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Lecture 10

随机（参量）信号检测

LECTURE9

- **高斯白噪声下，观测波形与信号波形进行相关运算**
  - 波形似然函数：任意坐标轴，K-L展开，N→∞，构建似然函数

$$L\left[y(t)\right]=\frac{f\left(y(t)\setminus H_1\right)}{f\left(y(t)\setminus H_0\right)}=\exp\left[\frac{2}{N_0}\int_0^T y(t)s(t)dt-\frac{E_s}{N_0}\right]\overset{H_1}{\geq} th$$

  - 充分统计量：以信号为基础，通过Gram-Schmidt方法构建坐标轴，有限维系数的似然表达
- **高斯有色噪声下，观测波形与信号波形根据噪声自相关函数的特征值“预白化”后进行相关运算（广义匹配滤波器）**

$$\int_0^T h_i(\tau)R_z(t,\tau)d\tau=u_i(t)$$

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检测场景

- **随机参量信号**

$$\begin{cases} H_1: y(t) = s_1\left(t;\vec{\beta}_1\right) + n(t) \\ H_0: y(t) = s_0\left(t;\vec{\beta}_0\right) + n(t) \end{cases}$$

  - ✓ β随机，且PDF已知
  - ✓ β随机，但PDF未知
  - ✓ β为未知参量
  - ✓ 观测连续/离散

- **随机信号**

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窄带通信

- **带通信号**

$$s_i(t)=a_i(t)\cos[2\pi(f_i+f_c)t+\Phi_i(t)]$$

$$s_i(t)=\mathrm{Re}\{u_i(t)exp[j(2\pi f_c t)]$$

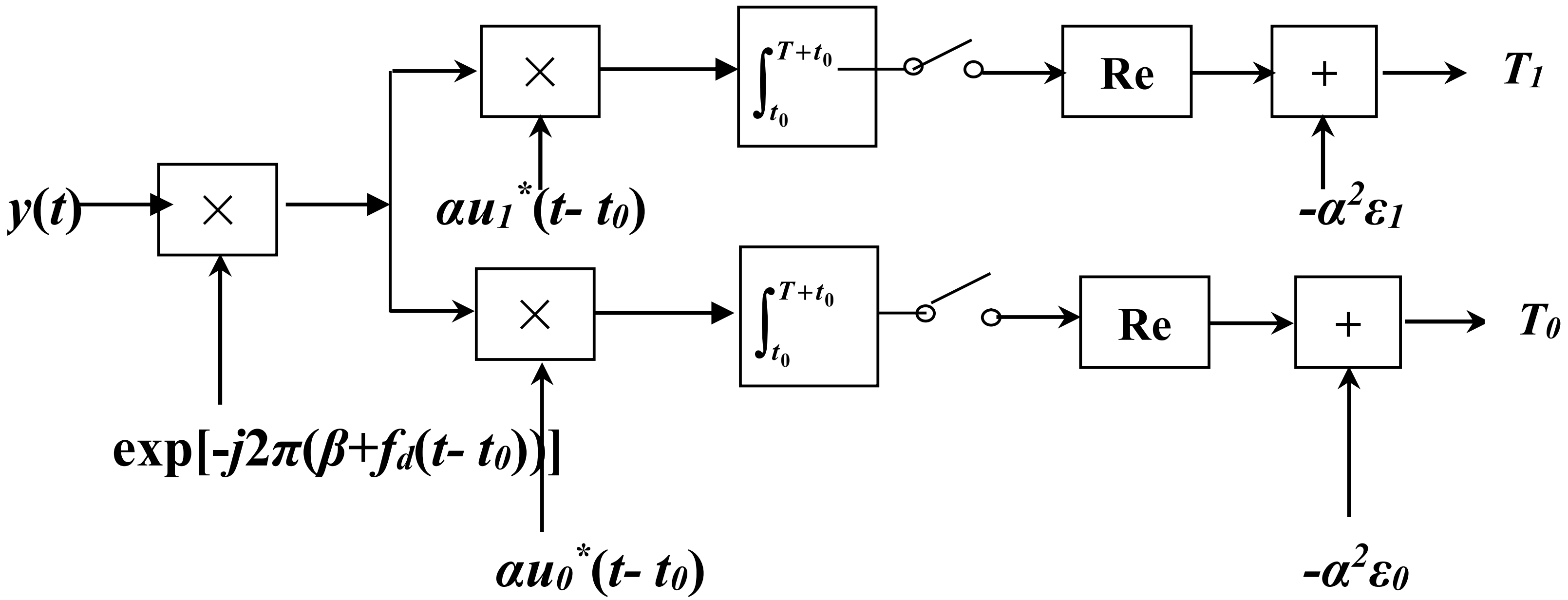
- **基带信号**

$$u_i(t)=a_i(t)exp[j(2\pi f_i t)+\Phi_i(t)]$$

- **接收信号（复包络）**

$$y(t)=\alpha a_i(t-t_0)exp\{j[2\pi(f_i+f_d)(t-t_0)+\Phi_i(t-t_0)+\beta]\}+z(t)$$

最佳检测器



$y(t)$
$\times$
$\alpha u_1^*(t-t_0)$
$\int_{t_0}^{T+t_0}$
Re
+
T_1
$-\alpha^2\varepsilon_1$
$\exp[-j2\pi(\beta+f_d(t-t_0))]$
$\alpha u_0^*(t-t_0)$
T_0
$-\alpha^2\varepsilon_0$

但：参量未知?

