

T-Test Vs Wilcoxon

Lukas Graz

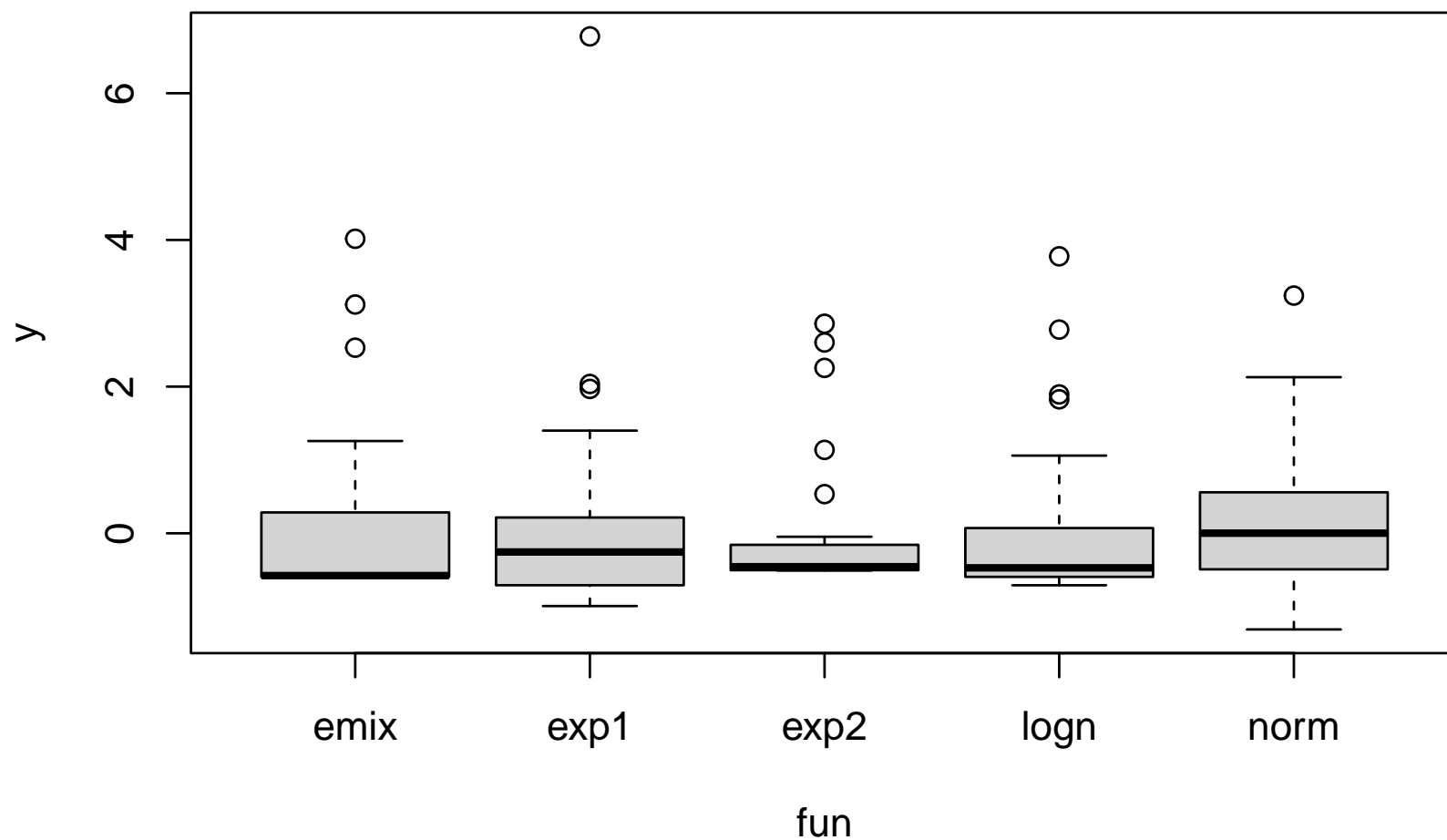
2023-09-20

Define a list of laws (i.e. distributions) with mean and sd.

```
# functions
n <- 40 # per group
laws <- list(
  norm = function(mean=0, sd=1) mean + sd * rnorm(n, 0, 1),
  logn = function(mean=0, sd=1) mean + sd * (rlnorm(n, sdlog=1) - 1.65) / 1.95,
  exp1 = function(mean=0, sd=1) mean + sd * (rexp(n) - 1),
  exp2 = function(mean=0, sd=1) mean + sd * (rexp(n)^2 - 2) / 3.94,
  emix = function(mean=0, sd=1) mean + sd * (c(rep(0, n/2), rexp(n/2)) - 0.5) / 0.84
)
```

```
set.seed(123)
get_data_all_laws <- function(){
  dat <- lapply(names(laws), function(fname){
    f <- laws[[fname]]
    data.frame(y=f(), fun=fname)
  })
  dat <- do.call(rbind, dat)
  dat
}
boxplot(y ~ fun, get_data_all_laws(), main= "Laws illustrated")
```

Laws illustrated



verify mean and standard deviation

```
sapply(laws, function(f) mean(replicate(10000, mean(f(0, 1)))))
```

```
##      norm      logn      exp1      exp2      emix
## 1.55e-03 -2.89e-03 -1.66e-03 -8.53e-05 -1.32e-03
```

```
sapply(laws, function(f) mean(replicate(10000, mean(f(1, 2)))))
```

```
##  norm  logn  exp1  exp2  emix
## 1.003 1.002 0.998 0.994 0.997
```

```
sapply(laws, function(f) mean(replicate(10000, sd(f(0, 1)))))
```

```
##  norm  logn  exp1  exp2  emix
## 0.994 1.008 0.977 0.999 1.000
```

```
sapply(laws, function(f) mean(replicate(10000, sd(f(1, 2)))))
```

```
## norm logn exp1 exp2 emix
## 1.99 2.01 1.96 1.99 2.00
```

Define the tests used

```
pval_t <- function(d) t.test(y ~ group, d)$p.value
pval_w <- function(d) coin::pvalue(coin::wilcox_test(y ~ group, d))
pval_m <- function(d) coin::pvalue(coin::median_test(y ~ group, d))
```

```
# returns data with f1() for group "A" and f2(mean2, sd2) for group "B"
get_data <- function(f1, f2, mean2=0, sd2=1){
  d <- rbind(
    data.frame(
      y=f1(),
      group="A"
    )
  )
}
```

```

    ),
    data.frame(
      y=f2(mean2, sd2),
      group="B"
    ))
d$group <- as.factor(d$group)
d
}

# get power of tests
get_power <- function(f1, f2, nsim=1000, mean2=0, sd2=1) {
  data_list <- replicate(nsim, get_data(f1, f2, mean2=mean2, sd2=sd2), simplify = FALSE)
  c(t = mean(sapply(data_list, pval_t) < 0.05),
    w = mean(sapply(data_list, pval_w) < 0.05),
    m = mean(sapply(data_list, pval_m) < 0.05))
}

# get all combinations of functions
fun_comb <- expand.grid(names(laws), names(laws)) |> as.matrix()
rnames <- apply(fun_comb, 1, paste0, collapse="_")

sim <- function(nsim=1000, mean2=0, sd2=1) {
  fun_comb_list <- split(fun_comb, row(fun_comb))
  coverage <- parallel::mclapply(fun_comb_list, function(f_names){
    f1 <- laws[[f_names[1]]]
    f2 <- laws[[f_names[2]]]
    get_power(f1, f2, nsim=nsim, mean2=mean2, sd2=sd2)
  })
  coverage <- do.call(rbind, coverage)
  rownames(coverage) <- rnames
  colnames(coverage) <- paste0(
    colnames(coverage), " ", as.character(mean2), " ", as.character(sd2))
  coverage |> as.data.frame()
}

