Module-01
Mathematical model of Experiment

1. define sample space, S
2. define event → favourable outcomes
3. assign probability of the event → axioms are satisfied

Eg: Two dice are thrown, find the probability of getting of sum of 6.

solu:

$A = \{(1,5), (5,1), (2,4), (4,2), (3,3)\}$

$\therefore P(A) = \frac{\text{No of favourable outcomes}}{\text{total outcome}} = \frac{5}{36}$

Axiom 1.

$P(A) \geq 0$

$P(S) = 1$

$\bigcup_{n=1}^{N} (A_n) = \sum_{n=1}^{N} P(A_n)$

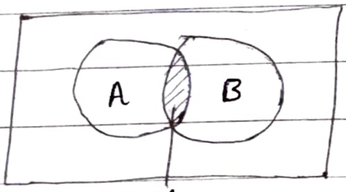
Joint probability

if mutual exclusive

$P(A \cap B) = 0$

if not mutually exclusive

$P(A \cap B) \neq 0$



common elements

$A \cap B$ → not mutually exclusive events

according to axioms

$P(A \cup B) = P(A) + P(B)$

↳ mutually exclusive

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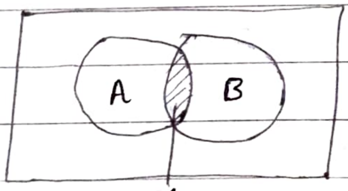

common elements

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↳ mutually exclusive

throwing a die,

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$ -> getting even

$B = \{1, 3, 5\}$ -> getting odd

=> joint probability is if two events are not mutual exclusive then union of events is

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Q1. A card may be selected at random from a deck of 52 cards

consider the event,

1. A = card is a heart
2. B = card is a face

obtain the probability that card is a heart or face card.

=> $P(A) = \frac{13}{52}$ $P(B) = \frac{12}{52}$ $P(A \cap B) = \frac{3}{52}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$

$= \frac{25}{52} - \frac{3}{52}$

$= \frac{22}{52} = \frac{41}{56}$

2. Suppose a student is selected at random from 80 students where 30 are taking maths, 20 are taking chemistry & 10 are taking both maths & chemistry. find the probability that the student is taking maths or chemistry.

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$\rightarrow$ $P(A) = \dfrac{30}{80}$ $P(B) = \dfrac{20}{80}$ $P(A \cap B) = \dfrac{10}{80}$

$$P(A \cup B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{30}{80} + \frac{20}{80} - \frac{10}{80}$$

$$= \frac{40}{80}$$

$$= \frac{1}{2}\%.$$

conditional probability,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) \neq 0$$

similarly $P(B/A) = \dfrac{P(B \cap A)}{P(A)}$, where $P(A) \neq 0$

Now $P(B \cap A) = P(A \cap B)$

Q1. A pair of fair dice is tossed. find the probability that one of the die is 2 when the sum is 6.

$\Rightarrow$ $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$P(B) = \{(1,5), (2,4), (3,3), (4,2), 5,1\}$

$$= \frac{5}{36}$$

$$P(A \cap B) = \frac{2}{36}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{36} \times \frac{36}{5}$$

$$= \frac{2}{5}$$

$$= 40\%$$

$\dfrac{2/36}{5/36} = 2/5$

2. in a box there are 100 resistors having resistence & tolerance as shown,

Resistence (Ω)	tolerance (5 %)	tolerance (10%)	total
22 Ω	10	14	24
47 Ω	28	16	44
100 Ω	24	8	32
	62	38	100

three events are defined, let a resistor be selected from the box & assume each resistor has the same liklihood of being chosen.

A: draw a 47 Ω resistor

B: draw a resistor with 5% tolerence

C: draw a 100 Ω resistor

determine all joint & conditional probabilities

-> joint probabilities $P(A\cap B)$, $P(B\cap C)$, $P(A\cap C)$

conditional probabilities $P(A/B)$, $P(A/C)$, $P(B/C)$

$P(A) = \frac{44}{100}$ $\quad P(B) = \frac{62}{100}$ $\quad P(C) = \frac{32}{100}$

$P(A\cap B) = P(47\Omega \cap 5\%) = 28/100$

$P(A\cap C) = P(47\Omega \cap 100\Omega) = 0$

$P(B\cap C) = P(5\% \cap 100\Omega) = 24/100$

$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{28}{100} \times \frac{100}{62} = \frac{\cancel{28}}{\cancel{62}}$

$P(A/C) = \frac{P(A\cap C)}{P(C)} = 0 \times \frac{100}{32} = 0$

$P(B/C) = \frac{P(B\cap C)}{P(C)} = \frac{24}{100} \times \frac{100}{32} = \frac{24}{32}$

Total probability theorem,

consider a sample space where there are N no// of mutual exclusive events B_n where $n = 1, 2, 3, \ldots N$.

$B_n \cap B_n = \phi$ where i.e, $B_1 \cap B_2 = 0 = \phi$

$$\bigcup_{n=1}^{N} B_n = S \qquad \sum_{n=1}^{N} B_n = S$$

consider a event A on the sample space S where A is subset of S.

$$P(A) = \sum_{n=1}^{N} P\left(\frac{A}{B_n}\right) P(B_n)$$

$$\rightarrow P\left(\frac{A}{B_1}\right) P(B_1) + P\left(\frac{A}{B_2}\right) + \ldots P\left(\frac{A}{B_N}\right) \ldots P(B_N)$$

proof,

the sample space S of N mutually exclusive events,

B_n, $n = 1, 2, 3 \ldots$

i.e, $B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_N = S$

Let an event A be defined on sample space S. Since A is a subset of S, then $A \cap S = A$

$\therefore$ $A \cap S = A \cap \left[\bigcup_{n=1}^{N} B_n\right] = A$ $\therefore$

$$A = \bigcup_{n=1}^{N} (A \cap B_n)$$

applying probability $P(A) = P\left[\bigcup_{n=1}^{N} (A \cap B_n)\right] =$

$$= \bigcup_{n=1}^{N} P(A \cap B_n)$$

since the events $(A \cap B_n)$ are mutually exclusive, by applying axiom 3 of probability we get,

$$P(A) = \sum_{n=1}^{N} P(A \cap B_n)$$

from the defination of conditional probability

$$P(A \cap B_n) = P\left(\frac{A}{B_n}\right) P(B_n)$$

$$P(A) = \sum_{n=1}^{N} P\left(\frac{A}{B_n}\right) P(B_n)$$

Baye's theorem:

it states that if a sample space S has N mutually exclusive events B_n, $n = 1, 2, 3 \ldots N$, such that $B_m \cap B_n = \phi$ for $m = 1, 2, 3 \ldots N$ & any event A is defined ∂ this sample space then the conditional probability of B_n & A can be expressed as,

$$P(B_n / A) = \frac{P\left(\frac{A}{B_n}\right) P(B_n)}{P\left(\frac{A}{B_1}\right) P(B_1) + P\left(\frac{B}{B_2}\right) P(B_2) + \cdots + P\left(\frac{A}{B_n}\right) P(B_n)}$$

proof,

$$P\left(\frac{B_n}{A}\right) = \frac{P(B_n \cap A)}{P(A)} \quad \text{where, } P(A) \neq 0 \rightarrow (1)$$

$$P\left(\frac{A}{B_n}\right) = \frac{P(A \cap B_n)}{P(B_n)} \quad \text{where, } P(B_n) \neq 0 \rightarrow (2)$$

$$A \cap B_n = B_n \cap A$$

from eq^n (2)

