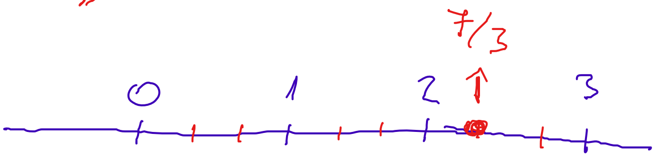
Soluciones

Ficha de Repaso

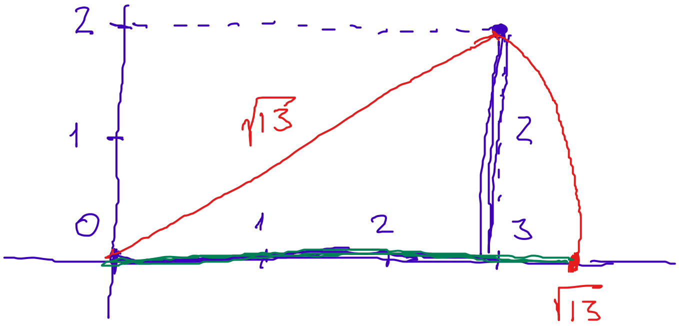
1.- $\sqrt[3]{-8} = -2 \to \mathbb{Z}$ $\qquad \sqrt[5]{\sqrt{1024}} = 2 \to \mathbb{N}$ $\qquad 16^{1/5} = \sqrt[5]{16} = 2\sqrt{2} \to \mathbb{R}(\mathbb{I})$

$0'123 \to \mathbb{Q}$ $\qquad \pi \to \mathbb{R}(\mathbb{I})$ $\qquad 0'101001000\ldots \to \mathbb{R}(\mathbb{I})$

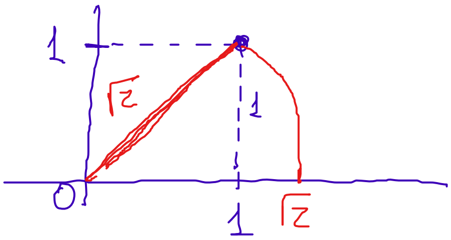
2.- a) $\frac{7}{3} \Rightarrow$



b) $\sqrt{13} \Rightarrow$



c) $\sqrt{2} \Rightarrow$



3.- $A = E(2,3) = (2-3, 2+3) = (-1,5)$

$|x-6| < 4 \begin{cases} x-6 < 4 \to x < 10 \\ x-6 > -4 \to x > 2 \end{cases} \to B = (2,10)$

$A \cup B = (-1,10)$

$A \cap B = (2,5)$

4.- a) $|x| < 6 \begin{cases} x < 6 \\ x > -6 \end{cases} \to (-6,6)$

b) $|x| \geq 4 \begin{cases} x \geq 4 \\ x \leq -4 \end{cases} \to (-\infty,-4) \cup (4,+\infty)$

c) $|x-5| \leq 5 \begin{cases} x-5 \leq 5 \to x \leq 10 \\ x-5 \geq -5 \to x \geq 0 \end{cases} \to [0,10]$

d) $|2x+5| < 11 \begin{cases} 2x+5 < 11 \to x < 3 \\ 2x+5 > -11 \to x > -8 \end{cases} \to (-8,3)$

5.-

$R_T = 6378'1 \text{ Km}$

$R_{ap.} = 6378 \text{ Km}$

$$\left\{ \begin{array}{l} \varepsilon_a = |R_T - R_{ap.}| = 0'1 \text{ Km} \\ \varepsilon_r = \dfrac{\varepsilon_a}{R_T} = 1'5\cdot 10^{-5} = 0'002\,\% \end{array} \right.$$

$$V_T = \frac{4}{3}\pi \cdot R_T^3 = 1'086832412 \cdot 10^{12} \text{ Km}^3$$

$$V_{ap} = \frac{4}{3}\pi R_{ap}^3 = 1'086781293 \cdot 10^{12} \text{ Km}^3$$

$$\left\{ \begin{array}{l} \varepsilon_a = 51119394'74 \text{ Km}^3 \\ \varepsilon_r = 4'7 \cdot 10^{-5} = 0'0047\,\% \end{array} \right.$$

