MECE 5397

Project A – Poisson Equation

Write a computer code to solve the two-dimensional Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -F(x,y) \tag{1}$$

The domain of interest is the rectangle

$$a_x < x < b_x, \qquad a_y < y < b_y \tag{2}$$

and the boundary conditions

$$u(x = a_x, y) = f_a(y), \qquad u(x = b_x, y) = g_a(y), \tag{3}$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=a_y} = 0, \qquad u(x, y = b_y) = f_a(b_y) + \frac{x - a_x}{b_x - a_x}[g_a(b_y) - f_a(b_y)] \tag{4}$$

$$a_x = a_y = 0, \qquad b_x = b_y = 2\pi \tag{5}$$

$$g_a(y) = (y - a_y)^2 \cos y, \qquad f_a(y) = y(y - a_y)^2 \tag{6}$$

$$F(x,y) = \cos\left[\frac{\pi}{2}\left(2\frac{x - a_x}{b_x - a_x} + 1\right)\right] \sin\left[\pi\frac{y - a_y}{b_y - a_y}\right] \tag{7}$$

Use ghost node(s) for Neumann condition(s).
After carrying out all the simulations needed for the report, run one last simulation with $F = 0$ and include the results in the report.