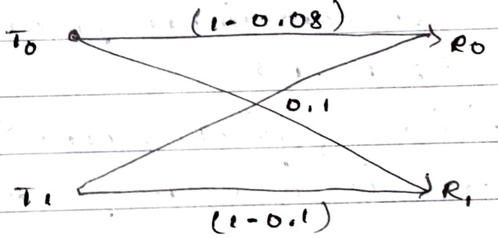
$$P(B_n \cap A) = P\left(\frac{A}{B_n}\right) \cdot P(B_n) \rightarrow ③$$

Substitute eqⁿ ③ in eqⁿ ①

$$P\left(\frac{B_n}{A}\right) = \frac{P\left(\frac{A}{B_n}\right) \cdot P(B_n)}{P(A)}$$

PROBLEMS,

1. In a binary communication system a zero & a one is transmitted with probability 0.6 & 0.4 respectively. due to error in the communication system a zero becomes a one -4 with probability 0.08. determine the probabili-
   1. of receiving a one &
   2. that a one was transmitted when the received message is one.



$P(T_0) = 0.6$

$P(T_1) = 0.4$

zero becomes '1'

$$P\left(\frac{R_1}{T_0}\right) = 0.1$$

one becomes '0'

$$P\left(\frac{R_0}{T_1}\right) = 0.08$$

1. probability of receiving 1:

$$P(A_1) = \sum_{n=1}^{N} P(A/B_n) \cdot P(B_n)$$

$$P(R_1) = P(R_1/T_0)\, P(T_0) + P(R_1/T_1) \cdot P(T_1)$$

$$= 0.1\,(0.6) + (1-0.1)\,(0.4)$$

$$= 0.1\,(0.6) + (0.9)\,0.4$$

$$= 0.42$$

2. $P\left(\frac{T_1}{R_1}\right) = \frac{P\left(\frac{R_1}{T_1}\right) \cdot P(T_1)}{P(R_1)}$

$= \frac{0.9\ (0.4)}{0.42}$

$= 0.857$

Independent events,

consider two events A, B with non-zero probabilities of occurance P(A) & P(B) i.e, $P(A) \neq 0$ & $P(B) \neq 0$. They are said to be staticaly independent if the probability of occurance of one event does not affect the occurance of other event.

$P(A|B) = P(A)$

$P(B|A) = P(B)$

Substituting the defination for statical independence in conditional probability eqⁿ

$P(A \cap B) = P(A) \cdot P(B)$

The probability of joint (venture)ˣ occurance of two events is equal to product of the events.

Multiple events,

in case of three events A_1, A_2, A_3 they are said to be independent if & only if they are independent by all pairs & are also independent as a triple.

$P(A_1 \cap A_2) = P(A_1)\ P(A_2)$

$P(A_2 \cap A_3) = P(A_2)\ P(A_3)$

$P(A_1 \cap A_3) = P(A_1)\ P(A_3)$

$P(A_1 \cap A_2 \cap A_3) = P(A_1)\ P(A_2)\ P(A_3)$

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PROBLEM,

1. in an exp one card is selected from an ordinary 52 card deck. define events A. as " select a king", " &
B. " select a jack or queen"
C. " select a heart"

$\Rightarrow$ $P(A) = \frac{4}{52}$ $P(B) = \frac{8}{52}$ $P(C) = \frac{13}{52}$

joint probabilities.

$P(A \cap B) = 0$

$P(A \cap C) = 1/52$

$P(B \cap C) = \frac{2}{52}$

determine whether A, B & C are independent by pairs.

1. $P(A \cap B) = 0$

$P(A) \cdot P(B)$

$= \frac{4}{52} \cdot \frac{8}{52}$

$= \frac{32}{2704}$

$P(A \cap B) \neq P(A) \cdot P(B)$

$\therefore$ A & C are independent as pair as are B & C A & B are not independent, A & B are mutually exclusive events.

2. $P(A \cap C) = \frac{1}{52}$

$P(A) \cdot P(C)$

$\frac{4}{52} \cdot \frac{13}{52} = \frac{1}{52}$

$P(A \cap C) = P(A) \cdot P(C)$

3. $P(B \cap C) = \frac{2}{52}$

$P(B) \cdot P(C) = \frac{8}{52} \cdot \frac{13}{52}$

$= 2/52$

2. A missile can be accidently launched if two relays A & B both have failed. the probability of A & B failing are known to be 0.01 & 0.03, respectively. it is also known that B is more likely to fail (probability 0.06) if A has failed.

a. what is the probability of an accidental missile launch?

b. what is the probability that A will fail if B has failed?

c. are the events 'A fails' & 'B fails' statically independent?

$\Rightarrow$ P (A fails) = 0.01

P (B fails) = 0.03

P (c fails) = 0.06

a. P (accidental missile launch)

= P (A fails $\cap$ B fails)

$= P\left(\frac{B}{A}\right) \cdot P(A)$ } conditional probability

= (0.06) (0.01)

= 0.0006 //

b. $P\left(\frac{A \text{ fails}}{B}\right) = \frac{P(B \cap A)}{P(B)}$

$= \frac{0.0006}{0.03}$

= 0.02

c. $P(A \cap B) = 0.0006$

$P(A) \cdot P(B) = 0.01 \times 0.03 = 0.0003$

$P(A \cap B) \neq P(A) \cdot P(B)$

$\therefore$ Not statically independent event.

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3. A factory uses three machines x, y, z to produce certain items, suppose,
1. machine x produces 50% of the time, of which 3% are defective.
2. machine y produces 30% of the items, of which 4% are defective,
3. machine z produces 20% of the items of which 5% are defective.

~~4. machine~~

1. find the probability that a randomly selected item is defective.
2. Suppose a defective item is found among the output. find the probability that it came from each of the machines.

=> $P(X) = 0.5$

$P(Y) = 0.3$

$P(Z) = 0.2$

$P\left(\frac{D}{X}\right) = 0.03$

$P\left(\frac{D}{Y}\right) = 0.04$

$P\left(\frac{D}{Z}\right) = 0.05$

1. $P(D) = P(X) \cdot P\left(\frac{D}{X}\right) + P(Y) \cdot P\left(\frac{D}{Y}\right) + P(Z) \cdot P\left(\frac{D}{Z}\right)$

$= (0.5)(0.03) + 0.3(0.04) + 0.2(0.05)$

$= 0.015 + 0.012 + 0.01$

$= 0.037$

$= 3.7\%$

2. $P\left(\frac{X}{D}\right) = \frac{P(D/X) \cdot P(X)}{P(D)}$

$= \frac{0.03(0.5)}{0.037} = 0.405 = 40.5\%$

$$P\left(\frac{Y}{D}\right) = \frac{P(D/Y)\,P(Y)}{P(D)}$$

$$= \frac{0.04\,(0.3)}{0.037} = 0.324 = 32.4\%$$

$$P(Z/D) = \frac{P(D/Z)\,(P(Z)}{P(D)}$$

$$= \frac{0.05\,(0.2)}{0.037}$$

$$= 0.270$$

$$= 27\%$$

4. In a certain college town, 25% of the students failed maths, 15% failed chemistry & 10% fails both maths & chemistry.
A student is selected at random.
a. if the student failed chemistry, what is the probability that he/she failed maths
b. if the student failed maths, what is the probability that he/she failed chemistry
c. what is the probability that the student failed maths & chemistry?
d. what is the probability that student failed neither maths & chemistry?

