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Lecture 9

波形检测

波形接收

- 观测信号为连续随机信号
- 白噪声下波形检测
- 有色噪声下波形检测

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KL展开假设表达

- 零均值平稳噪声$n(t)$，功率谱密度$N_0/2$
- 自相关函数$r_n(t-u)=(N_0/2)\delta(t-u)$
- 展开系数的观测模型

$$\begin{cases} H_0: y_i = n_i \\ H_1: y_i = s_i + n_i \end{cases}, \quad i = 1, 2, \ldots N$$

$$y_i = \int_0^T \tilde{y}(t) f_i^*(t) dt, i = 1, \ldots N$$

$$s_i = \int_0^T \tilde{s}(t) f_i^*(t) dt, i = 1, \ldots N$$

$$n_i = \int_0^T \tilde{n}(t) f_i^*(t) dt, i = 1, \ldots N$$

展开系数不相关，且高斯分布

似然表达

$$E\left\{y_i \backslash H_0\right\} = E\left\{\int_0^T n(t)\, f_i(t)dt\right\} = \left\{\int_0^T E\left[n(t)\right] f_i(t)dt\right\} = 0$$

$$V\left\{y_i \backslash H_0\right\} = E\left\{n_i^2\right\} = E\left\{\int_0^T n(t)\, f_i(t)dt \int_0^T n(u)\, f_i(u)du\right\}$$

$$= \int_0^T f_i(t)\left[\int_0^T E\left\{n(t)n(u)\right\} f_i(u)du\right]dt$$

$$= \frac{N_0}{2}\int_0^T f_i(t)\left[\int_0^T \delta(t-u)\, f_i(u)du\right]dt = \frac{N_0}{2}\int_0^T f_i^2(t)dt = \frac{N_0}{2}$$

$$f\left(Y \backslash H_0\right) = \left(\frac{1}{\pi N_0}\right)^{N/2} exp\left[-\sum_{i=1}^{N}\frac{y_i^2}{N_0}\right]$$

似然表达

$$E\{y_i \backslash H_1\} = E(s_i + n_i) = s_i + E(n_i) = s_i$$

$$V\{y_i \backslash H_1\} = V\{y_i \backslash H_0\} = \frac{N_0}{2}$$

$$f(Y \backslash H_1) = \left(\frac{1}{\pi N_0}\right)^{N/2} exp\left[-\sum_{i=1}^{N} \frac{(y_i - s_i)^2}{N_0}\right]$$

波形的似然函数

$$s(t)=\lim_{N\to\infty}\sum_{i=1}^{N}s_i f_i(t),\quad n(t)=\lim_{N\to\infty}\sum_{i=1}^{N}n_i f_i(t)$$

$$f\left(y(t)\backslash H_1\right)=\lim_{N\to\infty}\left\{\left(\frac{1}{\pi N_0}\right)^{N/2}\exp\left[-\sum_{i=1}^{N}\frac{\left(y_i-s_i\right)^2}{N_0}\right]\right\}$$

令$F=\lim_{N\to\infty}\left(\frac{1}{\pi N_0}\right)^{N/2}$

$$f\left(y(t)\backslash H_1\right)=F\exp\lim_{N\to\infty}\left[-\frac{1}{N_0}\sum_{i=1}^{N}\left(y_i-s_i\right)^2\right]$$

波形的似然函数

$$f\left(y(t)\backslash H_1\right)=F\exp\left(-\frac{1}{N_0}\right)\left\{\lim_{N\to\infty}\left[\sum_{i=1}^{N}y_i\int_0^T y(t)f_i(t)dt\right]\right.$$

$$\left.-2\lim_{N\to\infty}\left[\sum_{i=1}^{N}s_i\int_0^T y(t)f_i(t)dt\right]+\lim_{N\to\infty}\left[\sum_{i=1}^{N}s_i\int_0^T s(t)f_i(t)dt\right]\right\}$$

$$=F\exp\left[-\frac{1}{N_0}\int_0^T y^2(t)dt+\frac{2}{N_0}\int_0^T y(t)s(t)dt-\frac{1}{N_0}\int_0^T s^2(t)dt\right]$$

$$=F\exp\left[-\frac{1}{N_0}\int_0^T y^2(t)dt+\frac{2}{N_0}\int_0^T y(t)s(t)dt-\frac{E_s}{N_0}\right]$$

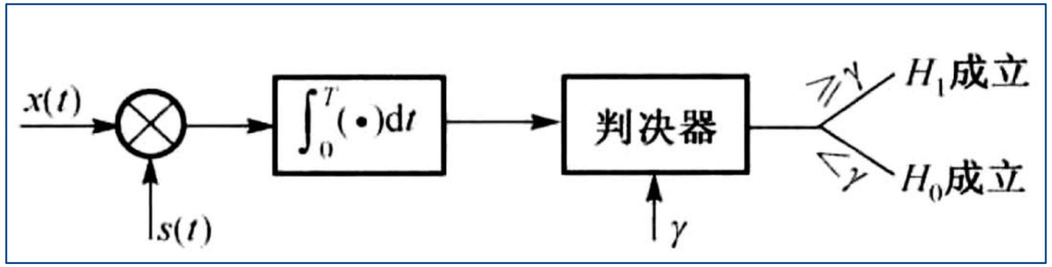
同理$f\left(y(t)\backslash H_0\right)=F\exp\left[-\frac{1}{N_0}\int_0^T y^2(t)dt\right]$

