

Evaporative Cooling

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1 Theoretical Models for Evaporative Cooling

1.1 General Scaling Laws

First, evaporative cooling happens on an **exponential scale**: Within a certain time interval (naturally measured in units of collision times or relaxation times), all relevant parameters (number of atoms, temperature, density) change by a certain factor. The characteristic quantities for the evaporation process are therefore logarithmic derivatives such as:

$$\alpha = \frac{d(\ln T)}{d(\ln N)} = \frac{\dot{T}/T}{\dot{N}/N}$$

Or, if evaporative cooling is described as a process with finite steps, we have:

$$\alpha = \frac{\ln(T'/T)}{\ln(N'/N)}, \quad T' = T + \Delta T, \quad N' = N + \Delta N$$

If α is constant during the evaporation process, the temperature drops with function:

$$\frac{T(t)}{T(0)} = \left(\frac{N(t)}{N(0)}\right)^\alpha$$

In a power law potential in d dimensions, $U(r) \propto r^{d/\delta}$, **all relevant quantities** scale as $[N(t)/N(0)]^x$ during evaporative cooling, where x depends only on δ and α . δ is defined in such a way that the volume scales as T^δ . All other quantities are products of powers of temperature, number, and volume, and their scaling is listed in Table 1.

Table 1: Scaling Laws For Evaporative Cooling In
A d Dimension Potential $U(r) \propto r^{d/\delta}$

Quantity	Exponent ^a , x
Number of atoms, N	1
Temperature, T	α
Volume, V	$\delta\alpha$
Density, n	$1 - \delta\alpha$
Phase-space density, D	$1 - \alpha(\delta + 3/2)$
Elastic collision rate, $n\sigma v$	$1 - \alpha(\delta - 1/2)$

Phases space density D is defined as $n\lambda_{dB}^3$ with the thermal de Broglie wavelength $\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mkT}}$, for an atom with mass m .

The key parameter of the whole cooling process is α , which **expresses the temperature decrease per particle lost**. Technically, evaporation is controlled by limiting the depth of the potential to ηkT . **The average energy of the escaping atoms** is $(\eta + \kappa)kT$, where κ is a small number usually between 0 and 1, depending both on η and on the dimension of the evaporation.

For large η , the energy distribution of the trapped atoms is close to a Boltzmann distribution with an average energy of $(\delta + \frac{3}{2})kT$, and there is **a simple relation between the average energy of an escaping atom and α** :

$$\alpha = \frac{\eta + \kappa}{\delta + \frac{3}{2}} - 1 = \frac{(\eta + \kappa - \delta - \frac{3}{2}) kT}{(\delta + \frac{3}{2}) kT}$$

This expression has an obvious meaning: α is a dimensionless quantity, characterizing how much more than the average energy $(\delta + \frac{3}{2}) kT$ is removed by an evaporating atoms. These considerations show that in principle there is

no upper bound for α or the efficiency of evaporative cooling. Therefore, efficiency of evaporation, and comparison between different trap geometries, can only be made if the trade-off between efficiency and cooling speed is considered. This is done by specifying loss mechanisms that are unavoidable in practice.

1.2 The Speed of Evaporation Loss Processes

We now want to extend the discussion and consider the speed of evaporation, introduce time as a parameter. The situation we have in mind is forced evaporation at a constant η parameter; the threshold for evaporation is lowered in proportion to the decreasing temperature. Constant η ensures that the energy distribution is only rescaled during the cooling and does not change its shape. This assumption is reasonably well fulfilled in experiments.

The principle of detailed balance can be used in kinetic systems which are decomposed into elementary processes (collisions, or steps, or elementary reactions). It states that at equilibrium, each elementary process is in equilibrium with its reverse process. $Rate(A \rightarrow B) = Rate(B \rightarrow A)$

We consider particles at density n_0 in a box potential, and we assume that η is large. The rate of evaporating atoms can then be obtained as follows: In an untruncated Maxwell-Boltzmann distribution, almost every collision involving an atom in the high energy tail removes the atom from the high energy tail. By detailed balance, elastic collisions produce atoms with energy larger than ηkT at a rate that is simply the number of atoms with energy larger than ηkT divided by their collision time. For a large value of η , the rate of evaporation in a truncated Boltzmann distribution is identical to the production rate of atoms with energy larger than ηkT in the untruncated distribution.