=) given,

$P(M) = 0.25$ $\quad 25/100$

$P(C) = 0.15$ $\quad 15/100$

$P(M \cap C) = 0.1$

a. $P(M/C) = \frac{P(M \cap C)}{P(C)} = \frac{0.1}{0.15} = 0.666$

b. $P(C/M) = \frac{P(C \cap M)}{P(M)} = \frac{0.1}{0.25} = 0.4$

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c. $P(M \cup C) = P(M) + P(C) - P(M \cap C)$

$= 0.25 + 0.15 - 0.1$

$= 0.3$ //.

d. P (neither maths nor chemistry)

$= 1 - P(M \cup C)$

$= 1 - 0.3$

$= 0.7$ //.

5. In an electronics lab, there are identically looking capacitors of 3 types, A_1, A_2, A_3 in the ratio of 2:3:4, it is known that 1% of A_1, 1.5% of A_2, 2% of A_3 are defected.

a. what percentage of capacitors in the lab are defective?

b. if a capacitor picked at random is found to be defective, what is the probability from A3 it is of type A3.

=) solution,

given

$P(A_1) = 2/9$

$P(A_2) = 3/9$

$P(A_3) = 4/9$

$P\left(\frac{D}{A_1}\right) = 0.01$

$P\left(\frac{D}{A_2}\right) = 0.015$

$P\left(\frac{D}{A_3}\right) = 0.02$

a. $P(D) = P(D/A_1) \cdot P(A_1) + P(D/A_2) \cdot P(A_2) + P(D/A_3) \cdot P(A_3)$

$= (0.01)(0.22) + 0.015(0.33) + (0.02)(0.44)$

$= 0.0022 + 0.00495 + 0.0088 = 0.015$

b. $P\left(\frac{A_3}{D}\right) = \frac{P(D/A_3) \cdot P(A_3)}{P(D)}$

$= \frac{(0.02) \cdot (0.49)}{0.015}$

$= 0.53$

6 in a lot of 100 semiconductor chips contain 20 are defective, 2 chips selected at random without replacement from the lot.

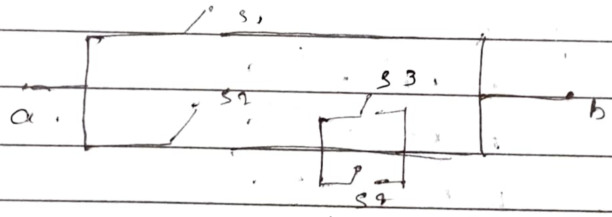
a. what is the probability that the 1st one selected is defective

b. what is the probability that the 2nd one is selected defective given that the 1st one was defective.

c. what is the probability that the both are defective.

=) solution,

7. consider the switching network



it is equally likely that a switch will or will not work. find the probability that a closed path will exists b/w terminals a and b.

solution,

S_1	S_2	S_3	S_4
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1

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0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

(closed path will exists b/w a & b).

$= P(S_1) \cup P(S_2 \cap S_3) \cup P(S_2 \cap S_4)$.

$= P\left[(S_1) \cup (S_2 \cap S_3) \cup (S_2 \cap S_4)\right]$.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C + P(A \cap B \cap C$

$= P(S_1) + P(S_2 \cap S_3) + P(S_2 \cap S_4) - P(S_1 \cap (S_2 \cap S_3) - P(S_1 \cap S_2 \cap S_4) - P(S_2 \cap S_3 \cap S_2 \cap S_4) + P(S_1 \cap S_2 \cap S_3 \cap S_4$

$= P(S_1) + P(S_2 \cap S_3) + P(S_2 \cap S_4) - P(S_1 \cap S_2 \cap S_3) - P(S_1 \cap S_2 \cap S_4) - P(S_2 \cap S_3 \cap S_4) + P(S_1 \cap S_2\ S_3 \cap S_4)$

$P(S_1) = \frac{8}{16}$

$P(S_2 \cap S_3) = 4/16$

$P(S_2 \cap S_4) : 4/16$

$P(S_1 \cap S_2 \cap S_3) : 2/16$

$P(S_1 \cap S_2 \cap S_4) = 2/16$

$P(S_2 \cap S_3 \cap S_4) = 2/16.$

$P(S_1 \cap S_2 \cap S_3 \cap S_4) = 1/16$

$\Rightarrow \frac{8}{16} + \frac{4}{16} + \frac{4}{16} - \frac{2}{16} - \frac{2}{16} - \frac{2}{16} + \frac{1}{16}$

$$= \frac{11}{16}$$

MODULE - 02

~~WHY~~ RANDOM VARIABLE

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-> random variable,

it is always a numerical quantity, regardless of the random exp from which it, derives.

Eg, Throwing a die

$S = \{1, 2, 3, 4, 5, 6\}$

1. $X(s) = s^2$

↓ maping outcome of the sample space to real no//. using

$X(s) = \{1, 4, 9, 16, 25, 36\}$

2. $X(s) = 2s + 3$

$X(s) = \{5, 7, 9, 11, 13, 15\}$

Q1. consider the sample tossing a coin twice

$S = \{HH, HT, TH, TT\}$.

when no head assigning values -> 0

one head -> 1

two head -> 2

$= \{2, 1, 1, 0\}$

0, 1, 2 -> range of the random variable

Q2. Throwing a die

consider one is getting odd no//. -> 0

" ——— " even no//. -> 1

solution,

$S = \{1, 2, 3, 4, 5, 6\}$

$= \{0, 1, 0, 1, 0, 1\}$

range of random variable 0, 1

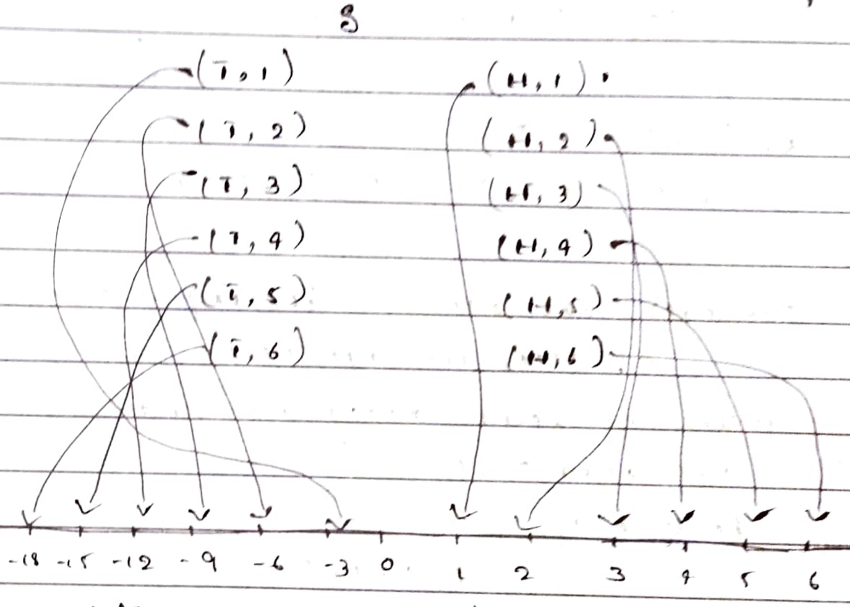
Q3. Let the random variable be a function of x chosen such that.