GLRT(Generalized Likelihood-ratio test)

- 最大似然估计（MLE）

$$\max_{\vec{\beta}_1} f_1\left(\vec{y} \setminus \vec{\beta}_1\right); \max_{\vec{\beta}_0} f_0\left(\vec{y} \setminus \vec{\beta}_0\right)$$

- 广义似然比

$$L_G\left(\vec{y}\right) = \frac{\max_{\vec{\beta}_1} f_1\left(\vec{y} \setminus \vec{\beta}_1\right)}{\max_{\vec{\beta}_0} f_0\left(\vec{y} \setminus \vec{\beta}_0\right)} = \frac{f_1\left(\vec{y} \setminus \hat{\vec{\beta}}_1\right)}{f_0\left(\vec{y} \setminus \hat{\vec{\beta}}_0\right)}$$

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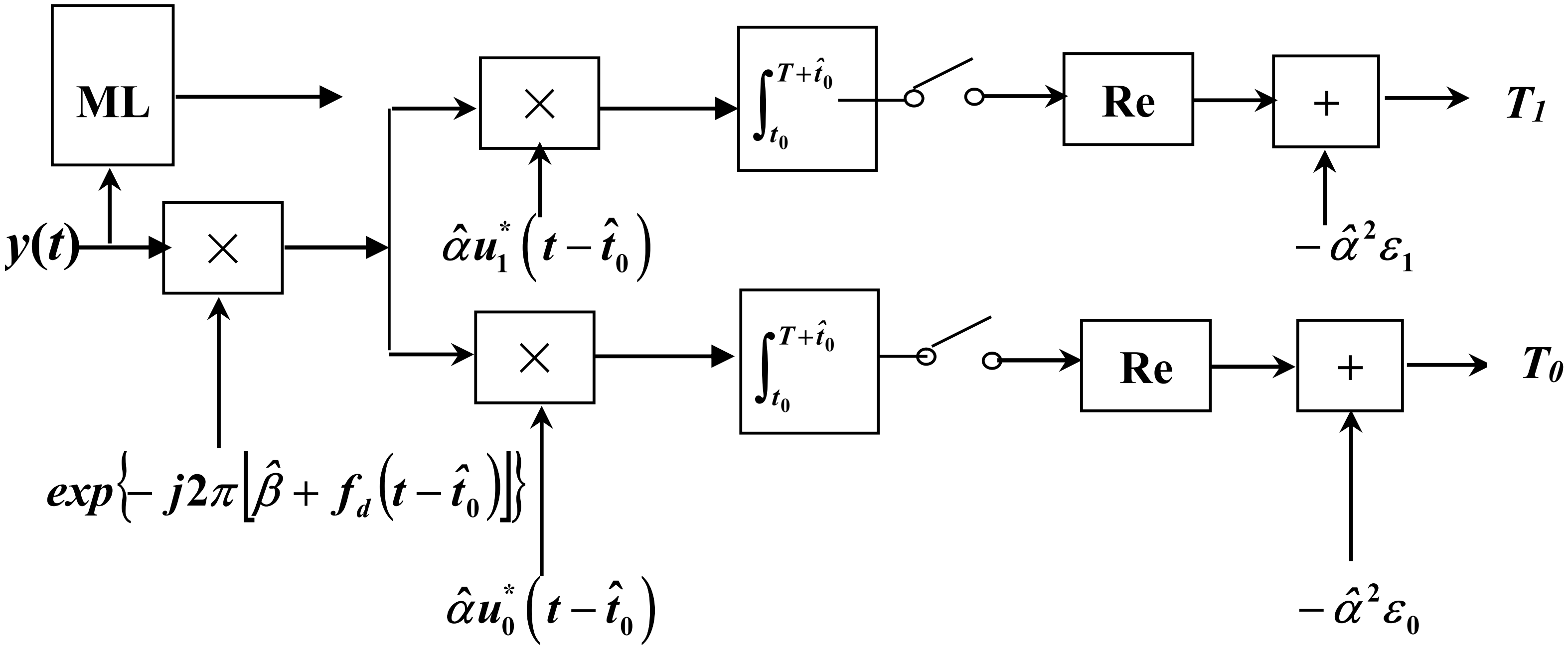
广义似然比检测器



ML
Re
Re
+
+
T_1
T_0
$y(t)$
$\hat{\alpha} u_1^*(t-\hat{t}_0)$
$\hat{\alpha} u_0^*(t-\hat{t}_0)$
$exp\{-j2\pi[\hat{\beta}+f_d(t-\hat{t}_0)]\}$
$\int_{t_0}^{T+\hat{t}_0}$
$\int_{t_0}^{T+\hat{t}_0}$
$-\hat{\alpha}^2\varepsilon_1$
$-\hat{\alpha}^2\varepsilon_0$

H_1:$Y=A+N$；H_0:$Y=N$。A未知。高斯白噪声。求检验准则。

解:

$$L_G\left(\vec{y}\right)=\frac{f\left(\vec{y}\backslash\hat{A},H_1\right)}{f\left(\vec{y}\backslash H_0\right)}>th$$

$$f\left(\vec{Y}\setminus A\right)=\left(\prod_{k=1}^{N}\frac{1}{\sqrt{2\pi}\sigma_n}\right)exp\left\{\sum_{k=1}^{N}-\frac{1}{2\sigma_n^2}\left(y_k-A\right)^2\right\}$$

$$ML方程:\frac{\partial}{\partial A}ln\,f\left(\vec{Y}\setminus A\right)\Big|=0,\ 即\sum_{k=1}^{N}\frac{1}{\sigma_n^2}\left(y_k-A\right)\Big|=0\Rightarrow\hat{A}=\frac{1}{N}\sum_{k=1}^{N}y_k$$

