Типовой Расчёт

Задание №1

$$
\begin{aligned}
&\int \frac{e^{\tan x}+2}{\cos^2 x}\mathrm{d}x \\
=&\int (e^{\tan x}+2)\sec^2 x\mathrm{d}x \\
=&\int (2\sec^2(x)+e^{\tan x}\sec^2 x)\mathrm{d}x \\
=&\int e^{\tan x}\sec^2 x\mathrm{d}x + 2\int \sec^2(x)\mathrm{d}x \\
&\text{Пусть } u=\tan x \text{ и } \mathrm{d}u=\sec^2 x \\
=&\int e^u\mathrm{d}u + 2\int \sec^2(x)\mathrm{d}x \\
=&e^u + 2\int \sec^2(x)\mathrm{d}x + C \\
=&e^{\tan x} + 2\tan(x) + C
\end{aligned}
$$

Задание №2

$$\int \sin^2(2x+1)\cdot\cos^2(2x+1)\mathrm{d}x$$

Пусть $u = 2x + 1$ и $\mathrm{d}u = 2\mathrm{d}x$

$$=\frac{1}{2}\int \sin^2(u)\cos^2(u)\mathrm{du}$$

$$=\frac{1}{2}\int \sin^2(u)(1-\sin^2(u))\mathrm{d}u$$

$$=\frac{1}{2}\int (\sin^2(u)-\sin^4(u))\mathrm{d}u$$

$$=\frac{1}{2}\int \sin^2(u)\mathrm{d}u-\frac{1}{2}\int \sin^4(u)u$$

$$=\frac{1}{8}\sin^3(u)\cos(u)+\frac{1}{8}\int \sin^2(u)\mathrm{d}u$$

$$=\frac{1}{8}\sin^3(u)\cos(u)+\frac{1}{8}\int \frac{1-\cos(2u)}{2}\mathrm{d}u$$

$$=\frac{1}{8}\sin^3(u)\cos(u)-\frac{1}{16}\int \cos(2u)\mathrm{d}u+\frac{1}{16}\int 1\mathrm{d}u$$

$$=\frac{1}{8}\sin^3(u)\cos(u)-\frac{1}{32}\int \cos(2u)\mathrm{d}(2u)+\frac{1}{16}\int 1\mathrm{d}u$$

$$=-\frac{\sin(2u)}{32}+\frac{1}{8}\sin^3(u)\cos(u)+\frac{1}{16}\int 1\mathrm{d}u$$

$$=-\frac{\sin(2u)}{32}+\frac{u}{16}+\frac{1}{8}\sin^3(u)\cos(u)+C$$

$$=-\frac{\sin(4x+2)}{32}+\frac{2x+1}{16}+\frac{1}{8}\sin^3(2x+1)\cos^3(2x+1)+C$$

Задание №3

$$\int \frac{(2-x)\mathrm{d}x}{\sqrt{3x^2+2x-5}}$$

$$\mathrm{d}(3x^2+2x-5)=(6x+2)\mathrm{d}x$$

$$=-\frac{1}{6}\int \frac{6x+2}{\sqrt{3x^2+2x-5}}\mathrm{d}x+\frac{7}{3}\int \frac{\mathrm{d}x}{\sqrt{3x^2+2x-5}}$$

Пусть $u=3x^2+2x-5, \mathrm{d}u=(6x+2)\mathrm{d}x$

$$=-\frac{1}{6}\int \frac{\mathrm{d}u}{\sqrt{u}}+\frac{3}{7}\int \frac{\mathrm{d}x}{\sqrt{3x^2+2x-5}}$$

$$=-\frac{1}{3}\sqrt{u}+\frac{3}{7}\int \frac{\mathrm{d}x}{\sqrt{3x^2+2x-5}}+C$$

$$=-\frac{1}{3}\sqrt{u}+\frac{3}{7}\int \frac{\mathrm{d}x}{\sqrt{(\sqrt{3}x+\frac{1}{\sqrt{3}})^2-\frac{16}{3}}}+C$$

Заменим впоследнем интеграле $t=\sqrt{3}x+\dfrac{1}{\sqrt{3}}, k^2=\dfrac{16}{3}, \mathrm{d}x=\dfrac{1}{\sqrt{3}}\mathrm{d}t$

$$=-\frac{1}{3}\sqrt{u}+\frac{3}{7\sqrt{3}}\int \frac{\mathrm{d}x}{\sqrt{t^2-k^2}}+C$$

$$=-\frac{1}{3}\sqrt{u}+\frac{3}{7\sqrt{3}}\arcsin(\frac{k}{t})+C$$

$$=-\frac{1}{3}\sqrt{u}+\frac{3}{7\sqrt{3}}\arcsin(\frac{k}{t})+C$$

$$=-\frac{1}{3}\sqrt{3x^2+2x-5}+\frac{3}{7\sqrt{3}}\arcsin(\frac{\sqrt{\frac{16}{3}}}{\sqrt{3}x+\frac{1}{\sqrt{3}}})+C$$

$$=-\frac{1}{3}\sqrt{3x^2+2x-5}+\frac{3\sqrt{3}}{21}\arcsin(\frac{\sqrt{16}}{3x+1})+C$$