6.-

a) $$\frac{2x^{-7/3}}{\sqrt{2x}} = \frac{2\sqrt[3]{x^{-7}}}{\sqrt{2x}} = \frac{2\sqrt[6]{x^{-14}}}{\sqrt[6]{2^3x^3}} = 2\sqrt[6]{\frac{x^{-14}}{2^3x^3}} = 2\sqrt[6]{\frac{1}{2^3x^{17}}} = 2\cdot\frac{1}{\sqrt[6]{2^3\cdot x^{17}}} =$$

$$= 2\cdot\frac{1\cdot\sqrt[6]{2^3x}}{\sqrt[6]{2^3x^{17}}\cdot\sqrt[6]{2^3\cdot x}} = \frac{\cancel{2}\sqrt[6]{2^3x}}{\cancel{2}x^3} = \frac{\sqrt[6]{2^3x}}{x^3}$$

b) $$\sqrt[3]{9ab^2}\cdot\sqrt[6]{18a^3b^2} = \sqrt[3]{3^2ab^2}\cdot\sqrt[6]{2\cdot3^2a^3b^2} = \sqrt[6]{3^4a^2b^4}\cdot\sqrt[6]{2\cdot3^2a^3b^2} =$$

$$= \sqrt[6]{2\cdot3^6\cdot a^5\cdot b^6} = 3b\sqrt[6]{2a^5}$$

c) $$\frac{\left(8x^2\sqrt{a^3}\right)^2}{42a^{-2}\sqrt{x}} = \frac{\left(2^3x^2\sqrt{a^3}\right)^2}{2\cdot3\cdot7a^{-2}\sqrt{x}} = \frac{2^6x^4\sqrt{a^6}}{2\cdot3\cdot7\,a^{-2}\sqrt{x}} = \frac{2^6x^4\cdot a^3}{2\cdot3\cdot7\,a^{-2}\sqrt{x}} =$$

$$= \frac{2^5\cdot x^4\cdot a^5}{3\cdot7\cdot\sqrt{x}} = \frac{2^5x^4a^5\sqrt{x}}{3\cdot7\cdot\sqrt{x}\cdot\sqrt{x}} = \frac{2^5x^4a^5\sqrt{x}}{3\cdot7\cdot x} = \frac{2^5x^3a^5\sqrt{x}}{3\cdot7}$$

Racionalizar

d) $$\left(\sqrt[3]{\sqrt[5]{a^{12}}}\cdot\sqrt[3]{\frac{1}{a^2}}\right):\left(a\sqrt[4]{a^{-2}}\right) = \frac{\sqrt[15]{a^{12}}\cdot\sqrt[3]{\frac{1}{a^2}}}{a\sqrt[4]{a^{-2}}} = \frac{\sqrt[15]{a^2}}{a\sqrt[4]{a^{-2}}} = \frac{\sqrt[15]{a^2}}{a\sqrt{a^{-1}}} =$$

$$= \frac{\sqrt[30]{a^4}}{a\sqrt[30]{a^{-15}}} = \frac{\sqrt[30]{a^{19}}}{a}$$

7.-

a) $-\sqrt[3]{16x} - \frac{3}{2}\sqrt[3]{2000x} = -\sqrt[3]{2^4x} - \frac{3}{2}\sqrt[3]{2^4\cdot 5^3 x} = -2\sqrt[3]{2x} - \frac{3}{2}\cdot 2\cdot 5\sqrt[3]{2x} =$