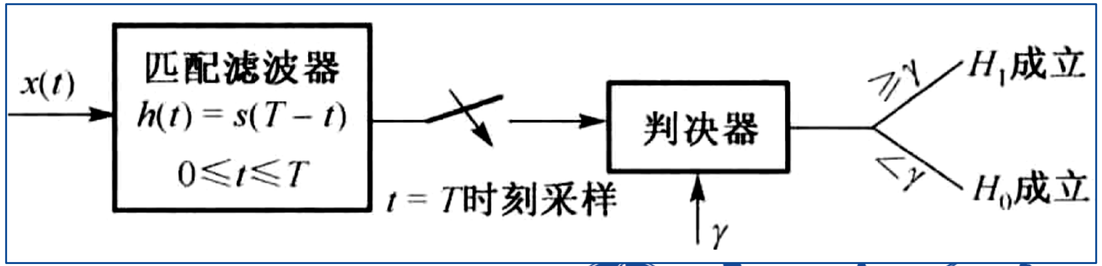
检验准则

$$L\left[y(t)\right]=\frac{f\left(y(t)\backslash H_1\right)}{f\left(y(t)\backslash H_0\right)}=\exp\left[\frac{2}{N_0}\int_0^{\mathrm{T}}y(t)s(t)dt-\frac{E_s}{N_0}\right]\overset{H_1}{\geq} th$$

等效判决

$$l\left[y(t)\right]=\int_0^{\mathrm{T}}y(t)s(t)dt\overset{H_1}{\geq}\gamma\left(=\frac{N_0}{2}\ln th+\frac{E_s}{2}\right)$$





Q: $s_0(t)$和$s_1(t)$情况?

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白噪声下正交函数集的任意性

$$C_{y_i y_j} = E\left\{\left[y_i - E\left(y_i\right)\right]\cdot\left[y_j - E\left(y_j\right)\right]\right\}$$

$$= \int_0^T f_i\left(t\right)\left[\int_0^T r_n\left(t-u\right) f_j\left(u\right)du\right]dt$$

$$= \int_0^T f_i\left(t\right)\left[\int_0^T \frac{N_0}{2}\delta\left(t-u\right) f_j\left(u\right)du\right]dt = \frac{N_0}{2}\delta_{ij}$$

满足齐次积分方程，即白噪声下可取任意正交函数集对平稳随机信号$y(t)$进行KL展开，系数之间互不相关。

正交展开

- 二元假设检验 $H_1: y(t) = s(t) + n(t)$

  $H_0: y(t) = n(t),\ t \sim [0, T]$

- 第一个坐标函数 $f_1(t) = \dfrac{1}{\sqrt{E_s}} s(t), 0 \le t \le T$

- 其余坐标函数与$f_1(t)$正交且两两正交

- 第一个展开系数 $y_1 = \int_0^T y(t) f_1(t) dt = \dfrac{1}{\sqrt{E_s}} \int_0^T y(t) s(t) dt$

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展开系数

- **第一个展开系数为充分统计量**

$$H_0: y_1 = \int_0^T y(t) f_1(t) dt = \frac{1}{\sqrt{E_s}} \int_0^T n(t) s(t) dt = n_1$$

$$H_1: y_1 = \frac{1}{\sqrt{E_s}} \int_0^T \left[s(t) + n(t) \right] s(t) dt = \sqrt{E_s} + n_1$$

- **其余展开系数**

$$H_0: y_k = \int_0^T n(t) f_k(t) dt = n_k, k \geq 2$$

$$H_1: y_k = \int_0^T \left[s(t) + n(t) \right] f_k(t) dt = n_k, k \geq 2$$

检验准则

- 以高斯r.v. y_1 构成似然比检验

$$E\left\{y_1 \backslash H_0\right\}=E\left\{n_1\right\}=E\left\{\frac{1}{\sqrt{E_s}}\int_0^T n(t)s(t)dt\right\}=0$$

$$V\left\{y_1 \backslash H_0\right\}=E\left\{n_1^2\right\}=E\left\{\left(\frac{1}{\sqrt{E_s}}\int_0^T n(t)s(t)dt\right)^2\right\}$$

$$=\frac{1}{E_s}\int_0^T s(t)\left[\int_0^T E\left\{n(t)n(u)\right\}s(u)du\right]dt$$

$$=\frac{N_0}{2E_s}\int_0^T s^2(t)dt=\frac{N_0}{2}$$

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检验准则

- 同理 $E\left\{y_1 \backslash H_1\right\} = E\left\{\sqrt{E_s} + n_1\right\} = \sqrt{E_s}$

$$V\left\{y_1 \backslash H_1\right\} = E\left\{n_1^2\right\} = N_0/2$$

- 似然函数

$$f\left(y_1 \backslash H_0\right) = \left(\frac{1}{\pi N_0}\right)^{1/2} exp\left[-\frac{y_1^2}{N_0}\right]$$

$$f\left(y_1 \backslash H_1\right) = \left(\frac{1}{\pi N_0}\right)^{1/2} exp\left[-\frac{\left(y_1 - \sqrt{E_s}\right)^2}{N_0}\right]$$

