Computer Animation 2022
Exercise 1 - Time Integration

Digital ART Laboratory

Deadline:2022.3.17, 03:00

1 Goals and Rules

In this exercise, you need to complete three tasks. The first one helps you get familiar with the framework code. In the following two tasks, you need to apply what you learned about different time integration schemes.

Setup Please download (or clone) the framework code at git repository and follow the instruction to compile and run the code. (repo link: http://dalab.se.sjtu.edu.cn/gitlab/courses/ca-framework-2022)

Submission. Compress your solution and name it in form **NAME_ID_ex1.zip**, then submit it in Canvas. Extra points will be given if you provide a document presenting your solution and comparing the result you get. Note: You only need to submit the part of the code that you modified, i.e. *1-0_earch, 1-1_cannonball, 1-2_spring* directories.

2 Earth

The scene contains two sphere objects, the bigger one represents earth and the other is moon. Your task is to make the moon orbiting around the earth.

Relevent Functions

- EarthSim::advance()
- p_moon → setPosition()

3 Cannonball

We will shoot a cannonball having the mass m, the velocity v_t, the position p_t at time t. In this problem, you are required to implement the following integrators in order to update the position $p_t + \Delta t$ and the velocity $v_t + \Delta t$ of the cannonball with the time step Δt and the gravity g:

- Analytic Solution
- Explicit Euler
- Symplectic Euler
- RK4(this case need to be extended in the framework)

Most of parts have been implemented in the framework, except **advance()** method in CannonBallSim.cpp.

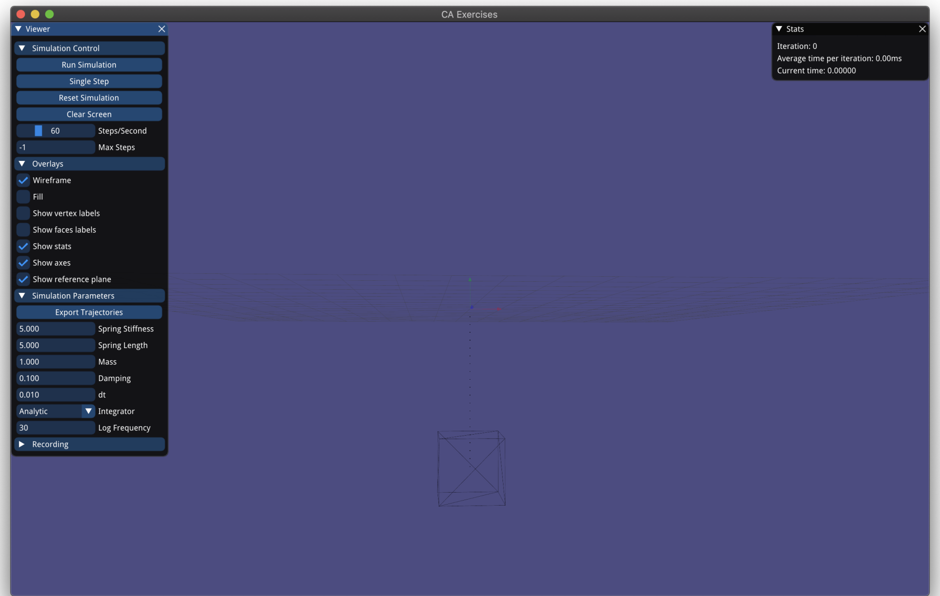

Figure 1 *Scene of the spring problem*

Relevent Functions

- p_ball → setPosition(), getPosition()
- p_ball → setLinearVelocity(), getLinearVelocity()

4 Spring

In this problem, a simple mass-spring system will be simulated as shown in Fig 1. A cube has the mass m and the velocity $\mathbf{v}$ and is connected to a spring at the position $\mathbf{p}$. One end-point of the spring is fixed at the position $\mathbf{p_0}$, and the other one falls due to gravity $\mathbf{g}$. It is characterized by its stiffness k, initial length L, and damping coefficient γ. These parameters are provided as function arguments. For the damping force, we use a linear model: $\mathbf{f_d} = \gamma * \mathbf{v}$. So at time t, the resulting force at point $\mathbf{p}$ is:

$$\mathbf{n} = \frac{\mathbf{p} - \mathbf{p_0}}{||\mathbf{p} - \mathbf{p_0}||}$$
$$\mathbf{f_t} = -k(||\mathbf{p} - \mathbf{p_0}|| - L)\mathbf{n} + m\mathbf{g} - \mathbf{f_d}$$

Similar to **Cannonball** problem, you are required to complete several integrator in **advance()** function at file *SpringSim.cpp*:

- Analytic Solution
- Explicit Euler
- Symplectic Euler
- Midpoint

- RK4(this case need to be extended in the framework)

For the analytic solution, see the appendix for the derivation.

Relevent Functions and Variables

- p_cube → setPosition(), getPosition()
- p_cube → setLinearVelocity(), getLinearVelocity()
- p_cube → getMass()
- m_spring
- m_time, m_gravity, m_dt

Appendix A Second Order Linear Differential Equations

The general from of the second order linear differential equation is

$$y'' + ay' + by = g(x)$$

When $g = 0$, we call the equation as the homogeneous equation. If the auxiliary equation for the homogeneous equation, $q^2 + aq + b = 0$, has the complex roots $\alpha \pm \beta i$, then every solution of the homogeneous differential equation is of the form

$$y(x) = e^{\alpha x}(Acos(\beta x) + Bsin(\beta x))$$

where A, B are determined by initial conditions.

For more details about second order linear differential equation, please refer to document (*ODE-ch12.pdf*).

Appendix B Analytic Solution for the Spring Problem

The problem can be reduced to 1D by looking at the y-axis, shown in Fig 2. We define the position of the cube by a scalar x, with the origin at spring rest position. Following Newton's law, we have $ma = F = mg - kx - \gamma v$. Rewrite it as differential form respect to x, we get

$$mx'' + \gamma x' + kx - mg = 0$$
$$\frac{m}{k}x'' + \frac{\gamma}{k}x' + x - \frac{mg}{k} = 0$$

Let $f = x - \frac{mg}{k}$, which has $f' = x'$ and $f'' = x''$. Substitute f to the above equation, we get

$$\frac{m}{k}f'' + \frac{\gamma}{k}f' + f = 0$$

which is a standard form of the homogeneous differential equation in Appendix A. So we have solution

$$x = \frac{mg}{k} + f = \frac{mg}{k} + e^{\alpha t}(Acos(\beta t) + Bsin(\beta t))$$
$$\alpha = -\frac{\gamma}{2m}, \beta = \frac{\sqrt{4mk - \gamma^2}}{2m}$$
$$A = -\frac{mg}{k}, B = \frac{-\alpha A}{\beta}$$

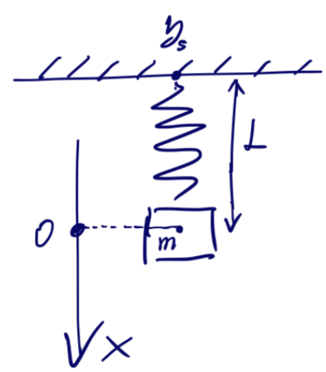

Figure 2 *Spring Problem*

The $x(t)$ we get has the origin at the spring rest position, so the real position and velocity of the cube is

$$
\begin{aligned}
x_{cube} &= -x(t) + y_s - L \\
v_{cube} &= -x'(t)
\end{aligned}
$$