令 $\hat{A}=\overline{y}$

$$L_G(\vec{y})=\frac{f\left(\vec{y}\backslash\hat{A},H_1\right)}{f\left(\vec{y}\backslash H_0\right)}=\frac{\left(\prod_{k=1}^{N}\frac{1}{\sqrt{2\pi}\sigma_n}\right)\exp\left\{\sum_{k=1}^{N}-\frac{1}{2\sigma_n^2}\left(y_k-\overline{y}\right)^2\right\}}{\left(\prod_{k=1}^{N}\frac{1}{\sqrt{2\pi}\sigma_n}\right)\exp\left\{\sum_{k=1}^{N}-\frac{1}{2\sigma_n^2}y_k^2\right\}}$$

$$\ln L_G(\vec{y})=-\frac{1}{2\sigma_n^2}\left(\sum_{k=1}^{N}y_k^2-2\overline{y}\sum_{k=1}^{N}y_k+N\overline{y}^2-\sum_{k=1}^{N}y_k^2\right)$$

$$=-\frac{1}{2\sigma_n^2}\left(-2N\overline{y}^2+N\overline{y}^2\right)=\frac{N\overline{y}^2}{2\sigma_n^2}>th'$$

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CLRT (Conditional Likelihood-Ratio Technique)

$$L\left(\vec{y} \setminus \vec{\beta}_0, \vec{\beta}_1\right) = \frac{f_1\left(\vec{y} \setminus \vec{\beta}_1\right)}{f_0\left(\vec{y} \setminus \vec{\beta}_0\right)}$$

- **独立于具体概率分布函数**
- **均匀最大势（Uniformly Most Powerful）**

H_1:Y=A+N；H_0:Y=N。A>0。高斯白噪声。求检验准则。

$$L_G(\vec{y}) = \frac{\left(\prod_{k=1}^{N}\frac{1}{\sqrt{2\pi\sigma_n}}\right)\exp\left\{\sum_{k=1}^{N}-\frac{1}{2\sigma_n^2}(y_k - A)^2\right\}}{\left(\prod_{k=1}^{N}\frac{1}{\sqrt{2\pi\sigma_n}}\right)\exp\left\{\sum_{k=1}^{N}-\frac{1}{2\sigma_n^2}y_k^2\right\}} > th$$

$$\ln L_G(\vec{y}) = -\frac{1}{2\sigma_n^2}\left(2A\sum_{k=1}^{N}y_k + NA^2\right) > \ln th$$

$$A\sum_{k=1}^{N}y_k > \sigma_n^2 \ln th + \frac{NA^2}{2} \Leftrightarrow T(Y) = \frac{1}{N}\sum_{k=1}^{N}y_k > \frac{\sigma_n^2}{NA}\ln th + \frac{A}{2} = th'$$

$$H_0 : T(Y) \sim N\left(0, \sigma_n^2 / N\right)$$

$$P_{fa} = \Pr\left\{T(Y) > th'; H_0\right\} = Q\left(\frac{th'}{\sqrt{\sigma_n^2 / N}}\right) \Rightarrow th' = \sqrt{\frac{\sigma_n^2}{N}} Q^{-1}\left(P_{fa}\right)$$

$$P_D = \Pr\left\{T(Y) > th'; H_1\right\} = Q\left(\frac{th' - A}{\sqrt{\sigma_n^2 / N}}\right)$$

Q: A>0?

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Bayes平均风险最小准则

$$\text{平均风险}\overline{C} = P(H_0)\int_{(\vec{\beta}_0)}\left[\int_{R_0} f_0(\vec{y}\setminus\vec{\beta}_0)C_{00}(\vec{\beta}_0)d\vec{y}\right]f_0(\vec{\beta}_0)d\vec{\beta}_0$$

$$+P(H_0)\int_{(\vec{\beta}_0)}\left[\int_{R_1} f_0(\vec{y}\setminus\vec{\beta}_0)C_{10}(\vec{\beta}_0)d\vec{y}\right]f_0(\vec{\beta}_0)d\vec{\beta}_0$$

$$+P(H_1)\int_{(\vec{\beta}_1)}\left[\int_{R_1} f_1(\vec{y}\setminus\vec{\beta}_1)C_{11}(\vec{\beta}_1)d\vec{y}\right]f_1(\vec{\beta}_1)d\vec{\beta}_1$$

$$+P(H_1)\int_{(\vec{\beta}_1)}\left[\int_{R_0} f_1(\vec{y}\setminus\vec{\beta}_1)C_{01}(\vec{\beta}_1)d\vec{y}\right]f_1(\vec{\beta}_1)d\vec{\beta}_1$$

$$\frac{\int_{(\vec{\beta}_1)}\left[C_{01}(\vec{\beta}_1)-C_{11}(\vec{\beta}_1)\right]f_1(\vec{y}\setminus\vec{\beta}_1)f_1(\vec{\beta}_1)d\vec{\beta}_1}{\int_{(\vec{\beta}_0)}\left[C_{10}(\vec{\beta}_0)-C_{00}(\vec{\beta}_0)\right]f_0(\vec{y}\setminus\vec{\beta}_0)f_0(\vec{\beta}_0)d\vec{\beta}_0} \geq \frac{P(H_0)}{P(H_1)}$$

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先验PDF

- 已有先验$f(\beta)$
- 构建$f(\beta)$
  - 均匀分布
  - 正态分布$\sim N(0，\sigma^2)$，$\sigma^2 \rightarrow \infty$

