

Evaporative Cooling

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1 Theoretical Models for Evaporative Cooling

1.1 General Scaling Laws

First, evaporative cooling happens on an **exponential scale**: Within a certain time interval (naturally measured in units of collision times or relaxation times), all relevant parameters (number of atoms, temperature, density) change by a certain factor. The characteristic quantities for the evaporation process are therefore logarithmic derivatives such as:

$$\alpha = \frac{d(\ln T)}{d(\ln N)} = \frac{\dot{T}/T}{\dot{N}/N}$$

Or, if evaporative cooling is described as a process with finite steps, we have:

$$\alpha = \frac{\ln(T'/T)}{\ln(N'/N)}, \quad T' = T + \Delta T, \quad N' = N + \Delta N$$

If α is constant during the evaporation process, the temperature drops with function:

$$\frac{T(t)}{T(0)} = \left(\frac{N(t)}{N(0)}\right)^\alpha$$

In a power law potential in d dimensions, $U(r) \propto r^{d/\delta}$, **all relevant quantities** scale as $[N(t)/N(0)]^x$ during evaporative cooling, where x depends only on δ and α . δ is defined in such a way that the volume scales as T^δ . All other quantities are products of powers of temperature, number, and volume, and their scaling is listed in Table 1.

Table 1: Scaling Laws For Evaporative Cooling In
A d Dimension Potential $U(r) \propto r^{d/\delta}$

Quantity	Exponent ^a , x
Number of atoms, N	1
Temperature, T	α
Volume, V	$\delta\alpha$
Density, n	$1 - \delta\alpha$
Phase-space density, D	$1 - \alpha(\delta + 3/2)$
Elastic collision rate, $n\sigma v$	$1 - \alpha(\delta - 1/2)$

Phases space density D is defined as $n\lambda_{dB}^3$ with the thermal de Broglie wavelength $\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mkT}}$, for an atom with mass m .

The key parameter of the whole cooling process is α , which **expresses the temperature decrease per particle lost**. Technically, evaporation is controlled by limiting the depth of the potential to ηkT . **The average energy of the escaping atoms** is $(\eta + \kappa)kT$, where κ is a small number usually between 0 and 1, depending both on η and on the dimension of the evaporation.

For large η , the energy distribution of the trapped atoms is close to a Boltzmann distribution with an average energy of $(\delta + \frac{3}{2})kT$, and there is **a simple relation between the average energy of an escaping atom and α** :

$$\alpha = \frac{\eta + \kappa}{\delta + \frac{3}{2}} - 1 = \frac{(\eta + \kappa - \delta - \frac{3}{2}) kT}{(\delta + \frac{3}{2}) kT}$$

This expression has an obvious meaning: α is a dimensionless quantity, characterizing how much more than the average energy $(\delta + \frac{3}{2}) kT$ is removed by an evaporating atoms. These considerations show that in principle there is

no upper bound for α or the efficiency of evaporative cooling. Therefore, efficiency of evaporation, and comparison between different trap geometries, can only be made if the trade-off between efficiency and cooling speed is considered. This is done by specifying loss mechanisms that are unavoidable in practice.

1.2 The Speed of Evaporation Loss Processes

We now want to extend the discussion and consider the speed of evaporation, introduce time as a parameter. The situation we have in mind is forced evaporation at a constant η parameter; the threshold for evaporation is lowered in proportion to the decreasing temperature. Constant η ensures that the energy distribution is only rescaled during the cooling and does not change its shape. This assumption is reasonably well fulfilled in experiments.

The principle of detailed balance can be used in kinetic systems which are decomposed into elementary processes (collisions, or steps, or elementary reactions). It states that at equilibrium, each elementary process is in equilibrium with its reverse process. $Rate(A \rightarrow B) = Rate(B \rightarrow A)$

We consider particles at density n_0 in a box potential, and we assume that η is large. The rate of evaporating atoms can then be obtained as follows: In an untruncated Maxwell-Boltzmann distribution, almost every collision involving an atom in the high energy tail removes the atom from the high energy tail. By detailed balance, elastic collisions produce atoms with energy larger than ηkT at a rate that is simply the number of atoms with energy larger than ηkT divided by their collision time. For a large value of η , the rate of evaporation in a truncated Boltzmann distribution is identical to the production rate of atoms with energy larger than ηkT in the untruncated distribution.

The velocity of atoms with energy ηkT is $\sqrt{\frac{2\eta kT}{m}} = \sqrt{\eta\pi} \cdot \frac{\bar{v}}{2}$, where \bar{v} denotes the average thermal velocity. For large η , the fraction of atoms with energy larger than ηkT approaches $2e^{-\eta}\sqrt{\frac{\pi}{\pi}}$. The rate of evaporating atoms is thus:

$$\dot{N} = -Nn_0\sigma\bar{v}\eta e^{-\eta} = \frac{-N}{\tau_{ev}}$$

Where we have introduced the elastic collision cross section σ and the time constant for evaporation τ_{ev} . It's ratio to the elastic collision time τ_{el} is expressed by $\lambda = \frac{\tau_{ev}}{\tau_{el}}$ Using $\frac{1}{\tau_{el}} = n_0\sigma\bar{v}\sqrt{2}$, where $\bar{v}\sqrt{2}$ is the average relative velocity between two atoms, we obtain in the limit of large η ,

$$\lambda = \frac{\sqrt{2}e^{\eta}}{\eta}$$

1.2.1 Runaway Evaporation

For alkali atoms, where the dominant loss mechanism is background gas collisions, an important criterion for sustained evaporation is to maintain or increase the elastic collision rate $n\sigma v$. It follows that *the elastic collision rate varies* as:

$$\frac{d(n\sigma v)}{dt}/n\sigma v = \frac{1}{\tau_{el}} \left(\frac{\alpha(\delta - 1/2) - 1}{\lambda} - \frac{1}{R} \right)$$

In the temperature range of interest, σ is the s-wave cross section and is independent of temperature. $R = \frac{\tau_{loss}}{\tau_{el}}$ is the number of elastic collisions per trapping time (also called the ratio of good to bad collisions) where τ_{loss} is the time constant for trap loss due to background gas collisions. Evaporation at constant or increasing collision rate ("runaway evaporation") requires:

$$R \geq R_{\min} = \frac{\lambda}{\alpha(\delta - 1/2) - 1}$$

In the absence of any loss process ($R = \infty$), the minimum η for runaway evaporation is determined by:

$$\alpha > \frac{1}{\delta - 1/2}$$

The increase of phase space density with time is given by:

$$\beta = 100\tau_{\text{el}} \frac{d}{dt} (\log_{10} D) = \frac{100}{\ln 10} \left(\frac{\alpha(\delta + 3/2) - 1}{\lambda} - \frac{1}{R} \right)$$

1.2.2 Maximizing Phase-Space Density

α describes only the change in temperature. If we regard evaporation in a linear potential as analogous to evaporation in a harmonic potential with continuous adiabatic compression, we realize that α does not provide the most meaningful comparison, because adiabatic compression trades in temperature against density. We therefore now focus on phase-space density D , which is invariant with respect to adiabatic changes of the potential.

The relative increase in phase space density with decreasing number N :

$$\gamma = -\frac{d(\ln D)}{d(\ln N)} = \frac{\alpha(\delta + 3/2)}{1 + \lambda/R} - 1$$

END