⇒ a coin head (H) → outcomes corresponds to '+ve' value of 'x' that are equal to no. that are shown on the die.

⇒ a coin tail (T) → outcome corresponds to '-ve' value of 'x' that are equal to thrice the numbers shown on the die.

solution,

exp; throwing a die and coin together



→ conditions for a function to be random variable

1. it is not multivalued
2. the set $\{X \le x\}$ shall be an event for any real no. it is denoted by $P(X \le x)$
3. $P\{X = \infty\} = 0$ & $P\{X = -\infty\} = 0$

→ classification of random variables

1. continuous " "
2. mixed " "

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DRV -> discrete random variable
-> it is one having only discrete value.
-> the sample space for discrete r.v can be discrete continuous or even a mixture of discrete & continuous points.
-> Eg; the wheel of chance experiment has a continuous sample space.

CRV -> continuous random variable
-> A continuous r.v is one having a continuous range.
-> it cannot be produced from a discrete sample space becz all r.v are single valued functions, of all sample space points
eg:- temperature.

Mixed random variable,
-> A MRV is one for which some of its values are discrete.
Eg: Q. A sample space s. defined by
$s = \{1, 2, 3, 4\}$
Random variable $x = x(s) = s^3$
$x = \{1, 8, 27, 64\}$.
if the probabilities of elements of s are
$p(1) = \frac{4}{24}$, $p(2) = \frac{3}{24}$, $p(3) = \frac{7}{24}$, $p(4) = \frac{10}{24}$

solution,
probabilities of random variable
$p\{x=1\} = \frac{4}{24}$ $\quad$ $p(x=27) = \frac{7}{24}$

$p\{x=8\} = \frac{3}{24}$ $\quad$ $p\{x=64\} = \frac{10}{24}$.

$P\{x \le 27\} = P\{x=1\} + P\{x=8\} + P\{x=27\}$

$= \frac{4}{24} + \frac{3}{24} + \frac{7}{24}$

$= \frac{14}{24}$

$P\{x \le 64\} = \frac{14}{24} + \frac{10}{24}$

$= \frac{24}{24}$

$= 1$

Q2. consider an experiment of throwing a fair die, let x be the r.v which assign 1 if the no. that appears is even & 0 if the no. that appears is odd.

1. what is the range of x,
2. find the probability $P(x=1)$ & $P(x=0)$.

Sol:
1. Range of $x = \{0, 1\}$ 1 -> even, 0 -> odd.
2. $P(x=1) = \frac{3}{6} = \frac{1}{2}$

$P(x=0) = \frac{3}{6} = \frac{1}{2}$

Q3. consider the exp of tossing a coin 3 times, let, x be the r.v giving the no. of heads obtained. we assume the tosses are independent, & the probability of a head is p.

a. what is the range of x?

b. find the probability $P(x=0)$, $P(x=1)$, $P(x=2)$ & $P(x=3)$.

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a. Range = $\{0, 1, 2, 3\}$.

b. $P(\cancel{x=0}) =$

$S = \{ (HHH, HHT, HTH, THH, TTH, THT, HTT, TTT \}$

b. $P(H \cap H \cap H) = P(H) \cdot P(H) \cdot P(H) = P \cdot P \cdot P = P^3$

$P(H \cap H \cap T) = P(H) \cdot P(H) \cdot P(T) = P^2 (1-P) = P^2 - P^3$

$P(H \cap T \cap H) = P(H) \cdot P(T) \cdot P(H) = P^2 (1-P) = P^2 - P^3$

$P(T \cap H \cap H) = P(T) \cdot P(H) \cdot P(H) = (1-P) \cdot P^2 = P^2 - P^3$

$P(T \cap T \cap H) = P(T) \cdot P(T) \cdot P(H) = (1-P) \cdot (1-P) \cdot P$

$= (1-P)^2 \cdot P$

$P(T \cap H \cap T) = P(T) \cdot P(H) \cdot P(T) = (1-P) \cdot P (1-P)$

$= (1-P)^2 \cdot P$

$P(T \cap T \cap T) = (1-P)$

1. $P\{x=0\} = P\{TTT\} = (1-P)^3$

2. $P\{x=1\} = P\{THT\} + P\{HTT\} + P(T, T, H\}$

$= (1-P)^2 \cdot P + P \cdot (1-P)^2 + P \cdot (1-P)^2$

$= 3P(1-P)^2$

3. $P(x=2) = P(HHT) + (HTH) + (THH)$

$= (P^2 - P^3) + (P^2 - P^3) + (P^2 - P^3)$

$= 3(P^2 - P^3)$ //.

4. $P(x=3) = (HHH)$

$= P^3$

Q. In above exp what is the value of $P(x<2)$.

sol :- $P(x=0) + P(x=1)$

$= (1-P^3) + 3P(1-P)^2$

$= (1-P^2) ((1-P) + 3P)$ //.

Q4. An information source generate symbols at random a from a four letter alphabet $\{a, b, c, d\}$ with probabilities $P(a) = 1/2$,

P/b

$P(b) = 1/4$, $P(c) = $ & with probabilities $P(d)$ $1/8$ accordingly. A coding scheme encodes these symbol into binary code with

if $a = 0$

$b = 10$

$c = 110$

$d = 111$

Let x be the r.v denoting the length of the code, i.e, no% of binary bits. what is range of R.V.? assuming generation of symbols are independent.

2. find the probabilities $P(x=0)$, $P(x=1)$, $P(x=2)$, $P(x=3)$

solution,

1. Range of R.V = $\{1, 2, 3\}$

2. $S = \{\overset{a}{0}, \overset{b}{10}, \overset{c}{110}, \overset{d}{111}\}$

$\quad\ \downarrow \quad \downarrow \quad \downarrow$

$\quad\ 1 \quad 2 \quad 3$

$P(x=1) = P\{0\} = 1/2 = P\{a\}$

$P(x=2) = P\{10\} = 1/4 = P\{b\}$

$P(x=3) = P\{110, 111\} =$

$= P\{c \cup d\}$

$= \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

$P(x > 3) = 0 \rightarrow$ Null event.