The velocity of atoms with energy ηkT is $\sqrt{\frac{2\eta kT}{m}} = \sqrt{\eta\pi} \cdot \frac{\bar{v}}{2}$, where \bar{v} denotes the average thermal velocity. For large η , the fraction of atoms with energy larger than ηkT approaches $2e^{-\eta}\sqrt{\frac{\pi}{2}}$. The rate of evaporating atoms is thus:

$$\dot{N} = -Nn_0\sigma\bar{v}\eta e^{-\eta} = \frac{-N}{\tau_{ev}}$$

Where we have introduced the elastic collision cross section σ and the time constant for evaporation τ_{ev} . It's ratio to the elastic collision time τ_{el} is expressed by $\lambda = \frac{\tau_{ev}}{\tau_{el}}$ Using $\frac{1}{\tau_{el}} = n_0\sigma\bar{v}\sqrt{2}$, where $\bar{v}\sqrt{2}$ is the average relative velocity between two atoms, we obtain in the limit of large η ,

$$\lambda = \frac{\sqrt{2}e^{\eta}}{\eta}$$

1.2.1 Runaway Evaporation

For alkali atoms, where the dominant loss mechanism is background gas collisions, an important criterion for sustained evaporation is to maintain or increase the elastic collision rate $n\sigma v$. It follows that *the elastic collision rate varies* as:

$$\frac{d(n\sigma v)}{dt}/n\sigma v = \frac{1}{\tau_{el}} \left(\frac{\alpha(\delta - 1/2) - 1}{\lambda} - \frac{1}{R} \right)$$

In the temperature range of interest, σ is the s-wave cross section and is independent of temperature. $R = \frac{\tau_{loss}}{\tau_{el}}$ is the number of elastic collisions per trapping time (also called the ratio of good to bad collisions) where τ_{loss} is the time constant for trap loss due to background gas collisions. Evaporation at constant or increasing collision rate ("runaway evaporation") requires:

$$R \geq R_{\min} = \frac{\lambda}{\alpha(\delta - 1/2) - 1}$$

In the absence of any loss process ($R = \infty$), the minimum η for runaway evaporation is determined by:

$$\alpha > \frac{1}{\delta - 1/2}$$

The increase of phase space density with time is given by:

$$\beta = 100\tau_{\text{el}} \frac{d}{dt} (\log_{10} D) = \frac{100}{\ln 10} \left(\frac{\alpha(\delta + 3/2) - 1}{\lambda} - \frac{1}{R} \right)$$

1.2.2 Maximizing Phase-Space Density

α describes only the change in temperature. If we regard evaporation in a linear potential as analogous to evaporation in a harmonic potential with continuous adiabatic compression, we realize that α does not provide the most meaningful comparison, because adiabatic compression trades in temperature against density. We therefore now focus on phase-space density D , which is invariant with respect to adiabatic changes of the potential.

The relative increase in phase space density with decreasing number N :

$$\gamma = -\frac{d(\ln D)}{d(\ln N)} = \frac{\alpha(\delta + 3/2)}{1 + \lambda/R} - 1$$

The most important figure of merit of evaporative cooling in atom traps is to ***achieve the maximum increase in phase-space density with the smallest loss in the number*** (to reach BEC with the largest number of atoms possible). This would mean that the goal is to achieve the largest value of the global parameter:

$$\gamma_{\text{tot}} = \frac{\ln(D_{\text{final}}/D_{\text{initial}})}{\ln(N_{\text{final}}/N_{\text{initial}})}$$

This global parameter is optimized by optimizing γ at any moment. To explain this, we first note that the elastic collision rate at any moment can be expressed by the phase-space density and the number of atoms:

$$n\sigma v \propto D^{(\delta-1/2)/(\delta+3/2)} N^{2/(\delta+3/2)}$$

Then we divide the increase in phase-space density in many small steps of size ΔD . The efficiency γ of the second step is optimized by maximizing R . According to last equation, this is achieved by maximizing the number of atoms left after the first step. As a result, following an evaporation path that maximizes γ at any given point maximizes γ_{tot} and, thus, the number of atoms left at the final phase-space density.

The same conclusion applies for cooling of atomic hydrogen, where the dominant loss process is inelastic binary collision. R is then independent of density and proportional to \sqrt{T} , resulting in:

$$R \propto D^{-1/[2(\delta+3/2)]} N^{1/[2(\delta+3/2)]}$$

There is ***one major difference*** between the loss due to background gas collisions and that due to inelastic collisions. Inelastic collisions heat up the sample because they happen most frequently in regions of high density, where the potential energy is small. As a result, atoms lost due to ***inelastic collisions*** carry away less than their share in total energy, thus increasing the average energy of the trapped atoms. This effect decreases the effective α , which expresses the average energy of a lost particle. Actually, α even changes sign as a function of temperature when the heating due to ***dipolar relaxation*** dominates over the evaporative cooling. The criterion $\alpha = 0$ determines the lowest temperature that can be achieved in evaporative cooling due to relaxation heating, whereas a model considering only background gas collisions allows cooling to arbitrarily small temperatures.