检验准则

- **似然比检验** $$L(y_1)=exp\left\{\frac{2}{N_0}\left(\sqrt{E_s}y_1-E_s\right)\right\}\overset{H_1}{\geq}\eta$$

- **等效准则** $$y_1\overset{H_1}{\geq}th'\left(=\frac{N_0}{2\sqrt{E_s}}\ln\eta+\frac{1}{2}\sqrt{E_s}\right)$$

即 $$\frac{1}{\sqrt{E_s}}\int_0^T y(t)s(t)dt\overset{H_1}{\geq}th'$$

$$\Leftrightarrow\int_0^T y(t)s(t)dt\overset{H_1}{\geq}\gamma\left(=\frac{N_0}{2}\ln\eta+\frac{1}{2}E_s\right)$$

- **与之前结论完全一致**

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检测性能

$$E(l\backslash H_0)=E\left[\int_0^{\mathrm{T}} n(t)s(t)dt\right]=\int_0^{\mathrm{T}} E\left[n(t)\right]s(t)dt=0$$

$$V(l\backslash H_0)=E\left\{\left[(l\backslash H_0)-E(l\backslash H_0)\right]^2\right\}=E\left\{\left[\int_0^{\mathrm{T}} n(t)s(t)dt\right]^2\right\}$$

$$=E\left\{\int_0^T n(t)s(t)dt\int_0^T n(u)s(u)du\right\}$$

$$=\int_0^T s(t)\left[\int_0^T E\left\{n(t)n(u)\right\}s(u)du\right]dt$$

$$=\frac{N_0}{2}\int_0^T s(t)\left[\int_0^T \delta(t-u)s(u)du\right]dt$$

$$=\frac{N_0}{2}\int_0^T s^2(t)dt=\frac{N_0E_s}{2}$$

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检测性能

$$E(l\backslash H_1)=E\left\{\int_0^{\mathrm{T}}\left[n(t)+s(t)\right]s(t)dt\right\}$$

$$=\int_0^T s^2(t)dt+\int_0^{\mathrm{T}}E\left[n(t)\right]s(t)dt=E_s$$

$$V(l\backslash H_1)=V(l\backslash H_0)=\frac{N_0E_s}{2}$$

偏移系数$d^2=\frac{\left[E(l\backslash H_1)-E(l\backslash H_1)\right]^2}{V(l\backslash H_0)}=\frac{E_s^2}{N_0E_s/2}=\frac{2E_s}{N_0}$

虚警概率$P_{fa}=Q(\ln\eta/d+d/2)$

发现概率$P_d=Q(\ln\eta/d-d/2)=Q\left(Q^{-1}(P_{fa})-d\right)$

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Gram-Schmide Orthonormalization

- 二元假设检验 $H_1: y(t) = s_1(t) + n(t)$

  $H_0: y(t) = s_0(t) + n(t),\ t \sim [0, T]$

- 第一个坐标函数 $f_1(t) = \dfrac{1}{\sqrt{E_{s_0}}} s_0(t), 0 \le t \le T$

$$\rho = \frac{\int_0^T s_1(t) s_0(t) dt}{\sqrt{E_{s_1}}\sqrt{E_{s_0}}}$$

- 令 $q(t) = s_1(t) - \int_0^T s_1(t) f_1(t) dt \cdot f_1(t)$

$$= s_1(t) - \int_0^T s_1(t) \frac{1}{\sqrt{E_{s_0}}} s_0(t) dt \cdot \frac{1}{\sqrt{E_{s_0}}} s_0(t)$$

$$= s_1(t) - \rho\sqrt{E_{s_1}/E_{s_0}}\, s_0(t), 0 \le t \le T$$

Gram-Schmide Orthonormalization

- 第二个坐标函数

$$E_{S_i}=\int_0^T s_i^2(t)\,dt,\quad \rho=\frac{\int_0^T s_1(t)s_0(t)\,dt}{\sqrt{E_{S_1}}\sqrt{E_{S_0}}}$$

$$f_2(t)=\frac{q(t)}{E_q}$$

$$=\frac{s_1(t)-\rho\sqrt{E_{S_1}/E_{S_0}}\,s_0(t)}{\sqrt{\int_0^T\left[s_1^2(t)+\left(\rho^2E_{S_1}/E_{S_0}\right)s_0^2(t)-2\rho\sqrt{E_{S_1}/E_{S_0}}\,s_0(t)s_1(t)\right]dt}}$$

$$=\frac{1}{\sqrt{\left(1-\rho^2\right)E_{S_1}}}\left[s_1(t)-\rho\sqrt{E_{S_1}/E_{S_0}}\,s_0(t)\right],0\le t\le T$$