$= -2\sqrt[3]{2x} - 15\sqrt[3]{2x} = -17\sqrt[3]{2x}$

b) $-\sqrt[6]{128x^4} - \frac{3}{2}\sqrt[6]{4000000x^4} = -\sqrt[6]{2^7x^4} - \frac{3}{2}\sqrt[6]{2^8\cdot 5^6x^4} =$

$= -2\sqrt[6]{2x^4} - \frac{3}{2}\cdot 2\cdot 5\sqrt[6]{2^2x^4} = -2\sqrt[6]{2x^4} - 15\sqrt[3]{2x^2}$

8.-

a) $\frac{2}{3-\sqrt{3}} = \frac{2(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{2(3+\sqrt{3})}{3^2-(\sqrt{3})^2} = \frac{2(3+\sqrt{3})}{9-3} = \frac{2\cdot(3+\sqrt{3})}{6} =$

$= \frac{3+\sqrt{3}}{3}$

b) $\frac{4+\sqrt{6}}{2\sqrt{3}} = \frac{(4+\sqrt{6})\cdot\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}} = \frac{4\sqrt{3}+\sqrt{6}\cdot\sqrt{3}}{2\cdot(\sqrt{3})^2} = \frac{4\sqrt{3}+\sqrt{18}}{2\cdot 3} = \frac{4\sqrt{3}+\sqrt{2\cdot 3^2}}{6} =$

$= \frac{4\sqrt{3}+3\sqrt{2}}{6}$

c) $\frac{\sqrt{2x}}{\sqrt[3]{\sqrt{32x^{11}}}} = \frac{\sqrt{2x}}{\sqrt[6]{2^5x^{11}}} = \frac{\sqrt{2x}\cdot\sqrt[6]{2x}}{\sqrt[6]{2^5x^{11}}\cdot\sqrt[6]{2x}} = \frac{\sqrt{2x}\cdot\sqrt[6]{2x}}{2x^2} = \frac{\sqrt[6]{2^3x^3}\cdot\sqrt[6]{2x}}{2x^2} =$

$= \frac{\sqrt[6]{2^4x^4}}{2x^2} = \frac{\sqrt[3]{2^2x^2}}{2x^2}$

9.-

a) $\log_6 1296 = x \rightarrow 6^x = 1296 \rightarrow x = 4$

b) $\log_x \frac{1}{81} = 3 \rightarrow x^3 = \frac{1}{81} \rightarrow x = \sqrt[3]{\frac{1}{81}} = \frac{1}{\sqrt[3]{3^4}} = \frac{\sqrt[3]{3^2}}{3}$

c) $\log_{2/3} x = -2 \rightarrow \left(\frac{2}{3}\right)^{-2} = x \rightarrow x = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

d) $\log_x 2000 = -3 \rightarrow x^{-3} = 2000 \rightarrow \frac{1}{x^3} = 2000 \rightarrow x^3 = \frac{1}{2000}$

$\rightarrow x = \sqrt[3]{\frac{1}{2000}} = \frac{1}{\sqrt[3]{2\cdot 10^3}} = \frac{1}{10\sqrt[3]{2}} = \frac{\sqrt[3]{2^2}}{10\cdot 2} = \frac{\sqrt[3]{2^2}}{20}$

e) $\log_x\left(\frac{1}{16}\right)=3 \rightarrow x^3=\frac{1}{16} \rightarrow x=\sqrt[3]{\frac{1}{16}}=\frac{1}{\sqrt[3]{2^4}}=\frac{1}{2\sqrt[3]{2}}=\frac{\sqrt[3]{2^2}}{4}$

f) $3^{2x}=81 \rightarrow 3^{2x}=3^4 \rightarrow 2x=4 \rightarrow x=2$

10.- a) $\log_2 64+\log_2\frac{1}{4}-\log_3 9-\log_2\sqrt{2}=\log_2 2^6+\log_2 2^{-2}-\log_3 3^2-\log_2 2^{1/2}=$