```

Simulation

```
nsim <- 1000
```

```
set.seed(4321)
```

```
null <- sim(nsim=nsim)
```

```
set.seed(4321)
```

```
s_diff <- sim(nsim=nsim, sd2=2)
```

```
set.seed(4321)
```

```
s_diffiff <- sim(nsim=nsim, sd2=5)
```

```
set.seed(4321)
```

```
mu_diff <- sim(nsim=nsim, mean2 = 0.1)
```

```
set.seed(4321)
```

```
mu_diffiff <- sim(nsim=nsim, mean2 = 0.2)
```

```
results <- cbind(
  null,
  s_diff,
  s_diff,
  mu_diff,
  mu_diff
)
results * 100
```

| | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0.1 | w 0.1 | m 0.1 | t 0.2 | w 0.2 | m 0.2 |
|-----------|-----|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|
| | 1 | 1 | 1 | 2 | 2 | 2 | 5 | 5 | 5 | 1 | 1 | 1 | 1 | 1 | 1 |
| norm_norm | 4.2 | 4.3 | 3.8 | 4.0 | 6.0 | 5.1 | 5.0 | 7.8 | 10.0 | 6.3 | 5.7 | 4.9 | 14.7 | 14.6 | 10.0 |
| logn_norm | 6.3 | 15.1 | 32.4 | 4.1 | 6.7 | 17.2 | 4.2 | 8.7 | 12.2 | 10.9 | 24.5 | 44.8 | 21.7 | 41.7 | 62.7 |
| exp1_norm | 5.6 | 8.9 | 18.3 | 5.2 | 6.5 | 12.5 | 4.7 | 8.6 | 13.3 | 8.2 | 18.2 | 30.8 | 16.3 | 32.0 | 43.4 |
| exp2_norm | 5.2 | 15.4 | 45.6 | 5.5 | 9.3 | 24.5 | 5.0 | 8.4 | 14.0 | 11.7 | 29.0 | 65.5 | 21.9 | 45.4 | 78.4 |
| emix_norm | 3.6 | 11.5 | 48.6 | 5.0 | 7.1 | 22.5 | 5.0 | 8.2 | 10.9 | 6.8 | 24.3 | 64.6 | 14.8 | 40.7 | 75.8 |
| norm_logn | 5.4 | 12.5 | 28.9 | 7.4 | 40.3 | 52.9 | 9.6 | 77.3 | 76.7 | 5.4 | 7.3 | 19.4 | 9.2 | 6.6 | 9.8 |
| logn_logn | 5.4 | 4.9 | 4.3 | 6.7 | 57.4 | 32.5 | 8.6 | 77.6 | 68.2 | 7.7 | 18.2 | 10.1 | 14.6 | 55.0 | 27.9 |
| exp1_logn | 3.6 | 6.7 | 5.2 | 7.1 | 47.6 | 26.8 | 9.8 | 77.1 | 67.0 | 6.5 | 20.0 | 6.9 | 11.0 | 38.7 | 16.1 |
| exp2_logn | 3.3 | 8.7 | 6.1 | 6.4 | 57.6 | 33.8 | 8.9 | 79.4 | 73.1 | 6.7 | 10.0 | 25.2 | 17.1 | 51.0 | 54.1 |
| emix_logn | 2.2 | 2.2 | 9.6 | 7.3 | 54.6 | 0.7 | 9.4 | 76.4 | 15.3 | 3.7 | 34.0 | 32.1 | 12.1 | 91.3 | 58.8 |
| norm_exp1 | 5.1 | 8.8 | 18.5 | 6.3 | 28.9 | 35.0 | 6.7 | 50.7 | 52.1 | 6.8 | 5.8 | 11.2 | 11.5 | 6.9 | 7.6 |
| logn_exp1 | 5.4 | 7.9 | 6.0 | 5.5 | 35.2 | 19.7 | 7.3 | 50.6 | 44.0 | 9.5 | 8.2 | 10.3 | 19.6 | 24.3 | 21.7 |
| exp1_exp1 | 3.8 | 4.3 | 4.0 | 6.5 | 30.8 | 15.1 | 6.4 | 51.5 | 40.1 | 7.4 | 11.0 | 6.8 | 14.3 | 29.1 | 13.1 |
| exp2_exp1 | 5.2 | 12.5 | 7.8 | 4.8 | 33.9 | 17.2 | 5.7 | 51.4 | 44.6 | 10.4 | 6.1 | 20.2 | 20.4 | 18.3 | 44.9 |
| emix_exp1 | 3.0 | 4.4 | 11.8 | 3.7 | 35.2 | 1.6 | 6.7 | 49.5 | 5.1 | 5.1 | 6.8 | 24.0 | 13.2 | 27.5 | 46.0 |
| norm_exp2 | 6.6 | 15.6 | 47.3 | 10.2 | 52.2 | 83.3 | 12.9 | 95.2 | 95.3 | 4.3 | 7.8 | 29.4 | 10.0 | 6.0 | 16.6 |
| logn_exp2 | 3.9 | 9.4 | 7.8 | 8.7 | 85.1 | 68.5 | 13.6 | 96.1 | 94.0 | 6.5 | 36.3 | 10.0 | 15.7 | 67.7 | 27.1 |
| exp1_exp2 | 4.3 | 10.1 | 7.7 | 10.4 | 73.7 | 61.4 | 12.8 | 98.1 | 93.2 | 6.0 | 28.7 | 7.3 | 12.9 | 47.8 | 15.3 |
| exp2_exp2 | 4.2 | 5.3 | 5.0 | 9.3 | 91.2 | 70.6 | 12.9 | 96.0 | 93.9 | 6.9 | 66.0 | 27.1 | 18.0 | 93.2 | 64.9 |
| emix_exp2 | 2.3 | 62.9 | 10.1 | 8.2 | 84.2 | 5.7 | 12.3 | 96.7 | 49.8 | 3.5 | 85.0 | 36.6 | 12.2 | 93.5 | 64.0 |
| norm_emix | 3.7 | 11.3 | 50.4 | 2.9 | 49.5 | 77.9 | 4.5 | 90.7 | 89.1 | 4.3 | 5.9 | 31.1 | 7.8 | 4.4 | 16.5 |
| logn_emix | 2.1 | 3.8 | 12.1 | 3.4 | 85.7 | 64.3 | 4.0 | 90.6 | 86.3 | 4.5 | 9.7 | 2.9 | 17.4 | 45.6 | 0.4 |
| exp1_emix | 2.5 | 4.8 | 11.6 | 2.9 | 88.2 | 58.4 | 4.2 | 91.4 | 86.8 | 5.5 | 13.9 | 4.5 | 11.3 | 32.6 | 1.0 |
| exp2_emix | 2.1 | 60.4 | 9.6 | 2.1 | 85.3 | 65.2 | 4.5 | 91.1 | 88.3 | 6.0 | 1.4 | 1.1 | 17.1 | 69.9 | 0.3 |
| emix_emix | 1.3 | 0.0 | 0.0 | 2.1 | 88.7 | 2.3 | 4.8 | 91.5 | 16.9 | 2.8 | 83.7 | 0.0 | 9.1 | 94.9 | 0.4 |