1.2.3 Strategies for Evaporation Cooling

It might appear that the two previous subsections on phase-space density increase and runaway evaporation describe different strategies or even conflicting goals. We now want to discuss qualitatively how they depend on each other and emphasize *the major difference between evaporative cooling in alkalis and in atomic hydrogen*.

The ultimate goal is the achievement of high phase-space densities. As was pointed out, maximizing γ is the optimum strategy for this goal. However, for small R , γ becomes negative; no increase in phase-space density is possible. Therefore, a phase-space density increase can only be sustained as long as R is larger than a critical value.

For loss due to *background gas collisions* (the dominant limitation for alkalis), R changes according to:

$$-\frac{d(\ln R)}{d(\ln N)} = \frac{\alpha(\delta - 1/2)}{1 + \lambda/R} - 1$$

R varies exponentially with $\frac{1}{N}$, with the exponent given by the right hand side of equation. If the exponent is negative at some point during the cooling, R decreases, resulting in an even more negative exponent later on. The consequence is an accelerated decrease of R and an accelerated reduction of the efficiency parameter γ , until eventually γ reaches zero and no further increase in phase-space density is possible. Work in *alkali systems* concentrated on realizing an initial situation with a sufficiently large R , so that R increased during evaporation. In principle, cooling with a (slowly) decreasing R is possible, as long as the desired phase-space density is reached before γ becomes zero. However, when the threshold of *runaway evaporation* is reached, R stays constant or increases. This guarantees *both fast and efficient cooling to very high phase-space densities*. The phase-space density increase is only limited by the onset of other loss processes such as *three-body recombination and dipolar relaxation*, which inevitably reduce R and throttle the cooling process.

In *atomic hydrogen*, the situation is quite different. All experiments in atomic hydrogen were done in a cryogenic environment where *background gas collisions were negligible*, and the dominant loss mechanism was *inelastic binary collisions*. R is *independent of density and decreases proportionally to \sqrt{T}* . Runaway or increasing collision rates speed up the cooling process, but in contrast to alkali atoms, they do not increase R nor improve the efficiency of evaporative cooling. In all experiments, the initial temperature, and therefore R , was large enough to allow efficient cooling in the beginning. However, during the cooling process the efficiency decreased, and the highest phase-space densities were reached when either γ became small or the number of atoms in the sample reached the detection limit.

1.3 Models

1.3.1 Amsterdam

The Amsterdam group modeled evaporative cooling within a kinetic theory involving a numerical solution of the Boltzmann equation. These results confirmed that *the energy distribution can be very well approximated by a truncated Boltzmann distribution*. Subsequently, a closed set of differential equations for evaporative cooling was derived under the assumption of a truncated Boltzmann distribution, that all particles with energy larger than ηkT leave the trap.

1.3.2 Davis and Coworkers (MIT)

It approximated the evaporation process as a discrete series of truncation and rethermalization processes, and it arrived at simple analytical results. These were used to discuss the threshold for runaway evaporation for different potentials.

1.3.3 Doyle and Coworkers (MIT)

John Doyle and collaborators derived a set of coupled differential equations that described various cooling, heating, and loss processes and included adiabatic changes of the potential during the cooling process. Furthermore, optimized trajectories in phase space were determined aiming at the highest final phase-space density for a given initial temperature and density combination.

1.3.4 Monte Carlo Simulations

Two groups developed an elegant Monte Carlo trajectory technique that allowed an efficient solution of the classical kinetic equation. This technique solves the Boltzmann equation directly without any assumptions such as partial equilibrium or the presence of a truncated Boltzmann distribution.

1.4 The Dimension of Evaporation

Evaporative cooling relies on the selection of energetic particles that leave the trap. This selection can be based on the total energy E or on the energy for the motion in a particular direction E_z . We call the selection one-dimensional if it is based on $E_z > \eta kT$, two-dimensional for $E_x + E_y > \eta kT$, and three-dimensional for $E > \eta kT$. This distinction is rigorous only for separable potentials. In the context of evaporative cooling the relevant timescale is the time between elastic collisions. Thus, if the ergodic mixing time is longer than the elastic collision time, the potential can be regarded as separable.