$=6-2-3-\frac{1}{2}=1-\frac{1}{2}=1/2$

b) $\log_2\frac{1}{32}+\log_3\frac{1}{27}-\log_2 1=\log_2 2^{-5}+\log_3 3^{-3}-\log_2 1=-5-3-0=-8$

c) $\ln e^{-1/4}+\ln\sqrt[4]{e^3}-\ln 1=\ln e^{-1/4}+\ln e^{3/4}-\ln 1=-\frac{1}{4}+\frac{3}{4}+0=\frac{2}{4}=1/2$

d) $\ln\sqrt[4]{\frac{1}{e^3}}=\ln e^{-3/4}=-3/4$

11.- $\log_a\frac{A}{a^3\sqrt{B}}=\log_a A-\log_a(a^3\sqrt{B})=\log_a A-\left(\log_a a^3+\log_a\sqrt{B}\right)=$

$=\log_a A-\log_a a^3-\log_a B^{1/2}=3-3-\frac{1}{2}\cdot(-2)=1$

12.- $\log_b\left(A^3\cdot\sqrt{\frac{1}{B}}\right)=\log_b A^3+\log_b\sqrt{\frac{1}{B}}=3\cdot\log_b A+\log_b B^{-1/2}=$

$=3\log_b A-\frac{1}{2}\log_b B=3\cdot\frac{1}{2}-\frac{1}{2}\cdot\left(\frac{-1}{3}\right)=\frac{3}{2}+\frac{1}{6}=\frac{10}{6}=\frac{5}{3}$

13.- $\log_2 2\sqrt{2\sqrt{2\sqrt{2}}}=\log_2 2\sqrt{2\sqrt{\sqrt{2^2\cdot 2}}}=\log_2 2\sqrt{2\sqrt[4]{2^3}}=\log_2 2\sqrt{\sqrt[4]{2^4\cdot 2^3}}=$

$=\log_2 2\sqrt[8]{2^7}=\log_2\sqrt[8]{2^8\cdot 2^7}=\log_2\sqrt[8]{2^{15}}=\log_2 2^{15/8}=15/8$

14.- a) $2\log x=2+\log\left(\frac{x}{10}\right) \Rightarrow \log x^2-\log\left(\frac{x}{10}\right)=2 \Rightarrow \log\left(\frac{x^2}{x/10}\right)=2$

$\Rightarrow \log(10x)=2 \Rightarrow 10^2=10x \Rightarrow x=10$ ⇝ Se comprueba la sol. y OK

b) $\log(11-x^2) = 2\log(5-x) - \log 2 \Rightarrow \log(11-x^2) = \log(5-x)^2 - \log 2$

$\Rightarrow \cancel{\log}(11-x^2) = \cancel{\log}\dfrac{(5-x)^2}{2}$

TRUCO: Cuando tenemos

$\log(\text{"algo"}) = \log(\text{"algo"})$

podemos "tachar" los logaritmos

$\Rightarrow 11 - x^2 = \dfrac{(5-x)^2}{2}$

$\Rightarrow 22 - 2x^2 = 25 - 10x + x^2$

$\Rightarrow 3x^2 - 10x + 3 = 0$

$x = \dfrac{10 \pm \sqrt{100-36}}{2\cdot 3} = \dfrac{10 \pm \sqrt{64}}{6} = \dfrac{10 \pm 8}{6} \begin{cases} \dfrac{18}{6} = 3 \\ \dfrac{2}{6} = 1/3 \end{cases} \rightarrow$ Comprobamos las soluciones

$\boxed{x=3} \rightarrow \log(11-3^2) = 2\log(5-3) - \log 2 \rightarrow \log 9 = 2\cdot\log 2 - \log 2$