Задание №4

$$\int \frac{x(x+4)\mathrm{d}x}{(x+2)^2(x^2-3x+8)}$$

Пусть $\dfrac{x(x+4)\mathrm{d}x}{(x+2)^2(x^2-3x+8)} = \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{Cx+D}{x^2-3x+8}$

$$\begin{cases} 0 = 16A + 8B + 4D \\ 4 = 2A - 3B + 4C + 4D \\ 1 = -A + B + 4C + D \\ 0 = A + C \end{cases} \Rightarrow \begin{cases} A = -\frac{7}{81} \\ B = -\frac{2}{9} \\ C = \frac{7}{81} \\ D = \frac{64}{81} \end{cases}$$

$$= \int \frac{7x+64}{81(x^2-3x+8)} - \frac{7}{81(x+2)} - \frac{2}{9(x+2)^2}\mathrm{d}x$$

$$= \frac{1}{81}\int \frac{7x+64}{x^2-3x+8}\mathrm{d}x - \frac{7}{81}\int \frac{1}{x+2}\mathrm{d}x - \frac{2}{9}\int \frac{1}{(x+2)^2}\mathrm{d}x$$

$$= \frac{1}{81}\int \frac{7x+64}{x^2-3x+8}\mathrm{d}x - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

$$= \frac{7}{162}\int \frac{2x-3}{x^2-3x+8}\mathrm{d}x + \frac{149}{162}\int \frac{\mathrm{d}x}{x^2-3x+8} - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

Пусть $u = x^2 - 3x + 8, \mathrm{d}u = 2x - 3\mathrm{d}x$

$$= \frac{7}{162}\int \frac{\mathrm{d}u}{u} + \frac{149}{162}\int \frac{\mathrm{d}x}{x^2-3x+8} - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

$$= \frac{149}{162}\int \frac{\mathrm{d}x}{x^2-3x+8} + \frac{7}{162}\ln(u) - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

$$= \frac{149}{162}\int \frac{\mathrm{d}x}{x^2-3x+8} + \frac{7}{162}\ln(x^2-3x+8) - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

$$= \frac{149}{162}\int \frac{\mathrm{d}x}{(x-\frac{3}{2})^2+\frac{23}{4}} + \frac{7}{162}\ln(x^2-3x+8) - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

Пусть $s = x - \dfrac{3}{2}, \mathrm{d}s = \mathrm{d}x$

$$= \frac{149}{162}\int \frac{\mathrm{d}s}{(s)^2+\frac{23}{4}} + \frac{7}{162}\ln(x^2-3x+8) - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

$$= \frac{298}{1863}\int \frac{\mathrm{d}s}{\frac{4s^2}{23}+1} + \frac{7}{162}\ln(x^2-3x+8) - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

Пусть $v = \dfrac{2u}{\sqrt{23}}, \mathrm{d}v = \dfrac{2}{\sqrt{23}}\mathrm{d}u$

$$= \frac{298}{1863}\frac{2}{\sqrt{23}}\int \frac{\mathrm{d}v}{v^2+1} + \frac{7}{162}\ln(x^2-3x+8) - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

$$= \frac{298}{1863}\frac{2\sqrt{23}}{23}\arctan(s) + \frac{7}{162}\ln(x^2-3x+8) - \frac{7}{81}\ln(x+2) + \frac{2}{9(x+2)} + C$$

Задание №5

$$\int \frac{\mathrm{d}x}{2\sqrt{x+5}-\sqrt[3]{x+5}-\sqrt[4]{x+5}}$$

Здесь $k_1 = 3, k_2 = 2,$ поэтому $p = 12.$

Пусть $x + 5 = t^{12}$ Тогда $x = t^{12} - 5, \mathrm{d}x = 12t^{11}\mathrm{d}t$

$$= \int \frac{12t^{11}\mathrm{d}t}{2t^6 - t^4 - t^3}$$

$$=12 \int \frac{t^{11}\mathrm{d}t}{2t^6 - t^4 - t^3}$$

$$=12 \int \frac{t^8\mathrm{d}t}{2t^3 - t - 1}$$

разделить $\frac{t^8\mathrm{d}t}{2t^3 - t - 1}$, получим :

