Note on the mutipole expansion of a heaving hemisphere   
on the free surface

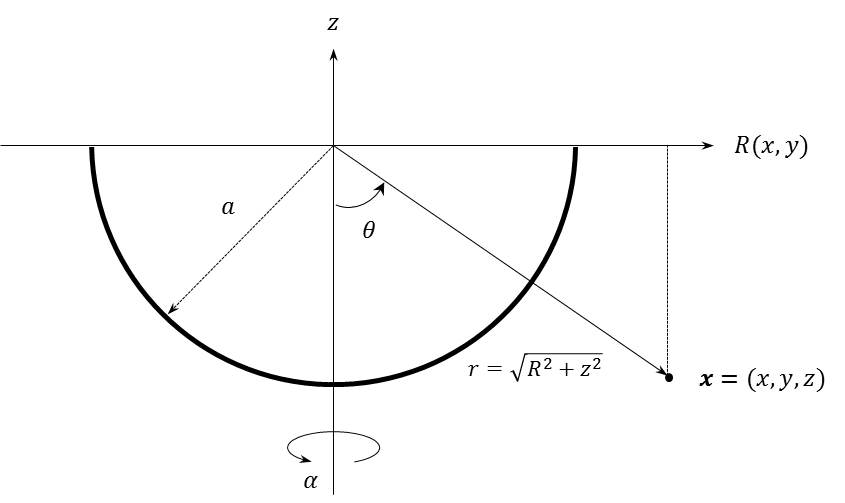
YoungMyung Choi

LHHEA, Ecole Centrale de Nantes, Nantes, France

**Heaving hemisphere problem**

An heaving hemisphere problem was first solved by Havelock(1955). Havelock uses the multipole expansion method similarly to what done by Ursell(1949). He expressed the velocity potential with multipoles composed of wave source and wave free terms. Later, Hulme(1982) modified the expression of Havelock to be more rigorous.

The coordinates for the heaving hemisphere are defined in the figure below.

****

**Multipole expansion of heaving hemisphere**

The multipole expansion for the heaving hemisphere is given by Hulme (1982). The multipoles satisfy the Laplace equation, the free surface boundary condition, the bottom boundary condition and the radiation condition. As the heave motion is symmetric, the velocity potential is expressed as follows:

where

and

Wave-pole is expressed in an alternative form. Please remind that the expression of di-gamma function in the following form is correct (different with Hulme original notation).

As it can be derived as follows:

**Multipole amplitudes**

The body boundary condition is applied to calculate the multipole amplitudes.

Integrating the above expression with respect to *μ* over (0, 1), and with the Hulme(1982) assumption that *c*0 is not zero:

where

Let

Substituting

Recalling the body boundary condition

The following equations are obtained after some manipulations:

Then multiplying ) and integrating with respect to *μ* over (0, 1).

Using the orthogonality of Legendre functions

The system of equations of the heaving hemisphere expressed with the multipoles is given as :

where

An algebraic form is given as:

The value of *c*0 is given as:

**Force acting on the heaving hemisphere**

The acting force is then given as

The nondimensionalized force is given as:

**Wave elevation and fluids velocity**

Wave elevation

Velocity

As the heave motion is symmetric,

The components to be evaluated for the velocity computation are:

**Derivatives with respect to r (Wave-term)**

considering

**Derivatives with respect to r (Wave-free-term)**

considering

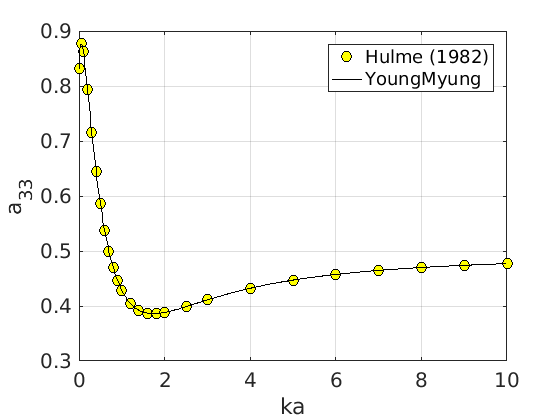
**Derivatives with respect to theta (Wave-term)**

**Derivatives with respect to theta (Wave-free-term)**

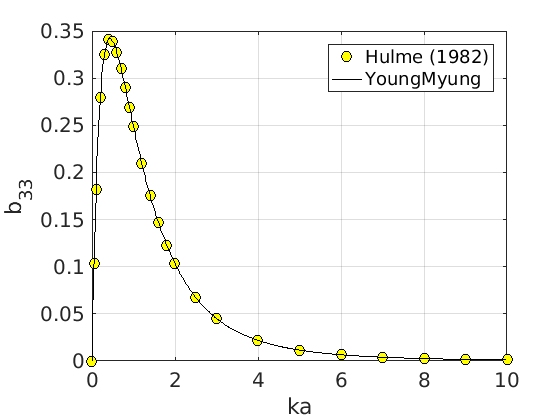
**Jacobian (Gasirowicz 1974, pp: 167-168, Arfket 1985; p.108)**