$P(x \geq 3) = 1 - P\{x < 3\}$

Distribution function,

the probability of the event $\{X \leq x\}$ is given by,

$P\{X \leq x\} = F_X(x)$

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Let $X = \{-1, -0.5, 0.7, 1.5, 3\}$,
$P = (0.1, 0.2, 0.1, 0.4, 0.2)$

$\rightarrow$ $F_X(-1) = P(X \leq -1) = 0.1$

$F_X(-0.5) = P(X \leq -0.5) = P(X = -1) + P(X = -0.5)$

$= 0.1 + 0.2$

$= 0.3$

$F_X(0.7) = P(X \leq 0.7)$

$= P(X = -1) + P(X = 0.5) + P(X = 0.7)$

$= 0.1 + 0.2 + 0.1$

$= 0.4$

$F_X(1.5) = P\{X \leq 1.5\}$

$= P\{x = -1\} + P\{x = -0.5\} + P\{x = 0.7\} + P\{x = 1.5\}$

$= 0.1 + 0.2 + 0.1 + 0.4$

$= 0.8$

$F_X(3) = P\{x \leq 3\}$

$= P\{x = -1\} + P\{x = -0.5\} + P\{x = 0.7\} + P\{x = 1.5\} + P\{x = 3\}$

$= 0.1 + 0.2 + 0.1 + 0.4 + 0.2$

$= 1$

Properties of cummulative distributive fn.

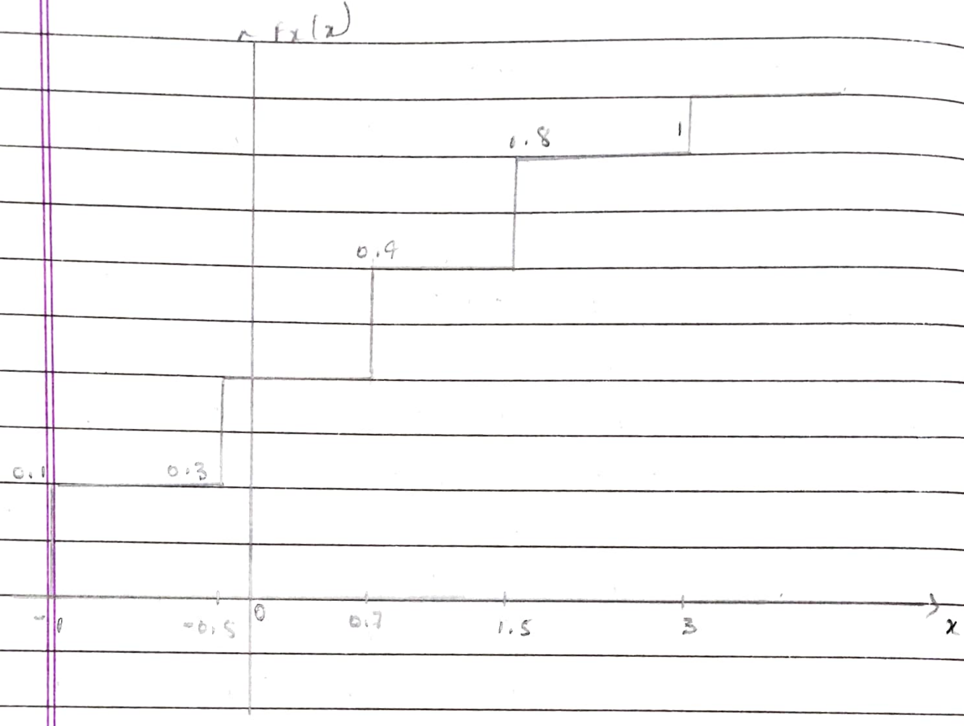
1. $F_X(-\infty) = 0$
2. $F_X(\infty) = 1$
3. $0 \leq F_X(x) \leq 1$
4. $F_X(x_1) < F_X(x_2)$ if $x_1 < x_2$
5. $P\{x_1 < x \leq x_2\} = F_X(x_2) - F_X(x_1)$
6. $F_X(x^+) = F_X(x)$

for eg,

$F_X(-1) = P\{x \leq -1\}$.

graph,

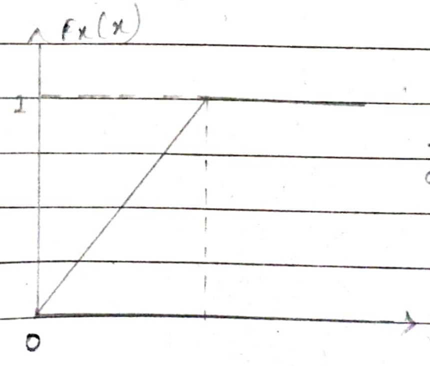
$F_X(x)$ vs x if x is discrete random variable it is in staircase form.



let x_i is the real no. value where $i = 1$, to N,

$$\boxed{F_X(x) = \sum_{i=1}^{N} P(X = x_i) \cdot u(x - x_i)}$$

if x is continuous R.V $F_X(x)$ whol of fortune



$\frac{d}{dx}$ unit impulse

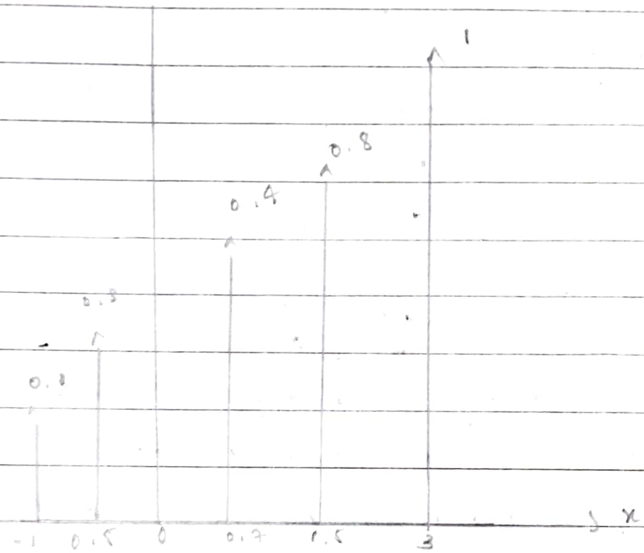
$\frac{d}{dx}$ ramp - unit

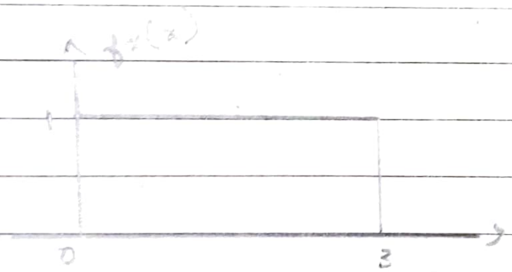
density function,

$$f_x(x) = \frac{d}{dx} F_x(x)$$

$$f_x(x) = \sum_{i=1}^{N} p(x_i)\,\delta(x - x_i)$$

plot the graph of following function





1. a person plays a game of tossing a coin thrice. For each head, is given ₹2 by the organizer of the game & for each tail he have to give ₹1.50 to the organizer. let 'X' denotes the amount got or lost by the person. show that 'X' is a RV & exhibit it as function on the sample space of the experiment.

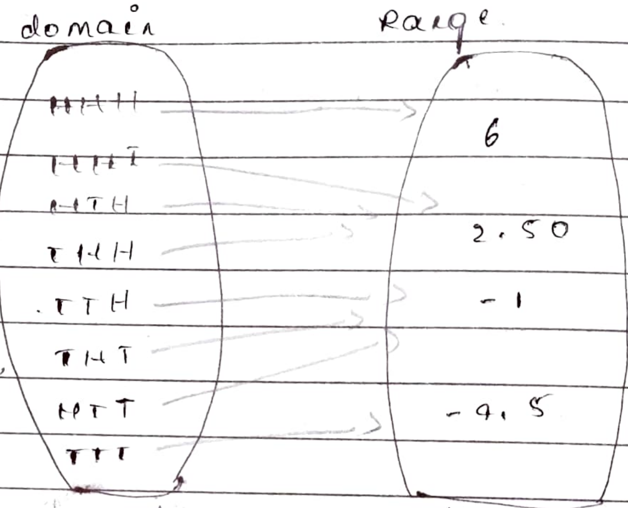
:) solution,

$S = \{(HHH),\ HHT,\ HTH,\ THH,$
$TTT,\ TTH,\ THT,\ HTT\}.$

H = Rs 2.
T = Rs 1.50.