$\rightarrow \log 9 = \log 2^2 - \log 2$

$\rightarrow \log 9 = \log\frac{4}{2}$ ¡Falso! No vale

$\boxed{x = 1/3} \rightarrow \log\left(11 - \left(\frac{1}{3}\right)^2\right) = 2\log\left(5 - \frac{1}{3}\right) - \log 2 \rightarrow \log\left(11 - \frac{1}{9}\right) = 2\log\frac{14}{3} - \log 2$

$\rightarrow \log\frac{98}{9} = \log\frac{14^2}{3^2} - \log 2 \rightarrow \log\frac{98}{9} = \log\frac{196/9}{2} \rightarrow \log\frac{98}{9} = \log\frac{98}{9}$

¡OK! Solución válida

c) $\ln(x+1) = \ln(x+19) - \ln(x-1) \rightarrow \cancel{\ln}(x+1) = \cancel{\ln}\dfrac{x+19}{x-1} \rightarrow x+1 = \dfrac{x+19}{x-1}$

$\rightarrow (x+1)(x-1) = x+19 \rightarrow x^2 - 1 = x + 19 \rightarrow x^2 - x - 20 = 0$

$x = \dfrac{1 \pm \sqrt{1+80}}{2} = \dfrac{1 \pm \sqrt{81}}{2} = \dfrac{1 \pm 9}{2} \begin{cases} \dfrac{10}{2} = 5 \\ \dfrac{-8}{2} = -4 \end{cases} \rightarrow$ Comprobamos

$\boxed{x=5} \rightarrow \ln(5+1) = \ln(5+19) - \ln(5-1) \rightarrow \ln 6 = \ln 24 - \ln 4 \rightarrow \ln 6 = \ln\frac{24}{4}$

OK Sol. válida

$\boxed{x=-4} \rightarrow \ln(-4+1) = \ln(-4+19) - \ln(-4-1) \rightarrow$ No válida

No existe   No existe

d) $2\ln(x+1)=\ln x+\ln(x+3) \to \cancel{\ln}(x+1)^2=\cancel{\ln}\, x\cdot(x+3) \Rightarrow (x+1)^2=x(x+3)$

$\to x^2+2x+1=x^2+3x \to 2x+1=3x \to x=1 \to$ Comprobamos

$\boxed{x=1} \to 2\ln(1+1)=\ln 1+\ln(1+3) \to 2\ln 2=0+\ln 4 \to \ln 2^2=\ln 4$ OK

15.- a) $P=P_0\cdot e^{kt}$

$\left.\begin{array}{l} t=5 \text{ años} \\ P=34 \text{ millones} \\ K=0'2 \end{array}\right\} \to P_0=?$

$34=P_0\cdot e^{0'2\cdot 5}$

$P_0=\frac{34}{e}\approx 12'51$ millones de habitantes

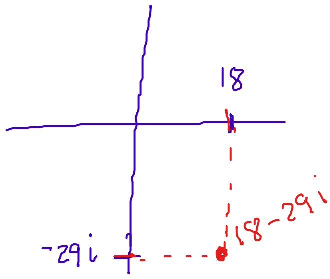
b) $\left.\begin{array}{l} P_0=10 \text{ mill.} \\ K=0'2 \\ P=30 \text{ mill.} \end{array}\right\} \to t=?$

$30=10\cdot e^{0'2t} \to 3=e^{0'2t}$

$\to \ln 3=0'2t \to t=\frac{\ln 3}{0'2}\approx 5'5$ años

16.- a) $-\frac{1}{2}-\frac{7}{5}i$ c) $\frac{3}{2}-\frac{i}{2}$ e) i

b) $-4i$ d) $\frac{77}{36}-i$ f) $\frac{6\sqrt{3}}{5}+\frac{3\sqrt{3}}{5}i$



17.- $2z^2-3\bar{w}\cdot z=2(z-i)^2-3(\overline{-3-2i})\cdot(z-i)=\dots=18-29i$

- Módulo de $18-29i \to r=\sqrt{18^2+(-29)^2}\approx 34'13$
- Argumento $\to \operatorname{arctg}\left(\frac{-29}{18}\right)\approx -58'17^\circ \Rightarrow \alpha=360-58'17=301'83^\circ$