$$\frac{t^8\mathrm{d}t}{2t^3 - t - 1} = \frac{t^5}{2} + \frac{t^4}{4} + \frac{t^2}{4} + \frac{t}{8} + \frac{1}{5(t-1)} + \frac{-t-2}{40(2t^2+2t+1)} + \frac{1}{4}$$

$$=12 \int \frac{t^5}{2} + \frac{t^4}{4} + \frac{t^2}{4} + \frac{t}{8} + \frac{1}{5(t-1)} + \frac{-t-2}{40(2t^2+2t+1)} + \frac{1}{4}\mathrm{d}t$$

$$=\frac{3}{10} \int \frac{-t-2}{2t^2+t+1}\mathrm{d}t + 6\int t^5\mathrm{d}t + 3\int t^3\mathrm{d}t + 3\int t^2\mathrm{d}t + \frac{3}{2}\int t\mathrm{d}t + \frac{12}{5}\int \frac{\mathrm{d}t}{t-1} + 3\int \mathrm{d}t$$

$$=\frac{3}{10} \int \left(-\frac{4t+2}{4(2t^2+2t+1)} - \frac{3}{2(2t^2+2t+1)}\right)\mathrm{d}t + 6\int t^5\mathrm{d}t + 3\int t^3\mathrm{d}t + 3\int t^2\mathrm{d}t + \frac{3}{2}\int$$

$$= -\frac{3}{40} \int \frac{4t+2}{2t^2+2t+1}\mathrm{d}t - \frac{9}{20}\int \frac{\mathrm{d}t}{2t^2+2t+1} + 6\int t^5\mathrm{d}t + 3\int t^3\mathrm{d}t + 3\int t^2\mathrm{d}t + \frac{3}{2}\int t\mathrm{d}t$$

$$= -\frac{3}{40} \int \frac{\mathrm{d}(2t^2+2t+1)}{2t^2+2t+1} - \frac{9}{20}\int \frac{\mathrm{d}t}{(\sqrt{2}t+\frac{1}{\sqrt{2}})^2+\frac{1}{2}} + 6\int t^5\mathrm{d}t + 3\int t^3\mathrm{d}t + 3\int t^2\mathrm{d}t + \frac{3}{2}$$

$$= -\frac{3}{40} \int \frac{\mathrm{d}(2t^2+2t+1)}{2t^2+2t+1} - \frac{9}{10\sqrt{2}}\int \frac{\mathrm{d}(\sqrt{2t}+\frac{1}{\sqrt{2}})}{2(\sqrt{2}t+\frac{1}{\sqrt{2}})^2+1} + 6\int t^5\mathrm{d}t + 3\int t^3\mathrm{d}t + 3\int t^2\mathrm{d}t$$

$$= -\frac{3}{40}\ln(2t^2+2t+1) - \frac{9}{20}\arctan(2t+1) + t^6 + \frac{3}{4}t^4 + t^3 + \frac{3}{4}t^2 + \frac{12}{5}\ln(t-1) + 3t + C$$

$(t = \sqrt[12]{x+5})$

Задание №6

$$\begin{aligned}
&\int \frac{\sqrt{(1-x^2)^3}}{x^2}\mathrm{d}x \\
&= \int \frac{(1-x^2)\sqrt{1-x^2}}{x^2}\mathrm{d}x \\
&= \int \frac{\sqrt{1-x^2}}{x^2}\mathrm{d}x - \int \sqrt{1-x^2}\mathrm{d}x \\
&\quad a = 1, x = a\sin t = \sin t, \mathrm{d}x = \cos t\mathrm{d}t \\
&= \int \frac{\sqrt{1-\cos^2 t}}{\sin^2 t}\cos t\mathrm{d}t - \int \sqrt{1-\sin^2 t}\cos t\mathrm{d}t \\
&= \int \frac{\cos t}{\sin t}\mathrm{d}t - \int \cos^2 t\mathrm{d}t \\
&= \int \frac{\mathrm{d}(\sin t)}{\sin t} - \frac{1}{2}\int (\cos(2t)+1)\mathrm{d}t \\
&= \ln(\sin t) - \frac{1}{4}\int \cos(2t)\mathrm{d}(2t) - \frac{1}{2}\int \mathrm{d}t + C \\
&= \ln(\sin t) - \frac{1}{4}\sin(2t) - \frac{t}{2} + C \\
&\quad t = \arcsin x \\
&= \ln x - \frac{1}{2}(x\sqrt{1-x^2}) - \frac{1}{2}\arcsin x + C
\end{aligned}$$