X : Amount gained / lost.

$X(HHH) = 2 \times 3 = 6$
$X(HHT) = X(HTH) = X(THH) = 2 \times 2 - 1 \times 1.50 = 4 - 1.50$
$X(TTH) = X(THT) = X(HTT) = 1 \times 2 - 3 = -1$
$X(TTT) = 0 - 4.5 = -4.5$



∴ X is a fn on the sample space whose range is $\{6, 2.50, -1, -4.5\}$.

2. A bag contains 2 white balls & 1 red balls. One ball is drawn at random and the put ball in the box after noting the colour. The process is repeated again. If 'X' denotes the no. of red balls recorded in the two draws, describe X.

∴ Solution,

$S = \{W_1W_1,\ W_1W_2,\ W_1R_1,\ W_2W_2,\ W_2W_1,$
$W_1R_1,\ RR,\ RW_1,\ RW_2\}.$

$X(R, R) = 1$

$X(W_1W_1) = X(W_1W_2) = X(W_2W_1) = X(W_2W_2)$

$X(W_1R_1) = X(RW_1) = X(RW_2) = 1$

X is a function of sample space whose range is 0, 1, 2.

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probability distribution of r.v

2.50 exp-) $f_1, f_2, f_3 \dots f_{10} = 3, 4, 3, 2, 5, 4, 3, 6, 4, 5.$
ex -) selecting one family from the 10 families.
Let x is a r.v which represents selecting a family and note down the no// of family members.

$\Rightarrow \quad S = \{ f_1, f_2, f_3 \dots f_{10} \}$

$x(f_1) = 3 \qquad x(f_6) = 4$
$x(f_2) = 4 \qquad x(f_7) = 3$
$x(f_3) = 3 \qquad x(f_8) = 6$
$x(f_4) = 2 \qquad x(f_9) = 4$
$x(f_5) = 5 \qquad x(f_{10}) = 5$

where,

$x = 3 = \{ f_1, f_3, f_7 \}$
$x = 2 = \{ f_4 \}$
$x = 4 = \{ f_2, f_6, f_9 \}$
$x = 5 = \{ f_5, f_{10} \}$
$x = 6 = \{ f_8 \}$

$p(x=3) = 3/10$
$p(x=2) = 1/10$
=0 $p(x=4) = (3/10)$
$p(x=5) = 2/10$
$p(x=6) = 1/10.$

R.V	2	3	4	5	6
PDRV	$1/10$	$3/10$	$3/10$	$2/10$	$1/10$

=) conditions of probability distribution

1. $p_i > 0$

2. $\sum p_i = 1$

formal definata =) probability distribution of RV 'x' with a system of NO's

$x = x_1 \ x_2 \dots x_n$

$p(x) = p_1, p_2 \dots p_n$

when $p_i > 0$

$\sum_{i=1}^{n} p_i = 1$

1. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. find the probability distribution of the no of aces.

solution,

x = no's of aces = 0, 1, 2

$P(x=0) = p(\text{non ace } \& \text{ non ace}) = P(NA) \times P(NA)$

$= \frac{48}{52} \times \frac{48}{52}$

$= \frac{2304}{2704}$

$= \frac{144}{169}.$

$p(x=1) = p((A \& NA) \text{ OR } (NA \& A)) =$

$= p(A).(p(NA) + p(NA).p(A)$

$= \frac{\cancel{4}^{1}}{\cancel{52}_{13}} \cdot \frac{\cancel{48}^{12}}{\cancel{52}_{13}} + \frac{\cancel{48}}{\cancel{52}} \cdot \frac{\cancel{4}}{52}$

$= \frac{12}{169} + \frac{12}{169}.$

$= \frac{24}{169}.$

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$P[X=2] = P(\text{Ace } \& \text{ Ace})$
$= P(A) \cdot P(A) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}.$

0	1	2
144/169	24/169	1/169.

2. find the probability distribution of no. of doublets in 3 throws of a pair of dice.

=> 'X' denotes the no. of doublets

$X = 0, 1, 2, 3.$

$P(X=0) = P(ND) = P(ND \times P(ND) \times P(ND)).$

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}.$

$= \frac{125}{216}.$

$P(X=1) = P(D) \times P(ND)$
$P(D)\,P(ND)\,P(ND) + P(ND)\,P(D)\,P(ND) +$
$P(ND)\,P(ND)\,P(D).$

$= 1/6 \times 5/6 \times 5/6 + 5/6 \times 1/6 \times 5/6 + 5/6 \times 5/6 \times 1/6.$

$= \frac{25}{216} + \frac{25}{216} + \frac{25}{216} = \frac{75}{216}.$

$P(x=2) = P(D) \times P(D) \times P(ND) + P(ND)\,P(D)\,P(D) +$
$P(D)\,P(ND)\,P(D).$

$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6}.$

$\times \frac{1}{6}$

$= 5/216 + 5/216 + 5/216 = 15/216.$

$P(x=3) = P(D)\,P(D)\,P(D)$
$= 1/6 \times 1/6 \times 1/6 = 1/216$

0	1	2	3
125/216	75/216	15/216	1/216.

let x denotes the no. of hours you studied during a selected school days. the probability that x can take the value x has the following form, where k is a unknown function

$$p(X=x) = \begin{cases} 0.1, & \text{if } x=0 \\ kx, & \text{if } x=1 \text{ or } 2 \\ k(5-x), & \text{if } x=3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

a. find the value k,

b. what is the probability that you study atleast 2 hours? exactly 2 hours? atmost 2 hrs

=) solution,

x	0	1	2	3	4	otherwise
$p(x)$	0.1	kx	kx	$k(5-x)$	$k(5-x)$	0.

$x \quad 0.1 + kx + kx + k(5-x) + k(5-x) = 1$

$0.1 + 2kx + 2k(5-x) = 1$

$0.1 + 2kx + 10k - 2kx = 1$

$10k = 1 - 0.1$

$k = \frac{0.9}{10}$

$\boxed{k = 0.09}$

$0.1 + k + 2k + 2k + k = 1$

$0.1 + 6k = 1$

$6k = 1 - 0.1$

$k = 0.9/6$

$\boxed{k = 0.15}$

$p(x \geq 2) = p(x=2) + p(x=3) + p(x=4)$

$= kx + k(5-x) + p(5-x)$

$= 2k + 2k + k$

$= 2(0.15) + 2(0.15) + 0.15$

$= 0.3 + 0.3 + 0.15$

$= 0.75$//.