Confidence intervals of ratios

```
prop <- function(ratio, nsim){
  confint_ <- prop.test(round(ratio*nsim), nsim)$conf.int[1:2]
  names(confint_) <- c("lower", "upper")
  c(
    ratio= ratio,
    confint_
  )
}
sapply(c(0:4/40, 3:10/20), prop, nsim) |> as.data.frame() * 100
```

| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 | V9 | V10 | V11 | V12 | V13 |
|-------|-------|------|------|------|-------|------|------|------|------|------|------|------|------|
| ratio | 0.000 | 2.50 | 5.00 | 7.50 | 10.00 | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 | 50.0 |
| lower | 0.000 | 1.66 | 3.77 | 5.98 | 8.24 | 12.9 | 17.6 | 22.4 | 27.2 | 32.1 | 37.0 | 41.9 | 46.9 |
| upper | 0.477 | 3.72 | 6.59 | 9.36 | 12.07 | 17.4 | 22.6 | 27.8 | 33.0 | 38.1 | 43.1 | 48.1 | 53.1 |

this gives an idea of the uncertainty of a ratio given 1000 simulations

redo analysis but with groupwise equal medians

Define a list of laws (i.e. distributions) with median and sd

```
# functions
n <- 40 # per group
# mean == to keep notation consistent
```

```
laws <- list(
  norm = function(mean=0, sd=1) mean + sd * rnorm(n, 0, 1)
  ,logn = function(mean=0, sd=1) mean + sd * (rlnorm(n, sdlog=1) - 1.02) / 1.95
  ,exp1 = function(mean=0, sd=1) mean + sd * (rexp(n) - 0.706)
  ,exp2 = function(mean=0, sd=1) mean + sd * (rexp(n)^2 - 0.523) / 3.94
  ,emix = function(mean=0, sd=1) mean + sd * (c(rep(0, n/2), rexp(n/2)) - 0.025) / 0.84
)
```

verify **median** and standard deviation

```
sapply(laws, function(f) mean(replicate(10000, median(f(0, 1)))))
```

```
##      norm      logn      exp1      exp2      emix
## -0.000142 -0.000216  0.002144  0.000204  0.000246
```

```
sapply(laws, function(f) mean(replicate(10000, median(f(1, 2)))))
```

```
## norm logn exp1 exp2 emix
## 1.000 1.001 0.998 1.000 0.999
```

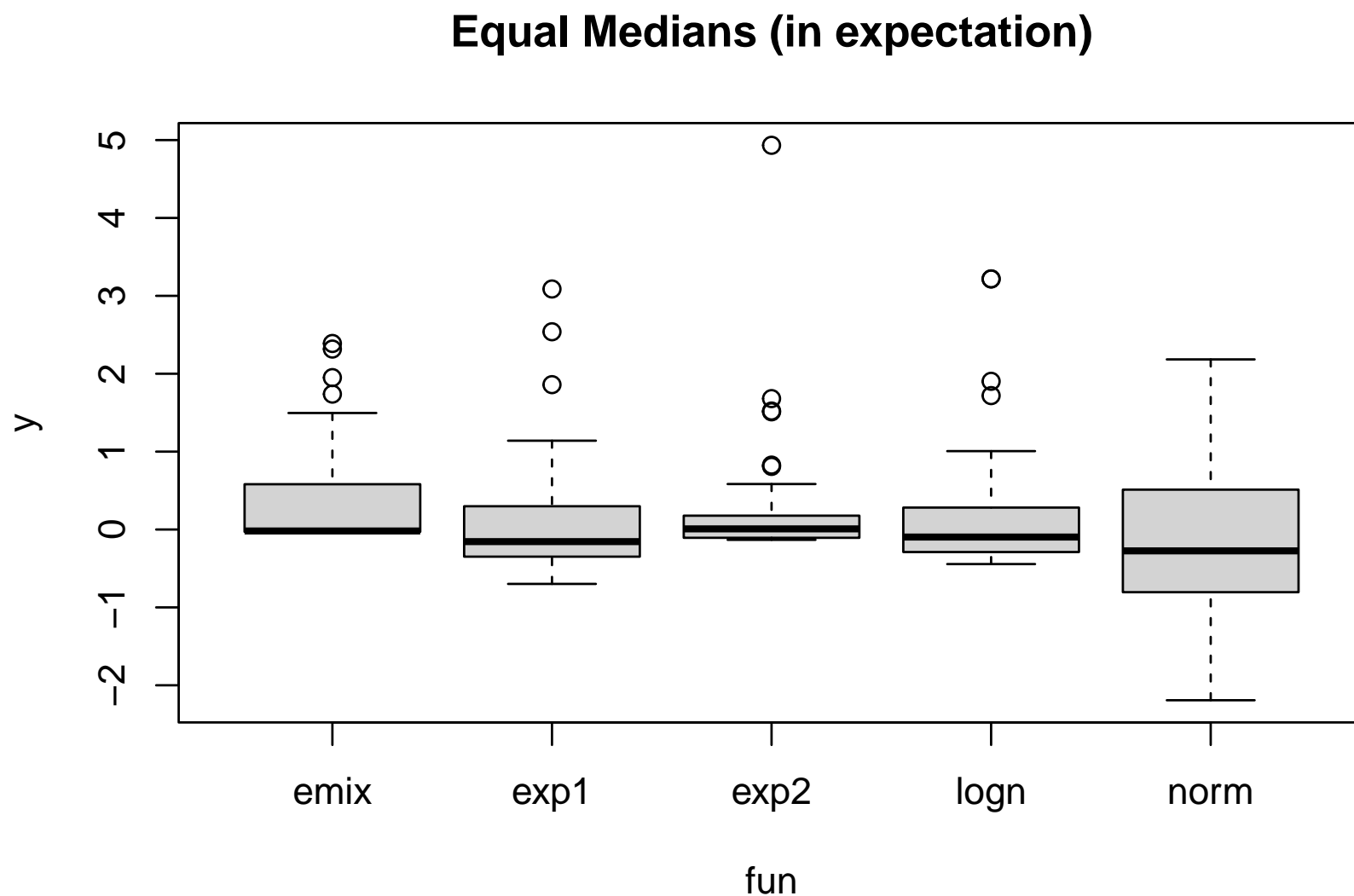
```
sapply(laws, function(f) mean(replicate(10000, sd(f(0, 1)))))
```

```
## norm logn exp1 exp2 emix
## 0.996 1.006 0.978 1.010 1.003
```

```
sapply(laws, function(f) mean(replicate(10000, sd(f(1, 2)))))
```

```
## norm logn exp1 exp2 emix
## 1.99 2.00 1.95 1.99 2.01
```

```
boxplot(y ~ fun, get_data_all_laws(), main= "Equal Medians (in expectation)")
```



```

set.seed(4321)
null_median <- sim(nsim=nsim)

set.seed(4321)
s_diff_median <- sim(nsim=nsim, sd2=2)

set.seed(4321)
s_difff_median <- sim(nsim=nsim, sd2=5)

set.seed(4321)
mu_diff_median <- sim(nsim=nsim, mean2 = 0.1)

set.seed(4321)
mu_difff_median <- sim(nsim=nsim, mean2 = 0.2)

results_median <- cbind(
  null_median,
  s_diff_median,
  s_difff_median,
  mu_diff_median,
  mu_difff_median
)
results_median * 100