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H_0假设下展开系数

$$y_1 = \int_0^T \left[s_0(t)+n(t)\right] f_1(t)dt = \frac{1}{\sqrt{E_{s_0}}}\int_0^T \left[s_0(t)+n(t)\right] s_0(t)dt = \sqrt{E_{s_0}} + n_1$$

$$y_2 = \int_0^T \left[s_0(t)+n(t)\right] f_2(t)dt$$

$$= \frac{1}{\sqrt{\left(1-\rho^2\right)E_{S_1}}}\int_0^T \left[s_0(t)+n(t)\right]\left[s_1(t)-\rho\sqrt{E_{S_1}/E_{S_0}}\,s_0(t)\right]dt$$

$$= \frac{1}{\sqrt{\left(1-\rho^2\right)E_{S_1}}}\left[\rho\sqrt{E_{s_1}E_{s_0}} - \rho\sqrt{E_{s_1}E_{s_0}}\right] + n_2 = n_2$$

$$y_k = \int_0^T \left[s_0(t)+n(t)\right] f_k(t)dt = n_k,\quad k = 3,4,\cdots$$

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H_1假设下展开系数

$$y_1=\int_0^T\left[s_1(t)+n(t)\right]f_1(t)dt=\frac{1}{\sqrt{E_{s_0}}}\int_0^T\left[s_1(t)+n(t)\right]s_0(t)dt=\rho\sqrt{E_{s_1}}+n_1$$

$$y_2=\int_0^T\left[s_1(t)+n(t)\right]f_2(t)dt$$

$$=\frac{1}{\sqrt{\left(1-\rho^2\right)E_{S_1}}}\int_0^T\left[s_1(t)+n(t)\right]\left[s_1(t)-\rho\sqrt{E_{S_1}/E_{S_0}}\,s_0(t)\right]dt$$

$$=\frac{1}{\sqrt{\left(1-\rho^2\right)E_{S_1}}}\left[E_{s_1}-\rho^2E_{s_1}\right]+n_2=\sqrt{\left(1-\rho^2\right)E_{S_1}}+n_2$$

$$y_k=\int_0^T\left[s_1(t)+n(t)\right]f_k(t)dt=n_k,\quad k=3,4,\cdots$$

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似然函数

充分统计量$Y=\begin{pmatrix} y_1 & y_2 \end{pmatrix}^T$高斯分布，且两分量互不相关

$$E\left(y_1 \backslash H_0\right)=E\left(\sqrt{E_{s_0}}+n_1\right)=\sqrt{E_{s_0}},\quad V\left(y_1 \backslash H_0\right)=E\left(n_1^2\right)=N_0/2$$

$$E\left(y_2 \backslash H_0\right)=E\left(n_2\right)=0,\quad V\left(y_2 \backslash H_0\right)=E\left(n_2^2\right)=N_0/2$$

$$f\left(Y \backslash H_0\right)=\left(\frac{1}{\pi N_0}\right)exp\left[-\frac{\left(y_1-\sqrt{E_{s_0}}\right)^2+y_2^2}{N_0}\right]$$

$$f\left(Y \backslash H_1\right)=\left(\frac{1}{\pi N_0}\right)exp\left[-\frac{\left(y_1-\rho\sqrt{E_{s_1}}\right)^2+\left(y_2-\sqrt{\left(1-\rho^2\right)E_{s_1}}\right)^2}{N_0}\right]$$

检验准则

$$L(Y)=\frac{f(Y\backslash H_1)}{f(Y\backslash H_0)}=\frac{\left(\frac{1}{\pi N_0}\right)exp\left[-\frac{\left(y_1-\rho\sqrt{E_{s_1}}\right)^2+\left(y_2-\sqrt{\left(1-\rho^2\right)E_{s_1}}\right)^2}{N_0}\right]}{\left(\frac{1}{\pi N_0}\right)exp\left[-\frac{\left(y_1-\sqrt{E_{s_0}}\right)^2+y_2^2}{N_0}\right]}$$

$$l(Y)=\frac{1}{N_0}\left[2y_1\left(\rho\sqrt{E_{s_1}}-\sqrt{E_{s_0}}\right)+E_{s_0}-\rho^2E_{s_1}+2y_2\sqrt{\left(1-\rho^2\right)E_{s_1}}-\left(1-\rho^2\right)E_{s_1}\right]$$

检验准则

$$\begin{cases} y_1 = \dfrac{1}{\sqrt{E_{s_0}}}\displaystyle\int_0^T y(t)s_0(t)dt \\ y_2 = \dfrac{1}{\sqrt{(1-\rho^2)E_{s_1}}}\displaystyle\int_0^T y(t)\left[s_1(t)-\rho\sqrt{E_{s_1}/E_{s_0}}\,s_0(t)\right]dt \end{cases}$$

$$l(Y) = \frac{1}{N_0}\left[2y_1\left(\rho\sqrt{E_{s_1}}-\sqrt{E_{s_0}}\right)+E_{s_0}+2y_2\sqrt{(1-\rho^2)E_{s_1}}-E_{s_1}\right]$$

