

Recursion

Key note:

- a) Find the relationship between current function and the next function.
- b) Find the end condition.

Q1: If a frog can jump 1 step or 2 steps. How many possible ways the frog can jump X steps?

- a) Relationship between each step: Assume we have n steps. Totally, there are f(n) possible ways to jump. Each time there are two possible: 1 step or 2 steps.

Case 1, if jump 1 step first, then we will have n-1 steps remaining which is f(n-1) possible ways.

Case 2, if jump 2 step first, then we will have n-2 steps remaining which is f(n-2) possible ways

Therefore: $f(n) = f(n-1) + f(n-2)$

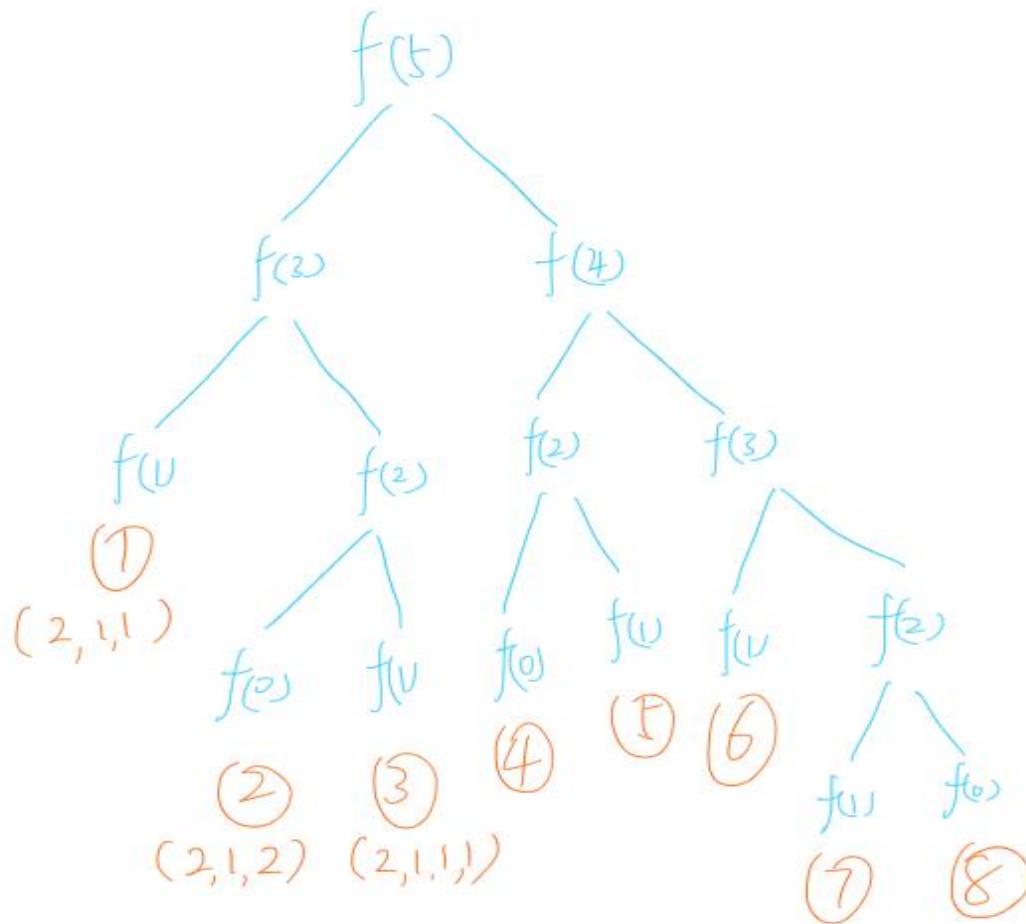
- b) End condition:

When $n < 0$. There are 0 possible ways to jump, so $f(n) = 0$ *** (Not count, since we limit $n=1$, so do not need to worry here.)

When $n == 0$. There are 0 possible ways to jump, so $f(n) = 1$ *** (This count, since it is a way to jump)

When $n = 1$. There are 1 possible way to jump, so $f(n) = 1$

```
public int recursion(int a) {  
    if(a <= 0){  
        return 1;  
    }  
    else if(a == 1){  
        return 1;  
    }  
    else{  
        return recursion(a - 1) + recursion(a-2);  
    }  
}
```



Improvement: The problem here is too many repeated calculations. Expensive!

Here we can use **dynamic programming**

Key note:

a) Save unique result in a map or other structure. Directly use without calculate again.