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$\rightarrow P(X=2) = kx$

$= 0.15(2)$

$= 0.3$

$\rightarrow P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.1 + kx + kx$

$= 0.1 + 0.15(1) + 0.15(2)$

$= 0.1 + 0.15 + 0.3$

$= 0.55$

probability density function [continuous] (PDF)

A function $f(x)$ is called as PDF if

1. $f(x) \geq 0$, where $-\infty \leq x < \infty$
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Q. if x is a continuous random variable with a following PDF,

$$F(x) = \begin{cases} \alpha(2x - x^2), & 0 < x < 2 \\ 0, & \text{otherwise,} \end{cases}$$

answer next page →

find

1. α

2. $P(x>1)$

solution,

1. $$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)dx = 1$$

$$= \int_{0}^{2} f(x)\,dx.$$

$$= \int_{0}^{2} \alpha\,(2x-x^2)\,dx = 1$$

$$= \alpha\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$= \alpha\left[\frac{4.8}{2} - \frac{8}{3}\right] - \left[0-0\right] = 1$$

$$= \alpha\left[\frac{12-8}{3}\right] = 1$$

$$= \alpha\left[\frac{4}{3}\right] = 1$$

$$\boxed{\alpha = \frac{3}{4}}$$

2. $P(x>1)$

$$= \int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_{1}^{2} f(x)\,dx$$

$$= \alpha \int_{1}^{2} (2x-x^2)\,dx$$

$$= \alpha\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_1^2$$

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$$= \alpha \left[x^2 - \frac{x^3}{3} \right]_1^2$$

$$= \alpha \left[4 - \frac{8}{3} - \left[1 - \frac{1}{3} \right] \right]$$

$$= \alpha \left[\frac{12-8}{3} - \left\{ \frac{3-1}{3} \right\} \right]$$

$$= \alpha \left[\frac{4}{3} - \left\{ \frac{2}{3} \right\} \right]$$

$$= \alpha \left[\frac{2}{3} \right]$$

$$= \frac{\cancel{3}}{\cancel{4}\,2} \times \frac{\cancel{2}^{1}}{\cancel{3}}$$

$$= \frac{1}{2}.$$

$$\boxed{p(x>1) = \frac{1}{2}}$$

Q. The Random variable x has got a PDF

$$f(x) = \begin{cases} Kx^2, & -3 < x < +3 \\ 0, & \text{otherwise} \end{cases}$$

find K, P $1 \leq x \leq 2$, $P(x \leq 2)$, $P(x > 1)$.

=> solution,

$$= \int_{-3}^{+3} Kx^2 \, dx = 1$$

$$= K \left[\frac{x^3}{3} \right]_{-3}^{+3} = 1$$

$$= \frac{K}{3} \left[\frac{\cancel{-9}}{3} \quad x^3 \right]$$

$$= \frac{K}{3} \left[27 + 1 \right]$$

$$=$$

$P(1 \le x \le 2)$

$= \int_1^2 kx^2 \, dx$

$= k \left. \frac{x^3}{3} \right]_1^2$

$= \frac{k}{3} [8 - 1]$

$= \frac{k}{3} [7]$

$= \frac{7k}{3}$

$= \frac{7}{3} \times \frac{1}{18}$

$= \frac{7}{54}$

$P(x \le 2)$

$= \int_{-3}^{2} kx^2 \, dx$

$= k \left. \frac{x^3}{3} \right]_{-3}^{2}$

$= \frac{k}{3} [8 + 27]$

$= \frac{k}{3} [35]$

$= \frac{35k}{3}$

$= \frac{35}{3} \times \frac{1}{18}$

$= \frac{35}{54}$

$P(x > 1)$

$= \int_1^3 kx^2 \, dx$

$= \frac{k}{3} \left. \frac{x^3}{3} \right]_1^3$

$= \frac{k}{3} [27 - 1]$

$= \frac{26k}{3}$

$= \frac{26}{3} \frac{1}{18}$

$= \frac{26}{54}$ //.

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Q. A 'x' continuous to be random variable in the following PDF,

$$f_X(x) = \begin{cases} Ce^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

find i, C
ii. $P(1 < X < 3)$

solution,

i. $\int_0^{\infty} f(x)\, dx = 1$

$= \int_0^{\infty} ce^{-x} = 1$

$\boxed{e^{-\infty} = 0}$

$= c \left. \frac{e^{-x}}{-1} \right]_0^{\infty} = 1$

$= \frac{c}{-1} \left[e^{-\infty} - e^{0} \right] = 1$

$= -c \left[0 - 1 \right] = 1$

$= \boxed{c = 1}$

ii. $\int_1^3 c\, e^{-x}\, dx.$

$= \frac{c}{-1} \left. e^{-x} \right]_1^3$

$= -c \left[e^{-3} - e^{-1} \right]$

$= -c \left[0.04 - 0.36 \right]$

$= -c \left[-0.32 \right]$

$= +c\, 0.32$

$= 0.32$ //.

=> PROPERTIES OF probability density function

1. $f_X(x) \geq 0$
2. $F_X(x) = \int_{-\infty}^{x} f_X(u)\, du$

3. $\int_{-\infty}^{\infty} f_x(x)\, dx = 1$

4. $P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} f_x(x)\, dx$

=> Bernoulli's triels,

the trails of random exp are called as bernoulies triels.

1. Number of trials must be finite.
2. trails must be independent
3. Each trail as exactly two outcomes (either success & failure)
4. the probababiliy of successs remains the same in each trail.

Q1. 6 balls are drawn successively from a bag containing 7 Red & 9 black balls tell whither & not the traels of drawing balls are bernouli trails when after each draw the ball drawn is

1. Replaced
2. not replaced in the bag

solution,

1. finite trails — finite trails
independent — dependent
true — true

$\frac{7}{16}$, $\frac{9}{16}$ — $\frac{7}{16}$, $\frac{6}{15}$

success, failure — success, success

1. with replacement is considered as bernouli traial
2. without Replacement

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Suppose 'x' is a random variable that takes two values 0 & 1, when probability mass function $P_x(1) = P\{x=1\} = P$ and $P_x(0) = 1-P$, $0 \le P \le 1$,

Such a random variable x is a Bernouli rv, bcoz its describe a outcome of Bernouli trail

- Bernouli random variable

Suppose 'x' is a descrete variable taking values from the set, $\{0, 1, 2 \ldots n\}$. x is called Bernouli rv with parameters n & $0 \le P \le 1$ if $\boxed{P_x(K) = {}^nC_K \, P^K \, (1-P)^{(n-K)}}$ $(K = 0, 1, 2 \ldots n)$ where,

$$\boxed{{}^nC_K = \frac{n!}{K!\,(n-K)!}}$$

The trails must meet the Requirements as Bernouli trail.

Cummulative distrebution

Let 'x' a contenuous random variable having PDF, $f(x)$ then $F_x(x)$ will be a continuous CDF of x if $F_x(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$

1. the probability density fn of a rv x is

$$f_x = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

1. find $P(x \ge 1.5)$
2. find the CDF.

$\Rightarrow$ i. $P(x \geq 1.5) = \int_{1.5}^{2} (2-x)\, dx$

$= \left[2x - \frac{x^2}{2} \right]_{1.5}^{2}$

$= \left[4 - 2 - \left[3 - \right.\right.$

ii. CDF

$x \leq 0$

$P(x \leq 0) = \int_{-\infty}^{0} 0\, dx = 0$

$P(0 < x < 1) = \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{x} f(x)\, dx$

$= \int_{0}^{x} x\, dx$

$= \frac{x^2}{2}$ //.