Задание №7

$$\int \frac{\cos x \mathrm{d}x}{(1-\cos x)^3}$$

Пусть $t=\tan\frac{x}{2}$, то $x=2\arctan t, \mathrm{d}x=\dfrac{2\mathrm{d}t}{1+t^2}$

$$\sin x = 2\frac{\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{2t}{1+t^2}; \cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{1-t^2}{1+t^2}$$

$$=\int \frac{2(1-t^2)}{(1+t^2)^2(1-\frac{1-t^2}{1+t^2})^3}\mathrm{d}t$$

$$=\int \frac{1-t^4}{4t^6}\mathrm{d}t$$

$$=\frac{1}{4}\int \frac{1-t^4}{t^6}\mathrm{d}t$$

$$=\frac{1}{4}\int \frac{1}{t^6}\mathrm{d}t-\frac{1}{4}\int \frac{1}{t^2}\mathrm{d}t$$

$$=\frac{1}{4t}-\frac{1}{20t^5}+C$$

$$t=\tan\frac{x}{2}$$

$$=\frac{1}{4}\cot\frac{x}{2}-\frac{1}{20}\cot^5\frac{x}{2}+C$$

Задание №8

$$\int_{\frac{1}{3}}^{\frac{2}{3}}(x^2-x+1)e^{3x}\mathrm{d}x$$

$$=\int_{\frac{1}{3}}^{\frac{2}{3}}(e^{3x}x^2-e^{3x}x+e^{3x})\mathrm{d}x$$

$$=\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}x^2\mathrm{d}x-\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}e\mathrm{d}x+\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}\mathrm{d}x$$

$$u=x^2,\mathrm{d}u=2x\mathrm{d}x$$

$$\mathrm{d}v=e^{3x},v=\frac{e^{3x}}{3}$$

$$=\frac{1}{3}e^{3x}x^2\Big|_{\frac{1}{3}}^{\frac{2}{3}}-\frac{5}{3}\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}x\mathrm{d}x+\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}\mathrm{d}x$$

$$=\frac{1}{27}e(4e-1)-\frac{5}{3}\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}x\mathrm{d}x+\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}\mathrm{d}x$$

$$u'=x,\mathrm{d}u'=|rmdx$$

$$\mathrm{d}v'=e^{3x},v'=\frac{e^{3x}}{3}$$

$$=\frac{1}{27}e(4e-1)+(-\frac{5}{9}e^{3x})\Big|_{\frac{1}{3}}^{\frac{2}{3}}+\frac{14}{9}\int_{\frac{1}{3}}^{\frac{2}{3}}e^{3x}\mathrm{d}x$$

$$=\frac{1}{27}e(4e-1)-\frac{5}{27}e(2e-1)+\frac{14}{9}e^{3x}\mathrm{d}x$$

$$=\frac{1}{27}e(4e-1)-\frac{5}{27}e(2e-1)+\frac{14}{27}e^{3x}\Big|_{\frac{1}{3}}^{\frac{2}{3}}$$

$$=\frac{1}{27}e(4e-1)-\frac{5}{27}e(2e-1)+\frac{14}{27}e(e-1)$$

$$=\frac{2}{27}e(4e-5)$$

Задание №9

$$\int_0^{\frac{\pi}{2}} \sin\varphi\sqrt{\cos\varphi}\mathrm{d}\varphi$$

$$\text{Пусть } \phi = \cos\varphi, \mathrm{d}\phi = \mathrm{d}\varphi$$

$$= -\int_1^0 \sqrt{\phi}\mathrm{d}\phi$$

$$= \int_0^1 \sqrt{\phi}\mathrm{d}\phi$$

$$= \frac{2}{3}\phi^{\frac{3}{2}}\Big|_0^1$$

$$= \frac{2}{3}$$

Задание №10

Найдите площадь области, ограниченной кривыми, заданными в декартовых координатах