In this example, we can save $f(4)$, $f(3)$, $f(2)$ when we first calculate. Therefore we can use anytime without do a repeat calculation.

```

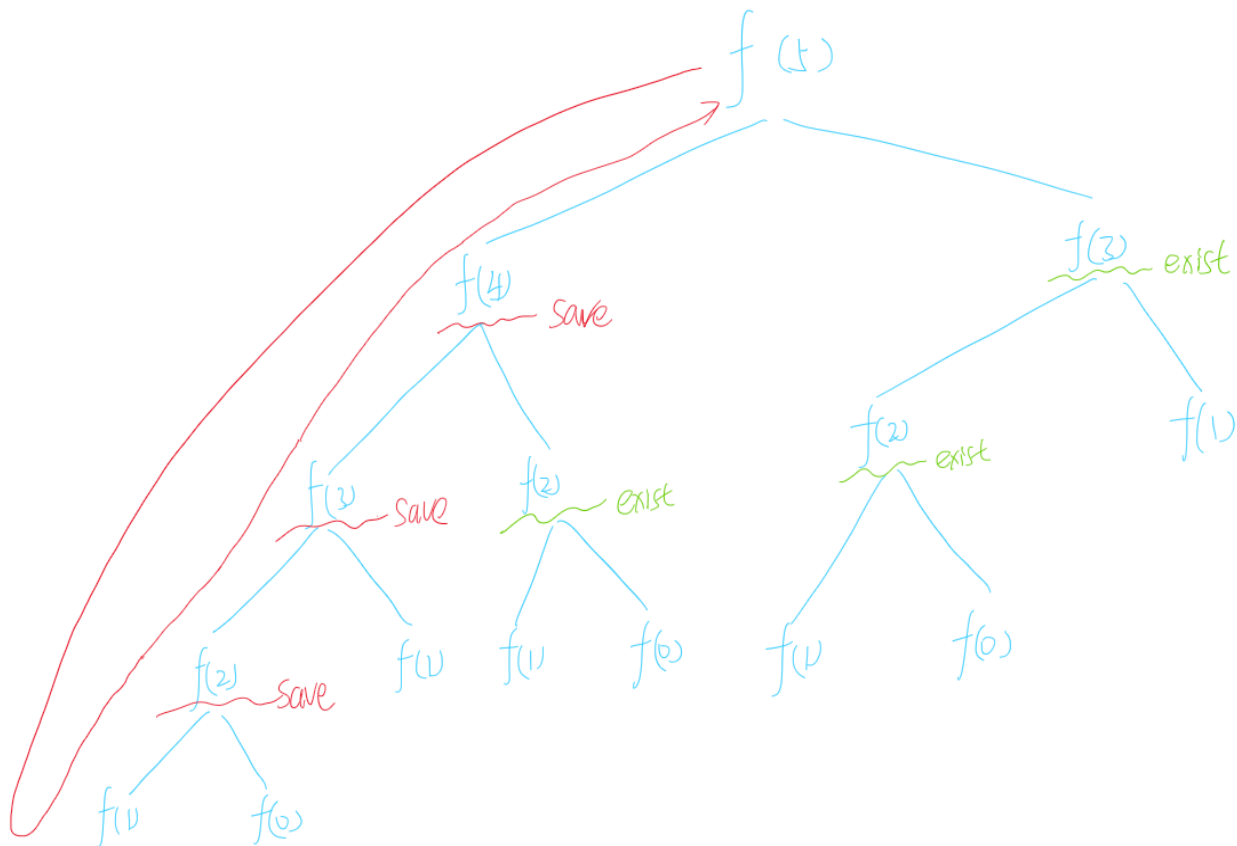
Map<Integer, Integer> map = new HashMap<>();
public int dynamic( int a ){
    if(a <= 0){
        return 1;
    }
    else if(a == 1){
        return 1;
    }
    else{
        if(map.containsKey(a)){

```

```

    return map.get(a);
}
else{
    int val = dynamic(a - 1) + dynamic(a-2);
    map.put(a,val);
    return val;
}
}

```



Q1: If a frog can jump 1 step, 2 steps, 3 steps or X step each times. How many possible ways the frog can jump X steps?

```

Map<Integer, Integer> map = new HashMap<>();
public int dynamic( int a ){
    if(a <= 0){
        return 1;
    }
    else if(a == 1){
        return 1;
    }
}

```

```
else{
    if(map.containsKey(a)){
        return map.get(a);
    }
    else{
        int val = 0;
        for(int i = 1 ;i<=a; i++){
            val += dynamic(a - i);
        }
        map.put(a,val);
        return val;
    }
}
```