```

| | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0.1 | w 0.1 | m 0.1 | t 0.2 | w 0.2 | m 0.2 |
|-----------|------|------|-----|------|------|------|-------|------|------|-------|-------|-------|-------|-------|-------|
| | 1 | 1 | 1 | 2 | 2 | 2 | 5 | 5 | 5 | 1 | 1 | 1 | 1 | 1 | 1 |
| norm_norm | 6.3 | 5.6 | 5.2 | 5.9 | 6.3 | 5.5 | 4.4 | 9.5 | 11.3 | 7.3 | 6.8 | 5.5 | 12.8 | 13.1 | 11.0 |
| logn_norm | 27.6 | 18.3 | 5.6 | 12.2 | 13.7 | 7.7 | 6.9 | 10.5 | 11.0 | 11.7 | 7.0 | 7.4 | 5.1 | 5.8 | 15.8 |
| exp1_norm | 23.8 | 12.6 | 4.2 | 12.0 | 12.2 | 6.7 | 5.8 | 9.3 | 9.8 | 10.9 | 6.0 | 6.5 | 5.5 | 6.1 | 12.8 |
| exp2_norm | 34.2 | 23.6 | 7.7 | 14.6 | 16.5 | 10.7 | 7.6 | 11.3 | 14.3 | 16.1 | 9.1 | 9.4 | 7.3 | 6.9 | 18.4 |
| emix_norm | 72.4 | 54.0 | 0.3 | 35.7 | 31.6 | 1.7 | 11.7 | 12.6 | 4.8 | 54.8 | 34.8 | 2.0 | 29.9 | 16.1 | 6.0 |
| norm_logn | 23.0 | 15.9 | 6.4 | 39.5 | 14.9 | 5.2 | 46.4 | 8.8 | 7.2 | 45.1 | 32.7 | 8.2 | 62.4 | 52.7 | 14.5 |
| logn_logn | 5.3 | 6.3 | 4.9 | 7.7 | 12.3 | 6.0 | 24.5 | 10.4 | 9.1 | 6.0 | 18.7 | 10.9 | 16.8 | 52.7 | 26.2 |
| exp1_logn | 3.8 | 9.4 | 4.3 | 8.7 | 6.5 | 4.6 | 23.6 | 8.7 | 5.9 | 8.9 | 25.5 | 8.7 | 14.8 | 47.6 | 20.8 |
| exp2_logn | 4.8 | 22.9 | 5.4 | 6.4 | 19.1 | 8.3 | 21.4 | 15.0 | 11.5 | 6.3 | 6.5 | 11.8 | 12.0 | 27.9 | 39.2 |
| emix_logn | 19.8 | 59.4 | 0.2 | 4.4 | 40.1 | 0.3 | 11.8 | 20.8 | 2.2 | 8.8 | 20.4 | 1.0 | 3.5 | 2.7 | 6.0 |
| norm_exp1 | 25.1 | 15.0 | 4.2 | 34.8 | 10.2 | 5.0 | 40.0 | 6.4 | 7.5 | 41.3 | 26.7 | 7.0 | 58.7 | 41.6 | 10.9 |
| logn_exp1 | 3.9 | 8.7 | 4.6 | 10.1 | 12.1 | 8.1 | 22.6 | 9.4 | 10.2 | 6.5 | 4.9 | 6.3 | 15.4 | 17.6 | 18.1 |
| exp1_exp1 | 5.3 | 5.7 | 4.9 | 9.6 | 9.5 | 5.2 | 24.4 | 9.2 | 8.7 | 9.7 | 12.8 | 7.2 | 15.1 | 27.0 | 12.2 |
| exp2_exp1 | 4.2 | 23.3 | 8.1 | 6.4 | 15.4 | 8.9 | 21.7 | 11.3 | 12.1 | 5.2 | 7.4 | 11.1 | 11.5 | 5.8 | 21.2 |
| emix_exp1 | 21.4 | 50.5 | 0.8 | 4.7 | 31.2 | 2.9 | 13.4 | 19.4 | 3.9 | 8.7 | 32.1 | 1.1 | 3.5 | 12.0 | 5.7 |
| norm_exp2 | 33.2 | 23.7 | 8.9 | 51.1 | 29.9 | 4.7 | 71.6 | 27.8 | 4.2 | 51.9 | 40.7 | 12.6 | 72.3 | 59.7 | 21.6 |
| logn_exp2 | 3.4 | 21.2 | 5.6 | 15.1 | 13.2 | 4.2 | 43.1 | 9.8 | 7.6 | 9.3 | 51.7 | 16.7 | 22.7 | 81.8 | 44.9 |
| exp1_exp2 | 5.7 | 24.2 | 7.3 | 15.6 | 22.9 | 3.9 | 43.6 | 6.0 | 5.6 | 11.3 | 43.8 | 15.0 | 21.4 | 64.6 | 29.7 |
| exp2_exp2 | 3.0 | 5.5 | 5.2 | 9.4 | 19.2 | 7.2 | 37.0 | 17.3 | 11.9 | 6.5 | 65.5 | 25.0 | 16.3 | 93.9 | 65.7 |
| emix_exp2 | 16.4 | 64.3 | 0.2 | 2.3 | 48.3 | 0.4 | 23.2 | 28.9 | 1.0 | 6.3 | 6.8 | 1.3 | 2.2 | 63.4 | 13.2 |
| norm_emix | 73.2 | 52.6 | 1.0 | 97.4 | 76.7 | 0.5 | 100.0 | 87.9 | 0.0 | 87.2 | 72.3 | 0.8 | 96.4 | 88.8 | 2.9 |
| logn_emix | 21.9 | 58.6 | 0.4 | 66.5 | 73.8 | 0.0 | 97.9 | 70.5 | 0.0 | 38.8 | 87.9 | 2.5 | 52.0 | 95.9 | 10.5 |
| exp1_emix | 21.9 | 52.2 | 0.6 | 68.9 | 72.2 | 0.5 | 99.4 | 81.9 | 0.0 | 39.1 | 73.2 | 1.5 | 59.1 | 86.6 | 4.3 |
| exp2_emix | 14.4 | 60.6 | 0.1 | 58.9 | 66.2 | 0.0 | 97.1 | 0.0 | 0.0 | 30.9 | 94.4 | 4.8 | 46.6 | 99.2 | 25.7 |
| emix_emix | 1.2 | 0.0 | 0.0 | 22.7 | 7.3 | 0.0 | 95.8 | 0.2 | 0.0 | 3.3 | 83.1 | 0.0 | 10.6 | 93.4 | 0.1 |