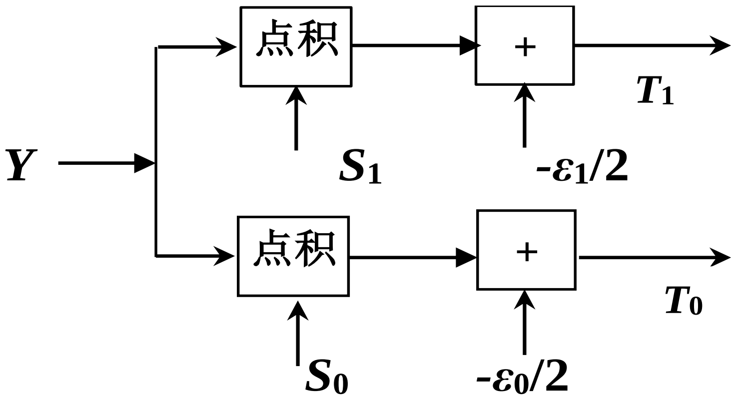
$$= \frac{1}{N_0}\left[2\int_0^T y(t)s_1(t)dt - E_{s_1}\right] - \frac{1}{N_0}\left[2\int_0^T y(t)s_0(t)dt - E_{s_0}\right] \overset{H_1}{\geq} \ln\eta$$

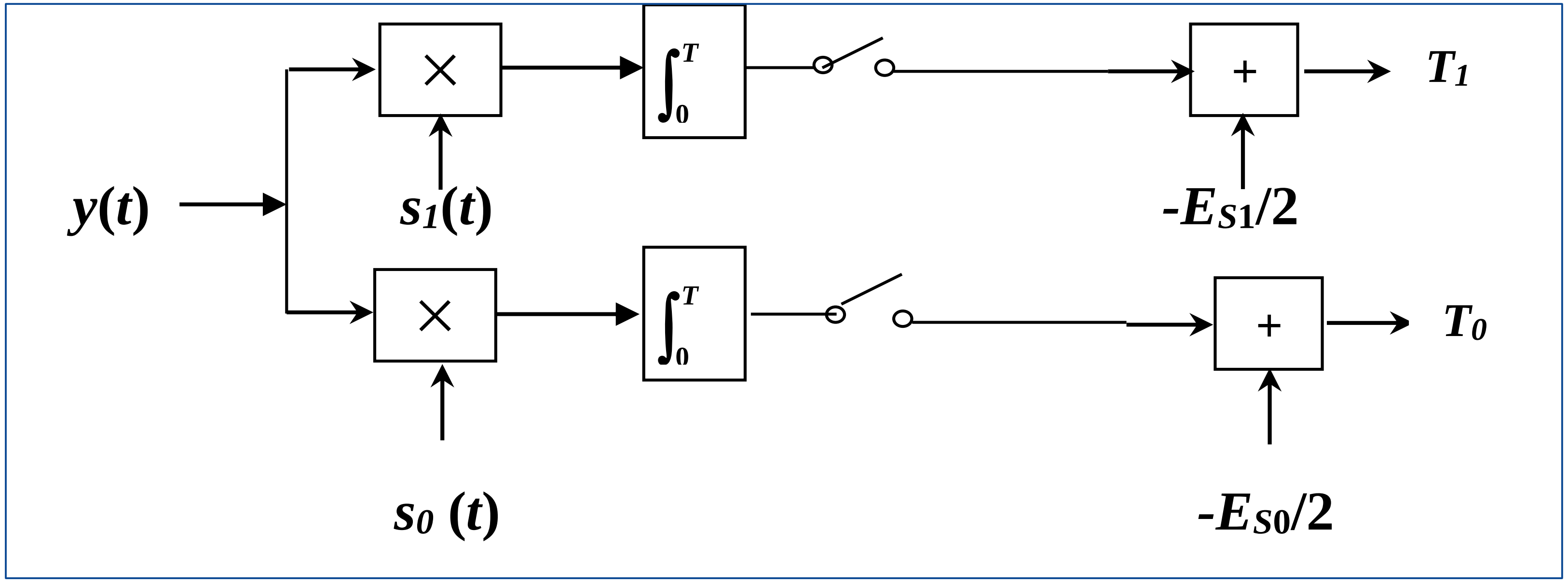
检验准则

$$T(Y)=T_1(Y)-T_0(Y)$$

$$=\left[\int_0^T y(t)s_1(t)dt-\frac{E_{s_1}}{2}\right]-\left[\int_0^T y(t)s_0(t)dt-\frac{E_{s_0}}{2}\right]\overset{H_1}{\geq}\frac{N_0}{2}\ln\eta$$