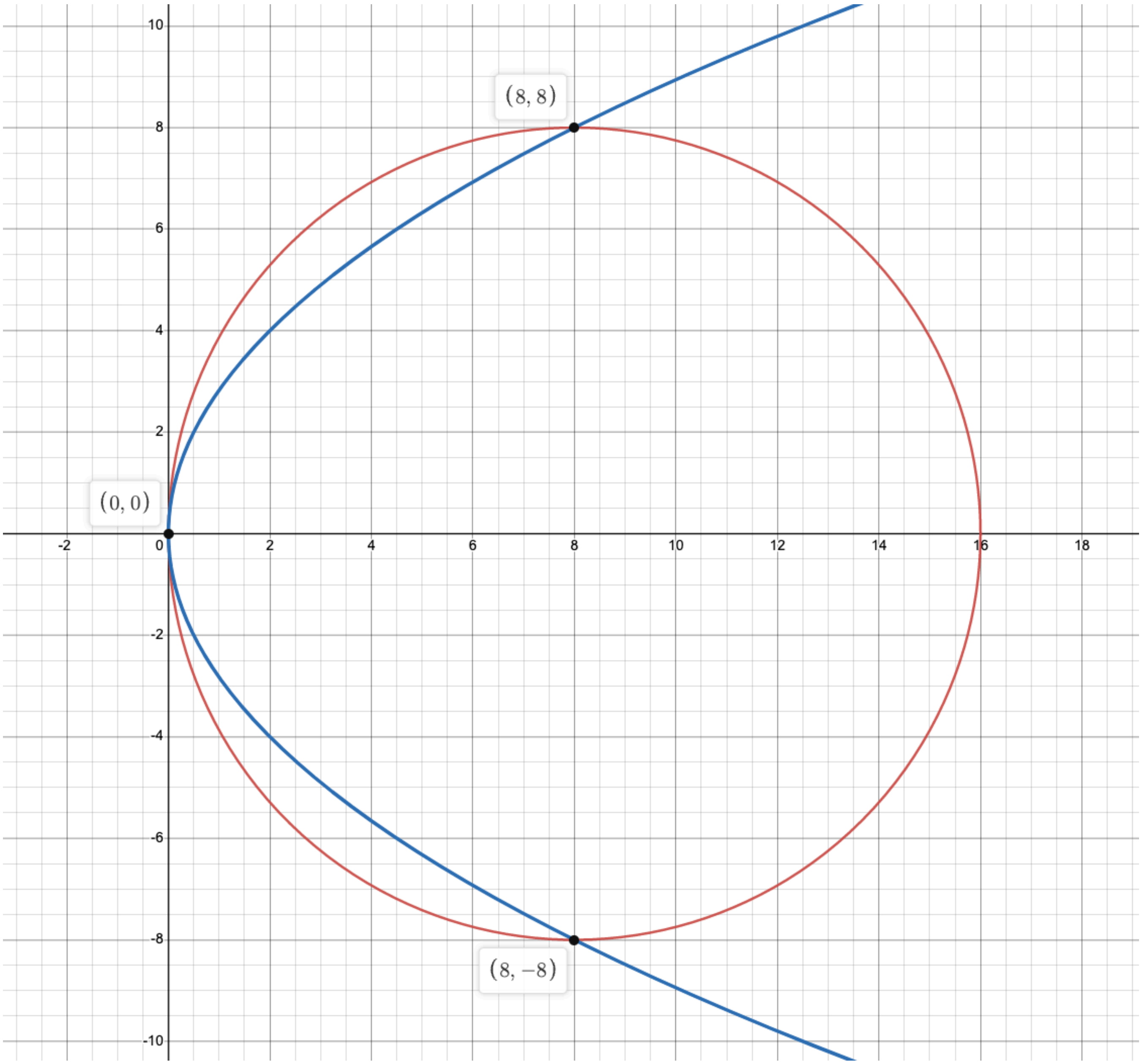

График Функция

$$\begin{cases} f_1 : x^2 + y^2 = 16x \\ f_2 : y^2 = 8x \end{cases} \Rightarrow \begin{cases} f_1 : y = \sqrt{16x - x^2} \\ f_2 : y = 2\sqrt{2x} \end{cases}$$

Очевидно, что f_1 - окружность,и Площать $S_1 = 64\pi$

$$
\begin{aligned}
\text{Пусть } S_\Delta &= \int_0^8 (f_1 - f_2)\mathrm{d}x = \int_0^8 (\sqrt{16x - x^2} - 2\sqrt{2x})\mathrm{d}x \\
&= 16\pi - \int_0^8 2\sqrt{2x}\mathrm{d}x \\
&= 16\pi - 2\sqrt{2}\int_0^8 \sqrt{x}\mathrm{d}x \\
&= 16\pi - \frac{4}{3}\sqrt{2}x^{\frac{3}{2}}\Bigg|_0^8 \\
&= 16\pi - \frac{128}{3}
\end{aligned}
$$

Площадь области,ограниченной заданными кривыми S:

$$S = S_1 - 2S\Delta = 64\pi - 2(16\pi - \frac{128}{3}) = 32\pi + \frac{256}{3}$$

