DS-288 Numerical Methods
UE-201 Introduction to Scientific Computing
Due date: August 28, 2022 (Sunday 11:59 PM)

Homework-1

Total 100 points   Weight 5%

<div align="center">Answer **All** Questions.</div>

1. Using the **bisection method**, find the solution of $f(x) = 0$ by considering the interval $[2, 4]$. Iterate until you reach a relative tolerance of 10^{-3} between successive iterates. Report the root found and the number of iterations required.

$$f(x) = x^3 - 9x + 1 = 0$$

[20 points]

2. Using the **Secant and Regula-Falsi methods,** find the solution of $f(x) = 0$ by considering initial approximations at $x_0 = 0$ and $x_1 = 1$. Iterate until you reach a relative tolerance of 10^{-6} between successive iterates. Report the root found and the number of iterations needed for each method.

$$f(x) = \cos x - xe^x = 0.$$

[15 × 2 = 30 points]

3. Using the **Newton's method, Secant method, and Modified Newton's method,** find the solution of $f(x) = 0$ for the functions listed. Iterate until you reach a relative tolerance of 10^{-6} between successive iterates. Report the root found and the number of iterations needed for each method.

   (a) $f(x) = x + e^{-x^2} \cos x$.
   (b) $f(x) = (x + e^{-x^2} \cos x)^2$.

   Comment on the observed convergence rates in these cases.   [25 × 2 = 50 points]

   Note: er

   - Use initial guess $x_0 = 0$ for Newton's method and modify Newton's method.
   - Use initial guesses $x_0 = 0$ and $x_1 = 1$ for the Secant method.
   - Relative tolerance $= \frac{|x_n - x_{n-1}|}{|x_n|}$, where x_n is the nth iteration approximation.