平均似然函数

$$\begin{cases} H_1 : y(t) = s_1(t;\beta) + n(t) \\ H_0 : y(t) = n(t) \end{cases} \overset{\text{高斯}}{\Rightarrow}$$

$$f\left(y(t) \backslash H_0\right) = F\,exp\left[-\frac{1}{N_0}\int_0^T y^2(t)\,dt\right]$$

$$f\left(y(t) \backslash \beta; H_1\right) = F\,exp\left[-\frac{1}{N_0}\int_0^T y^2(t)\,dt + \frac{2}{N_0}\int_0^T y(t)\,s(t;\beta)\,dt - \frac{E_s}{N_0}\right]$$

$$f\left(y(t) \backslash H_1\right) = \int_{\{\theta\}} f\left(y(t) \backslash \beta; H_1\right) f(\beta)\,d\beta$$

$$= F\,exp\left[-\frac{1}{N_0}\int_0^T y^2(t)\,dt - \frac{E_s}{N_0}\right]\int_{\{\beta\}} exp\left[\frac{2}{N_0}\int_0^T y(t)\,s(t;\beta)\,dt\right] f(\beta)\,d\beta$$

$$\begin{cases} H_1 : y(t) = A\cos(\omega_0 t + \theta) + n(t) \\ H_0 : y(t) = n(t) \end{cases}, 0 \le t \le T$$

其中θ为均匀分布的随机相位。

解：设$\omega_0 T = 2m\pi$

$$E_s = \int_0^T s^2(t;\theta)dt = A^2\int_0^T \cos^2(\omega_0 t + \theta)dt = A^2T/2$$

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$$\int_{\{\theta\}} exp\left[\frac{2}{N_0}\int_0^T y(t)s(t;\theta)dt\right]f(\theta)d\theta$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi} exp\left[\frac{2A}{N_0}\int_0^T y(t)cos(\omega_0 t+\theta)dt\right]d\theta$$

$$\left(令 y_I=\sqrt{2/T}\int_0^T y(t)cos\,\omega_0 tdt,\ \ y_Q=\sqrt{2/T}\int_0^T y(t)sin\,\omega_0 tdt\right)$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi} exp\left[\frac{2\sqrt{E_s}}{N_0}\left(y_I\,cos\,\theta-y_Q\,sin\,\theta\right)\right]d\theta$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi} exp\left[\frac{2\sqrt{E_s}}{N_0}l\,cos(\theta+\varphi)\right]d\theta=I_0\left(\frac{2\sqrt{E_s}}{N_0}l\right)$$

其中$\begin{cases} l=\sqrt{y_I^2+y_Q^2} \\ \varphi=\arctan\dfrac{y_Q}{y_I} \end{cases} \Leftrightarrow \begin{cases} y_I=l\cos\varphi \\ y_Q=l\sin\varphi \end{cases} \quad l\ge 0, -\pi\le\varphi<\pi$

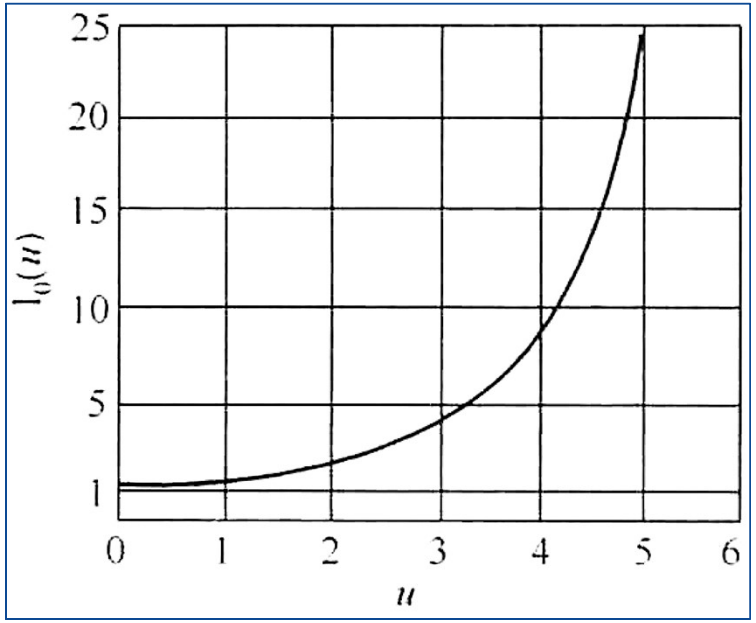
第一类零阶修正贝塞尔函数$I_0(x)=\dfrac{1}{2\pi}\int_{-\pi}^{\pi}e^{x\cos(\theta+\varphi)}d\theta$

$$\Rightarrow f\left(y(t)\backslash H_1\right)=F\exp\left[-\frac{1}{N_0}\int_0^T y^2(t)dt\right]I_0\left(\frac{2\sqrt{E_s}}{N_0}l\right)\exp\left(-\frac{E_s}{N_0}\right)$$

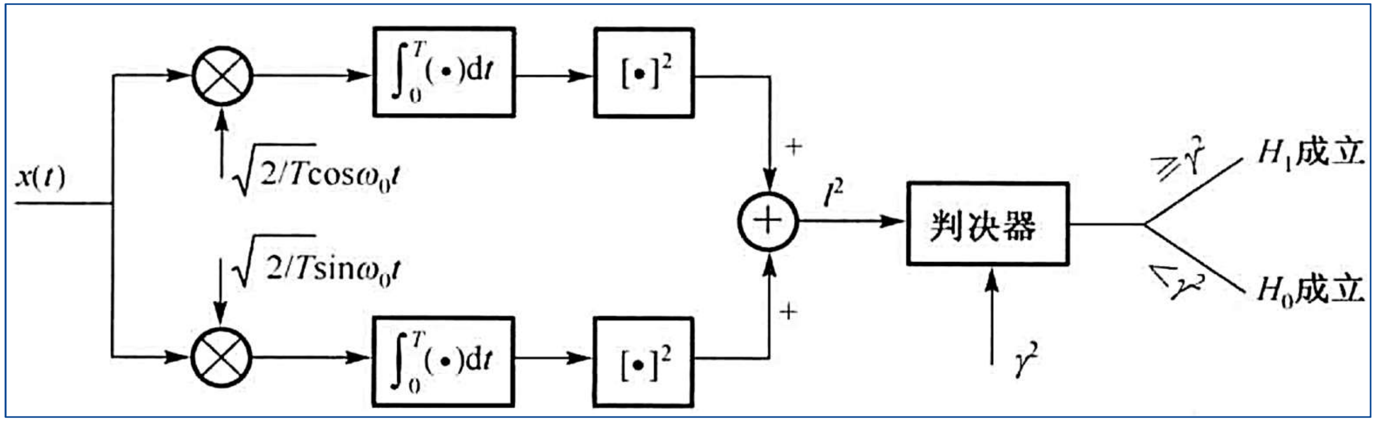
$$\Rightarrow L=\frac{f\left(y(t)\backslash H_1\right)}{f\left(y(t)\backslash H_0\right)}=I_0\left(\frac{2\sqrt{E_s}}{N_0}l\right)\exp\left(-\frac{E_s}{N_0}\right)\overset{H_1}{\geq}\eta$$