: See Wolfram alpha ( definition is opposite ,

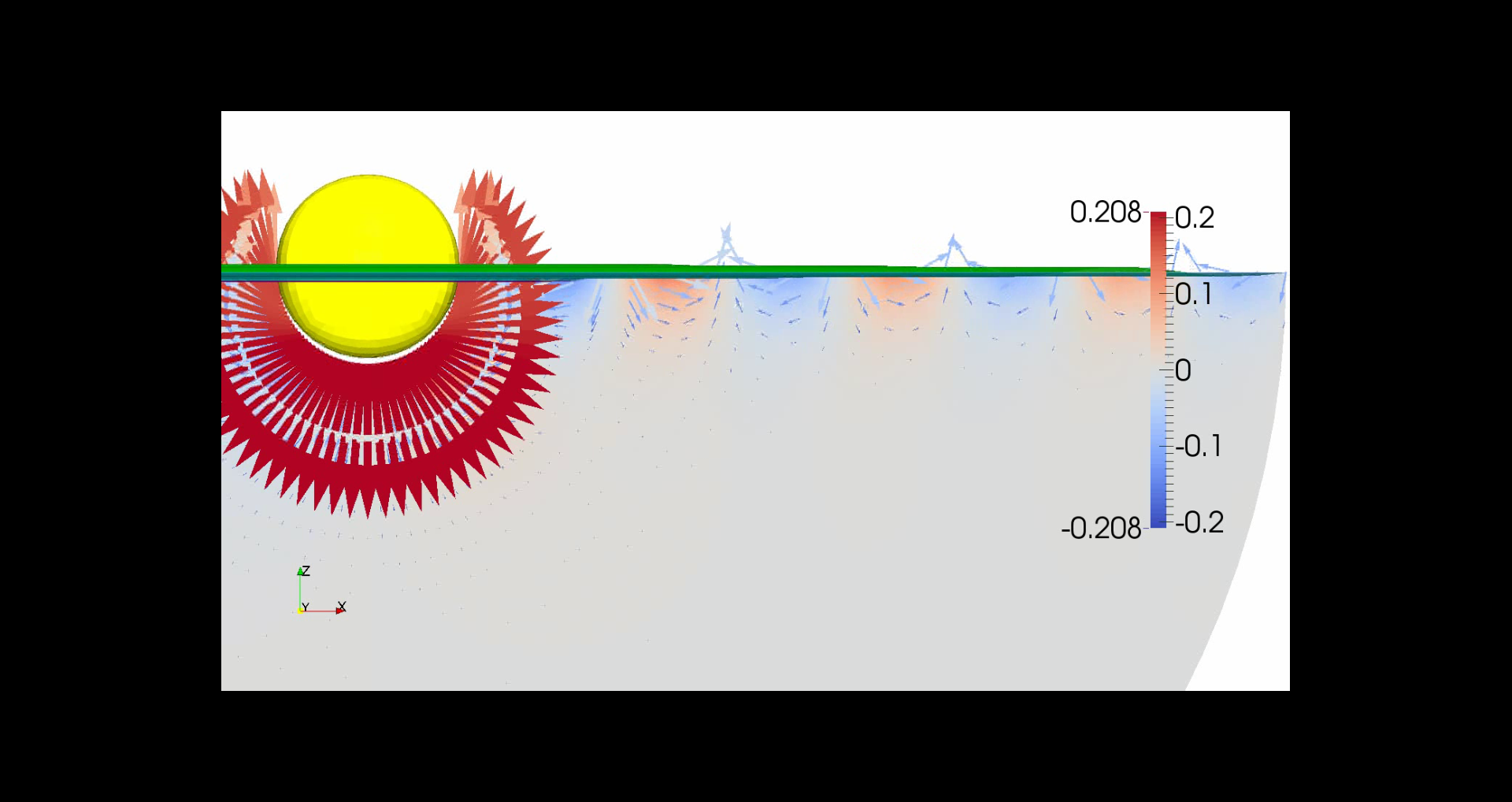
**Results**



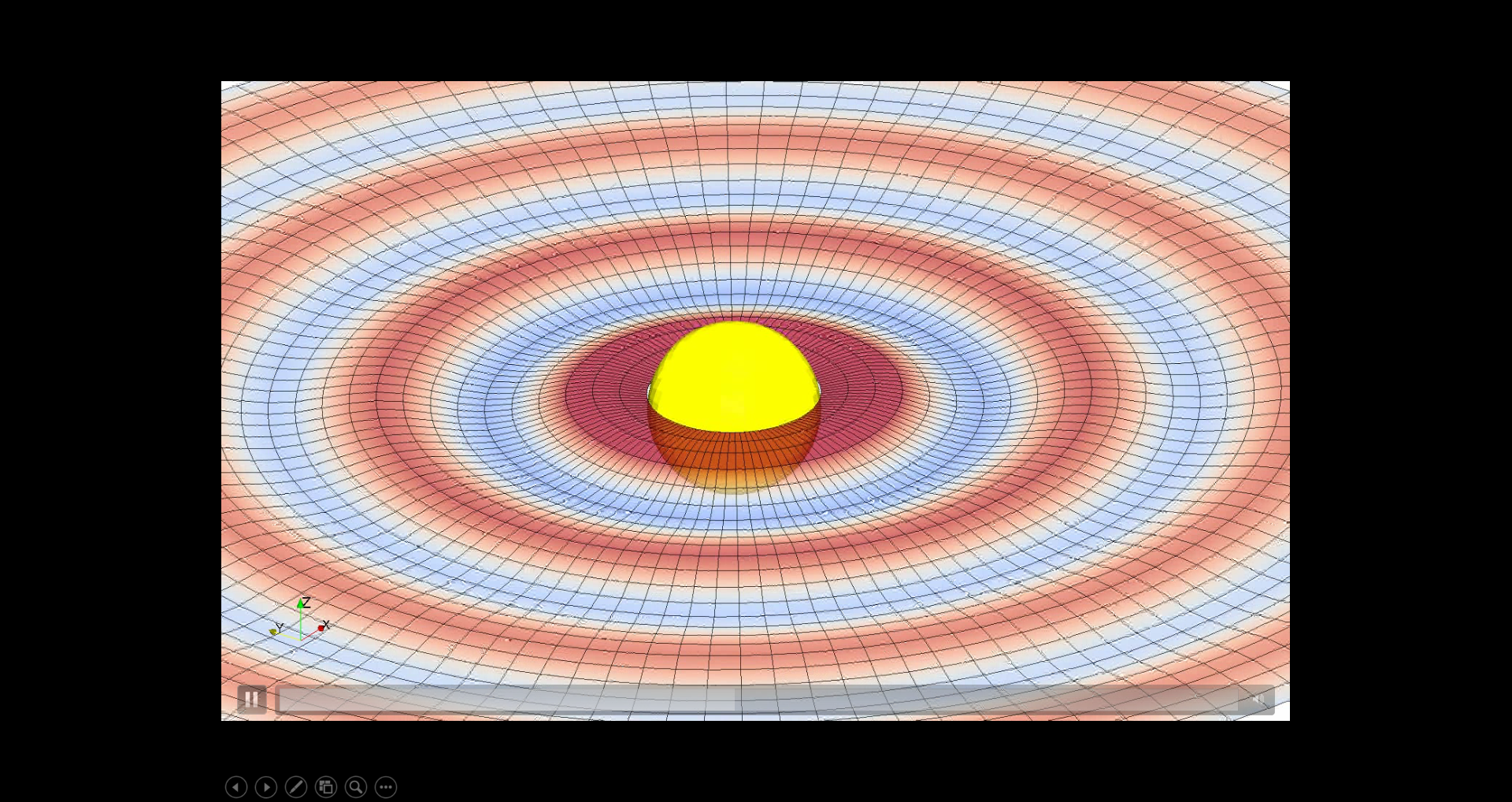
Nondimensionalized Added Mass



Nondimensionalized Damping



Velocity field of heaving hemisphere ()



Wave field of heaving hemisphere ()

**References**

Hulme A.,1982. The wave forces acting on a floating hemisphere undergoing forced periodic oscillations, *Journal of Fluid Mechanics*, Vol 121, pp. 443-463.

Ursell F.,1949. On the heaving motion of a circular cylinder on the surface of a fluid, *Quart. J. Mech. Appl. Math.* Vol 2, pp. 218-231.

Szmytkowski R.,2011. On the derivative of the associated Legendre function of the first kind of integer order with respect to its degree (with applications to the construction of the associated Legendre function of the second kind of integer degree and order), *Journal of Mathematical Chemistry*, Vol 49 (7), pp. 1436-1477.

Havelock T. 1955. Waves due to a floating hemi-sphere making periodic heaving oscillations. *Proc. R. Soc. Lond*. A 231. pp.1-7.

**Appendix A.** Legendre Function and Multipoles

Evaluate

* Velocity and Potential, Wave Elevation

**Derivatives with respect to order (n)**

The derivaties with respect to order of Legendre function is given in Szmytkowski(2011).

where

**Residual Form (Safe and Efficient)**

**Schelknuoff Representation**

**Appendix B.** Auxiliary Integrals

***Jm* Term**

***Imn* term**

**The derivatives of *Imn*  with respect to order**

if *m* = *n*

where

Therefore

Integral with log is integrated with adaptive numerical quadrature

if *m* != *n*

If *m* is even

If *m* is odd

where