$P(x \leq 2) = \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{1} f(x)\, dx +$

$\int_{1}^{x} f(x)\, dx$

$= 0 + \frac{x^2}{2} + \int_{1}^{2} (2-x)\, dx$

$= \frac{x^2}{2} + \left. 2x - \frac{x^2}{2} \right]_{1}^{x}$

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$$= \frac{x^2}{2} + 2x^2 - \frac{x^3}{2} - 2 + \frac{1}{2}$$

$$= \frac{x^2 + 4x^2 - x^3 - 4 + 1}{2}$$

$$= \frac{5x^2 - x^3 - 3}{2}$$

$$F_X(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{x^2}{2}, & 0 \le x \le 1 \\ -1 + 2x - \frac{x^2}{2} & \end{cases}$$

2. Let 'X' be any RV and $\phi(x)$ be ~~any~~ function of X. Then the expectation of $\phi(x)$ is denoted by $E[\phi(x)]$ defined by

$E[\phi(x)]$

- discrete RV: $E(\phi(x) P_i(x))$
- continuous R.V: $\int_{-\infty}^{\infty} \phi(x) f(x)\, dx$

Mean $\bar{X} = E(X)$

variance $\sigma = E(X^2) - [E(X)]^2$

Find the mean & variance of probability distribution given by the table

X	1	2	3	4	5
P(X)	0.2	0.35	0.25	0.15	0.05

$\Rightarrow$ solution,

mean $\bar{X} = E(X) = \sum_{i=1} x\, p_i(x)$.

$= 1(0.2) + 2(0.35) + 3(0.25) + 4(0.15) + 5(0.\ldots$

$= 0.2 + 0.7 + 0.75 + 0.6 + 0.25$

$= 2.5$

$$E(x^2) = \sum_{i=1}^{5} x^2 \; p_i(x)$$

$= 1(0.2) + 4(0.35) + 9(0.25) + 16(0.15)$
$25(0.05)$

$= 0.2 + 1.4 + 2.25 + 2.4 + 1.25$

$= 7.5$

~~variance,~~

~~$(\sigma = E(x)^2 -)x$~~

variance,

$\sigma = E(x^2) - (E(x))^2$

$= 7.5 - 6.25$

$= 1.25$

(HW). Q. a continuous RV of density function given by $f(x) = 2e^{-2x}$, $x > 0$

0,

find out the expected value of x $E(x)$ & variance of x.

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5) 1. the probability that man aged 60 will live upto 70 is 0.65 out of 10 men now aged 60. find probability.

i. atleast 7 will live upto 70

ii. exactly 9 will live upto 70.

iii. atmost 9 will live upto 70.

2. Out of 800 families with 5 children, How many families would be expected to have,

1. 3 boys
2. 5 girls
3. either 2 or 3 boys
4. atleast 2 girls

solution,

$n = 5$ Success = boy

$p = 0.5$ failure = girl

$q = 0.5$

1. $P(3) = {}^5C_3 (0.5)^3 (0.5)^2$

$= 0.3125$

2. $P(X=0) = {}^5C_0 (0.5)^0 (0.5)^5$

$= \cancel{{}^5C_3}\ 0.03125$

$= 0.03125$

3. $P(X=2) + P(X=3) = {}^5C_2 (0.5)^2 (0.5)^3 + {}^5C_3 (0.5)^3 (0.5)^2$

$= 10 (0.5)^2 (0.5)^3 + 10 (0.5)^3 (0.5)^2$

$=$

4. $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= {}^5C_0 (0.5)^0 (0.5)^5 + {}^5C_1 (0.5)^1 (0.5)^4 + {}^5C_2 (0.5)^2 (0.5)^3 + {}^5C_3 (0.5)^3 (0.5)^2$

$=$

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3. 10 eggs are drawn successively with replacement in lot 10% deffective eggs. find the probability that atleast one defective egg.

solution,

$n = 10$ $\quad q = 1 - 0.1$

$\qquad\qquad\qquad = 0.9$

$p = 10\%$

$= \frac{1\cancel{0}}{10\cancel{0}} = 0.1$

$(P(X \geq 1) = 1 - P(X = 0))$

$= 1 - (0.1)^0 \, {}^{10}C_0 \, (0.9)^{10}$

$= 1 \times$

$P(X \geq 1) = 1 - P(X = 0$

$= 1 - {}^{10}C_0 (0.1)^0 (0.9)^{10}$

$= 1 - 1 (0.348)$

$= 1 - 0.348$

$= 0.652$

4. A die is thrown 6 times, if getting an odd no., is the success. what is the probability of,

1. 5 success
2. atleast 5 success
3. atmost 5 success.

solution,

$n = 6$

$p = \frac{3}{6} = \frac{1}{2} = 0.5$

$q = 1 - 0.5$

$= 0.5$

i. $(P=5)$ $= {}^6C_5 \, (0.5)^5 \, (0.1)^1$

$=$

$= 0.09.$

ii. $(X \geq 5)$ $= P(5) + P(6)$

$= {}^6C_5 \, (0.5)^5 \, (0.5)^1 + {}^6C_6 \, (0.5)^6 \, (0.5)$

$= 6 \, (0.03125) \times 0.5 + 0.01$

$= 0.09 + 0.01$

$= 0.1$

3. $P(X \leq 5)$ $= P(0) + P(1) + P(2) + P(3) + P(4) + P(5).$

$= 1 - P(X \geq 5)$

$= 1 - 0.1$

$= 0.9$ //

5. ~~find~~ five cards are drawn successively with replacement from a well shuffled deck of 52 cards, what is the probability that.

a. all 5 cards spades.

b. only 3 cards are spades

c. none is a spades.

solution,

$n = 5$

$p = \frac{13}{52} = 0.25$ $\quad q = 1 - \frac{13}{52}.$

$= 0.75$

i. $P(X=5)$ $= {}^5C_5 \, (0.25)^5 \, (0.75)^0$

$= 0.00097$

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ii. $P(x=3) = {}^5C_3 (0.25)^3 (0.75)^2$
$= 10 \times 0.015 \times 0.56$
$= 0.084$

iii. $P(x=0) = {}^5C_0 (0.25)^0 (0.75)^5$
$= 1(0.237)$
$= 0.237$

6. the probability that a ball produced by a factory will be after 150 days of usage is 0.05. find the probability that out of 5 such balls.

1. only one
2. not more than 1
3. more than 1
4. atleast one will free.

solution,

$p = 0.05$ $n=5$
$q = 1-0.05$
$= 0.95$

1. $P(x=0) = {}^5C_0 (0.05)^0 (0.95)^5$
$= 1(0.793)$
$= 0.773$

2. $P(x \le 1) = P(x=0) + P(x=1)$
$= {}^5C_0 (0.5)^0 (0.95)^5 + {}^5C_1 (0.5)^1 (0.5)^4$
$= 0.773 + 0.05 (0.819)$
$= 0.976$

3. $P(x>1) = 1 - (P x \le 1)$
$= 1 - 0.976$
$= 0.024$