$I_0(x)$为单调递增函数

$$\Rightarrow l^2 \overset{H_1}{\geq} \left\{ \frac{N_0}{2\sqrt{E_s}} I_0^{-1}\left[\eta \exp\left(\frac{E_s}{N_0}\right)\right]\right\}^2$$



正交检测器



Q:匹配滤波器实现?

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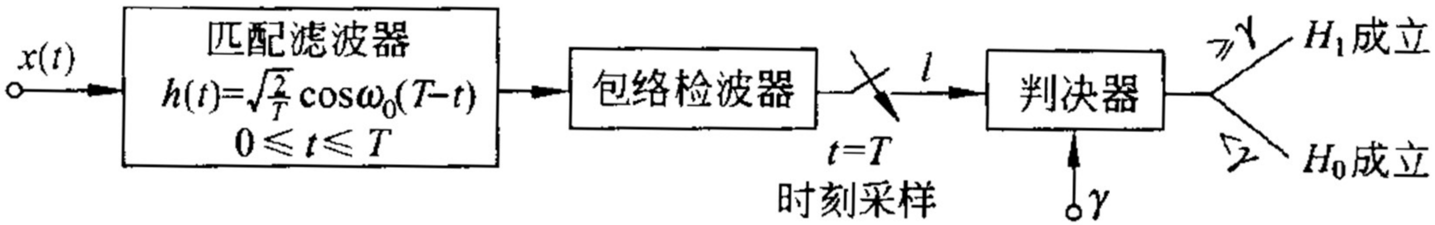
$s(t)$的匹配滤波器输出

$$y_o(t)=\int_{-\infty}^{\infty} y(\tau)h(t-\tau)d\tau=\int_0^T\sqrt{\frac{2}{T}}y(\tau)\cos\omega_0\left(T-t+\tau\right)d\tau$$

$$=\cos\omega_0\left(T-t\right)\int_0^T\sqrt{\frac{2}{T}}y(\tau)\cos\omega_0\tau d\tau-\sin\omega_0\left(T-t\right)\int_0^T\sqrt{\frac{2}{T}}y(\tau)\sin\omega_0\tau d\tau$$

包络

$$\boxed{\left[\left(\int_0^T\sqrt{\frac{2}{T}}y(\tau)\cos\omega_0\tau d\tau\right)^2+\left(\int_0^T\sqrt{\frac{2}{T}}y(\tau)\sin\omega_0\tau d\tau\right)^2\right]^{1/2}}$$



非相干检测

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复信号，离散检测，高斯白噪声，相位未知:

$$y(t)=\alpha a_i(t-t_0)e^{j\left[2\pi(f_i+f_d)(t-t_0)+\Phi_i(t-t_0)+\beta_i\right]}+z(t)$$

$$y_j=\alpha a_i(t_j)e^{j\left[2\pi f_i t_j+\Phi_i(t_j)+\beta_i\right]}+z_j$$

$$=\alpha u_{ij}e^{j\beta_i}+z_j$$

$$\vec{y}=\alpha\vec{u}_i e^{j\beta_i}+\vec{z}$$

$$f_i\left(\vec{y}\backslash\beta_i\right)=\frac{1}{\left(2\pi\sigma_z^2\right)^k}exp\left(-\frac{1}{2\sigma_z^2}\left[\vec{y}-\alpha\vec{u}_i e^{j\beta_i}\right]^T\left[\vec{y}^*-\alpha\vec{u}_i^* e^{-j\beta_i}\right]\right)$$

$$=\frac{1}{\left(2\pi\sigma_z^2\right)^k}exp\left(-\frac{1}{2\sigma_z^2}\sum_{j=1}^{k}\left|y_j\right|^2\right)exp\left(\frac{\alpha}{\sigma_z^2}\left|\sum_{j=1}^{k}y_j u_{ij}^*\right|cos\left(\beta_i-\xi_i\right)\right)exp\left(-\frac{\alpha^2}{2\sigma_z^2}\sum_{j=1}^{k}\left|u_{ij}\right|^2\right)$$