数字最佳检测





Q:匹配滤波器形式？M元假设？

二维矢量空间

- **二元假设检验：**

$$\begin{cases} H_1: \tilde{y}(t) = \tilde{s}_1(t) + \tilde{n}(t) \\ H_0: \tilde{y}(t) = \tilde{s}_0(t) + \tilde{n}(t) \end{cases}$$

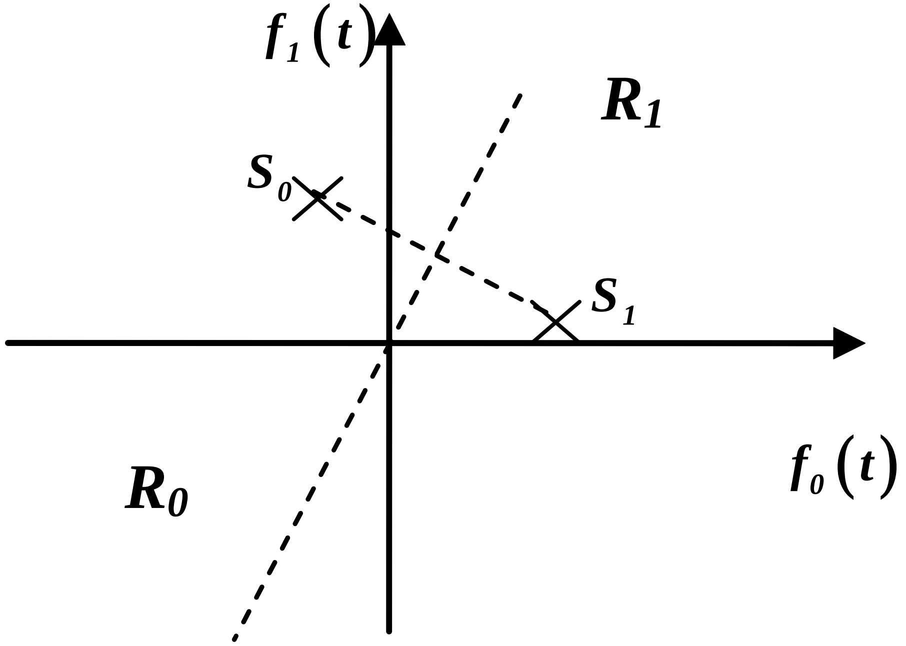
- **复矢量形式**

归一化正交基底坐标函数$f_1(t)$和$f_0(t)$

$$\begin{cases} H_0: Y = S_0 + N \\ H_1: Y = S_1 + N \end{cases}$$

其中$Y = [y_0, \ y_1], S_i = [s_{i0}, \ s_{i1}], N = [n_0, n_1]$

最小距离判断



经典最佳接收

- **似然函数**

$$f\left(Y\backslash H_i\right)=\frac{1}{\sqrt{2\pi\sigma}}exp\left\{-\frac{1}{2\sigma^2}\left\|Y-S_i\right\|^2\right\}$$

$$l\left(Y\right)=ln\frac{f\left(Y\backslash H_1\right)}{f\left(Y\backslash H_0\right)}=-\frac{1}{2\sigma^2}\left\{\left\|Y-S_1\right\|^2-\left\|Y-S_0\right\|^2\right\}$$

$$=\frac{1}{\sigma^2}Re\left(Y^T,S_1-S_0\right)-\frac{1}{2\sigma^2}\left\|S_1\right\|^2+\frac{1}{2\sigma^2}\left\|S_0\right\|^2$$

- **最大似然**

$$Re\int_0^T\tilde{y}\left(t\right)\cdot\left(\tilde{s}_1\left(t\right)-\tilde{s}_2\left(t\right)\right)^*dt\geq\frac{1}{2}\left(\left\|S_1\right\|^2-\left\|S_0\right\|^2\right)$$，判为H_1