T-Test Vs Wilcoxon

Lukas Graz

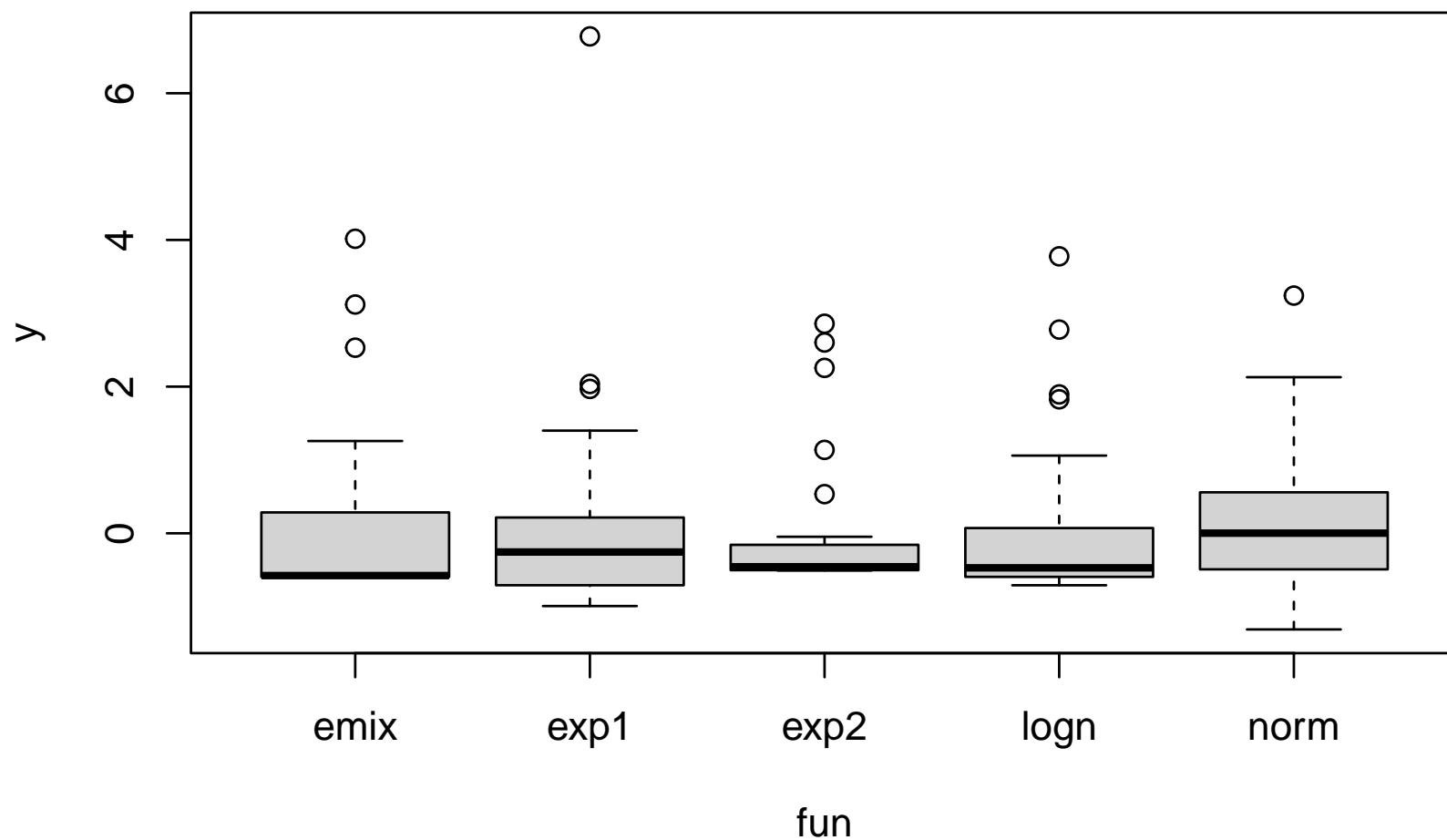
2023-09-20

Define a list of laws (i.e. distributions) with mean and sd.

```
# functions
n <- 40 # per group
laws <- list(
  norm = function(mean=0, sd=1) mean + sd * rnorm(n, 0, 1),
  logn = function(mean=0, sd=1) mean + sd * (rlnorm(n, sdlog=1) - 1.65) / 1.95,
  exp1 = function(mean=0, sd=1) mean + sd * (rexp(n) - 1),
  exp2 = function(mean=0, sd=1) mean + sd * (rexp(n)^2 - 2) / 3.94,
  emix = function(mean=0, sd=1) mean + sd * (c(rep(0, n/2), rexp(n/2)) - 0.5) / 0.84
)
```

```
set.seed(123)
get_data_all_laws <- function(){
  dat <- lapply(names(laws), function(fname){
    f <- laws[[fname]]
    data.frame(y=f(), fun=fname)
  })
  dat <- do.call(rbind, dat)
  dat
}
boxplot(y ~ fun, get_data_all_laws(), main= "Laws illustrated")
```

Laws illustrated



verify mean and standard deviation

```
sapply(laws, function(f) mean(replicate(10000, mean(f(0, 1)))))
```

```
##      norm      logn      exp1      exp2      emix
## 1.55e-03 -2.89e-03 -1.66e-03 -8.53e-05 -1.32e-03
```

```
sapply(laws, function(f) mean(replicate(10000, mean(f(1, 2)))))
```

```
## norm logn exp1 exp2 emix
## 1.003 1.002 0.998 0.994 0.997
```

```
sapply(laws, function(f) mean(replicate(10000, sd(f(0, 1)))))
```

```
## norm logn exp1 exp2 emix
## 0.994 1.008 0.977 0.999 1.000
```

```
sapply(laws, function(f) mean(replicate(10000, sd(f(1, 2)))))
```

```
## norm logn exp1 exp2 emix
## 1.99 2.01 1.96 1.99 2.00
```

Define the tests used

```
pval_t <- function(d) t.test(y ~ group, d)$p.value
pval_w <- function(d) coin::pvalue(coin::wilcox_test(y ~ group, d))
pval_m <- function(d) coin::pvalue(coin::median_test(y ~ group, d))
```

```
# returns data with f1() for group "A" and f2(mean2, sd2) for group "B"
get_data <- function(f1, f2, mean2=0, sd2=1){
  d <- rbind(
    data.frame(
      y=f1(),
      group="A"
    )
  )
}
```



```

    ),
    data.frame(
      y=f2(mean2, sd2),
      group="B"
    ))
d$group <- as.factor(d$group)
d
}

# get power of tests
get_power <- function(f1, f2, nsim=1000, mean2=0, sd2=1) {
  data_list <- replicate(nsim, get_data(f1, f2, mean2=mean2, sd2=sd2), simplify = FALSE)
  c(t = mean(sapply(data_list, pval_t) < 0.05),
    w = mean(sapply(data_list, pval_w) < 0.05),
    m = mean(sapply(data_list, pval_m) < 0.05))
}

# get all combinations of functions
fun_comb <- expand.grid(names(laws), names(laws)) |> as.matrix()
rnames <- apply(fun_comb, 1, paste0, collapse="_")

sim <- function(nsim=1000, mean2=0, sd2=1) {
  fun_comb_list <- split(fun_comb, row(fun_comb))
  coverage <- parallel::mclapply(fun_comb_list, function(f_names){
    f1 <- laws[[f_names[1]]]
    f2 <- laws[[f_names[2]]]
    get_power(f1, f2, nsim=nsim, mean2=mean2, sd2=sd2)
  })
  coverage <- do.call(rbind, coverage)
  rownames(coverage) <- rnames
  colnames(coverage) <- paste0(
    colnames(coverage), " ", as.character(mean2), " ", as.character(sd2))
  coverage |> as.data.frame()
}

