^{NR}}t_t^{NR} - \frac{\tau_c}{1+\tau_c}\hat{\tau}_{c,t} \tag{178}$$

The equation of aggregate consumption is therefore

$$\begin{aligned}\mu\tau_c\frac{C}{Y}\hat{C}_t = (1-\mu)\left[\frac{m}{Y\sigma_m}\left[\hat{\pi}_t + \frac{\beta}{1-\beta}\hat{R}_t - \frac{1}{1-\beta}\hat{R}_{t-1} - \left(\hat{\varepsilon}_t^B - \hat{\varepsilon}_{t-1}^B\right) - \left(\hat{\varepsilon}_t^M - \hat{\varepsilon}_{t-1}^M\right)\right]\right] \\ -\mu t_t - \mu\frac{\tau_w(1-\alpha)\theta}{(1+\lambda_p)}\hat{\tau}_{w,t} - \mu\frac{\tau_c C}{Y}\hat{\tau}_{c,t} + \mu\hat{w}_t\frac{(1-\tau_w)(1-\alpha)\theta}{(1+\lambda_p)}\end{aligned} \tag{179}$$

Equation 126 becomes

$$\begin{aligned}\Theta_k\hat{K}_t = (1+\beta)\Theta_k E_t\hat{K}_{t+1} - \beta\Theta_k E_t\hat{K}_{t+2} - \beta(1-\tau_k)r^k E_t\hat{r}_{t+1}^k - \beta\left(\delta - r^k\right)\tau_k E_t\hat{\tau}_{k,t+1} \\ +\hat{R}_t - E_t\hat{\pi}_{t+1} + \beta(1-\delta)E_t\hat{x}_{t+1} - \hat{x}_t\end{aligned} \tag{180}$$

Based on a zero-debt level at the steady state and a balanced budget, the log-linearized government budget constraint is

$$\begin{aligned}t_t + \tau_c\frac{C}{Y}\hat{C}_t + \tau_k\frac{Div}{Y}\widehat{Div}_t - \tau_k\frac{\delta K}{Y}\hat{K}_t + \tau_c\frac{C}{Y}\hat{\tau}_{c,t} + \tau_w\frac{wL}{Y}\hat{\tau}_{w,t} + \tau_k\frac{\left(Div - \delta K + r^k K\right)}{Y}\hat{\tau}_{k,t} \\ +\left(\tau_w\frac{(1-\alpha)}{\alpha}\right)\frac{wL}{Y}\left(\hat{w}_t + \hat{L}_t\right) + \tau_k\frac{r^k K}{Y}\left(\widehat{r^k}_t + \hat{K}_t\right) = g_t + b_{t-1} - \frac{b_t}{R_t}\end{aligned} \tag{181}$$

Or

$$\begin{aligned}t_t + \tau_c\frac{C}{Y}\hat{C}_t + \tau_k\left[\frac{\lambda_p}{1+\lambda_p}\hat{Y}_t - \frac{\theta}{1+\lambda_p}\left((1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{\varepsilon}_t^a\right)\right] - \tau_k\frac{\delta K}{Y}\hat{K}_t + \tau_c\frac{C}{Y}\hat{\tau}_{c,t} + \tau_w\frac{(1-\alpha)}{1+\lambda_p}\theta\hat{\tau}_{w,t} \\ +\tau_k\left[\frac{1+\lambda_p-\theta}{1+\lambda_p} + \frac{1-\beta}{\beta(1-\tau_k)}\frac{K}{Y}\right]\hat{\tau}_{k,t} + \left(\tau_w\frac{(1-\alpha)}{\alpha} + \tau_k\right)\frac{\alpha\,\theta}{1+\lambda_p}\left(\hat{w}_t + \hat{L}_t\right) = g_t + b_{t-1} - \frac{b_t}{R}\end{aligned} \tag{182}$$

The real wage equation becomes

$$\begin{aligned}\hat{w}_t = \frac{\beta}{(1+\beta)}E_t\hat{w}_{t+1} + \frac{1}{(1+\beta)}\hat{w}_{t-1} + \frac{\beta}{(1+\beta)}E_t\hat{\pi}_{t+1} - \frac{(1+\beta\gamma_w)}{(1+\beta)}\hat{\pi}_t + \frac{\gamma_w}{(1+\beta)}\hat{\pi}_{t-1} \\ +\left[\hat{\varepsilon}_t^L - \frac{\tau_w}{(1-\tau_w)}\hat{\tau}_{w,t} + \sigma_l\hat{L}_t + \frac{\sigma_c}{(1-h)}\left(\hat{C}_t - h\hat{C}_{t-1}\right) + \eta_t^w - \hat{w}_t\right]\frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\beta)(1+\sigma_l)\left(\frac{1+\lambda_w}{\lambda_w}\right)}\end{aligned} \tag{183}$$