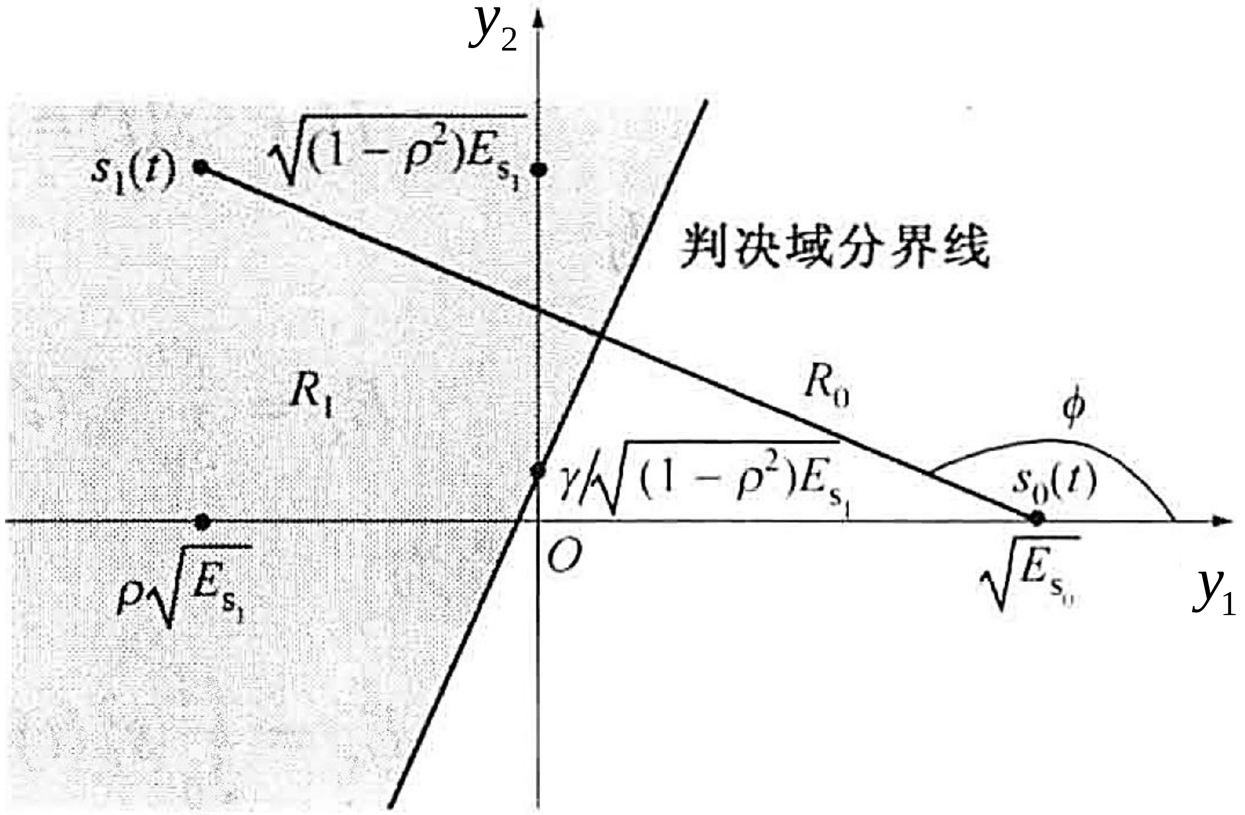
判决区域

$$l(Y)=\left(\rho\sqrt{E_{s_1}}-\sqrt{E_{s_0}}\right)y_1-\sqrt{\left(1-\rho^2\right)E_{s_1}}\,y_2 \overset{H_1}{\geq} \gamma\left(=\frac{N_0}{2}\ln\eta+\frac{E_{s_1}-E_{s_0}}{2}\right)$$

$$\begin{cases} f_1(t)=\dfrac{1}{\sqrt{E_{s_0}}}s_0(t) \\ f_2(t)=\dfrac{1}{\sqrt{\left(1-\rho^2\right)E_{s_1}}}\left[s_1(t)-\rho\sqrt{E_{s_1}/E_{s_0}}\,s_0(t)\right] \end{cases}$$

$$\Rightarrow s_1(t)=\rho\sqrt{E_{s_1}}\cdot f_1(t)+\sqrt{\left(1-\rho^2\right)E_{s_1}}\cdot f_2(t)$$

分界线$y_2=\dfrac{\rho\sqrt{E_{s_1}}-\sqrt{E_{s_0}}}{\sqrt{\left(1-\rho^2\right)E_{s_1}}}y_1+\dfrac{\gamma}{\sqrt{\left(1-\rho^2\right)E_{s_1}}}$



Q:检测性能？最佳波形？

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K-L系数

$y(t)=u_i(t)+z(t)$, $i=0,1$, $t \sim [0,T]$

$$y(t)=\lim_{N\to\infty}\sum_{j=1}^{N} y_j f_j(t)$$

$$y_j = u_{ij} + z_j, i = 0,1$$

$$u_{ij} = \int_0^T u(t) f_j^*(t)dt$$

$$z_j = \int_0^T z(t) f_j^*(t)dt$$

$$E\left\{\left[y_i - E(y_i)\right]\cdot\left[y_j - E(y_j)\right]^*\right\} = \lambda_j\delta_{ij}$$

$$y_j \sim N(u_{ij}, \lambda_j)$$

$$\int_0^T \operatorname{cov}\{y(t_1)y(t_2)\} f_j(t_2)dt_2 = \lambda_j f_j(t_1), 0 \le t_1 \le T$$

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似然比

$$f_N(\vec{y}\backslash H_i)=\frac{1}{(2\pi)^N\det(C)}\exp\left(-\frac{1}{2}[\vec{y}-\vec{u}_i]^T C^{-1}[\vec{y}-\vec{u}_i]^*\right)$$

$$\det(C)\to\prod_{j=1}^{N}\lambda_j$$

$$l_N(\vec{y})=\ln\frac{f_N(\vec{y}\backslash H_1)}{f_N(\vec{y}\backslash H_0)}$$

$$=-\sum_{j=1}^{N}(y_j-u_{1j})\frac{1}{\lambda_j}(y_j^*-u_{1j}^*)+\sum_{j=1}^{N}(y_j-u_{0j})\frac{1}{\lambda_j}(y_j^*-u_{0j}^*)$$

$$=\sum_{j=1}^{N}\frac{2}{\lambda_j}\left[\mathrm{Re}(y_j u_{1j}^*)-\frac{1}{2}|u_{1j}|^2\right]-\sum_{j=1}^{N}\frac{2}{\lambda_j}\left[\mathrm{Re}(y_j u_{0j}^*)-\frac{1}{2}|u_{0j}|^2\right]$$