```

Simulation

```
nsim <- 1000
```

```
set.seed(4321)
```

```
null <- sim(nsim=nsim)
```

```
set.seed(4321)
```

```
s_diff <- sim(nsim=nsim, sd2=2)
```

```
set.seed(4321)
```

```
s_diffiff <- sim(nsim=nsim, sd2=5)
```

```
set.seed(4321)
```

```
mu_diff <- sim(nsim=nsim, mean2 = 0.1)
```

```
set.seed(4321)
```

```
mu_diffiff <- sim(nsim=nsim, mean2 = 0.2)
```

```
results <- cbind(
  null,
  s_diff,
  s_diff,
  mu_diff,
  mu_diff
)
results * 100
```

| | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0.1 | w 0.1 | m 0.1 | t 0.2 | w 0.2 | m 0.2 |
|-----------|-----|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|
| | 1 | 1 | 1 | 2 | 2 | 2 | 5 | 5 | 5 | 1 | 1 | 1 | 1 | 1 | 1 |
| norm_norm | 4.2 | 4.3 | 3.8 | 4.0 | 6.0 | 5.1 | 5.0 | 7.8 | 10.0 | 6.3 | 5.7 | 4.9 | 14.7 | 14.6 | 10.0 |
| logn_norm | 6.3 | 15.1 | 32.4 | 4.1 | 6.7 | 17.2 | 4.2 | 8.7 | 12.2 | 10.9 | 24.5 | 44.8 | 21.7 | 41.7 | 62.7 |
| exp1_norm | 5.6 | 8.9 | 18.3 | 5.2 | 6.5 | 12.5 | 4.7 | 8.6 | 13.3 | 8.2 | 18.2 | 30.8 | 16.3 | 32.0 | 43.4 |
| exp2_norm | 5.2 | 15.4 | 45.6 | 5.5 | 9.3 | 24.5 | 5.0 | 8.4 | 14.0 | 11.7 | 29.0 | 65.5 | 21.9 | 45.4 | 78.4 |
| emix_norm | 3.6 | 11.5 | 48.6 | 5.0 | 7.1 | 22.5 | 5.0 | 8.2 | 10.9 | 6.8 | 24.3 | 64.6 | 14.8 | 40.7 | 75.8 |
| norm_logn | 5.4 | 12.5 | 28.9 | 7.4 | 40.3 | 52.9 | 9.6 | 77.3 | 76.7 | 5.4 | 7.3 | 19.4 | 9.2 | 6.6 | 9.8 |
| logn_logn | 5.4 | 4.9 | 4.3 | 6.7 | 57.4 | 32.5 | 8.6 | 77.6 | 68.2 | 7.7 | 18.2 | 10.1 | 14.6 | 55.0 | 27.9 |
| exp1_logn | 3.6 | 6.7 | 5.2 | 7.1 | 47.6 | 26.8 | 9.8 | 77.1 | 67.0 | 6.5 | 20.0 | 6.9 | 11.0 | 38.7 | 16.1 |
| exp2_logn | 3.3 | 8.7 | 6.1 | 6.4 | 57.6 | 33.8 | 8.9 | 79.4 | 73.1 | 6.7 | 10.0 | 25.2 | 17.1 | 51.0 | 54.1 |
| emix_logn | 2.2 | 2.2 | 9.6 | 7.3 | 54.6 | 0.7 | 9.4 | 76.4 | 15.3 | 3.7 | 34.0 | 32.1 | 12.1 | 91.3 | 58.8 |
| norm_exp1 | 5.1 | 8.8 | 18.5 | 6.3 | 28.9 | 35.0 | 6.7 | 50.7 | 52.1 | 6.8 | 5.8 | 11.2 | 11.5 | 6.9 | 7.6 |
| logn_exp1 | 5.4 | 7.9 | 6.0 | 5.5 | 35.2 | 19.7 | 7.3 | 50.6 | 44.0 | 9.5 | 8.2 | 10.3 | 19.6 | 24.3 | 21.7 |
| exp1_exp1 | 3.8 | 4.3 | 4.0 | 6.5 | 30.8 | 15.1 | 6.4 | 51.5 | 40.1 | 7.4 | 11.0 | 6.8 | 14.3 | 29.1 | 13.1 |
| exp2_exp1 | 5.2 | 12.5 | 7.8 | 4.8 | 33.9 | 17.2 | 5.7 | 51.4 | 44.6 | 10.4 | 6.1 | 20.2 | 20.4 | 18.3 | 44.9 |
| emix_exp1 | 3.0 | 4.4 | 11.8 | 3.7 | 35.2 | 1.6 | 6.7 | 49.5 | 5.1 | 5.1 | 6.8 | 24.0 | 13.2 | 27.5 | 46.0 |
| norm_exp2 | 6.6 | 15.6 | 47.3 | 10.2 | 52.2 | 83.3 | 12.9 | 95.2 | 95.3 | 4.3 | 7.8 | 29.4 | 10.0 | 6.0 | 16.6 |
| logn_exp2 | 3.9 | 9.4 | 7.8 | 8.7 | 85.1 | 68.5 | 13.6 | 96.1 | 94.0 | 6.5 | 36.3 | 10.0 | 15.7 | 67.7 | 27.1 |
| exp1_exp2 | 4.3 | 10.1 | 7.7 | 10.4 | 73.7 | 61.4 | 12.8 | 98.1 | 93.2 | 6.0 | 28.7 | 7.3 | 12.9 | 47.8 | 15.3 |
| exp2_exp2 | 4.2 | 5.3 | 5.0 | 9.3 | 91.2 | 70.6 | 12.9 | 96.0 | 93.9 | 6.9 | 66.0 | 27.1 | 18.0 | 93.2 | 64.9 |
| emix_exp2 | 2.3 | 62.9 | 10.1 | 8.2 | 84.2 | 5.7 | 12.3 | 96.7 | 49.8 | 3.5 | 85.0 | 36.6 | 12.2 | 93.5 | 64.0 |
| norm_emix | 3.7 | 11.3 | 50.4 | 2.9 | 49.5 | 77.9 | 4.5 | 90.7 | 89.1 | 4.3 | 5.9 | 31.1 | 7.8 | 4.4 | 16.5 |
| logn_emix | 2.1 | 3.8 | 12.1 | 3.4 | 85.7 | 64.3 | 4.0 | 90.6 | 86.3 | 4.5 | 9.7 | 2.9 | 17.4 | 45.6 | 0.4 |
| exp1_emix | 2.5 | 4.8 | 11.6 | 2.9 | 88.2 | 58.4 | 4.2 | 91.4 | 86.8 | 5.5 | 13.9 | 4.5 | 11.3 | 32.6 | 1.0 |
| exp2_emix | 2.1 | 60.4 | 9.6 | 2.1 | 85.3 | 65.2 | 4.5 | 91.1 | 88.3 | 6.0 | 1.4 | 1.1 | 17.1 | 69.9 | 0.3 |
| emix_emix | 1.3 | 0.0 | 0.0 | 2.1 | 88.7 | 2.3 | 4.8 | 91.5 | 16.9 | 2.8 | 83.7 | 0.0 | 9.1 | 94.9 | 0.4 |