Задание №11

Найдите длину кривой L, заданной в декартовых координатах

$$y^2 - 2y = 4x \quad -1 \le x \le 0$$

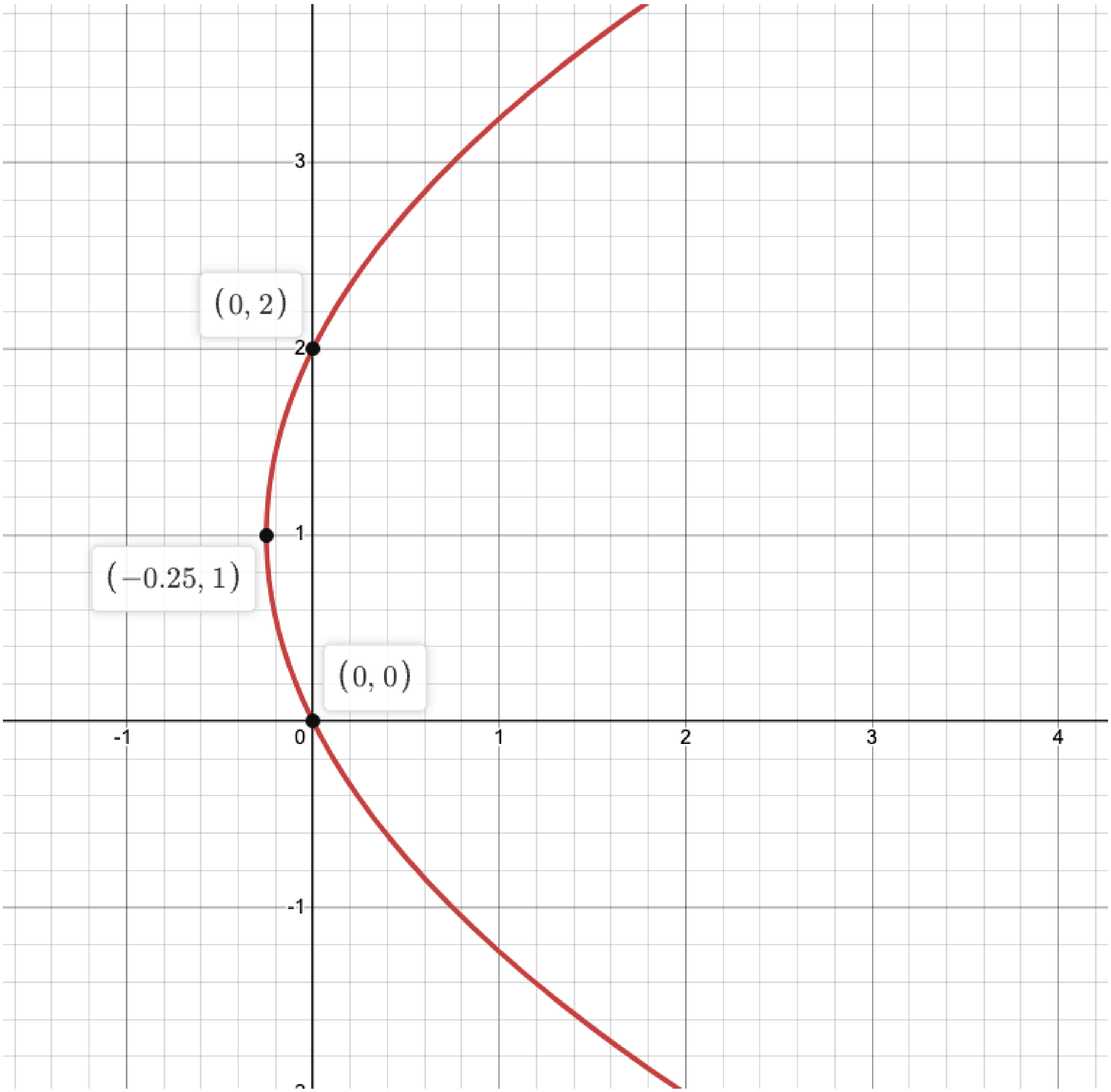

График Функция

Очевидно, что L равна его обратной фукцией в части $(-1 \leq y \leq 0)$:

$$x^2 - 2x = 4y \quad -1 \leq y \leq 0$$

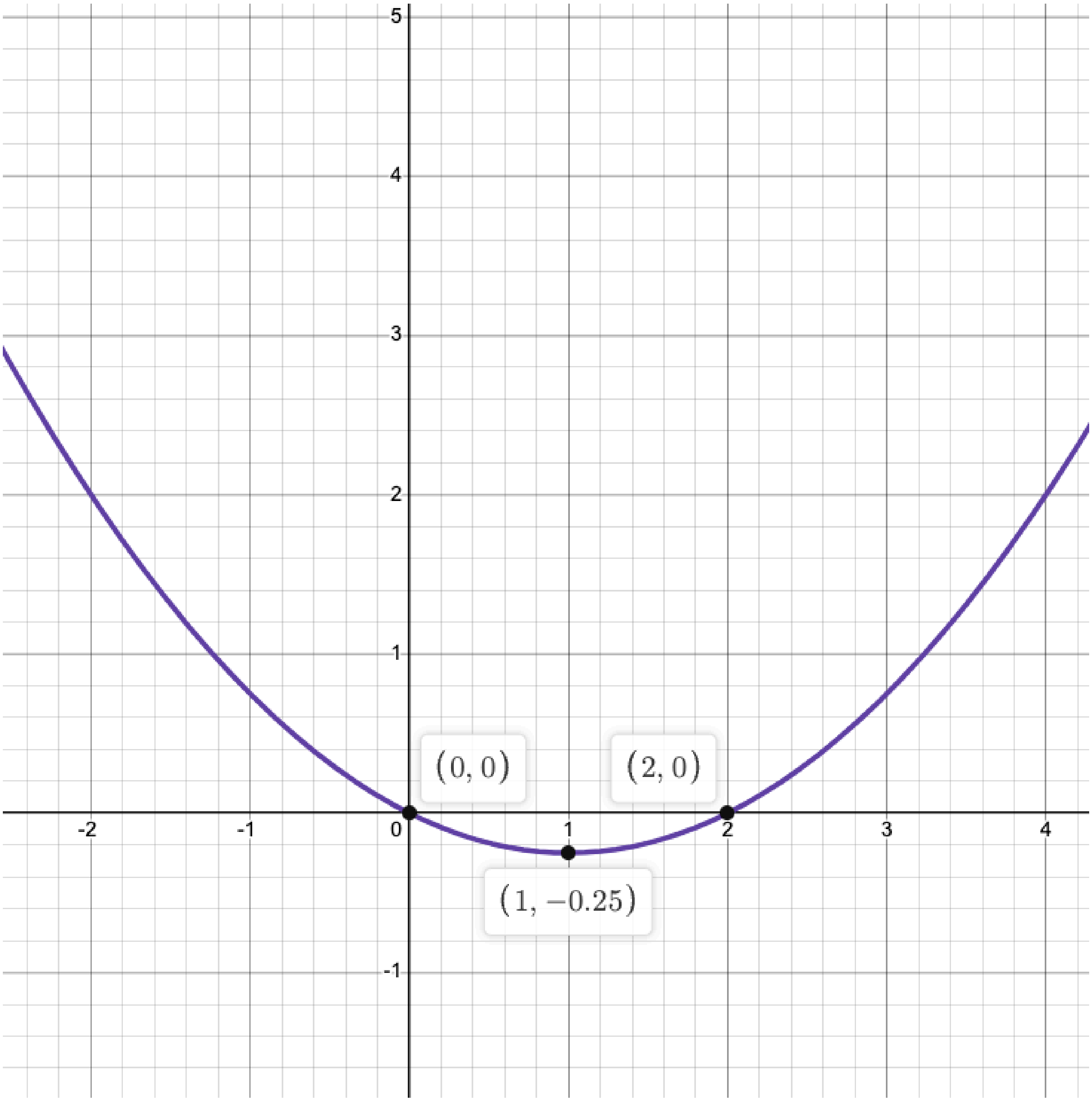

График Функция

$$
\begin{aligned}
x^2 - 2x &= 4y \\
\Rightarrow y &= \frac{x^2}{4} - \frac{x}{2} \\
y' &= \frac{x}{2} - \frac{1}{2} \\
L &= \int_{-1}^{0} \sqrt{1 + y'^2} \mathrm{d}x \\
&= \int_{-1}^{0} \sqrt{1 + (\frac{x}{2} - \frac{1}{2})^2} \mathrm{d}x \\
&= \frac{1}{2} \int_{-1}^{0} \sqrt{x^2 - 2x + 5} \mathrm{d}x \\
\text{Пусть } u &= x - 1, \mathrm{d}u = \mathrm{d}x \\
L &= \frac{1}{2} \int_{-2}^{-1} \sqrt{u^2 + 4} \mathrm{d}u \\
\text{Пусть } u &= 2\tan(t), \mathrm{d}u = 2\sec^2(t)\mathrm{d}t \\
L &= 2 \int_{-\frac{\pi}{4}}^{-\arctan\frac{1}{2}} \sec^3(t)\mathrm{d}t \\
&= \tan(t)\sec(t)|_{-\frac{\pi}{4}}^{-\arctan\frac{1}{2}} + \int_{-\frac{\pi}{4}}^{-\arctan\frac{1}{2}} \sec(t)\mathrm{d}t \\
&= \sqrt{2} - \frac{\sqrt{5}}{4} + \int_{-\frac{\pi}{4}}^{-\arctan\frac{1}{2}} \sec(t)\mathrm{d}t \\
&= \sqrt{2} - \frac{\sqrt{5}}{4} + \ln(\tan(t) + \sec(t))|_{-\frac{\pi}{4}}^{-\arctan\frac{1}{2}} \\
&= \sqrt{2} - \frac{\sqrt{5}}{4} - \ln(\sqrt{2} - 1) + \ln(\frac{1}{2}(\sqrt{5} - 1))
\end{aligned}
$$

