Fisher LDA

$y = w^T x$

$m_1 = \frac{1}{N_1} \sum_{n \in C_1} x_n$

$m_2 = \frac{1}{N_2} \sum_{n \in C_2} x_n$

$m_k = w^T m_k$ $\quad$ m_k는 특정 class k에 대한 평균

$m_2 - m_1 = w^T (m_2 - m_1)$ $\quad$ 극대화 해야 됨

$y_n = w^T x_n$

$s_k^2 = \sum_{n \in C_k} (y_n - m_k)^2 = \sum_{n \in C_k} \left(w^T (x_n - m_k) \right) \left(w^T (x_n - m_k) \right)^T$

$= \sum_{n \in C_k} w^T (x_n - m_k)(x_n - m_k)^T w$

$$J(w) = \frac{(m_2 - m_1)^2}{s_1^2 + s_2^2} = \frac{w^T (m_2 - m_1)(m_2 - m_1)^T w}{\sum_{n \in C_1} w^T (x_n - m_1)(x_n - m_1)^T w + \sum_{n \in C_2} w^T (x_n - m_2)(x_n - m_2)^T w}$$

$$= \frac{w^T S_B w}{w^T S_W w}$$

$S_B = (m_2 - m_1)(m_2 - m_1)^T$ $\quad$ between class

$S_W = \sum_{n \in C_1} (x_n - m_1)(x_n - m_1)^T + \sum_{n \in C_2} (x_n - m_2)(x_n - m_2)^T$ $\quad$ within class

multiclass로 바꾸기

$S_T = \sum_{n=1}^{N} (x_n - m)(x_n - m)^T$ $\quad$ 전체 class 평균

$= S_B + S_W$

$S_B = \sum_{k=1}^{K} N_k (m_k - m)(m_k - m)^T$

$S_W = \sum_{k=1}^{K} \sum_{n \in C_k} (x_n - m_k)(x_n - m_k)^T$ $\qquad \frac{1}{x} \qquad -\frac{1}{x^2}$

$$\frac{dJ(w)}{dw} = \frac{d}{dw} \left(\frac{w^T S_B w}{w^T S_W w} \right)$$

$$= \frac{1}{(w^T S_W w)^2} \left\{ \frac{d}{dw}(w^T S_B w) \, w^T S_W w - w^T S_B w \, \frac{d}{dw}(w^T S_W w) \right\}$$

$$= \frac{1}{(w^T S_W w)^2} \left(2 S_B w \cdot w^T S_W w - w^T S_B w \cdot 2 S_W w \right)$$

$$= \frac{2 S_B w}{w^T S_W w} - 2 \frac{w^T S_B w}{w^T S_W w} \frac{S_W w}{w^T S_W w} = 0$$

$$S_B w = \frac{w^T S_B w}{w^T S_W w} S_W w$$

$\frac{w^T S_B w}{w^T S_W w} = \lambda$

$S_B w = \lambda S_W w$

$\lambda w = S_W^{-1} S_B w$ $\qquad$ $S_B w = (m_2 - m_1)(m_2 - m_1)^T$

$\qquad\qquad\qquad\qquad\qquad \propto (m_2 - m_1)$

$w \propto S_W^{-1} (m_2 - m_1)$