In the case of negligible ergodic mixing, the depletion of the high-energy tail of the distribution depends on the selection method. In the opposite case of fast ergodic mixing the distribution is depleted for total energy $E > \eta kT$, independent of the dimensionality of the selection. Since the dynamics of the evaporation are determined by the dimension of the depletion, we will call this *the dimension of evaporation*. This distinction is rigorous only for separable potentials. In the context of evaporative cooling the relevant timescale is the time between elastic collisions. Thus, if the ergodic mixing time is longer than the elastic collision time, the potential can be regarded as separable.

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Most evaporative cooling experiments on hydrogen used evaporation over a saddle point of the potential which is a 1D selection scheme. Radiative evaporation using rf is a three-dimensional selection scheme in a dc magnetic trap. However, in the TOP trap, it is only two-dimensional, and if gravitational forces become essential, it is only one-dimensional.

The problem with evaporation in lower dimensions is the dramatic reduction in efficiency for an Ioffe-Pritchard (IP) trap. The reason is that a major fraction of the atoms that have enough total energy to escape collide with other

atoms and lose the high excitation energy. Therefore, several attempts are necessary before an atom leaves the trap.

As long as there are no loss processes, this affects only the time for evaporation. However, if a time limitation is set by inelastic collision processes and background gas collisions, one- or two-dimensional evaporation strongly decreases the efficiency of evaporative cooling. Since loss processes enter in the ratio $\frac{\lambda}{R}$, an increase in λ has an effect similar to a decrease in the ratio of good to bad collisions.

1.5 The Number of Collisions to Bose-Einstein Cindensation

All the models discussed here show that evaporative cooling is a fast and efficient way to reduce the temperature of trapped atoms and increase the phase-space density. A possible scenario is as follows: harmonic potential, η parameter of 6, removal of 0.7% of the atoms per collision time, increase of phase-space density by a factor of 10^6 in 600 collision times, and a loss in the number by a factor of 100. Reduction of the η parameter to 5 gives the same phase-space density increase in half the number of collisions, but with 5 times fewer atoms left.

Since the phase-space density of laser-cooled atoms is typically 10^{-6} lower than is required for Bose-Einstein condensation (BEC), one can summarize the potential of evaporative cooling in the following way: About 500 collision times suffice to achieve BEC with 1% of the atoms remaining.

Real experiments have performed somewhat worse, probably due to nonoptimized cooling, lower dimension of the evaporation, and additional loss and heating processes. However, the few experiments done so far have already impressively confirmed the potential of evaporative cooling.

1.6 Desirable Extensions of The Models

The theoretical foundations of evaporative cooling are well established. The different models we have discussed describe the salient features of the evaporation process. Extensions in the following directions appear worth- while:

- The calculation of optimized trajectories in phase space. There are obviously trade-offs between efficiency and cooling time, and some detailed treatment is desirable.
- So far, all models assume that all particles with a certain (1D or 3D) energy will escape. However, all methods of evaporation rely on spatial selection of atoms.
- All models discuss evaporation in the dilute regime. Dilute means here collisionally thin, or that the mean free path is longer than the size of the sample.
- Most models have treated the atomic motion classically, ignoring quantum statistical effects. For the description of the cooling process, this is justified because evaporative cooling has been used to increase the phase-space density by six orders of magnitude, whereas quantum statistical effects are only pronounced in the vicinity of BEC.

2 The Role of Collisions for Real Atoms

2.1 Elastic And Inelastic Collisions

Evaporative cooling is driven by elastic collisions. High density and the presence of collisions are therefore prerequisites for evaporative cooling. However, this means that inelastic collisions are unavoidable. We have already

mentioned background gas collisions and dipolar relaxation as loss processes, and we wish to give a more systematic account of collision processes in this section.

In a magnetic trap, three kinds of inelastic collisions are relevant for evaporative cooling experiments: *dipolar relaxation and spin relaxation, which are binary collisions, and three-body recombination (or dimerization)*.

If atoms are trapped in the upper hyperfine state, hyperfine state changing collisions are important, and the loss processes are similar to the situation in a magnetic trap.

Spin relaxation has a rather large rate coefficient, typically $10^{-12} \text{ cm}^3/\text{sec}$. It involves an exchange of angular momentum between the electron spin and the nuclear spin. This process is usually important only after the initial loading of the trap and leads to a self-purification of the sample in the sense that there are only atoms left in a hyperfine state, which does not undergo spin relaxation. This usually happens in the first few seconds after loading a magnetic trap with atomic hydrogen.

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