18.- $\frac{(3-2i)^2-(1+i)(2-i)}{-3+i}=\frac{(9-12i+4i^2)-(2-i+2i-i^2)}{-3+i}=$

$=\frac{9-12i-4-2+i-2i-1}{-3+i}=\frac{2-13i}{-3+i}=\frac{(2-13i)(-3-i)}{(-3+i)(-3-i)}=$

$=\frac{-6-2i+39i+13i^2}{(-3)^2-i^2}=\frac{-19+37i}{9+1}=\frac{-19}{10}+\frac{37}{10}i$

19.- $\dfrac{i^{1347}}{1+i^{27}} = \dfrac{-i}{1-i} = \dfrac{-i(1+i)}{(1-i)(1+i)} = \dfrac{-i-i^2}{1^2-i^2} = \dfrac{-i-(-1)}{1-(-1)} = \dfrac{1-i}{2} = \dfrac{1}{2}-\dfrac{1}{2}i$

→ Potencias de i: $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$

• $i^{1347} \rightarrow \begin{array}{r|l} 1347 & 4 \\ \hline 14 & 336 \\ 27 & \\ 3 & \end{array} \rightarrow \boxed{i^{1347}=i^3=-i}$

• $i^{27} \rightarrow \begin{array}{r|l} 27 & 4 \\ \hline 3 & 6 \end{array} \rightarrow \boxed{i^{27}=i^3=-i}$

20.- Calculamos primero $\rightarrow \dfrac{1+3xi}{3-4i} = \dfrac{(1+3xi)(3+4i)}{(3-4i)(3+4i)} = \dfrac{3+4i+9xi+12xi^2}{3^2-(4i)^2} =$

$$= \frac{3-12x+4i+9xi}{9+4} = \underbrace{\frac{3-12x}{13}}_{a} + \underbrace{\frac{4+9x}{13}}_{b} i$$

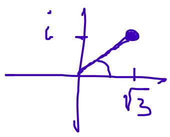
a) Si queremos que $\dfrac{3-12x}{13}+\dfrac{4+9x}{13}i$ sea real $\Rightarrow$ Su parte imaginaria (la b) tiene que valer 0 $\Rightarrow$

$\Rightarrow \dfrac{4+9x}{13}=0 \Rightarrow 9x=-4 \Rightarrow \boxed{x=\dfrac{-4}{9}}$

b) Si queremos que $\dfrac{3-12x}{13}+\dfrac{4+9x}{13}i$ sea imaginario puro $\Rightarrow$ Su parte real (la a) tiene que valer 0 $\Rightarrow$

$\Rightarrow \dfrac{3-12x}{13}=0 \Rightarrow 12x=3 \Rightarrow \boxed{x=\dfrac{3}{12}=\dfrac{1}{4}}$

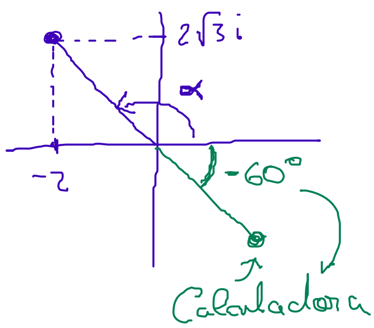
21.- • $\sqrt{3}+i$ → $r=\sqrt{(\sqrt{3})^2+1^2}=\sqrt{4}=2$; → $\operatorname{arctg}\left(\dfrac{\sqrt{3}}{1}\right)=60^\circ$



→ Al estar en el 1er cuadrante el ángulo es bueno $\Rightarrow 2_{60^\circ}$

• $-3i$ → Directamente → 3_{270°

• $z=-2+2\sqrt{3}i$ → $r=\sqrt{(-2)^2+(2\sqrt{3})^2}=\sqrt{4+12}=4$; → $\operatorname{arctg}\left(\dfrac{2\sqrt{3}}{-2}\right)=-60^\circ$ →