平均似然函数

$$f_i\left(\vec{y}\right)=\frac{1}{\left(2\pi\sigma_z^2\right)^k}exp\left(-\frac{1}{2\sigma_z^2}\sum_{j=1}^{k}\left|y_j\right|^2\right)I_0\left(\frac{\alpha}{\sigma_z^2}\left|\sum_{j=1}^{k}y_j u_{ij}^*\right|\right)exp\left(-\frac{\alpha^2}{2\sigma_z^2}\sum_{j=1}^{k}\left|u_{ij}\right|^2\right)$$

$$l\left(\vec{y}\right)=ln\,L\left(\vec{y}\right)$$

$$=ln\,I_0\left(\frac{\alpha}{\sigma_z^2}\left|\sum_{j=1}^{k}y_j u_{1j}^*\right|\right)-ln\,I_0\left(\frac{\alpha}{\sigma_z^2}\left|\sum_{j=1}^{k}y_j u_{0j}^*\right|\right)$$

$$-\frac{\alpha^2}{2\sigma_z^2}\sum_{j=1}^{k}\left|u_{1j}\right|^2+\frac{\alpha^2}{2\sigma_z^2}\sum_{j=1}^{k}\left|u_{0j}\right|^2$$

$$T_i=\ln I_0\left(\frac{\alpha}{\sigma_z^2}\left|\sum_{j=1}^{k}y_j u_{ij}^*\right|\right)-\frac{\alpha^2}{2\sigma_z^2}\sum_{j=1}^{k}\left|u_{ij}\right|^2$$

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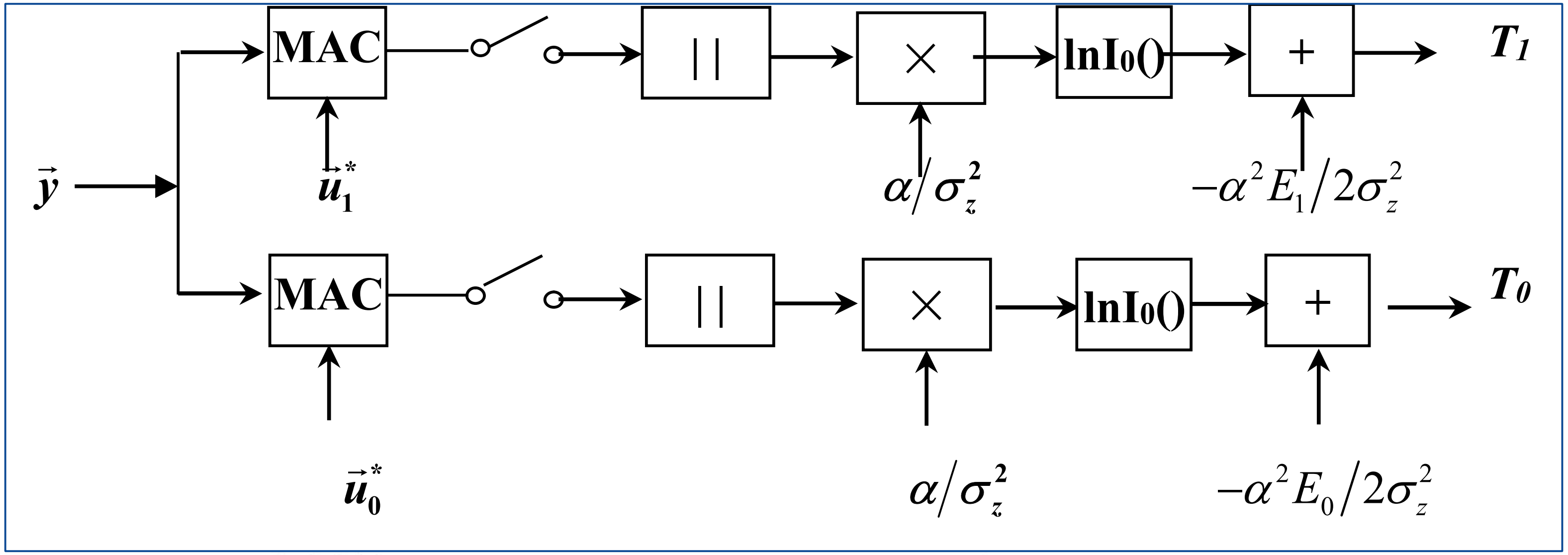
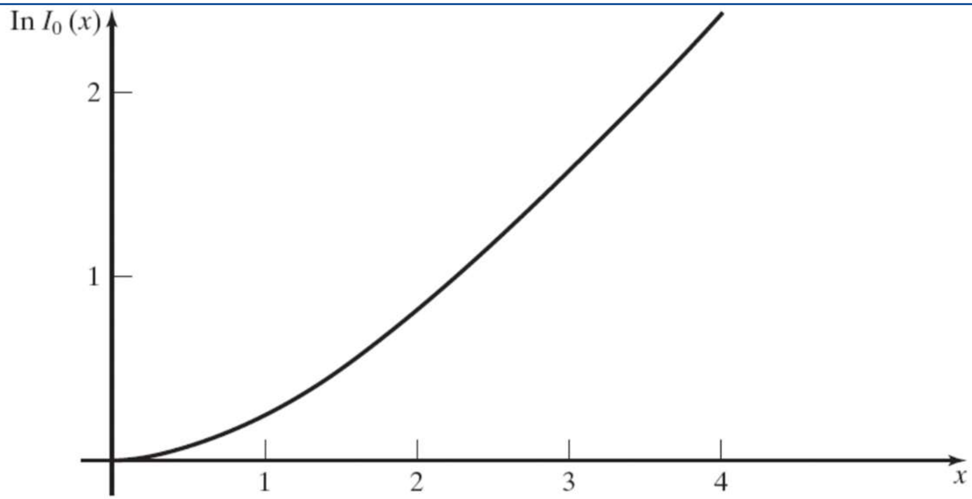

$\vec{y}$
MAC
MAC
$\vec{u}_1^*$
$\vec{u}_0^*$
| |
| |
×
×
α/σ_z^2
α/σ_z^2
lnI0()
lnI0()
+
+
$-\alpha^2 E_1/2\sigma_z^2$
$-\alpha^2 E_0/2\sigma_z^2$
T_1
T_0


ln $I_0(x)$
2
1
1
2
3
4
x

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零均值白色高斯信号

- 白信号高斯分布，均值为0，方差σ_s^2
- 白噪声高斯分布，均值为0，方差σ^2
- 信号与噪声相互独立
- H_i： $y_i = s_{ji} + n_i$, $j=0,1$; $i=1\ldots.M$
- H_0条件下： $Y \sim N(0, \sigma^2 I)$
- H_1条件下： $Y \sim N(0, (\sigma^2+\sigma_s^2)I)$