Confidence intervals of ratios

```
prop <- function(ratio, nsim){
  confint_ <- prop.test(round(ratio*nsim), nsim)$conf.int[1:2]
  names(confint_) <- c("lower", "upper")
  c(
    ratio= ratio,
    confint_
  )
}
sapply(c(0:4/40, 3:10/20), prop, nsim) |> as.data.frame() * 100
```

| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 | V9 | V10 | V11 | V12 | V13 |
|-------|-------|------|------|------|-------|------|------|------|------|------|------|------|------|
| ratio | 0.000 | 2.50 | 5.00 | 7.50 | 10.00 | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 | 50.0 |
| lower | 0.000 | 1.66 | 3.77 | 5.98 | 8.24 | 12.9 | 17.6 | 22.4 | 27.2 | 32.1 | 37.0 | 41.9 | 46.9 |
| upper | 0.477 | 3.72 | 6.59 | 9.36 | 12.07 | 17.4 | 22.6 | 27.8 | 33.0 | 38.1 | 43.1 | 48.1 | 53.1 |

this gives an idea of the uncertainty of a ratio given 1000 simulations

redo analysis but with groupwise equal medians

Define a list of laws (i.e. distributions) with median and sd

```
# functions
n <- 40 # per group
# mean == to keep notation consistent
```

```
laws <- list(
  norm = function(mean=0, sd=1) mean + sd * rnorm(n, 0, 1)
  ,logn = function(mean=0, sd=1) mean + sd * (rlnorm(n, sdlog=1) - 1.02) / 1.95
  ,exp1 = function(mean=0, sd=1) mean + sd * (rexp(n) - 0.706)
  ,exp2 = function(mean=0, sd=1) mean + sd * (rexp(n)^2 - 0.523) / 3.94
  ,emix = function(mean=0, sd=1) mean + sd * (c(rep(0, n/2), rexp(n/2)) - 0.025) / 0.84
)
```

verify **median** and standard deviation

```
sapply(laws, function(f) mean(replicate(10000, median(f(0, 1)))))
```

```
##      norm      logn      exp1      exp2      emix
## -0.000142 -0.000216  0.002144  0.000204  0.000246
```

```
sapply(laws, function(f) mean(replicate(10000, median(f(1, 2)))))
```

```
## norm logn exp1 exp2 emix
## 1.000 1.001 0.998 1.000 0.999
```

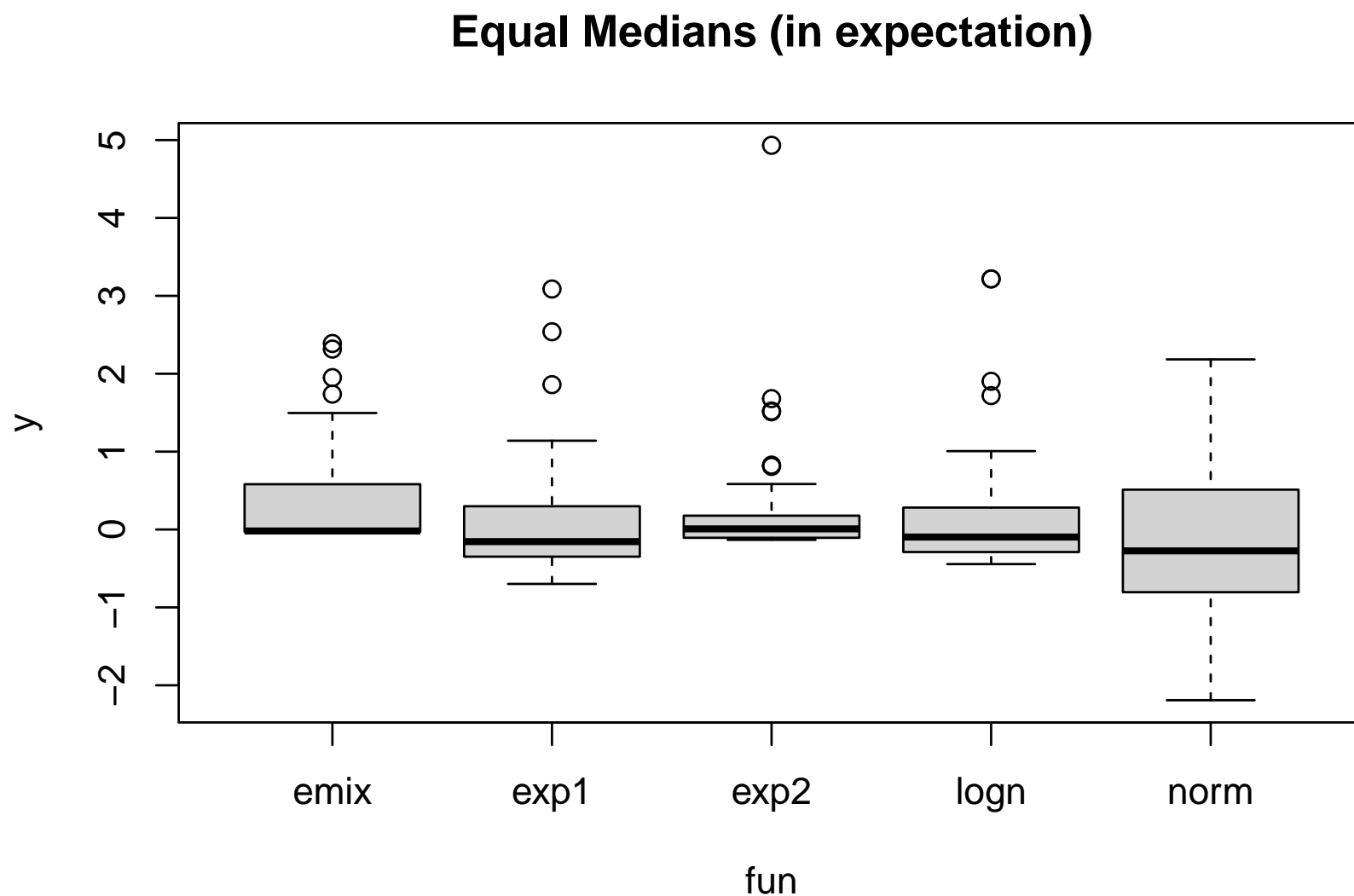
```
sapply(laws, function(f) mean(replicate(10000, sd(f(0, 1)))))
```

```
## norm logn exp1 exp2 emix
## 0.996 1.006 0.978 1.010 1.003
```

```
sapply(laws, function(f) mean(replicate(10000, sd(f(1, 2)))))
```

```
## norm logn exp1 exp2 emix
## 1.99 2.00 1.95 1.99 2.01
```

```
boxplot(y ~ fun, get_data_all_laws(), main= "Equal Medians (in expectation)")
```



```

set.seed(4321)
null_median <- sim(nsim=nsim)

set.seed(4321)
s_diff_median <- sim(nsim=nsim, sd2=2)

set.seed(4321)
s_difff_median <- sim(nsim=nsim, sd2=5)

set.seed(4321)
mu_diff_median <- sim(nsim=nsim, mean2 = 0.1)

set.seed(4321)
mu_difff_median <- sim(nsim=nsim, mean2 = 0.2)

results_median <- cbind(
  null_median,
  s_diff_median,
  s_difff_median,
  mu_diff_median,
  mu_difff_median
)
results_median * 100