$\Rightarrow \alpha = 180^\circ-60^\circ=120^\circ$

$\Rightarrow z = 4_{120^\circ}$

Para calcular z^4 usamos la forma polar:

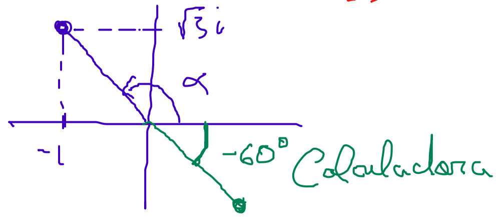
$$z^4 = \left(4_{120^\circ}\right)^4 = 4^4_{4\cdot 120^\circ} = 256_{480^\circ} = 256_{120^\circ}$$

NOTA: $480^\circ = 360^\circ + 120^\circ =$ 1 vuelta $+ 120^\circ = 120^\circ$

(22.-) $z = -1 + \sqrt{3}\,i$

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\operatorname{arctg}\left(\frac{\sqrt{3}}{-1}\right) = -60^\circ$$



$$\Rightarrow \alpha = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow z = 2_{120^\circ} = 2\cdot\left(\cos 120^\circ + i\cdot \operatorname{sen} 120^\circ\right)$$

$$\Rightarrow z^4 = \left(2_{120^\circ}\right)^4 = 2^4_{4\cdot 120^\circ} = 16_{480^\circ} = 16_{120^\circ}$$

(23.-) $w^3 = \left(3_{330^\circ}\right)^3 = 9_{990^\circ} = 9_{270^\circ}$

$$\begin{array}{r|l} 990^\circ & 360^\circ \\ -720 & 2 \\ \hline 270 & \end{array}$$

$\rightarrow 990^\circ = 360^\circ\cdot 2 + 270^\circ =$ "2 vueltas" $+ 270^\circ = 270^\circ$

$$z\cdot w = 6_{120^\circ}\cdot 3_{330^\circ} = 18_{450^\circ} = 18_{90^\circ}$$

$(450^\circ = 360^\circ + 90^\circ)$

$$\frac{z}{w} = \frac{6_{120^\circ}}{3_{330^\circ}} = 2_{-210^\circ} = 2_{150^\circ}$$

$-210^\circ \rightarrow 360^\circ - 210^\circ = 150^\circ$

$$2_{150^\circ} = 2\left(\cos 150^\circ + i \operatorname{sen} 150^\circ\right) = 2\cdot\left(\frac{-\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = -\sqrt{3} + i$$

(24.-) a) $x^2 - 3x + 7 = 3 - 3x$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \sqrt{-4} = \sqrt{4}\cdot\sqrt{-1} = \pm 2i$$

b) $x^2 - 4x + 6 = 0$

$$x = \frac{4 \pm \sqrt{16-24}}{2} = \frac{4 \pm \sqrt{-8}}{2} = \frac{4 \pm \sqrt{8}\,i}{2} = \frac{4 \pm 2\sqrt{2}\,i}{2} = 2 \pm \sqrt{2}\,i$$

c) $2x^2 + 6x + 5 = 0$

$$x = \frac{-6 \pm \sqrt{24-40}}{4} = \frac{-6 \pm \sqrt{-16}}{4} = \frac{-6 \pm \sqrt{16}\,i}{4} = \frac{-6 \pm 4i}{4} = \frac{-6}{4} \pm \frac{4i}{4} = -\frac{3}{2} \pm i$$

d) $x^2 - 8x + 20 = 0$

$$x = \frac{8 \pm \sqrt{64-80}}{2} = \frac{8 \pm \sqrt{-16}}{2} = \frac{8 \pm 4i}{2} = 4 \pm 2i$$