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零均值白色高斯信号

- **判决准则：** $L(Y)=\dfrac{f(Y\backslash H_1)}{f(Y\backslash H_0)}>th$，判为$H_1$；

$$L(Y)=\frac{\dfrac{1}{\left[2\pi\left(\sigma^2+\sigma_s^2\right)\right]^{\frac{M}{2}}}\exp\left\{-\dfrac{1}{2\left(\sigma^2+\sigma_s^2\right)}\sum_{i=1}^{M}y_i^2\right\}}{\dfrac{1}{\left[2\pi\sigma^2\right]^{\frac{M}{2}}}\exp\left\{-\dfrac{1}{2\sigma^2}\sum_{i=1}^{M}y_i^2\right\}}$$

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零均值白色高斯信号

$$l(Y)=\frac{M}{2}\ln\left(\frac{\sigma^2}{\sigma^2+\sigma_s^2}\right)-\frac{1}{2}\left(\frac{1}{\sigma^2+\sigma_s^2}-\frac{1}{\sigma^2}\right)\sum_{i=1}^{M}y_i^2$$

$$=\frac{M}{2}\ln\left(\frac{\sigma^2}{\sigma^2+\sigma_s^2}\right)+\frac{1}{2}\frac{\sigma_s^2}{\sigma^2\left(\sigma^2+\sigma_s^2\right)}\sum_{i=1}^{M}y_i^2$$

能量检测器：

$$T(Y)=\sum_{i=1}^{M}y_i^2\overset{H_1}{\geq}th'$$

零均值白色高斯信号

- $T(Y)$是M个IID高斯r.v.的平方和

$$H_0: \frac{T(Y)}{\sigma^2} \sim \chi_M^2; \quad H_1: \frac{T(Y)}{\sigma^2+\sigma_s^2} \sim \chi_M^2$$

$$P_{fa} = \Pr\left(T(Y) > th' \backslash H_0\right) = \Pr\left(\frac{T(Y)}{\sigma^2} > \frac{th'}{\sigma^2} \backslash H_0\right) = Q_{\chi_M^2}\left(\frac{th'}{\sigma^2}\right)$$

$$P_D = \Pr\left(T(Y) > th' \backslash H_1\right) = Q_{\chi_M^2}\left(\frac{th'}{\sigma^2+\sigma_s^2}\right) = Q_{\chi_M^2}\left(\frac{th'/\sigma^2}{1+\sigma_s^2/\sigma^2}\right)$$

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零均值有色高斯信号

$$Y \sim \begin{cases} N\left(0, \sigma^2 I\right) & \text{在} H_0 \text{条件下} \\ N\left(0, C_s + \sigma^2 I\right) & \text{在} H_1 \text{条件下} \end{cases}$$

$$L(Y) = \frac{\dfrac{1}{(2\pi)^{\frac{M}{2}}\left[\det\left(C_s + \sigma^2 I\right)\right]^{\frac{1}{2}}} \exp\left\{-\dfrac{1}{2} Y^T \left(C_s + \sigma^2 I\right)^{-1} Y\right\}}{\dfrac{1}{\left(2\pi\sigma^2\right)^{\frac{M}{2}}} \exp\left\{-\dfrac{1}{2\sigma^2} Y^T Y\right\}}$$

$$\Leftrightarrow -\frac{1}{2} Y^T \left[\left(C_s + \sigma^2 I\right)^{-1} - \frac{1}{\sigma^2} I\right] Y > th'$$

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零均值有色高斯信号

$$T(Y)=\sigma^2 Y^T\left[\frac{1}{\sigma^2}I-\left(C_s+\sigma^2 I\right)^{-1}\right]Y>2th'\sigma^2$$

$$\left(\because\left(C_s+\sigma^2 I\right)^{-1}=\frac{1}{\sigma^2}I-\frac{1}{\sigma^4}\left(C_s^{-1}+\frac{1}{\sigma^2}I\right)^{-1}\right)$$

$$=Y^T\left[\frac{1}{\sigma^2}\left(C_s^{-1}+\frac{1}{\sigma^2}I\right)^{-1}\right]Y=Y^T\hat{S}=Y^T C_s\left(C_s+\sigma^2 I\right)^{-1}Y$$

其中$\hat{S}=\left[\frac{1}{\sigma^2}\left(C_s^{-1}+\frac{1}{\sigma^2}I\right)^{-1}\right]Y=\frac{1}{\sigma^2}\left[\frac{1}{\sigma^2}\left(C_s+\sigma^2 I\right)C_s^{-1}\right]^{-1}Y$

$$=C_s\left(C_s+\sigma^2 I\right)^{-1}Y$$

Q：估计器-相关器？

注：$(A+BCD)^{-1}=A^{-1}-A^{-1}B(DA^{-1}B+C^{-1})^{-1}DA^{-1}$

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$N=2$，$C_s=\sigma_s^2\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}$。$\rho$是信号间相关系数。

解：

$$C_s=\sigma_s^2\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}\Rightarrow\begin{cases}\lambda=1\pm\rho\\ V=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\end{cases}$$

令$X=V^TY$，则$\Lambda_s=V^TC_sV=\sigma_s^2\begin{bmatrix}1+\rho & 0\\ 0 & 1-\rho\end{bmatrix}$