```

| | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0 | w 0 | m 0 | t 0.1 | w 0.1 | m 0.1 | t 0.2 | w 0.2 | m 0.2 |
|-----------|------|------|-----|------|------|------|-------|------|------|-------|-------|-------|-------|-------|-------|
| | 1 | 1 | 1 | 2 | 2 | 2 | 5 | 5 | 5 | 1 | 1 | 1 | 1 | 1 | 1 |
| norm_norm | 6.3 | 5.6 | 5.2 | 5.9 | 6.3 | 5.5 | 4.4 | 9.5 | 11.3 | 7.3 | 6.8 | 5.5 | 12.8 | 13.1 | 11.0 |
| logn_norm | 27.6 | 18.3 | 5.6 | 12.2 | 13.7 | 7.7 | 6.9 | 10.5 | 11.0 | 11.7 | 7.0 | 7.4 | 5.1 | 5.8 | 15.8 |
| exp1_norm | 23.8 | 12.6 | 4.2 | 12.0 | 12.2 | 6.7 | 5.8 | 9.3 | 9.8 | 10.9 | 6.0 | 6.5 | 5.5 | 6.1 | 12.8 |
| exp2_norm | 34.2 | 23.6 | 7.7 | 14.6 | 16.5 | 10.7 | 7.6 | 11.3 | 14.3 | 16.1 | 9.1 | 9.4 | 7.3 | 6.9 | 18.4 |
| emix_norm | 72.4 | 54.0 | 0.3 | 35.7 | 31.6 | 1.7 | 11.7 | 12.6 | 4.8 | 54.8 | 34.8 | 2.0 | 29.9 | 16.1 | 6.0 |
| norm_logn | 23.0 | 15.9 | 6.4 | 39.5 | 14.9 | 5.2 | 46.4 | 8.8 | 7.2 | 45.1 | 32.7 | 8.2 | 62.4 | 52.7 | 14.5 |
| logn_logn | 5.3 | 6.3 | 4.9 | 7.7 | 12.3 | 6.0 | 24.5 | 10.4 | 9.1 | 6.0 | 18.7 | 10.9 | 16.8 | 52.7 | 26.2 |
| exp1_logn | 3.8 | 9.4 | 4.3 | 8.7 | 6.5 | 4.6 | 23.6 | 8.7 | 5.9 | 8.9 | 25.5 | 8.7 | 14.8 | 47.6 | 20.8 |
| exp2_logn | 4.8 | 22.9 | 5.4 | 6.4 | 19.1 | 8.3 | 21.4 | 15.0 | 11.5 | 6.3 | 6.5 | 11.8 | 12.0 | 27.9 | 39.2 |
| emix_logn | 19.8 | 59.4 | 0.2 | 4.4 | 40.1 | 0.3 | 11.8 | 20.8 | 2.2 | 8.8 | 20.4 | 1.0 | 3.5 | 2.7 | 6.0 |
| norm_exp1 | 25.1 | 15.0 | 4.2 | 34.8 | 10.2 | 5.0 | 40.0 | 6.4 | 7.5 | 41.3 | 26.7 | 7.0 | 58.7 | 41.6 | 10.9 |
| logn_exp1 | 3.9 | 8.7 | 4.6 | 10.1 | 12.1 | 8.1 | 22.6 | 9.4 | 10.2 | 6.5 | 4.9 | 6.3 | 15.4 | 17.6 | 18.1 |
| exp1_exp1 | 5.3 | 5.7 | 4.9 | 9.6 | 9.5 | 5.2 | 24.4 | 9.2 | 8.7 | 9.7 | 12.8 | 7.2 | 15.1 | 27.0 | 12.2 |
| exp2_exp1 | 4.2 | 23.3 | 8.1 | 6.4 | 15.4 | 8.9 | 21.7 | 11.3 | 12.1 | 5.2 | 7.4 | 11.1 | 11.5 | 5.8 | 21.2 |
| emix_exp1 | 21.4 | 50.5 | 0.8 | 4.7 | 31.2 | 2.9 | 13.4 | 19.4 | 3.9 | 8.7 | 32.1 | 1.1 | 3.5 | 12.0 | 5.7 |
| norm_exp2 | 33.2 | 23.7 | 8.9 | 51.1 | 29.9 | 4.7 | 71.6 | 27.8 | 4.2 | 51.9 | 40.7 | 12.6 | 72.3 | 59.7 | 21.6 |
| logn_exp2 | 3.4 | 21.2 | 5.6 | 15.1 | 13.2 | 4.2 | 43.1 | 9.8 | 7.6 | 9.3 | 51.7 | 16.7 | 22.7 | 81.8 | 44.9 |
| exp1_exp2 | 5.7 | 24.2 | 7.3 | 15.6 | 22.9 | 3.9 | 43.6 | 6.0 | 5.6 | 11.3 | 43.8 | 15.0 | 21.4 | 64.6 | 29.7 |
| exp2_exp2 | 3.0 | 5.5 | 5.2 | 9.4 | 19.2 | 7.2 | 37.0 | 17.3 | 11.9 | 6.5 | 65.5 | 25.0 | 16.3 | 93.9 | 65.7 |
| emix_exp2 | 16.4 | 64.3 | 0.2 | 2.3 | 48.3 | 0.4 | 23.2 | 28.9 | 1.0 | 6.3 | 6.8 | 1.3 | 2.2 | 63.4 | 13.2 |
| norm_emix | 73.2 | 52.6 | 1.0 | 97.4 | 76.7 | 0.5 | 100.0 | 87.9 | 0.0 | 87.2 | 72.3 | 0.8 | 96.4 | 88.8 | 2.9 |
| logn_emix | 21.9 | 58.6 | 0.4 | 66.5 | 73.8 | 0.0 | 97.9 | 70.5 | 0.0 | 38.8 | 87.9 | 2.5 | 52.0 | 95.9 | 10.5 |
| exp1_emix | 21.9 | 52.2 | 0.6 | 68.9 | 72.2 | 0.5 | 99.4 | 81.9 | 0.0 | 39.1 | 73.2 | 1.5 | 59.1 | 86.6 | 4.3 |
| exp2_emix | 14.4 | 60.6 | 0.1 | 58.9 | 66.2 | 0.0 | 97.1 | 0.0 | 0.0 | 30.9 | 94.4 | 4.8 | 46.6 | 99.2 | 25.7 |
| emix_emix | 1.2 | 0.0 | 0.0 | 22.7 | 7.3 | 0.0 | 95.8 | 0.2 | 0.0 | 3.3 | 83.1 | 0.0 | 10.6 | 93.4 | 0.1 |