PHYS 3710 Short Experimental Project 1

Verification of special relativity & β spectroscopy

LIU HAOYANG

Chinese University of Hong Kong
1155173894@link.cuhk.edu.hk

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In classical mechanics the kinetic energy K, momentum $\vec{p}$ and velocity$\vec{v}$ of a particle are related by equations

$$\vec{p} = m_0 \vec{v} \tag{1}$$

$$K = \frac{p^2}{2m_o} \tag{2}$$

Where $p^2 = \vec{p} \cdot \vec{p}$ and m_o is the inertial mass of the particle.

In special relativity the kinetic energy K, momentum $\vec{p}$ and velocity$\vec{v}$ of a particle are related by equations

$$\vec{p} = \gamma m_o \vec{v} \tag{3}$$

$$E^2 = p^2c^2 + m_o^2c^4 \tag{4}$$

$$E = K + m_oc^2 = \gamma m_oc^2 \tag{5}$$

Where $\gamma = \frac{1}{\sqrt{1-\left(\frac{|v|}{c}\right)^2}}$, c is the velocity of light, E is the total energy and m_o is the inertial mass of the particle.
From equation (4) and (5), we could get

$$K = m_oc^2\left(\sqrt{1+\left(\frac{p}{m_oc}\right)^2} - 1\right) \tag{6}$$

$$\frac{p^2c^2}{2K} = m_oc^2 + \frac{K}{2} \tag{7}$$

In the limit of high velocities where $m_o c \lll p$, equation (6) approaches

$$K = pc \tag{8}$$

In this experiment, we could measure the momentum $\vec{p}$ and kinetic energy K of β particle. Then plot $\frac{p^2c^2}{2K}$ vs $\frac{K}{2}$. If we get a straight line with slope $= 1$ and y-intercept $= 511$ keV($m_o c^2 = 511 keV$), then we could prove special relativity is correct.

In part I *Momentum calibration*, we use a calibrated Hall sensor to measure the magnetic field B. Then we could get a relation between B and the Hall voltage V like $B = a(V - V_o)$, where a is a constant and V_o is the offset voltage. Using this relation, we could calculate B at different V.
We know $p = \frac{Bed}{2}$, where d is distance between the source and the detector of the spectrometer. So after knowing the magnetic field B and the distance d, we could get the momentum $\vec{p}$ of β particle.

In part II *Energy calibration*, we use some experimental devices to get rough values of kinetic energy, then we could get a straight line $T = b(k - k_o)$, where $b = \frac{624}{k_{624} - k_o}$. By using this formula, the accurate value of kinetic energy will be calculated.

In part III *Energy spectrum* and part IV *Momentum spectrum*, we use two different source, ^{133}Ba and ^{204}Tl ,to get different β particles. Then two energy spectrum will be collected, and momentum spectrum will be converted from those energy spectrum.

In part V *Verification of special relativity*, using a ^{204}Tl source, we could calculate p at different magnet current. After collecting an energy spectrum of these selected electrons, we will observe a peak. Then calculate it kinetic energy by using calibration curve. Plot K vs p, and then plot $\frac{p^2c^2}{2K}$ vs $\frac{K}{2}$ to verify special relativity