注：正交矩阵$V^T=V^{-1}$

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$$H_1: C_X = E\left(XX^T\right) = E\left(V^T YY^T V\right) = V^T C_Y V = V^T\left(C_s + \sigma^2 I\right)V = \Lambda_s + \sigma^2 I$$

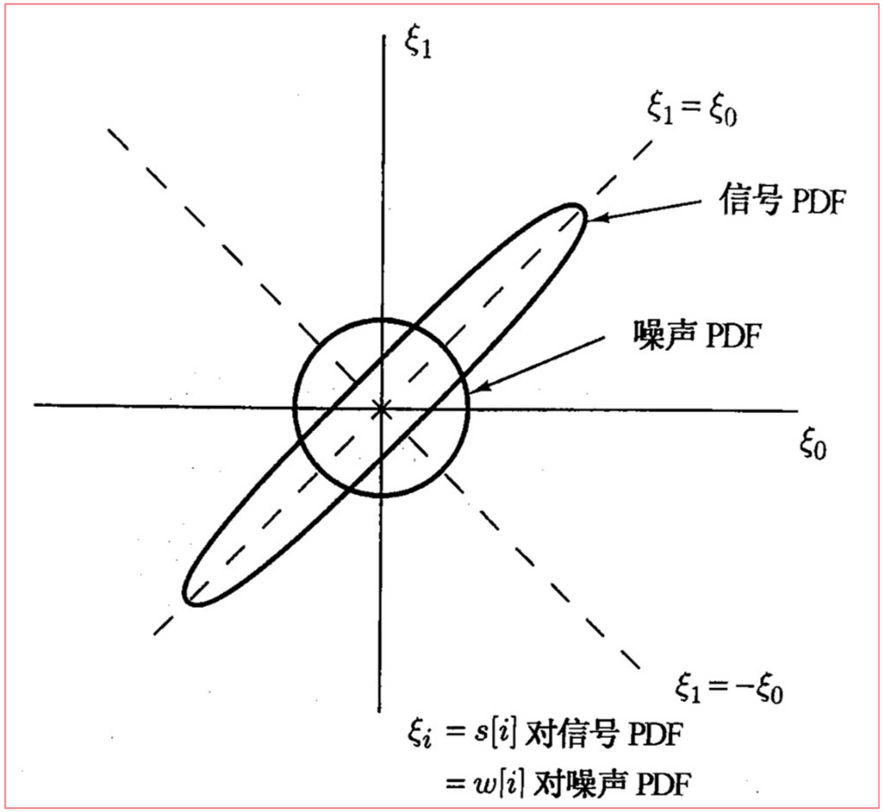
$$\Lambda_s\left(\Lambda_s + \sigma^2 I\right)^{-1} = \sigma_s^2 \begin{bmatrix} 1+\rho & 0 \\ 0 & 1-\rho \end{bmatrix} \begin{bmatrix} \sigma_s^2(1+\rho)+\sigma^2 & 0 \\ 0 & \sigma_s^2(1-\rho)+\sigma^2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \dfrac{\sigma_s^2(1+\rho)}{\sigma_s^2(1+\rho)+\sigma^2} & 0 \\ 0 & \dfrac{\sigma_s^2(1-\rho)}{\sigma_s^2(1-\rho)+\sigma^2} \end{bmatrix}$$

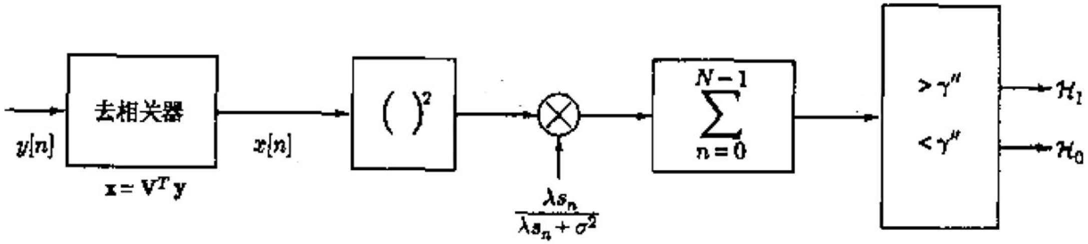
$$\Rightarrow T(X) = X^T \Lambda_s\left(\Lambda_s + \sigma^2 I\right)^{-1} X = \frac{\sigma_s^2(1+\rho)}{\sigma_s^2(1+\rho)+\sigma^2} x_1^2 + \frac{\sigma_s^2(1-\rho)}{\sigma_s^2(1-\rho)+\sigma^2} x_2^2$$

$$T(Y) = Y^T C_s \left(C_s + \sigma^2 I\right)^{-1} Y$$
$$= Y^T V V^T C_s V V^{-1} \left(C_s + \sigma^2 I\right)^{-1} V V^T Y$$
$$= \left(V^T Y\right)^T \left(V^T C_s V\right)\left(V^T C_s V + \sigma^2 I\right)^{-1} V^T Y$$
$$= \left(V^T Y\right)^T \left(V^T C_s V\right)\left[V^{-1}\left(C_s + \sigma^2 I\right) V\right]^{-1} V^T Y$$

$$T(X) = X^T \Lambda_s \left(\Lambda_s + \sigma^2 I\right)^{-1} X$$
$$= \sum_{i=1}^{N} \frac{\lambda_{s_i}}{\lambda_{s_i} + \sigma^2} x_i^2$$



$$\rho \approx 1, \quad \sigma_s^2 >> \sigma^2$$



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summary

- **随机参量信号检测**
  - 有PDF（先验或估计），则复合假设检验，计算平均似然函数/似然比
  - 无PDF，则估计+检测（GLRT）或UMPT
- **随机信号检测，能量检测/估计器-相关器**

Ref：§4.6(赵版)、第五章-第七章 (KAY版)

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