检验统计量

$$T_i(N)=\sum_{j=1}^{N}\mathrm{Re}\left[\frac{u_{ij}^*}{\lambda_j}\left(y_j-\frac{1}{2}u_{ij}\right)\right]$$

$$=\mathrm{Re}\left[\int_0^T\left(y(t)-\frac{1}{2}u_i(t)\right)\sum_{j=1}^{N}\frac{u_{ij}^* f_j^*(t)}{\lambda_j}dt\right]$$

$$h_{i,N}(t)=\sum_{j=1}^{N}\frac{u_{ij} f_j(t)}{\lambda_j}$$

$$h_i(t)=\lim_{N\to\infty}h_{i,N}(t)$$

$$\int_0^T h_i(\tau)R_z(t,\tau)d\tau=\sum_{j=1}^{\infty}\frac{u_{ij}}{\lambda_j}\int_0^T R_z(t,\tau)f_j(\tau)d\tau=u_i(t)$$

Q:平稳噪声过程?

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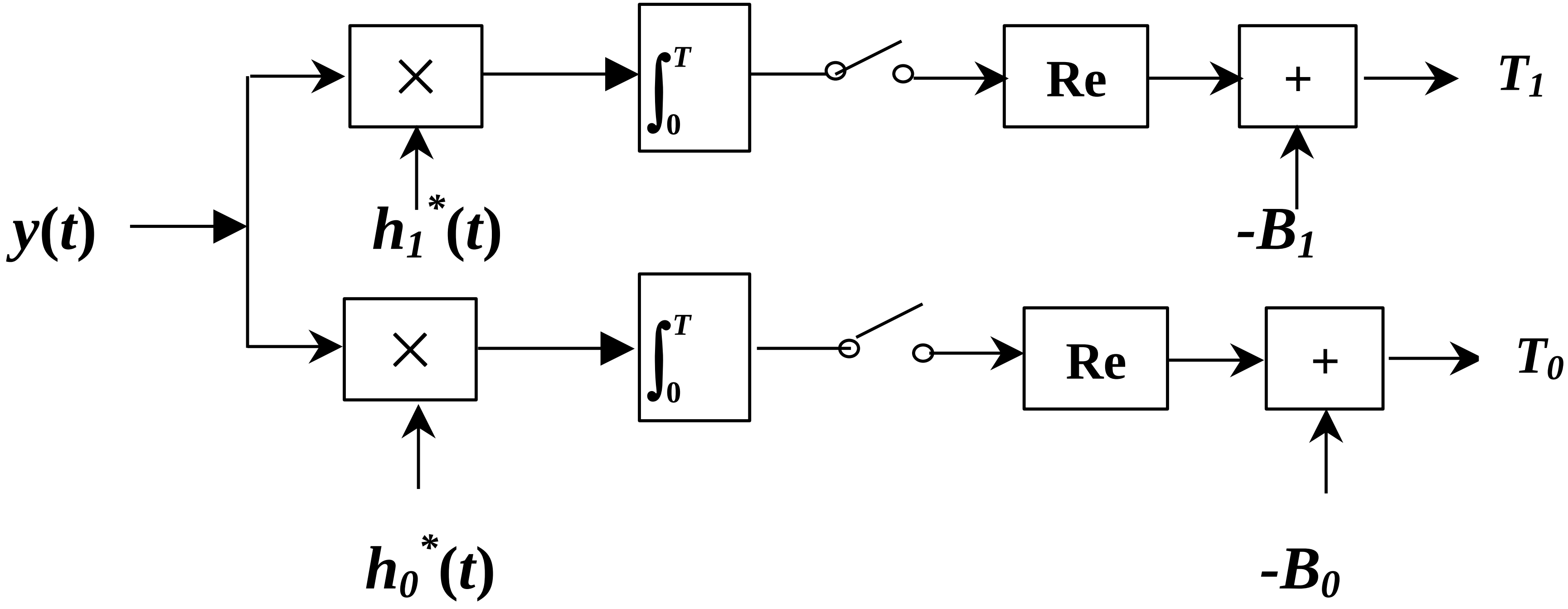
检测器



summary

- **高斯白噪声下，观测波形与信号波形进行相关运算**
  - 任意坐标轴，K-L展开，N→∞，构建似然函数
  - 以信号为基础，通过Gram-Schmide方法构建坐标轴，有限维系数的似然表达
- **高斯有色噪声下，观测波形与信号波形根据噪声自相关函数的特征值“预白化”后进行相关运算**

Ref：§4.4-§4.5(赵版)

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