Задание №12

Вычислите

- а) Площадь, Органиченную осью абсцисс и верзиерой

$$\begin{cases} x = 2t \\ y = \frac{8}{1+t^2} \end{cases}$$

Из формулы x получим $t = \frac{x}{2}$, подставляем в y:

$$\begin{aligned} y &= \frac{8}{1 + (\frac{x}{2})^2} = \frac{32}{x^2 + 4} \\ S &= \int_{-\infty}^{+\infty} \frac{32}{x^2+4} \mathrm{d}x \\ &= 32 \int_{\infty}^{+\infty} \frac{1}{x^2+4} \mathrm{d}x \\ &= 64 \int_{0}^{\infty} \frac{1}{x^2+4} \mathrm{d}x \\ &= 32 \int_{0}^{\infty} \frac{1}{\frac{x^2}{4}+1} \mathrm{d}(\frac{x}{2|}) \\ &= \lim_{a \to \infty} 32 \arctan(\frac{x}{2}) \Big|_0^a \\ &= 16\pi \end{aligned}$$

- б) Длину дуги кривой

$$r = 6 \sin^3 (\frac{\varphi}{3})$$

$$r' = 6 \sin^2(\frac{\varphi}{3}) \cos(\frac{\varphi}{3})$$

$$\begin{aligned} L &= \int_{\varphi_1}^{\varphi_2} \sqrt{r^2 + r'^2} \mathrm{d}\varphi \\ &= \int_{\varphi_1}^{\varphi_2} \sqrt{36 \sin^6(\frac{\varphi}{3}) + 36 \sin^4(\frac{\varphi}{3}) \cos^2(\frac{\varphi}{3})} \mathrm{d}\varphi \\ &= \int_{\varphi_1}^{\varphi_2} 6 \sin^2(\frac{\varphi}{3}) \mathrm{d}\varphi \end{aligned}$$

потому что $r \geq 0$, мы получим $\sin^3(\frac{\varphi}{3}) \geq 0$, то $\varphi_1 = 0, \varphi_2 = 3\pi$, поэтому

$$
\begin{aligned}
L &= \int_0^{3\pi} 6\sin^2(\frac{\varphi}{3})\mathrm{d}\varphi \\
&= 6\int_0^{3\pi} \sin^2(\frac{\varphi}{3})\mathrm{d}\varphi \\
&= 18\int_0^{\pi} \sin^2(\frac{\varphi}{3})\mathrm{d}(\frac{\varphi}{3}) \\
&= 18\int_0^{\pi} (\frac{1-\cos(\frac{2\varphi}{3})}{2})\mathrm{d}(\frac{\varphi}{3}) \\
&= 9\int_0^{\pi} \mathrm{d}(\frac{\varphi}{3}) - 9\int_0^{\pi} \cos(\frac{2\varphi}{3})\mathrm{d}(\frac{\varphi}{3}) \\
&= 9\int_0^{\pi} \mathrm{d}(\frac{\varphi}{3}) \\
&= 9\pi
\end{aligned}
$$

Задание №13

Найдите значение несобственного интеграла или установите его расходимость.

- a)

$$
\begin{aligned}
&\int_0^{+\infty} (4-3x)e^{-3x}\mathrm{d}x \\
=&4\int_0^{+\infty} e^{-3x}\mathrm{d}x - 3\int_0^{+\infty} xe^{-3x}\mathrm{d}x \\
=&4\lim_{b\to\infty}\int_0^{b} e^{-3x}\mathrm{d}x - 3\lim_{b\to\infty}\int_0^{b} xe^{-3x}\mathrm{d}x \\
=&3\lim_{b\to\infty}\int_0^{b} e^{-3x}\mathrm{d}x + \lim_{b\to\infty} xe^{-3x}\Big|_0^b \\
=&3\lim_{b\to\infty}\int_0^{b} e^{-3x}\mathrm{d}x + \lim_{b\to\infty} be^{-3b} \\
=&3\lim_{b\to\infty}\int_0^{b} e^{-3x}\mathrm{d}x + 0 \\
=& e^{-3b}\Big|_{-\infty}^{0} \\
=&1
\end{aligned}
$$

- б)

$$
\begin{aligned}
&\int_0^{\frac{1}{2}} \frac{x^3 \mathrm{d}x}{\sqrt{1-16x^4}} \\
\text{Пусть } u =& 1-16x^4 \quad \mathrm{d}u = -64x^3 \mathrm{d}x \\
=& -\frac{1}{64}\int_1^0 \frac{1}{\sqrt{u}} \mathrm{d}u \\
=& \frac{1}{64}\int_0^1 \frac{1}{\sqrt{u}} \mathrm{d}u \\
=& \lim_{b\to 0^+} \frac{\sqrt{u}}{32} \Big|_b^1 \\
=& \frac{\sqrt{1}}{32} - \frac{\sqrt{0}}{32} \\
=& \frac{1}{32}
\end{aligned}
$$